Macroeconomic Forecasting in Small Open Economies Using Dynamic Factor Models

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Abstract

This paper proposes a methodology for macroeconomic forecasting in a small open economy environment based on dynamic factor models (DFMs). Unlike in existing forecasting applications of DFMs, the suggested approach exploits the basic theoretical restriction that external variables can be taken as exogenous for domestic variables in a small open economy. The approach is applied to forecast real GDP growth and consumer price inflation in Chile using a large dataset of domestic and external time series. Out-of-sample forecast comparisons suggest considerable gains in terms of forecast accuracy with respect to an unrestricted model, especially for medium-term real GDP growth forecasts.

Keywords: Business cycles; Econometric models; Inflation forecasting; Multivariate time series; Open-economy forecasting.

JEL classification: C32; C51; C52; C53; E37.

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1 Introduction

In this paper we propose a methodology for macroeconomic forecasting in a small open economy environment based on dynamic factor models (DFMs). DFMs are often employed for macroeconomic forecasting (see Eickmeier and Ziegler, 2008). These models allow to capture the dynamics of a large number of variables through a few latent factors. Thereby, the number of estimated parameters does not increase as quickly with the number of variables as in other multivariate time series frameworks, implying potential gains in terms of forecast accuracy. However, most existing applications of DFMs for forecasting in small open economies do not impose the basic theoretical restriction that foreign variables can be taken as (block) exogenous for domestic variables in a small open economy, and might thus leave additional accuracy gains unexploited. We therefore aim to assess the benefits of explicitly incorporating this kind of small open economy restrictions through an application of the proposed methodology for Chile.

The application focuses on forecasting quarterly real gross domestic product (GDP) growth and consumer price index (CPI) inflation in Chile. Our dataset starts with the introduction of the Chilean inflation targeting monetary framework in 2001, and includes a broad set of macroeconomic indicators with 110 domestic and 63 international variables. These variables are modelled using a DFM. Only 3 domestic and 3 foreign factors are sufficient to explain about 50% of the variability of the underlying indicators. However, while the unrestricted version of our specified DFM would have more than 1,000 parameters to be estimated, the small open economy restrictions that we impose allow to reduce the number of parameters by about 200 and thereby yields a significantly more parsimonious structure.

Our benchmark exercise consists of comparing the restricted DFM with its unrestricted version alongside several simple univariate benchmark models in terms of their out-of-sample forecasting performance. In particular, we conduct recursive regressions and compute the implied forecasts up to 8 quarters ahead and associated forecast errors against the ex-post data in a pseudo real-time setup. We find that the restricted DFM significantly outperforms the unrestricted model in terms of forecast accuracy for real GDP growth forecasts, according to standard statistical tests on the root mean squared errors for the point forecasts, and to a lesser extent for CPI inflation. In addition, the restricted DFM generates significantly smaller errors than the univariate benchmarks for real GDP forecasts at medium-term horizons, while it produces similar forecast errors as the benchmarks for CPI inflation. An analysis of the forecast errors for other relevant variables suggests that the improved real GDP forecasts are associated with better forecasts particularly for aggregate demand components and employment. The analysis is further complemented by a number of alternative exercises, including a more rigorous real-time exercise using vintages of GDP and sample-by-sample seasonal adjustment. The overall results from the benchmark exercise are maintained in those alternative exercises.

Several previous studies have applied DFM frameworks and other types of multivariate factor models for macroeconomic forecasting in small open economies; see, for instance, Marcellino, Stock, and Watson (2003) for several euro area countries, Brisson, Campbell, and Galbraith (2003), Cheung and Demers (2007) and Gosselin and Tkacz (2010) for Canada, Camacho and Sancho (2003) for Spain, Artis, Banerjee, and Marcellino (2005) for the United Kingdom, Ferreira, Bierens, and Castelar (2005) for Brazil, Matheson (2006) and Giannone and Matheson (2007) for New Zealand, Moser, Rumler, and Scharler (2007) for Austria, Bandt, Michaux, Bruneau, and Flageollet (2007) for France, Chow and Choy (2009) for Singapore, Gupta and Kabundi (2010) for South Africa, Martinsen, Ravazzolo, and Wulfsberg (2014) for Norway, and Aguirre and Céspedes (2004) and Echavarría and González (2011) for Chile. However, while theoretical small open economy (i.e., block exogeneity) restrictions have often been applied in studies based on structural factor-augmented vector autoregressive (FAVAR) models (see, e.g., Boivin and Giannoni, 2007; Charnavoki and Dolado, 2014; Kamber, Karagedikli, Ryan, and Vehbi, 2016), to our knowledge none of the available studies on forecasting has explored the usefulness of block exogeneity restrictions in a small open economy environment, as we do in this paper.

The result that (restricted) DFMs can be useful devices for macroeconomic forecasting at medium-term horizons, even in a relatively short sample such as ours, is especially relevant for a small open emerging economy with an inflation targeting monetary framework such as Chile. In particular, the Central Bank of Chile (CBCH) is conducting forecasts of real GDP and inflation up to a two-year horizon, which is the maximum period during which the CBCH normally attempts to take the projected annual consumer price inflation rate back to 3% (CBCH, 2007), an operational definition of price stability that is paralleled (with different specified targets and policy horizons) in many other small open economies (see Hammond, 2012). Reliable forecasts of macroeconomic variables are therefore a key requirement for a successful implementation of inflation targeting frameworks. However, given the relatively short history available for time series in many emerging economies compared to most advanced economies and the relatively recent adoption of inflation targeting regimes in most cases, it seems important that the models used for forecasting in such an environment are specified in a parsimonious way, for instance, by exploiting potentially useful theoretical (e.g., small open economy) restrictions in popular forecasting models such as DFMs. Our results support that hypothesis for the case of Chile.

The rest of the paper is structured as follows. Section 2 describes the model and its estimation. Section 3 presents our empirical application and results. Finally, Section 4 concludes.

2 Econometric Model

In this section we describe the dynamic factor model with block exogeneity restrictions and its estimation. The model assumes that the latent factor dynamics follow a vector autoregressive (VAR) process and that the (explicit) relationship between the latent factors and the observed set of macroeconomic indicators is contemporaneous. Since our goal is to produce forecasts for a small open economy, we consider two blocks: domestic and external. We impose a structure that allows spillover effects from the external block onto the domestic block, but not vice-versa.

2.1 The Dynamic Factor Model (DFM) with block exogeneity restrictions

Let X_t^D represent an $N^D \times 1$ vector of domestic macroeconomic time series (domestic block) and X_t^E an $N^E \times 1$ vector of external macroeconomic time series (external block). Data is available for $t = 1, ..., T$, and all elements in X_t^D and X_t^E are assumed to be standardized and properly differentiated, such that they are rendered to stationary processes. Also, let the information in X_t^D and X_t^E be summarized by the latent factors F_t^D and F_t^E, respectively, where F_t^D is a $K^D \times 1$ vector of domestic factors and F_t^E is a $K^E \times 1$ vector of external factors. The dimensions of F_t^D and F_t^E are such that $K^D \ll N^D$ and $K^E \ll N^E$. We assume that X_t^D and X_t^E admit a factor model representation given by:

$$\begin{bmatrix} X_t^E \\ X_t^D \end{bmatrix} = \begin{bmatrix} \Lambda^{EE} & 0 \\ \Lambda^{ED} & \Lambda^{DD} \end{bmatrix} \begin{bmatrix} F_t^E \\ F_t^D \end{bmatrix} + \begin{bmatrix} e_t^E \\ e_t^D \end{bmatrix}, \tag{1}$$

where Λ^{EE}, Λ^{ED} and Λ^{DD} are conformable matrices of factor loadings, and the idiosyncratic innovations in e_t^E and e_t^D are allowed to follow univariate autoregressions as in Stock and Watson (2010):

$$\alpha_i^j (L)\, e_{i,t}^j = \gamma^j + v_{i,t}^j, \qquad \forall i = 1, \ldots, N^j, \tag{2}$$

where $j \in \{E, D\}$, $\alpha_i^j (L)$ are conformable lag polynomials of finite order, γ^j are constants, and $v_{i,t}^j$ are i.i.d. disturbances such that $v_t \sim N(0, \Sigma_v)$ with Σ_v diagonal. The presence (and structure) of serial correlation in the idiosyncratic innovations of (1) will be determined on a case-by-case basis, using standard univariate serial correlation tests. This type of specification can be thought of as a version of the exact static factor model of Connor and Korajczyk (1986), which allows for serial correlation in the error terms of the observation equation.

We further assume that the joint dynamics of the latent factors are given by the following reduced-form VAR:

$$\begin{bmatrix} \beta\left(L\right)^{EE} & 0 \\ \beta\left(L\right)^{ED} & \beta\left(L\right)^{DD} \end{bmatrix} \begin{bmatrix} F_t^E \\ F_t^D \end{bmatrix} = \begin{bmatrix} \delta^E \\ \delta^D \end{bmatrix} + \begin{bmatrix} \varepsilon_t^E \\ \varepsilon_t^D \end{bmatrix}, \tag{3}$$

where $\beta(L)^{EE}$, $\beta(L)^{ED}$ and $\beta(L)^{DD}$ are conformable lag polynomials of finite order p, δ^E and δ^D are constant terms, and the reduced-form innovations ε_t^E and ε_t^D may be cross-correlated, such that $[\varepsilon_t^{E'}, \varepsilon_t^{D'}]' \sim N(0, \Sigma_\varepsilon)$. Finally, we assume that $E[e_t \varepsilon_{t-s}^{'}] = 0 \; \forall s$.

Equations (1), (2) and (3) represent our DFM framework, which we will use to construct forecasts for the Chilean economy. The specification of (1) ensures that we capture the contemporaneous effects that the external factors exert upon the domestic block of observables through the matrix Λ^{ED}. In turn, the block of zeros in the upper right region of the factor loadings matrix reflects our small open economy assumption, imposing that the evolution of the domestic factors has no impact on the external block of observables. Similarly, the specification of (3) ensures that there is no dependence of the external factors with respect to the domestic ones. The imposition of block exogeneity restrictions on both equations (i.e., simultaneously) eliminates any potential (spurious) effects from past domestic factors onto future external observables.

Most of the available literature for small open economies that employs DFMs with block exogeneity restrictions such as in (1) and (3) addresses structural impulse response analysis based on identified FAVARs. However, the imposition of block exogeneity restrictions may also be useful in a forecasting application of DFMs: by focusing on a small open economy we may assume absence of spillovers (both past and present) from the domestic factors onto the external factors, and from the domestic factors onto the external block of macroeconomic indicators. This should result in efficiency gains for the parameter estimates, due to the associated dimensionality reduction of the econometric problem. On the other hand, the risk of potential model misspecification (and thus presence of biased and inconsistent estimates) seems relatively low given the factual nature of the small open economy assumption.

