Multiproduct Competition in Markets with Search Costs*

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Abstract

We develop a model of multiproduct competition in markets with search costs. Consumers differ on both their private valuation of quality and their search costs. In contrast to existing vertical-differentiation models, which predict retailers' specialization as a way to soften competition, we show that search frictions, no matter how small, lead retailers to carry both high and low quality products in equilibrium. When pricing these products, firms face a tension between competition and information-rent extraction: pricing products more aggressively to attract informed consumers requires to give up information rents on high-valuation consumers. This tension generates a testable prediction: during sales (i.e., when prices hit rock bottom) the price drop in high-quality products is larger than in low-quality products.

Keywords: price discriminaiton, search, vertical differentiation, retail competition.

1 Introduction

Main conclusion: search costs are at the heart of the existence of multiproduct retailers. In the absence of search costs, firms carry only one product; in order to mitigate competition, they avoid having overlapping products.

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Stress/look at:

- Ours is a model of single-product search, not multiproduct search, given that each consumer only buys one product. Our products are substitutes, not complements. One-stop shopping plays no role in this paper

- Multi product and single product firms charge different relative prices because they are affected by IC differently. They also choose different price distributions, leading to different degrees of price dispersion as well dispersion in relative prices

- How do relative prices depend on search frictions (the price gaps are larger when search frictions are smaller)

- Emphasize product heterogeneity and consumer heterogeneity

- Because with search costs, firms are multiproduct, and given that the specialization equilibrium falls apart, competition is more intense than otherwise [compare expected prices in the cases with multi-product firms and asymmetric single-product firms]. Hence, for comparative statics with respect to search costs, it is important to consider endogenous product line choices

- Quality is taken as given: this is a model of competition among retailers, it is not a model of competition among vertically integrated firms. This applies to many settings (e.g. supermarkets, book retailers). Models with exogenous quality are not a case of more general models with endogenous quality. Both models lead to distinct conclusions.

We study retailers selling products of different qualities when consumers must search. One may say that this is an "extension" of Varian's model of sales to vertical differentiation. I think there is a lot more than a mere extension. In fact, I would probably start by saying that current models of vertical differentiation (where consumers are perfectly informed) predict that firms specialize to soften competition (it's like in horizontal differentiation models where firms locate in the extremes of Hotelling's linear city; true, when transportation costs are quadratic). Take the model of Shaked and Sutton that is in Tirole (1988), for instance. In that model one firm sells the high quality product and the other the low quality product, which allows them to make positive profits. Our model makes the exact the same prediction if we add an early stage where firms decide which products to carry (see below) and all consumers are perfectly informed. But this specialization only holds in our model when all consumers are fully informed, as soon as you introduce a "bit of search" (i.e., an arbitrarily small fraction of uninformed consumers) that "specialization" equilibrium falls apart. Instead we have an equilibrium with both firms carrying both products.

To me that should be the starting point or the selling point of the paper...That could be the observational motivation we are looking for:....that we often see firms carrying products of different qualities (i.e., retailers are the best examples). In addition, and based on we have found so far, our model predicts that when firms go for sales....they offer larger discounts (in relative terms) on the high quality products..

I think we have something very good and novel here....and which is not trivial to solve...

We want to understand the influence of competition and market frictions on product line choices

We want to understand the interaction between the opportunities to price discriminate and search frictions, and how this manifests through the product line choices.

Our paper is related to two strands of the literature: competition with search costs, and product line decisions and price discrimination by firms under imperfect competition.

Relatively little is known about competition and product lines choices of multiproduct firms, not least about competition and product lines choices of multiproduct firms in markets with search costs.

Compare results regarding product line choices and prices under monopoly and duopoly with and without search costs.

2 The Model

Consider a market in which there are two products available for sale, a high quality product and a low quality product. Qualities are denoted by q^H and q^L for the high and low quality products respectively, with $\Delta q \equiv q^H - q^L > 0$. The marginal costs of producing each product are denoted by c^H and c^L respectively, with $\Delta c \equiv c^H - c^L > 0$.[1] In the market, there are two competing retailers. These might be either single-product stores and just sell one of the two products, or multi-product stores and thus sell the two products.

There is a mass of consumers normalized to one, each of which buys at most one good. Consumers differ in their preferences over quality. A fraction $\lambda > 0$ of them have a low valuation for quality, θ^L, while the remaining $1 - \lambda$ fraction have a high valuation θ^H, with $\Delta\theta \equiv \theta^H - \theta^L > 0$. A consumer of type $i = L, H$ who purchases product $j = L, H$

[1] We can also think of these costs as the wholesale prices at which retailers buy the products from a monopoly manufacturer. For now, we take both the qualities as the costs as exogenously given to the retailers.

at price p^j obtains the following utility

$$u^i = \theta^i q^j - p^j.$$

The timing of the game is as follows. First, retailers simultaneously decide which product(s) to sell. Once chosen, these become public information. Second, retailers choose the prices for the good(s) they sell. Consumers have to visit the stores in order to know those prices. Following Varian (1980),[2] we assume that there is a fraction $\mu < 1$ of consumers (the *informed* ones) who always visit the two stores, and hence know where to find the cheapest product of each quality type. The remaining $1 - \mu$ fraction of consumers (the *uninformed* ones) only visit one store. Thus, they can compare the prices of the goods sold within the store they have visited, but not across stores . Which store uninformed consumers visit depends on their type: we assume that an uniformed low-type (high-type) consumer visits the store that carries the low (high) quality product; if the two stores carry the same good, the consumer visits one of the two stores randomly. Once consumers have visited the store, they buy the product that gives them higher utility; in case of indifference, low-type (high-type) consumers buy the low (high) quality product.[34]

2.1 Preliminaries

In order to solve the model, it is useful to introduce two pieces of information.

First, consider the Incentive Compatibility constraints: for high-type consumers to be willing to buy the high-quality product,

$$\theta^H q^H - p^H \geq \theta^H q^L - p^L, \qquad (IC^L)$$

and for low-type consumers to be willing to buy the low-quality product,

$$\theta^L q^L - p^L \geq \theta^L q^H - p^H. \qquad (IC^L)$$

[2]SEE ALSO XXX

[3]An implicit assumption is that the fractions μ and λ are uncorrelated. THINK WHETHER AND HOW THIS WOULD AFFECT THE RESULTS

[4]As we will see, single and multi-product firms charge different prices. One could argue that uninformed consumers are thus not indifferent between visiting one single-product or one multi-product firm that carries their preferred product if they anticipate that on average their prices are different. We assume this is not the case, e.g. an uninformed consumer visits the closest store that sells its product without any further considerations.

Incentive compatibility requires that the price difference need not be neither too large nor too small; in particular, $\Delta p \equiv p^H - p^L \in \left[\theta^L \Delta q, \theta^H \Delta q\right]$. If the price difference was above $\theta^H \Delta q$, high-type consumers would rather buy the low-quality product, while if it was below $\theta^L \Delta q$, low type consumers would rather buy the high-quality product.

