The Concatenation of Forecasts

Javier Finot & Pablo Pincheira Brown

School of Business

Universidad Adolfo Ibáñez

May 2017

Abstract

In this paper we analyze the concatenation of point forecasts. The idea of a concatenation is very similar to that of interpolation or splicing. In a nutshell, when two sets of multistep ahead forecasts are available for the same target variable, and one of these sets is generated using a recursive strategy, it is possible to generate a third sequence of multistep ahead forecasts using the one-step-ahead forecast of method A, and the recursive strategy of method B. In other words we "feed" the sequence of multistep ahead forecasts of method B with a different one-step ahead forecast. We find theoretical conditions under which the concatenation of forecasts offers gains in Mean Squared Prediction Errors (MSPE). We illustrate the benefits of concatenation with an empirical application for inflation forecasts in an emerging economy. We show that the concatenation of survey-based forecasts with SARIMA models renders important gains in out-of-sample MSPE.

JEL Codes: C220, C530, E170, E270, E370, F370, L740, O180, R310.

Keywords: Forecasting, Inflation, Time Series, Combination of Forecasts

I. Introduction

Nowadays it is fairly common to find several different forecasts for some key variables of interest. In macroeconomics, for instance, central banks and other public and international institutions usually publicly announce forecasts for GDP growth, inflation and other relevant variables. In addition, it is now a long tradition in many countries to conduct forecasting surveys, like the Survey of Professional Forecasters in the USA. As expected, different forecasters usually disagree in their forecasts, which has been an area of active research in the last years. See for instance Andrade, Crump, Eusepi and Moench (2014) and Patton and Timmermann (2010).

In a context in which a number of forecasts are available for a given variable, the literature on forecast combinations has flourished. Combination gains arise from different sources like diversification, different reactions of the forecasts to structural breaks, and misspecification bias that could affect individual models. These benefits are pointed out by several authors including the excellent review by Timmermann (2006)[1].

In this paper, we analyze a method to generate new forecasts from series of existing ones. We call this method "concatenation". The idea of a concatenation is very similar to interpolation or splicing. In a nutshell, when two sets of multistep ahead forecasts are available for the same target variable, and one of these sets is generated using a recursive strategy, it is possible to generate a third sequence of multistep ahead forecasts using the one-step-ahead forecast of method A, and the recursive strategy of method B. In other words we "feed" the sequence of multistep ahead forecasts of method B with a different one-step ahead forecast. While this idea is not new, and private conversations with some forecasters reveal that this is a common practice amongst practitioners, to our knowledge a serious empirical or theoretical analysis of the possible advantages of this strategy has not been carried out yet.

We find theoretical conditions under which the concatenation of forecasts offers gains in Mean Squared Prediction Errors (MSPE). We illustrate the benefits of concatenation with an empirical application for inflation forecasts in an emerging economy. We show that the concatenation of survey-based forecasts with SARIMA models renders important gains in out-of-sample MSPE.

A natural interesting application of the concatenation of forecasts arises in the context of surveys. Faust and Wright (2013) point out that in some contexts judgmental forecast are remarkably hard to beat. This and the fact that judgmental forecasts are not always

[1] Empirical results show that the discussion about how to combine is far from settled, especially because optimal weights are often beaten when compared with an equally weighted forecast combination in a pseudo out of sample evaluation. See, for instance, Stock and Watson (2004).

available at every forecasting horizon suggest that a concatenation strategy could take the accuracy of judgmental forecasts to multiple forecasting horizons.

The rest of the paper is organized as follows. In section 2 we present a brief literature review. Section 3 contains a simple mathematical example to explain how the concatenation of forecasts could be useful. Section 4 describes the data used in our empirical illustration. Section 5 displays in-sample results. Section 6 describes our prediction evaluation strategy. Section 7 presents our out-of-sample empirical results and, finally, section 8 concludes.

II. Literature Review

The concatenation of forecasts is similar in spirit to the concept of forecast combination. That is the reason why we find interesting to present a quick review of this literature. Since Bates and Granger (1969) published his seminal work about forecast combinations, a number of articles have reported gains in forecast accuracy stemming from the combination process. Timmermann (2006) is an excellent starting point in this literature. In his work, he points out that (among other things) the empirical evidence shows that simple combination schemes are hard to beat and combination often dominates the best individual forecast in out-of-sample forecasting experiments. Similar results are also reported by Stock and Watson (2004) and Elliot and Timmermann (2005). In a different avenue, Pincheira (2012) shows that convex forecast combinations generate an inefficient forecast. Our interpretation of these results is that a combination is a good option when faced to a number of different forecasts for the same target variable, but there is no a unique way to combine and improvements probably can still be made with new approaches.

III. A simple example

In this section, we develop a simple example of a concatenation. We assume the existence of two sequences of multistep ahead forecasts at each point in time t, namely $\pi_t^{fA}(h)$ and $\pi_t^{fB}(h)$. Here $\pi_t^{fA}(h)$ denotes the h-step ahead forecast for the variable π_{t+h} using the information available in time t.

Let us assume that multistep ahead forecasts for the sequence A, satisfy the following recursion:

$$\pi_t^{fA}(h) = \rho * \pi_t^{fA}(h-1)$$

$$\pi_t^{fA}(0) = \pi_t$$

This is the case, for instance, of optimal forecasts coming from an AR(1) model, or the case of recursively generated forecasts in a VAR system.

In our simple setup we have that

$$\pi_t^{fA}(h) = \rho^h * \pi_t$$

Then the forecast error for the one-step-ahead forecast is

$$e_t^A(1) = \pi_{t+1} - \pi_t^{fA}(1)$$

$$e_t^A(1) = \pi_{t+1} - \rho * \pi_t$$

When $h = 2$ the forecast error is

$$e_t^A(2) = \pi_{t+2} - \pi_t^{fA}(2)$$

$$e_t^A(2) = \pi_{t+2} - \rho^2 * \pi_t$$

$$e_t^A(2) = \pi_{t+2} - \rho * \pi_{t+1} + \rho * \pi_{t+1} - \rho^2 * \pi_t$$

$$e_t^A(2) = e_{t+1}^A(1) + \rho * e_t^A(1)$$

And for the h-step-ahead forecast, the forecast error is

$$e_t^A(h) = \sum_{i=1}^{h} \rho^i \, e_{t+h-i}^A(1)$$

Analogously, let

$$\pi_t^{fB}(h)$$

Be another (e.g. survey from experts) h-step ahead forecast for π_t using the information available in t. We assume no iterative rule for the generation of multistep ahead forecasts with method B. Consequently the sequence of forecasts is simple given by

$$\pi_t^{fB}(1), \pi_t^{fB}(2), \ldots, \pi_t^{fB}(h) \ldots$$

And the sequence of forecasting errors is given by

$$e_t^B(1) = \pi_{t+1} - \pi_t^{fB}(1); e_t^B(2) = \pi_{t+2} - \pi_t^{fB}(2); \ldots; e_t^B(h) = \pi_{t+h} - \pi_t^{fB}(h) \ldots.$$

Now we define the concatenated method $\pi_t^{fC}(h)$ as follows

$$\pi_t^{fC}(h) = \pi_t^{fB}(h) \; if \; h \le h_0$$

And

$$\pi_t^{fC}(h) = \rho * \pi_t^{fC}(h-1) if \; h > h_0$$

where ρ is the same parameter as in the forecast method A.