2.2 Estimation

We now discuss the main issues regarding parameter estimation involving (1), (2) and (3). For this, it is useful to define $X^E = [X_1^{E'}, \ldots, X_T^{E'}]'$ as the $T \times N^E$ matrix of external time series, $F^E = [F_1^{E'}, \ldots, F_T^{E'}]'$ as the $T \times K^E$ matrix of external factors, $X^D = [X_1^{D'}, ..., X_T^{D'}]'$ as the $T \times N^D$ matrix of domestic time series, and $F^D = [F_1^{D'}, ..., F_T^{D'}]'$ as the $T \times K^D$ matrix of domestic factors. Furthermore, for the version of the model without block exogeneity restrictions, to which we will refer to as unrestricted DFM, we define a $T \times N$ matrix $X = [X_1', \ldots, X_T']'$ including the full set of external and domestic observables in $X_t = [X_t^{E'}, X_t^{D'}]'$ for $t = 1, \ldots, T$ (with $N = N^E + N^D$), and accordingly a $T \times K$ factor matrix $F = [F_1', ..., F_T']'$ and an unrestricted $N \times K$ matrix of factor loadings Λ.

2.2.1 Identification

The model defined by (1), (2) and (3) is econometrically not identified and thus cannot be estimated. To see this, without loss of generality let $\{\hat{\mathit{\Lambda}}, \hat{F}_t\}_{t=1}^T$ be a solution to the estimation of the unrestricted version of (1). This implies that there always exists an arbitrary non-singular matrix H such that the set $\{\tilde{\mathit{\Lambda}}, \tilde{F}_t\}_{t=1}^T$ defined by $\tilde{F}_t = H^{-1}\hat{F}_t$ for $t = 1, \ldots, T$ and $\tilde{\mathit{\Lambda}} = \hat{\mathit{\Lambda}}H$ is also a solution to the estimation problem. Therefore, an additional set of restrictions is required for proper identification. Stock and Watson (2002) demonstrated that the estimation of factor models via principal components resolves this identification problem by providing the required identifying restrictions. Principal component estimation consists of associating $\hat{\mathit{\Lambda}}$ with the ordered orthonormal eigenvectors of the (estimated) covariance matrix of the observables (after standardization). The factors $\hat{F}_t$ are then obtained as the projection of the observables over $\hat{\mathit{\Lambda}}$. Accordingly, this statistical procedure imposes non-singular limiting values (as both $N, T \to \infty$) for the norm of $\mathit{\Lambda}$ and the variance of F_t, which together with other necessary conditions (namely, bounded loadings and consistency for the covariance estimator of F_t), reduce H to an orthonormal diagonal matrix with elements equal to ± 1. This identifies the factors up to a change of sign. We refer to Stock and Watson (2002) for a more detailed discussion.

Hence, we use the restrictions implied by principal components estimation for $\hat{F}_t$ and $\hat{\Lambda}$, following most the DFM literature. A useful feature of this identification scheme is that, even though the contemporary factors are orthogonal, serial and lagged cross-correlation between them is still present, maintaining therefore their suitability for VAR forecasting.[1]

2.2.2 External block

The estimation via principal components of the external factors and their associated loadings relies on the covariance matrix of the observables. More specifically, the covariance matrix of the standarized variables X^E is estimated as $\widehat{Cov}(X^E) = X^{E'}X^E/(T-1)$. Then, $\hat{\Lambda}^{EE}$ is estimated as the eigenvectors of $\widehat{Cov}(X^E)$, normalized and ordered decreasingly. It can be shown that the eigenvalues associated with these eigenvectors represent the portion of the total variance explained by each of the factors.[2] Once $\hat{\Lambda}^{EE}$ is ready, the factors are estimated as linear combinations of X^E, where the weights correspond to the factor loadings: $\hat{F}^E = X^E\hat{\Lambda}^{EE}$.

Up to this point, $\hat{F}^E$ spans the same space as X^E. The previous procedure simply constitues a rotation of X^E, and thus, generates the same number of factors as observables (i.e., $K^E = N^E$). However, since our goal is to summarize information, it is necessary to remove factors and loadings, while minimizing the loss of explained variance. Since the eigenvalues associated with the eigenvectors in $\hat{\Lambda}^{EE}$ represent the portion of the total variance in X^E explained by each of the factors, the first obvious candidate for elimination is the eigenvector associated with the lowest eigenvalue (which corresponds to the last column of $\hat{\Lambda}^{EE}$ and $\hat{F}^E$, respectively). This elimination step is repeated until the desired number of factors is achieved (see Section 3.2.1).

[1] For further references regarding identification see Bai and Ng (2013).

[2] Since $\widehat{Cov}(X^E)$ is by construction a real symmetric matrix, its eigenvectors will be real and orthogonal, which alongside their normalization, render $\hat{\Lambda}^{EE}$ to be an orthonormal basis of the space spanned by $\widehat{Cov}(X^E)$. Furthermore, since $\widehat{Cov}(X^E)$ is positive definite, all its eigenvalues will be positive.

2.2.3 Domestic block

Once the external factors and loadings are estimated, it is possible to estimate the domestic block imposing the block exogeneity restrictions present in (1). This is done following the iterative approach proposed by Boivin and Giannoni (2007); see also Charnavoki and Dolado (2014) and Kamber et al. (2016). The idea behind this approach is that the estimated external factors can be used to orthogonalize (or "externally correct") the estimated domestic factors, so as to control for the effect of foreign factors on the domestic block. This guarantees that the estimated domestic factors capture only the dynamics in the domestic variables not captured by the foreign factors. Formally, this is achieved using the following iterative approach:

1. Estimate $\{\hat{\Lambda}^{EE}, \hat{F}_t^E\}_{t=1}^T$ and $\{\hat{\Lambda}^{DD(0)}, \hat{F}_t^{D(0)}\}_{t=1}^T$ via principal components independently following the estimation procedure described in Section 2.2.2 (suitably adapted for the domestic block) to obtain K^E external and K^D domestic factors.

2. For $i = 0$, regress X_t^D on $\hat{F}_t^E$ and $\hat{F}_t^{D(i)}$ using Ordinary Least Squares (OLS) to obtain $\hat{\Lambda}^{ED(i)}$ (which corresponds to the OLS coefficients associated with $\hat{F}_t^E$).

3. Compute $\hat{X}_t^{D(i)} = X_t^D - \hat{\Lambda}^{ED(i)}\hat{F}_t^E$.

4. Estimate $\{\hat{\Lambda}^{DD(i+1)}, \hat{F}_t^{D(i+1)}\}_{t=1}^T$ using the first K^D principal components of $\hat{X}_t^{D(i)}$.

5. Repeat steps 2-4 for $i = 1, 2, 3, \ldots,$ until convergence of $\{\hat{F}_t^{D(i+1)}\}_{t=1}^T$.

2.2.4 Vector autoregressions

The block exogeneity restrictions present in the VAR from (3) are imposed by a reparametrization of the unrestricted version of (3) that contains the desired linear constraints (i.e., exogeneity of the external factors with respect to the domestic factors). As a consequence of this reparametrization, not all the regressors are shared throughout all the equations present in the system. Thus, standard equation-by-equation OLS does not yield efficient parameter estimates.[3] This issue can

[3]See Zellner (1962) for a complete discussion.

be tackled by applying Feasible Generalized Least Squares (FGLS), which yields asymptotically efficient and consistent parameter estimates by estimating the covariance matrix of the residuals via an iterative approach. Also, since the VAR under consideration is stationary, standard asymptotic inference continues to apply. For further details see, e.g., Lütkepohl (2007).

2.2.5 Unrestricted DFM

The estimation of the unrestricted DFM encompasses similar steps as the ones described previously. Under this case, however, the estimated loadings $\hat{\Lambda}$ are extracted from the complete standarized dataset X, which includes all external and domestic observables. This yields the resulting estimated eigenvalues (and their corresponding eigenvectors) to be associated with the joint dispersion of all the variables under consideration. Consequently, $\hat{F}$ constitutes linear combinations of both external and domestic observables. Once the factors $\hat{F}$ are available, the unrestricted VAR which models their joint dynamics is estimated. Unlike for the VAR with block exogeneity restrictions, equation-by-equation OLS yields asymptotically efficient and consistent parameter estimates, given that the regressors are shared across all equations (Zellner, 1962).

2.3 Forecasts

Forecasts from (3) are obtained using a standard recursive scheme, in which the point forecast corresponds to the conditional mean of each variable given the current information set. The sample under consideration is quarterly, and forecasts for time horizons ranging from 1 up to 8 quarters ahead are computed. Point forecasts for the observables are then obtained by projecting the forecasts from (3) onto the observables using the estimated loadings matrix, and adding the forecasts for the idiosyncratic innovations from (2).

Formally, consider for simplicity the forecasts of the unrestricted DFM, in which the lag polynomials, the idiosyncratic innovations of (1) and the cross-correlated error terms of (3) are given by $\beta(L)$, e_t and ε_t respectively, and are analogous to their counterparts from the restricted

DFM. Then, the forecasts of the observable i are determined by:

$$\begin{aligned} E\left[X_{i,t}|X_{i,t-1},...\right] &= E[\Lambda_i F_t + e_{i,t}|X_{i,t-1},...] \\ &= E[\Lambda_i \beta(L) F_t|X_{i,t-1},...] + E[\alpha_i(L)(X_{i,t} - \Lambda_i \beta(L) F_t)|X_{i,t-1},...], \end{aligned} \tag{4}$$

The second equality above follows from the assumption that $E[e_t \varepsilon'_{t-j}] = 0 \; \forall j$, alongside the i.i.d. assumptions for $v_{i,t}$ and ε_t. As (4) shows, under this framework, the correlation among the series of observables in X_t only occurs through the factors F_t. However, in those cases in which $e_{i,t}$ is serially correlated, the term $E\left[e_{i,t}|X_{t-1},...\right]$ leads to the appearance of lagged terms of $X_{i,t}$ in the forecast equation.

3 Empirical Implementation and Results

In this section we present the empirical implementation and results for Chile, focusing on real GDP and CPI forecasts. Our analysis consists of a benchmark recursive exercise presented in Section 3.3 and a number of robustness checks documented in Section 3.4. The model specification for the benchmark exercise, which includes determining the number of factors in (1), the number of lags in (3) and the autocorrelation structure of (2) is determined considering the complete sample. In all recursive estimations conducted for the forecast accuracy assessment, we maintain the model specification selected for the complete sample.