We make the following standard assumption:

$(A1)$ $\Delta c \in \left[\theta^L \Delta q, \theta^H \Delta q\right]$.

In words, this assumption says that high-type consumers are willing to pay for the costs of the quality upgrade, while low-types consumers are not. In turn, this implies that the competitive solution is incentive compatible given that the difference in marginal costs, Δc, is below the high-types' maximum willingness to pay for the improved quality, $\theta^H \Delta q$. In what follows, we will sometimes refer to Δc as the *competitive separation* and to $\theta^H \Delta q$ as the *monopoly separation.*

Second, it is also useful to define monopoly prices. If the firm could perfectly discriminate between the two types of consumers, it would simply extract all their surplus. In particular, the (unconstrained) monopoly prices are $p^i = \theta^i q^i$ and the per unit profits are $\pi^i = \theta^i q^i - c^i$, for $i = L, H$. However, these prices are not incentive compatible as high-type consumers would rather buy low-quality product to obtain positive surplus. In order to assure separation, the monopolist has to leave information rents $q^L \Delta\theta$ to the high-type consumers. Thus, the (constrained) monopoly prices are $p^L = \theta^L q^L$ and $p^H = \theta^H q^H - q^L \Delta\theta$.[5] The associated per unit monopoly profits are π^L and $\pi^H - q^L \Delta\theta$.

Alternatively, the firm could decide to only serve high type consumers at the unconstrained monopoly price, $p^H = \theta^H q^H$. This latter can be ruled out if the share of low-type consumers λ is large enough. In particular, if λ is such that[6]

$$(1-\lambda)(\pi^H - q^L \Delta\theta) + \lambda \pi^L > (1-\lambda)\pi^H$$

This leads to our second assumption,

$(A2)$ $\lambda \pi^L > (1-\lambda)\Delta\theta q^L$.

In words, the rents from serving the low-types are higher than the information rents that have to be given to the high-types when the low-types are served. In turn, this guarantees that the share of low-type consumers λ is sufficiently large so that the monopolist optimally serves both consumer types.

[5] It will sometimes be useful to write the (constrained) monopoly price for product H as $p^H = \theta^L q^L + \theta^H \Delta q$.

[6] Note that the monopolist never finds it profitable to only sell product H to all consumers at $\theta^L q^H$ because of $(A1)$.

We are now ready to solve the same. We start by analyzing the game in which all consumers are informed ($\mu = 1$), which is equivalent to assuming that there are no search costs. We then move on to analyzing the game with strictly positive search costs.

3 No search costs

Assume that all consumers are informed ($\mu = 1$, i.e., there are no search costs). The following Proposition characterizes the unique (up to a reversal in firms' indexes) Subgame Perfect Equilibrium of the game.

Proposition 1 *Assume $\mu = 1$. There is a unique Subgame Perfect Equilibrium: in the first stage, firms choose to be asymmetric single-product retailers; in the second stage, a pure-strategy equilibrium does not exist.*

In order to prove this result, we proceed by backward induction. In particular, we have to characterize equilibrium pricing for every possible combination of product lines choices in the first stage. We thus need to consider six possible configurations, depending on whether both firms are multiproduct retailers, whether one is single-product while the other one is multi-product, or whether they are either symmetric or asymmetric single-product retailers.[7]

Suppose first that they are both multiproduct retailers. It is easy to see that, in the absence of search costs, Bertrand competition drives prices down to marginal cost, i.e., $p^L = c^L$ and $p^H = c^H$, and firms' profits to zero. From $(A1)$, equilibrium prices are incentive compatible, i.e., the high-type consumers purchase the high quality product and low type consumers purchase the low quality product. Since there are no information rents involved, a firm is absolutely indifferent as to which consumer to serve.[8]

Next, suppose that one retailer is single-product while the other one is multi-product. If the two firms carry the low quality product but only one carries the high quality one, the low quality product is priced at marginal cost, $p^L = c^L$, while the high quality product is sold at the highest price that satisfies the high-types' incentive compatibility constraint, i.e., $p^H = c^L + \theta^H \Delta q$. Note that it is never a best reply to charge $p^H = c^L + \theta^L \Delta q$ so as to also attract the low-types given that $\theta^L \Delta q < \Delta c$, by $(A1)$.

Alternatively, if the two firms carry the high quality product but only one carries the low quality one, the high quality product is priced at marginal cost, $p^H = c^H$, while the

[7] THIS SECTION CAN BE WRITTEN IN A MORE CONCISE WAY ONCE WE UNDERSTAND WHICH RESULTS WE WANT TO HIGHLIGHT. FOR NOW, I PREFER TO LEAVE IT AS IT IS TO HAVE DETAILS ON ALL POTENTIAL EQUILIBRIA.

[8] Under competitive separation, this holds true more generally, even when prices are above marginal costs.

low quality product is sold at at the highest price that satisfies the low-types' incentive compatibility constraint, i.e., $p^L = c^H - \theta^L \Delta q$. Again, it is never a best reply to charge $p^L = c^H - \theta^H \Delta q$ so as to also attract the high-types given that $\theta^H \Delta q > \Delta c$, by $(A1)$.

Last, suppose that both retailers are single-product. If they sell the same product, Bertrand competition again drives prices down to marginal cost and profits to zero. In contrast, if they sell different products, a pure strategy equilibrium does not exist. To see this, note that firms have to choose prices that are incentive compatible: otherwise, one firm would sell nothing and would thus be better off reducing its price until incentive compatibility is achieved. If the price difference is such that both incentive compatibility constraints are satisfied with slack, either firm could increase its price until its consumers' incentive compatibility constraint is just biding. However, if one of the two constraints is binding, the other firm would be able to profitable attract all customers by slightly reducing its own price. This destroys any candidate pure-strategy equilibrium. Thus, the equilibrium has to be in mixed strategies. In the Appendix we provide lower bounds to firms' profits at any mixed strategy equilibrium, and show that these profits are higher for both firms as compared to those in the subgames with one single-product and one multi-product retailer.

Given this, it follows immediately that here exists a SPE with asymmetric single-product retailers. To see that it is unique, note that the existence of an arbitrarily small fixed cost of carrying a product rules out any other candidate. In particular, there are no SPE in which firms sell overlapping products: given that in these cases Bertrand competition drives profits to zero, they are always better off dropping one of the two products. This result is in line with the existing literature, which showed that firms resort to product differentiation in order to relax competition.

In the next section we will assess the robustness of this result in the presence of search costs.

4 Positive Search Costs

In this section, we analyze the case in which not all consumers are informed, i.e., $\mu < 1$, i.e., search costs are strictly positive, and they are higher the lower is μ. We again proceed by backward induction by first characterizing equilibrium pricing for every possible combination and product lines.