So, if the concatenation of the forecast B with A occurs in the first forecast then

$$\pi_t^{fC}(1) = \pi_t^{fB}(1)$$

and

$$\pi_t^{fC}(h) = \rho * \pi_t^{fC}(h-1)$$

For all $h \geq 2$.

In this case the one step ahead forecast error is given by

$$e_t^C(1) = \pi_{t+1} - \pi_t^{fC}(1)$$

$$e_t^C(1) = \pi_{t+1} - \pi_t^{fB}(1)$$

The two step ahead forecast error is given by

$$e_t^C(2) = \pi_{t+2} - \pi_t^{fC}(2)$$

$$e_t^C(2) = \pi_{t+2} - \rho * \pi_t^{fB}(1)$$

$$e_t^C(2) = \pi_{t+2} - \rho\pi_t^{fA}(1) + \rho\pi_t^{fA}(1) - \rho * \pi_t^{fB}(1)$$

$$e_t^C(2) = e_t^A(2) + \rho(\pi_t^{fA}(1) - \pi_t^{fB}(1))$$

$$e_t^C(2) = e_t^A(2) + \rho(\pi_t^{fA}(1) - \pi_{t+1} + \pi_{t+1} - \pi_t^{fB}(1))$$

$$e_t^C(2) = e_t^A(2) + \rho(e_t^B(1) - e_t^A(1))$$

And using the fact that

$$e_t^A(2) = e_{t+1}^A(1) + \rho * e_t^A(1)$$

In the last equation we have that

$$e_t^C(2) = e_{t+1}^A(1) + \rho * e_t^A(1) + \rho(e_t^B(1) - e_t^A(1))$$

$$e_t^C(2) = e_{t+1}^A(1) + \rho * e_t^B(1)$$

Iterating forward we arrive to the following general expression

$$e_t^C(h) = \sum_{i=0}^{h-2} \rho^i * e_{t+h-1+i} + \rho^{h-1} * e_t^B(1)$$

Then, gains from concatenation exist if

$$E\left(e_t^A(h)\right)^2 - E\left(e_t^C(h)\right)^2 > 0$$

If h=1 the last expression is

$$E\left(e_t^A(1)\right)^2 - E\left(e_t^C(1)\right)^2 > 0$$

Which is the same as

$$E\left(e_t^A(1)\right)^2 - E\left(e_t^B(1)\right)^2 > 0$$

So, for the first step ahead, the concatenation method show gains only if the Mean Squared Prediction Error (MSPE) is lower for the method B.

For the two-step-ahead forecast, gains for concatenation exist if

$$E\left(e_t^A(2)\right)^2 - E\left(e_t^C(2)\right)^2 > 0$$

Replacing this last for the expression for $e_t^A(2)$ and $e_t^C(2)$,

$$E\left(e_{t+1}^A(1) + \rho * e_t^A(1)\right)^2 - E\left(e_{t+1}^A(1) + \rho * e_t^B(1)\right)^2 > 0$$

Using simple algebra and rearranging terms this can be expressed as

$$E(\rho^2 * (\, e_t^A(1)^2 - e_t^B(1)^2)) + 2 * E\left(\rho * e_{t+1}^A(1) * \left(\, e_t^A(1) - e_t^B(1)\right)\right) > 0 \quad (**)$$

This expression can be interpreted as the one-step-ahead differential between MSPE plus a covariance term between the one-step-ahead forecast error for method and the differential of one-step-ahead forecast errors between the two models in at time t.

This expression could be positive for three reasons: First term in (**) is positive, second term in (**) is positive and both terms are positive. The case in which the covariance term is positive but the first term is not, it is interesting. It is saying that one could start the sequence of forecasts worsening the one step ahead forecast. In spite of this initial worsening you get an improvement in the two step ahead forecasts due to the covariance term.

Generalizing the last expression for horizon *h* (proof in the appendix), we get

$$E\left(\rho^{2h-2} * (\, e_t^A(1)^2 - e_t^B(1)^2)\right) + 2E\left(\sum_{i=1}^{h-1} \rho^{h+i-2} * e_{t+h-i}^A(1) * (\, e_t^A(1) - e_t^B(1))\right) > 0$$

Analyzing this expression, we can see that the gains driven by the difference in MSPE are less important for long horizons and the result depend heavily on the covariance term. This implies that the gains of using a more accurate forecast could be less important if we want to forecast at longer horizons and it is far more relevant to choose a method with a positive covariance term.

IV. Empirical Illustration: Data

To illustrate the benefits from concatenation we rely on one of the conclusions of Faust and Wright (2013). They mention that judgmental forecasts are remarkably hard to beat. Accordingly, we use the Economics Expectations Survey (EES) of the Central Bank of Chile and some SARIMA models based on the work by Pincheira and García (2012) and Pincheira and Medel (2015) that pointed out that specific members of the SARIMA family perform well when forecasting year-on-year inflation in Chile and in a number of countries.

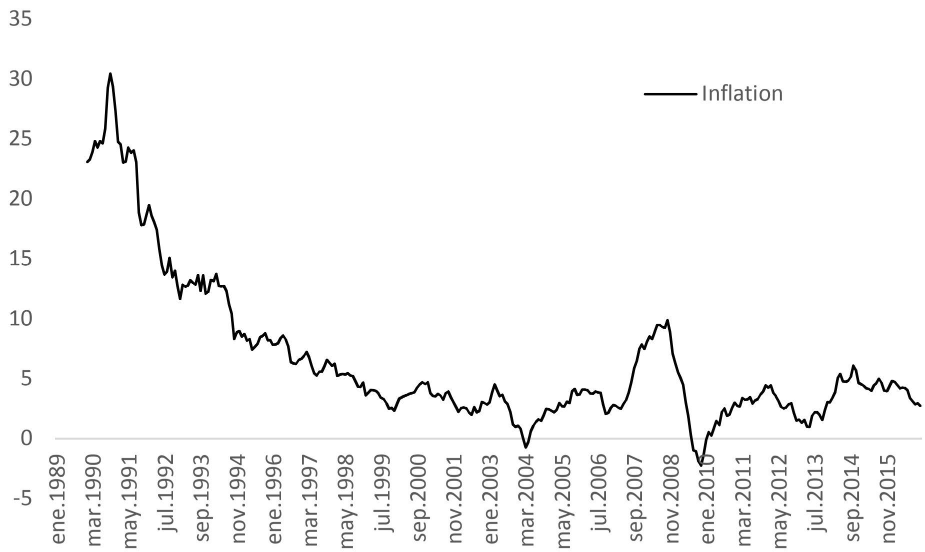
We use monthly data obtained from the Central Bank of Chile to obtain time series on inflation and time series from the Economic Expectation Survey (EES). For inflation, we construct the index using year-on-year variations of two-historic series for Consumer Price Index (CPI) with reference year 2008 and 2013. These series have been spliced to get a final series with observations from January 1990 to December 2016.

