

# Frequent Flyer Programs, Moral Hazard and Rewards per miles vs per dollar<sup>1</sup>

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## Abstract

We propose a theoretical model that considers the existence of two types of travelers, business and leisure, who are not distinguishable by the (monopoly) airline ex-ante, and which is the source of an adverse selection problem. We also consider that business travelers do not pay fully their air tickets, inducing a moral hazard problem between them and their employers, who actually pays. We model the airline's second-degree price discrimination problem, where it attempts to separate traveler types with the use of different menus of prices and rewards, while exploiting the moral hazard problem by 'bribing' business travelers. We find that the moral hazard problem induces an upward distortion in rewards for business travelers (distortion at the top) while deepening the typical distortion at the bottom caused by adverse selection (even smaller rewards for leisure travelers). We compare this 'optimal' program, where fares and rewards can be chosen independently, to real-life programs, where rewards are linked either to distance travelled or to the fare paid. We show that, at the design stage of the program, when demand parameter values are taken at its expected values (the ex-ante setting), the three programs are equivalent. Yet, ex-post, once the programs have already been designed and they cannot be modified, if demand parameter values differ from expected values, nor the per-distance nor the per-dollar program can mimic the optimal program. We show, however, that the per-dollar program allows the (monopoly) airline to respond to changes in a closer way to the optimal program than the per-distance program, which is consistent with the recent changes to per-dollar programs of a number of legacy carriers. Finally, we show that, ex-post, it is more likely that the lower classes are shut-down in a per-distance than in a per-dollar program, which is consistent with the data we collected from LATAM Airlines, Air Canada (rewards per distance) and Southwest Airlines (rewards per dollar).

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## 1. INTRODUCTION

Since their inception in the 80s, airlines' frequent flyer programs (FFP) have become one of the largest customer loyalty programs in the world economy. The aeronautical industry places unredeemed miles over 700,000 million dollars. However, despite their size and importance, these programs have remained relatively unexplored from a microeconomics point of view. Caminal & Claici (2007), Hartman & Viard (2008), Lederman (2007), Agostini et al (2013) have focused on the pro- or anti-competitive effects that FFPs might have, emphasizing the potential switching costs that they would create, namely, the benefit losses suffer by consumers that switch to another airline. The main effect that these switching costs would have is to allow firms to change the elasticity of demand and increase prices.

An alternative analysis of FFP's is provided by Basso, Clements & Ross (2009) who observe that, in many cases, it is not the traveler who pays his ticket but there exists a third-part payer. This is the main cases in most business-related traveler, where the employer is who actually pays the ticket. Their analysis emphasizes the fact that the employee might not always choose the cheapest ticket but will try to maximize individual benefits and it is there where FFP rewards play a major role: the miles or rewards that airlines offer, and which are received by the traveler, might be seen as a sort of bait –or bribe– to attract travelers towards higher price alternatives. We therefore have a moral hazard problem: employers, who pay for air tickets are not able to observe the actions –in the case, choice of airline– of their employees but, in many cases, can only accept or reject the proposal. Basso et al (2009) show that in such an environment, a lone airline can indeed take advantage of the third-party payer feature, increasing prices up to the employers' reservation price and defeating the competition. Yet, if all airlines use FFPs, rewards become simply a new competitive instrument which enables tougher competition: airlines will dissipate profits through FFP rewards in a standard Prisoner's Dilemma fashion. Prices will remain high though, showing that all benefits flow from employers and airlines to employees.

Basso et al (2009) suppose, however, that airlines are perfectly able to discriminate business from leisure travelers –who do pay for their tickets– and are, therefore, able to increase prices and miles to the former, without losing demand from the latter. In the economics literature, this is known as first degree price discrimination. In reality, however, firms in general and airlines in particular cannot easily distinguish one type of traveler from another, which implies that airlines face, also, an adverse selection problem, something actually well documented in the business literature.

If airlines recognize the existence of (at least) these two types of travelers and they have an idea about their actual share of total demand, then they would have two options when facing their decisions of prices and rewards. First, they can simply avoid attempting to separate them, in which case the airline will face the combined demand, choosing one price and one level of rewards (if positive) for all customers. A second option is to offer different menus or pair of prices and rewards, attempting to induce self-selection by travelers into the menu designed for them. In this case then, the bribe is not designed to attract consumers away from smaller prices of competitors but of the airline's own small prices. In reality, this case might seem the one that prevails. Air Canada, for example, has a "Tango" fare which, together with other restrictions, only gives 25% of the miles. The "Latitud" fare, however, while being larger, gives 125% of the miles. LATAM airlines offer something similar while Southwest, a low cost carrier, has also three classes, but in their case, the rewards are linked to the price paid: higher prices obtain a reward that is 12 times the price paid, the intermediate class obtains 9 times the price paid, while the lowest prices (the 'Wanna get away' class) obtains only 5 times the (lower) price paid. Other low cost carriers have also Southwest's system (JetBlue and Virgin America).

It is evident that the problem faced by airlines explained above is related to the second-degree price discrimination literature –in many cases, it has been used as a textbook example– where the firm offer heterogeneous consumers a well-crafted menu of options, with the intention to induce them to self-select and thus maximize profits. This literature is ample, starting with the seminal work by Mussa & Rosen (1978) and is well reviewed in Stole (2007). Our contribution is to consider simultaneously the moral hazard problem between employer and employee regarding the choice of airline and the adverse selection problem that airline faces.

Thus, in this paper we begin by, proposing a theoretical model that considers the existence of two types of travelers, business and leisure, who are not distinguishable by the (monopoly) airline ex-ante, and which is the source of the adverse selection problem. We also consider that business travelers do not pay fully their air tickets, inducing a moral hazard problem between them and their employers, who actually pays. The airline then faces a second-degree price discrimination problem, where it attempts to separate traveler types with the use of different menus of prices and rewards, while exploiting the moral hazard problem by ‘bribing’ business travelers. We look at the interplay of these two aspects of the airline problem, seeking to understand the trade-offs and the effects on the market (prices and rewards), efficiency and distribution of surpluses of all different agents involved (business travelers, leisure travelers, employers and airlines). We focus on the classical framework of second degree price discrimination where demand is of the all or nothing type; relaxing that assumption to allow for elasticities does not change the qualitative conclusions.

Our main results show that in the case of the full information benchmark, the airline finds it optimal to provide both types with positive rewards but in the case of the business traveler, this amount is inefficiently high, that is, airline’s marginal cost exceeds traveler’s marginal benefit. This is caused by the effect of the third party payer, which makes a business traveler less elastic with respect to price. When information is incomplete, the reward to the business traveler remains the same yet those assigned to a leisure traveler decreases. This effect is known as *no distortion at the top*, and happens because the airline needs to separate enough the two menus in order to induce self-selection by customers. In fact, the airline might find optimal not to offer any reward to leisure travelers but charge a low price, while having a different option with very high prices and a large number of miles. This is not the end of the story though, because business prices might end up being too high for what employers are willing to pay. To consider this we add the employer’s reservation price, which establishes a ceiling for airfares. This reservation price will induce a decrease in the rewards received by employees and an increase in the rewards received by leisure travelers, moving both rewards levels closer to their efficient levels.

This first analysis, however, may be understood as a point of reference: the airline is absolutely free to set prices and rewards yet, in practice, real programs link their rewards, in an explicit contract with their customers, to some other variable. Indeed, both Air Canada and LATAM link the rewards to the distance flown (hence, the old idea of obtaining ‘miles’) yet Southwest links the rewards to the price paid. The analysis of real FFPs and how they compare between them, and against what we will call the ‘optimal program’ is the subject of the second part of the paper. The importance of this analysis is further fueled by a couple of facts from the industry: first, in 2015 Delta followed the lead of Southwest and moved its Skymiles program to a per-dollar system.<sup>5</sup> United then followed the lead of Delta<sup>6</sup> and, very recently, LATAM announced that for

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<sup>5</sup> See e.g. <https://www.usatoday.com/story/travel/flights/2014/02/26/delta-frequent-flier/5815425/>.

<sup>6</sup> See e.g. <https://www.usatoday.com/story/todayinthesky/2014/06/10/united-fliers-to-earn-miles-based-on-fare-not-distance/10270819/>

local flights, rewards would be also linked to fares.<sup>7</sup> Is there, then, an advantage to using rewards linked to prices rather than distance flown? The second observation relates to class shutdowns. We collected data from three airlines, namely LATAM Airlines, Air Canada and Southwest Airlines, from October and November of 2014, referring to one-way travels for four pairs of cities in Chile, Canada and the U.S.<sup>8</sup> At the time of the data collection, LATAM Airlines and Air Canada offered a rewards-per-distance program, while Southwest used a rewards-per-dollar program. The data showed that the airlines effectively offered at least two fares with different rewards (three fares in the case of Air Canada and Southwest and four in the case of LATAM). Moreover, this data also revealed that LATAM and Air Canada often closed the lower classes: LATAM closed the fourth, third and second tier class 68%, 53% and 26% of the time respectively, while Air Canada closed the third and second tier class 34% and 4% of the time respectively. On the other hand, the data for Southwest showed that the lowest tier class was closed only once (<1% of the time). Is then that there is something structural about the rewards-per-dollar program that makes it more unlikely to shut down classes?

