

## 1 A Finer Measure of Manipulability

Arrow's possibility theorem and its many descendants tell us that to choose a voting rule is to trade off some desirable rationality properties against others because they are not mutually attainable. The Gibbard-Satterthwaite, Gardenfors and related theorems tell us that every election rule that one would consider to be reasonable is manipulable by a strategic voter. Therefore, we always sacrifice non-manipulability in lieu of other properties. But is it wise to take such an all-or-nothing position with respect to manipulability, especially since we always settle for nothing?

We introduce a measure of how much strategic voting could alter an election outcome with respect to the winning criterion. We call this measure the *price of deception*. The price of deception applies to systems that have central coordinators but where individuals may act strategically and misrepresent their true preferences. For example, define the *sincere Borda count* of a candidate in a strategic Borda election to be his Borda count with respect to the sincere voter preferences. Define a *sincere Borda winner* to be a candidate with largest sincere Borda count, i.e. a candidate who would win were all voters non-strategic. Suppose that the the sincere Borda count of a sincere Borda winner could not be more than twice the sincere Borda count of the winner of a strategic Borda election. Then the price of deception of Borda voting would be at most 2. As another example, suppose that approval voting had an infinitely large price of deception. Then it might be that a candidate sincerely approved by the fewest voters could win an approval vote election when voters are strategic.

We propose that the price of deception should be one of the criteria by which a voting rule is assessed. In general, a voting rule can be cast as a way to score each candidate and then select one with the best score. If a candidate's score has an intrinsic correspondence to his quality as the social choice, then the price of deception has a correspondence to the capability of a voting rule to select a quality candidate. Like the computational complexity of manipulation, the price of deception offers finer distinctions than simply between "manipulable" and "non-manipulable."

The price of deception is similar to a concept known in the computer science community as the "price of anarchy" [9]. However, the two concepts should not be viewed as equivalent. The price of anarchy measures the difference between the social values of an optimal solution selected by a central coordinator and an equilibrium solution when all agents act selfishly. We can view the price of deception as a price of anarchy only by assuming that the central coordinator forces individuals to be truthful, thus negating the existence of private information. Also, the term "anarchy" is an especially inapt descriptor of a democratic election.

In this paper we develop the concept of the price of deception. We show that the straightforward definition is not adequate because it permits arbitrarily bad but absurd outcomes. Similar research in [2] eliminates these spurious outcomes by examining only equilibria that can be obtained through an iterative process where each player selects her best

response with respect to the previous iteration given that everyone was truthful in the first iteration. However, we are able to show that, as the number of voters increases, the probability converges to 1 that this process terminates at the first iteration. Hence, although interesting, this refinement cannot provide an accurate model since strategic behavior is known to occur in large populations.

We introduce a different refinement, termed *minimal dishonesty*, which we argue is both plausible and supported in the main by experimental evidence [5, 6]. We then analyze the price of deception of several standard voting rules, including Borda count, approval voting, and plurality voting. Different rules turn out to have significantly different prices of deception. The results therefore support our proposal that the price of deception can help discriminate among different voting rules.

We also examine the price of deception when the social choice is the spatial location of a facility [7], and the cost to be minimized is the sum of  $L_1$  distances, the sum of squared Euclidean distances, and the maximum distance to the facility. Differences in costs, and in one case a difference in the tie-breaking rule, lead to greatly different prices of deception. Therefore, these results suggest that in some cases our proposed criterion ought to be the deciding factor between two spatial location mechanisms.

## 2 Defining the Price of Deception

The concept of the price of deception applies not only to voting, but to much of social choice in general. A centralized mechanism makes a decision that optimizes a measure of societal benefit based on private information submitted by individuals. However, the individuals have their own valuations of each possible decision. Therefore, they play a game in which they provide possibly untruthful information, and experience outcomes in accordance with their own true valuations of the centralized decision that is made based on the information they provide. The price of deception, or p.o.d. for short, is the worst-case ratio between the optimum possible overall benefit and the expected overall benefit resulting from a Nash equilibrium of the game. If the mechanism minimizes a societal cost – as it typically does when locating a facility – the price of deception is defined as the reciprocal of that ratio, so that its value is always at least 1.

We remark that the revelation principle is irrelevant to the p.o.d. This is because revelation elicits sincere information only by yielding the same outcome that strategic information would yield. The revelation principle can be a powerful tool for analyzing outcomes. But for our purposes, the elicitation of sincere preference information is not an end in itself.