The EES is a survey made by the Central Bank of Chile at a monthly basis since February - 2000. Around 50 professionals are surveyed each month (including scholars, managers and private consultants) and asked about their forecasts for several macroeconomics variables including inflation. Until 2010, since answering the survey is voluntary, the monthly mean of entries varies from 20 to 50. An excellent introductory note to this survey is summarized in Pedersen (2010).

From the EES, we use 2 series for different time horizons, the first one forecast inflation one month ahead and the second eleven months ahead. The first is in monthly variations so we transform it to annual variations. The second is already in yearly variation so no transformation is needed. It is important to mention that the forecast in 11 months are released before the actual data on inflation is released, so it is in fact 12-steps-ahead if we count the time since the last data is available (i.e. if the announced by the EES for 11 month it is on February, then the realization for inflation in that month is not available yet and the forecast is 12-step-ahead from the last data available on January).

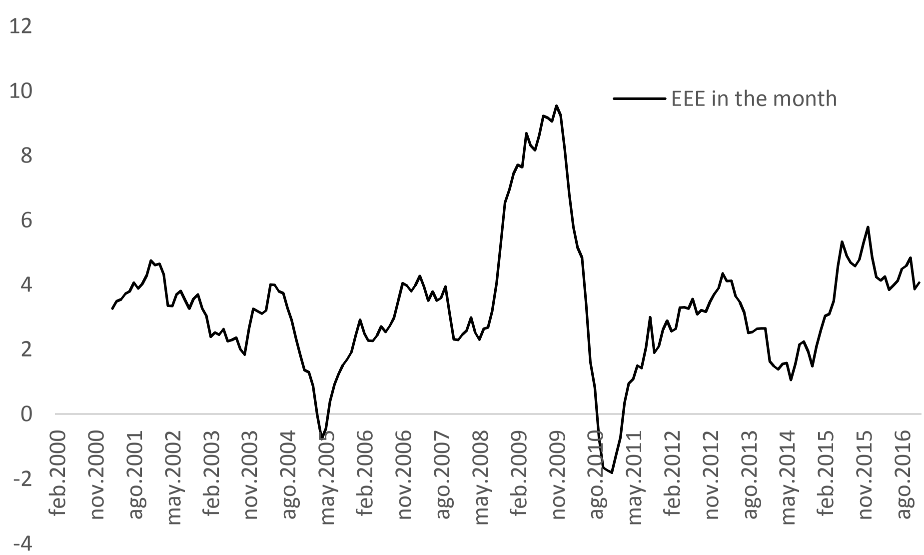
The Charts below show the time series from the year-on-year variation and the EES for the month and eleven months.

Chart 1: Year on year CPI inflation rate.



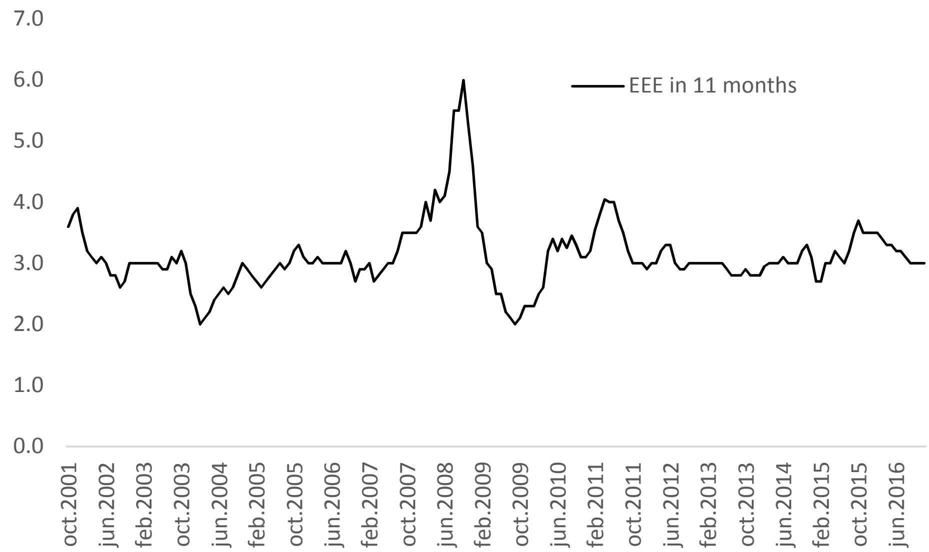
Source: Own calculations based on data from the Central Bank of Chile.

Chart 2: Survey based forecast of year on year CPI inflation rate.



Source: Own calculations based on data from the Central Bank of Chile.

Chart 3: EEE forecast for the inflation eleven months ahead.



Source: Own calculations based on data from the Central Bank of Chile.

