Input third-degree price discrimination in transport markets

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Abstract

We study the efficiency of input third-degree price discrimination in transport markets. Relevant cases are transport facilities such as airports, seaports and railway stations that sell access to the infrastructure to downstream firms. The two key distinctive features of these markets are the presence of negative consumption externalities and of public ownership (domestic-welfare maximizing sellers). We find that each of these features enlarges the extent to which input price discrimination is desirable. Our main result suggests that the current practice of enforcing a ban on input price discrimination by congestible facilities may be in place at the cost of efficiency.

Keywords: Price discrimination, Airport pricing, Transport regulation, Congestion

JEL Codes: L12, L93

1. Introduction

In trade and transport markets there is a often a special regulation enforcing a ban on input price discrimination. For example, the regulations of the World Trade Organization (WTO) through the General Agreement on Tariffs and Trade (GATT) basically do not allow price discrimination by ports. The EU Airport Charges directive (2009/12/EC) prohibits differentiated charges to airlines using the same service (i.e. terminal and level of service) and a similar ban holds for airports in the U.K. (Section 41 of the 1986 Airports Act) and in the US (2013 FAA's Policy Regarding Airport Rates and Charges). In these markets the sellers of intermediate goods normally deal with both national and international downstream markets and they can use the origin or the destination of the product to apply

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third-degree price discrimination.

A remarkable set of examples can be found in the aviation industry, where airports consistently try to differentiate linear landing charges based on the origin of the flight. In a case brought by two airlines in 2011, the Civil Aviation Authority, under the U.K. Airports Act 1986, ruled that the per-passenger landing charges proposed by London Heathrow Airport for the period 2011–2012 were unreasonably discriminatory against airlines operating domestic flights (CAA, 2014). This behavior from a fully private airport, the busiest in Europe, is not surprising and may be expected due to differences in demand elasticities between international and domestic markets.

In 2000, the European Commission ruled against a system of landing fees applied at Spanish airports owned by the State because they discriminated in favor of national airlines (Commission Decision 2000/521/EC OJ L 20). The different landing fees were set according to the origin of the flight, as prescribed in a royal decree by the Spanish government, and they were indirectly giving an advantage to national airlines. More recently, in 2012, the French Independent Supervisory Authority rejected the pricing regimes proposed by 5 airports (fully or by majority) owned by the French State because they were set based on the origin of the flight and therefore did not comply with the non-discriminatory legislation (DTA, 2012). What these cases show is a different rationale for price discrimination, namely governments trying to extract rents from foreign firms and to increase national surplus.

Besides the different ownership of the upstream firms attempting to price discriminate, there is a second characteristic of this industry that is relevant: the presence of a negative consumption externality. Due to congestion, an additional unit of output produced by any downstream firm imposes additional costs (delays) on consumers and firms of all markets. In the aviation industry, these costs are substantial; for example, Ball et al. (2010) estimate that the cost of US air transportation delays in 2007 to passengers and airlines was 25 billion US dollars.

This paper studies the efficiency of a policy that bans input price discrimination in the presence of the two distinctive features of this industry: (i) negative consumption externalities and (ii) public input sellers. We are interested in unveiling whether these features make price discrimination especially welfare decreasing so that an additional regulation, on top of national laws, is in place for efficiency purposes.

For this purpose we study the effects of input third-degree price discrimination on prices, output and welfare in the presence of each of these two features separately and together. First, we show that input third-degree price discrimination by a private supplier may increase total output and total welfare using a model where, in the absence of negative consumption externalities, input third-degree price discrimination always reduces total welfare. By finding benefits from price discrimination, we conclude that the presence of negative consumption externalities enlarges the extent to which price discrimination is desirable. Using the same model we show that in absence of externalities, price discrimination by a public supplier that maximizes domestic surplus (as opposed to total welfare), may increase aggregate quantity and total welfare. Therefore, the presence of public suppliers also enlarges the extent to which price discrimination is desirable. We finally show that the results also hold when both public sellers and negative externalities are present.

The results of the paper suggest that the current policy of enforcing a broad ban on input price discrimination in transport markets may be inefficient. The two key distinctive characteristics of these industries offer a reversal of some of the current results in the third-degree price discrimination literature and may make a ban on price discrimination welfare reducing.

Our results are relevant for airport pricing regulations, but are not limited to this market as the features and the implications of our model are representative of other industries as well. Rail stations are often publicly owned (e.g. by the State or Federal Governments), they sell access to the infrastructure to both foreign and national firms and they are prone to congestion, so there are negative consumption externalities. An example of this is a court case in 1997, in which the EU Commission ruled against the *Deutsche Bahn* for setting discriminatory prices for access to the German railway infrastructure in favor of a national company and in detriment of a Belgian rail company (see Geradin and Petit (2006) for a description of the case). The case of seaports and shipping companies is similar. In all these cases, for cargo as well as for passenger transport, input price discrimination may be more desirable due to the public ownership of the sellers or due to the presence of negative consumption externalities.

To highlight the differences with the case without externalities, our model considers a structure commonly used in the literature in that the input provider is an unrestricted monopolist and firms take the input price as given. There are two downstream markets and two downstream firms, one domestic and one foreign, that are equally efficient.[1] The domestic firm serves a domestic market and the foreign firm serves a market with a mix of foreigners and nationals. We assume that consumers incur a cost that increases in aggregate output, so there is a negative consumption externality, and its shadow price differs between markets. Each downstream firm is thus a monopoly in one market and the only interdependency is through the consumption externalities. Demands and costs are linear so, in the absence of externalities, input price discrimination by a private supplier does not change output and reduces total welfare (Arya and Mittendorf, 2010).

We use this setting and confine our analysis to the case of covered markets because it is unfavorable for price discrimination and because it provides an ideal framework to transparently isolate the effects of the externality and of the ownership form. Considering the opening of markets due to price discrimination by a private supplier would enhance its performance and a different functional form of the demand may have the same effect. Even if it the (concavity of the) demand function is more unfavorable for price discrimination there is no apparent reason to believe that this would undermine the effect of the externalities on the welfare effect of price discrimination. We believe that our message is stronger in a context of linear demands and covered markets because it is transparently defined in an adverse fashion for price discrimination and because the intuition in absence of our novel features is clear-cut.

When the supplier maximizes profit, the nationality of firms and consumers is not relevant, but the negative consumption externality plays a key role. It acts in a similar way as substitution does, because a price increase in one market increases the willingness to pay in the other market through decreased consumer costs as a result of decreased demand.

[1] There is a large stream of literature studying third-degree price discrimination in input markets when downstream firms have different levels of (cost) efficiency that shows benefits of enforcing uniform pricing (e.g., Katz, 1987; DeGraba, 1990; Yoshida, 2000; Valletti, 2003). Nevertheless, uniform pricing can be harmful when there is bargaining between buyers and suppliers (O'Brien and Shaffer, 1994), and when there is input demand-side substitution (Inderst and Valletti, 2009).

We show that this increase in willingness to pay is proportional to the shadow price of the externality and that it gives an incentive to increase the input price in the market with the low shadow price. Phrased differently, the decreased externality is more profitable in the market where it is valued more. Consequently, if the market with a lower shadow price has a higher demand elasticity (e.g. because of lower income), the presence of negative externalities gives incentives to raise the price in the high-elasticity market. This is opposite to the incentive given by the different elasticities in the absence of externalities. As a result, when the externality effect is stronger than the elasticity effect, something that we formalize in the analysis, input price discrimination raises the price in the high-elasticity market and decreases the price in the low-elasticity market. In this case, the traditional price effect of allowing price discrimination is reversed, and we find that price discrimination increases total output and can increase total welfare.[2]

On the other hand, under public ownership and in the absence of consumption externalities, the seller has an incentive to give price concessions in the domestic market and to raise the price in the market where the foreign firm is active and there are foreign consumers. The price concession is such that the final price in the domestic market equals the marginal social cost, which induces a large quantity increase and removes all inefficiencies there. In the foreign market, the input provider captures part of the downstream firm's profit also decreasing consumer surplus in that market. Nevertheless, we show that when this market is relatively more elastic, the losses are limited relative to the gains from inducing marginal social cost pricing in the domestic market. The overall effect is an increase in total output and total welfare. When negative consumption externalities are present, input price discrimination by a public supplier may also raise output and total welfare. This is particularly true if the domestic market, besides having a lower demand elasticity, exhibits a higher shadow price of the externality.

The main policy implication of the paper is that the current practice of banning price discrimination by congestible facilities, such as airports, may be in place at the cost of effi-

[2]Importantly, this result could not hold in our framework if downstream markets were substitutes and in absence of externalities, which highlights the key role of the negative consumption externalities.

ciency. While the theoretical predictions of our model are unambiguous, a natural concern is that they may not apply well to particular circumstances or markets. In the case of a profit-maximizing seller, our results rely on the assumption that consumers of the market with the lower demand elasticity have a higher willingness to pay to reduce delay costs. This is natural in transport markets and particularly true in aviation, as two main drivers of differences between markets are income and trip purpose. Markets (or routes) that are more intensive in business trips exhibit a lower elasticity (Brons et al., 2002; Ciliberto and Williams, 2014) and a higher willingness to pay to reduce delays (Koster et al., 2011; Kouwenhoven et al., 2014; Shires and De Jong, 2009). A stronger assumption is needed for the results under public ownership: the market with lower elasticity and higher value of time needs to be the domestic market. Nevertheless, a transportation industry in a country where the presence of foreign firms is higher in international markets, and the domestic market is more intensive in business trips is consistent with the assumption. For instance, the US aviation market broadly satisfies our assumptions as the vast majority of airports are publicly owned, business trips are more common for US destinations than for international destinations and foreign firms are not allowed to serve domestic markets. For example, at Los Angeles International Airport (LAX) the share of business trips in 2011 was 90% higher for US destinations than for international destinations (Unison Consulting, 2011). In addition, there is empirical evidence in US aviation markets that business travelers have a lower elasticity of demand and a higher willingness to pay to reduce airport delays (see e.g. Berry and Jia, 2010; Yan and Winston, 2014). The empirical evidence supports our claim that the analysis is relevant for transport and trade markets and, in particular, for airports.

We conclude the analysis by studying the case where downstream firms can perfectly price discriminate consumers. Downstream first-degree price discrimination, which also works as a proxy for a competitive downstream market, is useful to check the robustness of our results when the downstream inefficiency due to monopoly pricing is smaller. We find that the main result of the paper holds in this setting: input price discrimination can raise total output and total welfare in the presence of negative consumption externalities regardless of the supplier's ownership form. This reinforces our findings by showing that a large deadweight loss caused by monopoly downstream pricing is not a necessary condition and suggests that our results would also hold when downstream markets are oligopolistic.

The remainder of the paper is structured as follows. Section 2 introduces the model and main assumptions. Section 3 analyzes the effects of price discrimination by a private facility while Section 4 analyzes the case of a public facility. Section 5 extends the results to downstream perfect price discrimination and Section 6 concludes. The Appendix contain all the proofs required in the article.

2. The model and the downstream markets

There are two downstream markets, A and B, that are only related through a negative consumption externality, as an additional unit of output in any market imposes additional costs on all other consumers, but are otherwise independent. Downstream firms transform one unit of input into one unit of output. To simplify terminology and because transport markets have all the features in our model and are relevant examples, hereafter we refer to the upstream monopolist as transport facility, the negative consumption externality as congestion and its shadow price as value of time (as a shorthand for willingness to pay to reduce congestion delays). Nevertheless, the results are general to any intermediate good market where negative consumption externalities are important and/or public ownership is common. As mentioned above, a relevant example is airports setting per-passenger charges for airlines flying to different cities, where congestion occurs at the passengers' facilities.

There are two downstream firms and we denote them by the market in which they operate. Thus, firm i operates in market i with $i = \{A, B\}$. As one of our aims is to analyze the role of the negative consumption externality on the effects of input price discrimination, we follow much of the literature on input price discrimination and assume that demands are linear. The purpose is twofold, it is unfavorable for price discrimination and it allows us to compare our results to those in the previous literature more transparently. The linear inverse demand in market i, $P_i(q_i)$, is given by:

$$P_i(q_i) = A_i - b_i \cdot q_i \tag{1}$$

where A_i is the maximum reservation price (inverse demand intercept), b_i the slope, and q_i is the quantity set by firm i.

This is, up to here, conventional: there are two sources of heterogeneity between markets, namely the reservation price and the sensitivity of the demand. Both are arguably correlated through income, as a higher income would explain a higher reservation price and a less sensitive demand. In the absence of congestion externalities, the input price is higher in the less elastic market, which with linear demands is the market with the higher inverse demand intercept (Arya and Mittendorf, 2010). Hereafter we assume that market A is the high-income market, and B is the low-income market, so that $A_A > A_B$ holds in the remainder of the paper without loss of generality and in the absence of congestion the input price would be higher in market A.[3]

A consumer in market i faces a full price, which is the sum of the downstream firm's price (e.g. ticket), t_i, and the cost of congestion (e.g. delays at the airport). The delay due to congestion, $D(Q)$, is also linear and increases in the aggregate consumption ($Q = q_A + q_B$) to reflect within- and cross-market negative consumption externalities. Therefore, downstream firms are able to charge consumers a price equal to the marginal willingness to pay net of congestion delay costs:

$$t_i = A_i - b_i \cdot q_i - v_i \cdot D(Q) \tag{2}$$

where v_i is the value of time, which is assumed to be the same for all individuals in a market. As $D(Q)$ is assumed to be linear, without loss of generality we use $D(Q) = q_A + q_B$, so that v_i captures the heterogeneity in congestion delay costs.

