Market Liquidity and Financial Fragility

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Abstract

This paper proposes a model of the endogenous formation of financial networks, where government and central bank policies that enhance the liquidity of assets - government intervention for short - play a key role. Under government intervention, large and less liquid investments become more profitable, but to finance them banks need to resort to the interbank market. This makes the structure of the financial network - and its associated exposure to shocks, i.e., fragility - to be dependent on government intervention. The main result of the paper is to show that, despite improving the networth of banks, government intervention might lead to more financial fragility, as changes in network structure can create additional channels for contagion.

Keywords: Financial networks; intervention; liquidity; fragility.

JEL Classification: G1; G2; G3.

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1 Introduction

The global financial crisis of 2007-2009, the market turmoil in Europe in 2010, 2012 and 2015, and developments in the Chinese stock market in 2015, all questioned once again the role of governments and central banks in times of financial distress. If, on the one hand, there are those who advocate in favor of interventions of the like of bailouts and quantitative easing (QE) programs, on the other there are those who claim that corporations and markets should be let at their own fate since government or central bank assistance can only aggravate the moral hazard problem. It is into this discussion that this paper aims to shed some light.

Market intervention is an option considered by governments and central banks as a way of managing and mitigating the risk of financial contagion, which peaks whenever the failure of a financial institution, or a collapse in asset prices, can trigger a chain reaction compromising the stability of the whole financial system. A financial system more exposed to the risk of financial contagion can be said to be more fragile, and one that would be then more prone to intervention.

If the benefits of intervention in terms of a reduction in the risk of contagion are clear, the associated costs might not be so. There is an array of unintended consequences of intervention, and the one to be explored and modeled in this paper refers to changes in the interbank market structure, or connectivity, among financial institutions. The rationale behind it is that increases in connectivity have a direct effect on the likelihood of financial contagion. Figure 1, taken from Shin (2009), helps to illustrate the idea. The figure depicts how financial intermediaries adjust their balance sheets in response to changes in the value of their assets.

Referring to the figure, a positive change in fundamentals can be taken to be a new policy implemented by the government or the central bank aiming at increasing market liquidity, leading to an increase in the market value of assets and, ceteris paribus, in the equity of financial intermediaries. An increase in equity, in turn, would allow financial intermediaries to engage in more leverage, implemented by means of new borrowing and lending. A fraction of the new borrowing and lending might occur through the interbank market, which would change the structure of financial networks, hence indirectly affecting systemic risk.

Indeed, there is evidence not only that financial intermediaries adjust their balance sheets facing an increase in the value of their assets, but also that interbank lending is positively associated with measures of market liquidity. Regarding the first, Adrian and Shin (2010) present data showing that, contrary to households - which typically display a decrease in leverage following asset price movements that increase their equity - financial intermediaries do engage in more borrowing under
similar circumstances. Also, Adrian and Shin show that whereas commercial banks increase borrowing and lending in a way that keeps their leverage ratios constant, U.S. investment banks prior to the 2007-2009 financial crisis tended to increase even more their leverage ratios following an increase in the value of their assets.

Figure 2 shows the association between measures of market liquidity and interbank lending, from the beginning of the 2000’s until the start of the recession in the United States, by the end of the same decade. Considering as an example the U.S. housing market, the series of the normalized federal funds rate and of the agency and government sponsored enterprises (GSE)-backed mortgage pools capture policies that affect market liquidity. The impact of these policies on asset prices is reflected on the S&P/Case-Shiller 20-City Composite Home Price Index, also depicted on the graph.

Confronted against these series, the volume of interbank loans of all commercial banks is negatively associated with the federal funds rates, and positively associated with government participation in the housing market. In other words, interbank lending is positively associated with measures of market liquidity. On the other hand, if liquidity in the housing market is to have any effect on the decision of banks to engage in more interbank transactions, banks should be at the same time investing in more housing market-related assets, which is confirmed by the series representing the mortgage debt outstanding by depository institutions.

The crises’ episodes previously mentioned have shown that, as far as systemic risk
is concerned, the topology of financial markets matter, i.e., an understanding of how financial institutions are connected is of vital importance. It is this particular feature of systemic risk that makes the use of networks in the modeling of financial systems and in the study of financial crises very pertinent, and it comes as no surprise that such an approach has indeed been pursued by many authors.

If not the first using networks to study the issue of financial contagion, Allen and Gale (2000) were pioneers in showing that, in spite of leading to the same allocative efficiency goals, different financial network structures expose economies to corresponding different degrees of financial fragility, i.e., the possibility of contagion following negative shocks to financial intermediaries. For example, a complete financial network with all financial intermediaries connected would be as efficient as an incomplete one with financial intermediaries connected in a circular fashion, despite the later being much more prone to financial contagion and systemic risk than the former.

