Labor Markets, Inequality, and Hiring Selection*

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Abstract

Income inequality is largely driven by wage dispersion. Labor markets are obvious suspects for magnifying ex ante inequality. Everyday events such as hirings, layoffs, and promotions generate differential outcomes depending on the productivity of individuals. Yet, no much attention have been devoted to empirically an theoretically evaluate if and how labor markets act as a catalyst for wage inequality. To do so, first I show evidence of a large positive longitudinal correlation between wage inequality and unemployment using merged CPS monthly and ORG data. More detailed analysis shows that high inequality is often related to low job finding rates, suggesting that at least part of the phenomenon may be explained by delving deeper into the hiring mechanism of firms.

To rationalize this evidence, I construct a general equilibrium model of non-sequential employer search with hiring selectivity and heterogeneous workers, and characterize its equilibrium. The model departs from the standard search model by allowing firms to simultaneously meet several applicants and choose the best. Available independent evidence of the recruiting process shows that selective hiring is important: firms interview a median of five applicants per vacancy and spend 2.5% of their total labor cost –about US$4200 per recruit– in these activities (National Employer Survey, 1997).

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The model provides an endogenous matching process in which the job finding probability increases in worker productivity. Under selective hiring, lifetime inequality increases relative to the sequential search benchmark because low wage workers go through longer and more volatile unemployment spells, and have less valuable outside options to bargain with firms. I calibrate the model to replicate the positive correlation between wage dispersion and unemployment rate, and the negative correlation between wage dispersion and the average job finding rate.

Finally, I characterize a social planner solution and find that the optimal allocation could be implemented through a hiring subsidy schedule decreasing in productivity.

Keywords: Employer search, Nonsequential search, Recruiting selection, Duration dependence, Hazard rate heterogeneity, Inequality, Search externalities.

1 Introduction

Firms devote considerable resources to recruiting new workers. After posting a job opening, employers typically evaluate resumes and conduct interviews to identify applicants' qualifications. In the National Employer Survey 1997 (NES97) firms report that they interview a median of 5 applicants per vacancy and spend 2.5% of their total labor cost in recruiting activities, with an average close to US$4200 per recruited worker[1]. Even though such widespread recruiting activities affect outcomes of employers and workers in real labor markets, this feature is absent in most popular models of labor search. This paper embeds the recruiting selection process into an otherwise standard stationary general equilibrium search model of the labor market and characterizes its equilibrium.

I model the recruiting selection process as a *nonsequential* employer search strategy. Firms simultaneously meet various heterogenous workers to fill vacancies and screen them to select the best. This approach nests the sequential search model as a particular case and solves several of its shortcomings. First, the sequential approach empirically runs into trouble to replicate the fact that several applicants are interviewed before filling a vacancy. Second, sequential employer search makes selection a random process, ignoring the role of workers' heterogeneity in the job assignment.

[1] Reported statistics are controlled for nonresponse bias. A description of NES is in Cappelli (2001)

In contrast, with a screening technology available, firms optimally evaluate several candidates at the same time; this allows them to increase the productivity of their hiring from among their applicant pool. Therefore, the model provides microfoundations for an endogenous matching process with heterogenous workers. The approach is also justified empirically. Several papers[2] have documented that firms fill job openings by choosing an applicant from a pool that is formed shortly after the posting of the vacancy, and that almost no applications arrive afterwards. Vacancy durations are selection periods.

The recruiting selection model also generates important macroeconomic implications that benchmark models of sequential search (McCall 1970; Mortensen and Pissarides 1994) do not readily produce. My model qualitatively replicates and provides a simple explanation for cross-sectional features of CPS data. Specifically, the mean and variance of log re-employment wages decrease in unemployment duration, and the mean and variance of the unemployment duration are negatively correlated with log re-employment wages.

The economic environment of the model is deliberately simple to highlight the effect of recruiting selection. There is no aggregate uncertainty and jobs are homogeneous. Workers are risk-averse and heterogeneous only in time-invariant productivity. Their only decision is whether to submit a costly application per period. Since all vacant jobs are *ex ante* identical, in a symmetric equilibrium all firms receive the same expected number of applications.

To fill a vacancy, firms observe the number of applicants they receive and decide how many to interview considering screening costs. The employer perfectly observes the screened applicants' productivities and picks the best candidate. Rejected applicants remain in the unemployed pool. In equilibrium, the best workers get jobs faster on average and the unemployment pool worsens as the duration of unemployment increases. Therefore, the hazard rate out of unemployment –or job finding probability– decreases in productivity and the average productivity of workers is negatively correlated with the duration of unemployment.

Once a worker is chosen, the wage is determined by Nash-bargaining[3]. Because both the firm's profit and the value of worker's outside option increase in productivity, it is not readily clear that the most productive workers are also the most prof-

[2]Barron, Bishop, and Dunkelberg (1985), van Ours and Ridder (1992), Abbring and van Ours (1994), Weber (2000)

[3]In Villena-Roldan (2008) using a similar framework, I study the implications of an alternative wage determination mechanism

itable. I show conditions for the existence of the symmetrical "Coincidence Ranking" equilibrium in which the productivity and profitability rankings are the same for all workers. In the steady-state of the model, the joint distribution of productivities and unemployment durations and the hazard rates are *equilibrium objects* that arise from the employers' recruiting selection technology, worker-selection strategies of firms, and the exogenous distribution of workers' productivity types.

The presented model is consistent with empirical evidence on hazard rates and wages. Several empirical studies[4] conclude that after controlling for observable variables and allowing for unobserved idiosyncratic heterogeneity, the negative duration dependence of the hazard rate disappears, i.e. elapsed duration does not negatively impact the job finding rate. Additionally, variables that are usually associated with higher earnings are also related to higher hazard rates. Both facts are consistent with the recruiting selection model because (i) high productivity workers find jobs more easily and (ii) the negative duration dependence disappears once productivity determinants are controlled for. Using data generated by my model, an econometrician who does not perfectly observe productivity will find some negative duration dependence. A closely related finding is that the unobserved heterogeneity accounts for a great deal of hazard rate variation after controlling for elapsed duration and observables[5]. Instead of linking the unobserved heterogenous component to some idiosyncratic feature that affects the job finding rate, the recruiting selection model establishes a clear relationship between a worker's productivity and her hazard rate.

Related to this evidence, several empirical studies find that conditional on observables, re-employment wages exhibit significant negative duration dependence. Additionally, individuals who have experienced long unemployment durations tend to go through long unemployment spells frequently[6]. Stewart (2007) also finds that recurrence of unemployment spells is not only caused by past unemployment duration, but also by low past wages. The model has a simple explanation for all these phenomena: less productive workers have lower chances of being hired due to recruiting selection. In a labor market with search frictions, the recruiting selection model generates more lifetime inequality than a sequential search model. While in the latter case all workers are affected in the same way by the search frictions, in a model of recruiting selection low productivity workers experience longer unemployment spells

[4] Machin and Manning (1999) (survey), Abbring et al. (2002) and Cockx and Dejemeppe (2005).

[5] See for instance, van den Berg and van Ours (1996),van den Berg and Ridder (1998), Machin and Manning (1999)

[6] See Omori (1997) and Gregg (2001).

and earn lower wages because their outside options are less valuable. Since low productivity workers face more volatile unemployment spells, under recruiting selection lifetime welfare inequality is even greater if workers are risk-averse.

In the recruiting selection model, some labor market outcomes have untraditional meaning. Indeed, positive unemployment rate plays a role for the efficient assignment of jobs that is absent in sequential search models. Since employers do not observe productivities before meeting applicants, only with some level of unemployment can firms meet several candidates to hire the best one. Another labor market outcome, the median unemployment duration, reflects the degree of firms' screening effort in this model. In the model, long unemployment durations are due to effective employer screening, which tends to reject low productivity applicants *given some distribution of unemployed workers*.

However, more screening is not always better. Private firms' screening and entry decisions generate externalities on other firms. As a side-effect of interviewing a large number of applicants and taking out the best, firms indirectly deteriorate the productivity of the unemployment pool and make it harder for other employers to find good workers. On the other hand, if more vacancies are posted, the number of expected applicants decreases so that firms cannot hire the best available workers very effectively. For this reason, the average productivity of the unemployment pool increases, with positive consequences for all employers. If other firms do not post too many vacancies, then a given firm may not find it worth posting either because the unemployment pool may consist mostly of low productivity workers. If screening is very effective these externalities are strong enough to generate multiple equilibria.

In Section 4 I calibrate the model to target the unemployment rate, median number of interviewed applicants and the mean and variance of log wages in CPS data 1985-2006 for the whole labor market and for a segment of production workers. Using a reasonable parametrization, model-generated moments of the joint distribution of wages and unemployment durations qualitatively mimic their empirical counterparts in CPS data. However, because the simple model exaggerates the correlations observed in the data, I develop a slight variation of the model with stochastic screening costs that fits the data better. Using these calibrations, I also perform the counterfactual experiment of increasing the marginal cost of screening so that firms choose to search sequentially. This change substantially decreases both the average and the cross-sectional variation of welfare. However, because the higher screening cost shuts down selection, the loses due to lack of hiring selectivity are partially offset by the improvement of the unemployed pool. Considering that a more effective screening technology creates a worse unemployment pool composition, the increase in the marginal screening cost generates the largest welfare reduction in the economy with *less effective* screening technology.

The paper is organized as follows. Section 2 describes the model in detail and characterizes the most relevant equilibrium of the model. Section 5 discusses the model's implications. Section 4 contains a calibration and empirical evaluation of the model and runs some counterfactual experiments. The last section concludes.

2 The Model

In the model time is discrete and there is a continuum of homogenous risk-neutral firms or employers that post *ex ante* identical job vacancies. There is also a fixed mass of size 1 of workers who have a time-invariant productivity θ from a exogenous distribution with density $f(\theta)$. Workers can either be employed (with unemployment duration $\delta = 0$) or unemployed with $\delta > 0$. Workers cannot borrow nor save.

All jobs are *ex ante* identical. The general state of the economy is a tuple $\mathcal{X} \equiv (\mathcal{U}, \mathcal{V}, G(\theta, \delta))$, where $\mathcal{U}$ represents the mass of unemployed applicants in the economy, $\mathcal{V}$ is the mass of aggregate vacancies, and $G(\theta, \delta)$ the endogenous joint distribution of types and unemployment durations. I denote $G_U(\theta) = G(\theta|\delta > 0, a(\theta) = 1)$ as the endogenous distribution of unemployed workers. In this paper, I solely focus on the symmetric steady-state equilibrium of this economy. In equilibrium, employers optimally post $\mathcal{V}$ vacant jobs per period. The mass of $\mathcal{U}$ unemployed workers are indifferent among employers. Hence, the probability that the application of a particular worker arrives to a given vacancy is $1/\mathcal{V}$. Thus, the number of applications arrived to a vacancy, K, follows a binomial distribution.

$$Prob(K = k) = \binom{\mathcal{U}}{k} (1/\mathcal{V})^k (1 - 1/\mathcal{V})^{\mathcal{U}-k}$$

As both $\mathcal{U}, \mathcal{V} \to \infty$ with its ratio $q = \mathcal{U}/\mathcal{V}$ constant, the number of applicants received per vacancy K converges to a Poisson distribution with mean q, the queue size.

The matching process differs from the typical urn-ball approach in which coordination failure creates unemployment (Petrongolo and Pissarides 2001). If balls are heterogenous, firms have incentives to select which ball to pick. Although coordination failure of workers still plays a role for unemployment (bad luck of being sorted into a vacancy with strong competitors or vice versa), recruiting selection is the main determinant of job assignment.

2.1 Workers

At the beginning of the period, all unemployed workers receive an exogenous income $\rho\theta$ with $0 < \rho < 1$ and decide whether to submit a free application. By doing so, a worker is randomly allocated into some firm's applicant pool and faces an *equilibrium* job finding probability $p(\theta)$. In case the worker obtains the job, the worker gets the value of being employed $W(\cdot)$ earning a wage $w(\theta)$ bargained with a prospective employer that will by renegotiated period by period. If the worker receives no offers, she increases her unemployment spell by one unit of time and faces the participation decision again.

Workers have linear preferences over consumption and have a constant discount factor $\beta \equiv (1+r)^{-1}$. Hence, an unemployed worker's lifetime utility is $U(\theta) = \max\{U_1(\theta), U_0(\theta)\}$, where $U_1(\cdot)$ stands for the value of submitting an application and is given by

$$U_1(\theta) = \rho\theta + \beta p(\theta)W(\theta) + \beta(1 - p(\theta))U(\theta)$$

and U_0 is the value of not submitting an application given by

$$U_0(\theta) = \rho\theta + \beta U(\theta) \tag{1}$$

When hired, the worker produces her productivity θ and receives a bargained wage $w(\theta)$. Before the period ends, the worker faces the exogenous chance of separation, η. The value of a type θ worker employed at wage $w(\theta)$ is

$$W\left(w(\theta)\right) = w(\theta) + \beta((1 - \eta)W\left(w(\theta)\right) + \eta U(\theta)) \tag{2}$$

Since the hazard rate $p(\theta)$ does not change because in steady state, a worker of productivity type θ always makes the same participation decision denoted by $a(\theta) = 1$ if participates; otherwise $a(\theta) = 0$. Hence, if both $p(\theta)$ and $w(\theta)$ are nondecreasing in equilibrium, in general there is a productivity threshold below which the gain from obtaining a job does not compensate the searching cost $b(\theta)\zeta$. Hence, there is a $\underline{\theta}$ such that $U_1(\underline{\theta}) = U_0(\underline{\theta})$.

2.2 Firms

A filled job with a worker of productivity θ generates a flow profit $\theta - w(\theta)$. After production, the job can be destroyed with exogenous probability η, in which case the employer obtains the value of posting a vacancy V, which is described below. Hence, the value of a filled job equals

$$J(\theta) = \theta - w(\theta) + \beta((1-\eta)J(\theta)) + \eta V) \quad (3)$$

Idle employers observe the aggregate state and optimally create vacancies by paying a fixed flow cost κ. Vacancies simultaneously meet K applications drawn from the distribution of unemployed workers $G_U(\theta) = G(\theta|\delta > 0, a(\theta) = 1)$, which is the key departure assumption from sequential search and matching models. I show below that this matching setup nests the standard sequential framework.