The congestion externalities add a third source of heterogeneity between markets through the value of time (willingness to pay to reduce congestion delays), which is also arguably correlated with income. A natural expectation is that the value of time is higher in the high-income market ($v_A > v_B$), however, we do not restrict the analysis to this case. As we show later on, our main results rely on downstream markets being asymmetric both with respect to demand and value of time. Nevertheless, the particular case of asymmetry only

[3]This is traditionally called the "strong" market as the discriminatory price is higher. We use the high-income market as we find that the input price could be lower in this market.

in one aspect is also discussed.

We assume that downstream firms play a quantity setting game and that they have constant marginal costs, which we normalize them to 0.[4,5] In the analysis that follows, we study the case where downstream firms cannot discriminate consumers so, in equilibrium, the firm's price equals the expression in Eq. (2). Section 5 extends the analysis by studying downstream first-degree price discrimination. For a given input price, w_i, the downstream firm i maximizes:

$$\pi_i = q_i \cdot [P_i(q_i) - v_i \cdot D(Q) - w_i] \;\; \forall \; i \in \{A, B\} \; , \tag{3}$$

and the first-order condition leads to the following pricing rule:

$$\frac{\partial \pi_i}{\partial q_i} = 0 \Rightarrow \; t_i = w_i + q_i \cdot [b_i + v_i] \;\; \forall \; i \in \{A, B\} \; . \tag{4}$$

Eq. (4) shows that the firm's pricing rule has the facility charge (w_i), a traditional monopoly market power markup ($q_i \cdot b_i$) and the marginal congestion cost of firm i's own consumers ($q_i \cdot v_i$). The downstream firm realizes that an additional consumer raises congestion and reduces the price it can charge, but does not internalize the effect on the other firm's consumers. This internalization result was recognized by Daniel (1995) in the context of airport congestion pricing and explored theoretically by Brueckner (2002). Eq. (4) also shows that the quantity choices by the downstream firms are strategic substitutes.[6] The system of first-order conditions in Eq. (4) defines the derived demands faced by the input provider $q_A(w_A, w_B)$ and $q_B(w_A, w_B)$, whose closed form are in Appendix A.1.

Before moving into the facility's maximization problem, it is useful to compare the down-

[4]We assume that congestion does not affect the downstream firms' costs, but this could be readily included in our analysis without changing the main results and conclusions. The reason is that congestion does affect firms in that increased congestion raises the full price faced by consumers and therefore final good prices will be lowered by the increased congestion. In the downstream firms' profit function, whether congestion raises the costs or reduces the passengers' willingness to pay makes no difference. See Silva and Verhoef (2013) for a discussion.

[5]Following Singh and Vives (1984), this can be interpreted as if their costs were incorporated through the intercept of the inverse demand function. If a_i is the inverse demand intercept in market i and c_i the marginal cost, we may replace $a_i - c_i$ by A_i.

[6]It follows from Eq. (4) that the best response function is $q_i^*(q_j) = (A_i - w_i - v_i \cdot q_j)/(2 \cdot b_i + 2 \cdot v_i)$, and that $\partial q_i^* / \partial q_j < 0$.

stream pricing rule with the welfare maximizing pricing rule. In this model, normalizing the facility's costs to 0, total welfare is:

$$W = \sum_i \left[\int_0^{q_i} P_i(x)dx \right] - \left[\sum_i q_i \cdot v_i \right] \cdot D(Q) \ , \tag{5}$$

and the welfare maximizing downstream pricing rule is

$$\frac{\partial W}{\partial q_i} = 0 \Rightarrow \ t_i = q_A \cdot v_A + q_B \cdot v_B \ \ \forall \, i \in \{A, B\} \ . \tag{6}$$

A comparison between Eq. (4) and Eq. (6) reveals that, input prices aside, the prices set by downstream firms are not necessarily higher than optimal. If the demand is sufficiently elastic, i.e. the demand-related markup is low compared to the un-internalized externality (e.g. $q_A \cdot b_A < q_B \cdot v_B$), prices will be too low and output too high. This result and its policy implications have been discussed in the air transportation literature (see e.g. Pels and Verhoef, 2004) and with special emphasis on price discrimination by downstream firms (Czerny and Zhang, 2011, 2014).

The pricing regimes that we study are uniform pricing, where the facility is restricted to charge all firms the same price per unit of output, and third-degree price discrimination, where the facility is allowed to charge different unit prices.[7] To simplify notation we often refer to third-degree price discrimination as price discrimination.

We assume throughout the paper that all markets are always served under both pricing regimes. This assumption is not for simplification purposes. We are aware that the possibility of foreclosure is serious when uniform pricing is imposed, but this would not change our message. Price discrimination would be more desirable because of the presence of negative externalities and because it leads to less foreclosure. As the latter effect is well studied in the literature, we omit this advantage of allowing price discrimination. The equilibrium concept that we use is subgame-perfect Nash equilibrium, and we use backward induction to identify it. We first study the case of a profit maximizing facility.

[7] There is a distinction between price differentiation and price discrimination in congestible markets (see e.g. van der Weijde, 2014). As in our setting the marginal external cost ($\sum_i v_i \cdot q_i$) is the same for all consumers, there is no difference between discrimination and differentiation.

3. Private facility

3.1. Price discrimination

When price discrimination is allowed, the facility chooses w_A and w_B to maximize:

$$\Pi^{PD} = w_A \cdot q_A(w_A, w_B) + w_B \cdot q_B(w_A, w_B) \ , \tag{7}$$

as we normalize the facility's costs to 0. The first-order conditions lead to the closed-form solutions for w_A and w_B (see Appendix A.1) and imply the following pricing rules:

$$w_A = 2 \cdot q_A \cdot [b_A + v_A] + q_B \cdot v_B \ , \tag{8}$$

$$w_B = 2 \cdot q_B \cdot [b_B + v_B] + q_A \cdot v_A \ . \tag{9}$$

The input provider exerts market power and consumers face a double marginalization. In addition, the facility charges the marginal congestion cost that is not internalized by the firm (the last term on the right-hand side of Eqs. (8) and (9)). Therefore, under price discrimination, the final price in each market is higher than the socially optimal price and output is inefficiently low. This result is useful for the welfare analysis below and it is essentially different to the case of final good markets and congestion externalities, where the quantity under downstream price discrimination can be inefficiently high (Czerny and Zhang, 2015). This is because the downstream firm's markup is not necessarily higher than the marginal external congestion cost, but the sum of the downstream and upstream markup is.

As discussed above, in the absence of congestion externalities, the input price is higher in market A, the high-income market. We seek to understand what the effect of the congestion externality on this is. Assuming that the second-order conditions are satisfied,[8] the following proposition summarizes a main result of the analysis: under price discrimination the input price can be higher in the low-income market.

[8] A sufficient condition, for this case, is that time valuations are not too distinct in that $v_B/v_A > 7 - 4\sqrt{3} \approx 0.072$.

Proposition 1. *The input price under price discrimination is higher in the low-income market ($w_B > w_A$) if, and only if,*

$$\frac{A_B}{A_A} > \lambda_1 = \frac{8 \cdot b_A \cdot b_B + 5 \cdot v_A \cdot v_B + v_B^2 + 2 \cdot v_B \cdot [4 \cdot b_A + b_B] + 6 \cdot b_B \cdot v_A}{8 \cdot b_A \cdot b_B + 5 \cdot v_A \cdot v_B + v_A^2 + 2 \cdot v_A \cdot [4 \cdot b_B + b_A] + 6 \cdot b_A \cdot v_B}.$$

To understand the intuition behind the proposition, first consider the case where time valuations are the same in both markets ($v_A = v_B$). In this case, $\lambda_1 = 1$ holds and the result obtained in the absence of externalities goes through. When cross congestion effects are symmetric, the facility's incentive to charge a higher price in one market over the other does not change and the input price is higher in the high-income market (the less elastic market). In general, λ is different from 1, so the proposition reveals that the incentives provided by the externalities can overturn the incentives given by the elasticities. A necessary condition is that $\lambda_1 < 1$, as $A_B/A_A < 1$, which holds when $v_A > v_B$. Therefore, when the value of time is higher in the high-income market (which, as argued above, is a natural assumption), the input price can be lower in that market and higher in the low-income market.

To gain further insight, consider the case where the reservation price is the same in both markets ($A_B/A_A = 1$), a case where, in the absence of externalities, it is optimal for the facility to set a uniform price because the elasticities of the derived demand are the same. As $v_A > v_B$ implies $\lambda_1 < 1$, it follows directly that $w_B > w_A$ holds. Thus, asymmetric cross-congestion effects can provide, on their own, the incentives to set a higher price in the market with the lower value of time. In our setting, raising the price in one market causes a decrease in congestion costs through decreased demand, which, in turn, causes an increase in the profitability of the other market as the willingness to pay is increased. Consequently, when the reservation price is the same in both markets, it is optimal for the facility to set a higher price in the market with low time valuation. Phrased differently, for the facility, the decreased congestion is more profitable in the market with high time valuations because the increase in willingness to pay is higher.[9]

[9]Adachi (2005) shows in final good markets that when there are only within-market congestion externalities (i.e. absence of interrelation), it is optimal for the downstream firm to set a uniform price when demands are linear and reservation prices are equal. The reason is that a higher reservation price fully determines which market is less elastic when consumption externalities are linear in the quantity. In the case of input markets, this is also the case as it is straightforward to show that the differences in elasticity

In the general case of full asymmetry, the elasticity effect and the externality effect come into play as Proposition 1 reveals. A lower reservation price, through higher demand elasticity, gives incentives to decrease the input price and a lower value of time gives incentives to increase the price in that market because of cross congestion effects. It is straightforward to show that λ_1 decreases as the ratio v_B/v_A is lower, so that the more asymmetric the congestion effects are, the stronger the incentives to raise the price in market B. Therefore, Proposition 1 implies that for the congestion effects to overturn the incentives provided by different demand elasticities in the absence of congestion, the relative difference between time valuations must be higher than the relative difference between demand intercepts. For a given ratio of demand intercepts (A_B/A_A), a stronger heterogeneity in values of time extends the parameter region where the input price is higher in the low-income market.

As the novelty of our article lies essentially here, in showing that the presence of negative consumption externalities may lead to the facility to set a higher input price in the low-income market and a lower price in the high-income market, in the remainder of the paper we focus on this case, when Proposition 1 holds ($v_A > v_B$ and $A_B/A_A > \lambda_1$). The analyses for other cases are in Appendix A.4.

3.2. The effects of price discrimination

We first briefly discuss the uniform pricing regime as a benchmark, where the private facility maximizes:

$$\Pi^U = w \cdot [q_A(w,w) + q_B(w,w)] \ , \tag{10}$$

and the first-order condition leads to the following pricing rule:

$$\begin{aligned} w = \quad & 2 \cdot q_A \cdot [b_A + v_A] \cdot \frac{b_B + v_B}{b_A + b_B + \bar{v}} + 2 \cdot q_B \cdot [b_B + v_B] \cdot \frac{b_A + v_A}{b_A + b_B + \bar{v}} \\ & - \frac{(q_A + q_B)}{2} \cdot \frac{v_A \cdot v_B}{b_A + b_B + \bar{v}} \ , \end{aligned} \tag{11}$$

where $\bar{v} = (v_A + v_B)/2$ is the unweighted average value of time.

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of the input demand can be fully explained by differences in the reservation price due to the linear demand and congestion assumption.

The pricing rule in Eq. (11) includes a weighted sum of the markups that the facility charges when price discrimination is allowed and a negative term that is related to the marginal congestion costs. It is straightforward to show that the uniform price in Eq. (11) is not a weighted average of the differentiated prices in Eqs. (8) and (9). We study the relationship between uniform and discriminatory prices in detail below.

3.2.1. The effects of price discrimination on prices and output

To study the effect of price discrimination on input prices and on output, we use the price-difference constraint method used by Leontief (1940) and Schmalensee (1981). We assume that the facility maximizes profit subject to the constraint $w_B - w_A \leq t$. This is, the facility cannot differentiate prices more than an exogenous amount $t \geq 0$. When $t = 0$, the facility sets the uniform price derived above (Eq. (11)). As t gradually increases, the facility is gradually allowed to increase the price differentiation until it reaches a point, t^*, where it sets the prices w_A and w_B in Eqs. (8) and (9). The method consists of evaluating the marginal effect of relaxing the constraint on a variable, such as aggregate output. If the sign of the marginal effect does not change in the range $[0, t^*]$, the overall effect of price discrimination on the variable will have the same sign.[10] All the derivations needed for the results in this section are in Appendix A.1.