4.1 Pricing Behavior

In this section we characterize pricing behavior of multiproduct duopolists. For now, we assume that both firms carry the two products, while in the next section we shift our focus to product choice decisions.

We start by providing some general properties of equilibrium pricing that are common across all subgames, i.e., regardless of the initial product line choices.

Lemma 1 *At any possible pricing subgame, in equilibrium, multi-product firms choose incentive compatible prices for their products,* $\Delta p \in \left[\theta^L \Delta q, \theta^H \Delta q\right]$.

Lemma above shows that it is always optimal for multi-product retailers to choose prices that satisfy incentive compatibility. The intuition is simple. If the price of the high-quality product is too high so that all consumers buy the low-quality product, it is profitable for the firm to reduce p^H, while leaving p^L unchanged, so as to attract the high-type consumers and obtain a larger profit margin (by $(A1)$, we know that $\left(p^H - c^H\right) = \left(p^L + \theta^H \Delta q - c^H\right)$ is greater than $\left(p^L - c^L\right)$). Analogously, if the price of the low-quality product is too high so that all consumers buy the high-quality product, it pays the firm to reduce p^L, while leaving p^H unchanged, so as to attract the low-type consumers (by $(A1)$, we know that $\left(p^L - c^L\right) = \left(p^H - \theta^L \Delta q - c^L\right)$ is greater than $\left(p^H - c^H\right)$). These incentives are analogous to those in the monopoly case. However, the incentives are stronger in the oligopoly case given that the reduction in either p^H or p^L needed to achieve incentive compatibility might possibly attract consumers that would otherwise have bought from the rival firm.

This result constitutes an important difference with respect to Varian (1980), as it implies that the price of one product cannot be picked independently from the price of another product within the same store. In our setting, consumers' consumption decisions depend not only on the prices of the goods in the two stores (as in Varian), but also on the prices of the two goods *within* the same store.

4.1.1 Pure Strategy Equilibria

The following lemmas provides a useful result concerning the existence and characterization of pure-strategy equilibria.

Lemma 2 *Suppose that single-product firms charge the (unconstrained) monopoly prices,* $p^H = \theta^H q^H$ *or* $p^L = \theta^L q^L$, *and multi-product firms charge the (constrained) monopoly prices,* $p^H = \theta^H q^H - \theta^L \Delta q$ *and* $p^L = \theta^L q^L$. *If, at these prices, all informed consumers buy from a single firm, then those prices constitute the unique pure-strategy equilibrium of the subgame. Otherwise, no pure-strategy equilibrium exists.*

At any pure-strategy equilibrium, all informed consumers must buy the product(s) from a single firm. Otherwise, one of the two firms would find it profitable to undercut its rival's price(s) in order to attract all the informed consumers. Since one firm is selling its product(s) only to uninformed consumers, his optimal prices must be the unconstrained monopoly prices (if single-product) or (by Lemma xx) the constrained monopoly prices (if multi-product). If firms have incentives to deviate from these prices to attract the informed consumers, the equilibrium must be found in mixed strategies.

In Varian (1980), as well as in most search models, pure-strategy equilibria fail to exist as firms face countervailing incentives: (i) on the one hand they want to reduce their prices to attract the informed consumers; (ii) on the other hand, they want to extract all rents from the uninformed consumers who visit their store. In our problem, the fact that firms choose incentive compatible prices for their products, partly alleviates these countervailing incentives. In particular, a multi-product firm need not always compete head to head with a single product firm selling the high-quality product. The reason is that the highest prices the two firms are willing to charge differ by $\theta^L \Delta q$, given that the multi-product firm is constrained by incentive compatibility whereas the single-product firm is not. Hence, firms endogenously split the market among themselves: the single-product firm selling the high-quality product only serves high-type uninformed consumers at the monopoly price, whereas the multi-product firm serves all the rest, including all the informed consumers.

The same result does not hold when (i) the two firms are multi-product: since they are both constrained by incentive compatibility, there is no wedge between the highest prices that the two firms are willing to charge for each product; nor when (ii) the single-product firm sells the low-quality product: since there is no price difference between the constrained and unconstrained monopoly prices, there is no price wedge either and so firms are willing to undercut each other to attract the uninformed consumers.

Corollary 1 *There are no pure-strategy equilibria in the subgames in which (i) firms are symmetric and/or those in which (ii) both firms sell the low-quality product, i.e., subgames with product lines* (LH, LH), (L, L), (H, H) *and* (L, LH).

It follows that the only subgames in which existence of a pure-strategy equilibrium is not ruled out by Lemma xx are the ones in which one firms have asymmetric product lines with one firm selling the high-quality, i.e., (H, LH) and (H, L). The next two Propositions look at the existence of pure-strategy equilibria in these two subgames.

Proposition 2 *At the subgame* (H, LH), *there exists a pure-strategy equilibrium if and only if* μ: *the* H *firm charges the (unconstrained) monopoly price, and the* LH *firm* *charges the (constrained) monopoly prices. Equilibrium profits are*

$$\begin{aligned}\Pi(H, LH) &= \frac{1-\mu}{2}(1-\lambda)\pi^H \\ \Pi(LH, H) &= \lambda\pi^L + \left(\frac{1-\mu}{2} + \mu\right)(1-\lambda)(\pi^H - q^L\Delta\theta)\end{aligned}$$

Intuition for the PSE:

- Since LH is a multi-product firm, his prices have to be IC
- Since H is a single-product firm, his price is not constrained by IC
- This creates a wedge of $q^L\Delta\theta$ between the prices charged for product H by the two firms
- Firm H only serves have of the uninformed H-types (mass $(1-\mu)(1-\lambda)/2$), and could compete for the informed ones (mass $\mu(1-\lambda)$), but this would require reducing the price by $q^L\Delta\theta$
- For this deviation to be unprofitable, the mass of informed consumers μ has to be small enough

Proposition 3 *At the subgame (L, H), there exists a PSE iff μ small: the two firms charge the (unconstrained) monopoly prices. Equilibrium profits are*

$$\begin{aligned}\Pi(L, H) &= (\lambda + \mu(1-\lambda))\pi^L \\ \Pi(H, L) &= (1-\mu)(1-\lambda)\pi^H\end{aligned}$$

Intuition for the PSE:

- Firm H only serves the uninformed H-types (mass $(1-\mu)(1-\lambda)$), and could compete for the informed ones (mass $\mu(1-\lambda)$), but this would require reducing the price by $q^L\Delta\theta$
- For this deviation to be unprofitable, the mass of informed consumers μ has to be small enough.

4.1.2 Mixed Strategy Equilibria

When pure-strategy equilibria do not exist, the equilibrium has to be found in mixed strategies. The following Lemma provides properties that all mixed-strategy equilibria must satisfy.