To study the real FFP programs we consider two settings. First, in the 'ex-ante' setting, airlines design their programs for expected parameter values. A program design means fares and rewards in the optimal program, fares and percentage of miles in the per-distance program, and fares and fare-multipliers in the case of the per-dollar program. We show that in this case all three programs are equivalent, in terms of fares and resulting rewards. This is irrespective of whether separation (as in the standard adverse selection result) or shutting down the lowest class is better: ex-ante, both real programs can mimic the optimal program.

The second setting we analyze is the 'ex-post' setting. What we mean here is that after designing the program, demand parameter values may differ from their expected values.<sup>9</sup> In the optimal program, the airline would be able to adjust both rewards and fares yet in real programs, we assume the airline is tied to the ex-ante program design, that is, the airline cannot change the percentages in the per-distance program or the fare-multipliers in the per-dollar program. Thus, in the ex-post setting an airline can imperfectly adjust values of fares and reward, and the way this happens do depend on the type of program.

Our analysis of the ex-post setting shows that in the per-distance program, rewards cannot be changed, as they are tied to the distance, while in most cases what the airline would like to do optimally is to change all rewards and fares. The sub optimal solution is, in many cases to move fares in the opposite direction of what the airline would do if it had total freedom. The per-dollar program has rewards tied to prices, and therefore, both fares and rewards move, although at a fix rate. This characteristic, while being suboptimal, can be shown to, at least and under some conditions, allow the airline to move fares and rewards in the same direction as if it had total freedom. In other words, in the ex-post setting, the per-price program gives allows the airline to better adjust. This robustness of the per-dollar program may explain why it is being adopted by legacy carriers. Furthermore, we show that the condition for profitable class shutdown differs between programs and that, indeed, it is more easily reached in the per-distance program, i.e. our models predict that shutting down the lower classes is less likely with a per-dollar program, just as the data we collected shows.

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<sup>7</sup> See e.g. [https://www.latam.com/es\\_cl/laser\\_latam\\_pass/mailv/juntosmaslejosvoladores/](https://www.latam.com/es_cl/laser_latam_pass/mailv/juntosmaslejosvoladores/)

<sup>8</sup> Chicago-Boston (Southwest), Toronto-Vancouver (Air Canada), Santiago-Puerto Montt and Santiago-Antofagasta (LATAM).

<sup>9</sup> According to the marketing literature, a customer loyalty program, to be successful, need to be, among other things, simple and stable over time. Hence, the design of the program is very slow to adjust.

The rest of the paper is organized as follows: Section 2 contains the model formulation and the benchmark case of full information, that is, perfect discrimination. Section 3 considers the adverse selection and moral hazard problems, and characterizes what we have called ‘the optimal program’. Section 4 considers the formulation and analyses of real programs, namely rewards-per-distance and rewards-per-dollar, in the ex-ante setting. Section 5 show the analysis for real programs in the ex-post setting, with comparisons between programs and against the optimal program, including shutdown conditions. Section 7 concludes.

## 2. MODEL FORMULATION AND PERFECT DISCRIMINATION BENCHMARK

Our model considers two types of travelers, business and leisure, who make the purchase arrangements by themselves, and an airline that possess a frequent flyer program and that has all customers enrolled in. Miles or rewards flow to the traveler, and not to the payer of the ticket. The business traveler only pays, or perceives a fraction of the air ticket and is forced to travel as an employment condition. This ‘fraction’ attempts to capture that, while not paying, the employer is partially sensitive to the price, either because he may care about future travel, and therefore needs to be careful about the travel account, or because he fears he will be audited. The leisure traveler pays the ticket from his own pocket and flies if this leaves him with positive utility.

The airline knows the share of demand that each type of customer represents and has the ability to generate a menu of two pairs of prices and rewards, with the objective of inducing self-selection and thus discriminate between consumers. We assume there are no cost complementarities between the two menus for ease of exposition, but this is not central to the analysis.

Travelers utility functions are represented by an additive and separable function of price and reward benefits similar to the one used by Rochet & Stole (1997, 2002). To model the existence of business travel we follow Cairns & Galbraith (1990) and Basso et al (2009) where these types of consumers only pay a small fraction  $\alpha$  of the airfare. Therefore, utility functions are described by:

$$U_L(P_L, F_L, z) = U_0 + \theta_L V(F_L) - P_L$$

$$U_H(P_H, F_H) = \theta_H V(F_H) - \alpha P_H$$

where, L refers to leisure travel and H to business travel (*Low* type and *High* type), and  $U_0 > 0$ ,  $\theta_L \leq \theta_H$ ,  $0 \leq \alpha \leq 1$ ,  $0 < V(0)$ ,  $V'(F) \geq 0$  y  $V''(F) < 0$ . Total market demand is  $N_H + N_L$ , where  $N_H$  is the number of passengers of *High* type and  $N_L$  the corresponding of *Low* type. Note that the business traveler has no personal utility of travelling. This is because travelling is a condition of employment.

We set airline’s operational marginal cost to zero and suppose that its reward cost structure is separable, that is:

$$C(F_H; F_L) = C_L(F_L) + C_H(F_H)$$

Rewards marginal costs are increasing and convex, i.e.  $C_L'(F) > 0$ ,  $C_H'(F) > 0$  and  $C_L''(F) > 0$ .

The airline will generate a menu with two pairs of prices and miles, each targeted at each type of traveler. These pairs, in order to induce truthful self-selection must be designed such that they fulfill incentive compatibility constraints:

$$U_L(P_L, F_L, z) \geq U_L(P_H, F_H, z) \quad (1)$$

$$U_H(P_H, F_H) \geq U_H(P_L, F_L) \quad (2)$$

The monopoly airline must also ensure participation of customers. For leisure traveler, this simply requires that utility end up being positive. In the case of the business traveler, given that traveling is a condition for employment, his participation constraint is not her own but the employer's, which here is modeled through a reservation price  $R$ , which is the maximum price that she is willing to pay for an air ticket. Participation constraints are, then:

$$U_L(P_L, F_L, z) \geq 0 \quad (3)$$

$$P_H - R \geq 0 \quad (4)$$

Note that we are modelling all-or-nothing type of demands, as in the most common adverse-selection model: either all business (leisure) traveler fly, or none does. Introducing elasticities does not change the main qualitative results and insights.

We now establish the airline's optimal decisions for the full information benchmark, that is, a case in which the airline can perfectly discriminate between customers. Consider first the case without moral hazard, i.e.  $\alpha = 1$ . In that case, it is direct to obtain that the airline will offer rewards and prices such that the marginal utility of each customer equates the marginal cost, and will choose prices to extract all surplus.

$$\theta_L V'(F_L^*) = C_L'(F_L^*) \quad (5)$$

$$\theta_H V'(F_H^*) = C_H'(F_H^*) \quad (6)$$

$$P_L^* = \theta_L V(F_L^*) \quad (7)$$

$$P_H^* = \theta_H V(F_H^*) \quad (8)$$

It happens that this situation is efficient, that is, it coincides with the (social) first-best because, despite that the monopolist captures all consumer surplus, it is only a transfer which generate no welfare losses.

If we add the moral hazard problem, which in this model occurs when  $0 \leq \alpha < 1$  and the reservation price exists, leads to a different result:

$$\theta_L V'(F_L^*) = C_L'(F_L^*) \quad (9)$$

$$\frac{\theta_H}{\alpha} V'(F_H^*) = P_H^* = R \quad (10)$$

$$P_L^* = \theta_L V(F_L^*) \quad (11)$$

$$P_H^* = R \quad (12)$$

In this case the airline has an incentive to keep increasing the price for business travelers, up to the reservation price of the employer, while inducing self-selection through inflated rewards, which is shown by the fact that the marginal benefit for business travelers in (10) is divided by  $\alpha$ . This shows that the moral hazard problem

### 3. ADVERSE SELECTION

We consider now imperfect information in order to include the adverse selection problem. To build intuition we assume first that the employer's reservation price is "very large" ( $R = \infty$ ), so that, for the time being, the airline does not consider her participation constraint in its profit maximization problem. The problem the airline faces is

$$\text{Max}_{(P_H, P_L, F_H, F_L)} \pi = N_L(P_L - C_L(F_L)) + N_H(P_H - C_H(F_H))$$

s. t.

$$\theta_H V(F_H) - \alpha P_H \geq \theta_H V(F_L) - \alpha P_L$$

$$\theta_L V(F_L) - P_L \geq \theta_L V(F_H) - P_H$$

$$U_0 + \theta_L V(F_L) - P_L \geq 0$$

$$F_H \geq 0$$

$$F_L \geq 0$$

The two first constraint are incentive compatibility and the third is the participation constraint of leisure travelers (recall that employees must travel). At this point, it becomes very useful for solving the problem, to perform a change of variables (Rochet y Stole, 2002):

$$u_L = U_0 + \theta_L V(F_L) - P_L, \quad u_H = \frac{\theta_H}{\alpha} V(F_H) - P_H \quad (13)$$

This change of variables can be interpreted as the firm choosing the level of rewards, given by  $F_H$  y  $F_L$ , and a utility level for each customer, given by  $u_H$  y  $u_L$ . With this, we obtain the rules that the airline follows in its optimal policy. Calculations are presented in the Appendix A but, as is usual, the incentive compatibility constrain that is active is that of the high type (business traveler), while the participation constraint that is active is that of the low type (leisure traveler). The other two are slack. The *optimal program* is then characterized by:

$$u_L^* = 0 \quad (14)$$

$$u_H^* = V(F_L^*) \left( \frac{\theta_H}{\alpha} - \theta_L \right) - U_0 \quad (15)$$