3.1 Data

The data used for the estimation of the DFM is a balanced panel of 173 quarterly series for the period from 2001Q1 to 2016Q4. The starting point for this panel coincides with the kick-off of Chile's inflation targeting monetary framework. The data series are divided into external and domestic variables. The external block comprises 63 international series that are categorized into 14 groups: GDP, consumption, investment, government expenditure, prices, unemployment, foreign trade, monetary policy rates, real exchange rates, sovereign bond yields, stock market indicators, sectoral economic indicators, commodity prices, and volatility indexes. With the exception of the latter two, all other groups include variables (when available) from China, the United States, Japan and the euro area, which combined constitute roughly 61% of Chile's total foreign trade.[4] The domestic block includes 110 series, divided into 13 groups: GDP, sectoral economic indicators, expenditure, prices, employment, other labor market indicators, wages, credit, monetary aggregates, interest rates, stock market, exchange rates and economic surveys.

All data series are transformed to induce stationarity, demeaned and standardized prior to the estimation (sample-by-sample in the recursive exercise). In addition, in the benchmark exercise we use seasonally adjusted series from source whenever available, as well as the last available vintage for variables that are subject to revisions. In Section 3.4.2 we conduct an alternative exercise where we use real-time vintages of GDP and sample-by-sample seasonal adjustment of the raw data. Finally, the variables present in the external block were mostly retrieved using Bloomberg, whereas the domestic variables were obtained through the CBCH's statistical database.[5] As the raw data includes both monthly and quarterly series, the monthly series were aggregated into quarterly series using monthly averages. Seasonal adjustment of series that are not available in seasonally adjusted form directly from source (as well as the sample-by-sample seasonal adjustment for the exercise presented in Section 3.4.2) is conducted using the Census X11 multiplicative method. The complete list of data series and description of data transformations is provided in the appendix.

[4]Source: Ministry of Foreign Affairs of Chile, General Directorate for International Economic Relations (DIRECON), Foreign Trade Quarterly Report, May 2017.

[5]See `http://si3.bcentral.cl/Siete/secure/cuadros/home.aspx?Idioma=en-US`.

3.2 Model specification

3.2.1 Number of factors

Several approaches to determine the number of factors in a DFM framework have been used in the literature. Bai and Ng (2002) propose selecting the factor specification based on information criteria. The idea is to assess the trade-off between the benefit of increased in-sample fit through including an additional factor against the associated cost in parametric variability arising from its inclusion.[6] However, Bernanke, Boivin, and Eliasz (2005) argue that this criterion does not address the issue of how many factors to include in the VAR. Instead, they explore the sensitivity of their results under different factor specifications. An alternative approach is to select the minimum number of factors in the model so as to be able to explain some ad-hoc level of variance that is set *a priori*. Kamber et al. (2016) opt for a variation of that approach and choose the number of factors by examining the eigenvalues from the eigenvalue-eigenvector decomposition of the sample covariance. Here, the idea is to detect the specific number of factors at which the marginal contribution to the explained variance becomes reasonably negligible. We adopt this kind of approach in our benchmark exercise in Section 3.3 and analyze the sensitivity of our results under alternative factor specifications in Section 3.4.1.

Since the level of explained variance attained by the principal components estimator is a monotonically increasing and concave function of the number of factors, setting the threshold for the explained variance too high (say, 75% or higher) would quickly lead to an overparametrization of the model. On the other hand, setting the threshold too low (say, 25% or lower) would lead to potential presence of strong cross-correlations in the idiosyncratic innovations of (1). Both might negatively affect the forecasting ability of the model. Hence, we set the threshold for the explained variance to 50%. Figure 1 shows the level of explained variance for every combination of factors in each block of the restricted DFM and the complete set of observables in the unrestricted DFM, alongside the resulting explained variance for the selected specification (horizontal line). Our

[6] This is done by minimizing a penalized likelihood function.

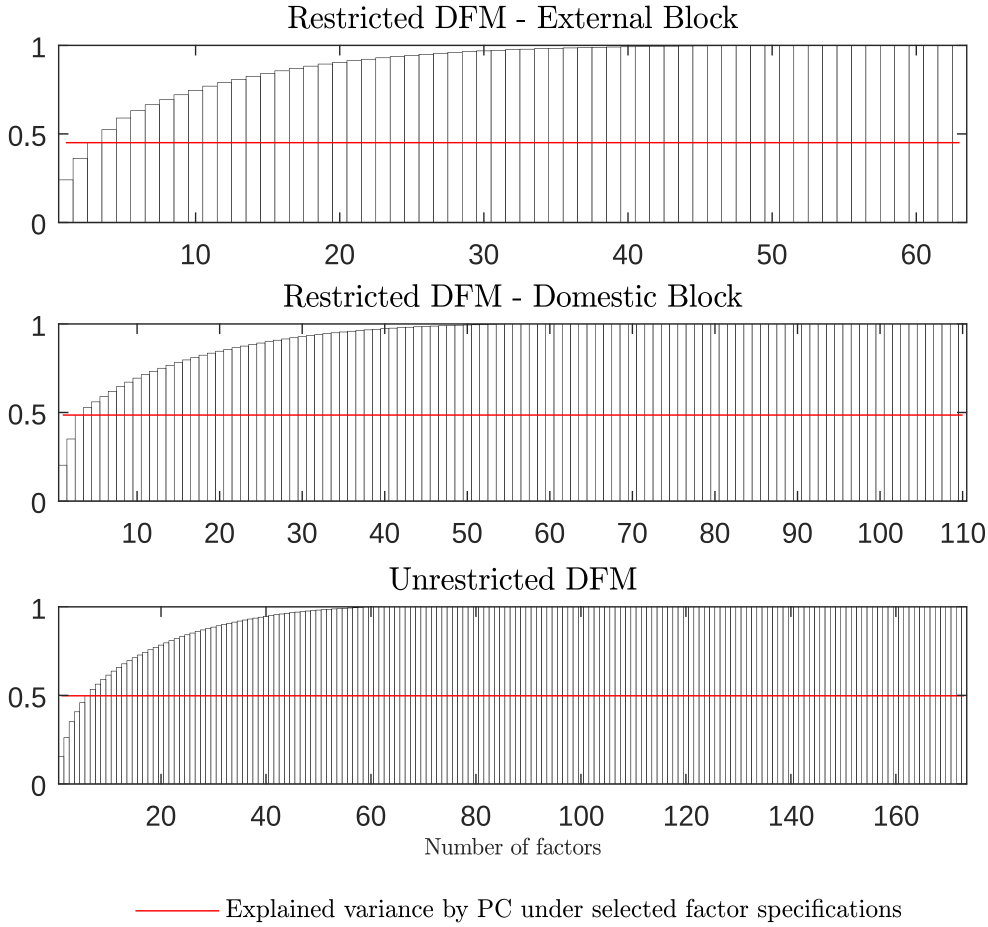

Figure 1: Explained variance for the external and domestic observables ($N^E = 63$ and $N^D = 110$) in the restricted DFM, and all observables ($N = 173$) in the unrestricted DFM. *Note*: The selected number of factors, highlighted by the horizontal lines, corresponds to $K^E = 3$, $K^D = 3$ and $K = 6$, respectively. Under these specifications, the amount of total explained variance is: 45.1% for the external block and 48.6% for the domestic ("externally-corrected") block in the restricted DFM, and 49.8% for the complete dataset in the unrestricted DFM.

criterion thereby yields $K^D = K^E = 3$ for the restricted model and $K = 6$ for the unrestricted model. Note that, similarly as in Kamber et al. (2016), in both models the marginal contribution of additional factors to the explained variance is relatively low at that level.

3.2.2 VAR order

The number of lags p considered for the VAR from (2) is determined using the Schwarz-Bayesian information criterion. We choose this criterion over other common alternatives present in the literature (e.g., Akaike information criterion) based on: (1) its consistency and (2) its tendency towards choosing parsimonious specifications. The results are documented in Table 1 and suggest

Table 1: Schwarz information criteria for vector autoregression.

	DFM_r	DFM_u
$p = 0$	13.14	15.30
$p = 1$	9.79	8.68
$p = 2$	10.25	10.14
$p = 3$	11.16	11.34
$p = 4$	10.70	12.03

Note: p indicates the lag order of the VAR, and DFM_r and DFM_u refer to the restricted and unrestricted DFM, respectively.

that the optimal specification includes only one lag for both versions of the model.

3.2.3 Autoregressive structure of idiosyncratic innovations

Once the factors and loadings of (1) are computed, the series of idiosyncratic innovations for observable i can be obtained as: $\hat{e}_{i,t}^E = X_{i,t}^E - \hat{\Lambda}_i^{EE} \hat{F}_t^E$ if i belongs to the external block (the expression for the unrestricted model is akin), or $\hat{e}_{i,t}^D = X_{i,t}^D - \hat{\Lambda}_i^{ED} \hat{F}_t^E - \hat{\Lambda}_i^{DD} \hat{F}_t^D$ if i belongs to the domestic block.[7] Then, standard serial correlation tests are conducted, so as to determine (on a case-by-case basis) if the imposition of an autoregressive structure for the idiosyncratic innovations is warranted. In that case, the parameters of the univariate model $\alpha_i(L)$ are estimated using OLS. We choose the Lagrange multiplier (LM) test, setting the lag order p_{LM} of the auxiliary regression to 4. We compute the LM-statistic,[8] which is asymptotically distributed as $\chi^2(p_{LM})$, and assess its statistical significance considering a 5% confidence level.[9]

Table 2 exhibits the LM test results under different specifications for the idiosyncratic innovations of the real GDP growth rate and the CPI inflation rate. The results for real GDP growth suggest that there is no need to impose additional innovation structure for that variable, as the null (absence of serial correlation) is not rejected for both versions of the DFM. Hence, we use a white noise specification for the idiosyncratic innovations of real GDP growth. In turn, the

[7]The subscript i used in the loadings matrices denotes their i-th row.

[8]The LM-statistic is defined as $obs \times R^2$, where obs denotes the number of observations and R^2 the uncentered coefficient of determination of the auxiliary regression.

[9]This confidence level is maintained throughout this paper.

Table 2: Serial correlation (Lagrange multiplier) tests for idiosyncratic innovations.