## 3 Minimal Dishonesty

### 3.1 Voting

Let  $V$  be a set of voters,  $C$  be a set of candidates,  $\pi_v$  be the preference ordering of voter  $v \in V$  over  $C$ , and  $\Pi = \{\pi_v : v \in V\}$  be the profile of voter preferences. Let  $f$  be a single-valued

voting rule that selects  $f(\Pi) \in C$  as the winning candidate given profile  $\Pi$ . (The definition can be slightly altered to permit  $f(\Pi)$  to be a subset of  $C$ .) Then the strategic voting game is the following:

Each voter  $v$  submits a purported preference ordering  $\pi'_v$  over  $C$ . Let  $\Pi'$  be the profile of purported voter preferences  $\pi'_v : v \in V$  and denote the resulting social choice  $c' = f(\Pi')$ . The outcome for voter  $v$  is  $c'$ , evaluated by  $v$  with respect to the true ordering  $\pi_v$ .

**Theorem 3.1.** *The p.o.d. of the strategic voting game can be arbitrarily large for the following non-dictatorial voting rules: Kemeny, any Condorcet-consistent rule, Borda, approval, plurality, Dodgson, STV (and many others).*

*Proof.* Let  $C = \{c_1, c_2, \dots, c_m\}$ . Let  $\pi_v = (c_1, c_2, \dots, c_m) \forall v \in V$ . Then for  $m \geq 2$  and  $|V| \geq 3$  unanimity implies that the submitted preference profile  $\Pi'$  where  $\pi'_v = (c_m, c_{m-1}, \dots, c_1) \forall v$  yields social choice  $f(\Pi') = c_m$ . Moreover, if any one voter  $v$  alters  $\pi'_v$ ,  $c_m$  remains the Condorcet winner, the Kemeny winner, the Borda winner, etc. Therefore,  $\Pi'$  corresponds to a pure Nash equilibrium of the strategic voting game. On the other hand, by unanimity  $f(\Pi) = c_1$ , the candidate most preferred by every voter, and hence  $\Pi$  corresponds to a pure Nash equilibrium of the game. For each voting rule listed,  $\Pi$  yields a best possible score,  $\Pi'$  yields a worst possible score, and the ratio of these scores can be arbitrarily large.  $\square$

The Nash equilibrium strategy set  $\Pi'$  in the proof of the theorem is absurd because the voters are dishonest to their own disbenefit. It is not plausible that voters would lie in order to achieve a less preferable outcome. Instead, we hypothesize that voters will only lie in order to achieve a more preferable outcome.

**Definition 3.2.** A player  $v \in V$  in the strategic voting game is “minimally dishonest” if for every  $\pi''_v$  that

1. is the same as  $\pi'_v$  except for the swap of two candidates in the preference order;
2. is more consistent with  $\pi_v$  than is  $\pi'_v$  in the number of pairwise preferences;

the player strictly prefers (with respect to sincere preference  $\pi_v$ ) the game outcome from submitting  $\pi''_v$  to the game outcome from submitting  $\pi'_v$  if all other players do not alter their submitted preferences.

A Nash equilibrium is minimally dishonest if every player is minimally dishonest.

We propose to refine the set of Nash equilibria to the set of minimally dishonest Nash equilibria.

## 3.2 Facility Location

Again let  $V$  be a set of voters and let  $S \subset \mathbb{R}^k$  be a compact convex space of possible facility locations. Each voter  $v \in V$  has some preferred location  $\pi_v \in S$  and let  $\Pi = \{\pi_v : v \in V\}$  be

the profile of preferred locations. Let  $f$  be a single-valued voting rule that selects  $f(\Pi) \in S$  as the optimal location. The strategic voting game is the following:

Each voter  $v$  submits strategic preference  $\pi'_v \in S$ . Let  $\Pi'$  be the set of strategic preferences and let  $c' = f(\Pi')$  be the resulting social choice. The outcome for voter  $v$  is  $c'$ , evaluated by  $v$  with respect to the sincere preferred location  $\pi_v$ .

**Theorem 3.3.** *The p.o.d. of the strategic facility location game can be arbitrarily large when minimizing either the  $L_1$  norm or squared euclidean distance.*

The proof of Theorem 3.3 follows in the same fashion as Theorem 3.1. There are equilibria where all voters submit ridiculous falsehoods even though they all agree on the ideal facility location. To avoid this, we define “minimally dishonest” in this setting and we examine only the minimally dishonest Nash equilibria.