Upon receiving applications, the employer optimally attaches a probability ϕ of interviewing or screening each applicant with marginal cost $\xi \geq 0$. In principle, this decision could be contingent on the number of arrived applicants k, although I mainly consider the case in which ϕ is set before learning the realized number of applicants. After each interview, the employer perfectly learns the applicant's type θ. Due to this assumption, we focus on selection issues, leaving aside informational effects. The employer offers the position to the most profitable worker, or posts a vacancy again, whatever is better. Thus, the value of posting a vacancy is

$$V = \max_{\phi \in [0,1]} \left\{ -\kappa + \beta \mathbb{E}_K \left[\max\{H(k), V\} \right] \right\}$$

where $H(k)$ is the maximum profit obtained out from a pool of k applicants following an optimal screening probability ϕ given a realization of K.

$$H(k) = \mathbb{E}_S \left[-\xi S + \max_j \left\{ J(\theta_j) \right\}_{j=1}^{S} \mid a(\theta_1) = ... = a(\theta_S) = 1, \delta_1, ..., \delta_S > 0, K = k, S \leq K \right]$$

2.3 Wages

A natural framework for wage determination is Nash bargaining. Once the firm has chosen some worker, both sides negotiate about how to split the generated surplus. For the worker, the outside option is the value of being unemployed next period, $U(\theta)$. For the firm, the outside option is to post vacancies again under free entry, i.e $V = \varphi \geq 0$ with φ being the employer's sunk cost of entering in the labor market.

Bargained wages are never turned down in equilibrium because (1) the value of the match is solely determined by θ, (2) all employers are identical and therefore pay the same wages, and (3) Workers only send one application per period. The Nash axiomatic solution solves the following problem.

$$\max_w \left\{ \; (W(w,\theta) - U(\theta)^{\alpha} (((J(w,\theta) - V))^{1-\alpha} \right\}$$

Substituting equations (2), and (3), and using the free entry condition $V = \varphi$ for an interior solution the first order condition is

$$w(\theta) = \alpha\theta + (1-\beta)((1-\alpha)U(\theta) - \alpha\varphi) \tag{4}$$

The worker wants to keep the match (i.e, the solution is interior) only if $w(\theta) > \rho\theta$. Because of the free-entry condition, employers will hire the worker as long as $\theta > w(\theta)$. One key consequence of Nash bargaining is that the surplus may not be strictly increasing in θ given a particular distribution $G(\theta, \delta)$. On one hand, higher productivity increases the total surplus but, on the other hand, the value of unemployment also grows, especially if the job finding probability $p(\theta)$ is increasing. Thus, employers may not prefer the top applicant in equilibrium because they could obtain larger profits from other candidates. The problem is complicated since the profitability ranking of applicants determines the hiring preferences in equilibrium, reflected through $p(\theta)$. I deal with this problem in the next sections.

2.4 Solving the Competitive Equilibrium

The model solution schematically involves four steps:

1. Conjecture an optimal hiring policy, for instance, a Coincidence Ranking Equilibrium (CRE)

2. Find out the job finding rate schedule $p(\theta)$ consistent with the conjecture of point 1.

3. Using the job finding rate schedule $p(\theta)$ to derive the invariant joint distribution $G(\theta, \delta)$ and $G^U(\theta)$ in terms of the equilibrium queue size q and optimal screening probability ϕ.

4. Having $p(\theta)$ and $G^U(\theta)$ find out the equilibrium queue size q using the free-entry condition $V = \varphi$

5. Verify is the solution obtained $(q, G^U(\theta, \delta))$ satisfies the conjectured form of the optimal hiring policy.

2.4.1 Conjecturing optimal hiring

A natural conjecture is "The more productive, the more profitable". Formally, the productivity and profitability rankings coincide. Thus, a **Coincidence Ranking Equilibrium** (CRE) holds if and only if $J'(\theta) > 0$ for all θ in equilibrium. Job productivity increases more than wages do on θ, so that employers optimally offer the position to the highest productivity worker arrived in equilibrium.

2.4.2 Equilibrium Job Finding Rate

A worker is hired whenever she generates a profit value $J(\theta)$ for the employer which is greater than that of other applicants to the same vacancy. Under the CRE conjecture, the highest productivity applicant yields the highest profit in equilibrium. In particular, if an employer screens s applicants, the top candidate gets the offer with probability $G_U(\theta)^{s-1}$. For the sake of simplicity, the probability of being screened ϕ is taken as exogenous and for all number of applicants. Given that ϕ is fixed, the number of screened applicants follows a binomial distribution. By Bayes' law, a worker of type θ has a probability of being hired

$$Prob(\theta \text{ hired}|k \text{ total applicants}, \theta) = \sum_{s=1}^{k} \binom{k}{s} \phi^{s-1}(1-\phi)^{k-s+1} G_U(\theta)^{s-1} = (\phi G_U(\theta) + 1 - \phi)^{k-1}$$

Nevertheless, at the moment a worker applies, she ignores how many applications are competing for the same job she applied to. Considering that the number of applicants follow a Poisson distribution, the probability of being hired $p(\theta)$ is

$$p(\theta) = \sum_{k=1}^{\infty} \frac{e^{-q} q^{k-1}}{(k-1)!} (\phi G_U(\theta) + 1 - \phi)^{k-1} = e^{-q\phi(1-G_U(\theta))} \tag{5}$$

Another potential extension is to allow for heterogeneous or stochastic screening costs, i.e. ξ follows a distribution over the population of open vacancies. This feature essentially flattens the job finding rate schedule per type, since low types have greater chances when the screening cost becomes large for a share of the vacancies. I focus in a special two-point distribution in which for the screening cost. Employers interview all arrived candidates with probability ϕ when they face the low screening cost $\underline{\xi}$ with probability $1 - \psi$. Otherwise, they draw a screening cost $\overline{\xi}$ with probability ψ, in which case they interview only one candidate with probability ϕ. Then, the hazard rate is

$$p(\theta) = \psi \frac{1 - e^{-\phi q}}{\phi q} + (1 - \psi) e^{-q\phi(1-G_U(\theta))} \tag{6}$$

It is possible to generalize the screening cost distribution and to endogenize the screening probability choice, and even make it contingent to the number of arrived applicants k. However, there is no much theoretical insight we gain by doing so. I adopt this somewhat form approach here to contrast scenarios with high screening and no-screening in an easy way. As proven in Lemma 6, the average hazard rate in this case equals

$$\mathbb{E}[p(\theta)] = \frac{1 - e^{-\phi q}}{\phi q}$$

The larger the value of ϕ, the flatter becomes the shape of the hazard rate across productivities because screening becomes less intensive and unemployment probabilities depend more on coordination failure –or plain luck– than on selection. The value of ψ plays a similar role although it generates substantially larger redistribution of job finding rates across the population. Lemma **??** also tells us that worker heterogeneity does not affect (in partial equilibrium) the size of the flow leaving unemployment, therefore the unemployment rate in steady state can easily be computed by equating inflows and outflows

$$\eta(1 - \mathcal{U}) = \mathbb{E}[p(\theta)]\mathcal{U} \quad \Rightarrow \mathcal{U} = \frac{\eta}{\eta + \frac{1-e^{-\phi q}}{\phi q}} \tag{7}$$

Generalizing this environment, I could allow for interviewing probabilities ϕ contingent on the number of applicants, which nests the standard sequential search model. Concretely, suppose that firms choose (perhaps suboptimally) $\phi_k \in (0, 1]$ if $k = 1$, and $\phi_k = 0$ otherwise, for a relatively large marginal screening cost, ξ. This scenario is essentially the standard sequential search model: the employer interviews only one applicant, who gets hired with chance $\frac{1-e^{-\phi_1 q}}{\phi_1 q}$, and ignores the rest. If the employer wants to interview some other candidate, he needs to post a vacancy again. I stick to the case in which ϕ is unconditionally set because its enough to highlight the main points of this paper.

2.4.3 Distributions

So far the analysis has been in partial equilibrium because the distribution of profits of unemployed workers $G_U(\theta)$ is given. However the recruiting selection process affects the distribution of unemployed workers. I show in this section how this distribution is endogenously determined in equilibrium.

In steady state, the mass of workers with unemployment duration $\delta + 1$ is the mass of workers of duration δ who do not find a job. Considering that workers who do not apply so that $(a(\theta) = 0)$ have no chance of being hired,

$$g(\theta, \delta + 1) = (1 - p(\theta))g(\theta, \delta) \quad \forall \delta \geq 1 \tag{8}$$

Individuals of duration 1 are those who just had a separation shock. Employed workers are those who were not hit by the separation shock and those who find a job regardless of duration. The laws-of-motion for agents with $\delta = 0$ or $\delta = 1$ are given by

$$g(\theta, 1) = \eta g(\theta, 0) \tag{9}$$

$$g(\theta, 0) = (1 - \eta)g(\theta, 0) + p(\theta) \sum_{i=1}^{\infty} g(\theta, \delta = i) \tag{10}$$

Doing some algebra and recalling that $l(\pi)$ is the marginal distribution of productivities[7] and that $f(\theta) - g(\theta, 0) = \sum_{i=1}^{\infty} g(\theta, \delta = i)$ I obtain that

$$g(\theta, \delta > 0) = g(\theta) - g(\theta, 0) = \frac{\eta f(\theta)}{\eta + p(\theta)}$$

Since the equilibrium unemployment rate equals the mass of workers with positive unemployment duration, i.e. $\mathcal{U} = \sum_{i=1}^{\infty} g(\delta = i | a(\theta) = 1)$, the cumulative distribution function of productivities of unemployed workers is

$$\begin{aligned} G_U(\theta) &= \int_{\underline{\theta}}^{\theta} g(v | \delta > 0, a(\theta) = 1) dv \\ &= \int_{\underline{\theta}}^{\theta} \frac{\eta f(v)}{\mathcal{U}(\eta + p(v))} dv = (\eta + \mathbb{E}[p(\theta)]) \int_{\underline{\theta}}^{\theta} \frac{f(v)}{\eta + p(v)} dv \end{aligned} \tag{11}$$

This is a Volterra nonlinear integral equation whose solution is the endogenous cumulative distribution function of the unemployed workers. Equation (11) can be written as

$$\frac{dG_U(\theta)}{d\theta} = (\eta + \mathbb{E}[p(\theta)]) \frac{f(\theta)}{\eta + \widetilde{p}(G_U(\theta))}$$

with $\widetilde{p}(G) = \psi \frac{e^{-\phi q}}{\phi q} + (1 - \psi) e^{-\phi q (1 - G)}$. This is also a separable differential equation such that AQUI

$$\int (\eta + \widetilde{p}(G_U(\theta))) dG_U(\theta) = (\eta + \mathbb{E}[p(\theta)]) \int f(\theta) d\theta$$

$$\eta G_U(\theta) + \frac{e^{-\phi q (1 - G_U(\theta))}}{\phi q} + C = (\eta + \mathbb{E}[p(\theta)]) F(\theta) \tag{12}$$

[7] Given our definitions, $f(\theta) = g(\theta) = \sum_{i=0}^{\infty} g(\theta, \delta = i)$.

The boundary conditions dictate that $G_U(\infty) = F(\infty) = 1$. Because $F_\theta(\infty) = 1$ we can determine that $C = -\frac{(1-\psi)e^{-\phi q}}{\phi q}$. Equation (12) shows us it is not generally possible to obtain a close-form solution for $G_U(\theta)$ in terms of the primitives. However, there is a close-form mapping between quantiles of the unemployed distribution and the quantiles of the original population, given an equilibrium queue size q and the optimal hiring probability, ϕ. Using the transformation $x = G_U(\theta) \rightarrow G_U^{-1}(x) = \theta$. Therefore, we obtain

$$F^{-1}(M(x;q)) = G_U^{-1}(x) = \theta \text{ with } M(x;q) \equiv \frac{(\eta\phi q + \psi(1-e^{-\phi q}))x + (1-\psi)(e^{-\phi q(1-x)} - e^{-\phi q})}{\eta\phi q + 1 - e^{-\phi q}} \tag{13}$$

The result in equation (13) is key to express the free-entry condition in a manner it does not depend on the unknown distribution $G_U(\theta)$.

2.4.4 Solving the hiring problem

Under CRE conjecture, I characterize the problem of (2.2) given a distribution of unemployed workers $G_U(\theta)$ which is exogenous from the viewpoint of an individual employer. First, notice that employers never choose to reject all applicants and repost a vacancy since no applicant whose value to the firm is lower than V will ever submit a costly application.

Generally, in Appendix B I show the free entry condition can be written as

$$\kappa + \beta\xi\phi q = \beta\phi q \int_0^1 J(G_U^{-1}(x))\widetilde{p}(x)dx \tag{14}$$

In this setup, quantiles of an unknown distribution $G_U(\theta)$ are mapped to the quantiles of a known distribution. By doing so, we obtain an equation that characterizes the equilibrium value of q without figuring out $G_U(\theta)$. Therefore, we obtain

$$\kappa + \beta\xi\phi q = \beta\phi q \int_0^1 J(F^{-1}(M(x;q)))\widetilde{p}(x)dx \tag{15}$$

The mapping $M(q,x)$ always maps quantiles x of the unemployed distribution to a lower quantiles of the population distribution x. As the queue size q increases, the recruiting selectivity intensifies. Moreover, the quantile mapping is key for showing the existence of a queue size in this economy, as established next

Proposition 1 *Under a CRE conjecture,*

- *There exists at least one solution in* (18) *for the queue size* q *in competitive equilibrium if* $\xi < \frac{(1-\alpha)(1-\rho)\mathbb{E}[\theta]+\alpha(1-\beta)\varphi}{1-\beta(1-\eta)} \equiv \overline{J}\left(\mathbb{E}(\theta)\right)$
- *For any solution of* q*, there exists a unique steady-state joint distribution of productivities and durations* $G(\theta,\delta)$ *with unique density* $g(\theta,\delta)$.

Proof. See Appendix A ■ Under CRE, the first point states a sufficient condition for the existence of at least one equilibrium q in the competitive economy. The marginal screening cost ξ has to be lower than an upper bound of the present value of profits generated by the worker with average productivity in the population.