For a given value of $t \in [0, t^*]$, the facility maximizes:

$$\Pi = w_A \cdot q_A(w_A, w_A + t) + (w_A + t) \cdot q_B(w_A, w_A + t) \ . \tag{12}$$

Totally differentiating the first-order condition $\partial\Pi/\partial w_A$, we can obtain the marginal

[10] This is true because we focus on the case where the price is higher in the low-income market ($w_B > w_A$). If the opposite holds, i.e. $w_B < w_A$, the overall effect of price discrimination will have the opposite sign of the marginal effect, because the price discrimination behavior is approached by making t negative.

effect on the aggregate output and input prices:

$$\frac{dQ}{dt} = \frac{[v_A - v_B]}{2 \cdot \Omega_1} > 0 \ , \tag{13}$$

$$\frac{dw_A}{dt} = \frac{-4 \cdot b_A - [3 \cdot v_A - v_B]}{\Omega_2} < 0 \ , \tag{14}$$

$$\frac{dw_B}{dt} = \frac{[3 \cdot v_B - v_A] + 4 \cdot b_B}{\Omega_2} \gtrless 0 \ , \tag{15}$$

where Ω_1 and Ω_2 are positive constants. The results that follow from Eqs. (13)–(15) are summarized in the following proposition.

Proposition 2. *When the facility sets a higher input price in the low-income market (i.e. $w_B > w_A$), price discrimination:*

(i) Increases aggregate output.

(ii) Decreases the input price in the high-income market (A).

(iii) Decreases both input prices if time valuations are sufficiently different in that $v_A - 3 \cdot v_B > 4 \cdot b_B$; otherwise, it increases the input price in the low-income market (B).

The result in part (i) of Proposition 2, i.e. that output changes with price discrimination when demands are linear is due to the negative externality. In our setting outputs are not substitutes nor complements, but the interdependency through congestion generates a similar effect as substitution. Raising the price in one market, induces an output increase in the other market that is proportional to the time valuations ($\partial q_i/\partial w_j \propto v_i > 0 \, \forall i \neq j$). It is therefore intuitive that the change in aggregate output is not 0 as long as the cross congestion effects are not symmetric ($v_A \neq v_B$).

The result that price discrimination decreases the input price in the high-income market (part (ii)) and that it can decrease both prices (part (iii)) are also due to the cross-congestion effects. An increase in the input price of one market increases the profitability of the other market, because willingness to pay is increased through decreased congestion costs. The decreased congestion costs are more profitable in the high-income market, which gives incentives to decrease the price in the high-income market. To see why price discrimination can move both prices in the same direction, recall that under uniform pricing the marginal profit of the input provider in each market has a different sign. Consider that the marginal profit is negative for market A under uniform pricing (consistent with $w_B > w_A$). If the marginal profit increases slowly toward zero, the decrease in price toward the optimally differentiated w_A will be large. This large decrease may cause a large reduction in the profitability in market B, which was positive at uniform prices, and can make it negative at $\{w_A, w\}$. This will therefore also cause a reduction in the price in market B. This is what happens when the facility sets a higher input price in the low-income market and time valuations are sufficiently different in that $v_A - 3 \cdot v_B > 4 \cdot b_B$. Importantly, our result that price discrimination decreases prices in both markets needs asymmetry both with respect to demand and to the negative externality. If there is asymmetry in only one of the two elements, one price must rise and the must other fall.

Before moving on to the welfere analysis, we discuss the relation between these results and those in two papers that are related to ours. Layson (1998) shows, in substitute final good markets, that prices may rise or fall and aggregate quantity may rise with price discrimination. Although our results may be seen as analogous to those in Layson (1998), he finds that a necessary condition for prices to move in the same direction is decreasing marginal costs. All of our results are found with constant marginal costs, and we can show that in this framework prices cannot move in the same direction under linear demand and (asymmetric) substitution. It is the (linear) negative externality that is essential for the result.[11] Finally, we focus in the cases where $w_B > w_A$ holds, but in our model the reverse may also be true, so there is no strong or weak market. Again, this cannot hold without consumption externalities, but with substitution instead.

Czerny and Zhang (2015), also studying price discrimination in final goods, focus on the role of the negative externality for independent markets, namely business and leisure passengers. They find that under linear demands, aggregate output cannot increase with price discrimination and that both prices cannot decrease. A contrast with our paper lies in the fact that we are studying input markets, and the properties of the derived demands can differ essentially with the final good demands. In their case, it is natural to assume

[11] Another difference with Layson (1998) is that the relative magnitude of the cross effects is not the only determinant of the sign of the output effect. That is, if $v_A > v_B$ holds, output may rise or fall.

that the price in the strong market rises with price discrimination while in our setting it is not. In our framework the assumption that there is a market where consumers pay a higher price in equilibrium is not necessarily a good proxy for the relative magnitude of the input prices under price discrimination. It is possible that $w_B > w_A$ holds and that the downstream equilibrium price is higher in market A.

The likelihood of price discrimination changing both prices in the same direction depends on how asymmetric the time valuations are. Price discrimination is likely to move prices in the opposite direction when the ratio of time valuations v_B/v_A is not too low (higher than 1/3 is sufficient) and to change prices in the same direction when it is sufficiently low (v_B/v_A at least lower than 1/3). As explained above, this is because when they are sufficiently different ($v_A - 3 \cdot v_B > 4 \cdot b_B$), the change in profitability in one market due to the change in the input price of the other is large. The likelihood of $v_A - 3 \cdot v_B > 4 \cdot b_B$ is somewhat difficult to assess. One way of casting light on its likelihood is by considering that the differences across markets are caused by differences in trip purpose. Koster et al. (2011), Kouwenhoven et al. (2014) and Shires and De Jong (2009) provide empirical evidence that the ratio of time valuations between business and other users in transport markets is not higher than 3. This suggests that $v_A - 3 \cdot v_B > 4 \cdot b_B$ is a rather stringent condition when differences between markets are caused by differences in the proportion of business and other types of travelers. In that case it is more likely that input price discrimination changes the prices in the opposite direction.

3.2.2. Welfare analysis

A fundamental result of the price discrimination literature is that an increase in aggregate output is a necessary condition for third-degree price discrimination to increase welfare (see Stole (2007) for a survey on the subject). Recent advances show that in the presence of negative consumption externalities, aggregate output expansion is not a necessary condition. In particular, Czerny and Zhang (2015) show that welfare can increase when aggregate output decreases. However, in their analysis, the aggregate output may be inefficiently high because of the presence of the negative externality. In our model, this cannot happen under price discrimination as there is a double marginalization that is absent in final good markets. As the novelty of our paper lies essentially in the case when the facility sets a higher price in the low-income market (Proposition 1), and this implies that aggregate quantity rises with price discrimination (Proposition 2), our welfare analysis is placed on the more traditional side. This is, we study the conditions under which welfare increases given that aggregate quantity increases with price discrimination.

We provide a partial characterization of the effect of price discrimination on welfare by deriving sufficient conditions for welfare improvement.[12] The marginal change in total welfare as more discrimination is allowed, using the same method as in the previous section, can be written as:

$$\frac{dW}{dt} = \frac{dq_A}{dt} \cdot [w_A - [q_B \cdot v_B - q_A \cdot b_A]] + \frac{dq_B}{dt} \cdot [w_A + t - [q_A \cdot v_A - q_B \cdot b_B]] , \qquad (16)$$

where the terms in square brackets multiplying the marginal quantity changes are the difference between the input price set by the facility and the socially optimal input price.

The welfare analysis can be divided into two cases, namely when price discrimination changes both quantities in the same direction (both either rise or fall) and when it increases the quantity in one market and it decreases it in the other. We first focus on the latter case. Opposite changes in quantities due to price discrimination are a consequence of opposite changes in prices. As discussed in Proposition 2, this happens when time valuations are not too different.

When aggregate output increases, the quantity decrease in the low-income market (B) is lower than the increase in the high-income market (A). As a consequence, from Eq. (16), if the difference in actual and socially optimal input price is positive in market A and higher than in market B for all values of t, then price discrimination increases welfare. The conditions for this are summarized in the following proposition:

Proposition 3. *When the facility sets a higher input price in the low-income market (i.e.* $w_B > w_A$*) and quantities move in the opposite direction (time valuations are similar in*

[12] A full characterization of the marginal welfare effect would be tedious. First, unlike the case of final good markets, under uniform pricing there is, in general, a misallocation of output between markets. This is because downstream firms charge a markup related to demand characteristics and time valuations, so that when the input price is uniform, the marginal willingness to pay is, generally, not the same in each market.

that $v_A - 3 \cdot v_B < 4 \cdot b_B$*), price discrimination increases welfare when:*

$$\frac{A_B}{A_A} < \lambda_2 = \frac{12 \cdot b_A \cdot b_B + 10 \cdot v_A \cdot v_B + 2 \cdot v_B^2 + 3 \cdot v_B \cdot [4 \cdot b_A + b_B] + 11 \cdot b_B \cdot v_A}{12 \cdot b_A \cdot b_B + 10 \cdot v_A \cdot v_B + 2 \cdot v_A^2 + 3 \cdot v_A \cdot [4 \cdot b_B + b_A] + 11 \cdot b_A \cdot v_B},$$

The reason why welfare increases is that the benefit in the high-income market from a decreased input price and increased quantity is larger than the loss in the low-income market, where the opposite happens. Therefore, it follows that demand in the low-income market (B) cannot be significantly larger than in the high-income market (A) for this to hold. This is why an upper bound on A_B/A_A is needed.[13] Thus, price discrimination is likely to increase welfare when time valuations are similar ($v_B/v_A > 1/3$ is sufficient) and the reservation prices are more similar. For example, when $1/3 < v_B/v_A < 1/2$, price discrimination can increase welfare when $2/3 < A_B/A_A < 1$. Figure 1 summarizes the effects of allowing price discrimination in our relevant case.

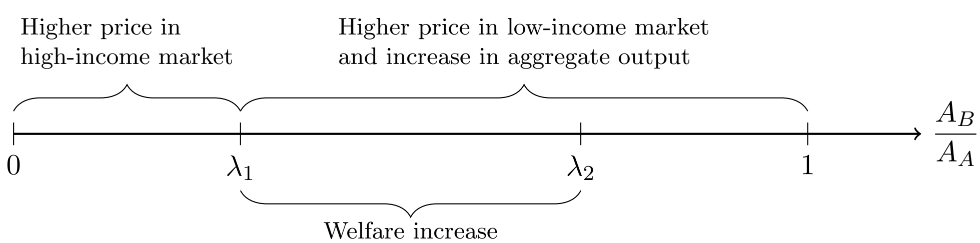

Figure 1: Effects of price discrimination by a private facility. $v_B/v_A < b_B/b_A$.

The welfare analysis when price discrimination changes both quantities in the same direction is in Appendix A.4. We choose not to discuss it here because the conditions that make price discrimination increase or decrease both quantities are rather stringent and not very informative. We show that both prices moving in the same direction are not sufficient for both quantities to move in the same direction. The conditions that make quantities either rise or fall involve an upper and lower bound on the ratio of time valuations and a restriction on the relationship between time valuations and demand slopes. Nevertheless, when price discrimination increases the quantity in both markets it increases consumer surplus in both markets and total welfare.

[13]When the market with high time valuations is also the market with low demand price sensitivity, so $v_A > v_B$ and $b_A > b_B$ hold, it is straightforward to show that $\lambda_2 < 1$. In addition, the interval $[\lambda_1, \lambda_2]$ is non-empty when the ratio of the inverse demand's slopes is less than the ratio of time valuations, i.e. $v_B/v_A < b_B/b_A$.

The results of this section show benefits from input price discrimination in the presence of negative consumption externalities and that price discrimination can increase consumer surplus. Importantly, the benefits are found in a setting where, in the absence of externalities, price discrimination yields lower social welfare. In the following section we analyze price discrimination by a public facility with the aim of understanding the effects of the ownership form of the supplier on the desirability of third-degree input price discrimination.

4. Public facility

We now study price discrimination by a public facility. If the facility were maximizing total welfare, allowing price discrimination would always be optimal as the facility would have one more degree of freedom in choosing the welfare maximizing prices. The prices would induce downstream marginal social cost pricing and the analysis of the effects of price discrimination on total welfare would be trivial. We introduce a source of divergence from total welfare maximization, namely that consumers and firms may be foreign and the facility is domestic. Among the many possible domestic-foreign structures, we assume that the high-income market (A) is fully domestic (consumers and firm A are domestic), and the firm B, together with a fraction of the consumers in the low-income market (B), are foreign. The assumption that the high-income market is the domestic market is, we believe, a realistic assumption if the differences in income across markets are a consequence of differences in trip purpose, as business travel is normally more frequent for domestic destinations than for international travel, where leisure travel is often predominant.[14]

We denote the fraction of foreign passengers in market B as α. The public facility maximizes the sum of its profit, firm A's profit, the consumer surplus in market A and the

[14]For example, in 2011, at Los Angeles International Airport (LAX), the share of business trips was 90% higher for US destinations than for international destinations (Unison Consulting, 2011). Our assumption may be less realistic for air transportation in high income countries with small domestic markets, such as the Netherlands or Switzerland. In those cases, our model may be representative of other transportation markets where congestible facilities provide an input to downstream firms, such as rail transportation.

fraction α of the consumer surplus in market B:

$$W_D = \left[\int_0^{q_A} P_A(x)dx - v_A \cdot q_A \cdot D(Q)\right] + \alpha \cdot \left[\int_0^{q_B} P_B(x)dx - q_B \cdot P_B(q_B)\right] + [w_B \cdot q_B] \ , \tag{17}$$

where the first term in square brackets is total welfare in market A (the sum of the consumer surplus, firm A's profit and the facility's revenue from market A), the second term in square brackets is the consumer surplus in market B and the third term is the facility's revenue from market B.

