# On Precautionary Money Demand

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Abstract

I solve a model of precautionary money demand where households face idiosyncratic shocks to their marginal utility of consumption. The household need to decide the portfolio allocation between a liquid asset; money and an illiquid one; shares on a fixed supply asset producing an stochastic dividend. Money has a liquidity property in the Cash-in-Advance tradition, once households have chosen their portfolio allocation, they then find out their actual marginal utility of consumption which defines how urgent their consumption is, at that stage only money can serve the purpose. In spite of the inherent heterogeneity of the model, it can be solved with standard methods as a representative agent is constructed. I take data on households portfolio allocation from the Flow of Funds, and examine if the model can mimic their behavior when the model is fed with actual data for dividends. I focus on the period 1995-2016, which covers the last two US recessions, when the economy's dividends of the stock market have fluctuated considerably. I find that in spite of the model's unique driver, fluctuating dividends, the fraction of liquid assets it delivers reasonably trace the data around the recessive periods. Households in the model are quite sophisticated in their optimization behavior, hence the result of this paper warns against some views risen since the great recession that traditional macro models emphasizing rationality are useless to understand complex financial decisions.

**Keywords: Precuationary money demand, Portfolio allocation, Heterogeneity.**

**JEL Classification: E10,E41.**

*E-mail: sergiosalaslan@gmail.com. This paper draws heavily in Professor Robert Lucas' model framework of Lucas [2010], which was developed in a course I audit in 2010. All errors are entirely my responsibility.

# 1 Introduction

During recent US recessions, asset markets became quite volatile. Portfolio decisions for both firms and households showed marked re-balances as compared to pre-recession levels. The academic discussion focused mainly on firms' financial decisions, but fewer studies have been conducted for households. It is important to understand households portfolio decisions because as documented by Wen [2015] and Mulligan and Sala-i-Martin [2000] a large fraction of households only have liquid assets such as cash and savings accounts.

This paper solves a model of portfolio decisions for households, borrowing in fundamental ways on a theory developed by Robert Lucas (Lucas [2010]). Lucas laid down the main foundations of the theory in a general set up. The main idea is that individuals may be buffeted by "urgencies to consume": stochastic changes in their marginal utility of consumption. Households need to make a decision each period about how much of their financial wealth is invested in a given asset whose dividend is stochastic and how much is kept in cash. There exists a financial friction in that once the portfolio decision is made the asset market is closed and only then the individuals find out how urgent is their consumption. At that point, the only liquid asset is money that is used to finance consumption, because a Cash-in-Advance constraint is in place. Money demand therefore is precautionary.

The model features ex-post heterogeneity because different individuals may face different consumption urgencies, and different individuals will have different wealth accumulation patterns. I show that using a standard CRRA utility function, the policy functions are linear in wealth and therefore aggregation is straightforward, and the model can be solved with a representative agent. Once the equilibrium is found, I take the model to the data by focusing on the recent US economic history. First, I compute the fraction of households' "liquid assets" from the US flow of funds and I also take the data of aggregate dividends from Robert Shiller's database. Second, I feed the model with this actual dividend data series since 1995. The model then gives the theoretical series of the fraction of liquid assets which can be compared to the data.

I focus on periods near the two last US recessions, of 2001 and 2008, where there have also been important fluctuations in the stock market. Data on dividends show that they start decreasing prior to the 2001 recession, increase strongly from 2004 to 2008 at which point they are almost flat to then decrease markedly since 2009 to 2010. Dividends are increasing afterwards. The constructed series from the Flow of Funds for the fraction of liquid assets also show volatile behavior around those periods, increasing during the 2001 recession with a pick in 2003, after which it decreases to rise markedly again for the 2008 recession and start declining since the end of 2009 onwards. While the model is unable to trace closely the actual series of the fraction of liquid assets since 1995, it performs reasonably well in tracing the general pattern. On one hand individuals in the model are highly sophisticated in their optimization and on the other hand the model leaves out potentially many drivers of household portfolio management, considering only aggregate dividends and the role of the liquidity of money due to urgencies to consume. Therefore I view the results as validating the main driver of portfolio re-balances of the theory, that is, individuals make an effort to economize on cash holdings based on the behavior of the stock market. This is important because since the last recession some analysts have raised concerns about the usefulness of macro models that emphasize individual rationality. This model shows that at least as an approximation, actual households in their portfolio decisions and money demand, behave as the sophisticated agents in the model.

**Related Literature**

The model presented in this paper fits into the class of heterogeneous agent models where idiosyncratic shocks induce heterogeneity in asset holdings. One class of models of this sort belongs to the Bewley [1977] tradition, such as Hugget [1993], Aiyagari [1994], and Krusell and Smith [1998]. Although these papers consider shocks to income, not to marginal utility of consumption, there is a large strand of the literature that focuses on this case. Lucas [1980], Taub [1988], Taub [1994], Wen [2015], Lucas [1992], and Atkenson and Lucas [1992] consider shocks to marginal utility of consumption. None of these contributions uses a simple model to investigate if the model's portfolio allocation is consistent with what we observe in the data. The closest previous work, to the best of my knowledge, is that of Wen [2015]. He uses a similar liquidity model to study the welfare cost of inflation. In this paper I do not pursue this venue. The model is constructed to give a specific prediction regarding the portfolio allocation of wealth among a liquid asset and an illiquid asset.