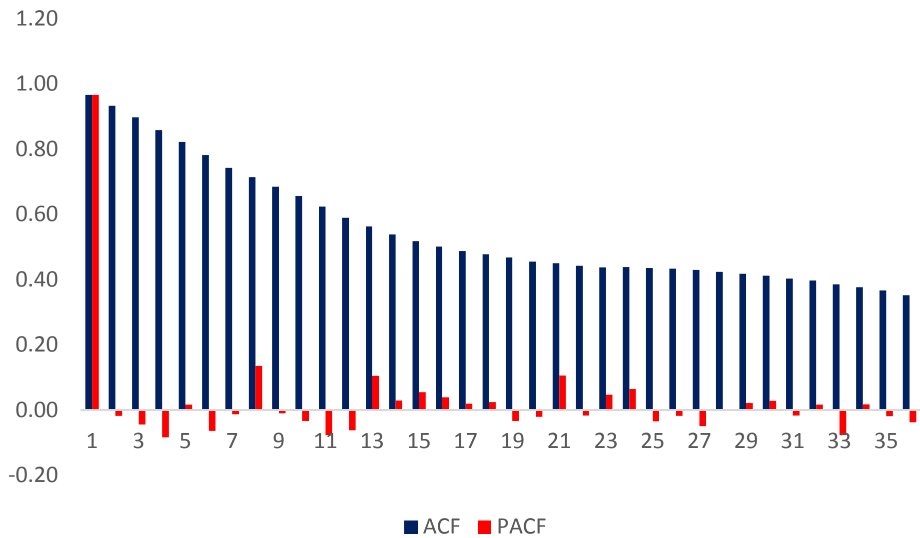
Our pseudo-out-of-sample analysis uses a rolling window approach. To maximize the use of the EES we use an initial estimation window length of 109 observations such that our first one-step-ahead forecast is made for February 2000 (the date of the first EES available), while the last one is made for December 2016. Additional to this forecast we construct 3 types of concatenation, the first is using the EES for the month, the second and third use the EES for both the month and 11 months and, in order to construct a fair comparison because the EES for 11 months start in September 2001, we change the window length to start with 128 observations, the first one-step-ahead forecast is made for September 2001 and the last is made for December 2016. All the equations were estimated with HAC Standard Errors using Newey West (1987, 1994).

V. In-sample analysis

For the in-sample analysis we use the Box and Jenkins (1976) strategy to get possible functions to forecast inflation. We choose the candidate with lower Akaike to carry out the out-of-sample exercise.

In Chart 4, we show the Autocorrelation Function (ACF) for the process and the Partial Autocorrelation Function (PACF).

Chart 4: ACF and PACF for Year on Year inflation in Chile.



Source: Own calculations based on data from the Central Bank of Chile.

Our plausible candidate models to forecast inflation are presented in the next table.

Table 1: Possible methods to forecast inflation.

N° Method	Representation
1:	$\pi_t^f(h) = \alpha + \rho_1 * \pi_t^f(h-1) + \rho_{12} * \pi_t^f(h-12)$
2:	$\Delta\pi_t^f(h) = \rho_1 * \Delta\pi_t^f(h-1) + \theta_1 * \epsilon_{t+h-1} + \theta_{12} * \epsilon_{t+h-12}$
3:	$\pi_t^f(h) = \alpha + \rho_1 * \pi_t^f(h-1) + \rho_2 * \pi_t^f(h-2) + \theta_1 * \epsilon_{t+h-1} + \theta_{12} * \epsilon_{t+h-12}$
4:	$\pi_t^f(h) = \alpha + \rho_1 * \pi_t^f(h-1) + \rho_2 * \pi_t^f(h-2) + \theta_{12} * \epsilon_{t+h-12}$
5:	$\pi_t^f(h) = \alpha + \rho_1 * \pi_t^f(h-1) + \rho_2 * \pi_t^f(h-2)$
6:	$\Delta\pi_t^f(h) = \rho_1 * \Delta\pi_t^f(h-1) + \rho_{12} * \Delta\pi_t^f(h-12)$
7:	$\Delta\pi_t^f(h) = \rho_1 * \Delta\pi_t^f(h-1) + \rho_{12} * \Delta\pi_t^f(h-12) + \theta_1 * \epsilon_{t+h-1} + \theta_{12} * \epsilon_{t+h-12} + \theta_{13} * \epsilon_{t+h-13}$
8:	$\pi_t^f(h) = \alpha + \rho_1 * \pi_t^f(h-1) + \rho_2 * \pi_t^f(h-2) + \theta_{12} * \epsilon_{t+h-12} + \theta_{13} * \epsilon_{t+h-13}$

In sample estimates for these models are displayed next:

Table 2: In-Sample Analysis of our candidate models.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Dependent Variable	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7	Model 8
constant	0.11*		0.06	0.06***	0.1*			0.06***
AR(1)	1.01***	0.54***	1.28	1.27***	1.17***	0.23***	0.97***	1.27***
AR(2)			-0.30	-0.29***	-0.19**			-0.29***
SAR(12)	-0.04**					-0.47***		
MA(1)		0	0				-0.88***	
SMA(12)		-0.97***	-0.97***	-0.97***			-0.97***	-0.97***
SMA(13)							0.86***	0
R-squared	0.99	0.50	0.99	0.99	0.99	0.32	0.54	0.99
N	311	311	311	311	311	311	311	311
Akaike Criterion	1.80	1.21	1.02	1.01	1.79	1.51	1.12	1.02
Schwarz Criterion	1.84	1.25	1.08	1.06	1.83	1.53	1.17	1.08
Durbin-Watson	1.64	1.23	2.04	2.04	1.98	1.52	1.14	2.04

Source: Own calculations based on data from the Central Bank of Chile. * p<10%, ** p<5%, *** p<1%. HAC standard errors according to Newey & West (1987) in parentheses. Each parameter corresponds to its analog in Table 1.

The models we use in the out of sample analysis are models 1 and 3. The first one is chosen because it is similar to our mathematical analysis and the SAR term allows further concatenation gains at long horizons. Model 3 was chosen on the basis of the Akaike criterion.

VI. Predictive Evaluation Strategy

In our baseline exercise, we evaluate the out of sample predictive ability of the standard models against their concatenated versions. We construct 3 types of concatenated forecasts. The first is based on a concatenation in the first-step-ahead with the second forecast method (i.e. Economic surveys), so the forecast are given by

$$\pi_t^f(1) = \pi_t^E(1)$$

$$\pi_t^f(2) = \alpha + \rho_1 * \pi_t^f(1) + \rho_{12} * \pi_{t-10}$$

$$\vdots$$

$$\pi_t^f(12) = \alpha + \rho_1 * \pi_t^f(11) + \rho_{12} * \pi_t$$

$$\pi_t^f(13) = \alpha + \rho_1 * \pi_t^f(12) + \rho_{12} * \pi_t^f(1)$$

$$\vdots$$

$$\pi_t^f(24) = \alpha + \rho_1 * \pi_t^f(23) + \rho_{12} * \pi_t^f(12)$$

where $\pi_t^f(h)$ is the h-step-ahead forecast using information available at time t and $\pi_t^E(h)$ is the h-step-ahead forecast with the second method using information available at time t.