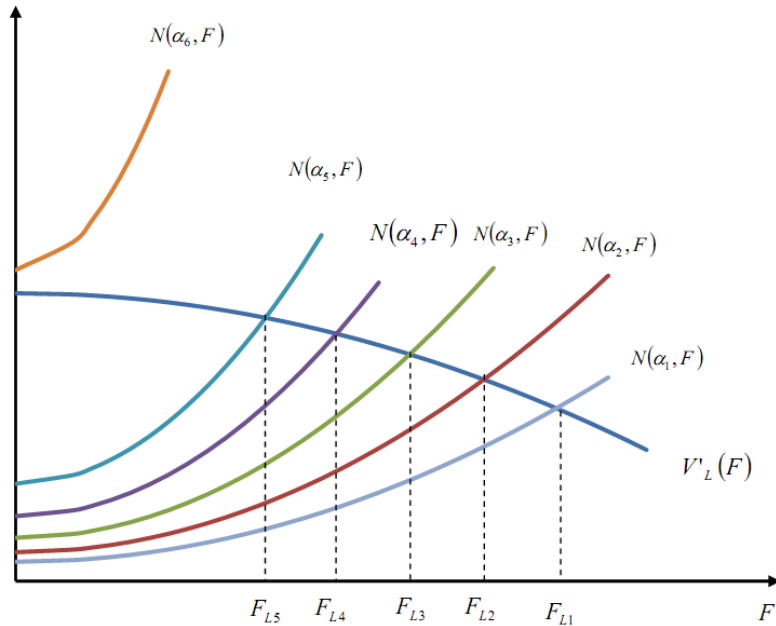
$$\frac{\theta_H}{\alpha} V'(F_H^*) = C'_H(F_H^*) \quad (16)$$

$$\theta_L V'(F_L^*) = \frac{C'_L(F_L^*) - B}{1 - \frac{N_H}{N_L} \frac{\left( \frac{\theta_H}{\alpha} - \theta_L \right)}{\theta_L}} \quad (17)$$

Where  $B$  is the Lagrange multiplier associated with the non-negativity of  $F_L$ .

Note that here business travelers obtain positive utility (equation 15), as opposed to what would happen under perfect discrimination. The airline has to leave this type of customers with informational rents in order to induce them to truthfully self-select. Regarding rewards, one can see that the upward distortion of business rewards still occurs (equation 16), just as in the case with full information and moral hazard. This inefficiency, therefore, occurs only because of the moral hazard problem induced by the third-party payer with imperfect monitoring and not because of adverse selection. On the other hand, comparing equations (17) and (9) one can see that the allocation of rewards to leisure travelers does change; they diminish in this case. The intuition is simple: since now the airline has to achieve customer self-selection, as it faces the adverse selection problem, it decreases the amount of rewards to the lowest type to make that option less attractive to business customers. In a nutshell, there will be now two distortions on the rewards side: upwards in the case of business travel, caused by the fact that they do not fully pay their fare, and downwards in the case of leisure travel to induce a larger difference (in terms of rewards) between the two options. This last effect, caused by adverse selection is well-known, but here we have an added effect caused by the moral hazard problem since  $F_L$  now depends on  $\alpha$ . This behavior of  $F_L^*$  with respect to  $\alpha$  can be seen graphically by considering the function

$$N(\alpha, F) = \frac{C_L'(F)}{\theta_L \left( 1 - \frac{N_H}{N_L} \frac{(\theta_H - \theta_L)}{\alpha} \right)}$$



**Figure 1.** Variation of  $F_L^*$  with respect to  $\alpha$  ( $\alpha_1 > \alpha_2 > \alpha_3 > \alpha_4 > \alpha_5 > \alpha_6$ ).

From Figure 1 one can observe that, as  $\alpha$  diminishes, the optimal value of  $F_L$  also does: the third-party payer feature increases the differences in mile rewards. Now, if  $\alpha$  is so low that the curves  $V'(F)$  and  $N(\alpha, F)$  no longer intersect, then the optimal airline's solution reaches a corner and then  $F_L = 0$  (as it will be captured by  $B$ ). It follows that there exist a critical  $\alpha^*$  such that, ceteris paribus,  $F_L^* = 0$  if  $\alpha$  is smaller than this value. Zero reward for leisure travel causes no problem for the solution of the rest of the problem though, since  $V(0)$  y  $V'(0)$  are positive.



We can now look at prices. The analysis for  $P_H$  shows that as  $\alpha$  decreases  $P_H$  will rise, that is, as expected, the moral hazard problem induces higher prices. Leisure travelers, on the other hand end up paying the price that allows the airline to fully extract their surplus, i.e.  $P_L = \theta_L V(F_L)$ . Thus, as  $\alpha$  diminishes,  $P_L$  diminishes and so does  $F_L$ . Note that these results help explain the observed behavior of different alternatives with high price and a large amount of rewards (100% or ‘more’ of the usual miles number) or low prices with small rewards (such as 25% of the usual miles number).

Now, the results described up to here are valid only when it is desirable for the airline to actually accommodate both types. This however, is not valid for all  $\alpha$ , not yet at least. In fact, if for given parameter values the following condition is satisfied

$$N_L(P_L^* - C_L(F_L^*)) + N_H(P_H^* - C_H(F_H^*)) < N_H(P_H^{**} - C_H(F_H^{**})) \quad (18)$$

where  $P_H^{**}$  and  $F_H^{**}$  are the fare and reward offered to business passengers in the absence of leisure passengers, then the low class is closed and only the business passengers are served.

From the equations above, tedious yet straightforward calculations mapped in Appendix B allow us to solve for comparative static results. These derivatives are summarized in the following table:

**Table 1.** Comparative statics for the optimal program.

$\frac{d}{dx}$	$F_H$	$F_L$	$P_H$	$P_L$
$\alpha$	$\leq 0$	$\geq 0$	$\leq 0$	$\geq 0$
$\theta_H$	$\geq 0$	$\leq 0$	$\geq 0$	$\leq 0$
$\theta_L$	$= 0$	$\geq 0$	Depends	$\geq 0$
$N_H$	$= 0$	$\leq 0$	$\geq 0$	$\leq 0$
$N_L$	$= 0$	$\geq 0$	$\leq 0$	$\geq 0$

These results will be helpful in the ex-post comparison, as they show how fares and rewards should change if the ex-ante expected values are, ex-post, different. Note that, in all cases, at least one of the rewards is adjusted.

#### 4. REAL FFPs EX ANTE

##### REWARDS PER DISTANCE

We may now analyze the performance of the programs observed in reality. The setting is ex-ante in that the airline solves for expected parameter values. We start by the classic and oldest per-distance program. In this case the rewards depend on distanced traveled, that is

$$F_L = k_L \cdot D \quad F_H = k_H \cdot D$$

where  $D$  is the distance flown. The problem where the airline can decide  $P_H, P_L, F_H, k$  (or equivalently  $k_L, k_H$ ) without reservation price on  $P_H$ , is written as

$$\text{Max}_{(P_H, P_L, F_H, k)} \pi = N_L(P_L - C_L(kF_H)) + N_H(P_H - C_H(F_H))$$

$$\theta_H V(F_H) - \alpha P_H \geq \theta_H V(kF_H) - \alpha P_L$$

$$\theta_L V(kF_H) - P_L \geq \theta_L V(F_H) - P_H$$

$$U_0 + \theta_L V(kF_H) - P_L \geq 0$$

$$F_H \geq 0$$

$$k \geq 0$$

That this program will, ex-ante, deliver the same rewards and fares as the optimal program can be easily seen because, if in the optimal program the airline chose  $(F_H^*, F_L^*, P_L^*, P_H^*)$ , it only needs to establish that

$$k_H^* = \frac{F_H^*}{D}, \quad k_L^* = \frac{F_L^*}{D}$$

Using these equalities, we can rewrite the following conditions for the optimum:

$$\frac{\theta_H}{\alpha} V'(F_H^*) = C'_H(F_H^*) \quad (19)$$

$$\theta_L V'(kF_H^*) = \frac{C'_L(kF_H^*)}{1 - \frac{N_H}{N_L} \left( \frac{\theta_H - \theta_L}{\alpha} \right)} \quad (20)$$

The last two expressions imply in general that  $k < 1$  ( $\Leftrightarrow k_L < k_H$ ): for example, if the marginal cost of providing rewards is constant and equal for the business and leisure passengers, then

$$V'(kF_H^*) = \frac{C'_L(kF_H^*)}{\theta_L \left( 1 - \frac{N_H}{N_L} \left( \frac{\theta_H - \theta_L}{\alpha} \right) \right)} > \frac{\alpha}{\theta_H} C'_H(F_H^*) = V'(F_H^*)$$

And using  $V' > 0, V'' < 0$ , we obtain  $kF_H^* < F_H^*$ .

Note that, if for the given parameter values it was more profitable under the optimal program to shut down the leisure class, it will be also the case here. And in that case, the airline will choose the same fare  $P_H^{**}$  while setting  $k_H^{**} = F_H^{**}/D$ . In other words, irrespective of whether separation or class shutdown is better, the per-distance program can mimic the optimal program ex-ante.