The incentive to price discriminate is to capture part of the foreign firm's profit and stimulate domestic production. The model can easily be extended to cases where there are foreign passengers in both markets, but results are not likely to change significantly. What matters is that there is a clear incentive to reduce the price in one market in detriment of the other, and not what the mechanism is that provides this incentive.

4.1. Price discrimination

The first-order conditions of maximizing W_D with respect to both input prices lead to the input prices w_A and w_B (see Appendix A.2 for the prices and all derivations of the results in this section). Here, as in the previous section, we present the pricing rules:

$$w_A = -q_A \cdot b_A + q_B \cdot v_B \ , \tag{18}$$

$$w_B = q_B \cdot b_B \cdot [2 - \alpha] + 2 \cdot q_B \cdot v_B + q_A \cdot v_A \ . \tag{19}$$

The input price for the domestic firm is a subsidy equal to the downstream markup ($-q_A \cdot b_A < 0$) and the marginal congestion cost that is not internalized by firm A ($q_B \cdot v_B$). This is the first-best pricing rule, as it makes the final price in the domestic market equal to the marginal social cost (see Eq. (6)). The input price in the foreign market is the same as the one charged by a private facility (see Eq. (8)) except for a discount in the demand related markup ($q_B \cdot b_B \cdot [2 - \alpha]$). As a fraction of consumers in market B are domestic, their surplus is taken into account and the markup is lower.

From comparing the pricing rules above, it follows that $w_B > w_A$ always holds in this case, a result of the assumed domestic-foreign structure. This explains the usual argument to enforce uniform pricing by a central authority that it protects consumers of foreign markets. The result is intuitive and important, as it reveals how the public ownership of the facility can provide incentives to set a lower input price in the high-income market.

4.2. The effects of price discrimination

The case where the facility is restricted to charge the same input price to both firms is useful to compare the discriminatory charges. Solving the facility's optimization problem we obtain the following pricing rule:

$$\begin{aligned} w = \quad & - q_A \cdot b_A \cdot \frac{2 \cdot [b_B + v_B] - v_A}{2 \cdot [b_A + b_B] + [v_A + v_B]} + q_B \cdot b_B \cdot \frac{2 \cdot [b_A + v_A] \cdot [2 - \alpha]}{2 \cdot [b_A + b_B] + [v_A + v_B]} \\ & + q_A \cdot v_A \cdot \frac{2 \cdot [b_A + v_A] - v_B}{2 \cdot [b_A + b_B] + [v_A + v_B]} + q_B \cdot v_B \cdot \frac{4 \cdot b_A + 3 \cdot v_A + \alpha \cdot b_B}{2 \cdot [b_A + b_B] + [v_A + v_B]} \, . \end{aligned} \tag{20}$$

The pricing rule in Eq. (20) includes a weighted sum of the subsidy for firm A and the markup for firm B present in the discriminating prices. It also includes a weighted sum of each of the firm-specific marginal external congestion costs that are also part of the differentiated input prices. We elaborate below on the relation between the uniform and the discriminating input prices and show that the uniform price is a weighted average of the discriminatory prices.

4.2.1. The effects of price discrimination on prices and output

Using the same price difference constraint method as in Section 3, we obtain the following results regarding the marginal effect of price discrimination on the aggregate output and on input prices:

$$\frac{dQ}{dt} = \frac{[b_A + v_A]\,[2 \cdot b_B \cdot [3 - \alpha] + 4 \cdot v_B - v_A] - v_B \cdot [b_B \cdot [2 - \alpha] + v_B]}{\Omega_3} \gtreqless 0 \, , \tag{21}$$

$$\frac{dw_A}{dt} < 0 \, , \tag{22}$$

$$\frac{dw_B}{dt} > 0 \, , \tag{23}$$

where Ω_3 is a positive constant. The results that follow from Eqs. (21)–(23) are summarized in the following proposition.

Proposition 4. *Price discrimination by a public facility:*

(i) Increases the aggregate output if time valuations are sufficiently similar in that $v_A - 3 \cdot v_B < b_B \cdot [4 - \alpha]$.

(ii) Decreases the input price in the high-income market (served by the domestic firm).

(iii) Increases the input price in the low-income market (served by the foreign firm).

This is intuitive, when the facility is allowed to price discriminate it reduces the price in the market served by the domestic firm and raises the price charged to the foreign firm to capture part of its profit. When the condition in part (i) of Proposition 4 holds, the output increase in the high-income domestic market is larger than the decrease in the low-income foreign market. Note that in the absence of negative consumption externalities, input price discrimination by a public facility always increases aggregate output, regardless of the value of α.

4.2.2. Welfare analysis

Unlike in the case of a private facility, we can analyze the welfare effect directly as opposed to using the price difference constraint method used in Section 3. Recall that in this section we look at how total welfare changes when a facility that maximizes domestic welfare is allowed to differentiate prices. In the general case, congestion effects come into play and patterns are complex. For this reason, we study the welfare effects when the low-income market (B) is fully foreign ($\alpha = 0$). We choose this particular value because it is, again, unfavorable for price discrimination. As α increases, the maximization problem of the facility approaches total welfare maximization, so it is more likely that price discrimination increases total welfare for greater values of α.[15] The following proposition summarizes the main result of this section: price discrimination by a public facility may increase total welfare in a setting in which price discrimination by a private facility always decreases total welfare.

[15]When $\alpha = 1$ the only difference between the objective function of the public facility and total welfare is the absence of firm B's profit.

Proposition 5. *When the low-income market (B) is fully foreign ($\alpha = 0$), price discrimination by a public facility increases total welfare if, and only, if $\frac{A_B}{A_A} < \lambda_4$, and it decreases total welfare when $\frac{A_B}{A_A} > \lambda_4$.*

Where λ_4 is a fraction whose numerator and denominator are a function of the demand sensitivity parameters (b_A, b_B) and of the congestion effects (v_A, v_B) in a similar way as λ_1 and λ_2 are. However, both the numerator and the denominator of λ_4 are polynomials to the 7th degree, so we omit the expression here (see Appendix A.2).

Proposition 5 also shows that when the facility is public, input price discrimination may increase total welfare in the presence of negative consumption externalities. Although less can be inferred analytically in this case, some results may be established without the need for numerical analysis. If markets are fully symmetric ($A_A = A_B$, $b_A = b_B$ and $v_A = v_B$), input price discrimination always increases welfare because $\lambda_4 > 1$ holds. More generally, when the high-income market is less price sensitive ($b_A > b_B$), the asymmetry between markets is the same in that $A_A/A_B = b_A/b_B = v_A/v_B$ and output increases ($v_B/v_A > 1/3$ is sufficient), input price discrimination increases total welfare.

To gain further insight, and to isolate the effect of the ownership form, we study the case without externalities ($v_A = v_B = 0$). The main result of the analysis is summarized in the following proposition.

Proposition 6. *When the low-income market (B) is fully foreign ($\alpha = 0$) and in the absence of negative consumption externalities, price discrimination by a public facility increases total welfare if, and only, if*

$$\frac{A_B}{A_A} < \lambda_5 = \frac{2 + 32 \cdot b_A/b_B}{5 + 8 \cdot b_A/b_B},$$

and it decreases total welfare when $\frac{A_B}{A_A} > \lambda_5$.

Several results follow from Proposition 6. First, it is also true in this case that if markets are fully symmetric ($A_A = A_B$ and $b_A = b_B$) input price discrimination increases welfare, because $\lambda_5 > 1$ holds. Second, if the inverse demand is steeper in the high-income market (i.e. $b_A > b_B$), $\lambda_5 > 1$ also holds, and therefore, total welfare increases with price discrimination. This result is natural, price discrimination raises the price in the low-income foreign market (B), which is more elastic and more price sensitive, so the welfare losses due to the double marginalization are limited compared to the gains of inducing marginal social cost pricing in the high-income domestic market. As argued above, a domestic market more intensive in business trips (relative to the foreign market) is consistent with the sufficient conditions for welfare improvement under price discrimination. More generally, $b_B/b_A > 8$ is sufficient for price discrimination to increase total welfare. If the inverse demand slope of the low-income (foreign) market is not too much larger (8 times) than the slope of the high-income (domestic) market, price discrimination increases total welfare.[16]

This section has shown how the public ownership of the provider enlarges the extent to which input price discrimination is desirable. In the following section we extend our analysis by allowing for downstream price discrimination.

5. Robustness: downstream first-degree price discrimination

The main aim of this section is to examine the robustness of our main results by studying the welfare effect of input third-degree price discrimination when downstream firms can apply perfect price discrimination. This is a theoretical extreme that is useful for studying a situation where there is no downstream inefficiency due to market power and works as a proxy for a perfectly competitive downstream market.

Downstream firms that perfectly discriminate consumers set a unit price equal to the marginal cost, which is the input price w_i plus the marginal congestion cost that is internal to the firm $q_i \cdot v_i$, and ask for a premium equal to the surplus of each individual (their willingness to pay net of the experienced delays). This changes the derived demands faced by the input provider, which are now a result of the following pricing rules:

$$P_i(q_i) - v_i \cdot D(Q) = w_i + q_i \cdot v_i \ \ \forall \, i \in \{A, B\} \ . \tag{24}$$

Following the same methodology as in Sections 3 and 4, it is possible to derive similar sufficient conditions for welfare improvement under third-degree input price discrimination

[16]If the low-income market is not fully foreign, we get $\lambda_5 = \frac{[4-\alpha]\cdot\left[b_B + b_A \cdot (\alpha-4)^2\right]}{b_B \cdot [10-3\cdot\alpha] + b_A \cdot [4-\alpha]\cdot[4-\alpha\cdot(2-\alpha)]}$.

for both ownership forms. The main aim of this extension is to analyze how the results in Propositions 3 and 5 change. Let $\lambda_i^{'}$ be the analogous boundary to λ_i derived in Sections 3 and 4. The following Proposition summarizes the results (see Appendix A.3 for the proof).

Proposition 7. *When downstream firms can perfectly discriminate consumers and in the presence of negative consumption externalities:*

(i) Input price discrimination increases total output when time valuations are similar in that $v_A - 3 \cdot v_B < 4 \cdot b_B$, for both a private and a publicly owned facility.

(ii) Input price discrimination by a private facility increases total welfare if $\lambda_1^{'} < \frac{A_B}{A_A} < \lambda_2^{'}$.

(iii) Input price discrimination by public facility, when market B is fully foreign, increases total welfare if $\frac{A_B}{A_A} < \lambda_4^{'}$.

Proposition 7 shows that allowing input providers to price discriminate can raise total output and increase total welfare even in the extreme case of downstream perfect price discrimination, regardless of the ownership form of the facility, and in a similar way as when downstream firms set a uniform price. The benefits of input price discrimination do not rely on the presence of downstream market power inefficiencies and, therefore, it can be expected that our main finding holds for imperfect downstream competition.

6. Conclusions

This paper has studied how the presence of congestion externalities influences the effects of input third-degree price discrimination. We have shown that the presence of downstream within- and cross-market negative externalities makes all demands interrelated in a way that is similar to the case where downstream firms offer substitute products. We find that aggregate output and welfare can increase when price discrimination is allowed in a setting in which in the absence of congestion, price discrimination by a private input provider leads to welfare deterioration and constant aggregate output. The results of the paper suggest that the presence of congestion externalities enlarges the extent in which input price discrimination by a private facility is desirable from a welfare standpoint.

We have also analyzed the effects of price discrimination when the input supplier maximizes domestic welfare, a common ownership form of transport facilities. The difference lies in that the public supplier wants to stimulate domestic production and capture foreign profit, so it gives price concessions in markets with a higher participation of domestic firms and consumers. In this case we also find that allowing price discrimination may cause an expansion of total output and an increase of total welfare. While policies that ban price discrimination in markets where this is a relevant issue, such as the EU Airport Charges directive (2009/12/EC) and the World Trade Organization's General Agreement on Tariffs and Trade (GATT), may be in place due to social reasons, we find that they may come at a cost to aggregate welfare.

Considering other demand and cost functions is a natural avenue for future research. Analyzing demand functions following Cowan (2007) may help in establishing sufficient conditions for welfare improvement with price discrimination for several other demand functions, but possibly at the cost of adding significant complexity to the analysis. Other important extensions to our analysis include the consideration of competition and cost regulations. However, given the robustness of our results to the presence of downstream perfect price discrimination and that the presence of firms in multiple markets also enhances the performance of input third degree price discrimination (Arya and Mittendorf, 2010), it may provide similar insights. The cost regulation issue may be tackled extending the framework of Armstrong and Vickers (1991) to upstream price regulation. Another avenue of future research is to study how a ban on price discrimination performs under private information by downstream firms. It is unlikely that the regulator will have such detailed information as we assume in the paper, so it is relevant to check the robustness of our results when, for example, firms' costs and the externality valuation are not known by the regulator.