	GDP		CPI		
	DFM_r	DFM_u	DFM_r		DFM_u
	WN	WN	WN	$AR(1)$	WN
LM-statistic	6.96	6.61	12.68	5.70	3.80
Prob. $\chi^2(4)$	0.14	0.16	0.01	0.22	0.43

Note: The null hypothesis of the Lagrange multiplier (LM) test is absence of residual serial correlation. The test statistic has a chi-squared distribution with four degrees of freedom. WN indicates a white noise specification, while $AR(1)$ indicates an autoregressive specification of order 1. DFM_r and DFM_u refer to the restricted and unrestricted DFM, respectively.

null is rejected in the case of the CPI inflation rate at a 5% confidence level. Consequently, an AR(1) structure is imposed for the idiosyncratic innovations of CPI inflation, which effectively renders the innovations to white noise according to the test results for that specification. Finally, the unrestricted DFM exhibits white noise innovations for both variables considered according to the LM test and, therefore, no further innovation structure is included in that case.

3.3 Recursive forecasts: benchmark exercise

Recursive forecasts are computed using 25 samples, going from 2001Q1-2010Q4 up to 2001Q1-2016Q4 (the latter corresponds to the complete dataset). We use an incremental scheme, given the relatively narrow set of observations at hand. The model is reestimated at every iteration, and recursive out-of-sample forecasts of 1 up to 8 quarters ahead are generated at each step. We compute the implied root mean squared errors (RMSEs) of the real GDP growth and CPI inflation forecasts for each of the time horizons under consideration. We conduct this recursive forecasting exercise for both versions of the DFM, as well as for five univariate benchmark models: AR(1), AR(2), AR(3), AR(4) and a random walk (RW) specification.

In order to formally assess the forecast accuracy of the aforementioned set of models, we apply the modified version of the Diebold-Mariano (DM) test proposed by Harvey, Leybourne, and Newbold (1997). Through simulation experiments, Diebold and Mariano (1995) showed that

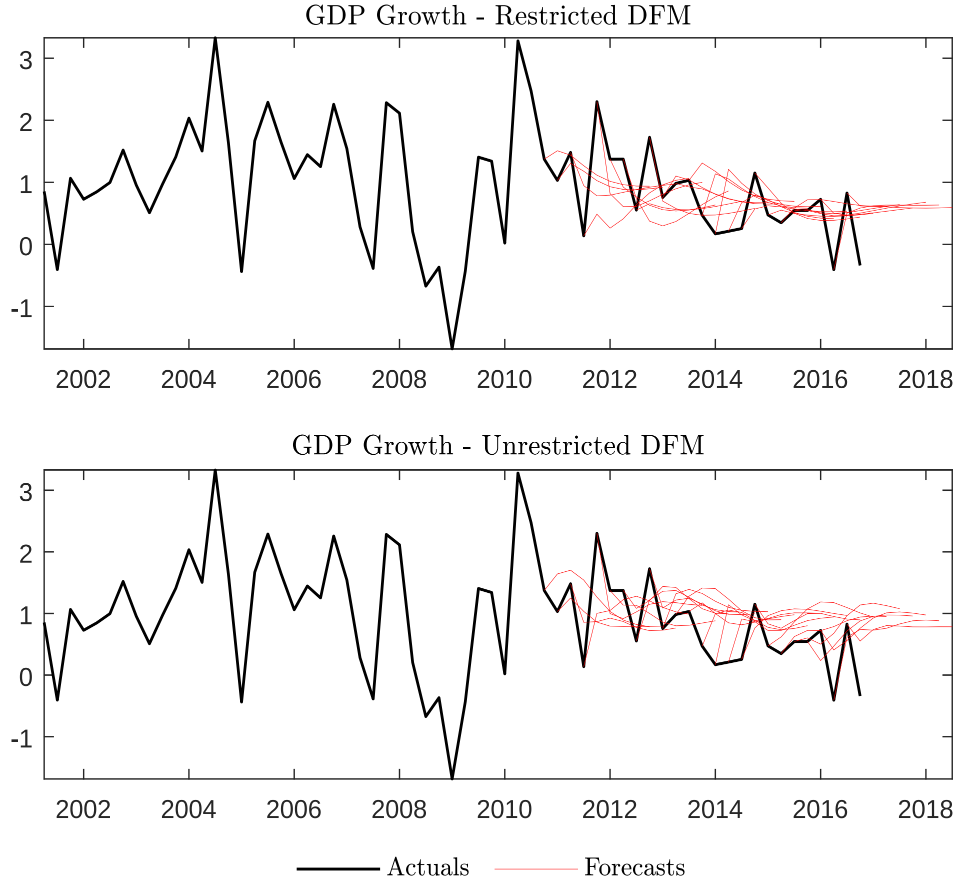

Figure 2: Recursive forecasts for real GDP growth with $K^E = 3$, $K^D = 3$ (restricted DFM) and $K = 6$ (unrestricted DFM). *Note*: Recursive forecasts are generated using an incremental window scheme for 1-8 steps ahead.

the normal distribution can be a very poor approximation of the DM test's finite-sample null distribution. Their results indicate that the DM test can have the wrong size and generate a bias towards type I errors depending on the degree of serial correlation among the forecast errors and the sample size. In turn, the modified DM test has better small-sample properties by: (1) making a bias correction to the DM test statistic and (2) comparing the corrected statistic with a Student's t-distribution. We refer to Harvey et al. (1997) for a complete discussion.

3.3.1 Results for real GDP growth

Figure 2 displays the results (point forecasts) from the recursive exercise for real GDP growth. The results show that the unrestricted DFM tends to overpredict the observed growth rates towards the end of the sample, unlike the restricted DFM. In general, the unrestricted model

Table 3: Root mean squared errors of recursive forecasts.

	GDP growth forecasts						
	DFM_r	DFM_u	$AR(1)$	$AR(2)$	$AR(3)$	$AR(4)$	RW
$h=1$	0.68	0.59	0.68	0.73	0.76	0.78	0.83
$h=2$	0.72	0.65	0.66	0.72	0.69	0.69	0.63
$h=3$	0.66	0.75	0.71	0.74	0.71	0.70	0.80
$h=4$	0.58	0.68	0.70	0.73	0.68	0.69	0.61
$h=5$	0.46	0.68	0.68	0.70	0.69	0.70	0.76
$h=6$	0.45	0.66	0.69	0.72	0.71	0.73	0.72
$h=7$	0.45	0.65	0.72	0.74	0.74	0.75	0.78
$h=8$	0.46	0.66	0.73	0.76	0.75	0.76	0.83
	CPI inflation forecasts						
	DFM_r	DFM_u	$AR(1)$	$AR(2)$	$AR(3)$	$AR(4)$	RW
$h=1$	0.44	0.47	0.43	0.45	0.45	0.45	0.50
$h=2$	0.41	0.46	0.42	0.43	0.43	0.43	0.52
$h=3$	0.40	0.47	0.43	0.42	0.42	0.42	0.54
$h=4$	0.40	0.46	0.47	0.45	0.44	0.44	0.68
$h=5$	0.46	0.49	0.46	0.45	0.44	0.44	0.68
$h=6$	0.48	0.49	0.47	0.46	0.45	0.45	0.69
$h=7$	0.47	0.50	0.46	0.45	0.45	0.45	0.65
$h=8$	0.47	0.47	0.45	0.45	0.45	0.45	0.70

Note: The table entries are the RMSEs (1-8 steps ahead) for real GDP growth and CPI inflation forecasts. These values where generated using an incremental window scheme ranging from 2001Q1-2010Q4 to 2001Q1-2016Q4.

seems to produce more dispersed forecasts compared to the restricted model, which might be due to the relatively heavy parameterization of the unrestricted model.

The implied RMSEs and prediction equality tests for real GDP growth are exhibited in the top panels of Tables 3 and 4, respectively.[10] These results suggest that for GDP forecasts, the restricted DFM (DFM_r) outperforms the unrestricted DFM (DFM_u) in terms of forecast accuracy at medium-term forecast horizons. In particular, for forecast horizons of more than a few quarters the RMSEs implied by the restricted DFM are considerably lower than the RMSEs implied by the unrestricted model. The p-values from Table 4 show that statistically significant differences start to appear for forecast horizons from 4 quarters onwards. Similarly, the restricted DFM beats all univariate benchmark models considered starting from 4-step-ahead forecasts and

[10] Table 4 exhibits blank values for forecast horizons in which the RMSEs implied by the restricted DFM are greater than the RMSEs from the respective benchmark model.

Table 4: Harvey et al. (1997) prediction equality tests for RMSEs, DFM_r vs. benchmarks.

	GDP growth forecasts					
	DFM_u	$AR(1)$	$AR(2)$	$AR(3)$	$AR(4)$	RW
$h = 1$			0.218	0.114	0.064	0.043
$h = 2$			0.499			
$h = 3$	0.088	0.303	0.254	0.311	0.335	0.002
$h = 4$	0.028	0.071	0.050	0.134	0.126	0.340
$h = 5$	0.020	0.008	0.007	0.005	0.006	0.006
$h = 6$	0.000	0.001	0.001	0.001	0.000	0.000
$h = 7$	0.000	0.000	0.000	0.000	0.000	0.003
$h = 8$	0.000	0.000	0.000	0.000	0.000	0.000
	CPI inflation forecasts					
	DFM_u	$AR(1)$	$AR(2)$	$AR(3)$	$AR(4)$	RW
$h = 1$	0.204		0.368	0.368	0.457	0.196
$h = 2$	0.144	0.407	0.329	0.313	0.428	0.048
$h = 3$	0.106	0.247	0.279	0.308	0.369	0.005
$h = 4$	0.251	0.002	0.028	0.109	0.177	0.000
$h = 5$	0.266	0.384				0.000
$h = 6$	0.466					0.000
$h = 7$	0.203					0.000
$h = 8$						0.000

Note: The table entries are the p-values for the Harvey et al. (1997) prediction equality tests. The null hypothesis is equal predictive ability between the restricted DFM and the respective benchmark. Blanks indicate that RMSE implied by benchmark resulted lower than RMSE from restricted DFM.

onwards. In addition, the results from Table 4 indicate that the improved performance of the restricted DFM with respect to the univariate benchmarks is statistically significant.

3.3.2 Results for CPI inflation

Figure 3 displays the results from the recursive exercise for CPI inflation. The results are similar to the ones for real GDP growth in that the restricted DFM seems to produce more accurate forecasts. The larger dispersion and excessive persistence of the forecasts from the unrestricted model is even more notorious in this case, generating apparent upward and downward biases in the first and second half of the evaluation sample, respectively. These issues are corrected in the more parsimonious restricted model specification.

