# USE OR LOSE IMPLIES ABUSE: THE SUPPLY SIDE OF WASTEFUL YEAR-END FISCAL EXPENDITURE

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`Preliminary and incomplete. Comments are welcome.`

Abstract. I study the behavior of firms participating in public procurement. It's been widely studied how year-end public expenditure is empirically much higher than average expenditure over the rest of any given fiscal year. Firms know this and may optimally condition their intertemporal pricing strategies to face this higher expenditure prospect. Using a two-period price-competition duopoly, I conclude that under the presence of capacity constrains, a mixed equilibrium exists in which a less limited firm profitably bids higher prices in public auctions. This result is driven by the possibility of participating in a single-offer auction when a competitor exhausts its capacity. A brief comment on the empirical relevance of this model is also presented.

## 1. Introduction

Public procurement has been subject to several critiques over the past years. These critiques point to different (in)efficiency considerations when it comes to assess what is accrued at the end of each fiscal year. Indeed, it's been vastly studied how different public-sector constrains, such as expiring budgets, generate *use it or lose it* incentives on public agencies (Liebman and Mahoney, 2016). When these agencies have considerable unspent resources near the end of a fiscal year, important expenditure spikes are observed (see Figure 1).

One might argue that this rises considerable interest in studying the demand side of this public procurement market. Indeed, there are few papers that examine how this behavior may be importantly conditioned by demand-side conditions (Engel et al., 2015). Nevertheless, this document focuses on the supply side implications of these reiterative patters that are observed on year-end public spending. The idea is simple: firms may anticipate higher levels of expenditure (and budget availability) at the end of any given year and behave strategically in order to maximize profits.

Nevertheless, there are non-trivial results behind this idea. Firstly, only might think that firms would set higher prices in order to extract the excess of budget that public agencies must spend. But then, if many firms are aware of this, they would also participate more on public procurement and introduce more competition into

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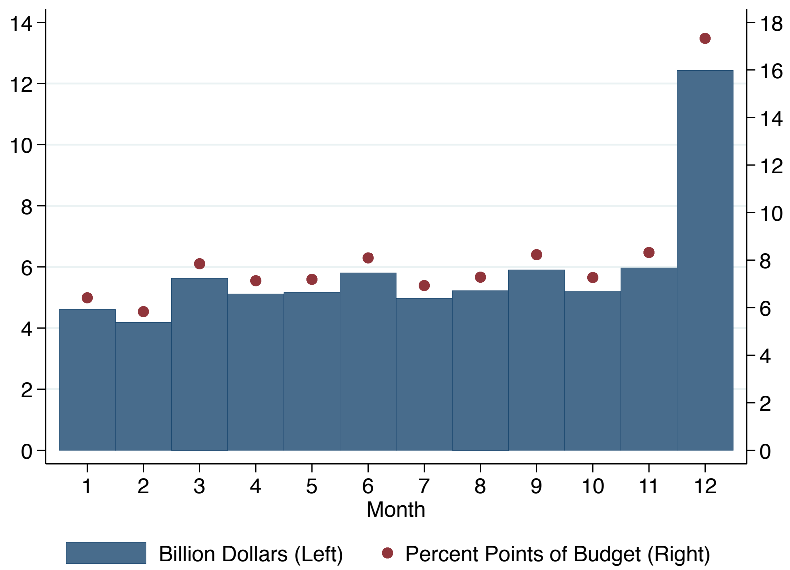
Figure 1. Average monthly public expenditure in Chile (2009-2011)




this market. The latter would drive prices down.

In order to study this setting in a simplified environment, I propose a price-competition duopoly model with capacity constrains represents the trade-off just described. Some classic results indicate that price competing duopolies with capacity constrains usually don't achieve an equilibrium (Levitan and Shubik, 1972). However, some settings such as those proposed by Dudey (1988) or Osborne and Pitchik (1986) do find equilibria.