In addition, I can prove that the choice of the optimal screening probability satisfies

$$\xi(1+\phi q)+\kappa = \beta\int_0^1 J\left(G_U^{-1}(x)\right)x\widetilde{p}(x)dx$$

Using the same rationale, there exists a quantile x^{**} such that the following holds

$$\frac{\xi(1+\phi q)+\kappa}{\beta\int_0^1 x\widetilde{p}(x)dx} = \frac{\left(\xi(1+\phi q)+\kappa\right)(\phi q)^2}{\beta(\phi q+e^{-\phi q}-1)} = J\left(G_U^{-1}(x^{**})\right) = J\left(F^{-1}(M(x^{**};q,\phi)\right)$$

To obtain a full solution, I characterize the functional form of $J(\theta)$ in equilibrium. As an intermediate step, I derive the equilibrium value of unemployment per type using the Nash wage equation in (4) as well as value functions in (2.1) and (2)

$$(1-\beta)U_1(\theta) = L(\theta)\rho\theta + (1-L(\theta))(\theta-(1-\beta)\varphi) \tag{16}$$

$$\text{with } L(\theta) = \frac{1-\beta(1-\eta)}{1-\beta(1-\eta)+\alpha\beta p(\theta)}$$

Given the stationary environment, once a nonemployed worker searches, it will always choose to search. Using the equilibrium value of $U_1(\theta)$, it is simple to show that jobless workers search provided its productivity satisfies

$$\theta > \underline{\theta} = \frac{(1-\beta)\varphi}{1-\rho}$$

Using the latter result, algebra shows that the value of $J(\theta)$ is

$$J(\theta) = \frac{(1-\alpha)L(\theta)((1-\rho)\theta-(1-\beta)\varphi)+(1-\beta)\varphi}{1-\beta(1-\eta)} \tag{17}$$

Finally, the equation determining the value of q in equilibrium is obtained by substituting equation (17) into (**??**)

$$\frac{\kappa+\beta\phi\xi q}{\beta(1-e^{-\phi q})} = \frac{(1-\alpha)L(F^{-1}(M(x^*;q)))((1-\rho)F^{-1}(M(x^*;q))-(1-\beta)\varphi)+(1-\beta)\varphi}{1-\beta(1-\eta)} \tag{18}$$

This equation characterizes a solution for q which *does not depend* on the unknown distribution of unemployed workers. In principle, it may have multiple solutions. Conditions for a solution of the above equation [**PENDING**] Proven the existence of q independently from the existence of G_U, it is relatively simple to show the existence of the invariant distribution.

Proposition **??** proves the existence of a solution under the conjecture for any solution equation (18) generates. Hence, the model could no equilibrium at if, for instance, κ is too large, but it could also delivers multiple equilibria. [**PENDING**] The above equation is useful to prove the existence of an equilibrium in the model, but not that much for finding the set of equilibria because the quantile x^* is unknown in principle. Despite of this difficulty, since some primitive parameters of the model are not readily observed, equation (18) is still important to numerically solve the model. A second observation is that the model would be quite amenable to quantile utility maximizer employers as apposed to expected utility maximizer ones. The latter has become the standard in economics, while the former has has been studied in decision theory only recently **?**.[CHECK THIS SECTION]

2.4.5 Verifying the conjecture

Finally, the solution of the model must meet the conjecture on the optimal hiring policy. Thus, the a Coincidence Ranking Equilibrium hold if $J'(\theta) > 0 \quad \forall\theta$. From equation (17) we can easily conclude that this condition holds if

$$-\frac{d\log L(\theta)}{d\log\theta} \leq \frac{\theta}{\theta - \varphi(1-\beta)}$$

In Appendix C I show that this condition holds if

$$\frac{\beta^{-1} - 1 + \eta}{\alpha} > p'(\theta) - p(\theta) = p(\theta)(\phi q g_U(\theta) - 1) \quad \forall\theta > \underline{\theta} \tag{19}$$

Inspecting the previous expression it is readily apparent that the CRE conjecture holds more easily when the interest rate $\beta^{-1} - 1$ and separation rate η are large. This occurs because the value of future matches decrease in this “effective” discount rate, reducing the value of the workers’ outside options. At the same time a small worker bargaining power α make it easier the CRE to hold because the employer can obtain larger profits.

Previous condition also shows that large values for the density of the unemployed at some θ make it hard the CRE to hold. The shape of $g_U(\theta)$ depends on the population’s productivity density and the screening technology. Intuitively, the relative productivity ranking of workers is what determines the job finding rate. If the density of productivities is large at some θ_0 (i.e. not enough local dispersion), a great difference in job finding probabilities can be due to a small increase in productivity. The outside option of a worker rises much more than her productivity does, making it unprofitable for the firm to prefer high productivity workers around θ_0.[8] Moen (1999) does something similar... [**PENDING**] A large queue size q –or equivalently a high unemployment rate– makes it difficult for a CRE to exist. Intuitively, if the average number of applicants per vacancy is high, available jobs are scarce and it is extremely easy for very good workers to get hired. Since the top applicant's outside option is too high, firms have to yield almost all the surplus to the worker, which creates incentives to hire less productive workers.

Moreover, it is possible to derive a condition under which the conjecture holds by maximizing the right-hand side of the inequality (19). The quantile x^{**} at which that expression is maximized[9] is given by

$$-\frac{f'(F^{-1}(M(x^{**};q)))}{f(F^{-1}(M(x^{**};q)))^2} = \frac{\eta(\phi\eta q + 1 - e^{-\phi q})}{\eta + e^{-q\phi(1-x^{**})}}$$

Hence, by computing x^{**} and checking if (19) evaluated at $\theta^{**} \equiv F^{-1}(M(x^{**};q))$, we obtain a sufficient condition for a Coincidence Ranking Equilibrium.

2.4.6 Stationary Symmetric Recursive Equilibrium

With the previous steps, we can formally define a Stationary Symmetric Recursive Equilibrium of this model as

i. A set of value functions $U(\theta) = \max\{U_1(\theta), U_0(\theta)\}$, $W(\theta)$, V and $J(\theta)$ defined in equations (2.1), (1), (2), (2.2), and (3) that describe the optimal choices by workers and firms given an aggregate state $\mathcal{X}$.

ii. Policy function $a(\theta) = \mathbb{I}[\theta > \underline{\theta}]$ that solves the worker's application problem in $U_1(\theta)$, given the aggregate state $\mathcal{X}$.

[8] There is a similar effect in Shimer (1999), but he shows that in his auction wage determination mechanism wages can be locally decreasing in productivity to achieve a coincidence ranking equilibrium in spite of the reduced local variability of productivities. Due to the Nash bargaining framework I assume here, locally decreasing wages are not possible.

[9] Rigourously, I only derive necessary first-order conditions. Multiple numerical simulations using a lognormal distribution of types show it is indeed a maximum.

iii. Equilibrium job finding probability function $p(\theta)$ as described in equation (5) conditional on the aggregate state $\mathcal{X}$.

iv. A wage schedule $w(\theta)$ that solves condition (4) given $\mathcal{X}$.

v. An aggregate state $\mathcal{X} = (\mathcal{U}, \mathcal{V}, G(\theta, \delta))$ consistent with individuals' behavior and free entry $V = \varphi$, in which $\mathcal{U}$ is defined according to $a(\theta)$ as in equation (7) and $G_U(\theta)$ is defined as in equation (**??**).

2.5 Characterization of the equilibrium

The equilibrium of the model generates several features of the cross-sectional distribution of wages and unemployment durations that are characterized in the following propositions[10]. In the first result, workers with long unemployment durations decrease their wages in expectation. This is a direct consequence of the recruiting selection process.

Proposition 2 *Conditional on the duration of an unemployment spell, the mean and variance of the wage are decreasing in this duration.*

1. *Given a distribution* $G_U(\theta)$*, for all* $\delta > 0$ *and* $a(\theta) = 1$, $\mathbb{E}[w(\theta)|\delta] > \mathbb{E}[w(\theta)|\delta + 1]$ *and* $\mathbb{V}[w(\theta)|\delta] > \mathbb{V}[w(\theta)|\delta + 1]$
2. *If* $\delta = 0$, $\mathbb{E}[w(\theta)|\delta] = \mathbb{E}[w(\theta)|\delta + 1]$ *and* $\mathbb{V}[w(\theta)|\delta] = \mathbb{V}[w(\theta)|\delta + 1]$

Proof. See Appendix A ■

The second result shows that expected durations are longer for individuals of low productivity.

Proposition 3 *The expected duration conditional on the productivity type is*

$$\mathbb{E}[\delta|\theta] = \frac{\eta}{(\eta + \pi(\theta))\pi(\theta)} \tag{20}$$

which is a differentiable and strictly decreasing function in θ.

Proof. See Appendix A ■

Since for a given worker the probability of leaving unemployment is $\pi(\theta)$, the duration of unemployment follows a geometric distribution conditional on θ. For that

[10]The propositions obviously hold only for individuals in the labor force ($a(\theta) = 1$)

reason, Proposition 3 shows that the unconditional duration of unemployment is the probability of being unemployed $\frac{\eta}{\eta+\pi(\theta)}$ multiplied by the expected duration conditional on being unemployed $\frac{1}{\pi(\theta)}$. Using the same rationale, the variance of the duration conditional on being unemployed is $\frac{1}{\pi(\theta)^2}$. The next result concerns the unconditional variance of unemployment durations

Proposition 4 *The variance of the unemployment duration conditional on the productivity type is*

$$\mathbb{V}[\delta|\theta] = \frac{\eta^2 + (2\eta - \eta^2)\pi(\theta) - \eta\pi(\theta)^2}{(\eta + \pi(\theta))^2\pi(\theta)^2} \tag{21}$$

which is a differentiable and strictly decreasing function in θ.

Proof. See Appendix A ■

3 Efficiency Analysis

As much of the literature assumes, the Social Planner (SP) is entitled to make workers' and employers' decisions, but she uses the same recruiting and screening technology that all individuals agents use. The SP maximizes the sum of all productions in the economy, including employed, unemployed and inactive workers net from the cost of posting vacancies and screening applicants.

Therefore, the SP can control the queue size margin $q = \mathcal{U}/\mathcal{V}$, through vacancy posting, and the participation decision. Given the Planner's objective, it is readily clear that she always assign the job to the most productive applicant for an open vacancy. This allows us to use the same machinery we develop for the competitive equilibrium solution under a CRE. Thus, the output-maximizer SP would like to find an optimal solution for the following steady state production:

$$\begin{aligned} Y(q) = &-(\kappa + (1-\psi)\underline{\xi}\phi q + \psi\overline{\xi}\phi)\mathcal{V} + F(\underline{\theta})\int_{-\infty}^{\underline{\theta}} \rho\theta f(\theta)d\theta \\ &+ (1 - F(\underline{\theta}))\left((1-\mathcal{U})\int_{\underline{\theta}}^{\infty} \theta g_E(\theta)d\theta + \mathcal{U}\int_{\underline{\theta}}^{\infty} \rho\theta g_U(\theta)d\theta\right) \end{aligned}$$

where $g_E(\theta) \equiv g(\theta|\delta = 0) = \frac{f(\theta)p(\theta)}{(1-\mathcal{U})(\eta+p(\theta))}$ is the density of productivities of employed workers.[11] Doing some algebra and considering that $q = \mathcal{U}/\mathcal{V}$, and that steady-state

[11] To derive the former result, just consider that $f(\theta) = g(\theta|\delta > 0)\mathcal{U} + g(\theta|\delta = 0)(1 - \mathcal{U})$.

unemployment equals $\mathcal{U} = \frac{\eta}{\eta + \frac{1-e^{-\phi q}}{\phi q}}$, we obtain

$$\begin{aligned} Y(q) &= -\frac{\eta(\frac{\kappa+\psi\overline{\xi}\phi}{q} + (1-\psi)\underline{\xi}\phi)}{\eta + \frac{1-e^{-\phi q}}{\phi q}} + \int_0^\infty \theta f(\theta)\frac{\rho\eta + p(\theta)}{\eta + p(\theta)}d\theta \\ &= \mathbb{E}[\theta] - \frac{\eta\phi(\kappa + \psi\overline{\xi}\phi + (1-\psi)\underline{\xi}\phi q)}{\eta\phi q + 1 - e^{-\phi q}} - (1-\rho)\eta\int_0^\infty \frac{\theta f(\theta)}{\eta + \psi\frac{1-e^{-\phi q}}{\phi q} + (1-\psi)e^{-\phi q(1-G_U^{SP}(\theta))}}d\theta \end{aligned} \tag{22}$$

where $\underline{\theta} = 0$, the infimum of the support of $F(\theta)$, since all workers are more productive at work. The term $G_U^{SP}(\theta)$ stands for the cumulative distribution of jobless workers when $\underline{\theta} = 0$ (as opposed to $\underline{\theta} = \varphi(1-\beta)/(1-\rho)$ in the competitive equilibrium).

The first term of equation (22) indicates the output obtained in a frictionless environment. The second is the direct cost of recruiting effort, that is, vacancy posting and candidate screening. The third term is the indirect cost associated to the screening process. As recruiting activity increases, the composition of the unemployment pool worsens, making it increasingly difficult to recruit high productivity workers.

In that the hiring policy of the SP is the same as an employer under CRE, we can use the insight of equation (13) (provided $\underline{\theta} = 0$) to write (22) as

$$Y(q) = \mathbb{E}[\theta] - \frac{\eta\phi(\kappa + \psi\overline{\xi}\phi + (1-\psi)\underline{\xi}\phi q)}{\eta\phi q + 1 - e^{-\phi q}} - \frac{(1-\rho)\eta\phi q}{\eta\phi q + 1 - e^{-\phi q}}\int_0^1 F^{-1}(M(x;q))dx \tag{23}$$

which is very amenable to numeric calculations because it does not depend on the unknown equilibrium distribution of unemployed workers $G_U^{SP}(\theta)$.

It is useful to understand the current discussion when compared to the standard sequential model. The previous formulation can be specialized to analyze this traditional bechmark by setting $\psi = 1$.

4 Empirical Assessment

4.1 The data

This model concerns about distributions of productivity types (for a very large economy), not necessarily individuals. The predictions of the model for labor markets hold if individuals change their type every period as long as the population distribution of types $F(\theta)$ remains invariant. Thus, the model is compatible with many visions of the productivity stochastic process at the individual level.

Taking the model to the data requires an appropriate matching between theoretical and empirical measured variables. Since productivity types can only be inferred through paid wages, it is necessary to impute counterfactual wages to unemployed workers. This is a task that requires additional assumptions. Using CPS March data, we have several ways to do this. One consists on looking at current wages of individuals who declare to were unemployed during last year for a given number of weeks in one stretch. Nevertheless, the productivity type of them may have changed since the last unemployment spell due to human capital acquisition. An alternative approach, pursued here consists on estimating a Tobit selection equation of elapsed unemployment duration, which will allow us to correct for potential productivity loses during the unemployment spell. The last consideration would be particularly important if there is duration dependent or scarring effects during unemployment spells.