**Definition 3.4.** A player  $v \in V$  in the strategic facility location game is “minimally dishonest” if for every  $\pi''_v$  such that  $\pi''_v$  is closer to  $\pi_v$  than  $\pi'_v$ , the player strictly prefers (with respect to sincere preference  $\pi_v$ ) the game outcome from submitting  $\pi'_v$  to the game outcome from submitting  $\pi''_v$  if all other players do not alter their submitted preferences.

## 4 Results and Conclusions

### 4.1 Voting

Table 1 summarizes our results for several voting mechanisms. Proofs are postponed to a later section.

Voting Mechanism	Price of Deception
Approval	2
Borda Count	$[\frac{m+2}{3}, \frac{m}{2}]$
Majority Judgment	$[\frac{m-1}{2}, \frac{m}{2}]$
1st Order Copeland	$[\frac{m-1}{2}, m]$
Plurality	$m$

Table 1: Price of Deception for various voting mechanisms

Plurality voting has received heavy criticism for encouraging tactical voting [3], limiting the number of political parties (Duverger’s law) [4], wasted votes resulting in lower voter turnout, and being vulnerable to the spoiler effect (i.e. grossly violating independence of irrelevant alternatives). We find it confirmatory that, of the voting mechanisms we have analyzed, plurality has the worst price of deception - a candidate is able to win at an equilibrium even if the candidate would receive no votes when everyone is honest.

On the other hand, approval voting has been used by organizations such as the Mathematical Association of America, the Institute for Operations Research and the Management

Sciences, the American Statistical Association, the Institute of Electrical and Electronics Engineers and the United Nations [8]. In addition, it is believed that approval voting improves voter turnout, deters the spoiler effect and reduces negative campaigning [1]. Of the voting mechanisms analyzed, approval voting has the lowest price of deception.

## 4.2 Facility Location

For the facility location problem, we always assume that every individual's preferred facility location can be anywhere in some convex compact subset of  $\mathbb{R}^k$ . Furthermore, when submitting a strategic location, the individual must also choose a point in this subset.

**Theorem 4.1.** *The p.o.d. for facility location when minimizing the sum of  $L_1$  norm distances (from the individuals to the facility), and breaking ties by selecting the centroid, is 1.*

**Theorem 4.2.** *The p.o.d. for facility location when minimizing the sum of  $L_1$  norm distances and breaking ties uniformly at random is  $\infty$ .*

**Theorem 4.3.** *The p.o.d. for facility location when minimizing the sum of squared distances is  $\Theta(n)$ .*

**Theorem 4.4.** *The p.o.d. for facility location when minimizing the sum of the Euclidean ( $L_2$  norm) distances while breaking ties by selecting the centroid or uniformly at random, is 1 in  $R^1$  and  $\infty$  in  $R^k$  for  $k \geq 2$ .*

**Theorem 4.5.** *The p.o.d. for facility location when minimizing the maximum  $L_2$  norm distance is 2 in  $R^1$  and  $\infty$  in  $R^k$  for  $k \geq 2$ .*

The most striking contrast among the results occurs when the sum of  $L_1$  distances is to be minimized, and the tie-breaking rule is the centroid or a random choice from among the optima. Theorems 4.1 and 4.2 tell us that the p.o.d. difference is as extreme as possible, 1 compared to infinity. Theorems 4.3, 4.4 and 4.5 show a large difference between minimizing functions of Euclidean distances and the sum of squared Euclidean distances. There are few situations for which the minimax objective would be considered appropriate by social planners, because it gives no consideration to all but a few of the individuals affected by the decision. Since few individuals can have a large impact on the final facility location, it is satisfying that Theorem 4.5 indicates that we may place the facility arbitrarily far away from any individual.

## 5 Selected Proofs

*Proof of Theorem 4.1.* Since optimizing over the  $L_1$  norm is separable over each of the coordinates, it suffices to only consider when the set of preferences is in  $\mathbb{R}$ .