The reason for complete network structures to be more resilient to shocks in the Allen and Gale model is that losses are spread among a larger number of counterparts, with no financial intermediary bearing alone the costs associated with negative shocks. It is the fact that banks co-insure each other against bad events that prevents contagion, and co-insurance is implemented by banks cross-sharing the deposits they take from households. The same co-insurance motive is used by Freixas, Parigi, and Rochet (2000) to model financial networks and systemic crises, in a model where households face uncertainty regarding not when but where their consumption needs will arise.
Despite appealing from a theoretical point of view, and advancing the fundamental question regarding the connection between efficiency and financial fragility, the model provided by Allen and Gale leave aside the question of how particular network structures come to be. Their interest is not as much in what leads to the formation of specific networks as it is in how shocks propagate in different networks that have the distinguishing feature of being all able to achieve the same allocative efficiency goal. In other words, by analyzing fixed network structures, their model is not suitable to capture the indirect effects on systemic risk arising from changes in financial networks due to government and central bank policies, the question that, as previously mentioned, the present paper is concerned with.

A more suitable approach to study the relation between government and central bank policies, financial networks and systemic risk must rely, therefore, on an endogenous model of network formation. Even though not concerned with the same questions as here, endogenous models of the formation of financial networks were developed, to cite a few, by Leitner (2005), Castiglionesi and Navarro (2011), and Babus (2015), all of them based on a co-insurance motive resembling that used by Allen and Gale. A striking result of the model by Babus, in particular, is the existence of a connectivity threshold above which no contagion ensues, directly in accordance with the model by Allen and Gale.

Despite its importance, therefore, the question of how the structure of interbank markets reacts in response to new policies, and the corresponding effects on the likelihood of systemic crises, seems to be largely overlooked. The current paper then aims at contributing to the literature in terms of providing a model of how financial networks are endogenously formed as a result of government and central bank policies, indirectly allowing for the study of the impact of these very same policies on systemic risk. The focus of the model to be presented will be on a stylized policy that affects market liquidity, and the impact on systemic risk will be obtained by comparing the propagation of shocks on financial networks where the government or central bank do enhance market liquidity, to that on networks where the government or central bank do not.

The choice for the focus of the analysis to be on the effects of intervention policies that alter the liquidity of markets is due to the fact that, if anything, the main impact from the response to recent crises episodes put forward by governments and major central banks around the world - bailout of banks and sovereigns, and quantitative easing programs - is on liquidity. Therefore, the effects of intervention on the structure of financial networks can be indirectly captured by understanding how the decision of financial institutions to become connected is affected by the extra liquidity provided by governments and central banks.
To be precise, the question to be addressed is whether government and central bank intervention policies might lead to a more fragile network, where fragility is viewed as the potential number of bank failures after banks’ assets are hit by shocks. This is done by first constructing a network through a banks’ interaction process, where financial intermediaries decide whether or not to be connected, i.e., make an interbank loan to finance investments. Following that and with the network in place, shocks are imposed on banks’ assets, and the number of banks in distress is calculated.

The structure of the network turns out to depend on the intervention policy and, therefore, so does fragility.

The model developed is thought of as an economy divided into several regions, each with its own idiosyncrasies in terms of investment opportunities and consumers’ preferences. There is a representative bank in each region, responsible for taking deposits and making investments. Investments can be made in two types of long-term assets, namely large and small projects. Large projects command a higher payoff than small ones, but at the same time demand an extra level of initial capital that can be secured only through a loan taken from another bank. This is the mechanism that creates connections between banks.

Embedded in the framework is banks’ traditional maturity mismatch problem, due to the financing of long-term assets with short-term liabilities. Banks are assumed to be short of capital to service depositors, which forces them to sell a fraction of their assets - projects or loans - before they are ripe. This happens at a fire-sale cost: there is a penalty applied to any fraction of an asset sold before maturity. The fire-sale cost is taken to be proportional to the size of the asset, making large projects to be sold at a higher discount - or, in other words, have a lower recovery rate - than small projects and loans.

Government or central bank policy - intervention for short - is modeled in reduced form as a mechanism that enhances market liquidity, in practice alleviating the fire-sale cost that banks face when they sell assets before maturity. A too-big-to-fail type of intervention is characterized as one where the intervention and its associated impact on market liquidity is more pronounced for large than for small projects.