Lemma 3 *At any possible pricing subgame, in any mixed-strategy equilibrium, firms choose prices* p^L *in* $\left[\underline{p}^H, \overline{p}^H\right]$ *and* p^L *in* $\left[\underline{p}^L, \overline{p}^L\right]$. *The bounds of the price support satisfy the following properties:*

(i) The upper bound is given by the (constrained) monopoly prices, $\overline{p}^H = \theta^H q^H - \theta^L \Delta q$ *and* $\overline{p}^L = \theta^L q^L$, *and the lower bound is strictly above marginal costs,* $\underline{p}^i > c^i$ *for* $i = L, H$.

(ii) At the upper bound, the high-types' incentive compatibility constraint is binding, $\Delta\overline{p} \equiv \overline{p}^H - \overline{p}^L = \theta^H \Delta q$, *while at the lower bound it is satisfied with slack,* $\Delta\underline{p} \in \left(\Delta c, \theta^H \Delta q\right)$.

(iii) If firms are symmetric, there are no mass points in the support.

At the upper bound, firms are not able to extract all the surplus from the informed consumers (as in Varian's model) because incentive compatibility would not be satisfied. This brings us back to the (constrained) monopoly problem. In particular, at the upper bound, firms extract all surplus $\theta^L q^L$ from the uninformed low-type consumers, but have to leave information rents $q^L \Delta\theta$ to the uninformed high-type consumers. Thus, the incentive compatibility for the high-types is binding at the upper bound.

Since firms have to be indifferent between charging any price in the support, including the upper bounds, expected profits at any mixed strategy equilibrium are unambiguously defined. How much each firm sells at the upper bound depends on both its own product lines as well as that of the rival.

If the two firms are multi-product firms, at the upper bound they split equally the demand of the uniformed consumers. Given that a pure-strategy equilibrium does not exist in this case (Corollary xx), equilibrium profits are unambiguously given by

$$\mathcal{E}[\Pi\left(LH, LH\right)] = \frac{1-\mu}{2}\left[\lambda\pi^L + (1-\lambda)(\pi^H - q^L\Delta\theta)\right]$$

However, even though equilibrium profits are unique and well defined, there might be multiplicity of mixed strategy equilibria. In particular, when characterizing the lower bounds of the price distribution, the problem has an extra degree of freedom: there is a continuum of price pairs satisfying incentive compatibility, $\underline{p}^H - \underline{p}^L \in \left[\theta^L \Delta q, \theta^H \Delta q\right]$ which also yield the same equilibrium profits. Consider for instance the subgame in which both firms are multi-product retailers. If a firm prices the two good at the lower bound of the price supports, it attracts the uninformed consumers that visit its store plus all the informed consumers of each type.

$$\mathcal{E}[\Pi\,(LH, LH)] = \frac{1+\mu}{2}\left[\lambda(\underline{p}^L - c^L) + (1-\lambda)(\underline{p}^H - c^H)\right]$$

Clearly, there are several price pairs satisfying this condition. In turn, this gives rise to a continuum of equilibrium price distributions.

Let us provide one such pairs of equilibrium price distributions. Given that the challenge is to discourage high-type consumers from buying product L, a natural equilibrium to consider is one in which firms keep on pricing product L as if they were just selling that product, but adjust their pricing for product H. The following Proposition characterizes such equilibrium:

Proposition 4 *There exists a mixed-strategy equilibrium in which firms choose p^L in $\left[\underline{p}^L, \overline{p}^L\right]$ according to*

$$F^L(p^L) = \frac{1+\mu}{2\mu} - \frac{1-\mu}{2\mu}\frac{(\overline{p}^L - c^L)}{(p^L - c^L)}$$

and such that, for given p^L, the price p^H is chosen in $\left[\underline{p}^H, \overline{p}^H\right]$ so that

$$\frac{p^H - c^H}{p^L - c^L} = \frac{\overline{p}^H - c^H}{\overline{p}^L - c^L} \tag{1}$$

where, for $i = L, H$,

$$\underline{p}^i = c^i + \frac{1-\mu}{1+\mu}(\overline{p}^i - c^i) > c^i.$$

The proposed equilibrium has several appealing features. While firms price product L as if they were just selling that product, on average they choose a lower price p^H as compared to when they only sell product H. This is a direct implication of the fact that $\overline{p}^H < \theta^H q^H$ because of the information rents left to the high-types. Indeed, the distribution of p^H,

$$F^H(p^H) = \frac{1+\mu}{2\mu} - \frac{1-\mu}{2\mu}\frac{(\overline{p}^H - c^L)}{(p^H - c^L)}$$

puts higher weight on lower prices all along the support than in the independent products case.

Under this equilibrium, the choice of p^L results in a unique choice of p^H such that the relative profit margin of the two products remains constant along the whole support; see equation (1). In particular, the relative markups of the two products are the same as under monopoly. That is, competition affects the price level but not the price structure within the firm.

The price difference that is embodied in this price structure can be expressed as

$$\Delta p = \alpha\theta^H \Delta q + (1-\alpha)\,\Delta c.$$

where $\alpha = \left(p^L - c^L\right) / \left(\overline{p}^L - c^L\right)$. Consistently with Lemma **??**, the price difference is a weighted average between $\theta^H \Delta q$ (i.e., the monopoly separation) and Δc (i.e., the competitive separation). Furthermore, the higher (lower) p^L, the higher the weight put on the monopoly separation (competitive separation). At the upper bound, when the incentive compatibility constraint of the high-types is binding, the price difference is maximal, $\Delta\overline{p} = \theta^H \Delta q$. As we move down the support, the incentive compatibility constraint is satisfied with slack and the price difference narrows down. The difference is minimal at the lower bound, when $\alpha = (1-\mu)/(1+\mu)$. Importantly, as μ approaches to 1 (no search costs), the prices at the lower bound converge to marginal cost, so that the price gap approaches the competitive separation. On the other extreme, μ approaches to 0, the prices at the lower bound converge to the monopoly prices so that the price gap approaches the monopoly separation.

5 Product Line Choices

In this section we analyze product choice decisions given the continuation equilibria characterized above. In the previous section we showed that in the absence of search costs ($\mu = 1$), there exists one Subgame Perfect Equilibrium (SPE) with single-product retailers, in which one retailer carries product H and the other carries product L. This is a standard result in the existing literature as quality pre-commitment allows firms to avoid head-to-head competition. However, this "specialization" equilibrium breaks down as soon as search costs matter, $\mu < 1$. Indeed, no matter how small search costs are, the unique SPE results in multi-product retailers, as assumed in the previous section.

Proposition 5 *If $\mu < 1$, there is a SPE in which one retailer carries the H product while the other retailer carries the L product.*

[THIS RESULT IS TRU FOR μ SMALL. WE STILL HAVE TO CHARACTERIZE SOME PROFITS FOR THE ASYMMETRIC CASES WHEN μ IS LARGE. WE CANNOT COMPLETE THE PROOF UNTIL WE HAVE ALL PROFUT EXPRESSIONS. DETAILS ON THIS BELOW.]