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Appendix A.

Appendix A.1. Calculations and proofs for Section 3

Derived demands

Solving simultaneously the first-order conditions of both downstream firms (see Eq. (4)) and denoting A_i the inverse demand intercept in market i, we obtain the derived demands:

$$q_A(w_A, w_B) = \frac{2 \cdot [b_B + v_B] \cdot [A_A - w_A] - v_A \cdot [A_B - w_B]}{\Omega_1} \tag{A.1}$$

$$q_B(w_A, w_B) = \frac{2 \cdot [b_A + v_A] \cdot [A_B - w_B] - v_B \cdot [A_A - w_A]}{\Omega_1} \tag{A.2}$$

$$\text{where } \Omega_1 = 2 \cdot [b_A + v_A] \cdot 2 \cdot [b_B + v_B] - [v_A \cdot v_B] > 0 \tag{A.3}$$

And deriving with respect to the input prices, we get:

$$\frac{\partial q_A}{\partial w_A} = \frac{-2 \cdot [b_B + v_B]}{\Omega_1} < 0 \tag{A.4}$$

$$\frac{\partial q_A}{\partial w_B} = \frac{v_A}{\Omega_1} > 0 \tag{A.5}$$

$$\frac{\partial q_B}{\partial w_A} = \frac{v_B}{\Omega_1} > 0 \tag{A.6}$$

$$\frac{\partial q_B}{\partial w_B} = \frac{-2 \cdot [b_A + v_A]}{\Omega_1} < 0 \tag{A.7}$$

Input prices

Solving the first-order conditions for the input supplier under price discrimination, $\partial \Pi^{PD} / \partial w_A$ and $\partial \Pi^{PD} / \partial w_B$, we get:

$$w_A = \frac{[b_A + v_A] \, [8 \cdot A_A \cdot [b_B + v_B] - 2 \cdot A_B \cdot [v_A - v_B]] - A_A \cdot v_B \cdot [v_A + v_B]}{16 \cdot [b_A + v_A] \cdot [b_B + v_B] - [v_A + v_B]^2} \tag{A.8}$$

$$w_B = \frac{[b_B + v_B] \, [8 \cdot A_B \cdot [b_A + v_A] + 2 \cdot A_A \cdot [v_A - v_B]] - A_B \cdot v_A \cdot [v_A + v_B]}{16 \cdot [b_A + v_A] \cdot [b_B + v_B] - [v_A + v_B]^2} \tag{A.9}$$

Solving $\partial \Pi^U / \partial w$, we obtain:

$$w = \frac{A_A \cdot [2 \cdot b_B + v_B] + A_B \cdot [2 \cdot b_A + v_A]}{4 \cdot [b_A + b_B] + 2 \cdot [v_A + v_B]} \tag{A.10}$$

Proof of Proposition 1

Using Eqs. (A.8) and (A.9), we get that $w_B - w_A$ equals:

$$\begin{aligned}&\frac{A_B \cdot \left[8 \cdot b_A \cdot b_B + 5 \cdot v_A \cdot v_B + v_A^2 + 2 \cdot v_A \cdot [4b_B + b_A] + 6 \cdot b_A \cdot v_B\right]}{16 \cdot [b_A + v_A] \cdot [b_B + v_B] - [v_A + v_B]^2}\\ -&\frac{A_A \cdot \left[8 \cdot b_A \cdot b_B + 5 \cdot v_A \cdot v_B + v_B^2 + 2 \cdot v_B \cdot [4b_A + b_B] + 6 \cdot b_B \cdot v_A\right]}{16 \cdot [b_A + v_A] \cdot [b_B + v_B] - [v_A + v_B]^2}\end{aligned} \tag{A.11}$$

where the denominator is positive by the second-order conditions of the supplier maximization problem ($v_B/v_A > 7 - 4\sqrt{3} \approx 0.0718$ is sufficient). Therefore, the condition in Proposition 1 follows straightforwardly as the terms multiplying A_A and A_B in Eq. (A.11) are positive.

Effect of price discrimination on output and prices

To simplify notation, we omit the arguments of the functions and let τ be the input price in market A and $\tau + t$ the charge in market B. For a given $t \in [0, t^*]$, the first-order condition of the supplier's maximization profit is:

$$\partial\Pi/\partial\tau = [q_A + q_B] + \tau \cdot \left[\frac{\partial q_A}{\partial \tau} + \frac{\partial q_B}{\partial \tau}\right] + t \cdot \frac{\partial q_B}{\partial \tau}\ . \tag{A.12}$$

This first-order condition defines implicitly τ as a function of t in the following way:

$$\frac{d\tau}{dt} = -\frac{\partial^2\Pi/\partial\tau\partial t}{\partial^2\Pi/\partial\tau^2} = -\frac{\left[\frac{\partial q_A}{\partial w_B} + 2 \cdot \frac{\partial q_B}{\partial w_B} + \frac{\partial q_B}{\partial w_A}\right]}{2 \cdot \left[\frac{\partial q_A}{\partial w_A} + \frac{\partial q_A}{\partial w_B} + \frac{\partial q_B}{\partial w_A} + \frac{\partial q_B}{\partial w_B}\right]} \tag{A.13}$$

$$= \frac{\left[-4 \cdot b_A - [3 \cdot v_A - v_B]\right]/\Omega_1}{\left[4 \cdot [b_A + v_A] + 4 \cdot [b_B + v_B] + 2 \cdot [v_A + v_B]\right]/\Omega_1} \tag{A.14}$$

The marginal output effect is given by:

$$\frac{dQ}{dt} = \frac{d\tau}{dt} \cdot \left[\frac{\partial q_A}{\partial w_A} + \frac{\partial q_A}{\partial w_B} + \frac{\partial q_B}{\partial w_A} + \frac{\partial q_B}{\partial w_B}\right] + \left[\frac{\partial q_A}{\partial w_B} + \frac{\partial q_B}{\partial w_B}\right] \tag{A.15}$$

which can be simplified using Eq. (A.13) to:

$$\frac{dQ}{dt} = \frac{1}{2} \cdot \left[\frac{\partial q_A}{\partial w_B} - \frac{\partial q_B}{\partial w_A}\right] = \frac{[v_A - v_B]}{2 \cdot \Omega_1} \tag{A.16}$$

where the last equality in Eq. (A.14) and in Eq. (A.16) use Eqs. (A.4)–(A.7).

The marginal effect on input prices follows from Eq. (A.14):

$$\frac{dw_A}{dt} = \frac{d\tau}{dt} = \frac{-4 \cdot b_A - [3 \cdot v_A - v_B]}{\Omega_2} \tag{A.17}$$

$$\frac{dw_B}{dt} = \frac{d\tau}{dt} + 1 = \frac{[3 \cdot v_B - v_A] + 4 \cdot b_B}{\Omega_2} \tag{A.18}$$

$$\text{where } \Omega_2 = 4 \cdot [b_A + v_A] + 4 \cdot [b_B + v_B] + 2 \cdot [v_A + v_B] > 0 \tag{A.19}$$

From which Proposition 2 follows directly.

Proof of Proposition 3

- Welfare improvement: $\frac{A_B}{A_A} > \lambda_1$

When $v_A - 3 \cdot v_B < 4 \cdot b_B$ holds, it follows from Eqs. (A.17) and (A.18) that $d\tau/dt < 0$ and $d\tau/dt + 1 > 0$. This implies that $dq_A/dt > 0$ and $dq_B/dt < 0$ (see Eqs. (A.62) and (A.63)). From Eq. (16), it follows then that showing that $w_A + q_A \cdot b_A - q_B \cdot v_B$, which is positive (see Proof of Proposition 8), is greater than $w_A + t + q_B \cdot b_B - q_A \cdot v_A$ for any value of $t \in [0, t^*]$ is sufficient for $dW/dt > 0$. Denote $f(t)$ the difference between these two terms; we prove that the condition in Proposition 3 is sufficient for $f(t) > 0$ to hold. Formally,

$$f(t) = q_A \cdot [b_A + v_A] - q_B \cdot [b_B + v_B] - t \tag{A.20}$$

$$\frac{df}{dt} = \frac{dq_A}{dt} \cdot [b_A + v_A] - \frac{dq_B}{dt} \cdot [b_B + v_B] - 1 \tag{A.21}$$

$$\frac{df}{dt} = -\frac{8[b_A + b_B][b_B \cdot v_A + b_A \cdot b_B + b_B \cdot v_B]}{\Omega_2} \tag{A.22}$$

$$- \frac{[v_A - v_B]^2[v_A + v_B] + [5v_A + v_B][b_B \cdot v_B + b_A \cdot v_A]}{\Omega_2} \tag{A.23}$$

As $df/dt < 0$, $f(t^*) > 0$ is sufficient for $dW/dt > 0$. Using that $t^* = w_B - w_A$ and Eq. (A.11), we get:

$$f(t^*) = \frac{A_A \cdot \left[12 \cdot b_A \cdot b_B + 10 \cdot v_A \cdot v_B + 2 \cdot v_B^2 + 3 \cdot v_B \cdot [4b_A + b_B] + 11 \cdot b_B \cdot v_A\right]}{16 \cdot [b_A + v_A] \cdot [b_B + v_B] - [v_A + v_B]^2} - \frac{A_B \cdot \left[12 \cdot b_A \cdot b_B + 10 \cdot v_A \cdot v_B + 2 \cdot v_A^2 + 3 \cdot v_A \cdot [4b_B + b_A] + 11 \cdot b_A \cdot v_B\right]}{16 \cdot [b_A + v_A] \cdot [b_B + v_B] - [v_A + v_B]^2} \tag{A.24}$$

from which the result in Proposition 3 follows directly, as the denominator is positive by the second-order conditions and the terms multiplying A_A and A_B in Eq. (A.24) are positive.

Finally, to show that the interval $[\lambda_1, \lambda_2]$ is non-empty when $\frac{b_A}{b_B} \leq \frac{v_A}{v_B}$ holds, we look at

$\lambda_2 - \lambda_1$:

$$\lambda_2 - \lambda_1 = \frac{L_1 \cdot [b_B \cdot v_A - b_A \cdot v_B]}{L_2 \cdot L_3} \tag{A.25}$$

$$\text{where } L_1 = 16 \cdot [b_A + v_A] \cdot [b_B + v_B] - [v_A + v_B]^2 > 0$$

$$L_2 = 8 \cdot b_A \cdot b_B + 5 \cdot v_A \cdot v_B + v_A^2 + 2 \cdot v_A \cdot [4b_B + b_A] + 6 \cdot b_A \cdot v_B > 0$$

$$L_3 = 12 \cdot b_A \cdot b_B + 10 \cdot v_A \cdot v_B + 2 \cdot v_A^2 + 3 \cdot v_A \cdot [4b_B + b_A] + 11 \cdot b_A \cdot v_B > 0$$

where $L_1 > 0$ follows from $\partial^2\Pi/\partial w_A^2 \cdot \partial^2\Pi/\partial w_B^2 > [\partial^2\Pi/\partial w_A \partial w_B]^2$, a second-order condition that we assume to hold. Therefore, $\lambda_2 - \lambda_1 \geq 0 \Leftrightarrow b_B \cdot v_A - b_A \cdot v_B > 0$, which proves the result.