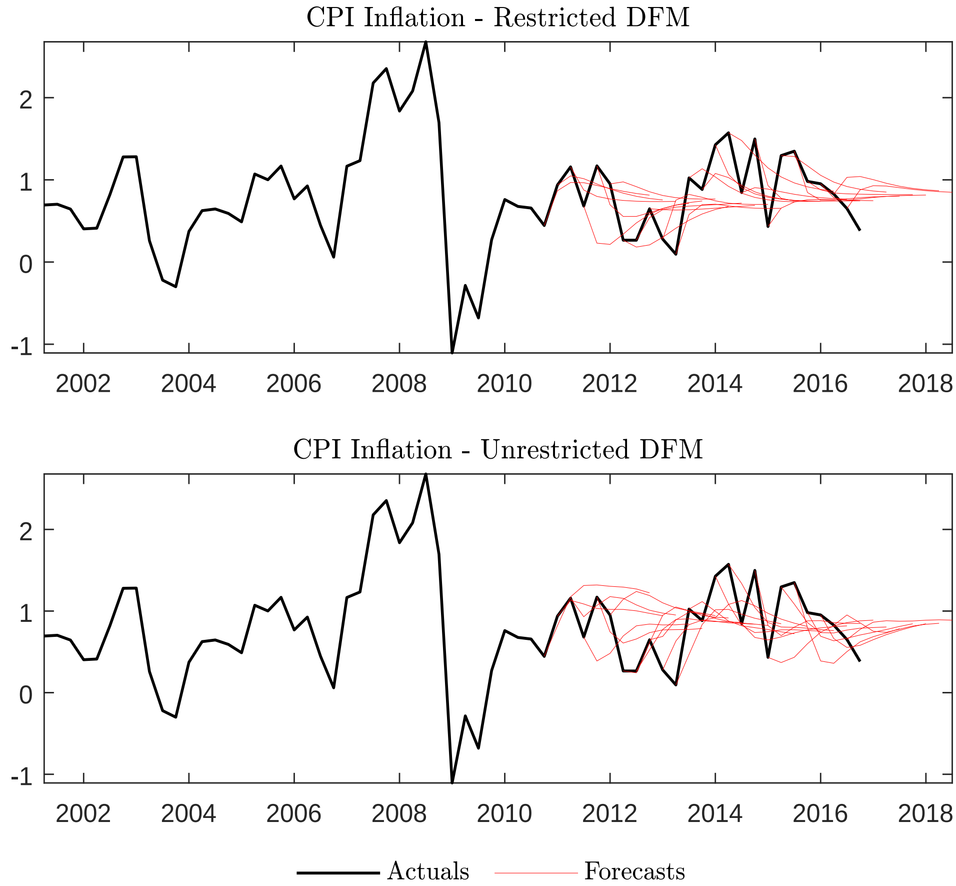

Figure 3: Recursive forecasts for CPI inflation with $K^E = 3$, $K^D = 3$ (restricted DFM) and $K = 6$ (unrestricted DFM). *Note*: See Figure 2.

The bottom panels of Tables 3 and 4 show the RMSEs and prediction equality tests for the case of CPI inflation forecasts. The results suggest that the restricted DFM beats the unrestricted DFM at almost every forecast horizon considered. However, the differences are not statistically significant, partly due to the relatively large dispersion of the RMSEs for CPI inflation which reduces the power of the prediction equality test. Similarly, the restricted DFM beats several of the univariate benchmarks at short horizons of less than a year, although the differences are not found to be statistically significant in most cases, with the exception of the random walk (where the differences are significant for horizons from 2 quarters onwards).

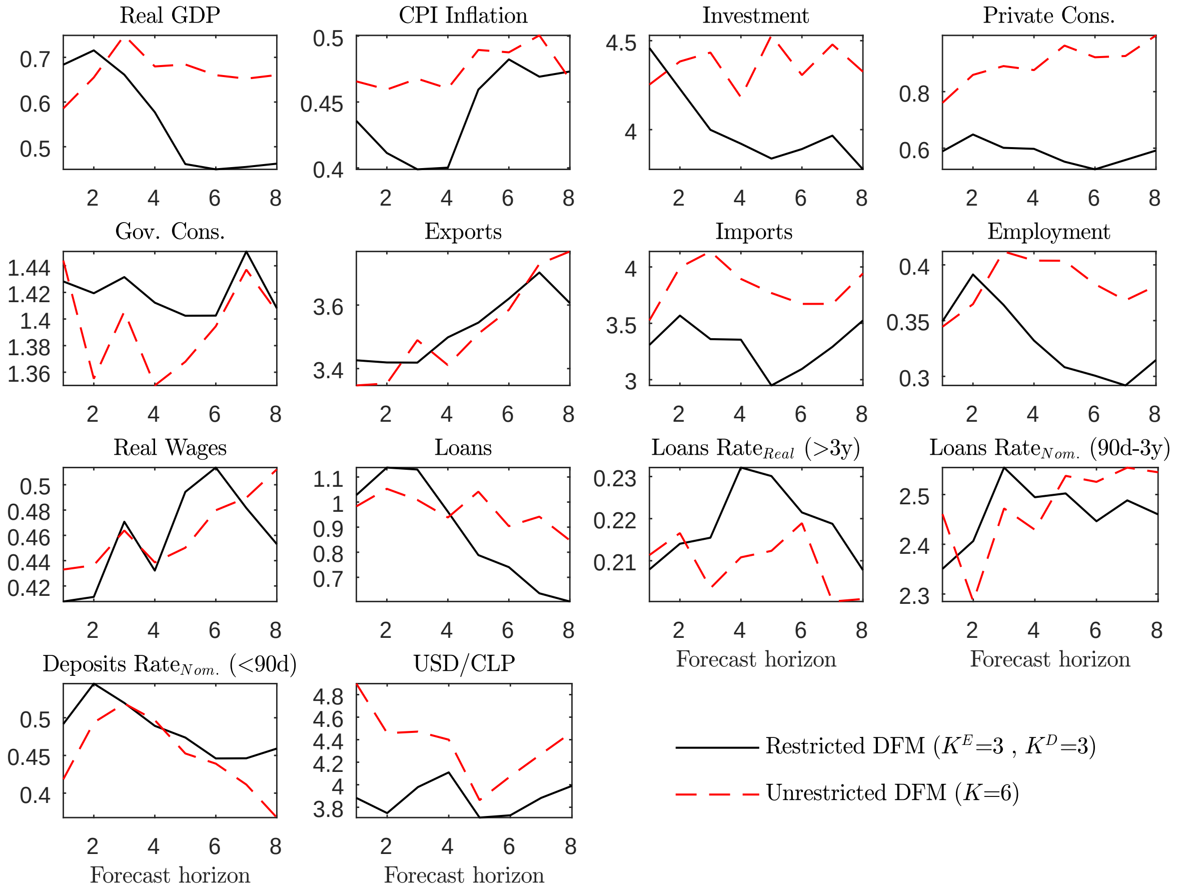

Figure 4: RMSEs (1-8 steps ahead) for selected variables with $K^E = 3$, $K^D = 3$ (restricted DFM) and $K = 6$ (unrestricted DFM). *Note*: These values where generated using an incremental window scheme ranging from 2001Q1-2010Q4 to 2001Q1-2016Q4.

3.3.3 Results for other variables

In order to better gauge how well the restricted model captures the small open economy's dynamics, we compute RMSEs for additional domestic observables and compare them with their unrestricted counterparts. We focus on aggregate demand, financial and labor market indicators, as well as the exchange rate. The results are plotted in Figure 4, and show the following: (1) the restricted DFM continues to outperform the unrestricted DFM when forecasting key aggregate demand indicators including investment and private consumptions, and also imports, which is consistent with the results for GDP obtained in Section 3.3; (2) for the labor market variables considered, the two models have similar forecasting performance for real wages, but the restricted DFM provides better forecasts of employment; (3) the results for the financial market indicators suggest, overall, a similar forecasting ability of the alternative specifications; and (4) the restricted DFM appears to better capture the dynamics of the nominal exchange rate. Overall, these results provide further evidence for the superior forecasting performance of the DFM with block exogeneity restrictions in our small open economy environment.

3.4 Robustness

In order to assess the robustness of the results presented in Section 3.3, we include two additional exercises. In the first exercise, we assess the sensibility of the restricted and unrestricted DFMs' forecasting performance when the combination of external and domestic factors is changed. In the second exercise, which follows Matheson (2006) and is thought to further approximate real-time forecasting conditions, the seasonal adjustment prior to the recursive estimation is made for each of the 25 sub-samples whenever non-adjusted series are available. Through seasonally adjusting the raw data sample-by-sample, that exercise mimicks the real-time problems associated with estimating seasonal factors. Also, we use real-time vintages of real GDP in the estimation, i.e., the data that were available when such forecasts would have been made.[11]

3.4.1 Varying the number of factors

To analyze the sensibility of the results to the number of factors, we compute the RMSEs for several variations of our baseline DFM specifications ($K^E = K^D = 3$ for the restricted version and $K = 6$ for the unrestricted version). To this end, we construct a grid of domestic and external factors with all possible factor combinations in the restricted DFM, ranging from 1 to 5 for each block, and compute the ratio of the RMSE under each combination with respect to the unrestricted DFM, maintaining the same total number of factors in both specifications.

The results are summarized in Table 5, which shows the ratios of the RMSEs for real GDP forecasts (top panel) and CPI forecasts (bottom panel) between the restricted and the unrestricted DFM. Thus, an entry below (above) 1 means that the restricted (unrestricted) DFM

[11] Our other forecast series of interest, the CPI inflation rate, is not subject to ex-post revisions.

Table 5: Ratios of RMSEs (restricted/unrestricted DFM) under different factor specifications.