The differences across regions in terms of projects’ payoffs and preferences of depositors, combined with the fire-sale cost and the prevailing intervention policy, will determine the profitability of investments. The profitability of investments will in general point to the direction where the money is flowing, i.e., which are the banks getting most of the loans being made. So as to isolate the possibility of financial contagion from reasons that are not specifically related to banks’ connectivity structure, e.g. asset commonality, financial intermediaries are not allowed to invest in assets (other than loans) not in their own regions. Intervention affects the profitability of
investments, and hence might lead to the creation of links that otherwise would be nonexistent, making the structure of the financial networks dependent on the policy chosen by the government or central bank.

After the formation of the network, the degree of financial fragility is assessed by the number of bank failures caused by having banks’ assets hit by shocks. Upon the shocks, the payoff of projects turn out to be only a fraction of what was originally assumed when projects were undertaken. These are perturbations of the network, in the sense of being probability zero events that banks are not prepared for and, therefore, they cause direct losses as soon as they take place. Financial contagion will ensue depending on the structure of the financial network that was formed.

Intervention makes the number of links in a network of banks to be at least as high as that of a network formed under no intervention. However, the effect on financial fragility as defined in the paper is not straightforward, since a higher number of links leads to a higher exposure of banks to the possibility of contagion at the same time that it increases their profitability, and hence their cushion to absorb shocks - the net worth. Simulations are performed in order to shed light on the factors that give rise to this fragility - net worth trade-off, and what are the effects of intervention on that.

In the simulations, 3 types of economies that one could think of as representing different stages of financial development are defined. These economies differ in terms of the support of the probability distribution used to generate their parameters, the level of development varying inversely with the level of dispersion of the support. Regardless of the economy type, however, the results show that government intervention leads to a much higher number of bank failures, despite concomitantly increasing the networth of the banking system, and this effect is stronger the less developed the economy is. The simulations also show that networks with very different degrees of fragility might have similar degrees of leverage, a result that highlights the importance of considering the intrinsic structure of the interbank market for the purpose of systemic risk monitoring: traditional measures might not capture the true exposure to systemic risk.

The paper is structured as follows: the following subsection discusses an example illustrating the main idea of the model that will subsequently be presented; the related literature is discussed in subsection 1.2; section 2 details the model; section 3 discusses the link formation process; section 4 derives some results related to the implications of government and central bank intervention for the network structure; section 5 gives the balance sheet characterization of the financial system represented by the network, and introduce the shocks to assets’ payoff that will be used to study financial fragility; section 6 presents the main result of the paper, i.e., the trade-off
between networth and financial fragility that arises through government intervention policies that aim at increasing the market liquidity of assets; section 7 offers some concluding remarks.

1.1 Example

Consider two banks, \( A \) and \( B \), representing distinct regions of an economy, in a 3-period world, \( t = 0, 1, 2 \). These banks have, at \( t = 0 \), the opportunity to invest in local projects paying \( r_A \) and \( r_B \), respectively, at \( t = 2 \). The other opportunity available is a 1-period riskless bond paying \( 1 + b \) at \( t + 1 \), for any $1 invested at \( t \). Without loss of generality, assume that \( r_A > r_B > 2 \). Both projects demand an initial investment of $2, which can be partially supplied by households, who deposit an amount of $1 at \( t = 0 \) - the other $1 required to start a project can only be obtained through a loan from the other bank, to be repaid at \( t = 1 \).

Households withdraw their money from banks at \( t = 1 \) and, after an investment is made, banks can obtain the $1 demanded by depositors only by selling a fraction of their projects. This premature sell, at \( t = 1 \), will be at a fire-sale price due to the market for shares in projects not being perfectly liquid. In this way, a project which is worth \( r_i \) at \( t = 2 \) can only be transacted at a price of \( \rho r_i \) at \( t = 1 \), for \( i = A, B \). Upon investing in projects, therefore, banks need to sacrifice, at \( t = 1 \), a fraction \( \alpha_i \) of their investments in order to obtain the amount to service depositors and pay back the loan:\(^1\)

\[
\alpha_i \rho r_i = \frac{1}{1 + 1 + b}, \quad \Leftrightarrow \quad \alpha_i = \frac{2 + b}{\rho r_i}, \quad i = A, B. \tag{1}
\]

Thus, the profit banks can realize out of projects, at \( t = 2 \), is

\[
\Pi_i = (1 - \alpha_i) r_i, \quad \Leftrightarrow \quad \Pi_i = r_i - \frac{2 + b}{\rho}, \quad i = A, B. \tag{2}
\]