## REWARDS PER DOLLAR

We now focus in the model where the rewards depend on the price paid, that is

$$\begin{aligned} F_L &= k_L \cdot P_L \\ F_H &= k_H \cdot P_H \end{aligned}$$

The problem where the airline can decide  $F_H, F_L, k_L, k_H$  without reservation price on  $P_H$ , is written as

$$\text{Max}_{(F_H, F_L, k_L, k_H)} \pi = N_L \left( \frac{F_L}{k_L} - C_L(F_L) \right) + N_H \left( \frac{F_H}{k_H} - C_H(F_H) \right)$$

$$\theta_H V(F_H) - \alpha \frac{F_H}{k_H} \geq \theta_H V(F_L) - \alpha \frac{F_L}{k_L}$$

$$\theta_L V(F_L) - \frac{F_L}{k_L} \geq \theta_L V(F_H) - \frac{F_H}{k_H}$$

$$U_0 + \theta_L V(F_L) - \frac{F_L}{k_L} \geq 0$$

$$F_L, F_H \geq 0$$

$$k_L, k_H \geq 0$$

Returning to the variables  $(F_H, F_L, P_L, P_H)$  by making  $\frac{F_H}{k_H} = P_H$  and  $\frac{F_L}{k_L} = P_L$  is easy to see that this problem is equivalent to the optimal program (and to the ex-ante rewards per-distance program). Intuitively, under this program the airline chooses  $(F_H^*, F_L^*, P_L^*, P_H^*)$  and then simply fixes

$$k_H^* = \frac{F_H^*}{P_H^*}, \quad k_L^* = \frac{F_L^*}{P_L^*}$$

where  $(F_H^*, F_L^*, P_L^*, P_H^*)$  are given again by equations (14), (15), (16) and (17). It is easy to show that if  $V$  is 'concave enough', then  $k_L^* < k_H^*$  (see Appendix C)

Note again that, if for the given parameter values it was more profitable under the optimal program to shut down the leisure class, it will be also the case here. And in that case, the airline will choose the same fare  $P_H^{**}$  while setting  $k_H^{**} = F_H^{**}/P_H^{**}$ . In other words, irrespective of whether separation or class shutdown is better, the per-dollar program can also mimic the optimal program ex-ante.

## 5. REAL PROGRAMS EX-POST

The 'ex-post' setting considers that after designing the program, parameter values may end-up differing from its expected values. In the optimal program, the airline would be able to adjust both rewards and fares yet in real programs, we assume the airline is tied to the ex-ante program

design, that is, the airline cannot change the percentages in the per-distance program or the fare-multipliers in the per-dollar program. Thus, in the ex-post setting the airline can imperfectly adjust values of fares and reward. To analyze this, we write the constraint problem of the airline and repeat the comparative statics exercise we did for the optimal program, but this time assuming that the program design is fixed.

#### REWARDS PER DISTANCE

Since the contract between the airline and consumers is now set, we consider that the parameter values change marginally, but the airline can no longer change the parameter  $k$  nor  $F_H$  (equivalently  $k_H, k_L$ ) as in the real-world case where the airline has already declared their multipliers for rewards, and the distances are obviously fixed. The most obvious implication is that the airline would like to change the rewards but it cannot. The airline thus solves:

$$\begin{aligned} \text{Max}_{(P_H, P_L)} \pi &= N_L \left( P_L - C_L(\bar{k}\bar{F}_H) \right) + N_H (P_H - C_H(\bar{F}_H)) \\ &\text{s.t} \\ \theta_H V(\bar{F}_H) - \alpha P_H &\geq \theta_H V(\bar{k}\bar{F}_H) - \alpha P_L \\ \theta_L V(\bar{k}\bar{F}_H) - P_L &\geq \theta_L V(\bar{F}_H) - P_H \\ U_0 + \theta_L V(\bar{k}\bar{F}_H) - P_L &\geq 0 \end{aligned}$$

This problem is easy to solve once we consider that the airline can only change the fares, and that increasing fares always increase profit. Again, the active constraints are the H-type incentive compatibility and the L-type participation constraint. We re-write these as

$$\begin{aligned} P_L &\leq U_0 + \theta_L V(\bar{k}\bar{F}_H) \\ P_H &\leq \frac{\theta_H}{\alpha} V(\bar{F}_H) - \frac{\theta_H}{\alpha} V(\bar{k}\bar{F}_H) + P_L \end{aligned}$$

Since higher fares is better, the first equation determines the leisure fare. The second equation then determines the business fare:

$$P_L^* = U_0 + \theta_L V(\bar{k}\bar{F}_H) \quad (21)$$

$$P_H^* = \frac{\theta_H}{\alpha} V(\bar{F}_H) - \frac{\theta_H}{\alpha} V(\bar{k}\bar{F}_H) + P_L^* \quad (22)$$

It's important to remark that if  $\bar{k}$  was chosen as the optimum of the optimal program of Section 3, we recover the same solution. This is the expected behavior of an airline: they choose the set of parameters  $(k_L, k_H)$  to achieve the optimal value of the optimal program ex-ante, but once they do that, they are locked in the restricted problem and a change in any of the values of  $(\theta_H, \theta_L, N_H, N_L, \alpha)$  will result in a suboptimal solution. The behavior of the fares ex-post when any of these parameters change is easy to analyze from equations (21) and (22)

$$\frac{dP_L}{d\alpha} = 0$$

$$\frac{dP_H}{d\alpha} = -\frac{\theta_H}{\alpha^2}(V(F_H) - V(F_L)) \leq 0$$

$$\frac{dP_L}{d\theta_H} = 0$$

$$\frac{dP_H}{d\theta_H} = \frac{1}{\alpha}(V(\bar{F}_H) - V(\bar{k}\bar{F}_H)) \geq 0$$

$$\frac{dP_L}{d\theta_L} = V(\bar{k}\bar{F}_H) \geq 0$$

$$\frac{dP_H}{d\theta_L} = \frac{dP_L}{d\theta_L} = V(\bar{k}\bar{F}_H) \geq 0$$

Additionally, it is direct to see that  $\frac{dP_L}{dN_L} = \frac{dP_L}{dN_H} = \frac{dP_H}{dN_L} = \frac{dP_H}{dN_H} = 0$ , while all these derivatives are non-zero in the optimal program.

The following table, where the first column shows the response in the optimal program, and the second the response in the rewards-per-distance program, allows us to show when the per-distance program can move, ex-post, the variable in the right direction (in green) and when it cannot (red), as compared to the optimal program. Note that a green cell only means that the variable moves in the right direction, not that it achieves the same values.

**Table 2.** Responses ex-post for the optimal problem and rewards-per-distance program.

$\frac{d}{dx}$	$F_H$		$F_L$		$P_H$		$P_L$	
$\alpha$	$\leq 0$	= 0	$\geq 0$	= 0	$\leq 0$	$\leq 0$	$\geq 0$	= 0
$\theta_H$	$\geq 0$	= 0	$\leq 0$	= 0	$\geq 0$	$\geq 0$	$\leq 0$	= 0
$\theta_L$	= 0	= 0	$\geq 0$	= 0	Depends	$\geq 0$	$\geq 0$	$\geq 0$
$N_H$	= 0	= 0	$\leq 0$	= 0	$\geq 0$	$\leq 0$	$\leq 0$	$\leq 0$
$N_L$	= 0	= 0	$\geq 0$	= 0	$\leq 0$	= 0	$\geq 0$	= 0

## REWARDS PER DOLLAR

We focus again in the Rewards per Dollar program. The logic is the same used in the previous subsection: the airline first solves this unrestricted problem, but then is locked to the problem where it cannot move the multipliers  $k_L, k_H$  anymore, that is:

$$Max_{(F_L, F_H)} \pi = N_L \left( \frac{F_L}{k_L} - C_L(F_L) \right) + N_H \left( \frac{F_H}{k_H} - C_L(F_H) \right)$$

s.t

$$\theta_H V(F_H) - \alpha \frac{F_H}{k_H} \geq \theta_H V(F_L) - \alpha \frac{F_L}{k_L} \quad (23)$$

$$\theta_L V(F_L) - \frac{F_L}{k_L} \geq \theta_L V(F_H) - \frac{F_H}{k_H} \quad (24)$$

$$U_0 + \theta_L V(F_L) - \frac{F_L}{k_L} \geq 0 \quad (25)$$

The main difference here, compared to the per-distance program, is that rewards do change when fares change, albeit at a fixed rate. This problem is not as easy to solve, because we cannot easily say which restriction is active for any set of parameters  $(\theta_H, \theta_L, N_H, N_L, \alpha)$ : in the optimal program fares and rewards can be changed independently, leading to only two out of four constraints being active in the optimum. In the ex-post per-distance program, only fares can be changed, which again makes it obvious that those same constraints are the ones active in the ex-post optimum. But here, fare and rewards move simultaneously at a fix rate, and thus which restriction is active at the optimum is far from obvious.

What we do, then, is start from the ex-ante problem, where the H-type incentive compatibility and the L-type participation constraint are active, and analyze small changes over the set of parameters, and their impact over the restrictions. We then try to move fares and rewards in the same directions that the optimal program would while satisfying all constraints, which lead to, in some cases, conditions on functional forms and evaluation points. All calculations are mapped in Appendix D.