The paper is organized as follows: Section 2 presents the model, Section 3 presents the analysis of the theoretical model, Section 4 presents the empirical analysis, and Section 5 concludes.

# 2 The Model

## 2.1 Environment

The economy is populated by a measure one of infinite lived agents indexed by $i$. They aim to maximize:[1]

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \theta_{it} u(c_{it}), \tag{2.1}$$

$\mathbb{E}$ is the expectation operator that refers to randomness induced by an aggregate and an idiosyncratic shock. The aggregate shock refers to the dividend of the asset, and it will be displayed later. $\theta_{it}$ is the idiosyncratic shock, an "urgency to consume" which was widely used in the "Bewley" tradition. In this context it is used to rationalize liquidity demand, as is explained shortly. I assume that $\theta_{it}$ are independent and identically draws with distribution $F(\theta)$, $\theta \in \Theta \equiv [1, \infty)$.

Each period is divided in two subperiods. There is a single asset in the economy in fixed supply, a "tree" that produces stochastic dividend $d_t$ each period. Agents start the first sub-period with nominal wealth $W_{it}$. Observing current dividend $d_t$, they make a portfolio decision. Let $Q_t$ be the nominal price of shares over the dividend. Individuals decide on how much of nominal wealth to hold as money, and how much of asset shares to buy at price $Q_t$. Let $A_{it}$ and $M_{it}$ be the number of shares and money the household decides to hold, respectively. Portfolio decisions must satisfy: $W_{it} = Q_t A_{it} + M_{it}, A_{it} \geq 0, M_{it} \geq 0$. The higher the amount of shares the household holds the less money is kept for the second subperiod.

[1] Throughout, nominal variables will be denoted by upper case letters, real variables with lower case letters, and aggregates without subindex $i$.

Purchases of shares are paid with cash that will increase money holdings of other households, those who sell shares. Once this portfolio decision is made, the asset market close and no further transactions of assets are allowed.

The portfolio decision in the first subperiod is not trivial even though the aggregate shock is observed. Because in the second sub-period agents will face $\theta_{it}$, and any amount of consumption must be purchased with money: there is a CIA constraint. In this second subperiod we can think of agents composed by two individuals, a producer, and a shopper. The producer collects the dividend share $A_{it}d_t$ and sell it at price $P_t$. The shopper carries money set in the previous subperiod and purchase the consumption good from other households that are selling dividend shares. As is usual in this CIA environment, money from selling dividend shares by the producer cannot be used to purchase the consumption good by the shopper. At the beginning of next period, the household holds the shares from last period: $A_{it} = (W_{it} - M_{it})/Q_t$ and the nominal value of sales of dividend shares plus unspent cash carried over: $P_t A_{it} d_t + M_{it} - P_t c_{it}$:

$$W_{it+1} = Q_t \frac{W_{it} - M_{it}}{Q_t} + P_t \frac{W_{it} - M_{it}}{Q_t} d_t - P_t c_{it} + M_{it} \tag{2.2}$$

Therefore, the feasibility set for the household is given by:

$$W_{it+1} = W_{it} + \frac{W_{it} - M_{it}}{Q_t} P_t d_t - P_t c_{it} \tag{2.3a}$$

$$P_t c_{it} \leq M_{it} \tag{2.3b}$$

I disregard money injections in the economy and express the constraint in real terms by dividing by the price level $P_t$. Furthermore, a convenient formulation is obtained by defining the fraction of "liquid" wealth as: $z_{it} = M_{it}/W_{it}$. Under such a formulation constraints are written as:

$$w_{it+1} = [1 + (1 - z_{it}) y_t] w_{it} - c_{it} \tag{2.4}$$

$$c_{it} \leq w_{it} z_{it} \tag{2.5}$$

where $y_t = d_t/q_t$, has been defined as the "aggregate yield" for the asset, the aggregate dividend divided by its real price, where $q_t = Q_t/P_t$ . The constraint (2.4) has an intuitive explanation. Next period *real* wealth is given by current period wealth $w_{it}$ plus any net additions. On one hand real wealth is increased by the fraction of wealth put in shares which result in extra wealth of $(1 - z_{it})w_{it}y_t$. Given a portfolio decision reflected in $z_t$, the higher the dividend, the more wealthy the household becomes for next period. The higher the price of shares, the lower is wealth for next period because a higher fraction of wealth is currently used to purchase shares. On the other hand, wealth is decreased by the amount of resources used to finance consumption, $c_{it}$.

Depending on the realization of $\theta_{it}$ a household may or may not bind the CIA constraint. We expect that a high urgency to consume will lead to a binding CIA constraint. Money demand is precautionary, money is kept in the portfolio to finance consumption in the second subperiod. Ex post, a household may find that it has excess cash for consumption and another household may find that it would have wanted to keep a higher fraction of liquid wealth. Note that unspent cash is already subsumed in $w_{it}$ in (2.4), therefore if consumption is actually low such that the CIA constraint (2.5) does not bind, then the household would have wanted to choose a lower $z_{it}$, which increases wealth for next period without sacrificing current consumption. It is clear that this model displays an economy with ex-post inefficiency.