On the other hand, if a bank chooses to invest the $1 from depositors in the risk-free bond, the profit at \( t = 2 \) is:

\(^1\)Since banks have the opportunity to invest in a risk-free asset paying \( 1 + b \), they demand an equivalent amount when lending.
\[ \Pi_i = [1 (1 + b) - 1](1 + b) \]
\[ \iff \Pi_i = b (1 + b), \quad i = A, B. \quad (3) \]

In this way, banks would consider undertaking a project only if it offers a higher profit than the risk-free bond, the condition for which is:

\[ r_i - \frac{2 + b}{\rho} > b (1 + b) \]
\[ \iff \rho > \rho_i := \frac{2 + b}{r_i - b (1 + b)}, \quad i = A, B. \quad (4) \]

Therefore, according to the parameters considered, there are the possible scenarios regarding the investment decision of banks:

- Both A and B prefer to borrow if \( \rho > \rho_B > \rho_A \);
- Only A prefers to borrow if \( \rho_B > \rho > \rho_A \); and
- None of the banks wants to borrow if \( \rho_B > \rho_A > \rho \).

Consider now an intervention by the government or central bank that increases the liquidity of the market for shares in projects, in practice meaning a subsidy of a fraction \( \gamma \) of the loss due to the fire-sale cost incurred by banks, \( 1 - \rho \). Upon intervention, thus, the recovery rate of projects increases from \( \rho \) to \( \rho + \gamma (1 - \rho) \), and a project that at \( t = 1 \) was worth \( \rho r_i \) is now priced at \( [\rho + \gamma (1 - \rho)] r_i \), for \( i = A, B \). The fraction of projects needed to be prematurely sold to service depositors and pay back the loan is now given by:

\[ \alpha_i^G \left[ \rho + \gamma (1 - \rho) \right] r_i = \frac{\text{Depositors}}{1} + \frac{\text{Loan}}{1 + b}, \]
\[ \iff \alpha_i^G = \frac{2 + b}{[\rho + \gamma (1 - \rho)] r_i}, \quad i = A, B. \quad (5) \]

Analogously to the previous case, the profit banks can realize out of projects, at \( t = 2 \), is:
\[
\Pi_i^G = \left(1 - \alpha_i^G\right) r_i \\
\iff \Pi_i^G = r_i - \frac{2 + b}{\rho + \gamma(1 - \rho)}, \quad i = A, B. \tag{6}
\]

The condition for banks to be willing to undertake projects rather than the risk-free bond is now:
\[\rho > \rho_i^G := \frac{1}{1 - \gamma} \left[ \frac{2 + b}{r_i - b(1 - b)} - \gamma \right], \quad i = A, B. \tag{7}\]

As before, the parameters will determine which of the following scenarios hold:

- Both \(A\) and \(B\) prefer to borrow if \(\rho > \rho_B^G > \rho_A^G\);
- Only \(A\) prefers to borrow if \(\rho_B^G > \rho > \rho_A^G\); and
- None of the banks wants to borrow if \(\rho_B^G > \rho_A^G > \rho\).

Given an enhanced market liquidity due to a government or central bank intervention, i.e., \(\gamma > 0\), the condition for banks to be willing to borrow to invest in a project becomes easier to be satisfied. Intervention in fact might lead to bank lending in circumstances where otherwise there would not be any. For instance, if:
\[
\frac{2 + b}{r_A - b(1 + b)} > \rho > \frac{1}{1 - \gamma} \left[ \frac{2 + b}{r_B - b(1 - b)} - \gamma \right], \quad \tag{8}
\]
then it follows that:
\[\rho_B > \rho_A > \rho > \rho_B^G > \rho_A^G. \tag{9}\]

The set of inequalities in (9) represent the case that, under no intervention, banks \(A\) and \(B\) would not be willing to borrow, whereas both would like to do so in case the liquidity of the market is enhanced. For example, a set of parameters that would lead to the inequalities in (9) is given by \(r_A = 4, r_B = 3, b = 0, \gamma = .415\) and \(\rho = .465\).