We show that under the following condition:

$$V'(F_H^*) < \frac{\alpha}{\theta_H k_H} \quad (26)$$

the airline can move  $F_H$  (and thus,  $P_H$ ) in the same direction of the optimal program when facing small changes over  $\alpha$  or  $\theta_H$ . If we have the stronger conditions

$$V'(F_H^*) < \frac{\alpha}{\theta_H k_H} - \delta \left( \frac{1}{\theta_L k_L} - \frac{\alpha}{\theta_H k_L} \right) \quad (27)$$

$$V'(F_L^*) \in \left[ \frac{1}{\theta_L k_L}, \frac{1}{\delta} \left( \frac{\alpha}{\theta_H k_H} - V'(F_H^*) \right) + \frac{\alpha}{\theta_H k_L} \right] \quad (28)$$

then the airline would also be able to also move  $F_L$  (and thus,  $P_L$ ) in the same direction of the optimal program, where  $\delta$  is the magnitude of the movement over  $F_L$  compared to the magnitude of the movement over  $F_H$  (in the optimal program, when facing a change over  $\alpha$  or  $\theta_H$  we obtain  $\delta \ll 1$ ). It is also easy to show that if  $U_0$  is large enough, (27) and (28) are naturally satisfied given the concavity of  $V$ .

The same conditions would allow the airline to move  $F_L$  (and thus,  $P_L$ ) in the same direction of the optimal program when facing small changes over  $\theta_L$ , but this requires in the ex-post setting to move  $P_H, F_H$  in the opposite direction of the movement in  $F_L, P_L$ , even though in the optimal program the airline would like to maintain the reward assigned to business passengers.

Finally, we show that under excluding conditions, the airline can move fares and rewards in a similar way to the optimal program when facing changes over the number of passengers of high class  $N_H$  or low class  $N_L$ . To be more precise, if we have

$$V'(F_L^*) \geq \frac{1}{\theta_L k_L} \quad (29)$$

as in the lower bound of (28), then the airline can react correctly to changes over  $N_L$ , but not over  $N_H$ . On the other hand, if

$$V'(F_L^*) \leq \frac{1}{\theta_L k_L} \quad (30)$$

then we have the opposite case: the airline can react correctly to changes over  $N_H$ , but not over  $N_L$ .

A summary table is provided, where the first column show the response in the optimal program, and the second the response in the rewards-per-dollar program. Green cells indicate that the change is in the same direction as in the optimal program, red cells indicate that it is not.

**Table 3.** Responses ex-post for the optimal problem and rewards-per-dollar program.

$\frac{d}{dx}$	$F_H$		$F_L$		$P_H$		$P_L$	
	$\leq 0$	$\leq 0$	$\geq 0$	$\geq 0$	$\leq 0$	$\leq 0$	$\geq 0$	$\geq 0$
$\alpha$	$\leq 0$	$\leq 0$	$\geq 0$	$\geq 0$	$\leq 0$	$\leq 0$	$\geq 0$	$\geq 0$
$\theta_H$	$\geq 0$	$\geq 0$	$\leq 0$	$\leq 0$	$\geq 0$	$\geq 0$	$\leq 0$	$\leq 0$
$\theta_L$	$= 0$	$\leq 0$	$\geq 0$	$\geq 0$	Depends	$\leq 0$	$\geq 0$	$\geq 0$
$N_H$	$= 0$	$\geq 0$	$\leq 0$	$\leq 0^*$ $(=0)$	$\geq 0$	$\geq 0$	$\leq 0$	$\leq 0^*$ $(=0)$
$N_L$	$= 0$	$\leq 0$	$\geq 0$	$\geq 0^*$ $(=0)$	$\leq 0$	$\leq 0$	$\geq 0$	$\geq 0^*$ $(=0)$

#### CLASS-SHUTDOWN COMPARISON

We now compare class-shutdown conditions ex-post. Recall that, ex-ante, if for given parameter values it was more profitable under the optimal program to shut down the leisure class, it was also the case in both programs and they were both able to mimic the optimal program. But this is no longer the case. Suppose that ex-ante, for expected values of the parameters, the airline decides to design its program to separate but, when facing ex-post parameter values, the airline has to reassess. The general shutdown condition is:

$$N_L(P_L^* - C_L(F_L^*)) + N_H(P_H^* - C_H(F_H^*)) < N_H(P_H^{**} - C_H(F_H^{**}))$$

Where \* denotes separation values and \*\* denotes what would be chosen if shutting down the L-type class. For the case of the per-distance rewards program tough, ex-post only the fare can

be adjusted, not the reward and, therefore,  $F_H^* = F_H^{**}$ . This implies that the shutdown condition for the per-distance program in the ex-post setting is:

$$N_L(P_L^* - C_L(F_L^*)) < N_H(P_H^{**} - P_H^*) \quad (31)$$

In the per-dollar program, however, ex-post, we have  $F_H^* = k_H^* P_H^* < k_H^* P_H^{**} = F_H^{**}$ . Thus, the shutdown condition for the per-dollar program in the ex-post setting is:

$$N_L(P_L^* - C_L(F_L^*)) \leq N_H(P_H^{**} - P_H^*) - \underbrace{N_H(C_H(k_H^* P_H^{**}) - C_H(k_H^* P_H^*))}_{+} \quad (32)$$

We can easily see that this condition is stronger than the one in the rewards per-distance program in (31): charging a higher price  $P_H^{**} > \frac{F_H^*}{k_H^*}$  to business passengers has a downside for the airline as they have to offer the correspondent rewards and face a cost  $C_H(k_H^* P_H^{**}) > C_H(F_H^*)$ . Hence, the models predict that in a per-dollar program, class shutdown will be less common than in a per-distance program. This matches the data we collected; we followed four pairs of local city pairs in Chile, Canada and the U.S. on October and November of 2014 and discovered that LATAM and Air Canada, both with per-distance programs often closed the lower classes. LATAM closed the fourth, third and second tier class 68%, 53% and 26% of the time respectively, while Air Canada closed the third and second tier class 34% and 4% of the time respectively. On the other hand, Southwest, with a per-dollar program, closed its lowest class only once, while the middle class was never shutdown.

## 6. CONCLUSIONS

Since their inception in the 80s, airlines' frequent flyer programs (FFP) have become one of the largest customer loyalty programs in the world economy. However, despite their size and importance, these programs have remained relatively unexplored from a microeconomics point of view. Basso et al (2009) study the FFP considering the existence of a business related traveler, where the employer is who actually pays the ticket, which impose a moral-hazard situation where the employee might not always choose the cheapest ticket but will try to maximize individual benefits and it is there where FFP rewards play a major role.

We extend this approach, proposing a theoretical model that considers the existence of two types of travelers, business and leisure, who are not distinguishable by the (monopoly) airline ex-ante, and which is the source of an adverse selection problem, and where business travelers do not pay fully their air tickets, inducing a moral hazard problem between them and their employers, who actually pays. The airline then faces a second-degree price discrimination problem, where it attempts to separate traveler types with the use of different menus of prices and rewards, while exploiting the moral hazard problem by 'bribing' business travelers.

We find that the moral hazard problem induces an upward distortion in rewards for business travelers (distortion at the top) while deepening the typical distortion at the bottom caused by adverse selection (even smaller rewards for leisure travelers).

We compare this 'optimal' program, where fares and rewards can be chosen independently, to real-life programs, where rewards are linked either to distance travelled or to the fare paid. We show that, at the design stage of the program, when demand parameter values are taken at its



expected values (the ex-ante setting), the three programs are equivalent. Yet, ex-post, once the programs have already been designed and they cannot be modified, if demand parameter values differ from expected values, nor the per-distance nor the per-dollar program can mimic the optimal program. We show, however, that the per-dollar program allows the (monopoly) airline to respond to changes in a closer way to the optimal program than the per-distance program, which is consistent with the recent changes to per-dollar programs of a number of legacy carriers. Finally, we show that, ex-post, it is more likely that the lower classes are shut-down in a per-distance than in a per-dollar program, which is consistent with the data we collected from LATAM Airlines, Air Canada (rewards per distance) and Southwest Airlines (rewards per dollar).

The model introduced in this study could be easily used for future research. We focused in a single monopoly airline serving the demand, so a natural step would be to include competition: Basso et al (2009) show that for the case of only business travelers, a lone airline can indeed take advantage of the moral hazard, increasing prices up to the employers' reservation price and defeating the competition. Yet, if all airlines use FFPs, rewards become simply a new competitive instrument which enables tougher competition: airlines will dissipate profits through FFP rewards in a standard Prisoner's Dilemma fashion. Prices will remain high though, showing that all benefits flow from employers and airlines to employees. These conclusions could translate to the two-classes case, generating different conditions under which an airline could find beneficial to use one or the other scheme of rewards.

Also, in this study we showed the results for only one homogenous market. Yet, when designing their programs, airline may need to use single FFP contracts (i.e. just one pair of multipliers) for many different city pair markets, with different distances and number of passengers of each class. It could be the case then, that one real program may be preferable for some circumstances of demands and markets, while the other program may be preferable for different circumstances, something worthy of research.