Finally, the only exogenous variable is dividends which follow the process:

$$\ln d_t = (1 - \rho) \ln d + \rho \ln d_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim (0, \sigma^2) \tag{2.6}$$

## 2.2 Equilibrium

Let $h_{it}(w), A_{it}(w), z_{it}(w)$ denote the optimal next period real wealth, nominal shares and the fraction of liquid wealth respectively at time $t$, of an individual with current real wealth $w_{it} = w$. Given next period wealth, optimal consumption $c_{it}$ is derived from the budget constraint (2.4). The transition for wealth is fully determined by the functions $h_{it}(w), A_{it}(w), z_{it}(w)$.

Assume for now that there exists a joint distribution of real wealth and urgencies to consume in the population $\Psi_t(w,\theta) = \psi_t(w)F(\theta)$, where the factorization holds because of the i.i.d. assumption on $\theta$. With these definitions and assumptions an equilibrium is defined.

**Definition 1.** *An equilibrium is a sequence of prices of shares $\{q_t\}_{t=0}^{\infty}$, a sequence of next period real wealth, nominal shares and fraction of liquid assets $\{h_{it}(w), A_{it}(w), z_{it}(w)\}_{t=0}^{\infty}$, a sequence of distributions $\{\psi_t\}_{t=0}^{\infty}$, such that, given the initial distribution $\psi_0$:*

1. *$\{h_{it}(w), A_{it}(w), z_{it}(w)\}_{t=0}^{\infty}$ are optimal*
2. *$\Psi_t$ is consistent with individuals' policies*
3. *The market on shares over the aggregate dividend, clears:*

$$\int_{\omega_t} A_{it}(w) d\psi_t(w) = 1 \tag{2.7}$$

*where:*

$$\omega_t \equiv \omega_t(w_{it+1}) = \{w_{it} : (1 - z_{it}) y_t w_{it} \leq w_{it+1} \leq [1 + (1 - z_{it}) y_t] w_{it}\}$$

## 3 Analysis

Even if agents start with the same level of wealth, because they are buffeted recurrently with idiosyncratic shocks, they will be, at any moment, heterogenous in their wealth. I show below that by assuming that the distribution of the idiosyncratic shock is continuous, and CRRA preferences:

$$u(c_{it}) = \frac{c_{it}^{1-\sigma}}{1-\sigma}, \tag{3.1}$$

then closed form solutions are found for policy functions that are linear in wealth. This result is related to Samuelson [1969]. The fact that policy functions turn out to be linear functions of wealth enable easy aggregation.

I begin by formulating the Bellman equation of the households' problem.

$$\mathcal{V}_{it}(w_{it}) = \max_{z_{it}} \int_{\Theta} \max_{c_{it}} \left[\theta_{it} u\left(c_{it}\right) + \beta \mathbb{E}_t \mathcal{V}_{it+1}(w_{it+1})\right] dF(\theta) \tag{3.2}$$

subject to (2.4) and (2.5).

I employ a Guess and Verify approach. First, I assume that all agents fix a threshold value $\theta_t$, which may depend on any information in the first sub-period. In the second subperiod, once $\theta_{it}$ is known, $\theta_t$ will determine which individuals will be desperate and not desperate. Desperate individuals will have $\theta_{it} > \theta_t$, binding their CIA constraint, all money they carried to that period will be spent on the consumption good. Non desperate individuals will face $\theta_{it} \leq \theta_t$, will not bind their CIA constraint and unspent money will be carried as wealth for next period. Note that the cutoff value is not idiosyncratic, it may depend on any aggregate variable, but not on individual's variables. This result is verified later, and it is inherited by the CRRA assumption on preferences.

To solve (3.2), I use the following guess for the value function:

$$\mathcal{V}_{it}(w_{it}) = \mathcal{V}_t(w_{it}) = D_t \frac{w_{it}^{1-\sigma}}{1-\sigma} \tag{3.3}$$

Note that under (3.3), the value function itself is the same across individuals, of course, individual wealth will differ among them and so their welfare. $D_t$ is an unknown stochastic parameter to be determined later.

Let me define:

$$\mathcal{U}_t(w_{it}, \theta_{it}) = \max_{c_{it}} \left[\theta_{it} u\left(c_{it}\right) + \beta \mathbb{E}_t \mathcal{V}_{t+1}(w_{it+1})\right] \tag{3.4}$$

subject to (2.4) and (2.5), as the value function for the inner problem in (3.2). Besides the idiosyncratic preference shock and individual wealth, which is made explicit, the function may depend on any other aggregate variables dated $t$. I further guess that $z_{it} = z_t$, hence the guess is that all individuals will allocate the same fraction of their wealth in money. The function $\mathcal{U}_t(w_{it}, \theta_{it})$ in (3.4) is therefore interpreted as being *conditional* on a given value $z_t$ chosen in the first subperiod.