The second forecast uses not only the first-step-ahead but another exogenous forecast for the 12-steps-ahead concatenation with the second forecast method. So, we get

$$\pi_t^f(1) = \pi_t^E(1)$$

$$\pi_t^f(2) = \alpha + \rho_1 * \pi_t^f(1) + \rho_{12} * \pi_{t-10}$$

$$\vdots$$

$$\pi_t^f(12) = \alpha + \rho_1 * \pi_t^f(11) + \rho_{12} * \pi_t$$

$$\pi_t^f(13) = \alpha + \rho_1 * \pi_t^E(12) + \rho_{12} * \pi_t^f(1)$$

$$\vdots$$

$$\pi_t^f(24) = \alpha + \rho_1 * \pi_t^f(23) + \rho_{12} * \pi_t^f(12)$$

And the third forecast, as the second, uses the first and the 12-steps ahead from the second method, but the difference between both is that in the third method we use the 12-step-ahead prediction in the same way that we use the first one. So

$$\pi_t^f(1) = \pi_t^E(1)$$

$$\pi_t^f(2) = \alpha + \rho_1 * \pi_t^f(1) + \rho_{12} * \pi_{t-10}$$

$$\vdots$$

$$\pi_t^f(12) = \pi_t^E(12)$$

$$\pi_t^f(13) = \alpha + \rho_1 * \pi_t^f(12) + \rho_{12} * \pi_t^f(1)$$

$$\vdots$$

$$\pi_t^f(24) = \alpha + \rho_1 * \pi_t^f(23) + \rho_{12} * \pi_t^f(12)$$

To evaluate the predictive ability, we perform the test attributed to Diebold & Mariano (1995) and West (1996) (hereafter DMW) to do a horse race between the standard model and the concatenated version. To do so we split the database and estimate the models in rolling windows of fixed size, then generate the first h-step-ahead forecasts and get the sample RMSPE (SRMSPE) and the sample MSPE.

To describe this test, let us suppose we have a data set of $\pi_t$with T+1 observations and that we want to determine if model 2 has better forecasting performance than model 1. We can use the first R observations of the data set to estimate both models. Then, with this estimation, we can build multi-step ahead forecasts from R+1 to R+24 and obtain forecast errors according to:

$$\pi_R^f(h) - \pi_{R+h} = e_R(h)$$

Where $h = 1, \dots, 24$, and as before $\pi_R^f(h)$ represent the h-step-ahead forecast using information available at time R and π_{R+h} is the actual value of inflation in the period *R+h*.

Next we update our estimation sample to start with the second observation and to finish with observation R+1. We repeat this process with this fixed window scheme until we reach the last possible rolling window of R observations between T+1-R and T+1. By doing so, we generate a total of $P(h)$ forecast for each horizon with $P(h)$ satisfying $R + (P(h) - 1) + h = T + 1$. So

$$P(h) = T + 2 - h - R$$

To measure forecast accuracy with RMSPE we use the sample analog of this population moment. This is:

$$SRMSPE = \sqrt{\frac{1}{P(h)} \sum_{t=R}^{T+1-h} e_R(h)^2}$$

For the DMW test, we use the difference in sample MSPE for the models $i = 1, 2$. This is

$$\bar{d} = SMSPE_i - SMSPE_j$$

with

$$SMSPE_i = \frac{1}{P(h)} \sum_{t=R}^{T+1-h} e_{Ri}(h)^2$$

Then the DMW t-statistic test is:

$$DM = \frac{\bar{d}}{\sqrt{\hat{V}^*}}$$

Where $\hat{V}^*$is an estimator from the long run variance of $\bar{d}$.

And the null hypothesis is

$$H_0: MSPE_1 - MSPE_2 \leq 0$$

Against the alternative

$$H_A: MSPE_1 - MSPE_2 > 0$$

As a practical matter, a simple way to proceed is to regress d_i on a constant and use a t-test with Newey and West (1987, 1994) Standard Errors to determine whether the constant is statistically significant.

VII. Out-of-Sample Results

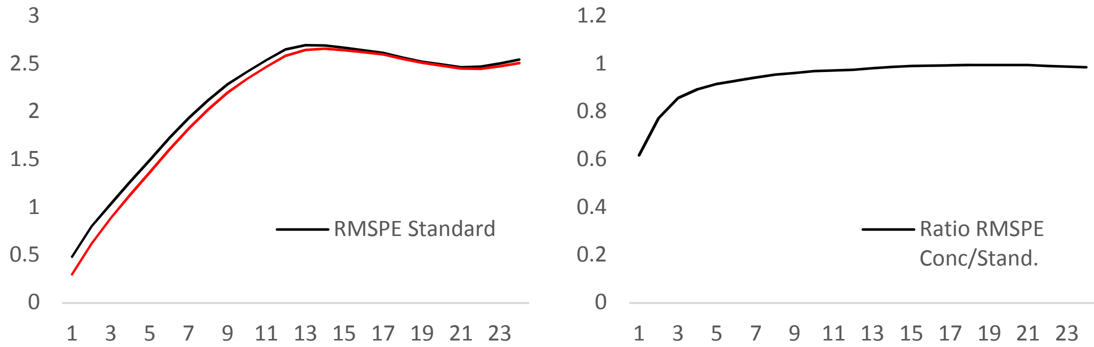
In this section, we present results from the DMW test for two forecast models, each model has been concatenated with the Survey of Economic Expectations using the 3 different types of concatenation presented in the previous section.

The forecast models were chosen using the Box & Jenkins (1976) strategy, and the DMW test was used to compare forecast accuracy between the two prediction models (standard models) and three possible concatenations for these models

Table 3: Standard Forecasting models

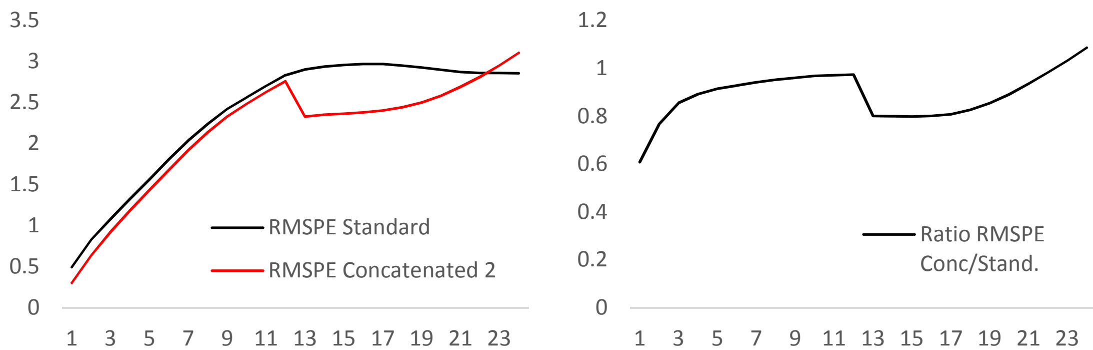
1:	$\pi_t^f(h) = \alpha + \rho_1 * \pi_t^f(h-1) + \rho_{12} * \pi_t^f(h-12)$
2:	$\pi_t^f(h) = \alpha + \rho_1 * \pi_t^f(h-1) + \rho_2 * \pi_t^f(h-2) + \theta_1 * \epsilon_{t+h-1} + \theta_{12} * \epsilon_{t+h-12}$

Chart 5: RMSPE and the RMSPE ratio between model 1 and the first concatenated forecast.