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## 8. APPENDIX

### A. DERIVATION OF OPTIMAL POLICY FOR SECTION 3 PROBLEM

We consider the model:

$$\text{Max}_{(P_H, P_L, F_H, F_L)} \pi = N_L(P_L - C_L(F_L)) + N_H(P_H - C_H(F_H))$$

s. t.

$$\theta_H V(F_H) - \alpha P_H \geq \theta_H V(F_L) - \alpha P_L$$

$$\theta_L V(F_L) - P_L \geq \theta_L V(F_H) - P_H$$

$$U_0 + \theta_L V(F_L) - P_L \geq 0$$

$$F_H \geq 0$$

$$F_L \geq 0$$

After the change of variables  $u_L = U_0 + \theta_L V(F_L) - P_L$  ,  $u_H = \frac{\theta_H}{\alpha} V(F_H) - P_H$ , incentive compatibility constraints (1) and (2) can be rewritten as

$$V(F_L) \left( \frac{\theta_H}{\alpha} - \theta_L \right) \leq u_H - u_L + U_0 \quad (33)$$

$$u_H - u_L + U_0 \leq V(F_H) \left( \frac{\theta_H}{\alpha} - \theta_L \right) \quad (34)$$

and therefore, the airlines problem finally is:

$$\text{Max}_{(u_H, u_L, F_H, F_L)} \pi = N_L(U_0 + \theta_L V(F_L) - u_L - C_L(F_L)) + N_H \left( \frac{\theta_H}{\alpha} V(F_H) - u_H - C_H(F_H) \right)$$

$$\text{s.t.} \quad V(F_L) \left( \frac{\theta_H}{\alpha} - \theta_L \right) \leq u_H - u_L + U_0$$

$$u_H - u_L + U_0 \leq V(F_H) \left( \frac{\theta_H}{\alpha} - \theta_L \right)$$

$$u_L \geq 0$$

$$F_L \geq 0$$

$$F_H \geq 0$$

We simplify the problem by showing that any optimal solution (or feasible vector close to it) must fulfill some conditions. First, any optimal solution of this problem  $(u_H^*, u_L^*, F_H^*, F_L^*)$  must fulfill that:

$$u_L^* = 0$$

$$u_H^* = V(F_L^*) \left( \frac{\theta_H}{\alpha} - \theta_L \right) - U_0$$

$$F_H^* > 0$$

which is easily proved by contradiction. For example, if  $u_L^* > 0$ , then the airline can decrease both  $u_L^*$  and  $u_H^*$ , thus increasing profits and satisfying all constraints. Using these simplifications, the problem can be presented compactly:

$$\text{Max}_{(u_H, F_H, F_L)} \pi = N_L (U_0 + \theta_L V(F_L) - C_L(F_L)) + N_H \left( \frac{\theta_H}{\alpha} V(F_H) - u_H - C_H(F_H) \right)$$

$$\text{s.t.} \quad u_H - V(F_L) \left( \frac{\theta_H}{\alpha} - \theta_L \right) + U_0 = 0$$

$$F_L \geq 0$$

Forming the Lagrangean, first order conditions are easily obtained as follows:

$$\left\{ \begin{array}{l} \frac{\partial \mathcal{L}}{\partial u_H^*} = -N_H + A = 0 \\ \frac{\partial \mathcal{L}}{\partial F_H^*} = N_H \left( \frac{\theta_H}{\alpha} V'(F_H^*) - C_H'(F_H^*) \right) = 0 \\ \frac{\partial \mathcal{L}}{\partial F_L^*} = N_L (\theta_L V'(F_L^*) - C_L'(F_L^*)) - A V(F_L^*) \left( \frac{\theta_H}{\alpha} - \theta_L \right) + B = 0 \\ \frac{\partial \mathcal{L}}{\partial A} = u_H^* - V(F_L^*) \left( \frac{\theta_H}{\alpha} - \theta_L \right) + U_0 = 0 \\ B \frac{\partial \mathcal{L}}{\partial B} = B F_L^* = 0 \text{ con } B \geq 0 \text{ ó } B = 0 \end{array} \right.$$

where A is the Lagrange multiplier associated with the incentive compatibility constraint and y B is the Lagrange multiplier associated with the non-negativity of  $F_L$ . From the first-order conditions we obtain the following rules that the airline follows in its optimal policy:

$$u_H^* = V(F_L^*) \left( \frac{\theta_H}{\alpha} - \theta_L \right) - U_0 \quad (35)$$

$$\frac{\theta_H}{\alpha} V'(F_H^*) = C_H'(F_H^*) \quad (36)$$

$$\theta_L V'(F_L^*) = \frac{C_L'(F_L^*) - B}{1 - \frac{N_H}{N_L} \frac{(\theta_H - \theta_L)}{\theta_L}} \quad (37)$$

## B. RESPONSES TO CHANGES IN PARAMETERS FOR SECTION 3 PROBLEM

We already showed that

$$\frac{dP_L^*}{d\alpha} \geq 0, \quad \frac{dP_H^*}{d\alpha} \leq 0$$

$$\frac{dF_L^*}{d\alpha} \geq 0, \quad \frac{dF_H^*}{d\alpha} \leq 0$$

From this, it is clear that

$$\frac{dP_L^*}{d\theta_H} \leq 0, \quad \frac{dP_H^*}{d\theta_H} \geq 0$$

$$\frac{dF_L^*}{d\theta_H} \leq 0, \quad \frac{dF_H^*}{d\theta_H} \geq 0$$

as an increase in  $\theta_H$  can be seen as a decrease in  $\alpha$  (and vice-versa).

Also,

$$\frac{dF_H^*}{d\theta_L} = 0, \quad \frac{dF_L^*}{d\theta_L} \geq 0$$

the last coming from the fact that

$$\frac{dV'(F_L^*)}{d\theta_L} = - \frac{1 + \frac{N_H}{N_L}}{\left( \theta_L \left( 1 - \frac{N_H}{N_L} \frac{(\theta_H - \theta_L)}{\theta_L} \right) \right)^2} < 0$$

and the fact that  $V'' < 0$ .

And we also have

$$\frac{dP_L^*}{d\theta_L} = V(F_L^*) + \theta_L \frac{dV(F_L^*)}{d\theta_L} \geq 0$$

$$\frac{dP_H^*}{d\theta_L} = - \frac{\theta_H}{\alpha} \frac{dV(F_L^*)}{d\theta_L} + V(F_L^*) + \theta_L \frac{dV(F_L^*)}{d\theta_L}$$

with that last sign depending on the magnitude of  $V(F_L^*)$ .

Finally, it's clear that

$$\frac{dF_H^*}{dN_L} = \frac{dF_H^*}{dN_H} = 0$$

and

$$\frac{dF_L^*}{dN_L} \geq 0, \quad \frac{dF_L^*}{dN_H} \leq 0$$

once we consider that

$$\frac{dV'(F_L^*)}{dN_H} = \frac{\frac{1}{N_L} \left( \frac{\theta_H}{\alpha} - \theta_L \right)}{\left( \theta_L \left( 1 - \frac{N_H}{N_L} \frac{\left( \frac{\theta_H}{\alpha} - \theta_L \right)}{\theta_L} \right) \right)^2} > 0$$

$$\frac{dV'(F_L^*)}{dN_L} = - \frac{N_H \frac{\left( \frac{\theta_H}{\alpha} - \theta_L \right)}{\theta_L}}{\left( N_L \theta_L \left( 1 - \frac{N_H}{N_L} \frac{\left( \frac{\theta_H}{\alpha} - \theta_L \right)}{\theta_L} \right) \right)^2} < 0$$

And the concavity of  $V$ .

Later,

$$\frac{dP_H^*}{dN_L} = - \left( \frac{\theta_H}{\alpha} - \theta_L \right) \frac{dV(F_L^*)}{dN_L} \leq 0$$

$$\frac{dP_H^*}{dN_H} = - \left( \frac{\theta_H}{\alpha} - \theta_L \right) \frac{dV(F_L^*)}{dN_H} \geq 0$$

$$\frac{dP_L^*}{dN_L} = \theta_L \frac{dV(F_L^*)}{dN_L} \geq 0$$

$$\frac{dP_L^*}{dN_H} = \theta_L \frac{dV(F_L^*)}{dN_H} \leq 0$$

### C. CONDITIONS FOR $k_L^* < k_H^*$ FOR REWARDS-PER-DOLLAR PROGRAM

As noted, this program will, ex-ante, deliver the same rewards and fares as the optimal program. This can be easily seen because, if in the optimal program the airline chose  $(F_H^*, F_L^*, P_L^*, P_H^*)$ , it only needs to establish that

$$k_H^* = \frac{F_H^*}{P_H^*}, \quad k_L^* = \frac{F_L^*}{P_L^*}$$

where  $(F_H^*, F_L^*, P_L^*, P_H^*)$  are given by equations (14) to (17). In particular

$$P_H^* = \frac{\theta_H}{\alpha} (V(F_H^*) - V(F_L^*)) + \theta_L V(F_L^*) + U_0$$

$$P_L^* = \theta_L V(F_L^*) + U_0$$

So that

$$\frac{1}{k_H^*} = \frac{\theta_H V(F_H^*)}{\alpha F_H^*} - \left( \frac{\theta_H}{\alpha} - \theta_L \right) \frac{V(F_L^*)}{F_H^*} + \frac{U_0}{F_H^*} \quad (38)$$

$$\frac{1}{k_L^*} = \theta_L \frac{V(F_L^*)}{F_L^*} + \frac{U_0}{F_L^*} \quad (39)$$

Noting that  $F_L^* < F_H^*$ , and  $\frac{\theta_H}{\alpha} > \theta_L$ , a sufficient condition to ensure that  $k_L^* < k_H^* \Leftrightarrow \frac{1}{k_H^*} < \frac{1}{k_L^*}$  is that

$$\frac{V(F_H^*)}{F_H^*} < \frac{\alpha}{\theta_H} \theta_L \frac{V(F_L^*)}{F_L^*} \quad (40)$$

But given the concavity of  $V$ , we always have

$$\frac{V(F_H^*)}{F_H^*} \leq \frac{V(F_L^*)}{F_L^*}$$

If we require  $V$  to be strongly concave, such that the stronger condition (40) holds, then we have  $k_L^* < k_H^*$ . Note that we could weaken this condition, because we need such inequality only in  $(F_H^*, F_L^*)$ , i.e. the ex-ante optimal rewards, not in every point. Also, the condition (40) obviously doesn't hold when  $\alpha \rightarrow 0$ , but in that case, we could require the weaker condition instead:

$$\frac{V(F_H^*)}{F_H^*} < \frac{\alpha}{\theta_H} \theta_L \frac{V(F_L^*)}{F_L^*} + \left( 1 - \frac{\alpha}{\theta_H} \theta_L \right) \frac{V(F_L^*)}{F_H^*} \quad (41)$$

#### D. REWARDS PER DOLLAR: EX-POST COMPARATIVE STATICS

We know that at the optimal value (ex-ante) (23) and (25) are active, while (24) is slack, because we showed that for the optimal program of Section 3. We'll now turn to analyze small changes over the set of parameters, and their impact over the restrictions, while we try to move prices and rewards in the directions given by the sensibility of the optimal problem. We'll study only the behavior for increases in those parameters, but the cases where there is a decrease are very similar.