Appendix A.2. Calculations and proofs for Section 4

Input prices

Solving the first-order conditions for the input supplier under price discrimination, $\partial W_D/\partial w_A$ and $\partial W_D/\partial w_B$, we get:

$$w_A = \frac{A_B \cdot [b_A \cdot [v_A + 2 \cdot v_B] + 2 \cdot v_A \cdot v_B]}{[b_A + 2 \cdot v_A] \cdot [b_B \cdot [4 - \alpha] + 4 \cdot v_B] - [v_A + v_B]^2} - \frac{A_A \cdot [v_B \cdot [v_A + v_B] + b_A \cdot [b_B \cdot [4 - \alpha] + 4 \cdot v_B]]}{[b_A + 2 \cdot v_A] \cdot [b_B \cdot [4 - \alpha] + 4 \cdot v_B] - [v_A + v_B]^2} \tag{A.26}$$

$$w_B = \frac{A_A \cdot [[b_B + v_B] \cdot [v_A - v_B] + \alpha \cdot b_B \cdot v_B]}{[b_A + 2 \cdot v_A] \cdot [7 \cdot b_B + 8 \cdot v_B] - 2 \cdot [v_A + v_B]^2} + \frac{A_B \cdot [[b_A + v_A] \cdot [b_B \cdot [2 - \alpha] + v_B] + v_A \cdot [2 \cdot v_B - v_A]]}{[b_A + 2 \cdot v_A] \cdot [7 \cdot b_B + 8 \cdot v_B] - 2 \cdot [v_A + v_B]^2} \tag{A.27}$$

Solving $\partial W_D/\partial w$, we obtain:

$$w = \Bigg[A_A \cdot \Big[[b_B + v_B][2b_A[4v_B - 3v_A + 2b_B] + [5v_A v_B - 4v_A^2]] + b_B[v_A v_B + \alpha v_B[v_B - v_A - b_A]\Big] - A_B \cdot \Big[v_A^2[-3b_A + 8b_B + [7v_B - 2v_A]] + 2\alpha b_B[v_A + b_A][2[v_A + b_A] - v_B]\Big] + 2b_A\left[4[b_B + v_B][b_A + 2v_A] + b_B v_A\right]\Bigg]/\Omega_3$$

$$\text{where } \Omega_3 = -4b_A b_B\left[b_A + 2v_A\right][4 - \alpha] + v_B\left[[-4b_A + v_B][4b_A + b_B\alpha + 2v_A]\right] - 2v_A\left[-2b_B[-2b_B + 5v_A] - [v_A^2 - 7v_A v_B] + b_A[12v_B + v_A]\right] + b_A[-2b_B - v_A]^2 \tag{A.28}$$

Effect of price discrimination on output and prices

To simplify notation, we again omit the arguments of the functions and let τ be the input price in market A and $\tau + t$ the charge in market B. The marginal effect on τ is:

$$\frac{d\tau}{dt} = -\frac{\partial^2 W_D/\partial\tau\partial t}{\partial^2 W_D/\partial\tau^2}\ , \tag{A.29}$$

where,

$$\begin{aligned}\frac{\partial^2 W_D}{\partial\tau\partial t} = & \left[\frac{\partial q_A}{\partial w_A} + \frac{\partial q_A}{\partial w_B}\right]\left[\frac{\partial q_A}{\partial w_B}[-b_A - v_A] - v_A\left[\frac{\partial q_A}{\partial w_B} + \frac{\partial q_B}{\partial w_B}\right]\right] \\ & + \left[\frac{\partial q_B}{\partial w_A} + \frac{\partial q_B}{\partial w_B}\right]\left[\frac{\partial q_B}{\partial w_B}\cdot[\alpha\cdot b_B] - \frac{\partial q_A}{\partial w_B}v_A + 1\right] + \frac{\partial q_B}{\partial w_B}\end{aligned} \tag{A.30}$$

$$\begin{aligned}\frac{\partial^2 W_D}{\partial\tau^2} = & \left[\frac{\partial q_A}{\partial w_A} + \frac{\partial q_A}{\partial w_B}\right]\left[\left[\frac{\partial q_A}{\partial w_A} + \frac{\partial q_A}{\partial w_B}\right][-b_A - v_A] - v_A\left[\frac{\partial Q}{\partial w_A} + \frac{\partial Q}{\partial w_B}\right]\right] \\ & + \left[\frac{\partial q_B}{\partial w_A} + \frac{\partial q_B}{\partial w_B}\right]\left[\left[\frac{\partial q_B}{\partial w_A} + \frac{\partial q_B}{\partial w_B}\right]\cdot[\alpha\cdot b_B] - \left[\frac{\partial q_A}{\partial w_A} + \frac{\partial q_A}{\partial w_B}\right]\cdot v_A + 2\right]\end{aligned} \tag{A.31}$$

Substituting Eqs. (A.4)–(A.7) in Eq. (A.29) yields:

$$\begin{aligned}\frac{d\tau}{dt} = & \frac{4b_A^2[4[b_B + v_B] - b_B\alpha] + b_A\left[[-v_A^2 + 17v_A v_B - v_B^2]\right] + 2b_B v_A(9 - 2\alpha)}{\Omega_3} \\ & + \frac{[v_A + b_A]\left[[-2v_A^2 + 13v_A v_B - 3v_B^2] + 2b_B[v_B(\alpha - 2) + 2v_A(4 - \alpha)]\right]}{\Omega_3}\end{aligned} \tag{A.32}$$

where Ω_3 is negative as we assume that the second-order conditions of the supplier's maximization problem under uniform pricing holds (i.e. $\partial^2 W_D/\partial w^2 < 0$). This, together with $v_A > v_B$ and $\partial^2 W_D/\partial w_A^2 \cdot \partial^2 W_D/\partial w_B^2 > [\partial^2 W_D/\partial w_A \partial w_B]^2$, which again holds as we assume that the second-order conditions of the supplier's maximization problem under price discrimination holds, imply that $d\tau/dt < 0$. As $dw_A/dt = d\tau/dt$, we get that price discrimination decreases the input price in market A.

The marginal effect on w_B is:

$$\begin{aligned}\frac{dw_B}{dt} = \frac{d\tau}{dt} + 1 = & \frac{b_B v_B \alpha\left[[2v_A - v_B] + 2b_A\right] + 2b_A\left[2b_B + v_A\right]\left[+b_B + v_B\right]}{-\Omega_3} \\ & + \frac{v_A\left[8b_B[b_B + 8v_A v_B] + v_B[v_A + v_B]\right]}{-\Omega_3} > 0\end{aligned} \tag{A.33}$$

which proves that price discrimination increases the input price in market B.

Using Eqs. (A.4)–(A.7), (A.15) and (A.32), we get:

$$\frac{dQ}{dt} = \frac{[v_B + b_B(2-\alpha)]\,[+b_A + [v_A - v_B]] + [v_A + b_A]\,[b_B(4-\alpha) + [3v_B - v_A]]}{-\Omega_3} \tag{A.34}$$

From Eq. (A.34) it follows that $v_A - 3 \cdot v_B < 4b_B$ is sufficient for $dQ/dt > 0$ to hold.

Proof of Proposition 5

Subtracting the value of the total welfare in Eq. (5) when evaluated at $\{w_A, w_B\}$ and at w, we obtain:

$$W(w_A, w_B) - W(w) = \frac{[w_B - w_A]}{4 \cdot \Omega_4} \cdot \frac{[A_A \cdot \lambda_4^N - A_B \lambda_4^D]}{\Omega_5^2} \tag{A.35}$$

$$\text{where } \Omega_4 = 4[v_B + b_B][b_A + 2v_A] - [v_A + v_B]^2$$

$$\text{where } \Omega_5 = 16b_A^2[v_B + b_B] + b_A\left[4b_B^2 + 36b_B v_A - [3v_A^2 - 32v_A v_B + 4v_B^2]\right]$$
$$-2v_A\left[-4b_B^2 + [v_A^2 - 7v_A v_B + v_B^2] - 2b_B[4v_A + v_B]\right]$$

As $w_B > w_A$ holds and Ω_4 is positive because we assume that the second-order conditions of the supplier's maximization problem hold, the sign of the welfare change is given by $[A_A \cdot \lambda_4^N - A_B \lambda_4^D]$, where λ_4^N and λ_4^D are given by Eqs. (A.36) and (A.37) respectively. As λ_4^N is the numerator of λ_4 and λ_4^D is the denominator of λ_4 and it is positive, the result proves Proposition 5.

$$
\begin{aligned}
\lambda_4^N =& \Bigg[4b_B^3 + 4b_B^2(85v_A - 6v_B) - b_B\left[7v_A^2 - 696v_Av_B + 32v_B^2\right] - 2v_B\left[7v_A^2 - 168v_Av_B + 8v_B^2\right]\Bigg] 4b_A^3(v_B - b_B) \\
&+ \Bigg[b_B\left[-6v_A^4 - 435v_A^3v_B + 7895v_A^2v_B^2 - 672v_Av_B^3 + 160v_B^4\right] \\
&\quad + b_B^2\left[-154v_A^3 + 8102v_A^2v_B - 516v_Av_B^2 + 344v_B^3\right] + 4b_B^3\left[664v_A^2 - 5v_Av_B + 59v_B^2\right] \\
&\quad - 2v_B^2\left[127v_A^3 - 1245v_A^2v_B + 136v_Av_B^2 - 8v_B^3\right] + 40b_B^4(3v_A + v_B)\Bigg] b_A^2 \\
&+ \Bigg[32b_B^4v_A(9v_A + 5v_B) + 8b_B^3\left[279v_A^3 + 67v_A^2v_B + 95v_Av_B^2 - 5v_B^3\right] \\
&\quad - 4b_B^2\left[66v_A^4 - 1664v_A^3v_B + 25v_A^2v_B^2 - 244v_Av_B^3 + 25v_B^4\right] \\
&\quad + 2b_B\left[v_A^5 - 312v_A^4v_B + 3162v_A^3v_B^2 - 344v_A^2v_B^3 + 227v_Av_B^4 - 34v_B^5\right] \\
&\quad + v_B\left[9v_A^5 - 325v_A^4v_B + 1947v_A^3v_B^2 - 331v_A^2v_B^3 + 64v_Av_B^4 - 12v_B^5\right]\Bigg] b_A + 256b_A^4(b_B + v_B)^3 \\
&- v_Av_B^3\left[80b_B^3 - 674b_B^2v_A + 205b_Bv_A^2 - 538v_A^3\right] + v_B^6(7b_B - 20v_A) + v_B^5\left[6b_B^2 - 103b_Bv_A + 54v_A^2\right] \\
&+ 2v_A^2v_B^2\left[288b_B^3 + 101b_B^2v_A + 878b_Bv_A^2 - 60v_A^3\right] + 4b_Bv_A^3\left[56b_B^3 + 156b_B^2v_A - 28b_Bv_A^2 - v_A^3\right] + 2v_B^7 \\
&+ 2v_A^2v_B\left[80b_B^4 + 256b_B^3v_A + 928b_B^2v_A^2 - 122b_Bv_A^3 + 3v_A^4\right] - v_Av_B^4\left[162b_B^2 - 321b_Bv_A + 124v_A^2\right]
\end{aligned}
\tag{A.36}
$$

$$
\begin{aligned}
\lambda_4^D =& 32b_A^4\left[b_B+v_B\right]\left[4v_B(3b_B+v_A)+b_B[2b_B+5v_A]+8v_B^2\right]\\
&+\Bigg[8v_B^2\left[18b_B^2+407b_Bv_A+60v_A^2\right]+2v_B\left[104b_B^3+1268b_B^2v_A+606b_Bv_A^2-21v_A^3\right]\\
&\quad+b_B\left[40b_B^3+452b_B^2v_A+758b_Bv_A^2-49v_A^3\right]+80v_B^3(15v_A-2b_B)-128v_B^4\Bigg]b_A^3\\
&-\Bigg[8v_B^3\left[10b_B^2+52b_Bv_A-261v_A^2\right]+v_B^2\left[40b_B^3-892b_B^2v_A-6026b_Bv_A^2-597v_A^3\right]\\
&\quad-v_Av_B\left[1132b_B^3+5140b_B^2v_A+1703b_Bv_A^2-120v_A^3\right]+4v_B^4(4b_B+103v_A)\\
&\quad-v_A\left[240b_B^4+1116b_B^3v_A+1204b_B^2v_A^2-153b_Bv_A^3+3v_A^4\right]-16v_B^5\Bigg]b_A^2\\
&-\Bigg[v_B^4\left[115b_Bv_A+394v_A^2-6b_B^2\right]-2v_B^5(2b_B+9v_A)+v_Av_B^3\left[308b_B^2+136b_Bv_A-1573v_A^2\right]\\
&\quad-v_Av_B^2\left[-160b_B^3+1778b_B^2v_A+4709b_Bv_A^2+269v_A^3\right]\\
&\quad+v_A^2v_B\left[-2032b_B^3-4288b_B^2v_A-902b_Bv_A^2+103v_A^3\right]\\
&\quad-v_A^2\left[480b_B^4+1088b_B^3v_A+732b_B^2v_A^2-140b_Bv_A^3+5v_A^4\right]\Bigg]b_A\\
&-\Bigg[v_B^4\left[-12b_B^2+141b_Bv_A+94v_A^2+v_Av_B^3\left[296b_B^2-189b_Bv_A-412v_A^2\right]\right]\\
&\quad-v_Av_B^2\left[-160b_B^3+1140b_B^2v_A+1233b_Bv_A^2+34v_A^3\right]-2v_B^6\\
&\quad+4v_A^2v_B\left[-300b_B^3-284b_B^2v_A-39b_Bv_A^2+7v_A^3\right]+v_B^5(8v_A-11b_B)\\
&\quad-2v_A^2\left[160b_B^4+152b_B^3v_A+76b_B^2v_A^2-20b_Bv_A^3+v_A^4\right]\Bigg]v_A
\end{aligned} \tag{A.37}
$$

Proof of Proposition 6

Setting v_A and v_B to 0 and subtracting the value of the total welfare in Eq. (5) when evaluated at $\{w_A, w_B\}$ and at w, we obtain:

$$W(w_A,w_B)-W(w)=\frac{A_A(\alpha-4)+A_B(\alpha-2)}{8\cdot\left[b_B-b_A(\alpha-4)\right]^2\left[\alpha-4\right]^2}\cdot\left[A_B\lambda_5^D-A_A\cdot\lambda_5^N\right] \tag{A.38}$$

$$\text{where } \lambda_5^N=[4-\alpha]\cdot\left[b_B+b_A(\alpha-4)^2\right]>0 \tag{A.39}$$

$$\text{and } \lambda_5^D=b_B\cdot[10-\alpha]+b_A\cdot[\alpha-4][\alpha(\alpha-2)-4]>0$$

As the fraction in the right-hand side of Eq. A.38 is negative, we get that price discrimination raises welfare if $A_B\lambda_5^D - A_A \cdot \lambda_5^N < 0$. Evaluating at $\alpha = 0$ proves Proposition 6.