Characterization of (3.4) and its associated policy functions are given in Proposition 1.[2]

**Proposition 1.** *Under* (3.3)*, individual's policy functions are:*[3]

$$h_{it}(w,\theta) = \begin{cases} \zeta_{it}[1+(1-z_t)y_t]w_{it} & \text{for} \quad \theta_{it} \leq \theta_t \\ (1+y_t)(1-z_t)w_{it} & \text{for} \quad \theta_{it} > \theta_t \end{cases} \tag{3.5}$$

*where:*

$$\zeta_{it} \equiv \frac{(\beta \mathbb{E}_t D_{t+1})^{\frac{1}{\sigma}}}{(\xi_t \theta_{it})^{\frac{1}{\sigma}} + (\beta \mathbb{E}_t D_{t+1})^{\frac{1}{\sigma}}} \tag{3.6}$$

*and:*

$$\mathcal{U}_t(w_{it},\theta_{it}) = \begin{cases} \mathcal{W}_t(\theta_{it})u(w_{it}) & \text{for} \quad \theta_{it} \leq \theta_t \\ \mathcal{W}_t(\theta_{it})u(w_{it}) & \text{for} \quad \theta_{it} > \theta_t \end{cases} \tag{3.7}$$

*where $\mathcal{W}_t$ is a piecewise function:*

$$\mathcal{W}_t(\theta_{it}) = \begin{cases} [\theta_{it}(1-\zeta_{it})^{1-\sigma} + \beta \mathbb{E}_t D_{t+1}\zeta_{it}^{1-\sigma}][1+(1-z_t)y_t]^{1-\sigma} & \text{for} \quad \theta_{it} \leq \theta_t \\ \theta_{it} z_t^{1-\sigma} + \beta \mathbb{E}_t D_{t+1}\left[(1+y_t)(1-z_t)\right]^{1-\sigma} & \text{for} \quad \theta_{it} > \theta_t \end{cases} \tag{3.8}$$

Results in Proposition 1 are very useful. In particular note that policy functions are linear in wealth, this is true independently of the actual urgency to consume individuals face. Also, the inner value function $\mathcal{U}_t(w_{it}, \theta_{it})$ is factorized as the product of a function $\mathcal{W}_t(\theta_{it})$ and $u(w_{it})$, where $u(\cdot)$ is the

[2] The proof of this Proposition is omitted as it follows the standard guess-and-verify method.

[3] I omit the policy functions for consumption, since they can be readily obtained from the budget constraint (2.4).

utility function in (3.1). Therefore is straightforward the result that:

$$\mathcal{V}_t(w_{it}) = \max_{z_t} \left[ \int_\Theta \mathcal{W}_t(\theta_{it}) dF(\theta) \right] u(w_{it}) \tag{3.9}$$

And then from (3.3), I obtain:

$$D_t = \max_{z_t} \int_\Theta \mathcal{W}_t(\theta_{it}) dF(\theta) \tag{3.10}$$

Obtained as part of the Guess and Verify method. Note that upon integration over $\Theta$ in (3.10), the optimal $z_t$ is obtained which is independent of any individual variable, which confirms the guess that the value function in (3.3) is the same for all individuals.

**Optimality Conditions**

The first order condition for the maximization problem in (3.10) gives:

$$\begin{aligned} (z_t)^{-\sigma} \int_{\theta_t}^{\infty} \theta_{it} dF(\theta) &= \left[ \int_1^{\theta_t} \theta_{it}(1-\zeta_{it})^{1-\sigma} dF(\theta) + \beta \mathbb{E}_t D_{t+1} \int_1^{\theta_t} \zeta_{it}^{1-\sigma} dF(\theta) \right] [1 + (1-z_t) y_t]^{-\sigma} y_t \\ &+ \beta \mathbb{E}_t D_{t+1} [(1+y_t)(1-z_t)]^{-\sigma} [1 - F(\theta_t)](1+y_t) \end{aligned} \tag{3.11a}$$

Making explicit $D_t$ in (3.10):

$$\begin{aligned} D_t &= \left[ \int_1^{\theta_t} \theta_{it}(1-\zeta_{it})^{1-\sigma} dF(\theta) + \beta \mathbb{E}_t D_{t+1} \int_1^{\theta_t} \zeta_{it}^{1-\sigma} dF(\theta) \right] [1 + (1-z_t) y_t]^{1-\sigma} \\ &+ (z_t)^{1-\sigma} \int_{\theta_t}^{\infty} \theta_{it} dF(\theta) + \beta \mathbb{E}_t D_{t+1} [(1+y_t)(1-z_t)]^{1-\sigma} [1 - F(\theta_t)] \end{aligned} \tag{3.11b}$$

Determination of $\theta_t$ is given by the fact that for the individual who faces a shock $\theta_{it}$ which is equal to $\theta_t$, policy functions derived above must coincide. For either policy, consumption or next period wealth, I get:

$$\frac{(\theta_t)^{\frac{1}{\sigma}}}{(\theta_t)^{\frac{1}{\sigma}} + (\beta \mathbb{E}_t D_{t+1})^{\frac{1}{\sigma}}} = \frac{z_t}{1 + (1-z_t) y_t} \tag{3.11c}$$