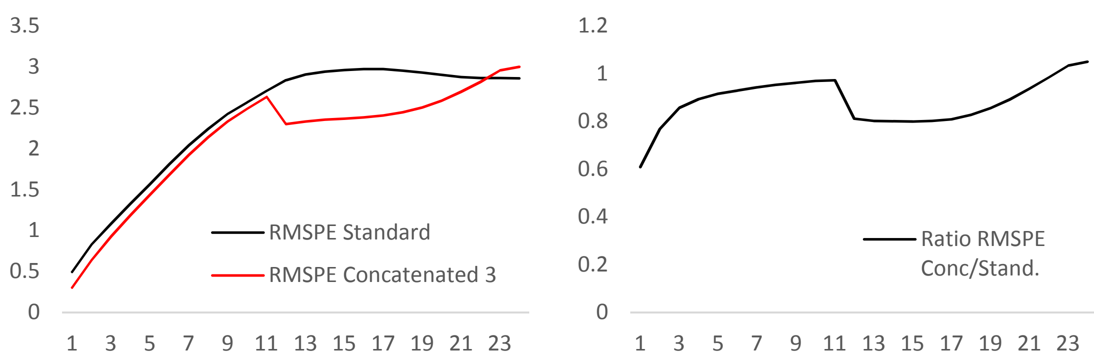
Source: Own calculations based on data from the Central Bank of Chile.

Chart 6: RMSPE and the ratio from forecast model 1 and second concatenated forecast.



Source: Own calculations based on data from the Central Bank of Chile.

Chart 7: RMSPE and the ratio from forecast model 1 and the third concatenated forecast.



Source: Own calculations based on data from the Central Bank of Chile.

Table 4: DMW test when comparing forecasting accuracy for model 1, horizons 1 to 6.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
		DMW and RMSPE					
		Forecast					
Model	DMW/RMSPE	h=1	h=2	h=3	h=4	h=5	h=6
Model 1	DMW	0,14***	0,26***	0,28***	0,32***	0,36***	0,4***
	RMSPE	0,30	0,62	0,89	1,13	1,37	1,60
	RMSPE benchmark	0,48	0,80	1,04	1,27	1,49	1,72
Model 2	DMW	0,15***	0,28***	0,31***	0,36***	0,39***	0,45***
	RMSPE	0,30	0,64	0,93	1,19	1,43	1,68
	RMSPE benchmark	0,50	0,83	1,08	1,33	1,57	1,81
Model 3	DMW	0,15***	0,28***	0,31***	0,36***	0,39***	0,45***
	RMSPE	0,30	0,64	0,93	1,19	1,43	1,68
	RMSPE benchmark	0,50	0,83	1,08	1,33	1,57	1,81

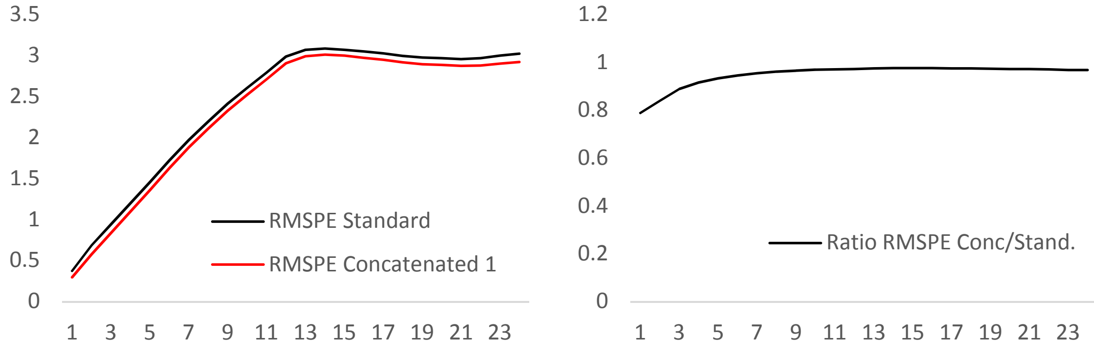
Source: Own calculations based on data from the Central Bank of Chile data. * p<10%, ** p<5%, *** p<1%. HAC standard errors according to Newey and West (1987) in parentheses.

Table 5: DMW test when comparing forecasting accuracy for model 1, horizons 12 to 17.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
		DMW and RMSPE					
Forecast							
Model	DMW/RMSPE	h=12	h=13	h=14	h=15	h=16	h=17
Model 1	DMW	0,35**	0,27**	0,19*	0,13	0,11	0,09
	RMSPE	2,58	2,65	2,66	2,64	2,62	2,60
	RMSPE benchmark	2,65	2,70	2,69	2,67	2,64	2,61
Model 2	DMW	0,4**	3,01**	3,12***	3,16**	3,15**	3,05**
	RMSPE	2,76	2,33	2,35	2,37	2,38	2,40
	RMSPE benchmark	2,83	2,91	2,94	2,96	2,97	2,97
Model 3	DMW	2,74**	3,01**	3,12***	3,16**	3,15**	3,05**
	RMSPE	2,30	2,33	2,35	2,37	2,38	2,40
	RMSPE benchmark	2,83	2,91	2,94	2,96	2,97	2,97

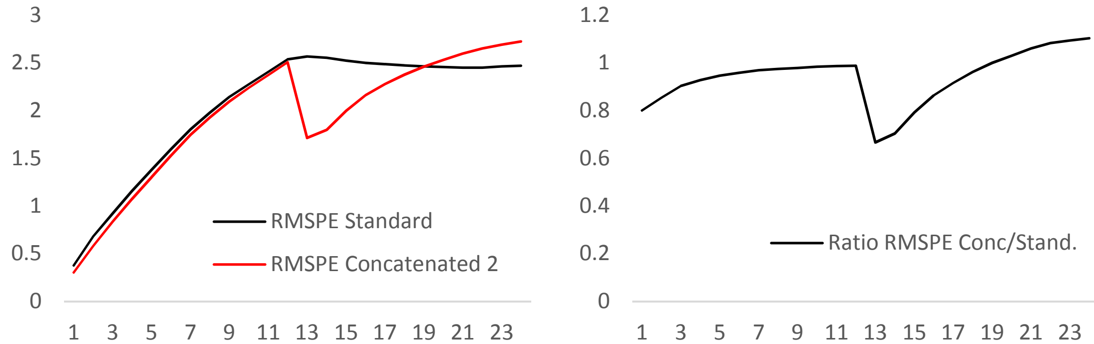
Source: Own calculations based on data from the Central Bank of Chile. * p<10%, ** p<5%, *** p<1%. HAC standard errors according to Newey and West (1987) in parentheses.