6 Details on pricing equilibria

6.1 Case $\{LH, LH\}$:

Equilibrium must be in mixed strategies (see Lemma xx).

$$\mathcal{E}[\Pi\,(LH, LH)] = \frac{1-\mu}{2}\left[\lambda(\theta^L q^L - c^L) + (1-\lambda)(\theta^H q^H - q^L \Delta\theta - c^H)\right]$$

6.2 Case $\{L, L\}$:

Equilibrium must be in mixed strategies (see Lemma xx). If the two firms carry only one product which is the same. This is similar to Varian's equilibrium: equilibrium in mixed strategies with upper bound $\theta^i q^i$, but it is not exactly the same as here there are two consumer types.

If all consumers served;

$$\mathcal{E}[\Pi\,(L, L)] = \frac{1-\mu}{2}(\theta^L q^L - c^L)$$

or if only high-types served.

$$\mathcal{E}[\Pi\,(L, L)] = \frac{1-\mu}{2}(1-\lambda)(\theta^H q^L - c^L)$$

6.3 Case $\{H, H\}$:

Equilibrium must be in mixed strategies (see Lemma xx). Equilibrium must be in mixed strategies (see Lemma xx).

If all consumers served;

$$\mathcal{E}[\Pi\,(H, H)] = \frac{1-\mu}{2}(\theta^L q^H - c^H)$$

or if only high-types served.

$$\mathcal{E}[\Pi\,(L, L)] = \frac{1-\mu}{2}(1-\lambda)(\theta^H q^H - c^H)$$

6.4 Case $\{L, LH\}$:

Equilibrium must be in mixed strategies (see Lemma xx). Upper bounds at constrined monopoly prices. Profits:

$$\begin{aligned}\mathcal{E}[\Pi(L, LH)] &= \frac{1-\mu}{2}\lambda\pi^L \\ \mathcal{E}[\Pi(LH, L)] &= \frac{1-\mu}{2}\lambda\pi^L + (1-\mu)(1-\lambda)(\pi^H - q^L\Delta\theta)\end{aligned}$$

Note that when firm L prices at the upper bound $\theta^L q^L$, informed H type consumers buy product H from the rival, as $\theta^H q^L - \theta^L q^L = \Delta\theta q^L < \theta^H q^H - p^H$ since the rival is charging $p^H < \theta^H q^H - \Delta\theta q^L$.

When firm H prices at the upper bound, the informed low and high types buy from the rival. [NO CONTINUITY WITH THE CASE $\mu = 1$??] When the firm charges $\theta^H q^H - \Delta\theta q^L$ the informed high types prefer to buy the rival's low quality product and obtain utility $\theta^H q^L - p^L > \Delta\theta q^L$ given that at the other store's price is $p^L < \theta^L q^L$. For the same reason, the informed low types also buy from the other firm.

Comparing the profits of firm L in case $\{L, LH\}$ with those in case $\{LH, LH\}$ shows that its profits are greater under the latter. The profit difference is for the L firm:

$$\mathcal{E}[\Pi(LH, LH)] - \mathcal{E}[\Pi(L, LH)] = \frac{1-\mu}{2}(1-\lambda)(\pi^H - q^L\Delta\theta) > 0$$

This already shows that **the case** (L, LH) **cannot be an equilibrium**.

Comparing the profits of firm H in case $\{L, LH\}$ with those in case $\{L, H\}$ for μ small (such that a PSE exists)

$$\begin{aligned}\mathcal{E}[\Pi(LH, L)] - \mathcal{E}[\Pi(H, L)] &= \frac{1-\mu}{2}\lambda\pi^L + (1-\mu)(1-\lambda)(\pi^H - q^L\Delta\theta) - (1-\mu)(1-\lambda)\pi^H \\ &= (1-\mu)\left(\frac{1}{2}\lambda\pi^L - (1-\lambda)q^L\Delta\theta\right)\end{aligned}$$

The sign of this expression is unclear. Assumtpion 2 guarantees $\lambda\pi^L > (1-\lambda)\Delta\theta q^L$ but in the expression above the first term is multiplied by one half. [This is not really relevant, at least for μ small. We will see below that $\{L, H\}$ cannot be an equilibrium in any case since the L firm would rather deviate and also sell H; see below]

6.5 Case $\{H, LH\}$:

PURE STRATEGY EQUILIBRIUM

There exists a PS strategy equilibrium for μ small enough [note that in this case there is no fight for th informed consumers since they all buy from the multi-product firm]

The H firm charges $\theta^H q^H$ and makes profits [note it gets half of the uninformed high types because the other firm also carries product H] and the multi-product firm charges the constrained monopoly prices, it sells all informed and its uninformed low types, and the informed high types and half of the uninformed high types.

$$\begin{aligned}\Pi(H, LH) &= \frac{1-\mu}{2}(1-\lambda)\pi^H \\ \Pi(LH, H) &= \lambda\pi^L + \left(\frac{1-\mu}{2}+\mu\right)(1-\lambda)(\pi^H - q^L\Delta\theta)\end{aligned}$$

For this to be an equilibrium, it must be the case that firm H does not want to charge slightly less than the constrained monopoly price to also attract the high-type informed . In such a case, it would get

$$\begin{aligned}\Pi(H, LH) &= \left(\frac{(1-\mu)}{2}(1-\lambda) + (1-\lambda)\mu\right)(\pi^H - q^L\Delta\theta) \\ &= \frac{(1+\mu)(1-\lambda)}{2}(\pi^H - q^L\Delta\theta)\end{aligned}$$

Comparing the two profit expressions, the PS equilibrium exists iff

$$\frac{(1-\mu)}{2}(1-\lambda)\pi^H - \frac{(1+\mu)(1-\lambda)}{2}(\pi^H - q^L\Delta\theta) > 0$$

$$\mu \le \frac{\Delta\theta q^L}{\pi^H + (\pi^H - \Delta\theta q^L)}$$

[Note this threshold is lower than in the case $\{L, H\}$; see below]

Lets compare the profits of firm L-H with those in the case $\{L, H\}$ (see below for profit expression). Assume $\mu \le \frac{\Delta\theta q^L}{\pi^H + (\pi^H - \Delta\theta q^L)}$ (this guarantees the equilibrium in the (L,H) subgame is also in PSE)

$$\begin{aligned}\Pi(LH, H) - \Pi(L, H) &= \lambda\pi^L + \left(\frac{1-\mu}{2}+\mu\right)(1-\lambda)(\pi^H - q^L\Delta\theta) - (\lambda + \mu(1-\lambda))\pi^L \\ &= (1-\lambda)\left(\frac{1-\mu}{2}(\pi^H - q^L\Delta\theta) + \mu(\pi^H - q^L\Delta\theta - \pi^L)\right) > 0\end{aligned}$$