In this case, there will exist a rather simple equilibrium in mixed strategies that will condition a high-capacity firm to bid higher prices than a more constrained firm. As opposed to Van Den Berg et al. (2012) or Anton, Biglaiser, and Vettas (2014), the model works with discrete units that are supplied *via* separate auctions. This paper also emphasizes on the strategic role of firms, more than the role of the public agency. I finally comment on the empirical relevance of this model and the real procurement scenarios that may fit it's setting.

## 2. Theoretical Model

Consider a single-good two-period economy with a representative public agency and two homogeneous firms which compete in public auctions. The public agency ought to purchase three non-divisible and non-stockable units of the good before period 2 ends and both firms have the capacity to supply two units each, at zero cost. Each purchased unit generates a social value of $V$, which is totally internalized by the public agency. This is public information for both firms, which are also aware of their zero-cost symmetry, their two-units capacity constraint and their common discount factor $\beta$. The winner of each auction is the firm with the lowest bid, and capacity is reduced only when a firm wins an auction (i.e. this is a dynamical price-competing duopoly with capacity constrains). If there's a tie, a coin is tossed and only one firm gets whatever demand is in dispute, leaving only the residual demand for the other firm. A firm with capacity $k$ can choose to participate in up to $k$ auctions and the set of chosen ones is private information, along with its bid. Nevertheless, the past history of auctions is common knowledge.

The setting of this model is very similar to that of Anton, Biglaiser, and Vettas (2014), but in this case we center our attention in optimal decisions of firms, which are capable of choosing the auctions they wish to bid on. Another key difference is that the public agency won't directly choose how much to buy from each firm, as it will be an outcome of the auction mechanism, i.e. there is no endogenous "split auction" possibility here (Anton and Yao, 1989; Anton and Yao, 1992).

First of all, in order to simplify all further analysis, I prove that firms will optimally choose only one price to characterize their bids.

**Proposition 1.** *Given any firm that participates in $n$ auctions in a single period, it is optimal for it to bid homogeneously in each of them, i.e. bids may be characterized by a single price strategy for all participating auctions of a given firm.*

*Proof.* First note that any bid above $V$ yields a payoff of zero. Thus, we may compactify the strategy space by limiting bids in the set defined by $p \in [0, V]$.

By contradiction, suppose a firm participates in two auctions in the same period and optimally bids two different prices $p_1 \neq p_2$. Then these prices must maximize the expected profit of the firm, i.e.

$$\mathbb{E}\pi(p_1, p_2) \equiv p_1 \cdot \mathbb{P}[win|p_1] + p_2 \cdot \mathbb{P}[win|p_2] \geq \mathbb{E}\pi(p_1', p_2') \quad \forall p_1', p_2' \geq 0.$$

It's easy to note that $\pi$ is not a continuous function over $(p_1, p_2)$, but $\mathbb{E}\pi$ is. Therefore, because of Berge's maximum theorem, an optimum exists.

Now consider the case where $p_1 \cdot \mathbb{P}[win|p_1] \neq p_2 \cdot \mathbb{P}[win|p_2]$. In this scenario, the firm could define $\bar{p} \equiv \arg\max_{p \in \{p_1, p_2\}} p \cdot \mathbb{P}[win|p]$, and bid $\bar{p}$ in each auction, obtaining

$$\mathbb{E}\pi(\bar{p}, \bar{p}) = 2\bar{p} \cdot \mathbb{P}[win|\bar{p}] > \mathbb{E}\pi(p_1, p_2). \quad \perp$$

The strict inequality is dropped when $p_1 \cdot \mathbb{P}[win|p_1] = p_2 \cdot \mathbb{P}[win|p_2]$. Thus, each pair of different prices is weakly dominated by a single price strategy and therefore optimal bids may be characterized by a subset of single price strategies. □

Now, using backwards induction, we know that in period 2 there are four different scenarios:

(1) The public agency purchased all three units of the good in period 1, so the game is over before period 2 starts.
(2) The public agency purchased two units of the good in period 1, so there are two possible (sub)scenarios:

   (a) The units were purchased from a single firm, so there is a monopolist in the second period which charges $V$.
   (b) The units were separately purchased from both firms, so firms compete *a la* Bertrand in the second period and drive their profits to 0.
(3) The public agency purchased only one unit of the good in period 1, so there is a firm with capacity 1 (the one who sold a unit in the first period) and another with capacity 2. Both will compete for two demanded units of the good.
(4) The public agency did not purchase any unit in period 1, so both firms have their full capacity available and complete for three demanded units of the good.