Let  $S = [a, b]$  be the set of optimal facility locations given the sincere preferences  $\Pi$  and let  $S' = [a', b']$  be the set of optimal facility locations with respect to submitted preferences

$\Pi'$ . First we claim that if  $\pi_v \leq a'$  then  $\pi'_v \leq a'$ . For contradiction, suppose this isn't the case and that  $\pi'_v > a'$ . Then  $v$  can be more honest by indicating that they prefer  $a'$  and get at least as good of a solution, a contradiction. Thus, if  $\pi_v \leq a'$ , then  $\pi'_v \leq a'$  and  $\{v : \pi_v \leq a'\} = \{v : \pi'_v \leq a'\}$ . It immediately follows that  $\pi_v \leq a$  if and only if  $\pi'_v \leq a'$ .

Next, we claim that the selected facility location  $\frac{b'+a'}{2} \in S$ . For contradiction we assume this is not the case and without loss of generality that  $\frac{b'+a'}{2} < a$ . Since  $S = [a, b]$ , there must be a voter  $v$  such that  $\pi_v = a$ . Furthermore,  $\pi'_v \leq a'$ . If  $b' < a$ , then  $v$  can be honest and get at least as good of a solution. The outcome will then be in the interval  $[b', a]$ . If  $b' \geq a$ , then  $v$  can indicate that they prefer  $2a - b'$  which will result in the outcome  $a$ ,  $v$ 's preferred facility location. In both cases, we have a contradiction and thus  $\frac{b'+a'}{2} \in S$  and the price of deception is one.  $\square$

*Proof of Theorem 4.2.* Suppose that every individual must select a location in  $[-1, 1]$ . In the event that there are an odd number of voters, the optimal location will always be unique and the price of deception will be one by Theorem 4.1. Thus we assume there are  $2k$  voters.  $k$  voters prefer the location  $\epsilon > 0$  while the other  $k$  voters prefer the location  $-\epsilon$ . The optimal location is then selected uniformly at random from  $[-\epsilon, \epsilon]$  yielding an expected total distance of  $2k\epsilon$ .

Suppose that each of the first  $k$  voters indicate that they prefer location  $-1$  while each of the remaining voters indicate that they prefer location  $1$ . These preferences form a minimally dishonest equilibrium. Furthermore, the expected total distance from the sincere preferences is  $k(1 - \epsilon^2)$  yielding a price of deception of  $\frac{1-\epsilon^2}{2\epsilon}$  which can be made arbitrarily large by selecting  $\epsilon$  sufficiently small.  $\square$

*Proof of Theorem 4.3.* Let  $P$  be the convex hull of the set of sincere preferred locations  $\Pi$  and let  $e$  be the location selected at equilibrium. We claim that  $e \in P$ . If not, then there is a separating hyperplane  $a^T x = b$  where  $a^T x < b$  for all  $x \in P$  and where  $a^T e > b$ . Since  $e$  is computed by minimizing the sum of squared distances, there must be some individual that submitted preference  $\pi'_v$  such that  $a^T \pi'_v > b$ . Let  $d$  be a direction normal to  $a^T x = b$  where  $a^T d < b$ . If  $v$  updates their preference to be  $\pi'_v + d\epsilon$  for small  $\epsilon > 0$ , then the facility location will move in the direction  $d$  toward  $a^T x = b$  causing the facility to be strictly closer to every point in  $P$  implying that we were not at an equilibrium. Thus,  $e \in P$ .

Let  $D$  be the diameter of  $P$ . It is straightforward to see that the sincere sum of squared distance is at least  $\frac{D^2}{4}$ . Furthermore, since  $e \in P$ , the sum of squared distances to  $e$  is at most  $nD^2$ . Thus, the price of deception is at most  $4n$ .

To see that the price of deception is in  $\Omega(n)$ , let  $S = [0, n]$ . There are  $n - 1$  voters who prefer location  $0$  and are honest. There is one voter that prefers location  $1$  but submits a preference for  $n$ . When individuals are honest, the location selected is  $\frac{1}{n}$  with sum of squared distance  $\frac{n-1}{n}$ . The set of submitted preferences form a minimally dishonest equilibrium and the selected location is  $1$  with sincere sum of squared distance  $n - 1$  yielding a price of deception of  $n$ .  $\square$

*Proof of Theorem 4.4.*  $4n$  players have sincere preferences given by  $(0, 0)$ ,  $(1, 0)$ ,  $(0, 1)$  and  $(1, 1)$  with an equal number preferring each point. The optimal location is  $(.5, .5)$  with total

distance of  $2n\sqrt{2}$ . Suppose that players instead submit the preferences  $(0, k)$  and  $(1, k)$  and the location is somewhere between  $(0, k)$  and  $(1, k)$  with probability one. This is a minimally dishonest equilibrium and has yields a sincere distance greater than  $4n(k - 1)$  yielding an arbitrarily high price of deception.  $\square$