Whenever both banks prefer to borrow, there has to be a mechanism deciding which bank will be the lender and which will be the borrower. Thus, if one assigns all the bargaining power to bank \(A\), Figure 3 depicts the two types of networks that would emerge under different intervention policies:
These two structures have different implications for financial fragility. For instance, with government intervention, if at $t = 1$ it is anticipated that bank $A$’s project will be hit by a shock $\delta_A$ so that its project will be paying only $r_A (1 - \delta_A)$, the fraction of the project that needs to be sold to service depositors and pay back the loan will be:

$$\alpha^G_A \left[ \rho + \gamma (1 - \rho) \right] r_A (1 - \delta_A) = \frac{1}{1 + 1 + b},$$

$$\Leftrightarrow \alpha^G_A = \frac{2 + b}{[\rho + \gamma (1 - \rho)] r_i (1 - \delta_A)}, \ i = A, B. \quad (10)$$

If the shock is sufficiently high, i.e.:

$$\delta_A > 1 - \frac{2 + b}{[\rho + \gamma (1 - \rho)] r_A}, \quad (11)$$

then $\alpha^G_A > 1$, and therefore bank $A$ cannot afford to pay back the loan and service depositors simultaneously, meaning that it is in default. Assuming that bank $B$ is a second claimant on bank’s $A$ assets - the first claimants are the depositors - if the scrap value of the liquidated bank is sufficiently high, i.e.:

$$[\rho + \gamma (1 - \rho)] r_A (1 - \delta_A) - 1 > 1$$

$$\Leftrightarrow \delta_A < 1 - \frac{2}{[\rho + \gamma (1 - \rho)] r_A} \quad (12)$$

then bank $B$ can avoid its own default, whereas otherwise it will fail too. Summarizing:

- No bank is bankrupt if:

$$\delta_A \in \left[ 0, 1 - \frac{2 + b}{[\rho + \gamma (1 - \rho)] r_A} \right); \quad (13)$$
• Bank $A$ is bankrupt but not bank $B$ if:

$$\delta_A \in \left[1 - \frac{2 + b}{\rho + \gamma (1 - \rho)} r_A, 1 - \frac{2}{\rho + \gamma (1 - \rho)} r_A \right]; \quad (14)$$

• Both banks are bankrupt if:

$$\delta_A \in \left[1 - \frac{2}{\rho + \gamma (1 - \rho)} r_A, 1 \right]. \quad (15)$$

Therefore, in case of a sufficiently high shock, the whole network is compromised, since the loss of bank $A$ will cause it to default on the loan taken from bank $B$, leading to contagion. This possibility is precluded in the network that emerges without government intervention, since in this case both banks are investing in the risk-free bond, which by construction makes the network immune to shocks. In this sense, thus, government intervention brings more financial fragility. With the parametrization used previously, any shock $\delta_A > 27.22\%$ leads to the demise of both banks, with no consequences ensuing otherwise. Also, measuring the total networth of the banking system as the sum of the networth of banks $A$ and $B$, i.e., $W = \Pi_A + \Pi_B$ for the case without intervention and $W^G = \Pi_A + \Pi_B$ otherwise, one obtains from (2), (3) and (6) that $W = 2$ and $W^G = 2.09$.

Thus, intervention in the example brings more financial fragility, even though it increases the wealth of the banking system - this is the crucial trade-off brought about by the decision of the government or central bank to whether or not enhance market liquidity, and it is the main idea to be explored by the paper.

1.2 Related Literature

The questions addressed by the paper are mainly related to the effects of government and central bank policies, in particular the ones affecting market liquidity, on systemic risk and financial fragility. This issue is studied through a model of endogenous network formation which, for the purpose of the paper, is to be considered the financial network representing the interbank market. As such, the paper is directly related to the literature on financial networks and systemic risk measurement that explore the role played by government and central bank policies.

In the financial networks literature, the starting point for the model that will subsequently be presented is the seminal work of Allen and Gale (2000). As mentioned in the introduction, the model put forward by Allen and Gale shows forcibly
how the structure of financial networks is crucial for the understanding of systemic risk and financial fragility. Allen and Gale do not consider, though, the role played by government and central bank policies in the emergence of financial networks and, hence, the consequences for systemic risk and financial fragility arising from that, which is the main goal of the present paper and in the same spirit of Allen, Carletti, and Goldstein (2014).

The global financial crisis of 2007-2009, and many other crises’ episodes from the past, have shown that an important factor in regard to whether negative shocks might translate into full-fledged financial crises is the way that financial market participants are connected. Indeed, the aftermath of the 2007-2009 global financial crisis has seen a push by regulatory agencies and central banks around the world towards the collection of detailed bilateral exposure data across financial institutions, and also the use of network stress-test methodologies, as typified by the 2015 financial sector assessment program conducted by the IMF in the United States.

Connections, or links between financial markets participants, usually arise due to them having similar exposures to assets’ portfolios, or else by the direct celebration of loan/credit contracts. The consequences of a sharp fall in the price of a particular asset, or the bankruptcy of a financial intermediary, will both depend on the topology of the connectivity between market participants, and this is why networks have been extensively used in the study of financial crises, and in particular financial contagion.