One might *a priori* discard case 2b because of Theorem 1 and the tie assumption, but this won't be necessary, as we'll further note that it won't be an equilibrium outcome. Let's start by analyzing cases 3 and 4, departing from the strategy spaces of firms in both scenarios.

**Proposition 2.** *Given a firm with capacity 2 and other firm with capacity 1, both in period 2, the effective strategy space for each firm is the set defined by* $p \in \left[\frac{V}{2}, V\right]$, *i.e. bidding any price above* $V$ *or below* $\frac{V}{2}$ *is strictly dominated by other price for each firm. This also holds when both firms have their full capacity available.*

*Proof.* We know that bidding above $V$ yields zero payoff, so any price above $V$ is strictly dominated by any price in $(0, V]$.

Now consider the less constrained firm: if it chooses a price $p$ below $\frac{V}{2}$, then it can profit up till a total of $2p < V$, whereas bidding $p = V$ will definitely yield at least $V$ (because the more constrained firm may win only one auction). Thus, any price below $\frac{V}{2}$ is strictly dominated by a price equal to $V$ for the less constrained firm.

Having this in mind, the more constrained firm will also find it profitable to bid at least $\frac{V}{2}$, as any price below this threshold is strictly dominated by it.

Note that a similar argument applies when both firms have full capacity: they obtain nothing by bidding above $V$ and they always prefer bidding $V$ instead of any $p < \frac{V}{2}$. □

Having this said, we may attempt to compute an equilibrium, starting with case 3. It's easy to note that there are no pure Nash equilibria, nevertheless...

**Proposition 3.** *Given a firm with capacity 2 and other firm with capacity 1, both in period 2, there will exist a mixed-strategy Nash equilibrium in which the distribution of the less constrained firm stochastically dominates in first order that of the* *more constrained firm.*

*Proof.* Note that there are no Nash equilibria in pure strategies, as firms will always have incentives to deviate by undercutting each other until they reach $\frac{V}{2}$ and then (re-)enter an Edgeworth cycle.

Nevertheless, a mixed strategy equilibrium exists. Let $p_L$ be the low-capacity firm's bid and $p_H$ be the high-capacity firm's bid and let $F(p_L)$ and $G(p_H)$ be their respective probability distributions, both sharing a common support equivalent to $\left[\frac{V}{2}, V\right]$.

Then, the high-capacity firm solves

$$\max_{p_H \in \left[\frac{V}{2}, V\right]} \mathbb{E}\pi^H(p_H) \equiv p_H \int_{V/2}^{p_H} dF(p_L) + 2p_H \int_{p_H}^{V} dF(p_L), \tag{1}$$

where the FONC is

$$F(p_H) + p_H F'(p_H) + 2[1 - F(p_H)] - 2p_H F'(p_H) = 0$$

and generates a first order linear differential equation restated as

$$F'(p_H) + \frac{F(p_H)}{p_H} = \frac{2}{p_H}.$$

By using an integrating factor of $\mu(p_H) \equiv \exp\left(\int \frac{dp_H}{p_H}\right) = kp_H$, with $k \in \mathbb{R}$, we know that the solution to the differential equation is

$$F(p_H) = \frac{\int kp_H \frac{2}{p_H} dp_H}{kp_H} = 2 + \frac{c}{p_H},$$

where $c$ is some real scalar.