		GDP growth forecasts							
		$h=1$	$h=2$	$h=3$	$h=4$	$h=5$	$h=6$	$h=7$	$h=8$
$K^E=1$	$K^D=1$	1.17	1.11	1.03	1.00	0.98	0.99	1.00	0.99
$K^E=1$	$K^D=2$	1.12	1.13	1.09	1.03	1.07	1.13	1.19	1.20
$K^E=1$	$K^D=3$	1.10	1.16	1.06	1.15	1.10	1.15	1.17	1.15
$K^E=1$	$K^D=4$	1.03	1.05	0.98	0.99	0.96	1.01	1.08	1.08
$K^E=1$	$K^D=5$	1.10	1.07	0.93	0.95	0.94	0.99	1.06	1.04
$K^E=2$	$K^D=1$	1.07	0.99	0.91	0.87	0.81	0.88	0.94	0.93
$K^E=2$	$K^D=2$	1.02	0.97	0.91	0.99	0.85	0.94	0.96	0.92
$K^E=2$	$K^D=3$	1.06	0.96	0.91	0.89	0.82	0.91	0.98	0.96
$K^E=2$	$K^D=4$	1.00	0.92	0.85	0.87	0.89	1.00	1.11	1.06
$K^E=2$	$K^D=5$	1.08	1.02	0.88	0.99	0.96	1.00	1.12	1.07
$K^E=3$	$K^D=1$	1.15	1.10	0.96	0.96	0.79	0.80	0.83	0.80
$K^E=3$	$K^D=2$	1.14	1.10	0.96	0.90	0.70	0.67	0.68	0.68
$K^E=3$	$K^D=3$	1.17	1.09	0.88	0.85	0.67	0.68	0.70	0.70
$K^E=3$	$K^D=4$	1.17	1.11	0.94	0.94	0.77	0.70	0.74	0.73
$K^E=3$	$K^D=5$	1.16	1.18	0.83	0.89	0.77	0.75	0.80	0.79
$K^E=4$	$K^D=1$	1.02	0.96	0.84	0.82	0.78	0.87	0.95	0.98
$K^E=4$	$K^D=2$	1.03	0.99	0.80	0.77	0.76	0.88	0.96	0.98
$K^E=4$	$K^D=3$	1.07	1.04	0.84	0.89	0.82	0.89	0.95	0.96
$K^E=4$	$K^D=4$	1.06	1.11	0.78	0.86	0.87	0.96	1.05	1.05
$K^E=4$	$K^D=5$	1.02	1.07	0.70	0.64	0.68	0.86	0.94	1.05
$K^E=5$	$K^D=1$	1.10	1.01	0.87	0.88	0.87	0.94	1.00	0.99
$K^E=5$	$K^D=2$	1.13	1.06	0.87	0.93	0.88	0.91	0.95	0.96
$K^E=5$	$K^D=3$	1.08	1.13	0.79	0.88	0.86	0.93	0.98	0.97
$K^E=5$	$K^D=4$	1.05	1.10	0.73	0.66	0.69	0.86	0.90	0.99
$K^E=5$	$K^D=5$	1.04	1.09	0.71	0.70	0.73	0.86	0.88	0.96
		CPI inflation forecasts							
		$h=1$	$h=2$	$h=3$	$h=4$	$h=5$	$h=6$	$h=7$	$h=8$
$K^E=1$	$K^D=1$	0.93	0.88	0.91	0.99	1.03	1.07	1.03	1.04
$K^E=1$	$K^D=2$	0.91	0.79	0.77	0.83	0.86	0.96	0.99	1.05
$K^E=1$	$K^D=3$	0.90	0.90	0.97	0.99	0.95	0.96	0.93	0.98
$K^E=1$	$K^D=4$	0.92	0.81	0.96	0.96	1.01	1.08	0.98	1.03
$K^E=1$	$K^D=5$	0.95	0.94	1.07	1.18	1.28	1.36	1.29	1.36
$K^E=2$	$K^D=1$	0.92	0.81	0.75	0.82	0.92	1.01	1.03	1.07
$K^E=2$	$K^D=2$	0.94	0.89	0.91	0.95	0.97	0.99	0.95	1.01
$K^E=2$	$K^D=3$	0.99	0.86	0.86	0.95	1.02	1.08	1.00	1.07
$K^E=2$	$K^D=4$	0.97	0.84	0.84	0.93	0.97	1.04	0.94	1.05
$K^E=2$	$K^D=5$	0.93	0.82	0.81	0.91	1.00	1.06	0.96	1.08
$K^E=3$	$K^D=1$	1.00	0.98	0.94	0.92	0.97	1.01	0.99	1.11
$K^E=3$	$K^D=2$	0.96	0.90	0.87	0.88	1.01	1.09	1.03	1.11
$K^E=3$	$K^D=3$	0.94	0.90	0.85	0.87	0.94	0.99	0.94	1.01
$K^E=3$	$K^D=4$	0.95	0.81	0.79	0.81	0.94	1.01	0.95	1.06
$K^E=3$	$K^D=5$	1.00	0.89	0.95	0.93	1.17	1.24	1.13	1.25
$K^E=4$	$K^D=1$	0.95	0.90	0.91	0.90	1.02	1.06	1.02	1.14
$K^E=4$	$K^D=2$	0.91	0.89	0.89	0.90	0.99	1.03	0.99	1.14
$K^E=4$	$K^D=3$	0.93	0.82	0.82	0.81	0.95	0.98	0.95	1.12
$K^E=4$	$K^D=4$	0.94	0.86	0.91	0.89	1.09	1.07	0.98	1.17
$K^E=4$	$K^D=5$	0.93	0.82	1.02	1.01	1.37	1.21	1.20	1.25
$K^E=5$	$K^D=1$	1.00	1.01	0.93	0.90	1.00	1.03	0.99	1.14
$K^E=5$	$K^D=2$	1.01	0.89	0.85	0.81	0.96	1.00	0.96	1.13
$K^E=5$	$K^D=3$	0.98	0.87	0.92	0.87	1.04	1.07	0.98	1.18
$K^E=5$	$K^D=4$	1.01	0.81	1.03	0.98	1.36	1.24	1.15	1.28
$K^E=5$	$K^D=5$	0.99	0.85	1.04	0.97	1.35	1.32	1.25	1.22

Note: The table entries are the ratios of the RMSEs (1-8 steps ahead) for different numbers of external and domestic factors, using $K = K^E + K^D$ in the unrestricted DFM. An entry below (above) 1 means that the restricted (unrestricted) DFM produces a relatively lower RMSE.

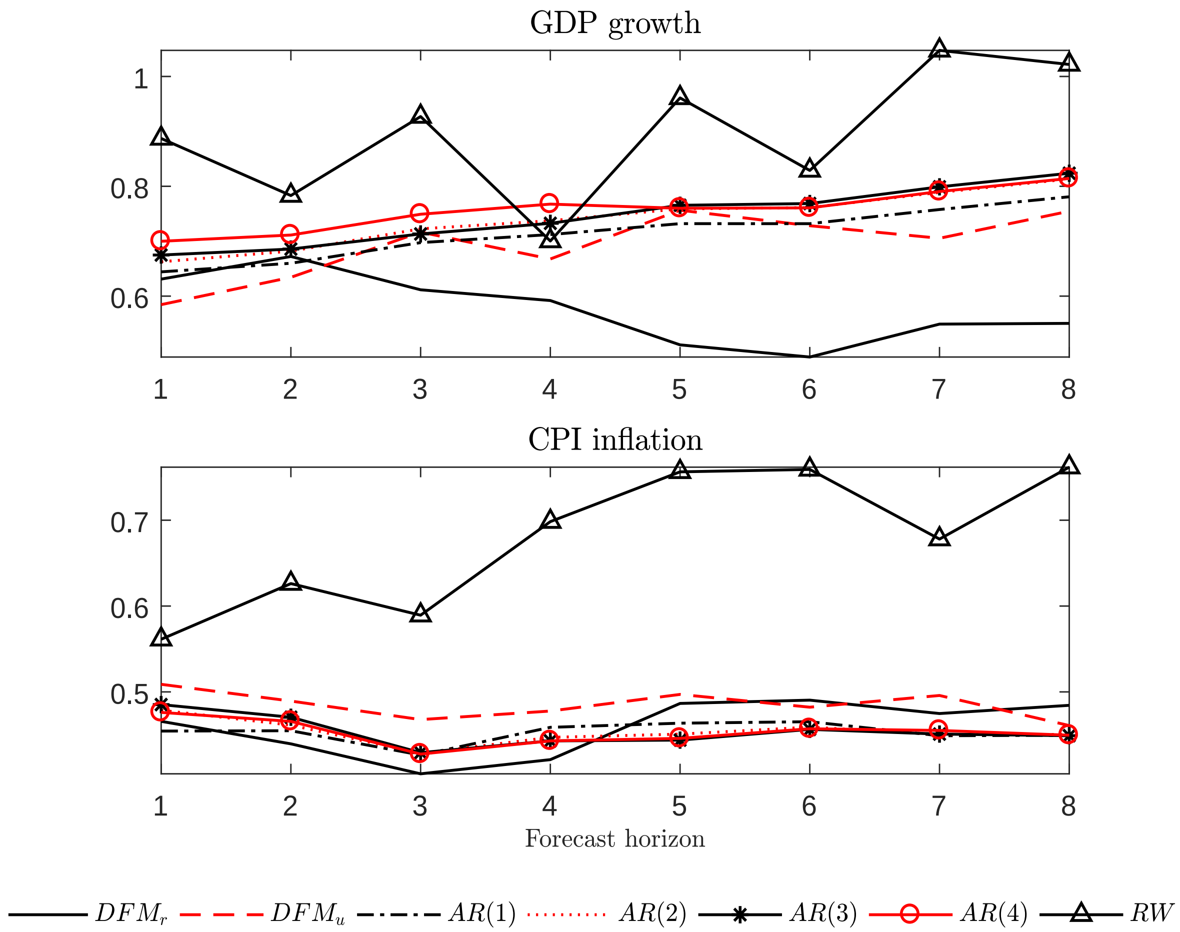

Figure 5: RMSEs (1-8 steps ahead) for DFMs and benchmark models, real-time forecasting exercise. *Note*: For this exercise, seasonal adjustment is made sample-by-sample and real-time vintages of GDP are used.

produces a relatively lower RMSE. Overall, the results from the benchmark exercise are maintained under the different factor combinations. As in the benchmark exercise, the RMSEs for real GDP at medium-term horizons (from $h = 3$ onwards) tend to be lower in the restricted DFM. The main exceptions occur under combinations where only one external factor is used, where the unrestricted DFM produces better forecasts for several numbers of domestic factors. However, in those cases the explained variance of the external observables is low (see Table 1), so these combinations would probably never be selected in an actual forecasting exercise. Regarding the CPI forecasts, the restricted version of the model also generates more accurate forecasts than the unrestricted version under the great majority of factor combinations.

Table 6: Prediction equality tests (p-values), real-time forecasting exercise.