As typified by the Allen and Gale model, the insurance motive is the main rationale present in the financial networks literature driving the creation of links between banks. In Freixas, Parigi, and Rochet (2000), credit lines across banks in different regions, due to depositors being uncertain of where their liquidity needs will take place, is the driver of the creation of links between banks. Brusco and Castiglionesi (2007) show how liquidity coinsurance might potentially bring down a bank in case it is paired with a not well capitalized institution that engages in excessive risk taking. Zawadowski (2013) uses a network framework to model bilateral over-the-counter contracts, arguing that banks underinsure against counterparty risk by not incorporating the network externalities they impose on third parties once they fail.

Financial institutions becoming connected due to the insurance motive could well be regarded as the extreme case where they would need to rely only on each other in case of an emergence, e.g. a liquidity crisis. This would be more among the lines of a bail-in, hence not capturing the effects of government and central bank policies that enhance market liquidity, which is more related to bailouts that in general can be of two types, market or institution-specific\(^2\). Being more concerned with the

\(^2\)The distinction between these two types of bailouts is not as clear cut as it might appears at first. For instance, whenever an institution facing problems is deemed as too-big-to-fail, a government
consequences for systemic risk and financial fragility arising from government and central bank policies, the present paper avoids having to rely on the insurance motive by focusing on a setting with no uncertainty, to be interpreted as an environment where policies adopted during turmoil periods persist and the economy is performing well.

Accordingly, the present paper is related to those where the possibility of financial contagion is only due to credit exposures, similarly to the work by Kiyotaki and Moore (1997) where a chain of firms engaging in borrowing and lending might give rise to systemic risk in case some of them become temporarily illiquid, causing others to run into difficulties as well. On the other hand, among others, Lagunoff and Schreft (2001) argue that diversification makes agents to have their portfolios linked, and Cifuentes, Ferrucci, and Shin (2005) show how contagion might be processed not only through direct balance sheet exposure among banks but also via asset prices\(^3\).

In fact, the paper explores a novel channel through which banks might become connected: investment opportunities. It is assumed that the economy is divided in distinct regions, each of them with a representative bank having access to local investment opportunities. Local banks are the only ones with the expertise necessary to invest in local projects, but they might lack the required capital to finance these investments, forcing them to borrow from banks located in other regions. Ultimately this is what makes banks to become connected, therefore giving rise to a financial network. A similar approach is pursued by Allen, Babus, and Carletti (2012), though in a set up where banks directly swap shares in projects, whereas here they are only indirectly exposed to other regions’ projects through the possibility of default of debtors exposed to investments that fail to succeed.

The financial network formed following the interbank borrowing/lending activities required to finance investments allows the study of the effects of negative shocks to be made in an input-output analysis fashion, similarly to Aldasoro and Angeloni (2015). Based on that, the main result of the paper shows that, despite increasing bailout has not only the direct effect of safeguarding the specific institution being bailed out, but also the indirect effect of salvaging the financial markets that otherwise would be compromised by the failure of that institution. Hence, whenever thinking of different types of intervention, the best way to differentiate them is by considering not the beneficiaries of the aforementioned intervention but rather the mechanism through which it is implemented.

\(^3\)It is also worth mentioning information-based models of financial contagion, in particular the work by Alvarez and Barlevy (2015) analyzing the impact from information disclosure on contagion, and also Caballero and Simsek (2013) and Pritsker (2013), both using a setting where Knightian uncertainty plays a role in the propagation of shocks. The results from these papers do not relate to the present as in the model to be presented there are no information issues, firstly due to the lack of uncertainty and secondly due to the fact that agents can freely observe the financial network.
the net worth of the system comprising all the banks, the increased connectivity of the financial system that results from government and central bank policies might increase the likelihood of contagion, depending on the size of the shocks. This result is similar to that of Acemoglu, Ozdaglar, and Tahbaz-Salehi (2015), who also show that more connectivity is associated to less financial fragility to the extent that shocks are small. The contribution is to show how government and central bank policies, by enhancing market liquidity, affect the trade-off between improving the net worth of the financial system and exposing it to systemic risk, i.e., financial contagion.

The model relies on a stylized market-intervention mechanism, according to which fire-sale costs that financial intermediaries are to incur due to the nature of their business, i.e., arising from the maturity mismatch between their assets and liabilities, are alleviated by either the government or central bank. The presence in the model of liquidation costs is motivated by the seminal work of Shleifer and Vishny (1992), and a similar approach to motivate government and central bank intervention is used by Gorton and Huang (2004), though their analysis is not performed within a network context as here.