Let's suppose first that  $\alpha$  increases marginally, it's clear that (24) and (25) are still slack and active in the original optimum, but (23) is no longer satisfied. To recover feasibility, we'd like to increase  $F_L, P_L$ , while decreasing  $F_H, P_H$ . To do the former and maintain the participation constraint (25), we must require

$$\frac{\partial}{\partial F_L} \left( U_0 + \theta_L V(F_L^*) - \frac{F_L^*}{k_L} \right) = \theta_L V'(F_L) - \frac{1}{k_L} \geq 0 \Leftrightarrow V'(F_L^*) \geq \frac{1}{\theta_L k_L} \quad (42)$$

To recover feasibility, we need to at least increase the left-hand size of (24) when we decrease  $F_H$ , i.e. we require

$$\frac{\partial}{\partial F_H} \left( \theta_H V(F_H^*) - \alpha \frac{F_H^*}{k_H} \right) = \theta_H V'(F_H^*) - \frac{\alpha}{k_H} < 0 \Leftrightarrow V'(F_H^*) < \frac{\alpha}{\theta_H k_H} \quad (43)$$

Now, given the first condition, the right-hand size of (23) increases when we increase  $F_L$ , so we must require a stronger condition to recover feasibility if we move both  $F_H, F_L$ . If we move  $F_H$

in a magnitude  $\varepsilon$ , while moving  $F_L$  in a magnitude  $\delta\varepsilon$ ,  $\delta \in (0, \infty)$ , a necessary condition to satisfy is:

$$\begin{aligned}
& \int_0^\varepsilon \left| \frac{\partial}{\partial F_H} \left( \theta_H V(F_H) - \alpha \frac{F_H}{k_H} \right) \right|_{F_H^* - x} dx > \int_0^{\delta\varepsilon} \left| \frac{\partial}{\partial F_L} \left( \theta_H V(F_L) - \alpha \frac{F_L}{k_L} \right) \right|_{F_L^* + x} dx \\
& \Leftrightarrow \int_0^\varepsilon \left( \frac{\alpha}{k_H} - \theta_H V'(F_H^* - x) \right) dx > \int_0^{\delta\varepsilon} \left( \theta_H V'(F_L^* + x) - \frac{\alpha}{k_L} \right) dx \\
& \Leftrightarrow \frac{\alpha\varepsilon}{k_H} + \theta_H (V(F_H^* - \varepsilon) - V(F_H^*)) > \theta_H (V(F_L^* + \delta\varepsilon) - V(F_L^*)) - \frac{\alpha\delta\varepsilon}{k_L} \\
& \Leftrightarrow \frac{\alpha}{k_H} + \theta_H \left( \frac{V(F_H^* - \varepsilon) - V(F_H^*)}{\varepsilon} \right) > \delta \theta_H \left( \frac{V(F_L^* + \delta\varepsilon) - V(F_L^*)}{\delta\varepsilon} \right) - \frac{\alpha\delta}{k_L}
\end{aligned}$$

When the movement over  $F_H$  is marginal, i.e.  $\varepsilon \rightarrow 0$ , the previous condition translates to

$$\begin{aligned}
& \frac{\alpha}{k_H} - \theta_H V'(F_H^*) > \delta \left( \theta_H V'(F_L^*) - \frac{\alpha}{k_L} \right) \\
& \Leftrightarrow V'(F_L^*) < \frac{1}{\delta} \left( \frac{\alpha}{\theta_H k_H} - V'(F_H^*) \right) + \frac{\alpha}{\theta_H k_L} \tag{44}
\end{aligned}$$

And then, to keep compatibility with  $V'(F_L^*) \geq \frac{1}{\theta_L k_L}$ , we must require

$$V'(F_H^*) < \frac{\alpha}{\theta_H k_H} - \delta \left( \frac{1}{\theta_L k_L} - \frac{\alpha}{\theta_H k_L} \right) \tag{45}$$

Let's remark that in the optimal problem,  $V'(F_H^*)$  and  $V'(F_L^*)$  are given by equations (16) and (17), and thus is easy to check that

$$\begin{aligned}
& \frac{\partial V'(F_H^*)}{\partial \alpha} = \frac{C'_H(F_H^*)}{\theta_H} \\
& \frac{\partial V'(F_L^*)}{\partial \alpha} = -\frac{N_L}{N_H} \frac{1}{\left(1 - \frac{\theta_L N_L}{\theta_H N_H} \alpha - \frac{\theta_L}{\theta_H} \alpha\right)^2} \frac{C'_L(F_L^*)}{\theta_H}
\end{aligned}$$

Where  $\frac{N_L}{N_H} \frac{1}{\left(1 - \frac{\theta_L N_L}{\theta_H N_H} \alpha - \frac{\theta_L}{\theta_H} \alpha\right)^2} > 1$  for  $\alpha$  small enough, so if we assume for example constant and equal marginal cost of providing rewards to leisure and business passengers, we have  $\left| \frac{\partial V'(F_H^*)}{\partial \alpha} \right| < \left| \frac{\partial V'(F_L^*)}{\partial \alpha} \right|$ . This gap is smaller (and we can even have  $\left| \frac{\partial V'(F_H^*)}{\partial \alpha} \right| > \left| \frac{\partial V'(F_L^*)}{\partial \alpha} \right|$ ) if the costs of providing rewards are increasing and/or larger for business travelers, but even if the derivate of  $F_L^*$  moves in a larger magnitude than the derivative of  $F_H^*$ , let's remember that we assume  $V$  concave, i.e.  $V'' < 0$ , so the change in slope of  $V$  is much bigger near  $F_L^*$  than near  $F_H^*$ , as is seen in figure 2. So, if we have  $|V''|$  large enough, then  $\delta \ll 1$ .

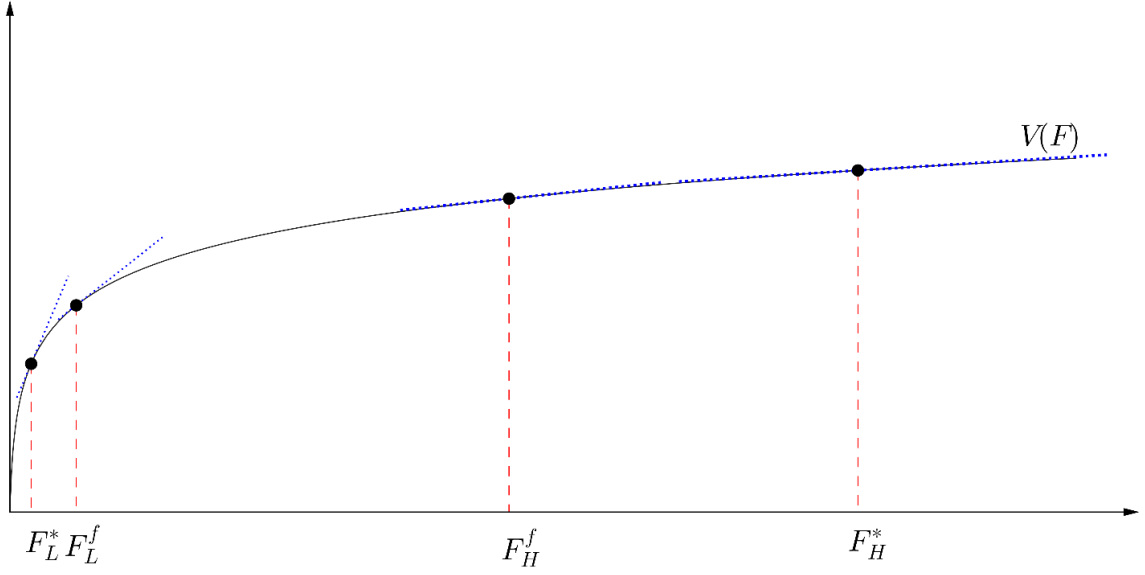


Figure 2. Even if we have  $\left| \frac{\partial V'(F_H^*)}{\partial \alpha} \right| < \left| \frac{\partial V'(F_L^*)}{\partial \alpha} \right|$ , the change in  $F_L^*$  tends to be smaller than the change in  $F_H^*$  when we move  $\alpha$ .