Another important consideration is the proper notion of "labor market" used to contrast the model to the data. Interpreting the model as the whole labor market is problematic since it would imply that very dissimilar workers –janitors vs neurosurgeons– compete for the same jobs. Therefore, the proper scope for the model is a representative "local" labor market defined by a occupation and industry.

it Therefore, because in the model jobs are *ex ante* identical, a reasonable step is to "clean" the data from any measurable influence of firm/match specific heterogeneity. However, different ways of controlling for these factors in the data does not perceptibly change the empirical conclusions[12]. To make conclusions as transparent as possible, I use unadjusted detrended log weekly earnings in dollars of 2000.

A second approach to matching the theoretical variables of the model is to focus on a narrower labor market. Calibrating the model to target aggregate labor market moments portrays surgeons and janitors competing for the same jobs. To avoid such criticisms, I also calibrate the model to a segmented labor market of production workers. Because the number of interviews in NES97 refers to production jobs, there is another reason to focus on this restricted labor market. In the NES97, a 45.5% of nonmissing employers' answers refer to typical job titles such as machine operators, assemblers, welders, laborers, line workers, etc. Using the Earnings Study of the CPS,

[12] I run several specifications of Mincer augmented regressions including polynomials of age; gender, race, education category, state and year dummies; industry and employer size category dummies; and last year weekly earnings March CPS. I "clean" the data by subtracting the estimated contribution of industry, employer size, state and year from the log weekly earnings. Besides a slight reduction of the unconditional standard deviation, the cross sectional distribution of adjusted log earnings and unemployment durations is very similar to the distribution of unadjusted log earnings and durations.

I obtain reasonable empirical counterparts of wages and unemployment durations for this specific labor market segment.

The reasonable data counterpart of the predicted relationship between (re-employment) wages and durations is the empirical joint distribution of completed unemployment spells and re-employment weekly earnings. Due to the cross-sectional nature of CPS data, I take the workers' current weekly earnings and the reported number of weeks of unemployment in one stretch to measure re-employment wages and the corresponding completed unemployment duration[13]. I do not include workers with more than one reported spell in the previous year because any particular way to impute duration is highly arbitrary. If separation is roughly exogenous as in the model, the sample is not biased. I also assume that the reported unemployment spell ended when the worker got the job she reports in the Earnings Study. Since earnings are reported in March, given the separation rates observed in the data, this assumption should not introduce a relevant bias. [14]

4.2 Calibration

The period is set to two weeks since this is the median time to fill a vacancy in NES97 data. The calibration matches data from CPS 1985-2006 such as the unemployment rate and mean and standard deviation of log weekly earnings of individuals who did not report being unemployed last year. I also target the median number of applicants interviewed from the NES97. Using parameters implied by this choice of targets, I compute the joint density of log wages and unemployment durations and compare the simulated data to their empirical CPS counterparts. I assume that the productivity distribution of the labor force $f_\theta(\theta|a(\theta) = 1)$ is lognormal truncated at the lowest productivity worker who earns the minimum wage.

[13]Although preferable, longitudinal data like NLSY79 or PSID do not have enough observations – specially for segmented labor market analysis– or are not representative enough of the whole US economy. CPS data provides a limited longitudinal structure that suffices for the empirical exercise in this paper.

[14]Individuals who had more than one unemployment spell in the previous year may be a non random sample of the population of individuals who report some unemployment. If separation rate depended on workers' productivity, the exclusion of these workers would generate sample-selection bias.

Targeting Unemployment and Recruiting Costs:

The double equality in equation (**??**) shows the relations among the average hazard rate, separation rate, unemployment rate and average number of applications. All these pieces of information come from independent sources. To measure the average job finding (hazard) and separation rates, I follow the procedures described in Shimer (2005). In steady state, the empirical job finding rate $\widehat{\mathbb{E}}[\pi(\theta)]$ equals the ratio between just-separated workers, $A_{\delta=1}$, and unemployed ones, A. Denoting by $\mathcal{L}$ the number of employed workers, the separation rate is computed as $\widehat{\eta} = A_{\delta=1}/\left(\mathcal{L}(1-0.5\widehat{\mathbb{E}}[\pi(\theta)])\right)$ to correct for the fact that some workers find jobs within the period. I use monthly flows to compute these statistics due to the zigzag pattern of the empirical distribution of durations, usually attributed to recall bias. Because of the different time span used, the numbers obtained are lower than those reported by Shimer (2005). The empirical unemployment rate $\widehat{\mathcal{U}}$ is traditionally measured.

Strictly speaking, NES97 reports *the number of interviewed candidates* instead of the number of applications. Taking the model literally, the average number of interviews is the mean of a Poisson random variable that is right-censored at the maximum number of applicants i^*. For calibration purposes i^* is treated as a parameter. The choice of i^* is crucial to mimicking the shape of the conditional distribution of unemployment durations $f_\delta(\delta)$ according to the discussion in Section **??**.

To choose i^* and λ, I use the computed separation and unemployment rates to calculate the implied number of applications per vacancy λ as the root of

$$\frac{1-e^{-\widehat{\lambda}}}{\widehat{\lambda}} = \widehat{\eta}\left(1/\widehat{\mathcal{U}}-1\right)$$

Using $\widehat{\lambda}$, the value of the maximum number of screened applicants i^* is the right-censoring point that equalizes the median number of interviewed candidates in NES97 data and in the model.[15]

After solving the model using the computed λ and i^*, the marginal screening cost is calculated to be consistent with the choice of i^*, for instance[16] $\xi = 0.5(\widetilde{J}(i^*+1) - \widetilde{J}(i^*-1))$. The vacancy posting cost κ is calibrated so that the numerical solution of

[15]In this case, targeting the median number of interviews instead of the mean is preferable for two reasons. First, the NES97 contains several outliers that reduce the reliability of the sample average. Second, trying to correct the nonresponse bias (17% in this question) by different strategies yields an adjusted average ranging between 4.8-6.2. In contrast, regardless of the adjustments, the median remains invariable.

[16]There are a range of compatible values.

$M(k) = \max\{\widetilde{J}(k) - \xi k, \widetilde{J}(i^*) - \xi i^*\}$ and $\widetilde{F}(\theta)$ is consistent with the free entry condition. Proposition **??** guarantees that the already computed λ is a unique solution for free entry condition.

Targeting unemployment benefits and minimum wage:

Assuming CRRA preferences with parameter γ, the Nash condition becomes

$$\theta = w(\theta) + \frac{1-\alpha}{\alpha(1-\gamma)} S(\theta) \left(w(\theta) - (b(1-z))^{1-\gamma} w(\theta)^{\gamma} \right) \tag{24}$$

A worker of the lowest type $\underline{\theta}$ earns the lowest wage $w(\underline{\theta})$. Moreover she has the lowest hazard rate $\pi(\underline{\theta}) = e^{-\lambda}$ because she only gets hired if she does not face any other competitor. Moreover, the type $\underline{\theta}$ must also be indifferent between being in the labor force or not, i.e.

$$Q_a(\underline{\theta}, w(\underline{\theta})) = Q_n \quad \Rightarrow \quad S(\underline{\theta}) u(b(1-z)) + (1 - S(\underline{\theta})) u(w(\underline{\theta})) = u(b)$$

Using these conditions, the unemployment income b is expressed in terms of other parameters.

$$b = w(\underline{\theta}) \left[\frac{1 - S(\underline{\theta})}{1 - S(\underline{\theta})(1-z)^{\gamma-1}} \right]^{\frac{1}{1-\gamma}} \tag{25}$$

Substituting (25) into (24) and doing some algebra,

$$\underline{\theta} = w(\underline{\theta}) \left(1 + \frac{1-\alpha}{\alpha(1-\gamma)} S(\underline{\theta}) \left(\frac{1 - (1-z)^{1-\gamma}}{1 - S(\underline{\theta})(1-z)^{1-\gamma}} \right) \right) \tag{26}$$

Two additional restrictions are needed: (i) the observed minimum wage $w(\underline{\theta})$ is greater than $b(1-z)$ and (ii) Unemployment income of an applicant $b(1-z)$ is positive. There are three cases to analyze. If $\gamma > 1$, by algebraic manipulation of (25), the conditions are met if $z < \widehat{z} \equiv 1 - S(\underline{\theta})^{\frac{1}{\gamma-1}}$. In case $\gamma < 1$, it is needed that $z > \widehat{z} \equiv 1 - S(\underline{\theta})^{\frac{1}{\gamma-1}}$. Finally, the restrictions are always satisfied if $\gamma = 1$.

Targeting wage distribution:

I choose the mean and variance of the log productivities to target the mean and variance of the log earnings of employed workers with no unemployment during the previous year. A Newton-Raphson procedure suffices to find a solution for the implied system of non-linear equations (see **??** for more details). Other parameters such as the relative risk aversion γ, the exogenous bargaining power α and the ratio application cost to unemployment income z are set at conventional values.

Solving the model:

Solving the model is to find a solution for the Volterra integral equation in (**??**), which is the cumulative distribution function of unemployed workers, $\widetilde{F}(\theta)$. The computational algorithm is simple and fast. I describe it in **??**. Using the numerical solution, it is straightforward to compute hazard rates, wages and value functions.

The results obtained are presented in the section 4.4. Although the model qualitatively replicates the features of the joint distribution of re-employment log wages and unemployment durations, the magnitudes are exaggerated. The ability of firms to distinguish high and low productivity workers seems overstated. As a result, compared to the data, high productivity workers leave unemployment too soon and low productivity ones leave too late. This is a direct consequence of two assumptions: (i) Firms can perfectly observe productivity once workers have been screened, and (ii) Wages entirely depend on market productivity. While it is possible to extend the basic model to include these features in several ways (leisure value heterogeneity, noisy productivity signals, measurement error of wages, etc.), I present next a very simple modification that can generate more realistic moments of the joint distribution of wages and durations.

4.3 A simple extension of the model: Stochastic screening costs

If workers' applications are noisy signals of their productivities, firms will exert greater screening effort –high marginal costs ξ– to assess noisier candidates. One way to capture this idea is to allow for a stochastic marginal screening cost[17]. In the simplest case, firms face only two possible cost realizations[18]. With probability ψ all applicants are interviewed becase $\xi = 0$ (hence, $i^* = \infty$). With probability $1 - \psi$ firms face high enough $\xi > 0$ that makes them optimally interview only one applicant ($i^* = 1$). The *ex ante* hazard rate becomes a mixture of two extreme cases: unrestricted non-sequential search and sequential search. It is expressed as

$$\pi(\theta) = \psi e^{-\lambda(1-\widetilde{F}(\theta))} + (1-\psi)\frac{1-e^{-\lambda}}{\lambda} \tag{27}$$

Compared to the baseline case, this modification flattens out the hazard rate and the endogenous outside value functions, which implies a milder relation between wages

[17]Explicitly introducing noisy signals would generate comparable results at the cost of introducing complicated and inessential modifications to the structure of the model.

[18]It is straightforward to create cases with more complicated random screening costs.

and productivities. The model is solved exactly as before. The modified hazard rate is plugged into the Volterra integral equation (**??**). The average hazard rate of the economy does not change.[19] Facing this new uncertainty, the modified firms' free-entry condition becomes[20]

$$0 = -\kappa + \beta\psi \sum_{k=1}^{\infty} \frac{e^{-\lambda}\lambda^k}{k!}\widetilde{J}_k + \beta(1-\psi)(1-e^{-\lambda})[\widetilde{J}_1 - \xi]$$

To calibrate the model, we can set ψ to target the observed (censored) expected unemployment duration of the lowest wage worker $\overline{\delta}_{min}$. Using Proposition 3, the following equality approximately holds

$$\overline{\delta}_{min} = \frac{\eta}{\left(\psi e^{-\lambda} + (1-\psi)\frac{1-e^{-\lambda}}{\lambda}\right)\left(\eta + \psi e^{-\lambda} + (1-\psi)\frac{1-e^{-\lambda}}{\lambda}\right)}$$

This is a quadratic equation in ψ. In the case of all occupations, ψ is targeted to match an unconditional expected duration of 0.9 fortnights. For production workers, I target an unconditional expected duration of 1.3 fortnights. The next section shows the results of the basic and the extended model. All parameters for Model 1 (deterministic screening costs) and Model 2 (stochastic screening costs) are summarized in Table 1. Parameters in Table 1 are set to exactly match the unemployment rate, mean and standard deviation of wages of the employed workers ($\delta = 0$) for all occupations and for production workers[21].

4.4 Results

Table 2 summarizes the main results of the model. For Model 1, the choice of i^* makes the model match the median number of interviewed applicants. For Model 2, the value of ψ targets the minimum unconditional expected duration. While the number of interviewed candidates is below the range suggested by NES97 data in the case

[19]The proof is analogous to the one of Lemma **??**.

[20]Since the right-hand size is strictly increasing in λ, it follows that there is a unique solution for this equation given some $\widetilde{F}(\theta)$, using the argument of Proposition **??**.

[21]Production workers category includes the following groups for 1985-2003: Supervisors, production (628-633); Precision metal working (634-655); other precision production (656-699); machine operators and tenders, not precision (703-779); fabricators, assemblers and hand working (783-795); production inspectors, testers, samplers and weighers (796-799). For 2004-2006, the category includes production occupations (7700-8960)

Table 1: Calibrated parameters

Parameter	All occupations		Production workers		Note
	Model 1	Model 2	Model 1	Model 2	
α : Worker barg. Power	0.5	0.5	0.5	0.5	Symmetric Nash
β: Discount rate	$\sqrt[26]{0.95}$	$\sqrt[26]{0.95}$	$\sqrt[26]{0.95}$	$\sqrt[26]{0.95}$	Standard
η: Separation rate	1.212%	1.212%	1.451%	1.451%	CPS data
γ: Relative risk aversion	2	2	2	2	Standard
z: Search cost (1)	0.01	0.01	0.01	0.01	
$\underline{w}$: Min wage	20	20	20	20	US$ in 2000
b: Unem. income	19.58	19.97	19.16	19.97	From eq. (25)
μ_θ: Mean of θ (2)	6.745	7.032	6.930	7.196	
σ_θ: SD of θ (2)	0.828	0.943	0.539	0.610	
ξ: Interview cost	3924	26729	2993	15828	Match i^* and λ
κ: Vac. post cost	23302	67166	26129	55318	FE condition
ψ: Prob of low ξ (3)		0.462		0.463	Target mean duration of $\underline{\theta}$

Notes: (1) Ratio application cost to unemployment income b; (2) Target mean and standard deviation of $\log w(\theta)$ according Algorithm 2 in **??**; (3) Marginal cost under high cost shock. Low cost shock is 0.

of all occupations, for production workers the average value matches the range of estimates.