To find out the equation of motion for wealth, I integrate the policy functions for next period wealth in (3.5). For this I define: $h_{it}(w) = \int_{\Theta} h_{it}(w,\theta)dF(\theta)$. Integrating (3.5):

$$h_{it}(w) \quad = \quad \left\{[1+(1-z_t)y_t]\int_1^{\theta_t} \zeta_{it} dF(\theta) + (1+y_t)(1-z_t)[1-F(\theta_t)]\right\} w_{it} \tag{3.11d}$$

Defining aggregate values for wealth as $w_{t+1} = \int_{\omega} h_{it}(w)d\psi_t(w)$ and $w_t = \int_{\omega} w_{it}d\psi_t(w)$, is straightforward to integrate (3.11d) to get:

$$w_{t+1} = \left\{[1+(1-z_t)y_t]\int_1^{\theta_t} \zeta_{it} dF(\theta) + (1+y_t)(1-z_t)[1-F(\theta_t)]\right\} w_t \tag{3.11e}$$

Finally, market clearing condition (2.7) applied to $w_{it} = q_t A_{it} + m_{it} = q_t A_{it} + z_{it} w_{it}$ gives:

$$w_t = \int_{\omega} w_{it} d\psi_t(w) = q_t \int_{\omega} A_{it} d\psi_t(w) + z_t \int_{\omega} w_{it} d\psi_t(w) = q_t + z_t w_t$$

from which:

$$w_t = \frac{q_t}{1-z_t} \tag{3.11f}$$

is obtained.

Equations (3.11a),(3.11b),(3.11c),(3.11e) and (3.11f), define a stochastic rational expectations dynamic system for the variables $z_t, D_t, \theta_t, w_t$ and $q_t$, along with (2.6). In Appendix A.2, I further examine the model performance by showing impulse response functions.

# 4 Empirical Analysis

## 4.1 The Data

I first construct the empirical analogue to $z_t$ from the US flow of funds. I consider liquid assets to be composed of checkable deposits and currency plus time and savings deposits plus money market mutual fund shares, the rest of the households assets are considered illiquid. Please see the Appendix A.1 for further details. I also consider real aggregate dividends from S&P 500 stocks, this series is taken from Robert Shiller's database.[4]

The following graphic shows the computed value for the fraction of liquid assets along with dividends from 1995 to 2016. I restrict the analysis to this period for two reasons. First, I intend to examine what the model has to say for the recent two recessive periods for the US. Second, many researchers have found that the financial sector had undergone some structural changes during the eighties.[5] In fact, when considering the empirical measure of the fraction of liquid assets as early as 1951, it is found that it persistently increases until late eighties. Thereafter decreasing during the nineties and from that decade onwards the series seem to fluctuate around a constant mean. The average value of the fraction of liquid assets since 1995 equals roughly 23%, this value will be used for calibration of the parameters of the model.[6]

Figure 1 show the data for the three series of interest. The fraction of liquid assets fluctuate around the average of 23%. It can be seen how the fraction of liquid assets increase near the recessive periods. The blue dotted line show real dividends which increase over time. Dividends start falling prior to the 2001 recession, since 2004 dividends start growing strongly and during the 2008 recession they cease to grow, only dropping strongly towards the end of the recession. Finally, the real price of the asset is

[4] Robert Shiller's data is available at http://www.econ.yale.edu/ shiller/data.htm. This data is an update of the data used in his monograph: Robert Shiller [1989].

[5] For example Urban and Quadrini [2012] in their study of financial frictions, focused on the period 1984 to 2011 for their empirical part that uses Flow of Funds. They argue that regulatory changes beginning 1980 had an impact in financial flows.

[6] The Appendix A.1 show data from 1951 to 2016.

seen to drop strongly and by almost equal magnitude during both recessions.