Now, returning to (45), when we move only  $F_H^*$  we naturally have  $\delta = 0$ , so the condition translates to

$$V'(F_H^*) < \frac{\alpha}{\theta_H k_H} \quad (46)$$

But we also have

$$\begin{aligned} \frac{\alpha}{\theta_H k_H} &= \frac{\alpha}{\theta_H} \frac{\frac{\theta_H}{\alpha} [V(F_H^*) - V(F_L^*)] + \theta_L V(F_L^*) + U_0}{F_H^*} \\ &= \frac{V(F_H^*)}{F_H^*} - \left(1 - \frac{\alpha \theta_L}{\theta_H}\right) \frac{V(F_L^*)}{F_H^*} + \frac{\alpha U_0}{\theta_H F_H^*} \\ &> \frac{V(F_H^*)}{F_H^*} - \left(1 - \frac{\alpha \theta_L}{\theta_H}\right) \frac{V(F_H^*)}{F_H^*} + \frac{\alpha U_0}{\theta_H F_H^*} \\ &= \frac{\alpha}{\theta_H} \left( \theta_L \frac{V(F_H^*)}{F_H^*} + \frac{U_0}{F_H^*} \right) \end{aligned}$$

So, a sufficient condition for (46) is

$$V'(F_H^*) < \frac{\alpha}{\theta_H} \left( \theta_L \frac{V(F_H^*)}{F_H^*} + \frac{U_0}{F_H^*} \right) \quad (47)$$

And using the concavity of  $V$ , we have that

$$V'(F_H^*) \leq \frac{V(F_H^*)}{F_H^*}$$



And a sufficient condition for (47) is thus

$$U_0 > \left( \frac{\theta_H}{\alpha} - \theta_L \right) V(F_H^*)$$

On the other hand, when we also move  $F_L^*$ , we require (45), but we know that

$$\frac{1}{\theta_L k_L} = \frac{V(F_L^*)}{F_L^*} + \frac{U_0}{\theta_L F_L^*}$$

$$\frac{\alpha}{\theta_H k_L} = \frac{\alpha}{\theta_H} \frac{\theta_L V(F_L^*) + U_0}{F_L^*}$$

And then

$$\begin{aligned} & \frac{\alpha}{\theta_H k_H} - \left( \frac{1}{\theta_L k_L} - \frac{\alpha}{\theta_H k_L} \right) \\ &= \frac{V(F_H^*)}{F_H^*} - \left( 1 - \frac{\alpha \theta_L}{\theta_H} \right) \frac{V(F_L^*)}{F_H^*} + \frac{\alpha U_0}{\theta_H F_H^*} - \delta \frac{V(F_L^*)}{F_L^*} - \delta \frac{U_0}{\theta_L F_L^*} \\ & \quad + \frac{\delta \alpha \theta_L V(F_L^*) + U_0}{\theta_H F_L^*} \\ &= U_0 \left( \frac{\alpha}{\theta_H} \frac{1}{F_H^*} - \frac{\delta}{\theta_L F_L^*} + \frac{\delta \alpha}{\theta_H} \frac{1}{F_L^*} \right) + \frac{V(F_H^*)}{F_H^*} - \frac{V(F_L^*)}{F_H^*} \left( 1 - \frac{\alpha \theta_L}{\theta_H} \right) - \frac{V(F_L^*)}{F_L^*} \delta \left( 1 - \frac{\alpha \theta_L}{\theta_H} \right) \\ &= U_0 \left( \frac{\alpha}{\theta_H} \frac{1}{F_H^*} - \frac{\delta}{\theta_L F_L^*} + \frac{\delta \alpha}{\theta_H} \frac{1}{F_L^*} \right) + \frac{V(F_H^*)}{F_H^*} - V(F_L^*) \left( \frac{\delta}{F_L^*} + \frac{1}{F_H^*} \right) \left( 1 - \frac{\alpha \theta_L}{\theta_H} \right) \end{aligned}$$

If  $\delta$  is small enough, then  $\left( \frac{\alpha}{\theta_H} \frac{1}{F_H^*} - \frac{\delta}{\theta_L F_L^*} + \frac{\delta \alpha}{\theta_H} \frac{1}{F_L^*} \right) > 0$  and a sufficient condition for (45) is

$$U_0 > \frac{V(F_L^*) \left( \frac{\delta}{F_L^*} + \frac{1}{F_H^*} \right) \left( 1 - \frac{\alpha \theta_L}{\theta_H} \right)}{\frac{\alpha}{\theta_H} \frac{1}{F_H^*} - \frac{\delta}{\theta_L F_L^*} + \frac{\delta \alpha}{\theta_H} \frac{1}{F_L^*}}$$

This concludes the analysis for  $\alpha$ . The case of changes in  $\theta_H$  is analogous, as it can be seen in a change in  $\alpha$ , so it will be omitted.

Now we focus in changes over  $\theta_L$ : when the valorization of rewards for the leisure passengers increases marginally, (23) is still active, while (24) continues being slack (the change is marginal), but (25) now is inactive. So, we're still in a feasible point of the problem, but we'd like to move fares and rewards as in the optimal problem, that is increase  $F_L, P_L$  while keeping fixed  $F_H$  and moving  $P_H$  in a direction that depends on  $V(F_L)$ . It's obvious that the last two conditions are incompatible under rewards-per-dollar program, because we move prices and rewards together.

Under the conditions of the previous case (a movement over  $\alpha$ ), when we increase  $F_L$ , the right-hand size of (23) grows, so we move out the feasible region. So, we'd like to return to it, and the only possibility under the conditions of the case of movement over  $\alpha$  is to decrease  $F_H$ , and now it's easy to see that the conditions that allows the airline to do that are exactly the conditions

found for the case of movements over  $\alpha$ . As (24) and (25) are slack, they don't generate any problem and we can move  $F_L, F_H$  marginally in any direction.

Now we focus in changes over  $N_H$ : when the number of high class passengers increases, the restrictions don't change for this problem, only the objective function does. So, the original optimum is still a feasible point, but very possible not the optimum for the new problem. In the optimal problem, this change causes a decrease in  $F_L, P_L$ , and an increase in  $P_H$  while keeping constant  $F_H$ . Again, the last the last two conditions are not achievable under this approach, so we'll try to increase  $P_H, F_H$ .

To maintain the participation constraint of leisure passengers (25) while we decrease  $F_L$ , we must require

$$\frac{\partial}{\partial F_L} \left( U_0 + \theta_L V(F_L^*) - \frac{F_L^*}{k_L} \right) = \theta_L V'(F_L) - \frac{1}{k_L} \leq 0 \Leftrightarrow V'(F_L^*) \leq \frac{1}{\theta_L k_L}$$

Which contradicts (42) and shouldn't be satisfied in general. And to keep the incentive compatibility constraint of the business passengers (23), we require at least

$$V'(F_L^*) > \frac{\alpha}{\theta_H k_L}$$

So that the right-hand size of that restriction decreases when  $F_L$  decreases. Also, we must include a restriction in a similar fashion to the one of the case when  $\alpha$  changes:

$$\begin{aligned} \int_0^\varepsilon \left| \frac{\partial}{\partial F_H} \left( \theta_H V(F_H) - \alpha \frac{F_H}{k_H} \right) \right|_{F_H^*+x} dx &\geq \int_0^{\delta\varepsilon} \left| \frac{\partial}{\partial F_L} \left( \theta_H V(F_L) - \alpha \frac{F_L}{k_L} \right) \right|_{F_L^*-x} dx \\ \Leftrightarrow \frac{\alpha}{k_H} - \theta_H \left( \frac{V(F_H^* + \varepsilon) - V(F_H^*)}{\varepsilon} \right) &\geq -\delta\theta_H \left( \frac{V(F_L^* - \delta\varepsilon) - V(F_L^*)}{\delta\varepsilon} \right) - \frac{\alpha\delta}{k_L} \end{aligned}$$

When the movement over  $F_H$  is marginal, i.e.  $\varepsilon \rightarrow 0$ , the previous condition translates to

$$\frac{\alpha}{k_H} - \theta_H V'(F_H^*) > \delta \left( \theta_H V'(F_L^*) - \frac{\alpha}{k_L} \right)$$

Which is the same condition found in the case of  $\alpha$ .

The case when  $N_L$  increases is very similar: in the optimal problem this causes a rise in  $F_L, P_L$ , a decrease of  $P_H$  while the rewards  $F_H$  is keep still. So, we aim to find conditions under which we can increase  $F_L, P_L$  and decrease  $F_H, P_H$ .

To maintain the participation constraint of leisure passengers (25) while we decrease  $F_L$ , we must require

$$\frac{\partial}{\partial F_L} \left( U_0 + \theta_L V(F_L^*) - \frac{F_L^*}{k_L} \right) = \theta_L V'(F_L) - \frac{1}{k_L} \geq 0 \Leftrightarrow V'(F_L^*) \geq \frac{1}{\theta_L k_L}$$

Condition that was obtained previously. And to keep the incentive compatibility constraint of the business passengers (23), we require at least

$$V'(F_L^*) > \frac{\alpha}{\theta_H k_L}$$

So that the right-hand size of that restriction decreases when  $F_L$  decreases. Also, we must include a restriction in a similar fashion to the one of the case when  $\alpha$  changes:

$$\int_0^{\varepsilon} \left| \frac{\partial}{\partial F_H} \left( \theta_H V(F_H) - \alpha \frac{F_H}{k_H} \right) \right|_{F_H^* - x} dx \geq \int_0^{\delta \varepsilon} \left| \frac{\partial}{\partial F_L} \left( \theta_H V(F_L) - \alpha \frac{F_L}{k_L} \right) \right|_{F_L^* + x} dx$$

Which is the same condition of the first case.