This already shows that **the $\{L, H\}$ cannot be an equilibrium when μ is small.**

Lets now compare the profits of firm H with those in the case $\{LH, LH\}$ (see above for profit expression)

$$\mathcal{E}\left[\Pi\left(LH,LH\right)\right]-\Pi\left(H,LH\right) = \frac{1-\mu}{2}\left[\lambda\pi^{L}+(1-\lambda)(\pi^{H}-q^{L}\Delta\theta)\right]-\frac{1-\mu}{2}\left(1-\lambda\right)\pi^{H}$$
$$= \frac{1-\mu}{2}\left(\lambda\pi^{L}-\left(1-\lambda\right)q^{L}\Delta\theta)\right)>0$$

and it is positive because of $(A2)$. Hence, **the case $\{H,LH\}$ cannot be an equilibrium when μ is small**

MIXED STRATEGY EQUILIBRIUM

Otherwise, for μ large, the equilibrium must be in mixed strategies.

Suppose firms choose prices in the supports $p^{L}\in[\underline{p}^{L},\overline{p}^{L}]$ and $p^{H}\in[\underline{p}^{H},\overline{p}^{H}]$.

Note that in this range, we have $\frac{(1+\mu)(1-\lambda)}{2}\left(\theta^{H}q^{H}-\Delta\theta q^{L}-c^{H}\right)>\frac{(1-\mu)}{2}\left(1-\lambda\right)\left(\theta^{H}q^{H}-c^{H}\right)$.

At the MS, firm L has to play a mass at the upper bound. Otherwise, when firm H charged $\theta^{H}q^{H}-\Delta\theta q^{L}$, since firm L would be charging prices below $\theta^{L}q^{L}$, and hence, it would also charge for product H prices below $\theta^{H}q^{H}-\Delta\theta q^{L}$, firm H would make profits $\frac{(1-\mu)}{2}\left(1-\lambda\right)\left(\pi^{H}-\Delta\theta q^{L}-c^{H}\right)$, which are stritly below the profits the firm could make when charging $\theta^{H}q^{H}$, i.e., $\frac{(1-\mu)}{2}\left(1-\lambda\right)\left(\pi^{H}-c^{H}\right)$

Accordingly, suppose firm L plays a mass point at $\overline{p}^{L}=\theta^{L}q^{L}$. Let $\omega=1-F^{L}\left(\theta^{L}q^{L}\right)$ be the size of the mass.

At the upper bound $\overline{p}^{H}=\theta^{H}q^{H}-\Delta\theta q^{L}$, firm H's profits are

$$\Pi\left(H,LH\right)=\left(\frac{\left(1+\mu\right)\left(1-\lambda\right)}{2}\omega+\frac{\left(1-\mu\right)\left(1-\lambda\right)}{2}\left(1-\omega\right)\right)\left(\pi^{H}-c^{H}\right)$$

Note that these profits are increasing in ω

$$\frac{\left(1+\mu\right)\left(1-\lambda\right)}{2}-\frac{\left(1-\mu\right)\left(1-\lambda\right)}{2}=\mu\left(1-\lambda\right)>0$$

Hence, an upper bound for these profits is when $\omega=1$,

$$\Pi\left(H,LH\right)<\left(\frac{\left(1+\mu\right)\left(1-\lambda\right)}{2}\right)\left(\pi^{H}-c^{H}\right).$$

**************INCOMPLETE

6.6 Case $\{H,L\}$:

Recall we are assuming that uninformed consumers visit the store that sells "their" product.

Consider the candidate equilibrium at which firms charge (unconstrained) monopoly prices, with profits

$$
\begin{aligned}
\Pi(H,L) &= (1-\mu)(1-\lambda)\pi^H \\
\Pi(L,H) &= (\lambda+\mu(1-\lambda))\,\pi^L
\end{aligned}
$$

Firm H serves the uniformed high-types and firm L serves uninformed low-types. The informed low types do not want to buy the H product, and the informed high types buy product L as this gives them positive utility.

For this to be an equilibrium, it must be the case the firm H does not want to charge the constrained monopoly price to also attract the high-type uninformed consumers. In such a case, it would get $(1-\lambda)\left(\pi^H - c^H\right)$. Comparing the two profit expressions, the PS equilibrium exists iff

$$
\mu \leq \frac{\Delta\theta q^L}{\pi^H}
$$

Otherwise, the equilibrium must be in mixed strategies.

Assume μ is such that no PSE exists. Let's characterize the MS equilibrium. Suppose firms choose prices in the supports $p^L \in [\underline{p}^L, \overline{p}^L]$ and $p^H \in [\underline{p}^H, \overline{p}^H]$. Note that in this range, we have $(1-\lambda)\left(\theta^H q^H - \Delta\theta q^L - c^H\right) > (1-\mu)(1-\lambda)(\theta^H q^H - c^H)$.

At the MS, firm L has to play a mass at the upper bound. Otherwise, when firm H charged $\theta^H q^H - \Delta\theta q^L$, since firm L would be charging prices below $\theta^L q^L$, firm H would make profits $(1-\mu)(1-\lambda)\left(\theta^H q^H - \Delta\theta q^L - c^H\right)$, which are stritly below the profits the firm could make when charging $\theta^H q^H$.

Accordingly, suppose firm L plays a mass point at $\overline{p}^L = \theta^L q^L$. Let $\omega = 1 - F^L\left(\theta^L q^L\right)$ be the size of the mass.

At the upper bound $\overline{p}^H = \theta^H q^H - \Delta\theta q^L$, firm H's profits are

$$
\Pi(H,L) = ((1-\lambda)\omega + (1-\mu)(1-\lambda)(1-\omega))\left(\theta^H q^H - \Delta\theta q^L - c^H\right)
$$

given that when firm H charges $\theta^H q^H - \Delta\theta q^L$ and firm L charges less than $\theta^L q^L$, all the informed low types buy product L. [Note: if this is correct, as μ tends to one, profits are positive as long as ω remains positive]

Note that these profits are increasing in ω

Hence, an upper bound for these profits is when $\omega = 1$,

$$
\Pi(H,L) < (1-\lambda)\left(\theta^H q^H - \Delta\theta q^L - c^H\right).
$$

Furthermore, since in this range we know that $(1-\lambda)\left(\theta^H q^H - \Delta\theta q^L - c^H\right) > (1-\mu)(1-\lambda)(\theta^H q^H - c^H)$, hence we have a lower and an upper bound on profits.

As it is reasonable to expect some profit continuity, at $\mu = \frac{\Delta\theta q^L}{\theta^H q^H - c^H}$, I believe the following condition need hold

$$(1-\mu)(1-\lambda)(\theta^H q^H - c^H) = ((1-\lambda)\omega + (1-\mu)(1-\omega))\left(\theta^H q^H - \Delta\theta q^L - c^H\right)$$

At this critical μ, $\omega = 1$ (indeed, firm L is playing at PS).