Imposing our support constrain over this probability distribution, we solve for $c$ and finally obtain $F(p_L) = 2 - \frac{V}{p_L}$.

We repeat the same process with the low-capacity firm's optimization problem:

$$\max_{p_L \in \left[\frac{V}{2}, V\right]} \mathbb{E}\pi^L(p_L) \equiv p_L \int_{p_L}^{V} dG(p_H).$$

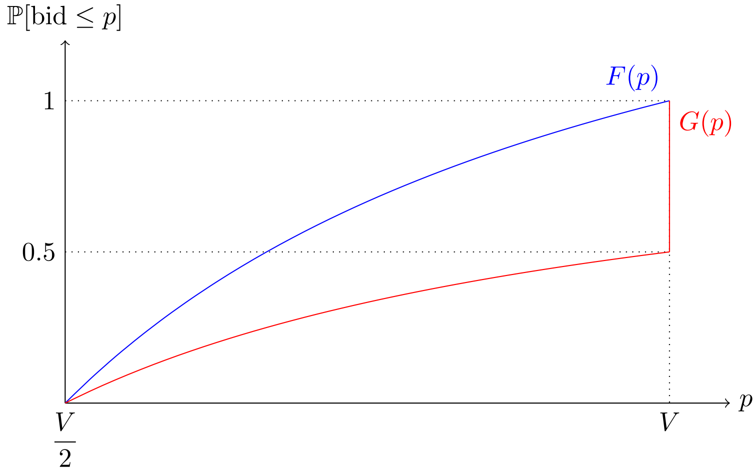
The FONC is now

$$[1 - G(p_L)] - p_L G'(p_L) = 0$$

and may be restated as another first order linear differential equation:

$$G'(p_L) + \frac{G(p_L)}{p_L} = \frac{1}{p_L}.$$

Figure 2. First-order stochastic dominance of $G$ over $F$




Again, $\mu(p_L) \equiv \exp\left(\int \frac{dp_L}{p_L}\right) = kp_L$ is the integrating factor that allows us to solve

$$G(p_L) = \frac{\int kp_L \frac{1}{p_L} dp_L}{kp_L} = 1 + \frac{c'}{p_L},$$

where $c'$ is another real scalar.

Imposing the support constrain, we obtain

$$G(p_H) = \begin{cases} 1 - \frac{V}{2p_H} & p_H \in \left[\frac{V}{2}, V\right) \\ 1 & p_H = V \end{cases}.$$

Note how the result is a tempered distribution, without a density defined at $p_H = V$. The reason behind this is that when a lower-bound support constrain is imposed we obtain the first section of the distribution. However, when we impose an upper-bound support constrain, the distribution degenerates into $G(p_H) = 1$, forcing a mass of 0.5 to be accumulated at $p_H = V$.

Thus, distribution $G$ first-order stochastically dominates distribution $F$. □

Case 4 is similar, as both firms now solve (1). Thus, there is a mixed strategy equilibrium in which both firms price according to $F(p) = 2 - \frac{V}{p}$.

Note how the expected payoff of firms in period two may be summarized in Table 1.

We now move on (or move back...) to period 1. Note that if the public agency does not call for tenders, then we are in case 4 and there is no action in period 1. However, there is an intresting result for the single-auction case in period 1.

Table 1. Expected payoffs in period 2

| Scenario | Firm 1 ($H$) | Firm 2 ($L$) |
|---|---|---|
| 1 | 0 | 0 |
| 2a | $V$ | 0 |
| 2b | 0 | 0 |
| 3 | $V$ | $\frac{V}{2}$ |
| 4 | $V$ | $V$ |

**Proposition 4.** *In case the public agency calls for a single auction in period 1, then there will be a single Nash equilibrium in pure strategies characterized by* $p = \frac{\beta V}{2}$ *for both firms.*

*Proof.* The expected discounted profit for any firm is

$$\max_p \mathbb{P}[win|p] \cdot \left(p + \frac{\beta V}{2}\right) + (1 - \mathbb{P}[win|p]) \cdot (0 + \beta V),$$

which is equivalent to solving

$$\max_p \mathbb{P}[win|p] \cdot \left(p - \frac{\beta V}{2}\right).$$

Note how this may be interpreted as maximizing the expected markup with respect to the minimum discounted profit that any firm will achieve: the one obtained in period 2 with a single-unit capacity. The firm who wins the auction in period 1 will be in this constrained situation in period 2, while the losing firm in period 1 will extract an expected profit of $V$ in period 2.