The networks used in the majority of the papers in the financial crises and contagion literature are not endogenously determined, hindering the analysis of the effects from government and central bank policies on financial fragility. The present paper aims to contribute to that, and other papers in the literature, among which some are discussed in the sequence, also provide and endogenous network formation mechanism, although with no scope for the role played by government and central bank policies.

In Leitner (2005), banks form links that allow transfers of endowments to be made, which in turn prevents the failure of less wealthy members that otherwise could cause the demise of the entire network. Castiglionesi and Navarro (2011) presents a model where banks decide whether or not to join a network that allows them to co-insure each other against liquidity shocks. Departing from the co-insurance motive, Cohen-Cole, Patacchini, and Zenou (2012) model the network formation process as a Cournot competition in the lending market, showing that banks prefer

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4The paper abstracts from the question of whether - if at all - a particular type of intervention is mostly preferable when financial institutions face fire-sale costs. Related to the pros and cons of institution-specific bailouts as opposed to market interventions, the reader is referred, regarding the former, to the paper by Keister (2015), and, regarding the later, to the papers by Farhi and Tirole (2012) and Diamond and Rajan (2012). In particular, Farhi and Tirole show that a policy whereby the government responds to crises decreasing the interest rate leads to more maturity mismatch in the economy and, hence, exposure to liquidity shocks, whereas Diamond and Rajan argue that such a policy turns out to be better than the alternatives, in particular the bailout of a specific institution.
to be linked with others that are more rather than less connected. Finally, Babus (2015) uses a game-theoretical approach to endogenously derive a network that can provide insurance to the possibility of contagion.

Although not making any contribution in terms of providing a new methodology, the paper is also related to the literature on systemic risk measurement. The shocks that might lead to contagion are similar to the ones considered by Allen and Gale (2000), i.e., perturbations of the network, or zero probability events. This precludes the use of more recently proposed measures of systemic risk, some of which are the ones developed by Acharya, Shin, and Yorulmazer (2011), Huang, Zhou, and Haibin (2011) and Adrian and Brunnermeier (2014), to cite a few. The measure of systemic adopted is simulation-based, calculated according to the number of bank failures after investments are hit by negative shocks, similarly to the exercise performed in Nier, Yang, Yorulmazer, and Alentorn (2007).

2 Model

Consider a 1-good ($), 3-period economy, $t = 0, 1, 2$, divided in an even number $N$ of regions, $N = \{1, \ldots, N\}$. Every region $i \in N$ has a representative bank $B_i \in \mathbb{B}$, with $\mathbb{B} = \{B_1, \ldots, B_N\}$ representing the set of banks.

Every region $i$ has $N - 1$ continuums of depositors, $D^i = \{D^i_1, \ldots, D^i_{N-1}\}$, each of them of unit mass. Depositors are endowed with $1$ and have Diamond-Dybvig preferences, i.e., they face uncertainty regarding the time of consumption, which is formalized by the following utility function:

$$U_i(c_1, c_2) = \begin{cases} 
  c_1, & \text{with probability } \omega_i, \\
  c_2, & \text{with probability } 1 - \omega_i.
\end{cases} \tag{16}$$

Any depositor from a continuum $D^i_j$ in an arbitrary region $i$ will, with probability $\omega_i$, face consumption needs at $t = 1$, denoted by $c_1$ (early depositor), whereas with probability $1 - \omega_i$ the consumption needs will arise only at $t = 2$, denoted by $c_2$ (late depositor). Uncertainty is resolved at $t = 1$, and the probability of consuming at either $t = 1$ or $t = 2$ varies across regions, but not within regions, i.e., the depositors of all the continuums $j \in \{1, \ldots, N - 1\}$ in region $i$ have the same probability or facing early or late consumption needs. Figure 4 illustrates the continuums of depositors for region $i$. A representative bank has available an infinite supply of two types of long-term investment opportunities, namely a large and a small project. At $t = 2$, the large project pays $r^*_1$, whereas the small yields $r_1$, the former for an investment of $2$ and
the later of $1, at $t = 0$. By assumption, $r_i^* > r_i$, for any $i \in N$. The cash flows of projects available to bank $B_i \in \mathbb{B}$ are represented in Figure 5.