The median and mean duration of ongoing unemployment spells is overestimated by Model 1, and underestimated by Model 2. A similar pattern is observed for completed durations although Model 1 is closer in this case. Figure **??** displays the marginal distribution of both ongoing and completed unemployment spells. The data show spikes at durations that are multiples of 4, which is usually attributed to recall or rounding bias. For ongoing durations (Panels A and C), both models replicate well the shape of the empirical distributions, except for the censoring point at 52 weeks (26 biweeks). While "all occupations" data show that about 8% of unemployed workers have durations longer than one year, Model 1 generates more than 25% and Model 2 only about 3%. The production workers distribution shows a similar pattern. The excessive number of long-term unemployed in Figure **??** generated by Model 1 can be reconciled with the evidence if unemployed workers include passive searchers, who want a job although they declare not to be actively searching[22]. Although there is no exact recorded nonemployment duration for these passive searchers, I compute an approximated measure with the available information[23], called duration A, that displays a spike at the censoring point that is similar to the one of Model 1.

The overall fit to the distributions of log wages of employed workers seems fine (see Figures **??** and **??**) although only the mean and variance are explicitly targeted. With respect to Model 1, Model 2 substantially improves the fit to the distributions of unemployed workers with 12 or less weeks, 26 or less weeks and all the unemployed. Intuitively, the random screening cost decreases firms' ability to select high productivity workers. For this reason, wages and hazard rates are less positively correlated and the composition of long-term unemployed workers does not deteriorate too much due to the limited screening power.

Other predictions of the model are those coming from Propositions 2, 3 and 4. In order to compare those predictions to CPS data, I computed the relevant statistics in the model and in the data. Figures **??** and **??** show the moments generated by the models and by the data for all occupations and production workers. Panels A show that while Model 1 exaggerates the negative correlation between log wages and durations, Model 2 generates a good fit to the data. This is partially unsurprising because the parameter ψ was calibrated to approximately hit the unconditional expected duration of the lowest wage. Panels B in Figures **??** and **??** show the negative relation between the unconditional variance of duration and log wages. Again, Model 2 generates a sensible improvement with respect to Model 1.

Panels C and D show that both Models have limitations in mimicking the magnitude of the negative relation between wages and durations conditional on unemployment, although Model 2 performs somewhat better. Comparing results of Panels C and A suggests that other factors such as separation rate and leisure value heterogeneity may play some role in explaining the wage gap between recently hired (workers with $\delta > 0$) and tenured workers ($\delta = 0$). Models' performance is somewhat better for the variance of unemployment duration in Panel D in Figures **??** and **??**.

[22] For instance, Yashiv (2006) considered this extended definition to assess the cyclical properties of the standard search and matching model

[23] This measure is constructed by using a CPS variable indicating how long ago the respondents out of the labor force worked for the last time. Individuals answering more than a year ago are included as top coded. For individuals out of the labor force, another variable registers how many weeks the respondent looked for a job last year. The nonemployment spell is the value of this variable plus ten weeks since the individual is interviewed in March.

Panels E and F depict the mean and variance of log wages decreasing in duration. For the expected wage, Model 2 performs relatively well, especially for production workers, although both models overstate the magnitude of the relation. For the variance of log wages, both models perform similarly. From these results, I conclude that the pure recruiting selection mechanism with perfect productivity observability generates a selection that is too strong. Since the simple modification in Model 2 makes the model perform better, it is possible the fit to the data possibly improves by introducing more realistic informational and stochastic structure. I see this mixed success in replicating data features as the price paid for having the simplest model with the recruiting selection mechanism.

Finally, I provide computations of the derivative of the wage function $w'(\theta)$ for all the models and samples in Figure **??**, Panel A. The derivative is below 1 in the entire domain, so the existence of a Coincidence Ranking equilibrium is verified for all the calibrations. In Panel B, I depict the equilibrium hazard rates $\pi(\theta)$ for all cases. As expected, Model 2, which allows for stochastic screening costs, generates flatter profiles. Panels C, D, E and F show different distributions of types conditional on duration. The recruiting selection makes the conditional distributions progressively shift to the left as unemployment duration increases. For Model 2, the differences between distributions are less pronounced than for Model 1. Since firms can sometimes not screen workers in Model 2, the recruiting selection is less effective and the average productivity of workers decreases less markedly with duration.

5 Related Literature

The recruiting selection model outlined here is related to the literature of nonsequential search models, whose tradition goes back to the seminal work of Stigler (1961), Wilde (1977) and Burdett and Judd (1983). Most empirical and theoretical contributions focus on the workers' side: after submitting multiple applications, the workers choose the best offer received from employers (Stern 1989; Acemoglu and Shimer 2000; Kandel and Simhon 2002; Albrecht, Gautier, and Vroman 2006). To my knowledge, only Shimer (1999) and Moen (1999) share an employer-side viewpoint that is similar to this paper[24]. Shimer (1999) proposes a matching mechanism for *ex ante* identical employers and heterogenous workers. He suggests job auctions as a wage setting

[24] I thank Daron Acemoglu and Espen Moen for pointing this out.

Table 2: Data vs. Model generated moments

Parameter	All occupations			Production workers		
Statistic	Data	Model 1	Model 2	Data	Model 1	Model 2
$\mathcal{U}$ Unem. rate	5.74%	5.74%	5.74%	7.46%	7.46%	7.46%
Median ongoing dur	5	7	4	5	8	4
Mean ongoing dur(1)	7.58	11.54	6.10	8.06	12.1372	6.80281
Std.Dev ongoing dur(1)	13.55	10.28	5.92	14.47	10.4988	6.48563
Median completed dur	6	7	4	7	8	4
Mean completed dur(2)	8.06	6.25	5.64	8.45	6.11	6.12
Std.Dev completed dur(2)	6.10	8.53	5.15	6.21	6.18	5.47
i^*, max # interview		8			5	
$\mathrm{med}(K)$ interview	5	5		5	5	
$\mathbb{E}[K]$ interview (3)	4.26-6.48	4.90	2.84	2.74-5.91	4.35	3.09
Mean $\log(w)$ Emp.	6.19	6.19	6.19	6.21	6.21	6.21
Std.Dev $\log(w)$ Emp.	0.77	0.77	0.77	0.57	0.57	0.57
Perc 10 $\log(w)$ Emp.	5.21	5.14	5.21	5.54	5.44	5.49
Perc 25 $\log(w)$ Emp.	5.76	5.70	5.62	5.86	5.87	5.76
Perc 50 $\log(w)$ Emp.	6.26	6.25	6.18	6.23	6.28	6.20
Perc 75 $\log(w)$ Emp.	6.71	6.74	6.76	6.60	6.62	6.65
Perc 90 $\log(w)$ Emp.	7.09	7.13	7.20	6.91	6.88	6.98
Mean $\log(w)$, $1 \leq \delta \leq 6$	5.90	5.75	5.96	6.05	5.93	6.04
Std.Dev $\log(w)$, $1 \leq \delta \leq 6$	0.72	0.79	0.72	0.50	0.61	0.52
Perc 10 $\log(w)$, $1 \leq \delta \leq 6$	4.94	4.07	5.00	5.47	4.44	5.37
Perc 25 $\log(w)$, $1 \leq \delta \leq 6$	5.50	4.40	5.34	5.73	4.71	5.57
Perc 50 $\log(w)$, $1 \leq \delta \leq 6$	5.94	5.01	5.74	6.01	5.29	5.83
Perc 75 $\log(w)$, $1 \leq \delta \leq 6$	6.38	5.79	6.26	6.34	5.97	6.23
Perc 90 $\log(w)$, $1 \leq \delta \leq 6$	6.78	6.46	6.80	6.73	6.46	6.65

Notes: (1) Model-generated statistics censored at 51 weeks to be compared to data statistics. (2) Model-generated statistics truncated at 51 weeks to be compared to data statistics. (3) Average number of interviews widely varies with different weighting and nonresponse bias correction procedures. Higher and lower estimates are reported.

mechanism, while I assume a standard Nash-bargaining coupled with screening technology, which makes easier to compare to the rest of the literature. In Shimer's paper there is no screening because workers self-reveal their types while bidding. In contrast, I assume firms devote resources to obtain information on prospective employees. In addition, I stress the consequences of the recruiting technology in aggregate labor market outcomes, and quantitatively compare the model's predictions to the data. Moen (1999) studies human capital acquisition in this environment, stressing that... Wolthoff (2012) develops an interesting model with two-sided nonsequential search (i.e. workers submit multiple applications and employers meet multiple workers) using a wage-posting framework. This generality comes at the cost of allowing homogenous agents. The fact that workers rarely turn down job offers, however may indicate that the multiple application margin seems less active in real labor markets.

5.1 Sequential vs. Nonsequential search

Somewhat unnoticed, most of the search models assume sequential search despite that Stigler (1961) seminal work portrays a nonsequential search. The sequential framework is a particular case of the general nonsequential framework. Facing costly screening, employers may choose to interview only one arrived applicant. Firms' optimally hire applicants of productivity above $\underline{\theta}$. All workers with productivity greater than $\underline{\theta}$ face the same job finding probability; for all productivities below $\underline{\theta}$, there is zero chance of being hired. If workers below $\underline{\theta}$ face an infinitesimal cost of applying, they do not search at all. Hence, the standard sequential framework (as in Burdett and Cunningham (1998)) cannot match two, theoretically related, empirical observations. First, the NES97 data shows (and other datasets as well) each employer interviews a median of five candidates to fill a vacancy, mostly for productive and sales occupations. Second, the evidence shows a large amount of heterogeneity in job finding rates among the unemployed.

One can construct augmented sequential search models that help rationalizing the multiple interviewing. Nevertheless, whatever this extension is, it cannot match NES97 data. A vacancy fills in 2.89 weeks on average whereas an average of 5 workers are interviewed during that time. This implies is found Although is virtually impossible to empirically distinguish a nonsequential search from a augmented sequential search, it is worth noticing that some extensions of the sequential framework that are able to explain multiple interviewing may generate unpalatable implications in other dimensions. I discuss these issues in greater detail in (**?**)

The sequential search model, when augmented with firm/match heterogeneity in the matching process, may replicate the number of applicants per vacancy observed in the data. In such a setup matches are rejected if the received shocks are not good enough, which explains that firms interview multiple candidates. However, estimated structural search models find that workers rarely turn down job offers and that the acceptance rate does not significantly vary with workers characteristics (van den Berg 1990; Devine and Kiefer 1991; Eckstein and van den Berg 2007). Barron et al. (1997) and Andrews et al. (2008) advocate the importance of employer search behavior in light of this evidence. Indeed, a sequential model with firm/match heterogeneous productivity replicates the data if workers on average turn down no less than 80% of job offers so that firms interview roughly 5 applicants in expectation, which is a realistic number according to NES97.

An objection to the previous argument is that whether the firm or the worker rejects the match is generally irrelevant in matching models because both sides will always agree or disagree on the acceptability of the match. However, some direct measures of job offer rejection are available. Lollivier and Rioux (2005) report that 8% of unemployed workers have rejected one or more offers in a month in France. Even Blau (1992), the most unfavorable evidence for my arguments I find in the literature, reports a job acceptance probability around 50%, implying two interviews per vacancy in a sequential search environment. In addition, Lollivier and Rioux (2005) and Blau (1992) find that the number of self-reported job offers substantially increases in education, as predicted by the recruiting selection model.

Since in the sequential search model firm/match heterogeneity is the driving force generating a realistic number of interviews, this mechanism is exposed to the criticism of Hornstein, Krusell, and Violante (2007). These authors point out that, under a realistic match/firm heterogeneity dispersion, workers should turn down an implausible large number of job offers because the option value of waiting for a good draw is huge. This argument puts even more strain on the sequential search model because the amount of match/firm productivity dispersion is only compatible with very low job offer arrival rates. The latter implies a ridiculously large number of interviewed candidates.[25]

Another argument in favor of nonsequential search is the empirical evidence of van Ours and Ridder (1992), who show that applications' arrival concentrates shortly after the vacancy is posted. A sequential employer strategy is only consistent with evenly spaced applications on average. Abbring and van Ours (1994) propose a different empirical test to distinguish among sequential and nonsequential employer search strategies. If the sequential case is prevalent we should observe that the duration of the vacancy decreases in the number of applicants per vacancy. Under sequential search, the position is filled if a sufficiently high mutually acceptable match quality realizes. When the number of interviews is large, it is more likely that such realization occurs. In contrast, in the nonsequential case, we should observe that a greater number of applicants does not decrease the vacancy duration. In fact, it is even possible that the vacancy spell increases because employers need more time to interview candidates. Using a similar argument, Abbring and van Ours (1994) find evidence for nonsequential search. In line with this, there is a significant positive correlation between vacancy durations and the number of applicants in NES97 data.[26] Thus, the recruiting selection model accounts for empirical evidence that is hard to reconcile with sequential employer search.

5.2 Unobserved heterogeneity and negative duration dependence

Another shortcoming of the simplest employer sequential model is that hazard rates out of unemployment are the same for all workers' productivities. Most studies on unemployment duration show that unobserved heterogeneity component accounts for a high proportion of the hazard variation (see for instance, Machin and Manning (1999), van den Berg (2001))[27]. Empirical studies try to disentangle true duration depen-

[25] Hornstein et al. (2007) conclude that an unemployment duration of 91 months is compatible with the empirical firm/match heterogeneity dispersion in the data. This exorbitant number suggests that more than 540 candidates are interviewed per vacancy in that employers make at least 1.5 interviews per week according to NES97.

[26] Non reported estimations of Mixed Proportional Hazard rate models for NES97 vacancy durations, show that (i) the vacancy filling hazard rate decreases in the number of applications, and (ii) the magnitude and significance of the effect holds for different choices of baseline hazard rates, unobserved heterogeneity distributions and covariates.