Appendix A.3. Calculations and proofs for Section 5

The calculations and proofs are more brief in this section as they follow the same logic as the ones in the previous sections. Under downstream perfect price discrimination the derived demands are:

$$q_A(w_A, w_B) = \frac{[b_B + 2 \cdot v_B] \cdot [A_A - w_A] - v_A \cdot [A_B - w_B]}{\Omega_1'} \tag{A.40}$$

$$q_B(w_A, w_B) = \frac{[b_A + 2 \cdot v_A] \cdot [A_B - w_B] - v_B \cdot [A_A - w_A]}{\Omega_1'} \tag{A.41}$$

$$\text{where } \Omega_1' = [b_A + 2 \cdot v_A] \cdot [b_B + 2 \cdot v_B] - v_A \cdot v_B > 0 \tag{A.42}$$

Private facility

Solving the first-order conditions for the private input supplier under price discrimination we get:

$$w_A = \frac{[b_A + 2 \cdot v_A]\,[2 \cdot A_A \cdot [b_B + 2 \cdot v_B] - A_B \cdot [v_A - v_B]] - A_A \cdot v_B \cdot [v_A + v_B]}{4 \cdot [b_A + 2 \cdot v_A] \cdot [b_B + 2 \cdot v_B] - [v_A + v_B]^2} \tag{A.43}$$

$$w_B = \frac{[b_B + 2 \cdot v_B]\,[2 \cdot A_B \cdot [b_A + 2 \cdot v_A] + A_A \cdot [v_A - v_B]] - A_B \cdot v_A \cdot [v_A + v_B]}{4 \cdot [b_A + 2 \cdot v_A] \cdot [b_B + 2 \cdot v_B] - [v_A + v_B]^2} \tag{A.44}$$

$$w = \frac{A_A \cdot [b_B + 2 \cdot v_B] + A_B \cdot [b_A + v_A]}{2 \cdot [b_A + b_B] + 2 \cdot [v_A + v_B]} \tag{A.45}$$

By subtracting both values we obtain that $w_B - w_A > 0$ if and only if $A_B/A_A > \lambda_1'$ where λ_1' is given by:

$$\lambda_1' = \frac{2 \cdot b_A \cdot b_B + 5 \cdot v_A \cdot v_B + v_B^2 + v_B \cdot [4b_A + b_B] + 3 \cdot b_B \cdot v_A}{2 \cdot b_A \cdot b_B + 5 \cdot v_A \cdot v_B + v_A^2 + v_A \cdot [4b_B + b_A] + 3 \cdot b_A \cdot v_B} \tag{A.46}$$

Using the same methodology as in Appendix A.1, we obtain:

$$\frac{dQ}{dt} = \frac{v_A - v_B}{2 \cdot \Omega_1^{'}} > 0 \,, \tag{A.47}$$

$$\frac{dw_A}{dt} = \frac{-2 \cdot b_A - [3 \cdot v_A - v_B]}{\Omega_2^{'}} < 0 \,, \tag{A.48}$$

$$\frac{dw_B}{dt} = \frac{[3 \cdot v_B - v_A] + 2 \cdot b_B}{\Omega_2^{'}} \,, \tag{A.49}$$

$$\Omega_2^{'} = 2 \cdot [b_B + b_A + v_A + v_B] > 0 \tag{A.50}$$

which proves that when $v_B/v_A \geq 1/3$, the prices move in opposite directions. As a consequence, quantities also move in the opposite direction with price discrimination.

The marginal welfare effect under downstream perfect price discrimination is:

$$\frac{dW}{dt} = \frac{dq_A}{dt} \cdot [w_A - [q_B \cdot v_B]] + \frac{dq_B}{dt} \cdot [w_A + t - [q_A \cdot v_A]] \;. \tag{A.51}$$

Also in this case it is straightforward to show that the bracketed terms are positive at the discriminating input prices and that $w_A - [q_B \cdot v_B]$ decreases with t, so that it is positive for all values of t. Moreover, just as in Appendix A.1 the difference between the two terms, $f^{'}(t)$, decreases with t. Then, when $A_B/A_A > \lambda_1^{'}$, $f^{'}(t^*) > 0$ is sufficient for $dW/dt > 0$. Using the differentiated prices above, we obtain:

$$f^{'}(t^*) = \frac{A_A \cdot [b_B + 2v_B] \cdot [2b_A + [5v_A + v_B]]}{4 \cdot [b_A + 2 \cdot v_A] \cdot [b_B + 2 \cdot v_B] - [v_A + v_B]^2} - \frac{A_B \cdot [b_A + 2v_A] \cdot [2b_B + [5v_B + v_A]]}{4 \cdot [b_A + 2 \cdot v_A] \cdot [b_B + 2 \cdot v_B] - [v_A + v_B]^2} \tag{A.52}$$

from which it follows that welfare increases when $\lambda_1^{'} < A_B/A_A < \lambda_2^{'}$, where

$$\lambda_2^{'} = \frac{[b_B + 2v_B] \cdot [2b_A + [5v_A + v_B]]}{[b_A + 2v_A] \cdot [2b_B + [5v_B + v_A]]} \tag{A.53}$$

Public facility

Solving the first-order conditions for the public supplier, we obtain the following prices:

$$w_A^{'} = \frac{v_B \cdot [A_B \cdot [b_A + 2v_A] - A_A \cdot [v_A + v_B]]}{\Omega_4^{'}} \tag{A.54}$$

$$w_B^{'} = \frac{A_A \cdot [b_B + 2v_B]\,[[v_A - v_B]]}{\Omega_4^{'}} + \frac{A_B \cdot [b_A \cdot [b_B + 2v_B] + v_A \cdot [2b_B + [3 \cdot v_B - v_A]]]}{\Omega_4^{'}} \tag{A.55}$$

$$w^{'} = \Big[A_A \cdot \Big[b_A[b_B + 2v_B][v_A - v_B] + v_A[2v_B + b_B][2v_A - 3v_B] + v_A v_B^2\Big] + A_B \cdot \Big[[b_A + 2v_A]\,[[b_A + 2v_A][b_B + 2v_B] - v_A[v_A + v_B]] + v_A^2 \cdot v_B\Big]\Big] / \Omega_5^{'}$$

where $\Omega_4^{'} = 2 \cdot [b_A + 2v_A] \cdot [b_B + 2v_B] - [v_A + v_B]^2 > 0$

and $\Omega_5^{'} = [b_A + 2v_A]\,[[b_B + v_B][b_B + 8v_A] - v_A[v_A + v_B]] + 2b_A^2\,[b_B + 2v_B] + [7b_A v_A + b_A b_B - 2v_A v_B]\,v_B$

Subtracting the value of the total welfare under discriminating and uniform prices, we obtain:

$$\Delta W = \frac{[w_B^{'} - w_A^{'}]}{\Omega_4^{'}} \cdot \frac{[A_A \cdot \lambda_4^{N^{'}} - A_B \lambda_4^{D^{'}}]}{4 \cdot \Omega_5^{'2}} \tag{A.56}$$

As $w_B^{'} > w_A^{'}$ holds, it follows that the sign of the welfare change is given by $[A_A \cdot \lambda_4^{N^{'}} - A_B \cdot \lambda_4^{D^{'}}]$ where $\lambda_4^{N^{'}}$ and $\lambda_4^{D^{'}}$ are defined below. As $\lambda_4^{D^{'}}$ is positive, the result proves that the sign of the welfare change has the same sign as $A_B/A_A - \lambda_4^{'}$, where $\lambda_4^{'} = \lambda_4^{'N}/\lambda_4^{'D}$.

$$
\begin{aligned}
\lambda_4^{'N} =& -8b_A^3[b_B+2v_B]^2\left[-b_Bv_A-3v_Av_B-v_B^2\right]-31v_Av_B^4[-b_B+4v_A][-b_B+v_A]\\
&+\Bigg[v_B^2\left[11b_B^2+39b_Bv_A+242v_A^2\right]+3b_Bv_A\left[b_B^2+16b_Bv_A-2v_A^2\right]\\
&\quad -v_B\left[-b_B^3-11b_B^2v_A-226b_Bv_A^2+18v_A^3\right]+3v_B^4-v_B^3[-21b_B-43v_A]\Bigg]b_A^2[b_B+2v_B]\\
&-\Bigg[v_B^4\left[17b_B^2-160b_Bv_A+15v_A^2\right]+v_B^3\left[3b_B^3-153b_B^2v_A-51b_Bv_A^2-859v_A^3\right]\\
&\quad -v_Av_B^2\left[49b_B^3+101b_B^2v_A+1283b_Bv_A^2-129v_A^3\right]-b_Bv_A^2\left[12b_B^3+95b_B^2v_A-24b_Bv_A^2+v_A^3\right]\\
&\quad -2v_B^5[19v_A-14b_B]-v_Av_B\left[4b_B^4+59b_B^3v_A+619b_B^2v_A^2-115b_Bv_A^3+3v_A^4\right]+12v_B^6\Bigg]b_A\\
&-v_Av_B^3\left[6b_B^3-139b_B^2v_A+113b_Bv_A^2-538v_A^3\right]+v_A^2v_B^2\left[46b_B^3+27b_B^2v_A+816b_Bv_A^2-120v_A^3\right]\\
&+v_B^6[3b_B-20v_A]+2b_Bv_A^3\left[6b_B^3+31b_B^2v_A-12b_Bv_A^2+v_A^3\right]+v_B^5\left[b_B^2-47b_Bv_A+54v_A^2\right]\\
&+2v_A^2v_B\left[2b_B^4+25b_B^3v_A+200b_B^2v_A^2-56b_Bv_A^3+3v_A^4\right]+2v_B^7
\end{aligned} \tag{A.57}
$$

$$
\begin{aligned}
\lambda_4^{'D} =& \Bigg[b_A^2b_B^4+2b_B^2v_A^2\left[3b_A^2+b_Ab_B+2b_B^2\right]+4b_Ab_B^4v_A+4b_B^2v_A^3[6b_A+b_B]+b_Bv_A^4[23b_B-5b_A]\\
&\quad -10b_Bv_A^5+v_A^6\Bigg][2v_A+b_A]-v_B^5\left[12b_A^2-3b_Ab_B+26b_Av_A-5b_Bv_A+8v_A^2\right]\\
&+\Bigg[6b_A^3-4b_A^2[7b_B+2v_A]+b_A\left[b_B^2-90b_Bv_A-86v_A^2\right]+v_A\left[2b_B^2-69b_Bv_A-94v_A^2\right]\Bigg]v_B^4\\
&+\Bigg[b_A^3b_B^2[8b_A+13b_B]-v_A^4\left[9b_A^2-164b_Ab_B-158b_B^2\right]+b_A^2b_B^2v_A[68b_A+77b_B]+v_A^5[98b_B-31b_A]\\
&\quad +b_Ab_Bv_A^2\left[18b_A^2+219b_Ab_B+152b_B^2\right]+b_Bv_A^3\left[94b_A^2+309b_Ab_B+100b_B^2\right]-28v_A^6\Bigg]v_B\\
&+\Bigg[4v_A^3\left[12b_A^2+254b_Ab_B+63b_B^2\right]+4b_Ab_Bv_A\left[65b_A^2+59b_Ab_B-3b_B^2\right]+v_A^4[67b_A+485b_B]\\
&\quad +b_A^2b_B\left[32b_A^2+43b_Ab_B-3b_B^2\right]+6v_A^2\left[2b_A^3+130b_A^2b_B+71b_Ab_B^2-2b_B^3\right]+34v_A^5\Bigg]v_B^2\\
&+\Bigg[-b_A^2\left[17b_B^2-207b_Bv_A-705v_A^2\right]-b_Av_A\left[63b_B^2-295b_Bv_A-881v_A^2\right]\\
&\quad -v_A^2\left[58b_B^2-125b_Bv_A-412v_A^2\right]+32b_A^4+b_A^3[45b_B+248v_A]\Bigg]v_B^3+2v_B^6[v_A+b_A]
\end{aligned} \tag{A.58}
$$

Appendix A.4. Complementary results, calculations and proofs

Welfare analysis when price discrimination by a private facility changes quantities in the same direction

Consider that price discrimination changes both prices and both quantities in the same direction (either fall or rise). When the input price in both markets is higher than socially optimal under both pricing regimes, the bracketed terms of Eq. (16) are always positive and a sufficient condition for welfare improvement is that output increases in both markets. If the quantity decreases in both markets, price discrimination causes welfare to deteriorate. Under price discrimination the input prices are higher than the welfare maximizing prices (see Eqs. (8) and (9)), but this is not necessarily true for the uniform price. Therefore the sufficient conditions for welfare improvement or deterioration in this case involve quantity changes (the sign of dq_A/dt and dq_B/dt) but also conditions so that output is contracted in both markets under uniform pricing. The following proposition summarizes the sufficient conditions that characterize the welfare change in this case.