	GDP growth forecasts					
	DFM_u	$AR(1)$	$AR(2)$	$AR(3)$	$AR(4)$	RW
$h = 1$		0.439	0.359	0.305	0.220	0.022
$h = 2$			0.462	0.445	0.352	0.102
$h = 3$	0.018	0.218	0.172	0.185	0.134	0.017
$h = 4$	0.023	0.055	0.037	0.032	0.028	0.150
$h = 5$	0.002	0.002	0.002	0.001	0.002	0.000
$h = 6$	0.000	0.000	0.000	0.000	0.000	0.000
$h = 7$	0.004	0.001	0.001	0.000	0.001	0.000
$h = 8$	0.002	0.002	0.001	0.001	0.001	0.000
	CPI inflation forecast					
	DFM_u	$AR(1)$	$AR(2)$	$AR(3)$	$AR(4)$	RW
$h = 1$	0.070		0.380	0.312	0.401	0.038
$h = 2$	0.073	0.342	0.266	0.179	0.185	0.003
$h = 3$	0.134	0.273	0.216	0.197	0.204	0.000
$h = 4$	0.268	0.020	0.130	0.204	0.284	0.000
$h = 5$	0.429					0.002
$h = 6$						0.000
$h = 7$	0.343					0.000
$h = 8$						0.000

Note: See Figure 5 and Table 3.

3.4.2 Real-time forecasting exercise

Our second robustness exercise is a recursive forecasting exercise with additional real-time elements, following Matheson (2006), where 1) the seasonal adjustment is made sample-by-sample whenever non-adjusted data is available, and 2) real-time vintages of domestic real GDP are used in the estimation. Regarding the seasonal adjustment, some data series are available only and/or used in source-adjusted form including several series for the US and some series for Japan, as well as most Eurozone data.[12] All domestic observables with seasonal patterns are available in non-adjusted form such that all of those series are adjusted sample-by-sample in this exercise. The real-time vintages of real GDP include a constant-price series with fixed base year 2005 available from 2001Q1 through 2011Q3, as well as two chain-linked series with reference years

[12] Although some Eurozone series are also available in non-adjusted form, we use source-adjusted data in those cases because the recommended method by Eurostat is the indirect (or decentralized) approach where Eurozone aggregates are derived from an aggregation of national seasonally adjusted series (see Eurostat, 2015).

2008 (after 2011Q3) and 2013 (after 2016Q3), respectively.

The results of that exercise are presented in Figure 5, which shows the RMSEs from the two versions of the DFM and the benchmark models over the forecast horizons considered, as well as Table 6, which displays the p-values from the corresponding Harvey et al. (1997) prediction equality tests. The results confirm the conclusions derived from the benchmark exercise: the restricted DFM significantly outperforms the unrestricted model and all univariate benchmark models at medium-term forecast horizons for real GDP growth, while it also generates more accurate forecasts of CPI inflation than the unrestricted model for some horizons considered, although the differences are mostly not statistically significant in this case. All in all, we can therefore conclude that the results are robust to relevant variations in the analysis.

4 Conclusions

This paper has proposed a methodology for macroeconomic forecasting in a small open economy environment based on dynamic factor models. The suggested approach allows to incorporate block exogeneity restrictions of external with respect to domestic observables, both in the observation equation and the transition equation of the model, in order to replicate the theoretical interaction among these two groups of variables in small open economies. In comparison to standard unrestricted DFMs, this requires several modifications in the estimation of the model.

An application for Chile focusing on forecasting real GDP growth and CPI inflation allows to highlight the following main findings: first, the restricted DFM outperforms the unrestricted version of the model in terms of the accuracy of point forecasts, especially for medium-term real GDP growth forecasts (and forecasts of several related macroeconomic variables). Second, by incorporating block exogeneity restrictions the DFM is also able to beat several simple univariate benchmark models for real GDP forecasts in the medium term and tends to produce similar forecast errors for CPI inflation, whereas without the restrictions the model would produce inferior forecasts than most of the benchmark models considered. These results are robust to several variations in the analysis. Hence, based on this application we may conclude that the imposition of theoretical small open economy restrictions can be a useful means to improve the forecasting performance of DFMs in a small open economy environment.

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A Data Appendix

The external series where directly taken from Bloomberg through its World Economic Statistics feature. The domestic series were taken from the CBCH's database. Table 7 summarizes this information. The transformation codes are: 0 - no transformation, 1 - first differences, 2 - natural logarithms, and 3 - first differences of natural logarithms. In the benchmark exercise, own seasonal adjustment is conducted using the Census X11 multiplicative method on the complete sample. Asterisks (*) indicate source-adjusted series that are also used in this form in the real-time forecasting exercise in Section 3.4.2, while all other series are obtained in non-adjusted form and adjusted sample-by-sample using the Census X11 multiplicative method for that exercise. The complete sample ranges from 2001Q1 to 2016Q4.

Table 7: Data description.

Id	Mnemonic	Transf.	SA	Series description
EXTERNAL VARIABLES				
Gross Domestic Product				
1	GDP USA	3	Source*	Real GDP, Bill. of Chained (2009) US Dollars, US
2	GDP JPN	3	Source	Real GDP, Bill. of Chained (2011) Yen, Japan
3	GDP EZ	3	Source*	Real GDP, Mill. of Chained (2010) Euros, Eurozone
4	GDP CHN	3	Own	Real GDP Index (2015Q6=150,000), China
Consumption				
5	CONS USA	3	Source*	Real Cons., Bill. of Chained (2009) US Dollars, US
6	CONS JPN	3	Source	Real Cons., Bill. of Chained (2011) Yen, Japan
7	CONS EZ	3	Source*	Real Cons., Mill. of Chained (2010) Euros, Eurozone
Investment				
8	INV USA	3	Source*	Real Inv., Bill. of Chained (2009) US Dollars, US
9	INV JPN	3	Source	Real Inv., Bill. of Chained (2011) Yen, Japan
10	INV EZ	3	Source*	Real Inv., Mill. of Chained (2010) Euros, Eurozone
11	INV CHN	3	Own	Nominal Inv., Bill. of Yuans, China
Government Expenditure				
12	GOV USA	3	Source*	Real Gov. Expend., Bill. of Chained (2009) US Dollars, US
13	GOV JPN	3	Source	Real Gov. Expend., Bill. of Chained (2011) Yen, Japan
14	GOV EZ	3	Source*	Real Gov. Expend., Mill. of Chained (2010) Euros, Eurozone
15	GOV CHN	3	Own	Nominal Gov. Expend., Bill. of Yuans, China
Prices				
16	CPI USA	3	Source	Consumer Price Index (1982-84=100), US
17	CPI JPN	3	Own	Consumer Price Index (2015=100), Japan
18	CPI EZ	3	Source*	Consumer Price Index (2015=100), Eurozone
19	CPI CHN	3	Own	Consumer Price Index (2016Q4=100), China

Table 7: Data description.

Id	Mnemonic	Transf.	SA	Series description
Unemployment				
20	UNEMP USA	3	Source	Unemployment, Thousands, US
21	UNEMP JPN	3	Source*	Unemployment, Ten Thousands, Japan
22	UNEMP EZ	3	Source*	Unemployment, Thousands, Eurozone
Foreign Trade				
23	X USA	3	Source*	Real Exports, Bill. of Chained (2009) US Dollars, US
24	X JPN	3	Source	Real Exports, Bill. of Chained (2011) Yen, Japan
25	X EZ	3	Source*	Real Exports, Mill. of Chained (2010) Euros, Eurozone
26	X CHN	3	Own	Nominal Exports, Bill. of Yuans, China
27	M USA	3	Source*	Real Imports, Bill. of Chained (2009) US Dollars, US
28	M JPN	3	Source	Real Imports, Bill. of Chained (2011) Yen, Japan
29	M EZ	3	Source*	Real Imports, Mill. of Chained (2010) Euros, Eurozone
30	M CHN	3	Own	Nominal Imports, Bill. of Yuans, China
Monetary Policy Rates				
31	MPR USA	0	None	Monetary Policy Rate, % Per Annum, US
32	MPR EZ	0	None	Monetary Policy Rate, % Per Annum, Eurozone
33	MPR CHN	0	None	Monetary Policy Rate, % Per Annum, China
Real Exchange Rates				
34	RER USA	2	None	Real Exch. Rate (Trade Weighted, 03/1973=100), US
35	RER JPN	2	None	Real Exch. Rate (Trade Weighted, 2010=100), Japan
36	RER EU	2	None	Real Exch. Rate (Trade Weighted, 2010=100), Eurozone
37	RER CHN	2	None	Real Exch. Rate (Trade Weighted, 2010=100), China
38	IPE	2	None	Ext. Price Index (Trade Weighted, 1986=100), Chile
39	IPE5	2	None	Ext. Price Index (5 Major Trade Part., 1986=100), Chile
Sovereign Bond Yields				
40	Y10 USA	1	None	Sovr. Bond Yields 10Yr (in US Dollars), % Per Ann., US
41	Y10 JPN	1	None	Sovr. Bond Yields 10Yr (in Yens), % Per Ann., Japan
42	Y10 EZ	1	None	Sovr. Bond Yields 10Yr (in Euros), % Per Ann., Eurozone
Stock Markets				
43	S&P500	2	None	Stock Price Index - S&P500, US
44	NIKKEI	2	None	Stock Price Index - Nikkei 225, Japan
45	EUROSTOXX	2	None	Stock Price Index - EURO STOXX 50, Eurozone
46	SHNGCOMP	2	None	Stock Price Index - SSE Composite, China
Sectoral Economic Indicators				
47	IP USA	3	Source	Industrial Production Index (2012=100), US
48	IP JPN	3	Source	Industrial Production Index (2010=100), Japan
49	IP EZ	3	Source*	Industrial Production Index (2010=100), Eurozone
50	IU CHN	3	Own	Industrial Utilities, Bill. of Yuans, China
51	RS USA	3	Source*	Retail Sales, Mill. of (2009) US Dollars, US

Table 7: Data description.