[27] These findings typically derive from the estimation of Mixed Proportional Hazard models in which the hazard rate is decomposed into three multiplicative components: a baseline hazard common to all individuals, observable and unobservable heterogeneity.

dence of the probability of leaving unemployment from compositional effects. Allowing for unobserved hazard rate heterogeneity takes into account that individuals with intrinsic low hazard rates (or longer unemployment spells) tend to be overrepresented in the stock of unemployed. In contrast, in this paper the hazard rate heterogeneity naturally arises from the interaction of workers' productivities and the recruiting technology of firms, rather than simply assuming an idiosyncratic exogenous worker characteristic.

In the model, negative duration dependence of the hazard rate in the data arises due to the imperfect observation of workers' productivities. My model suggests that as productivity heterogeneity is better captured by covariates, the negative duration dependence should fade away. Empirical evidence[28] indeed shows that after controlling for worker's observed and unobserved heterogeneity, the negative duration dependence pattern weakens or vanishes. In addition, consistent with the recruiting selection model, covariates that are associated to higher earnings are also related to higher hazard rates (with the exception of age).

Allowing for match/firm heterogeneity does not help the sequential model to explain the mentioned evidence. Long unemployment spells are due to high reservation wages, which in turn are positively related to high wages. To avoid this counterfactual implication, most sequential models allow for some non-stationarity such as decreasing unemployment benefits, and negative duration dependence of arrival rates and re-employment wages. Three leading examples are "stigma" effect of long unemployment spells, human capital depreciation and unemployment benefits exhaustion. In the first case, under imperfect observability of workers' productivity, a long duration becomes a bad signal encompassing information of multiple application rejections[29] (Lockwood 1991; Kollman 1994). In these models, firms' hiring selection creates informational externalities. In contrast, my paper isolates the importance of selection by shutting down informational issues. Since every model of stigma has employers selectively recruiting workers, if unemployment duration is

[28]Although there is evidence of negative duration dependence in the literature (van den Berg and van Ours(1996, 1999), Addison and Portugal (2003), Guell and Hu (2006) among many others), the findings are not generally robust. Machin and Manning (1999) assert that after controlling for readily observable workers' characteristics, the negative duration dependence pattern disappears in most European studies. Examples of the latter are Abbring et al. (2002),Cockx and Dejemeppe (2005).

[29]The ranking model of Blanchard and Diamond (1994) is slightly different because there is an equilibrium with arbitrary discrimination against long-term unemployed workers.

important *per se* for firm's hiring decision, then it should generate observationally different testable implications. To my knowledge, there is no such a distinctive prediction. To illustrate the point, consider the claim that stigma models predict that the informational content of long unemployment spells is greater in a boom than in a recession. Indeed, empirical evidence shows that past unemployment duration is a better predictor of current duration when the joblessness occurred during a recession (Omori 1997). However, this finding is also consistent with a simpler model of selection with irrelevant informational content of the unemployment duration. In a recession, the number of applicants per vacancy is higher so that the expected productivity of new hirings increases. On average, individuals experiencing long unemployment durations in booms are less productive than individuals of long durations in recessions. The prevailing cyclical condition when the worker's unemployment spell took place reveals some information to an econometrician who imperfectly observes productivity. For employers, however, that information may not be relevant in the likely case that recruiters observe productivity better than the econometrician.

The second approach, human capital depreciation, explains that wages decay as unemployment spell increases because workers become less productive. For the same reason, firms are reluctant to hire individuals of long unemployment durations. Keane and Wolpin (1997), in order to match the data, estimate a striking 30.5% yearly depreciation rate of human capital when white collars are nonemployed. Machin and Manning (1999) (p. 3119) argue that there is lack of direct evidence about human capital depreciation and that employer's surveys assess long-term unemployed workers no worse than the average recruit.

Third, although unemployment benefits exhaustion may theoretically generate a decreasing sequence of reservation wages that fit the hazard rate profile across productivities, it is unlikely that this effect dominates, especially for high wage workers. Hogan (2004) finds that unemployment benefits do not significantly impact reservation wages after controlling for previous and re-employment wages.

5.3 Permanent Income and Welfare

The recruiting selection technology influences the lifetime inequality in terms of permanent income and welfare. As a first step to discuss the relation between recruiting selection and inequality, I derive expressions for these variables. After doing the same algebra used to derive equations (**??**), the permanent income conditional on unemployment y^U is

$$y^U(\theta) = S(\theta)b(1-z) + (1-S(\theta))w(\theta) \tag{28}$$

where $S(\theta)$ is defined as in (**??**). It follows that the permanent income for an employed worker is

$$y^E(\theta) = \frac{(1-\beta)w(\theta) + \beta\eta y^U(\theta)}{1-\beta(1-\eta)} \tag{29}$$

The unconditional long-run probability of being employed is $p^E(\theta) = \frac{\pi(\theta)}{\pi(\theta)+\eta}$ while the one of being unemployed is $p^U(\theta) = 1 - p^E(\theta)$. Therefore, the unconditional permanent income is

$$y(\theta) = p^E(\theta)y^E(\theta) + (1-p^E(\theta))y^U(\theta) \tag{30}$$

With analogous logic, the unconditional welfare –certain equivalent[30]– measured in terms of consumption is

$$we(\theta) = u^{-1}\left((1-\beta)\left(p^E(\theta)W(w(\theta)) + (1-p^E(\theta))Q(\theta, w(\theta))\right)\right) \tag{31}$$

Given that employers are risk neutral and make zero profits in equilibrium, workers' certain equivalent is a correct measure of welfare for the whole economy.

In a sequential search model the *ex ante* inequality is simply determined by the workers' productivity because all the applicants have the same chance of being hired. In contrast, under recruiting selection individuals of low productivity forgo more labor earnings during their lifetimes than more productive workers do. Moreover, the joint effect of search frictions and Nash bargaining creates another channel to intensify inequality because high productivity workers obtain higher wages due to their more valuable outside options. The inequality amplification effect is even greater in terms of welfare because of workers' risk-aversion. Low productivity workers not only have longer unemployment durations, but also face greater uncertainty about the length of their unemployment spells according to Proposition 4.

The recruiting selection model predicts a negative relationship between permanent income and unemployment durations. There is evidence of negative duration dependence of re-employment wages[31] and substantial earnings losses due to job displacement.[32] Stewart (2007) finds that not only does past joblessness predict low

[30] This is the amount of fixed consumption per period that, if given forever to the worker, generates as much utility as his uncertain lifetime stream given his productivity θ.

[31] Addison and Portugal (1989),Belzil (1995),Rao Sahib (1998),Christensen (2002)

[32] Ruhm (1991),Kletzer (1998),Farber (2003)

wages and future unemployment but also low wages forecast future unemployment. In line with the recruiting selection model, low productivity workers appear to be trapped in a cycle of long unemployment spells and low wages.

An alternative explanation that may create similar effects on permanent income and welfare is the existence of endogenous separation in which low-quality workers are more likely to be fired. That would explain why the unemployed have lower average productivity and wages than the employed. However, if in such a hypothetical model employers search sequentially, all participating workers face the same chance of being hired, as shown above. The endogenous separation story fails to explain why, conditional on being unemployed, durations and wages are negatively related in cross-sectional data. If empirically relevant, endogenous separation manifests as reoccurrence of unemployment. Such a relation is not significant in the US data (Heckman and Borjas 1980; Corcoran and Hill 1985; Choi and Shin 2002) though it is for European countries (Arulampalam et al. 2000). Endogenous recruiting, on the other hand, should reveal persistency in the length of unemployment durations for individuals[33]. Omori (1997) and Gregg (2001) find this effect for the US (NLSY79) and the UK (NCDS), respectively.

5.4 Search Externalities

In a Walrasian world, firms observe all workers' productivities, so that the job assignment is efficient. Under search frictions, firms can only observe a limited number of workers' productivities by costly screening of applicants. Within the "local" labor market defined by one vacancy and its heterogenous applicants, the job assignment is efficient among the interviewed candidates (under CR equilibrium). This local competition for the job among workers can only be achieved with some positive unemployment rate. It follows that there is a crucial role of unemployment for the efficiency of job assignment process in heterogenous labor markets.

The recruiting selection model generates externalities that differ from those arising in search and matching models with homogenous workers. Previous literature –notably Hosios (1990)– has identified side-effects of individuals' actions that impact labor market transition probabilities. Allowing for workers' heterogeneity I show that firms' entry and screening margins not only affect the chances of job market transitions, but also the composition of the unemployment pool.

[33]Heckman and Borjas (1980) name this effect "lagged duration dependence".

Three new kinds of externalities arise in the recruiting selection model. First, there is a *selection externality*. The privately optimal decision of screening a number of applicants and choosing the best one decreases the expected productivity of new hirings for other vacancies. In general equilibrium, if firms interview more applicants, good workers are more easily taken out of the unemployment pool and the composition of unemployed workers worsens. Secondly, there is a *partial equilibrium entry externality*. Posting new vacancies not only makes it harder for other employers to find a worker and easier for workers to be hired, as typically occurs in search and matching models. Whenever a new vacancy is posted, the expected number of candidates decreases. Since the other employers receive fewer applications in expectation, the average productivity of their top applicants declines. Third, a *general equilibrium entry externality* arises when a firm posts a vacancy and indirectly reduces the expected number of applicants per vacancy. In this way, firms screen fewer applicants regardless of the maximum number of interviewed candidates i^* implied by the marginal cost of screening ξ and the distribution of unemployed workers $\widetilde{F}(\theta)$. But in general equilibrium, the diminished ability of firms to take out good applicants improves the productivity composition of the unemployed workers, which is a clear positive side-effect for other firms.

The previous analysis shows that from a macroeconomic perspective, some of the screening effort of firms is a waste of resources. If all firms could coordinate to reduce the number of applicants interviewed and/or to post more vacancies, they would receive less applicants per vacancy but those workers would be drawn from a distribution of higher average productivity. On the other hand, the existence of unemployment (i.e. a mass of vacancies that generates some $\lambda > 0$) and some degree of recruiting selection are important for improving the efficiency of the job assignment for heterogenous workers.

If the screening technology is powerful, that is, the marginal interview cost is not very high, these externalities can be so important that the economy has multiple equilibria. In a computed example in Section **??**, three Pareto-ranked equilibria exist with low, medium and high unemployment rates. The low unemployment rate holds when there is high entry of vacancies and the general equilibrium entry externality prevails. For instance, in this equilibrium, firms face a high productivity distribution of unemployed workers. Spurred by the high expected profits they may make, employers post a large number of vacancies. Consequently, few applicants per vacancy arrive and firms cannot effectively remove the best workers from the pool. The fact that other firms have posted a lot of vacancies so that the pool of unemployed contains attractive candidates on average is *ex ante* good news for a prospective employer. However, after posting a vacancy, having fierce competition is *ex post* bad news because the expected number of applications is low. In this way, the high-quality/low-quantity of applicants equilibrium is preserved because the firms' screening cannot deteriorate too much the quality of the unemployed pool.

Multiple equilibria portray high or low "unemployment traps" that may be a partial explanation for large differences in unemployment rates across otherwise similar economies. In high unemployment countries, firms do not post many vacancies because the productivity of unemployed workers is low. Since the unemployment rate is high, employers screen a large number of applicants and easily take out of unemployment the high productivity workers. In this way, firms' recruiting selection perpetuates high-quantity/low-quality equilibrium.

6 Conclusions

The model presented in this paper embeds a recruiting selection process into a benchmark sequential search model with heterogenous workers. The empirical relevance of nonsequential search by employers has been documented in several empirical papers. I also show that several augmentations of the sequential employer search model cannot satisfactorily explain why multiple applicants are interviewed to fill a vacancy. Recruiting selection, a highly realistic feature of the job search process, provides a simple explanation for documented empirical facts. In the data, high hazard rates and wages are positively correlated, there is substantial hazard heterogeneity after controlling for observables, and the negative duration of the hazard rate is generally found spurious once adjusted for workers' heterogeneity.

The model also explains the fact that workers with long unemployment spells have low permanent incomes. As shown, the more intense the recruiting selection is, the greater the welfare and permanent income inequality are. Although it is tempting to conclude that stronger screening leads to a more efficient job assignment, an improved selection technology does not guarantee that result at the aggregate level. Firms may devote too much effort to screen applicants and by doing so worsen the composition of the pool of unemployed workers. For this reason, the average productivity of recruited workers may improve if all the firms screen less intensively and/or post more vacancies.

The model suggests new ways to interpret labor markets outcomes. The existence of unemployment is crucial in this model for employers to assign jobs efficiently. From the perspective of a single firm, a low unemployment rate generates low chances of making good hirings. For the whole economy, low unemployment rate discourages overscreening of applicants, which would imply a great waste of resources.

Instead of being an indication of labor market rigidities or frictions –as some conventional wisdom may subscribe– long unemployment spells reflect the intensity of employers' recruiting selection technologies. Prevalence of long term unemployment may be an indication of a significant deterioration of the productivity of unemployed workers. If the model describes real labor markets to some extent, policy makers should worry about long unemployment durations, but for reasons that are different from the traditional labor market flexibility considerations.

Even though the calibration of the model overstates the features of the joint distribution of wages and unemployment durations observed in CPS data, this result seems to be driven by simplifying assumptions that make the model more tractable. The simple extension that allows for random screening costs substantially improves the empirical performance of the model. The tension arising from confronting a highly stylized model to the data is natural: models cannot usually make strong points without a great deal of simplification.

Appendices

Appendix A Proofs

Proof of Proposition ??. The first-order condition (interior optimum) for hiring policy in (**??**) are

$$0 = -\xi k + \sum_{s=0}^{k} \left(\frac{k!\phi^{s-1}(1-\phi)^{k-s}}{(s-1)!(k-s)!} - \frac{k!\phi^{s}(1-\phi)^{k-s-1}}{s!(k-s-1)!} \right) \widetilde{J}_s$$

Using the theorem of summation by parts[34] we obtain the following result

$$\xi k = k! \sum_{s=0}^{k-1} \frac{\phi^s (1-\phi)^{k-1-s}}{s!(k-1-s)!} (\widetilde{J}_{s+1} - \widetilde{J}_s)$$
$$\xi = \mathbb{E}[\widetilde{J}_{S+1} - \widetilde{J}_S | S < k] \equiv z(\phi, k) \tag{32}$$

The following lemma is useful since it shows $\widetilde{J}(k)$ is strictly concave and increasing. The result is intuitive: as the number of applicants increases it becomes progressively harder for a marginal applicant to be better than all the others.