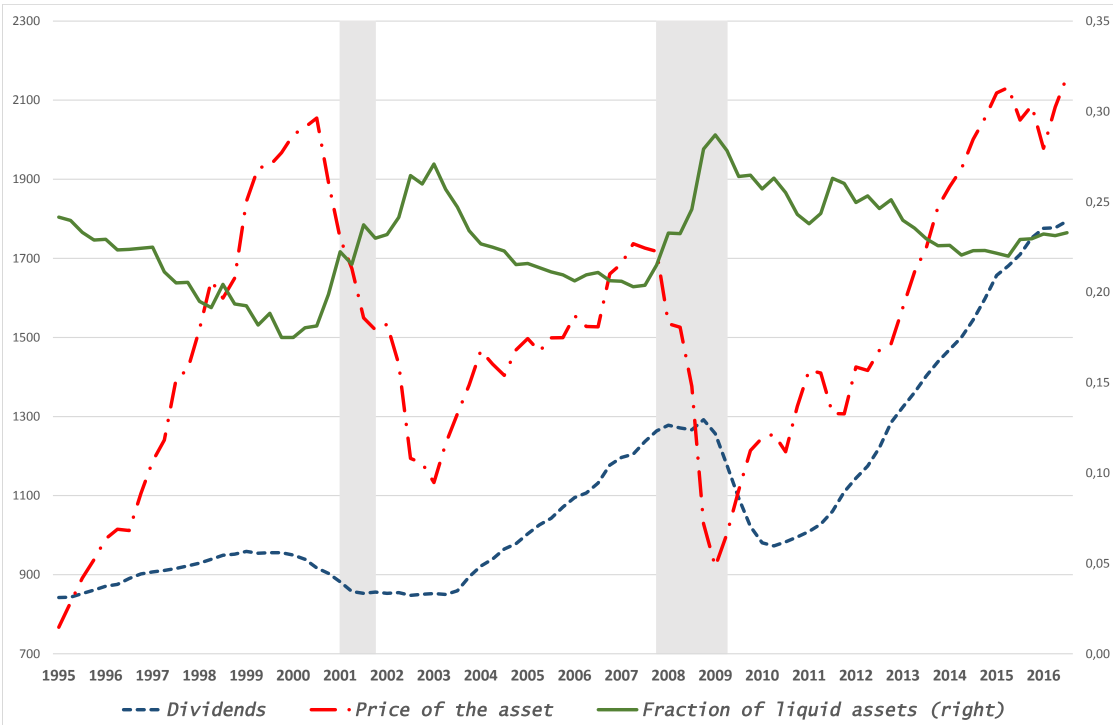


Figure 1: Dividends and price of assets taken from Robert Shiller's database. The fraction of liquid assets is computed from the Flow of Funds. The shaded areas depict the last two recessions.

My focus is in determining if the model is able to replicate the fraction of liquid assets as is seen in the figure, when it is fed with the real dividends shown in the graphic. The model also delivers the counterpart to the real price of assets in the data, but no further analysis is carried involving this series. Is a known fact that simple models as the one presented here are incapable of delivering realistic dynamics for asset prices.

To perform the exercise in mind, I need to calibrate the parameters of the model and make an assumption regarding the distribution of the urgencies to consume. I set $\sigma = 1$, that is logarithmic utility, I have also tried with values above unity, but for the task at hand, it made little difference. I assume that the distribution for idiosyncratic shocks is Pareto:

$$F(\theta) = 1 - \theta^{-\eta}, \eta > 1, \theta \in \Theta \equiv [1, \infty) \tag{4.1}$$

Such an assumption is also used by Wen [2015]. Despite the simplicity of this distribution, most of the integrals in the model cannot be obtained analytically, so I used numerical integration with Gauss-Legendre quadrature. The parameter $\eta$ is calibrated so that the steady state fraction of households' liquid assets is equal to the average in the data of 23%, which turn out to be $\eta = 1.26$. Finally, I set $\beta = 0.99$.

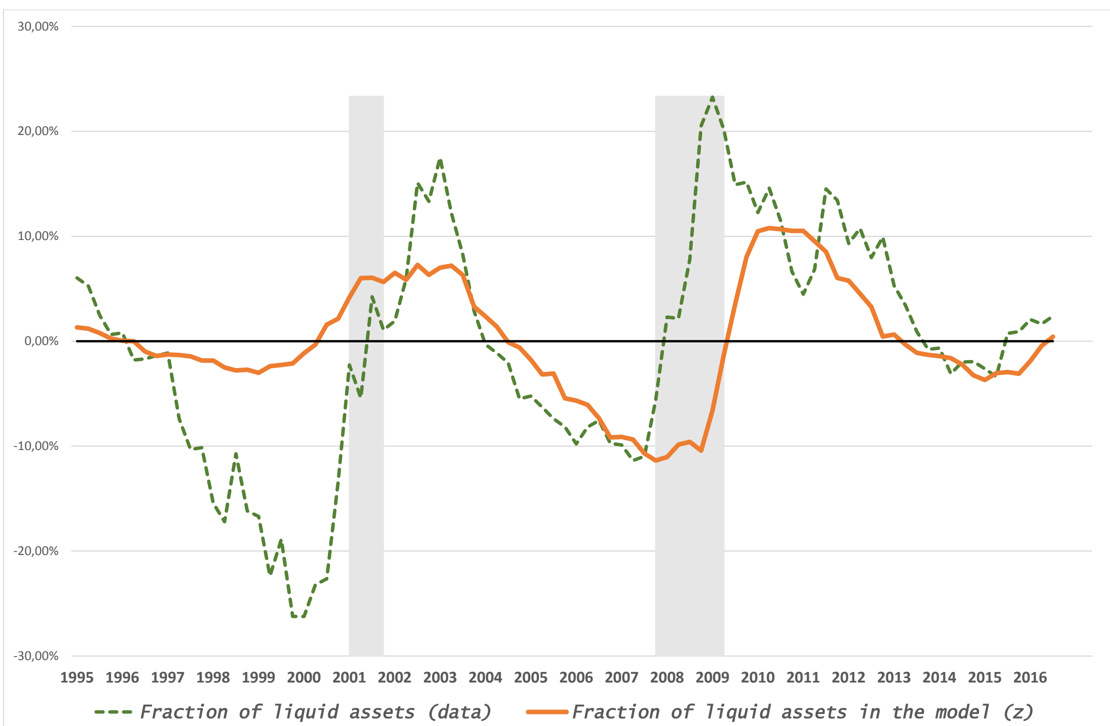


Figure 2: Dashed line show $z$ in the data, continuous line show $z_t$ as produced by the model.