Thus, firms will undercut each other until there is no such markup and both alternatives are equally attractive: winning or losing the auction in period 1. This is only possible when both bid $p = \frac{\beta V}{2}$.

Suppose that wasn't the case. Then some firm could profitably deviate from this alleged equilibrium. If any firm bids $p < \frac{\beta V}{2}$, then it would win the auction in period 1 at the cost of having a lower capacity in period 2. The loss of $\frac{V}{2}$ in period 2 profits surpasses the profit of $p$ obtained in period 1. Thus, lowering $p$ is not an optimal deviation.

If any firm tries to bid above $p = \frac{\beta V}{2}$, it will lose the auction in period 1 and will be indifferent with the case of winning it (in both cases it obtains an expected present value of $\beta V$).

Finally, one may think in the possibility of not participating in the auction as a mechanism to ensure a high capacity in period 2. Nevertheless, this is not an equilibrium, as both firms would rather be alone in period 1, extract $V$ as profits and still enjoy $\frac{V}{2}$ in period 2. Any firm that doesn't participate in the auction will find it profitable to do so and undercut its rival. □

Considering the case in which there are two open auctions in period 1, one might *a priori* think of a similar equilibrium, as, if both firms participate in both auctions, each of them solves

$$\max_p \mathbb{P}[win|p] \cdot (2p + 0) + (1 - \mathbb{P}[win|p]) \cdot (0 + \beta V)\,,$$

which, just as in the single-auction case, is also equivalent to solving

$$\max_p \mathbb{P}[win|p] \cdot \left(p - \frac{\beta V}{2}\right).$$

Thus, one might conclude that the equilibrium is given by both bids equating $p = \frac{\beta V}{2}$. This is not the case. The reason is simple: firms may decide not to participate in both auctions in order to, eventually, not compete with each other. The case in which both firms participate in only one of the auctions is sustainable as a mixed-strategy Nash equilibrium.

**Proposition 5.** *If there are two open auctions in period 1, a mixed strategy equilibrium exists in which both firms participate in only one auction. Moreover, this equilibrium will be enforced by both firms, as it is more profitable than the case in which they compete for both auctions.*

*Proof.* If each firm participates in only one auction, then there is a 50% chance they overlap in such auction and a 50% chance they don't. If they don't, each one will win the auction they chip in and then compete *a la* Bertrand, earning zero profits in period 2. Nevertheless, if they happen to chose the same auction, the hypotheses of Theorem 4 are replicated.

Thus, each firm solves

$$\max_p 0.5[p + 0] + 0.5\left\{\mathbb{P}[win|p] \cdot \left(p + \frac{\beta V}{2}\right) + (1 - \mathbb{P}[win|p]) \cdot (0 + \beta V)\right\},$$

which yields the following first order linear differential equation as FONC:

$$H'(p) + \frac{H(p)}{p - \frac{\beta V}{2}} = \frac{2}{p - \frac{\beta V}{2}},$$