**Theorem 5.1.** *The p.o.d. for Approval voting is at most 2.*

*Proof.* Assume that candidate  $c_1$  is a winner of the election when everyone is sincere and let  $A$  be the set of voters that approve of  $c_1$ . Suppose that  $c_2$  is a winner of the election at a minimally dishonest equilibrium and let  $B$  be the set of voters that sincerely approve of  $B$ . The price of deception is  $\frac{|A|}{|B|}$ . Since every candidate is minimally dishonest, every voter in  $A \setminus B$  will include candidate  $c_1$  in the submitted list of approved candidates. Furthermore, since every voter is minimally dishonest, the set of individuals that include  $c_2$  in their submitted list must be a subset of  $B$ . Since  $c_2$  is a winner at equilibrium,  $|B| \geq |A \setminus B|$  implying that  $\frac{|A|}{|B|} \leq 2$  completing the proof.  $\square$

**Theorem 5.2.** *The p.o.d. for Borda count voting is at least  $\frac{m+2}{3}$ .*

*Proof.* Let  $S(c_i)$  be the sincere Borda count for candidate  $c_i$ . Let  $S^e(c_i)$  be the Borda count for candidate  $c_i$  with respect to the submitted preferences. The voters vote in the following way:

$n$ voters have preferences	$c_1, c_3, c_4, c_5, \dots, c_m, c_2$ and are honest
$n - 1$ voters have preferences	$c_m, c_{m-1}, c_{m-2}, \dots, c_2, c_1$
but submit preferences	$c_2, c_m, c_{m-1}, \dots, c_3, c_1$
$m$ voters have preferences	$c_2, c_1, c_m, c_{m-1}, \dots, c_3$ and are honest

For all  $i$ ,  $S(c_i) \leq S(c_m) = nm + 2n + m^2 - 3m$  and  $S(c_2) = 3n - 2 + m^2$ . For all  $i \geq 3$ ,  $S^e(c_i) = nm + n + 1 - i + im - 2m \leq nm + n + 1 + m^2 - 3m$  and  $S^e(c_2) = S^e(c_1) + 1 = n + nm - m + m^2$  and thus  $c_2$  wins the election. Furthermore, no individual can alter their preferences to get a better solution and if anyone is more honest, then the individual would get a worse solution. Thus the p.o.d. is at least  $\frac{nm+2n+m^2-3m}{3n-2+m^2}$  which goes to  $\frac{m+2}{3}$  as  $n$  tends to infinity.  $\square$

**Theorem 5.3.** *The p.o.d. for Majority Judgment is at least  $\frac{m-1}{2}$ .*

*Proof.* For  $n$  odd, suppose that  $n - 1$  players give candidate  $c_1$   $m - 1$  points and that 1 player gives this candidate 1 point. Suppose that  $\frac{n-1}{2}$  players give candidate  $c_2$   $m$  points,  $\frac{n-1}{2}$  players give candidate  $c_2$  1 point, and the final player gives the candidate  $c_2$  2 points. Let all other candidate receive 1 point from all other players. Candidate  $c_1$  wins with a median score of  $m - 1$ . Note that candidate  $c_2$  has a median score of 2.

Now suppose that the player that gave candidate  $c_2$  2 points now gives this candidate  $m$  points. Candidate  $c_2$  is now the winner with a median score of  $m$ . These set of preferences form a minimally dishonest equilibrium and yield the desired price of deception.  $\square$

**Theorem 5.4.** *The p.o.d. for 1st Order Copeland voting is at least  $\frac{m-1}{2}$ .*

*Proof.* When defining the Copeland score, normally,  $c_i$  gets a point for each candidate  $c_j$  where the majority of individuals prefer  $c_i$  to  $c_j$ . We alter the definition such that each candidate gets one point plus the number of pairwise majority victories. By doing so, each candidate gets an integer score in  $[1, m]$ .