Lemma 5 *For a given distribution of unemployed workers $G_U(\theta)$, the expected lifetime profit conditional on receiving k applicants, $\widetilde{J}(k)$, is strictly increasing and strictly concave in k, that is, $\widetilde{J}(k) - \widetilde{J}(k-1) < \widetilde{J}(k-1) - \widetilde{J}(k-2)$ for all $k > 0$*

Proof of Lemma 5. Consider the lifetime firm's profit $J = J(\theta)$ generated by a worker of productivity θ, and denote $F_J(J)$ and $f_J(J)$ the cumulative distribution function and density of J, respectively. Remember from Section 2.2 that $\widetilde{J}(k+1) = \mathbb{E}[J^k | \delta > 0, a = 1, K = k]$ where J^i denotes the i-th highest profitability in a group of k applicants. Given the discreteness of the number of applicants, consider the following difference

$$\widetilde{J}(k+1) - \widetilde{J}(k) = \mathbb{E}[J^{k+1} | \delta > 0, a = 1, K = k+1] - \mathbb{E}[J^k | \delta > 0, a = 1, K = k]$$

Writing the expectations in terms of integrals I find that

$$\begin{aligned}
&\widetilde{J}(k+1) - \widetilde{J}(k) \\
&= \frac{1}{k+1} \left(\int J(k+1) f_J(v) F_J(v)^k dv - \int Jk(k+1) f_J(v) F_J(v)^{k-1} (1 - F_J(v)) dv \right) \\
&= \frac{1}{k+1} \left(\mathbb{E}[J^{k+1} | \delta > 0, a = 1, K = k+1] - \mathbb{E}[J^k | \delta > 0, a = 1, K = k+1] \right) > 0
\end{aligned}$$

Since the first term is the expected value of the best worker among $k+1$ applicants and the second term is the expected value of the second-best among $k+1$ applicants[35]

[34]It can be proven that

$$\sum_{n=0}^{N} a_n B_n = A_N B_N - A_0 B_0 - \sum_{n=0}^{N} A_{n-1} b_n$$

with $A_n = \sum_{n=0}^{N} a_n$ and $b_n = B_n - B_{n-1}$. Notice that both the first term in the parenthesis evaluated in $s = 0$ and the second term evaluated in the parenthesis in $s = k$ equal 0.

[35]In general, the density function of the m-th highest value within k elements is

$$f^m(x^m) = \frac{k!}{(m-1)!(k-m)!} F(x^m)^{m-1} (1 - F(x^m))^{k-m} f(x^m)$$

, the difference is always strictly positive, which proves that $\widetilde{J}(k)$ is strictly increasing in k.

To prove concavity, the function is differentiated twice

$$\left(\widetilde{J}(k+1)-\widetilde{J}(k)\right)-\left(\widetilde{J}(k)-\widetilde{J}(k-1)\right)$$
$$=\int J f_J(v)\left[F_J(v)^{k-1}(F_J(v)-k(1-F_J(v)))-F_J(v)^{k-2}(F_J(v)-(k-1)(1-F_J(v)))\right]dv$$
$$=\int J(v)f_J(v)F_J(v)^{k-1}\left[(k+2)F_J(v)^2-2(k+1)F_J(v)))+k\right]dv$$

Doing a bit of algebra, the last expression equals

$$-\frac{1}{k+1}\left[\int(k+2)(k+1)J(v)F_J(v)^{k-1}(1-F_J(v))f_J(v)dv\right.$$
$$\left.+\int k(k+1)J(v)f_J(v)F_J(v)^{k-1}(1-F_J(v))dv\right]$$
$$=-\frac{1}{k+1}\left[\mathbb{E}[J(\theta^k)|\delta>0,a=1,K=k+1]+\mathbb{E}[J(\theta^{k-1})|\delta>0,a=1,K=k]\right]<0$$

The last two terms are the expected value of the second-best worker when $k+1$ and k applicants arrive. The expression is always negative, which proves concavity. ■

Lemma 5 establishes that $\widetilde{J}_{s+1}-\widetilde{J}_s$ is strictly decreasing so that the right-hand side of equation (32) is a strictly decreasing function in ϕ while ξ is fixed. As long as $\widetilde{J}_1-\widetilde{J}_0=\widetilde{J}_1>\xi$ there is a unique ϕ for a given k that solves the previous problem. Since $\widetilde{J}_{s+1}-\widetilde{J}_s\to 0$ as $s\to\infty$, there must be a unique point of intersection by the intermediate value theorem. ■

Lemma 6 *The average job finding rate of the economy is*

$$\mathbb{E}[p(\pi)]=\psi\frac{1-e^{-\overline{\phi}q}}{\overline{\phi}q}+(1-\psi)\frac{1-e^{-\underline{\phi}q}}{\underline{\phi}q}$$

Proof of Lemma 6. Simply integrate equation (5) over the population of unemployed

with $F(\cdot)$, $f(\cdot)$ being the CDF and PDF of x.

workers expressing the exponential function as series. Hence, this integral equals

$$\begin{aligned}
\mathbb{E}[p(\pi)] &= \psi\frac{1-e^{-\overline{\phi}q}}{\overline{\phi}q} + (1-\psi)e^{-\underline{\phi}q}\sum_{k=0}^{\infty}\frac{q^k}{k!}\int_{\underline{\theta}}^{\infty} L^U(v)^{k-1}l^U(v)dv \\
&= \psi\frac{1-e^{-\overline{\phi}q}}{\overline{\phi}q} + (1-\psi)e^{-\underline{\phi}q}\sum_{k=0}^{\infty}\frac{q^k}{(k-1)!} \\
&= \psi\frac{1-e^{-\overline{\phi}q}}{\overline{\phi}q} + (1-\psi)\frac{1-e^{-\underline{\phi}q}}{\underline{\phi}q}
\end{aligned}$$

where the second step comes from the fact that $kL^U(v)^{k-1}l^U(v)$ is the density of the maximum profit worker among k applicants arrived. ■

Proof of Proposition 1. The first step is to show the existence of a real solution for the queue size q. First, applying the Mean Value Theorem for integrals to the equation (15), notice that the value of open a vacancy $V(q)$ can be written as

$$V(q) = J(F^{-1}(M(x^*;q))) - \frac{\kappa/\beta + \phi\xi q}{1-e^{-\phi q}}$$

where $x^* \in [0,1]$ is a quantile of the distribution of unemployed workers. It is simple to see that the last term $C(q) \equiv \frac{\kappa/\beta+\phi\xi q}{1-e^{-\phi q}}$ (total expected recruiting cost) is a function in q that satisfies

$$\lim_{q\to 0} C(q) = +\infty \quad \lim_{q\to +\infty} C(q) = +\infty$$

The function $C(q)$ has a global minimum. The first-order condition yields

$$\xi(1-e^{-\phi q}) - e^{-\phi q}(\kappa/\beta + \phi\xi q) = 0$$

The second order condition shows that $C''(q) = \frac{\phi e^{-\phi q}(\kappa/\beta+\phi\xi q)}{(1-e^{-\phi q})^2} > 0$. Hence, the minimizer q^* of the function $C(q)$ satisfies

$$\phi q^* = e^{\phi q^*} - 1 - \frac{\kappa}{\beta\xi}$$

Replacing the last result into $C(q)$ we obtain that

$$C(q^*) = \frac{\xi(e^{\phi q}-1)}{1-e^{-\phi q}}$$

Given the continuity of these functions and the fact that it is clear that the function $J(F^{-1}(M(x^*;q)))$ intersects $C(q)$ if the the minimum value of $C(q^*)$ is lower than the minimum value of $J(F^{-1}(M(x^*;q)))$. Under a CRE conjecture, $J(\cdot)$ is a strictly increasing function. Hence, it's minimum must be achieved at $\underline{\theta}$. By definition, this lower type make the employer indifferent between hiring the worker with $\underline{\theta}$ or to post vacancies for the next period. Hence,

$$C(q^*) = \frac{\kappa/betae^{\phi q} + \xi e^{2\phi q} - e^{\phi q}\frac{\kappa}{\beta\xi})}{1 - e^{-\phi q}} < J(\underline{\theta}) = \varphi$$

By means of the Volterra nonlinear integral equation in (**??**) I show the existence, uniqueness and differentiability[36] of the distribution of unemployed workers $\widetilde{F}(\theta)$ for a given hazard conjectured hazard rate function $\pi(\theta)$ and λ.

In equation (**??**) the hazard rate $\pi(\theta)$ depends on $\widetilde{F}(\theta)$. I define a function $\widetilde{\pi} : [0,1] \to [0,1]$ such that

$$\widetilde{\pi}(z) = \pi(\widetilde{F}(\theta)) = \sum_{k=1}^{i^*} \frac{\lambda^{k-1} e^{-\lambda}}{(k-1)!} z^{k-1} + \sum_{k=i^*+1}^{\infty} \frac{\lambda^{k-1} e^{-\lambda}}{(k-1)!} z^{i^*-1} \frac{i^*}{k}$$

Although I prove the existence of $\widetilde{F}(\theta)$ for the particular case of the Coincidence Ranking equilibrium, this Proposition holds for any conjectured hazard rate function satisfying the Lipschitz condition $|\widetilde{\pi}(\widetilde{F}(\theta_0)) - \widetilde{\pi}(\widetilde{F}(\theta_1))| \leq \Xi|\widetilde{F}(\theta_0) - \widetilde{F}(\theta_1)|$ with $\Xi < \infty$ for all θ_0, θ_1. Using the definition of $\widetilde{\pi(z)}$, the Volterra integral equation in (**??**) is expressed rewritten in terms of an operator $O(\cdot)$.

$$O(\widetilde{F})(\theta) = \int_{-\infty}^{\theta} \frac{\eta f_\theta(v)}{\mathcal{U}^*(\eta + \widetilde{\pi}(\widetilde{F}(v))} dv \tag{33}$$

Establishing that (33) is a contraction mapping, there exists a unique function $\widetilde{F}(\theta)$ that is a fixed point of $O(\cdot)$. A sufficient condition to show this is that the kernel function

$$K(v, \widetilde{F}(v)) = \frac{\eta f_\theta(v)}{\mathcal{U}^*(\eta + \widetilde{\pi}(\widetilde{F}(v))}$$

integrated in the right-hand side of (33) satisfies a global Lipschitz condition so that there exists a finite constant Λ for which holds

$$|K(v, z_0) - K(v, z_1)| \leq \Lambda |z_0 - z_1| \quad \forall v \in \mathbb{R} \quad \text{and } z_0, z_1 \in [0,1]$$

In what follows, I show that the Lipschitz condition is satisfied for the Coincidence

[36] This proof adapts a standard solution in Hackbusch (1995), chapter 2.

Ranking Equilibrium. Since $0 \leq z \leq 1$, the following result holds

$$|\pi(z_0) - \pi(z_1)| = \sum_{k=1}^{i^*} \frac{e^{-\lambda}\lambda^{k-1}}{(k-1)!}|z_0^{k-1} - z_1^{k-1}| + \sum_{k=i^*+1}^{\infty} \frac{e^{-\lambda}\lambda^{k-1}i^*}{(k!}|z_0^{i^*-1} - z_1^{i^*-1}|$$

$$\leq \sum_{k=1}^{\infty} \frac{e^{-\lambda}\lambda^{k-1}}{(k-1)!}|z_0^{k-1} - z_1^{k-1}| \leq \sum_{k=1}^{\infty} \frac{e^{-\lambda}\lambda^{k-1}}{(k-1)!}k|z_0 - z_1|$$

$$= |z_0 - z_1|\lambda \left[\sum_{k=2}^{\infty} \frac{e^{-\lambda}\lambda^{k-2}}{(k-2)!} + 1\right] \leq (\lambda + 1)|z_0 - z_1|$$

The second line follows from use of the standard factorization result $x^k - y^k = (x - y)(x^{k-1} + x^{k-2}y + ... + y^{k-1})$. Then, the absolute value of the difference of the two kernels is

$$|K(v, z_0) - K(v, z_1)| = \frac{\eta f_\theta(v)}{\mathcal{U}^*} \left| \frac{\pi(z_0) - \pi(z_1)}{(\eta + \widetilde{\pi}(z_0))(\eta + \widetilde{\pi}(z_1))} \right| \leq \frac{\lambda + 1}{\eta \mathcal{U}^*} \sup_\theta f_\theta(\theta)|z_0 - z_1|$$

Since $\sup_\theta f_\theta(\theta) < \infty$, the Lipschitz constant is $\Lambda = \sup_\theta f_\theta(\theta)\frac{\lambda+1}{\eta\mathcal{U}^*}$. Although the argument holds for the particular conjectured hazard rate based on CR equilibrium, in general it will hold for any conjecture that can satisfy the Lipschitz condition $|\pi(z_0) - \pi(z_1)| \leq \Xi|z_0 - z_1|$ with $\Xi < \infty$ as stated in the statement of the proposition. This condition is weaker than differentiability.

Let $\mathcal{C}$ be the space of continuous functions with the special norm $\|g\| = \max_v |\exp(-\phi\Lambda v)g(v)|$ and $\phi > 1$, which is equivalent to the sup-norm $\| \cdot \|_\infty$. As integration preserves continuity, the operator $O(\widetilde{F})$ maps from the space $\mathcal{C}$ into the same space because $f_\theta(\theta)$ and the kernel function $K(v, z)$ are continuous.