Chart 8: RMSPE and the ratio from forecast model 2 and first concatenated forecast.



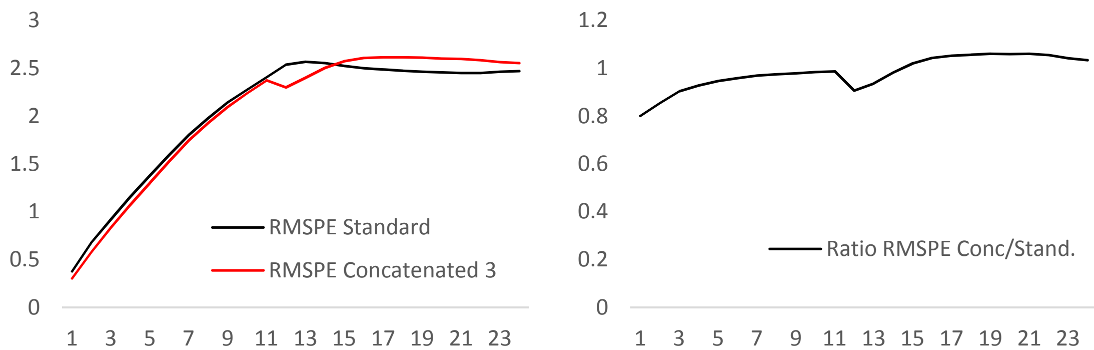
Source: Own calculations based on data from the Central Bank of Chile.

Chart 9: RMSPE and the ratio from forecast model 2 and second concatenated forecast.



Source: Own calculations based on data from the Central Bank of Chile.

Chart 10: RMSPE and the ratio from forecast model 2 and third concatenated forecast.



Source: Own calculations based on Central Bank of Chile data.

Table 6: DMW test when comparing forecasting accuracy for model 2, horizons 1 to 6.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
DMW and RMSPE							
Forecast							
Model	DMW/RMSPE	h=1	h=2	h=3	h=4	h=5	h=6
Model 1	DMW	0,05***	0,14***	0,19***	0,23***	0,28***	0,32***
	RMSPE	0,30	0,62	0,89	1,13	1,37	1,60
	RMSPE benchmark	0,48	0,80	1,04	1,27	1,49	1,72
Model 2	DMW	0,05***	0,12***	0,16***	0,19***	0,2***	0,21***
	RMSPE	0,30	0,64	0,93	1,19	1,43	1,68
	RMSPE benchmark	0,50	0,83	1,08	1,33	1,57	1,81
Model 3	DMW	0,05***	0,12***	0,16***	0,19***	0,2***	0,21***
	RMSPE	0,30	0,64	0,93	1,19	1,43	1,68
	RMSPE benchmark	0,50	0,83	1,08	1,33	1,57	1,81

Source: Own calculations based on Central Bank of Chile data. * p<10%, ** p<5%, *** p<1%. HAC standard errors according to Newey and West (1987) in parentheses.

Table 7: DMW test when comparing forecasting accuracy for model 2, horizons 12 to 17.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
DMW and RMSPE							
Forecast							
Model	DMW/RMSPE	h=12	h=13	h=14	h=15	h=16	h=17
Model 1	DMW	0,5**	0,47*	0,44*	0,44*	0,45*	0,47*
	RMSPE	2,58	2,65	2,66	2,64	2,62	2,60
	RMSPE benchmark	2,65	2,70	2,69	2,67	2,64	2,61
Model 2	DMW	0,15***	3,66***	3,29**	2,37*	1,59	1
	RMSPE	2,76	2,33	2,35	2,37	2,38	2,40
	RMSPE benchmark	2,83	2,91	2,94	2,96	2,97	2,97
Model 3	DMW	1,15**	0,83	0,25	-0,25	-0,55	-0,66
	RMSPE	2,30	2,33	2,35	2,37	2,38	2,40
	RMSPE benchmark	2,83	2,91	2,94	2,96	2,97	2,97

Source: Own calculations based on data from the Central Bank of Chile. * p<10%, ** p<5%, *** p<1%. HAC standard errors according to Newey and West (1987) in parentheses.

It is worth noticing that, for the prediction model 1, the first type of concatenation shows gains in accuracy relative to the standard model until 7-steps-ahead at the 1% significance level, until 13-steps-ahead at the 5% significance level and until 14-steps-ahead at the 10% significance level. Similar results are obtained for models 2 and 3 as well and when using the second and third concatenating strategies. The most important difference between the concatenation 2 and 3 is that for the 12-steps-ahead forecast, the third concatenation shows a greater difference in RMSPE relative to its standard counterpart, but the loss of RMSPE is accompanied with more variance. In summary, by concatenating our forecasting models it is possible to obtain substantial improvements in terms of accuracy measured by the DMW test relative to the same models without concatenation.

VIII. Concluding Remarks

In this paper, we provide an in-depth analysis of a strategy to construct a new sequence of multistep ahead forecasts based on the concatenation of two existing ones. In a nutshell, when two sets of multistep ahead forecasts are available for the same target variable, and one of these sets is generated using a recursive strategy, it is possible to generate a third sequence of multistep ahead forecasts using the one-step-ahead forecast of method A, and the recursive strategy of method B. In other words we "feed" the sequence of multistep ahead forecasts of method B with a different one-step ahead forecast

We show that concatenation may reduce the MSPE of the forecasts via two different sources: the first is the existence of a method with higher accuracy at short horizons. This is quite obvious as it is intuitive to expect higher accuracy even at longer horizons by improving the accuracy of your starting point. The second source of accuracy is related to an intertemporal covariance term. The intuition is that the two forecasting methods that are being concatenated tend to generate forecast errors in opposite directions. If this is so, long term forecasts of the concatenation strategy may benefit because short horizons errors in one direction are offset by long horizons errors in the opposite direction.

The first source is important because the accuracy of one forecast could be used to forecast at longer horizons even when the method is only available at short horizons (e.g. judgmental forecast). The second factor takes more importance in the long run and it is also important because it shows that this method could be useful even when there is not a clear short run winner in terms of forecast accuracy.

We illustrate our findings with an empirical application aimed at improving the forecast accuracy of inflation in Chile by concatenating the Economic Expectations Survey (EES) and univariate methods in two different ways. First, we concatenate the first step ahead forecast of the EES with different univariate forecasts and then we concatenate forecasts in the first and 12 steps ahead with the same univariate methods. We show that gains could be achieved at both short and long horizons in the two options (at least for 12-steps-ahead) and these gains are statistically significant in a pseudo-out-of-sample scheme.