Let  $S(c_i)$  be the sincere Copeland score and let  $S^e(c_i)$  be the Copeland score for candidate  $c_i$  with respect to the submitted preferences. A set of sincere preferences to achieve this lower bound are as follows:

voter 1 has preferences:  $c_3, c_4, c_5, \dots, c_{m-2}, c_{m-1}, c_2, c_1, c_m$   
voter 2 has preferences:  $c_4, c_5, c_6, \dots, c_{m-1}, c_m, c_2, c_1, c_3$   
voter 3 has preferences:  $c_5, c_6, c_7, \dots, c_m, c_3, c_2, c_1, c_4$   
 $\vdots$   
voter  $m - 2$  has preferences:  $c_m, c_3, c_4, \dots, c_{m-3}, c_{m-2}, c_2, c_1, c_{m-1}$   
voters in  $A$  has preferences:  $c_1, c_3, c_4, \dots, c_{m-3}, c_{m-2}, c_{m-1}, c_m, c_2$   
voters in  $B$  has preferences:  $c_1, c_m, c_{m-1}, \dots, c_6, c_5, c_4, c_3, c_2$

where  $|A| = |B| = \frac{m-3}{2}$  where  $m$  is odd. With these preferences,  $S(c_1) = m - 1$ ,  $S(c_2) = 2$  and  $S(c_i) = \frac{m+1}{2}$  for all other  $i$ . Candidate  $c_2$  only wins a majority election against  $c_1$  and thus would receive 1 points and thus  $c_1$  would be the winner if everyone was sincere. A minimally dishonest equilibrium is as follows:

voter 1 submit:  $c_2, c_3, c_4, c_5, \dots, c_{m-2}, c_{m-1}, c_1, c_m$   
voter 2 submit:  $c_2, c_4, c_5, c_6, \dots, c_{m-1}, c_m, c_1, c_3$   
voter 3 submit:  $c_2, c_5, c_6, c_7, \dots, c_m, c_3, c_1, c_4$   
 $\vdots$   
voter  $m - 2$  submit:  $c_2, c_m, c_3, c_4, \dots, c_{m-3}, c_{m-2}, c_1, c_{m-1}$   
voters in  $A$  submit:  $c_1, c_3, c_4, \dots, c_{m-3}, c_{m-2}, c_{m-1}, c_m, c_2$   
voters in  $B$  submit:  $c_1, c_m, c_{m-1}, \dots, c_6, c_5, c_4, c_3, c_2$

In this equilibrium,  $S^e(c_2) = S^e(c_1) + 1 = m$  and  $S^e(c_i) = \frac{m-1}{2}$  for all other  $i$ . Letting  $C$  be the directed cycle defined by  $c_3 \rightarrow c_4 \rightarrow \dots \rightarrow c_m \rightarrow c_3$ , candidate  $c_i$ , where  $3 \leq i \leq m$ , wins majority over the  $\frac{m-3}{2}$  candidates appearing before  $c_i$  and loses to the  $\frac{m-3}{2}$  appearing after to  $c_i$ . Furthermore, if  $c_j$  appears  $k \leq \frac{m-3}{2}$  positions prior to  $c_i$ , then  $c_i$  would lose a majority election to  $c_j$  by  $m - 2k$  votes. For  $i$  where  $3 \leq i \leq m$ , candidate  $c_i$  loses majority to  $c_2$  by 1 vote and to  $c_1$  by 1 vote. We now show that the solution is an equilibrium.

For voters in  $A$  and  $B$ , they cannot decrease  $c_2$ 's score, cannot increase  $c_1$ 's score and for all other  $i$ , can only increase  $c_i$ 's score by 1 point by placing  $c_i$  ahead of  $c_1$ . Therefore, for any permutation, candidate  $c_2$  still wins the election. Furthermore, voters in  $A$  and  $B$  are minimally dishonest.

By symmetry, it now suffices to show that voter 1 can do no better. The voter will only change their preferences if they can make  $c_i$  win where  $3 \leq i \leq m - 1$  or if they can be more honest and get at least as good of a result. However, for  $i$  where  $3 \leq i \leq m - 1$ , the voter can only increase the score of  $c_i$  by 1 since  $c_i$  already appears ahead of  $c_1$ . Since  $c_2$  has a score of  $m - 1$  and  $c_i$  has a score of  $\frac{m-3}{2}$ , the voter cannot make  $c_i$  win. Furthermore, the voter will be more honest only if they switch  $c_i$  with  $c_2$  for some  $i$  where  $3 \leq i \leq m - 1$ . However, this will cause the score of  $c_2$  to decrease by 1 and thus  $c_1$  and  $c_2$  will tie resulting in a strictly worse solution for the voter. Thus, the submitted preferences form a minimally dishonest equilibrium and the price of deception is at least  $\frac{m-1}{2}$ .  $\square$

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