Proposition 8. *When:*

(a) *Time valuations are sufficiently different in that* $v_A - 5 \cdot v_B > 4 \cdot b_B$ *and*

(b) *Congestion effects are not too high in that* $v_A < 5 \cdot b_B + \sqrt{[5 \cdot b_B]^2 + 8 \cdot b_A \cdot b_B}$ *or the time valuations are not too different in that* $v_B/v_A > [9 - \sqrt{73}]/4 \approx 0.114$*,*

Price discrimination changes the quantity in both markets in the same direction and:

(i) *Increases welfare if* $\frac{A_B}{A_A} > \lambda_1$ *(* $w_B > w_A$ *holds and both quantities increase).*

(ii) *Decreases welfare if* $\lambda_0 < \frac{A_B}{A_A} < \lambda_1$ *(* $w_B < w_A$ *holds and both quantities decrease), where* λ_0 *is defined in Appendix A.1.*

Proof: see below.

The conditions (a) and (b) imply that price discrimination changes both input prices and output in both markets in the same direction. That is, either both input prices fall and both quantities rise or *vice versa*. Proposition 8 (i) is intuitive. When the congestion effects and time valuations are such that conditions (a) and (b) in Proposition 8 hold, $A_B/A_A > \lambda_1$ implies that both prices fall and quantities rise. This, naturally, increases welfare because the input prices in both markets are higher than socially optimal in this case. Under these conditions, the consumer surplus in each market increases. As discussed above, this can only occur when the ratio of reservation prices A_B/A_A is higher than the ratio of time valuations v_B/v_A, which for conditions (a) and (b) to hold must be lower than 1/5. As a reference, λ_1 is greater than 1/2 when $v_B/v_A = 1/5$, so that the asymmetry of demand intercepts has to be significantly lower than the asymmetry of time valuations for welfare to increase with price discrimination.

In case (ii) of Proposition 8 the input prices rise and output falls in both markets, so that price discrimination decreases welfare if the prices were above socially optimal, which occurs when $\lambda_0 < A_B/A_A$ (see Appendix A.1 for the definition of λ_0). In this case, price discrimination increases the input prices and the differentiated prices are always higher than optimal (see Eqs. (8) and (9)), so the uniform price is not necessarily higher than the socially optimal price of each market. For example, when q_B is relatively low and q_A is relatively high, the differentiated input price set by the facility in market B is similar to the socially optimal price. Therefore, as the uniform price is lower than the differentiated prices in this case, w can be lower than the socially optimal price for market B. This is likely to happen when A_B/A_A is sufficiently small, which explains why a lower bound for this ratio is needed for the sufficient condition ($\lambda_0 < A_B/A_A$). Again, as a reference, when $v_B/v_A = 1/5$, $\lambda_0 < 1/2$, so $A_B/A_A = 1/2$ is already small enough for the condition in Proposition 8 (ii) to hold.

Proof of Proposition 8

The first step of the proof is to show that $\tau + q_A \cdot b_A - q_B \cdot v_B$ and $\tau + t + q_B \cdot b_B - q_A \cdot v_A$ are always positive. From Eqs. (8) and (9) it follows that they are positive under price discrimination, so we need to show that they are positive for any value of t between 0 and the optimal difference between input prices. As all functions are linear, the sign of the derivative of the terms does not change (see above that all derivatives are constant) and it is enough to show that the terms are positive under uniform pricing (at $t = 0$). For $\tau + q_A \cdot b_A - q_B \cdot v_B$, we show that the term decreases with t, which implies that when $A_B/A_A > \lambda_1$ holds the discriminatory price is approached by increasing t and therefore

$\tau + q_A \cdot b_A - q_B \cdot v_B$ has to be positive at any $t < t^*$:

$$\frac{d\left[\tau + q_A b_A - q_B v_B\right]}{dt} = \frac{8b_A^2\left[b_B + v_B\right] + \left[v_A - v_B\right]\left[-12b_B v_A + [11v_A - v_B]v_B\right]}{-2\left[2b_A + 2b_B + v_A + v_B\right]\left[4[b_A + v_A][b_B + v_B] - v_A v_B\right]} - \frac{b_A\left[v_A^2 - 2b_B[9v_A - 5v_B] + 17v_A v_B - 12v_B^2\right]}{-2\left[2b_A + 2b_B + v_A + v_B\right]\left[4[b_A + v_A][b_B + v_B] - v_A v_B\right]} < 0 \tag{A.59}$$

In the case where $A_B/A_A < \lambda_1$, the discriminatory price is approached by making t negative, so we need to asses directly $w + q_A \cdot b_A - q_B \cdot v_B$. Substituting the values of w, $q_A(w)$ and $q_B(w)$, we obtain the following condition:

$$w + q_A \cdot b_A - q_B \cdot v_B > 0 \Leftrightarrow \frac{A_B}{A_A} \cdot F_B > -F_A \tag{A.60}$$

$$\begin{aligned} \text{where } F_A =& 2b_B\left[[3v_A + 11v_B]\,b_A + v_B\left[7v_A + v_B\right] + 4b_A^2\right] + 4b_B^2\left[3b_A + 2v_A\right] \\ &+ v_B\left[[5v_A + 12v_B]\,b_A + v_B\left[7v_A + v_B\right] + 8b_A^2\right] > 0 \\ F_B =& b_A\left[\left[v_A^2 - 2\left[3v_A - 4v_B\right]b_B + 6v_B^2\right]\right] - 2b_A^2\left[v_A - 2b_B\right] \\ &+ v_A\left[v_B\left[v_A - 5v_B\right] + 4\left[v_A - 2v_B\right]b_B\right] \end{aligned}$$

If $F_B > 0$, the condition in Eq. (A.60) always holds. If $F_B < 0$, then the condition is equivalent to $\frac{A_B}{A_A} < -F_A/F_B$, and as $-F_A/F_B > \lambda_1$ holds, $A_B/A_A < \lambda_1$ is sufficient. Therefore, $\tau + q_A \cdot b_A - q_B \cdot v_B$ is positive for any value of t.

For $\tau + t + q_B \cdot b_B - q_A \cdot v_A$ to be positive under uniform pricing, we asses its sign directly. Replacing w, $q_A(w)$ and $q_B(w)$, we obtain that it is positive when $A_B/A_A > \lambda_0$, where:

$$\begin{aligned} \lambda_0 =& \Bigg[b_B\left[6v_A^2 + 2[4v_A - 3v_B]b_A + v_B^2\right] + 2b_B^2[-2b_A + v_B] \\ &+ v_B\left[4[2v_A - v_B]b_A + v_A[5v_A - v_B]\right]\Bigg] \\ &\cdot \Bigg[2b_A\left[[11v_A + 3v_B]b_B + v_A[v_A + 7v_B] + 4b_B\right] - 4b_A^2[2v_B + 3b_B] \\ &+ v_A\left[[12v_A + 5v_B]b_B + v_A[v_A + 7v_B] - 8b_B^2\right]\Bigg]^{-1} \end{aligned} \tag{A.61}$$

and, as $\lambda_0 < \lambda_1$, $A_B/A_A > \lambda_1$ is a sufficient condition for $\tau + t + q_B \cdot b_B - q_A \cdot v_A$ to be positive. In the case where $A_B/A_A < \lambda_1$, $A_B/A_A > \lambda_0$ is also needed.

The second part of the proof is to show that both quantities move in the same direction.

The marginal effect on downstream firm's quantities are:

$$\frac{dq_A}{dt} = \frac{\partial q_A}{\partial w_A} \cdot \frac{d\tau}{dt} + \frac{\partial q_A}{\partial w_B} \cdot \left[\frac{d\tau}{dt} + 1\right]$$
$$= \frac{2 \cdot [b_B + v_B] \cdot [4 \cdot b_A + [5 \cdot v_A - v_B]] - v_A \cdot [v_A + v_B]}{\Omega_2} \tag{A.62}$$
$$\frac{dq_B}{dt} = \frac{\partial q_B}{\partial w_A} \cdot \frac{d\tau}{dt} + \frac{\partial q_B}{\partial w_B} \cdot \left[\frac{d\tau}{dt} + 1\right]$$
$$= \frac{-2 \cdot [b_A - v_A] \cdot [4 \cdot b_B + [5 \cdot v_B - v_A]] + v_B \cdot [v_A + v_B]}{\Omega_2} \tag{A.63}$$

From Eq. A.63 it follows that part (a) of Proposition 8, $v_A - 5 \cdot v_B > 4 \cdot b_B$, is sufficient for $dq_B/dt > 0$ as all other terms are positive.

As $\Omega_2 > 0$, we focus on the numerator of (A.62) to determine the sign of dq_A/dt. Denote I the numerator and let $v_B = \phi \cdot v_A$ where ϕ is a constant in $[0, 1]$. This allows for focusing on a sufficient condition for v_A by taking into account that $v_B < v_A$ must always hold. Solving $I = 0$ for v_A, we get the following roots:

$$r_1 = \frac{b_B \cdot [5 - \phi] + 4 \cdot b_A \cdot \phi + \sqrt{[b_B \cdot [5 - \phi] + 4 \cdot b_A \cdot \phi]^2 + 8 \cdot b_A \cdot b_B \cdot [1 - 9 \cdot \phi + 2 \cdot \phi^2]}}{1 - 9 \cdot \phi + 2 \cdot \phi^2} \tag{A.64}$$
$$r_2 = \frac{b_B \cdot [5 - \phi] + 4 \cdot b_A \cdot \phi - \sqrt{[b_B \cdot [5 - \phi] + 4 \cdot b_A \cdot \phi]^2 + 8 \cdot b_A \cdot b_B \cdot [1 - 9 \cdot \phi + 2 \cdot \phi^2]}}{1 - 9 \cdot \phi + 2 \cdot \phi^2} \tag{A.65}$$

To prove that the condition in part (b) of Proposition 8 is sufficient for $dq_A/dt > 0$, we distinguish two cases. First, when $[1 - 9 \cdot \phi - 2 \cdot \phi^2] > 0$, which is equivalent to $\phi < [9 - \sqrt{73}]/4 \approx 0,114$, r_2 is negative, r_1 is positive, and $\partial^2 I/\partial v_A^2 < 0$. Therefore for all values of v_A in $[0, r_1]$, $dq_A/dt > 0$. The minimum value of r_1 when $[1 - 9 \cdot \phi + 2 \cdot \phi^2] > 0$ is achieved at $\phi = 0$, so that a sufficient condition is:

$$v_A < r_1 \mid_{\phi=0} = b_B \cdot 5 - \sqrt{[b_B \cdot 5]^2 + 8 \cdot b_A \cdot b_B} \tag{A.66}$$

which is the condition in part (b) of Proposition 8. In the case where $[1 - 9 \cdot \phi + 2 \cdot \phi^2] < 0$, which is equivalent to $\phi > [9 - \sqrt{73}]/4 \approx 0,114$, both roots are negative and $\partial^2 I/\partial v_A^2 > 0$ so that for all positive values of v_A, $dq_A/dt > 0$ holds. This completes the proof that the condition $v_A < r_1 \mid_{\phi=0}$ or $v_B/v_A > [9 - \sqrt{73}]/4 \approx 0,114$ is sufficient for $dq_A/dt > 0$ to hold.

Welfare deterioration when price discrimination by a private facility changes quantities in opposite directions

This extends the analysis in Proposition 3 when there is downstream uniform pricing (Section 3).

Proposition 9. *When time valuations are similar in that $v_A - 3 \cdot v_B < 4 \cdot b_B$, the quantities change in opposite directions with price discrimination and price discrimination decreases welfare if:*

$$\frac{A_B}{A_A} < min\,[\lambda_1, \lambda_3],\ where\ \lambda_3 = \frac{[b_A + v_A] \cdot [4 \cdot b_B + v_A + 5 \cdot v_B]}{[b_B + v_B] \cdot [4 \cdot b_A + v_B + 5 \cdot v_A]}$$

Proof: as $A_B/A_A < \lambda_1$, the price discriminating behavior is approached by making t negative in this case. Therefore, the effect of price discrimination on welfare, output and prices have the opposite sign than the marginal effect. That is, as $w_A > w_B$ holds, welfare decreases when $dW/dt > 0$. As $v_A - 3 \cdot v_B < 4 \cdot b_B$ holds, $dq_A/dt > 0$ and $dq_B/dt < 0$. Therefore, again, $w_A + q_A \cdot b_A - q_B \cdot v_B > w_A + t + q_B \cdot b_B - q_A \cdot v_A$ for any value of $t \in [-t^*, 0]$ is sufficient for $dW/dt > 0$ and thus for welfare deterioration. As $df/dt < 0$, the sufficient condition in this case is that $f(0) > 0$. Using Eqs. (A.1) and (A.2):

$$f(0) = \frac{A_A\,[b_B + v_B]\,[4 \cdot b_A + v_B + 5 \cdot v_A] - A_B\,[b_A + v_A] \cdot [4 \cdot b_B + v_A + 5 \cdot v_B]}{L_1} \tag{A.67}$$

from which the condition $\frac{A_B}{A_A} < \lambda_3$ follows straightforwardly.

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