Our finding is that Concatenation is a good alternative to make forecast for inflation in Chile. In further research, we hope to find the same gains when forecasting other relevant variables in an international context.

References

1. Andrade, P, R K Crump, S Eusepi, and E Moench (2014), "Fundamental disagreement", FRBNY Staff Report 655.
2. Bates, J. M., & Granger, C. W. (1969). The combination of forecasts. *Operations Research*, 451-468.
3. Box, G. E., & Jenkins, G. M. (1976). *Time series analysis: forecasting and control, revised ed*. Holden-Day.
4. Diebold, F. X., & Mariano, R. S. (1995). Comparing Predictive Accuracy. *Journal of Business & Economic Statistics*, 253-263.
5. Faust, J., & Wright, J. H. (2013). Forecasting inflation. *Handbook of economic forecasting*, *2*(Part A), 3-56.
6. Hansen, B. E. (2008). Least-squares forecast averaging. Journal of Econometrics, 146(2), 342-350.
7. Newey, W., & West, K. (1987). A Simple, Positive Semi-Definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix. *Econometrica, 55*(3), 703-708.
8. Patton, Andrew J., and Allan Timmermann (2010) "Why Do Forecasters Disagree? Lessons from the Term Structure of Cross-Sectional Disagreement," Journal of Monetary Economics, 57:7 (2010), pp. 803-820.
9. Pedersen, M. (2010). Una nota introductoria a la Encuesta de Expectativas Económicas. *Estudios Económicos Estadísticos, Banco central de Chile.*
10. Pincheira, P. M. (2012). Are Forecast Combinations Efficient? Working Paper N° 661, Central Bank of Chile.
11. Pincheira, P., & A. García, (2012). En busca de un buen marco de referencia predictivo para la inflación en Chile. *El Trimestre Económico*, 79(313), 85-123.
12. Pincheira, P. & C. A. Medel (2015). "Forecasting Inflation with a Simple and Accurate Benchmark: The Case of the US and a Set of Inflation Targeting Countries," *Czech Journal of Economics and Finance (Finance a uver)*, Charles University Prague, Faculty of Social Sciences, vol. 65(1), pages 2-29, January.
13. Stock, J. H., & Watson, M. W. (2004). Combination forecasts of output growth in a seven-country data set. *Journal of Forecasting*, *23*(6), 405-430.
14. Timmermann, A. (2006). Forecast combinations. *Handbook of economic forecasting*, *1*, 135-196.

Appendix

Proof from section 3.

$$E\left(\rho^{2h-2} * (\ e_t^A(1)^2 - e_t^B(1)^2)\right) + 2E\left(\sum_{i=1}^{h-1} \rho^{h+i-2} * e_{t+h-i}^A(1) * (\ e_t^A(1) - e_t^B(1))\right) > 0$$

To solve this expression, we use the iterative method to get the general expression

For h=1

$$E\left(e_t^A(1)\right)^2 - E\left(e_t^C(1)\right)^2 > 0$$

And because we are concatenating for the first-step-ahead this is equal to

$$E\left(e_t^A(1)\right)^2 - E\left(e_t^B(1)\right)^2 > 0$$

For h=2

$$E\left(e_t^A(2)\right)^2 - E\left(e_t^C(2)\right)^2 > 0$$

Replacing this last for the expression for $e_t^A(2)$ and $e_t^C(2)$,

$$E\left(e_{t+1}^A(1) + \rho * e_t^A(1)\right)^2 - E\left(e_{t+1}^A(1) + \rho * e_t^B(1)\right)^2 > 0$$

$$E(e_{t+1}^A(1)^2 + 2 * e_{t+1}^A(1) * \rho * e_t^A(1) + \rho^2 * e_t^A(1)^2) \\ - E\left(e_{t+1}^A(1)^2 + 2 * e_{t+1}^A(1) * \rho * e_t^B(1) + \rho^2 * e_t^B(1)\right) > 0$$

Then taking common factors and eliminating repeated terms we get

$$E\left(\rho^2 * (e_t^A(1)^2 - e_t^B(1)^2)\right) + 2 * E\left(\rho * e_{t+1}^A(1) * \left(e_t^A(1) - e_t^B(1)\right)\right) > 0$$

For h=3

$$E\left(e_t^A(3)\right)^2 - E\left(e_t^C(3)\right)^2 > 0$$

Replacing

$$E\left(e_{t+2}^A(1) + \rho * e_{t+1}^A(1) + \rho_1^2 * e_t^A(1)\right)^2 - E\left(e_{t+2}^A(1) + \rho * e_{t+1}^A(1) + \rho^2 * e_t^B(1)\right)^2 > 0$$

Operating the polynomial we get

$$E\left(e_{t+2}^{A}(1)^2+\rho^2*e_{t+1}^{A}(1)^2+\rho^4*e_{t}^{A}(1)^2+2*e_{t+2}^{A}(1)*\rho*e_{t+1}^{A}(1)+2*e_{t+2}^{A}(1)*\rho^2*e_{t}^{A}(1)+2*\rho^3*e_{t+1}^{A}(1)*e_{t}^{A}(1)\right)^2-E\left(e_{t+2}^{A}(1)^2+\rho^2*e_{t+1}^{A}(1)^2+\rho^4*e_{t}^{B}(1)^2+2*e_{t+2}^{A}(1)*\rho*e_{t+1}^{A}(1)+2*e_{t+2}^{A}(1)*\rho^2*e_{t}^{B}(1)+2*\rho^3*e_{t+1}^{A}(1)*e_{t}^{B}(1)\right)^2>0$$

Then re arranging terms.

$$E\left(\rho^4*(e_{t}^{A}(1)^2-e_{t}^{B}(1)^2)\right)+2*E\left(\rho^2*e_{t+2}^{A}(1)*\left(e_{t}^{A}(1)-e_{t}^{B}(1)\right)+\rho^3*e_{t+1}^{A}(1)*\left(e_{t}^{A}(1)-e_{t}^{B}(1)\right)\right)>0$$

$$E\left(\rho^4*(e_{t}^{A}(1)^2-e_{t}^{B}(1)^2)\right)+2*E\left(\left(\rho^2*e_{t+2}^{A}(1)+\rho^3*e_{t+1}^{A}(1)\right)*\left(e_{t}^{A}(1)-e_{t}^{B}(1)\right)\right)>0$$

Then the solution for any h-step-ahead is:

$$E\left(\rho^{2h-2}*(\,e_{t}^{A}(1)^2-e_{t}^{B}(1)^2)\right)+2E\left(\sum_{i=1}^{h-1}\rho^{h+i-2}*e_{t+h-i}^{A}(1)*(\,e_{t}^{A}(1)-e_{t}^{B}(1))\right)>0$$