Moreover, it follows that *for all* θ

$$|(O(\widetilde{F}_0) - O(\widetilde{F}_1))(\theta)| = \left| \int_{-\infty}^{\theta} (K(v, \widetilde{F}_0(v)) - K(v, \widetilde{F}_1(v)))dv \right|$$

$$\leq \int_{-\infty}^{\theta} \Lambda|\widetilde{F}_0(v) - \widetilde{F}_1(v)|dv$$

$$= \int_{-\infty}^{\theta} \Lambda \exp(\phi\Lambda v) \left|\exp(-\phi\Lambda v) \left(\widetilde{F}_0(v) - \widetilde{F}_1(v)\right)\right| dv$$

$$\leq \left|\max_{\widehat{v}} \left\{\exp(-\phi\Lambda\widehat{v})(\widetilde{F}_0(\widehat{v}) - \widetilde{F}_1(\widehat{v}))\right\}\right| \int_{-\infty}^{\theta} \Lambda \exp(\phi\Lambda v)dv$$

$$= \|\widetilde{F}_0 - \widetilde{F}_1\|\phi^{-1} \exp(\phi\Lambda\theta)$$

Since the former inequality holds for all possible θ, it also holds for the productivity that maximizes the value of the operator on the left-hand side. Therefore, the conclusion is that

$$\|(O(\widetilde{F}_0) - O(\widetilde{F}_1))(\theta)\| \leq \phi^{-1}\|\widetilde{F}_0 - \widetilde{F}_1\|$$

Since $\phi > 1$, the integral equation is a Contraction Mapping. Due to the Banach Fixed Point Theorem, existence, uniqueness and continuity of $\widetilde{F}(\theta)$ are proven. Moreover, thanks to the Fundamental Theorem of Calculus, the density $\widetilde{f}(\theta)$ also exists and is unique. Because of its definition ,$\pi(\theta) \equiv \widetilde{\pi}(\widetilde{F}(\theta))$ the hazard rate function also exists and it is unique. Then, using the fact that a positive λ implies an unemployment rate bounded away from 0 and 1, the existence and uniqueness of the density of unemployed workers $\widetilde{f}(\theta)$ is established. ■

Proof of Proposition 2. The second claim is obvious for expected values and variances since all separations are exogenous. For the case $\delta > 1$, we have that

$$\mathbb{E}[\theta|\delta] - \mathbb{E}[\theta|\delta+1] = \int \theta \left(\frac{f(\theta,\delta)}{f_\delta(\delta)} - \frac{f(\theta,\delta+1)}{f_\delta(\delta+1)} \right) d\theta$$

Substituting (**??**) into the latter equation, the right-hand side becomes

$$\int \theta \frac{f(\theta,\delta)}{f_\delta(\delta)} \left(1 - (1-\pi(\theta)a(\theta)) \frac{f_\delta(\delta)}{f_\delta(\delta+1)} \right) d\theta \tag{34}$$

Integrating both sides of equation (**??**) and considering the case when $a(\theta) = 1$ yields

$$\int f(\theta,\delta+1)d\theta = \int (1-\pi(\theta)) f(\theta,\delta) d\theta$$

$$f_\delta(\delta+1) = f_\delta(\delta) - f_\delta(\delta) \int \pi(\theta) \frac{f(\theta,\delta)}{f_\delta(\delta)} d\theta$$

Hence,

$$\frac{f_\delta(\delta)}{f_\delta(\delta+1)} = \frac{1}{1-\mathbb{E}[\pi(\theta)|\delta]}$$

Substituting this expression in equation (34) yields

$$\begin{aligned}
\mathbb{E}[\theta|\delta] - \mathbb{E}[\theta|\delta+1] &= \int \theta \frac{f(\theta,\delta)}{f_\delta(\delta)} \left(1 - \frac{1-\pi(\theta)}{1-\mathbb{E}[\pi(\theta)|\delta]} \right) d\theta \\
&= \mathbb{E}[\theta|\delta] - \frac{\mathbb{E}[\theta(1-\pi(\theta))|\delta]}{1-\mathbb{E}[\pi(\theta)|\delta]} \\
&= \mathbb{E}[\theta|\delta] - \frac{\mathbb{C}ov[\theta,(1-\pi(\theta))|\delta] - \mathbb{E}[\theta|\delta]\mathbb{E}[1-\pi(\theta)|\delta]}{1-\mathbb{E}[\pi(\theta)|\delta]} \\
&= \frac{\mathbb{C}ov[\theta,\pi(\theta)|\delta]}{1-\mathbb{E}[\pi(\theta)|\delta]} > 0
\end{aligned} \tag{35}$$

Since $\pi(\theta)$ is strictly increasing in θ, the covariance is positive, which proves the first part of the proposition.

To establish the inequality for the variance, I proceed by using an expression that is proven analogously to (35). Hence,

$$\mathbb{E}[\theta^2|\delta] - \mathbb{E}[\theta^2|\delta+1] = \mathbb{E}[\theta^2|\delta] - \frac{\mathbb{E}[\theta^2(1-\pi(\theta))|\delta]}{1-\mathbb{E}[\pi(\theta)|\delta]}$$

Thus, the difference of variances conditional on different durations is

$$\mathbb{V}[\theta^2|\delta] - \mathbb{V}[\theta^2|\delta+1] = \mathbb{E}[\theta^2|\delta] - \frac{\mathbb{E}[\theta^2(1-\pi(\theta))|\delta]}{1-\mathbb{E}[\pi(\theta)|\delta]} - \left(\mathbb{E}[\theta|\delta] - \frac{\mathbb{E}[\theta(1-\pi(\theta))|\delta]}{1-\mathbb{E}[\pi(\theta)|\delta]}\right)^2$$

Doing some algebra and using the fact that $0 \leq \mathbb{E}[\pi(\theta)|\delta] \leq 1$, I obtain that

$$\begin{aligned}
&\mathbb{V}[\theta^2|\delta] - \mathbb{V}[\theta^2|\delta+1] \\
&\geq \left(\mathbb{E}[\theta^2|\delta] - (\mathbb{E}[\theta|\delta])^2\right) - \underbrace{\left(\frac{\mathbb{E}[\theta^2(1-\pi(\theta))|\delta] - \mathbb{E}[\theta|\delta]\mathbb{E}[\theta(1-\pi(\theta))|\delta]}{1-\mathbb{E}[\pi(\theta)|\delta]}\right)}_{\text{Term 1}} \\
&- \underbrace{\left(\frac{(\mathbb{E}[\theta(1-\pi(\theta))|\delta])^2 - \mathbb{E}[\theta|\delta]\mathbb{E}[\theta(1-\pi(\theta))|\delta]}{(1-\mathbb{E}[\pi(\theta)|\delta])^2}\right)}_{\text{Term 2}}
\end{aligned}$$

Term 1 of the previous expression is equivalent to

$$\begin{aligned}
&\frac{\mathbb{E}[(\theta^2 - \theta\mathbb{E}[\theta|\delta])(1-\pi(\theta))|\delta]}{1-\mathbb{E}[\pi(\theta)|\delta]} \\
&= \frac{\mathbb{E}[(\theta-\mathbb{E}[\theta|\delta])^2(1-\pi(\theta))|\delta] - \mathbb{E}[\mathbb{E}[\theta|\delta]^2(1-\pi(\theta))|\delta]}{1-\mathbb{E}[\pi(\theta)|\delta]} \\
&= \frac{\mathbb{E}[(\theta-\mathbb{E}[\theta|\delta])^2(1-\pi(\theta))|\delta]}{1-\mathbb{E}[\pi(\theta)|\delta]} - \mathbb{E}[\theta|\delta]^2
\end{aligned}$$

Term 2 is always non-positive because $0 \leq \pi(\theta) \leq 1$. It can be written as

$$\frac{\mathbb{E}[\theta(1-\pi(\theta))|\delta]\,[\mathbb{E}[\theta(1-\pi(\theta))|\delta] - \mathbb{E}[\theta|\delta]]}{(1-\mathbb{E}[\pi(\theta)|\delta])^2} \leq 0$$

Using the previous expressions, a lower bound of the difference of variances is

$$\begin{aligned}
&\mathbb{V}[\theta^2|\delta] - \mathbb{V}[\theta^2|\delta+1] \\
&\geq \mathbb{E}[\theta^2|\delta] - \frac{\mathbb{E}[(\theta-\mathbb{E}[\theta|\delta])^2(1-\pi(\theta))|\delta]}{1-\mathbb{E}[\pi(\theta)|\delta]} \\
&\geq \mathbb{E}[\theta^2|\delta] - \frac{\mathbb{E}[\theta^2(1-\pi(\theta))|\delta]}{1-\mathbb{E}[\pi(\theta)|\delta]} \\
&= \frac{\mathbb{E}[\theta^2|\delta]\mathbb{E}[1-\pi(\theta)|\delta] - \mathbb{E}[\theta^2(1-\pi(\theta))|\delta]}{1-\mathbb{E}[\pi(\theta)|\delta]} = -\frac{\mathbb{C}ov[\theta^2(1-\pi(\theta))|\delta]}{1-\mathbb{E}[\pi(\theta)|\delta]} \geq 0
\end{aligned}$$

where the last expression is positive because $\pi(\theta)$ is increasing in θ. If the hazard rate is strictly increasing in θ as it is in a CR equilibrium, the inequality becomes strict, which is the desired result. ■

Proof of Proposition 3. Applying the definition of conditional expectation and equation (**??**)

$$\mathbb{E}[\delta|\theta] = \sum_{k=0}^{\infty} k \frac{f(\theta, \delta = k)}{f_\theta(\theta)} = f(\theta|\delta = 1) \sum_{k=0}^{\infty} k(1 - \pi(\theta)a(\theta))^{k-1}$$

Considering the case with $a(\theta) = 1$ and differentiating the absolutely summable series $\pi(\theta)^{-1} = \sum_{k=0}^{\infty}(1 - \pi(\theta))^k$ gives that

$$\sum_{k=0}^{\infty} k(1 - \pi(\theta))^{k-1} = \frac{1}{\pi(\theta)^2}$$

Therefore, using equations (**??**) and (**??**) yields

$$\mathbb{E}[\delta|\theta] = \frac{f(\theta|\delta = 1)}{\pi(\theta)^2} = \frac{\eta}{(\eta + \pi(\theta))\pi(\theta)}$$

Deriving the former expression, we obtain

$$\begin{aligned}\frac{d\mathbb{E}[\delta|\theta]}{d\theta} &= -\frac{\eta\pi'(\theta)(1 + 2\pi(\theta))}{(\eta + \pi(\theta))^2\pi(\theta)^2} \\ &= -\frac{\pi'(\theta)(1 + 2\pi(\theta))}{\eta}\mathbb{E}[\delta|\theta]^2\end{aligned}$$

which proves the claim. ■

Proof of Proposition 4. First consider that $\mathbb{V}[\delta|\theta] = \mathbb{E}[\delta^2|\theta] - \mathbb{E}[\delta|\theta]^2$. Applying the definition of conditional expectation and equation (**??**)

$$\mathbb{E}[\delta^2|\theta] = \sum_{k=0}^{\infty} k^2 \frac{f(\theta, \delta = k)}{f_\theta(\theta)} \tag{36}$$

$$= f(\theta|\delta = 1) \sum_{k=0}^{\infty} k^2(1 - \pi(\theta)a(\theta))^{k-1} \tag{37}$$

Assuming that $a(\theta) = 1$ and differentiating twice the absolutely summable series $\pi(\theta)^{-1} = \sum_{k=0}^{\infty}(1 - \pi(\theta))^k$ gives that

$$\sum_{k=0}^{\infty} k^2(1 - \pi(\theta))^{k-1} = \frac{2 - \pi(\theta)}{\pi(\theta)^3}$$

Replacing this expression into (38) yields

$$\mathbb{E}[\delta^2|\theta] = \frac{\eta(2-\pi(\theta))}{(\eta+\pi(\theta))\pi(\theta)^2} \tag{38}$$

By substituting equation (20) into the variance identity stated at the beginning of the proof, the expression for (21) is obtained

$$\mathbb{V}[\delta|\theta] = \frac{\eta^2 + (2\eta-\eta^2)\pi(\theta) - \eta\pi(\theta)^2}{(\eta+\pi(\theta))^2\pi(\theta)^2}$$

To prove that $\partial\mathbb{V}[\delta|\theta]/\partial\theta < 0$ involves tedious algebra. For this statement to be true it suffices to show that $\partial\mathbb{V}[\delta|\theta]/\partial\pi(\theta) < 0$ since $\pi'(\theta) > 0$. Hence,

$$\begin{aligned}\frac{\partial\mathbb{V}[\delta|\theta]}{\partial\pi(\theta)} &= \frac{2\eta-\eta^2-2\eta\pi(\theta)}{(\eta+\pi(\theta))^2\pi(\theta)^2} \\ &- \frac{(2(\eta+\pi(\theta))\pi(\theta)^2 + 2\pi(\theta)(\eta+\pi(\theta))^2)(\eta^2+(2\eta-\eta^2)\pi(\theta)-\eta\pi(\theta)^2)}{(\eta+\pi(\theta))^4\pi(\theta)^4}\end{aligned}$$

Since the denominator of the former expression is always positive, I focus only on the numerator, which after some simplifications becomes

$$\begin{aligned}&-(2\eta-\eta^2)(\eta+\pi(\theta))^2\pi(\theta)^2 - 2\eta^2(\eta+\pi(\theta))\pi(\theta)^2 - 2(2\eta-\eta^2)(\eta+\pi(\theta))\pi(\theta)^3 \\ &+ 2\eta(\eta+\pi(\theta))\pi(\theta)^4 - 2\eta^2(\eta+\pi(\theta))^2\pi(\theta) \\ &= -\eta(2-\eta)(\eta+\pi(\theta))\pi(\theta)^3 - 2\eta(\eta+\pi(\theta))\pi(\theta)^3 - 2\eta^2(\eta+\pi(\theta))\pi(\theta)^2(1-\pi(\theta)) \\ &- 2\eta(\eta+\pi(\theta))\pi(\theta)^3(1-\pi(\theta)) - 2\eta^2(\eta+\pi(\theta))\pi(\theta) < 0\end{aligned}$$

Therefore, the numerator is always negative for $\pi(\theta)$ between 0 and 1, which proves the proposition. ■

Appendix B Deriving Free-Entry Condition (14)

$$\begin{aligned}&\kappa + \beta(\psi\overline{\phi} + (1-\psi)\underline{\phi}q) = \\ &\beta\left(\psi(1-e^{-\overline{\phi}q})\int_{\underline{\theta}}^{\infty} J(v)g_u(v)dv + (1-\psi)\sum_{k=1}^{\infty}\frac{e^{-q}q^{k-1}}{(k-1)!}\int_{\underline{\theta}}^{\infty} J(v)(k-1)G_U(v)^{k-2}g_U(v)dv\right)\end{aligned}$$

Appendix C Deriving Coincidence Ranking Equilibrium Condition (14)

Starting from equation (19), we maximize the right-hand side by taking the first-order condition

$$
\begin{aligned}
p''(\theta) &= p'(\theta) \\
g_U'(\theta)p(\theta) + g_U(\theta)p'(\theta) &= g_U(\theta)p(\theta) \\
d\log g_U(\theta) &= -d\log p(\theta) \\
d\log f(\theta) &= -d\log p(\theta)\frac{\eta}{\eta + p(\theta)} \text{ (Using } g_U \text{ definition in (11))} \\
-\frac{f(\theta)}{f(\theta)^2} &= \frac{\eta(\phi q\eta + 1 - e^{-\phi q})}{\eta + e^{-\phi q(1-G_U(\theta))}}
\end{aligned}
$$

which is the desired result.

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