For $\mu > \frac{\Delta\theta q^L}{\theta^H q^H - c^H}$, we should have

$$(1-\mu)(1-\lambda)(\theta^H q^H - c^H) < ((1-\lambda)\omega + (1-\mu)(1-\omega))\left(\theta^H q^H - \Delta\theta q^L - c^H\right)$$

as otherwise profits at the MS would again be $(1-\mu)(1-\lambda)(\theta^H q^H - c^H)$. This requires $\omega < 1$.

What is the lower bound? My conjecture is that the price gap is never such that the low types prefer to buy product H [note that this requires that $\underline{p}^H - \overline{p}^L \geq \Delta\theta q^L$]. Hence, at the lower bound

$$\Pi(H, L) = (1-\lambda)\left(\underline{p}^H - c^H\right)$$

Equating both profits,

$$\underline{p}^H = c^H + \left(\frac{(1-\mu)(1-\omega)}{1-\lambda} + \omega\right)\left(\theta^H q^H - \Delta\theta q^L - c^H\right)$$

How to determine the size of the mass?

The expected profits of firm L are, at the upper bound, (assuming H plays no mass at the upper bound, as the two firms cannot have a mass)

$$\Pi(L, H) = \left(\theta^L q^L - c^L\right)\lambda$$

What is the lower bound for product L?

$$\Pi(L, H) = (1-\lambda)\left(\underline{p}^L - c^L\right)$$

$**************$INCOMPLETE

7 Testable Predictions

1. High quality products are relatively "cheaper" during *sales*

2. Prices for low quality products similar in single/multi-product firms

3. Prices for high quality products lower in multi-product firms

4. Wider product lines in less competitive markets[9]

8 What is next?

8.1 On baseline model

- finish characterizing the equilibrium, i.e., find $F()$
- explains what happens when the firm wants to serve only the high type as monopoly (λ small). I think the price support for the high-quality product starts from $\theta^H q^H$ and goes down gradually and when it reaches $\theta^H q^H - q^L \Delta\theta$ the firm starts selling both products....this is nice because in equilibrium we could see some firms just selling high quality products...but we should never see firms selling just low quality products.

9 Extensions beyond the baseline model

- Monopoly manufacturer: we discussed some of this...shouldn't be difficult to replicate what we did
- Quantity discounts (or making quality also endogenous)
- Should be easy to extend to N firms
- Our results should apply to other search models: sequential model of Stahl
- Continuum of types??? (not sure if adds much)
- Application to bundling (still pure pricing: Aquí va un ejemplo:
  - dos retailers vendiendo carne y vino a consumidores que pueden ser de dos tipos
  - tipo A valora la unidad de carne y de vino en lo mismo, v
  - el tipo B valora la carne en $2v - \alpha$ y el vino en α
  - cuando $\alpha = v$, hay correlación cero (o más bien, consumidores son idénticos)
  - cuando $\alpha = 0$, hay correlación perfectamente negativa entre valoraciones

[9] If "more competitive" menas more firms, be careful as the number of firms might be endogenous.

- cuando $\alpha = 2v$, hay correlación perfectamente positiva
- los retailers compran los productos a productores a precios mayoristas que normalizamos a cero
- vamos al caso $\alpha = 0$. Un retailer monopolista va a vender paquetes de carne y vino por $2v$, extrayendo todo el excedente. Con stand-alone pricing no puede extraer todo el excedente, ya que no puede ofrecer la carne a más de v.
- por otro lado, retailers compitiendo perfectamente (el caso de todos los consumidores informados), van a vender carne y vino con stand-alone prices iguales a cero.

References

[1] Johnson, J. and Myatt, D. (2003), Multiproduct Quality Competition: fighting brands and product line pruning, American Economic Review 749-774.

[2] Varian, H. (1980), A Model of Sales, American Economic Review.

Appendix: Proofs

Proof of Proposition 1. We first characterize equilibrium pricing for give product choices. We need to consider all potential product configurations. Because firms are symmetric, we can restrict attention to the four cases below:

1. Cases $\{(L,H),(L,H)\},\{(L),(L)\}$ or $\{(H),(H)\}$: Suppose both firms carry the same product(s). In equilibrium, $p^L = c^L$ and/or $p^H = c^H$ because of Bertrand competition. Firms make zero profits.

2. Case $\{(L),(L,H)\}$: Suppose firm 1 carries product L and firm 2 carries the two products. In equilibrium, $p^L = c^L$ because of Bertrand competition, and $p^H = c^L + \theta^H \Delta q$ because it is the highest price that allows to the firm to attract the H consumers. Charging $p^H = c^L + \theta^L \Delta q$ so as to attract all customers is not profitable given that $\theta^L \Delta q < \Delta c$ by $(A1)$. Firm 1 makes zero profits, while firm 2 makes profits $(1-\lambda)\left(\theta^H \Delta q - \Delta c\right)$.

3. Case $\{(H),(L,H)\}$: Suppose firm 1 carries product H and firm 2 carries the two products. In equilibrium, $p^H = c^H$ because of Bertrand competition, and $p^L = c^H - \theta^L \Delta q > c^L$ because it is the highest price that allows the firm to attract the L consumers. Charging $p^L = c^H - \theta^H \Delta q$ so as to attract all customers is not

profitable given that $\theta^H \Delta q > \Delta c$ by $(A1)$. Firm 1 makes zero profits, while firm 2 makes profits $\lambda\left(\Delta c - \theta^L \Delta q\right)$.

4. Case $\{(L), (H)\}$: Suppose firm 1 carries product L and firm 2 carries product H. A pure-strategy equilibrium does not exist.

    - Clearly we cannot have $\Delta p > \theta^H \Delta q$ nor $\Delta p < \theta^L \Delta q$ because either firm H or firm L would make no sales respectively.
    - We cannot have $\Delta p < \theta^H \Delta q$. If $p^H < \theta^H q^H$, firm H could increase its price without losing any customer. If $p^H = \theta^H q^H$, all customers would buy from firm L (in particular, the high types would prefer to buy product L and obtain a positive surplus $\theta^H q^L - p^L \geq \Delta\theta q^L > 0$) so the firm could increase its profits by reducing its price.
    - We cannot have $\Delta p > \theta^L \Delta q$. If $p^L < \theta^L q^L$, firm L could increase its price without losing any customer. If $p^L = \theta^L q^L$ and $\Delta p = \theta^H \Delta q$, firm L would be better off slightly undercutting his own price so as to serve all customers. If $p^L = \theta^L q^L$ and $\Delta p < \theta^H \Delta q$, firm H could increase its price without losing any customer.