Figure 2 show the results of the exercise. I de-trended the dividends series with a cubic trend, and I subtracted the average of 26% from the constructed series of the fraction of liquid assets. It can be seen that the model fails by large until prior to the 2001 recession. Afterwards however, the model delivers a series that is a reasonable approximation to the data. It can be seen that the theoretical model predicts an increase in $z_t$ prior to the 2001 recession, while data shows an increase during the recession, but the general pattern is comparable. From 2004 to 2008, the model is quite successful to trace the series. During the recession of 2008, the model lags actual data, because it increases above average at the end of the recession. Notwithstanding, the general pattern in the data since 2009 onwards is reasonably approximated by the model. Again, I must mention that the only driver of the model dynamics is actual dividends. Perhaps the introduction of other shocks may help to trace better the fraction of liquid assets in the data. But adding many shocks to match the data, in my opinion, can hardly be considered a reasonable explanation of a given phenomena. The main question of this paper was whether a simple model of portfolio management that focus on stochastic dividends and a motive for precautionary demand can account for what is observed in the data, and it was found that such a model performs reasonably well.

## 5 Conclusion

This paper solved a small model of households' portfolio management. The unique driver of dynamics in the model is aggregate dividends, which are assumed stochastic. The model displayed easy aggregation in spite of the inherent heterogeneity, and it was used to determine if it can deliver the time path of the actual fraction of households' liquid assets as it is observed in the data when fed with the actual dividends time series. The exercise is quite coarse and simple, but the model is capable to generate the general pattern of the fraction of liquid wealth as seen in the data in a large lapse of time since 1995. Simplicity of the model does not mean that households optimization problem in the model is simple, in fact it is quite sophisticated. Therefore the results of this model illustrate that macro models that emphasize individual rationality under complex financial decisions are not useless, as one often hear by some analysts since the last financial crisis.

# A Appendix

## A.1 Data

For stock market data, I use Robert Shiller's database, available at http://www.econ.yale.edu/ shiller/data.htm. To construct the fraction of liquid assets - cash - I resort to the Flow of Funds Accounts. Cash, is a broad definition that includes: checkable deposits and currency FL153020005, total time and savings deposits FL153030005, money market mutual fund shares FL153034005.

Risky assets are also a broad definition that includes: debt securities FL154022005, corporate equities LM153064105, mutual fund shares LM153064205, proprietors' equity in non-corporate business FL152090205. The single large item excluded from assets in this definition is pension entitlements FL153050005, which is excluded because pensions have their own dynamics not related to cash management addressed by the model of the paper. The whole amount of assets considered for these definitions account for roughly 70% of total assets for Households and Non-Profit Organizations. Figure 3 shows the data for the period 1951-2016

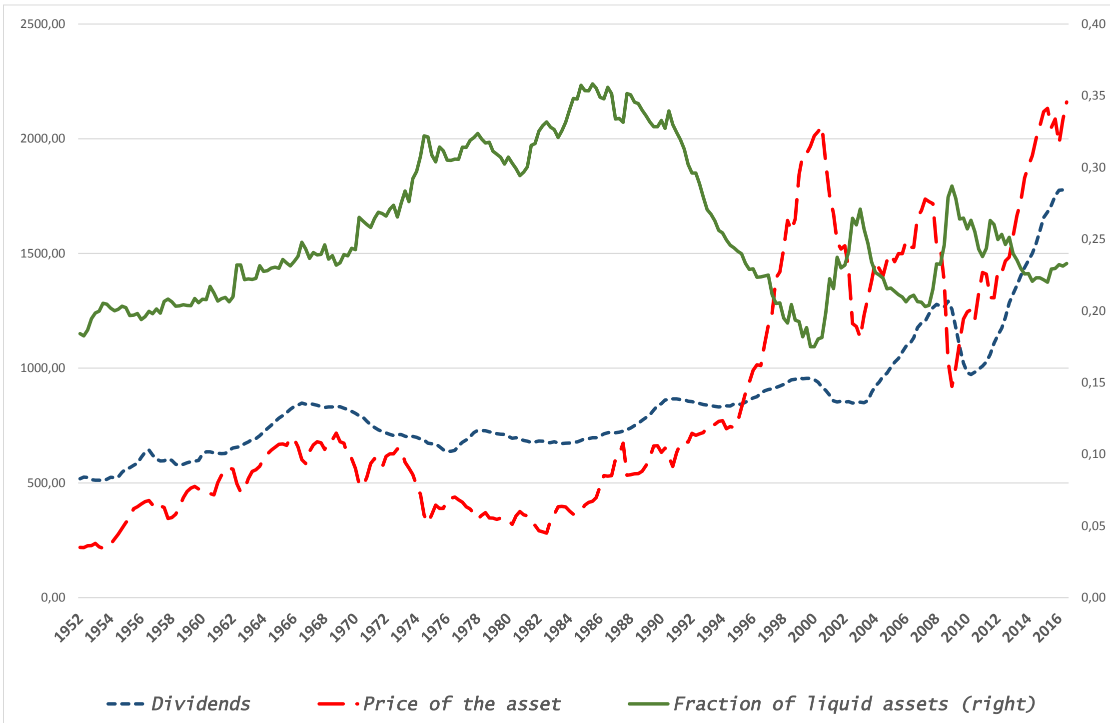


Figure 3: Data on dividends and prices taken form Robert Shiller's database. The fraction of liquid assets computed from the Flow of Funds.