Id	Mnemonic	Transf.	SA	Series description
52	RS EZ	3	Source*	Retail Sales Index (2010=100), Eurozone
53	RS CHN	3	Own	Retail Sales, Bill. of Yuans, China
Commodity Prices				
54	CU	3	None	Gen. 1st High Grade Copper Fut. Contr., US Dollars, CME
55	INV CU	3	None	Copper Inventories, Short Ton, Commod. Exch. Center
56	PULP	3	None	Gen. 1st Indust. Wood Pulp Fut. Contr., US Dollars, CME
57	WTI	3	None	Gen. 1st WTI Crude Oil Fut. Contr., US Dollars, CME
58	BRENT	3	None	Gen. 1st Brent Crude Oil Fut. Contr., US Dollars, CME
59	GAS	3	None	Gen. 1st Natural Gas Fut. Contr., US Dollars, CME
60	FAO FOOD	3	None	Food Price Index (2002-2004=100), FAO
Volatility Indexes				
61	VIX	2	None	Vix Volatility Index, Chicago Board Options Exch.
62	TED SPRD	2	None	3m Libor minus T-Bill (gen.) rate, basis pts., Bloomberg
63	EMBI GLBL	2	None	Emerg. Markets Bond Index Glob., US Dollars, J.P. Morgan
DOMESTIC VARIABLES				
Gross Domestic Product				
1	GDP	3	Source	Real GDP - Total, Mill. of Chained (2013) Pesos
2	GDP AGRIC	3	Source	Real GDP - Agriculture, Mill. of Chained (2013) Pesos
3	GDP FISH	3	Source	Real GDP - Fishing, Mill. of Chained (2013) Pesos
4	GDP MIN	3	Source	Real GDP - Mining, Mill. of Chained (2013) Pesos
5	GDP MANF	3	Source	Real GDP - Manufacturing, Mill. of Chained (2013) Pesos
6	GDP UTIL	3	Source	Real GDP - Utilities, Mill. of Chained (2013) Pesos
7	GDP CONS	3	Source	Real GDP - Construction, Mill. of Chained (2013) Pesos
8	GDP TRANS	3	Source	Real GDP - Transport, Mill. of Chained (2013) Pesos
9	GDP COMM	3	Source	Real GDP - Communcations, Mill. of Chained (2013) Pesos
10	GDP HS	3	Source	Real GDP - Housing, Mill. of Chained (2013) Pesos
11	GDP PERS	3	Source	Real GDP - Personal Services, Mill. of Chained (2013) Pesos
12	GDP PAD	3	Source	Real GDP - Public Admini., Mill. of Chained (2013) Pesos
Sectoral Economic Indicators				
13	HIDRO GEN	3	Source	Hydroelectric Generation, GWh
14	EE DISP	3	Source	Electrical Energy Dispatch, GWh
15	GEN SIC	3	Source	Gener. - Central Interconnected System (SIC), GWh
16	GEN SING	3	Source	Gener. - Norte Grande Interconnected System (SING), GWh
17	HS TOT	3	Source	Authorized Area for Building - Total, m^2
18	HS STGO	3	Source	Housing Sales, Santiago Metropolitan Area, Units
19	ISUP	3	Source	Supermarket Sales Index (2014=100)
20	IV ANAC	3	Source	Auto Sales - Total, Units
21	P SOFOFA	3	Own	Industrial Production Index (2003=100)
22	S SOFOFA	3	Own	Industrial Sales Index (2003=100)

Table 7: Data description.

Id	Mnemonic	Transf.	SA	Series description
23	X CU	3	Own	Copper Exports, Thousands of Tonnes of Fine Copper
24	BLD MAT	3	Own	Dispatch of Building Materials Index (2008=100)
Expenditure				
25	EXP INV	3	Source	Real Inv. - Total, Mill. of Chained (2013) Pesos
26	EXP INVI	3	Source	Real Inv. - Infrastructure, Mill. of Chained (2013) Pesos
27	EXP INVM	3	Source	Real Inv. - Mach.& Equip., Mill. of Chained (2013) Pesos
28	EXP PC	3	Source	Real Priv. Cons. - Total, Mill. of Chained (2013) Pesos
29	EXP PCD	3	Source	Real Priv. Cons. - Durable, Mill. of Chained (2013) Pesos
30	EXP PCND	3	Source	Real Priv. Cons. - Non Dur., Mill. of Chained (2013) Pesos
31	EXP GOV	3	Source	Real Gov. Expenditure, mill. of chained (2013) pesos
32	EXP X	3	Source	Real Exports, Mill. of Chained (2013) Pesos
33	EXP M	3	Source	Real Imports, Mill. of Chained (2013) Pesos
34	EXP STOCK	0	Source	Stock Variations to Real GDP, Rate
Prices				
35	CPI	3	Own	Cons. Price Index - Total (2013=100)
36	CPI CORE	3	Own	Cons. Price Index - Core (2013=100)
37	CPI COREG	3	Own	Cons. Price Index - Core ex Services (2013=100)
38	CPI CORES	3	Own	Cons. Price Index - Core ex Goods (2013=100)
39	CPI FOOD	3	Own	Cons. Price Index - Foodstuffs (2013=100)
40	CPI ALCOH	3	Own	Cons. Price Index - Alcoh. Beverages & Tobacco (2013=100)
41	CPI APPRL	3	Own	Cons. Price Index - Apparel (2013=100)
42	CPI COMB	3	Own	Cons. Price Index - House Rentals, Util. & Fuel (2013=100)
43	CPI HOUSE	3	Own	Cons. Price Index - Housing (2013=100)
44	CPI HLTH	3	Own	Cons. Price Index - Healthcare services (2013=100)
45	CPI TRANS	3	Own	Cons. Price Index - Transportation (2013=100)
46	CPI COMUN	3	Own	Cons. Price Index - Communication (2013=100)
47	CPI CULT	3	Own	Cons. Price Index - Culture (2013=100)
48	CPI EDUC	3	Own	Cons. Price Index - Education (2013=100)
49	CPI REST	3	Own	Cons. Price Index - Restaurants & Hotels (2013=100)
50	CPI OTHER	3	Own	Cons. Price Index - Other (2013=100)
Employment				
51	EMP	3	Source	Employment - Total, Thousands
52	EMP AGRIC	3	Source	Employment - Agriculture, Thousands
53	EMP RET	3	Source	Employment - Retail, Thousands
54	EMP CONS	3	Source	Employment - Construction, Thousands
55	EMP UTIL	3	Source	Employment - Utilities, Thousands
56	EMP IND	3	Source	Employment - Industrial, Thousands
57	EMP MIN	3	Source	Employment - Mining, Thousands
58	EMP COMS	3	Source	Employment - Community Services, Thousands

Table 7: Data description.

Id	Mnemonic	Transf.	SA	Series description
59	EMP FINS	3	Source	Employment - Financial Services, Thousands
60	EMP TRANS	3	Source	Employment - Transport, Thousands
Other Labor Market Indicators				
61	UNEMP	0	Source	Unemployment Rate - National
62	UNEMP MA	0	Source	Unemployment Rate - Santiago Metropolitan Area
63	WFORCE	3	Source	Work Force, Thousands
64	WHOURS	3	None	Effective Worked Hours, Hrs. per Week
65	VACANTS	2	Own	Job Vacancies Index (1995=100)
Wages				
66	WGS	3	Source	Real Wages - Total (2013=100)
67	WGS MIN	3	Source	Real Wages - Mining (2013=100)
68	WGS IND	3	Source	Real Wages - Industrial (2013=100)
69	WGS UT	3	Source	Real Wages - Utilities (2013=100)
70	WGS CONS	3	Source	Real Wages - Construction (2013=100)
71	WGS RET	3	Source	Real Wages - Retail (2013=100)
72	WGS TR	3	Source	Real Wages - Transport (2013=100)
73	WGS FINS	3	Source	Real Wages - Financial Services (2013=100)
74	WGS COMM	3	Source	Real Wages - Community Services (2013=100)
Credit				
75	LOANS	3	None	Nominal Loans - Total, Billions of Pesos
76	C LOANS	3	None	Nominal Loans - Consumption, Billions of Pesos
77	EXT LOANS	3	None	Nominal Loans - Foreign Trade, Billions of Pesos
78	MRG LOANS	3	None	Nominal Loans - Mortgages, Billions of Pesos
79	CRP LOANS	3	None	Nominal Loans - Corporate, Billions of Pesos
Monetary Aggregates				
80	MON M0	3	Source	Monetary Aggregates - M0, Billions of Pesos
81	MON M1	3	Source	Monetary Aggregates - M1, Billions of Pesos
Interest Rates				
82	MPR	0	None	Monetary Policy Rate, % Per Annum
83	DEPN 90D	1	None	Average Nominal Deposit Rate, 90 Days, % Per Annum
84	DEPN 1A	1	None	Average Nominal Deposit Rate, 1 Year, % Per Annum
85	DEPN 3A	1	None	Average Nominal Deposit Rate, 3 Years, % Per Annum
86	DEPR 90D	1	None	Average Real Deposit Rate, 90 Days, % Per Annum
87	DEPR 1A	1	None	Average Real Deposit Rate, 1 Year, % Per Annum
88	DEPR 3A	1	None	Average Real Deposit Rate, 3 Years, % Per Annum
89	LOANN 90D	1	None	Average Nominal Loan Rate, 90 Days, % Per Annum
90	LOANN 1A	1	None	Average Nominal Loan Rate, 1 Year, % Per Annum
91	LOANN 3A	1	None	Average Nominal Loan Rate, 3 Years, % Per Annum
92	LOANR 90D	1	None	Average Real Loan Rate, 90 Days, % Per Annum

Table 7: Data description.

Id	Mnemonic	Transf.	SA	Series description
93	LOANR 1A	1	None	Average Real Loan Rate, 1 Year, % Per Annum
94	LOANR 3A	1	None	Average Real Loan Rate, 3 Years, % Per Annum
Stock Market				
95	IPSA	2	None	Stock Price Index - IPSA
96	IGPA	2	None	Stock Price Index - IGPA
97	INTER10	2	None	Stock Price Index - INTER10
Exchange Rates				
98	RER	2	None	Real Exch. Rate - Multilat. (Tr. Weight., 1986=100)
99	RER5	2	None	Real Exch. Rate - 5 Major Tr. Part. (Tr. Weight., 1986=100)
100	FX	3	None	Nom. Exch. Rate, US Dollars
101	FXM	3	None	Nom. Exch. Rate, Multilat. (Tr. Weight., 1/2/1998=100)
Surveys				
102	UCH TDAY	2	Source	UChile Surv. - Consumer Confidence (2001Q1=100)
103	UCH 12M	2	Source	UChile Surv. - Exp. Situation in 12 months (2001Q1=100)
104	UCH 1YRA	2	Source	UChile Surv. - Current Situation - Personal (2001Q1=100)
105	UCH FAM	2	Source	UChile Surv. - Current Situation - Family (2001Q1=100)
106	UCH NAT	2	Source	UChile Surv. - Current Situation - Country (2001Q1=100)
107	EEE GDP	0	Source	CBCh Econ. Exp. Surv. - GDP growth for current quarter
108	EEE CPI	0	Source	CBCh Econ. Exp. Surv. - CPI infl. (annual) in 12 months
109	EEE MPR	0	Source	CBCh Econ. Exp. Surv. - MPR in 12 months, % Per Ann.
110	EEE FX	0	Source	CBCh Econ. Exp. Surv. - RER in 12 months