Suppose firms choose prices in the supports $p^L \in [\underline{p}^L, \overline{p}^L]$ and $p^H \in [\underline{p}^H, \overline{p}^H]$. We next provide some lower bounds to firms' profits in equilibrium. On the one hand, since firm L will never play anything lower than c^L, we must have $\underline{p}^H \geq c^L + \theta^H \Delta q$: if firm H plays $c^L + \theta^H \Delta q$ she will get the all the high types for sure and hence it does not pay to charge any lower price (recall that the firm doesn't want to price at $c^L + \theta^L \Delta q$ to get all consumers because $\theta^L \Delta q < \Delta c$). Hence, her equilibrium payoff must be

$$\pi^H \geq (1-\lambda)(\theta^H \Delta q - \Delta c) > 0, \tag{2}$$

which is her equilibrium payoff when she carries the two products and the rival only carries product L.

On the other hand, since firm H will never play anything lower than c^H, we must have $\underline{p}^L \geq c^H - \theta^L \Delta q > c^L$: if firm L plays $c^H - \theta^L \Delta q$ she will get the all the low types for sure and hence it does not pay to charge any lower price (recall that the firm doesn't want to price at $p^L = c^H - \theta^H \Delta q$ to get all consumers because $\theta^H \Delta q > \Delta c$). Hence, her equilibrium payoff must be

$$\pi^L \geq \lambda(\Delta c - \theta^L \Delta q) > 0, \tag{3}$$

which is her equilibrium payoff when she carries the two products and the rival only carries product H.

From the above, it is straightforward to characterize equilibrium product choice decisions. First, $\{(L,H),(L,H)\}$, $\{(L),(L)\}$ or $\{(H),(H)\}$ cannot be an equilibrium as firms make zero profits and hence do not cover the fixed costs of carrying the products, no matter how small these are. Second, $\{(L),(L,H)\}$ cannot be in equilibrium as from equation (2) firm 2 would be better off dropping product L. Third, $\{(H),(L,H)\}$ cannot be in equilibrium as from equation (3) firm 2 would be better off dropping product H. The only remaining candidate, $\{(L),(H)\}$, is an equilibrium given (2) and (3). ■

Proof of Lemma 1. Argue by contradiction. First, suppose that the firm chooses $\Delta p > \theta^H \Delta q$. Hence, all buyers visiting the store buy product L, and the firm makes a profit margin equal to $\left(p^L - c^L\right)$. If the firm reduced p^H to $p^H = p^L + \theta^H \Delta q > c^H$, it would not sell less than before (and could possibly attract consumers from the rival store) and could make more profits out of its current customers. In particular, it would still sell product L to the low-type customers at the same price, but would now sell product H to the high-type consumers with a higher profit margin $\left(p^H - c^H\right) = \left(p^L + \theta^H \Delta q - c^H\right) > \left(p^L - c^L\right)$, where the inequality follows from $\theta^H \Delta q > \Delta c$ by $(A1)$.

Second, suppose that the firm chooses $\Delta p < \theta^L \Delta q$. All buyers visiting the store buy product H, and the firm makes a profit margin equal to $\left(p^H - c^H\right)$. If the firm reduced p^L to $p^L = p^H - \theta^L \Delta q$, it would not sell less than before (and could possibly attract consumers from the rival store) and could make more profits out of its current customers. In particular, it would still sell product H to the high-type customers at the same price, but would now sell product L to the low-type consumers with a higher profit margin $\left(p^L - c^L\right) = \left(p^H - \theta^L \Delta q - c^L\right) > \left(p^H - c^H\right)$, where the inequality follows from $\theta^L \Delta q < \Delta c$ by $(A1)$. ■

Proof of Lemma 1. (ii) At the upper bound, prices are $\overline{p}^H = \theta^H q^H - \theta^L \Delta q$ and $\overline{p}^L = \theta^L q^L$. Hence, $\Delta \overline{p} = \theta^H \Delta q$. (iii) TO BE COMPLETED.

$$\frac{1+\mu}{2}\left[\lambda(\underline{p}^L - c^L) + (1-\lambda)(\underline{p}^H - c^H)\right] = \frac{1-\mu}{2}\left[\lambda(\overline{p}^L - c^L) + (1-\lambda)(\overline{p}^H - c^H)\right]$$

$$\frac{1+\mu}{2}\left[(\underline{p}^H - c^H) - \lambda\left(\theta^H \Delta q - \Delta c\right)\right] = \frac{1-\mu}{2}\left[(\overline{p}^H - c^H) - \lambda\left(\theta^H \Delta q - \Delta c\right)\right]$$

$$(\underline{p}^H - c^H) = \frac{1-\mu}{1+\mu}(\overline{p}^H - c^H) + \lambda\frac{2\mu}{1+\mu}\left(\theta^H \Delta q - \Delta c\right)$$

■

Proof of Proposition 4. We want to show that the equilibrium in the statement of Proposition is indeed an equilibrium. First, firms could deviate by playing the price pairs in the support with different probabilities, while still choosing price pairs that satisfy incentive compatibility. However, this is unprofitable given that all price-pairs in the support give equal expected profits. Indeed, the equilibrium has been constructed so that

$$(p^L - c^L)\left[\frac{1-\mu}{2} + \mu(1 - F^L(p^L))\right] = \frac{1-\mu}{2}(\overline{p}^L - c^L) = \frac{1+\mu}{2}(\underline{p}^L - c^L)$$

and

$$(p^H - c^H)\left[\frac{1-\mu}{2} + \mu(1 - F^H(p^H))\right] = \frac{1-\mu}{2}(\overline{p}^H - c^H) = \frac{1+\mu}{2}(\underline{p}^H - c^H)$$

with the ratio (1) derived in order for the price pair $\left(p^H, p^L\right)$ to satisfy $F^H(p^H) = F^L(p^L)$, i.e., the choice of p^L results in a choice of p^H, so that the prices satisfying that ratio are played with equal probability. Therefore, expected profits $\mathcal{E}[\pi(p^H, p^L)]$ at the proposed equilibrium are as in (**??**).

Second, firms could deviate by choosing p^L and p^H not satisfying equation (1) while still satisfying incentive compatibility. Again, these deviations are not profitable since all the prices in the support give equal profits. Deviating to prices that do not satisfy incentive compatibility is unprofitable because of Lemma **??**.

Last, firms could deviate by playing the price pairs outside the support. Choosing any prices above $\left(\overline{p}^L, \overline{p}^H\right)$ as defined above is unprofitable, as at these prices the firm is only selling to the uniformed consumers and $\left(\overline{p}^L, \overline{p}^H\right)$ are the optimal monopoly prices. Choosing any prices below $\left(\underline{p}^L, \underline{p}^H\right)$ as defined above is unprofitable, as at these prices the firm is inelastically selling to all consumers with probability one and would thus gain by rasing the price up to $\left(\underline{p}^L, \underline{p}^H\right)$. ■