## A.2 Impulse Response Functions

To understand the model's dynamics, I compute impulse response functions for the linearized model to a negative one percent dividend shock. Figure 4 shows the results. The calibration used is discussed in the text.

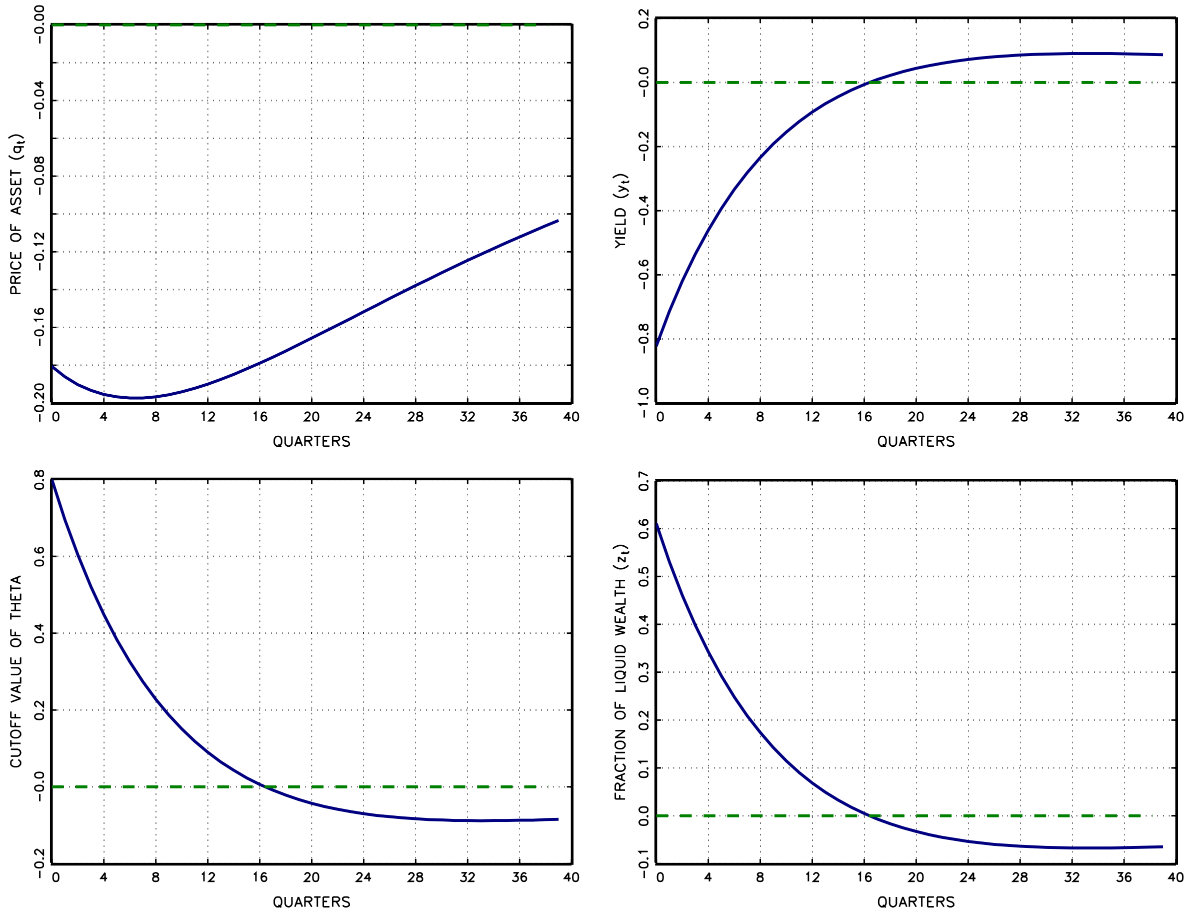


Figure 4: Percentage responses to a one percent *negative* shock to the dividend process, $\rho = 0.9$.

A negative dividend shock produces a negative wealth effect for the household, they tend to cut down on both consumption and savings. But at the same time, dividends are expected to be low for some time, therefore is not profitable to keep the same portfolio balance between shares and money. Low dividends will produce low consumption which makes a high urgency to consume later much more harmful. Households re-balance their portfolio towards more liquidity, this increases $z_t$. This means that there is lower demand for the asset whose price falls to clear the market. Note that the decrease in the price is only 0.2 percent at impact, therefore the yield decreases by 0.8 percent at impact. The cutoff value $\theta_t$ increases which means that the fraction of desperate households is reduced, this is a reflection of the fact that more cash is carried to the second subperiod, so the threshold above which households define themselves as desperate is reduced.

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