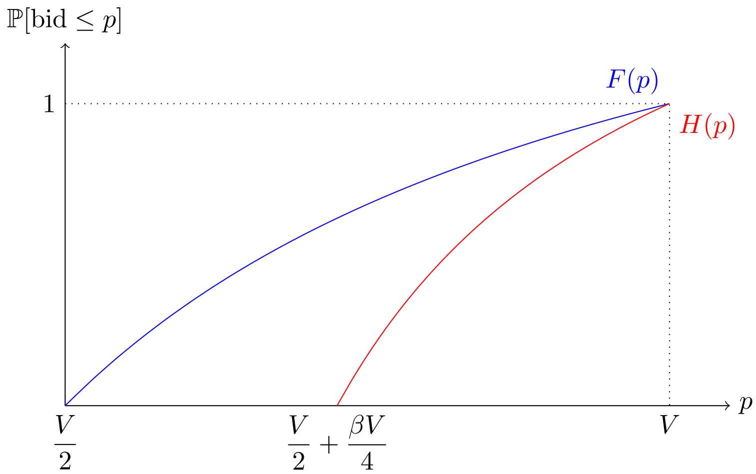
where $H(p)$ is the probability distribution of prices.

Solving for $H(p)$ and imposing $H(V) = 1$ we obtain

$$H(p) = \frac{2p - V\left(1 + \frac{\beta}{2}\right)}{p - \frac{\beta V}{2}}, \quad \forall p \in \left[\frac{V}{2} + \frac{\beta V}{4}, V\right].$$

It's worth noting that this distribution implies a less aggressive pricing strategy: firms will only price above $\frac{V}{2} + \frac{\beta V}{4}$. Intuitively, the price-undercutting argument is

FIGURE 3. First-order stochastic dominance of $H$ over $F$




no more valid because there's a chance firms won't meet in any auction and they'd rather price $V$. But, on the other hand, there is an equal chance they will meet and play a game where $p = \frac{\beta V}{2}$ characterizes the equilibrium. Stating the trade-off between these situations, it's easy to note how the minimum bid is a simple average of these two equilibria.

There is no pure Nash equilibrium because any firm would have incentives to slightly undercut it's rival until they reach the lower bound, where it would optimally choose to price $p = V$ and (re-)enter an Edgeworth cycle.

It's easy to see how this less aggressive price strategy yields a higher expected profit for the firms as compared to the situation where they both bid in two auctions. Having only one firm participating in two auctions is not sustainable as an equilibrium because it'd rather leave one. □

Finally, the case with three auctions is trivial: it replicates scenario 4 in period 1 if both firms participate in two auctions, but they won't, as they prefer to participate in only 1 auction each. Thus, they'll replicate Theorem 5, but with an even lower probability (1/3) of meeting in an auction which will generate incentives to reduce competition.

As a response to the latter, a fully rational public agency would rather open a single auction in period 1 and two auctions in period 2. Under this equilibrium, firms earn an expected present value of $\beta V$, and average prices are higher in period 2 than in period 1.

## 3. Concluding Remarks

In spite of departing from the real-life situation in which public agencies spend more funds at the end of a fiscal year *per se*, this model induces a particular finding: even if agencies would like to distribute their expenditure in a different way, firms will force them not to do so. In all cases we end up with firms that will behave asymmetrically and will price above standard competitive levels.

These higher final-period prices are skewed towards firms with higher relative capacity, as they know some auctions won't present any competitors due to their exhausted capacity. On the other hand, constrained firms won't have the ability to over-price that much. This is basically due to the risk of simply not selling at all: they replace their extensive competitive margin with intensive competitive margin.

This may have clear empirical relevance if higher-capacity firms are the ones that tend to abuse of expiring budgets at the end of a fiscal year. However, It's not easy to map the model's setting, particularly when interested in defining a firm's capacity constrain (Jofre-Bonet and Pesendorfer, 2003) or its implausibility to restore it over time. Besides that, firms must be capable of, for instance, desert a tender and roll it over to a next period.

All of the above would fit scenarios where auctions are held in highly concentrated sectors, where firms have a lot of market power and at the same time face binding capacity constrains (at least in the short run). Firms with such characteristics may be, for example, highly specialized consulting firms, large-scale engineering enterprises or firms with low liquid credit lines that won't allow them to participate in many public auctions that require upfront cash as a guarantee. Indeed, future research may be focused on evaluating the veracity and applicability of this model's implications on actual data.

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