Projects are available only to the representative bank of the respective region, i.e., a bank $i \in N$ cannot invest in projects other than the ones in its own region - cross-region investment is ruled out\(^5\). Projects can be partially sold before maturity, i.e., at $t = 1$, at a fire-sale price, as it will be explained in the sequence. Banks are also allowed to borrow (long-term) from other banks, and they also have available, at $t = 0$, a short-term bond that pays zero interest rate. Depositors do not have access to either long-term projects or short-term bonds, and are thus forced to deposit their money at local banks, which happens by means of a deposit contract that allows withdrawals at will. This implies that banks can borrow from depositors at a zero interest rate.

\(^5\)One way of thinking about this is that projects require an expertise that only local banks have.
2.1 Banks’ Interaction Process and Arrival of Depositors

Banks’ interaction process, i.e., the event whereby they meet, occurs in a random way at \( t = 0 \). The interaction protocol ensures that every bank will meet each other once, and only once. Given an even number \( N \) of banks, therefore, at \( t = 0 \) there will be \( N - 1 \) rounds of interaction. As an example, with four banks, \( N = 4 \), the rounds of interaction will be as depicted in Figure 6.

\[
\begin{align*}
\text{Round 1:} & \quad B_1 \leftrightarrow B_2 ; \quad B_3 \leftrightarrow B_4 \\
\text{Round 2:} & \quad B_1 \leftrightarrow B_3 ; \quad B_2 \leftrightarrow B_4 \\
\text{Round 3:} & \quad B_1 \leftrightarrow B_4 ; \quad B_2 \leftrightarrow B_3
\end{align*}
\]

Figure 6: Interaction process of banks for \( N = 4 \).

Synchronized with the interaction process is the arrival of depositors at their respective local banks. Each continuum of depositors arrives sequentially, in a time fashion that matches the way that banks meet. In every round of banks’ interaction process, therefore, one of the \( N - 1 \) continuums of depositors in each region will arrive at the respective local bank, and since the interaction process of banks is composed of \( N - 1 \) rounds, by the end of \( t = 0 \) all deposits will have been made at their respective banks.

In order to capture the differences in liquidity across the banks, it is also assumed that, at every round of interaction with others, banks have available an endowment of \( e_i > 0 \), for any bank \( B_i \in \mathbb{B} \). This endowment is to be thought of as equity that banks have available to invest, though their availability takes place only when banks meet, and cannot be used for later investments. As it will become clear when the budget constraints that banks have to face in each date are explained, this endowment will be mainly used to service early depositors, diminishing the fraction of long-term investments needed to be sold before maturity.

2.2 Network Structure

At every round of interaction at date \( t = 0 \), a bank then (i) receives deposits and a specific amount of endowment, and (ii) meets another bank, which is when the opportunity to decide on how to allocate the funds received arise. An important assumption is that every bank \( B_i \in \mathbb{B} \) needs to decide during the round of interaction
how to allocate the $1 received from depositors and the endowment of $e_i > 0$. This implies banks behaving in a myopic way, since they cannot keep the money received for more profitable transactions that might show up in the future. The endowment will straightforwardly be invested in the short-term bond, since small and large projects cannot take amounts different than 1 and 2, respectively. At each round of interaction, therefore, banks will have to choose mainly how to allocate the money received from depositors, one among the following three options:

(i) Invest $1 in a small project;
(ii) Borrow $1 and invest the total $2 in a large project;
(iii) Lend $1 to the other bank.

Figure 7 illustrates, for a particular round of interaction, the possibilities arising from a meeting between banks $i$ and $j$. The blue dashed line represents an investment in a small project, the green in a large project in region $j$ - by means of a loan agreement between bank $i$ (lender) and bank $j$ (borrower) - whereas the red an investment in a large project in region $i$, with the roles of bank $i$ and $j$ switched.

The condition for a loan agreement to take place is the payment by the borrower (interest plus principal) to be at least as large as the opportunity cost of the lender. The opportunity cost of the lender arises from the fact that, by lending $1$, the possibility of investing in a small project is foregone. Therefore, in any loan agreement, the borrower must pay to the lender at least the payoff the later would get by investing in a small project.

Upon the meeting of banks $i$ and $j$, the double-headed arrows in Figure 6 turn into one of the following:

(i) $B_i \rightarrow B_j$: $B_i$ lends to $B_j$;
(ii) $B_i \leftarrow B_j$: $B_i$ borrows from $B_j$;
(iii) $B_i \cdots B_j$: No loan agreement between $B_i$ and $B_j$.

2.3 Maturity Mismatch

To finance an investment in either a small project or a loan, banks need deposits and, in case of large projects, additionally borrow money from other banks. Since assets’
payoff are only due in the long-term whereas a fraction of deposits is withdrawn in the short, banks are exposed to a maturity mismatch problem, i.e., the use of short-term funds to finance long-term assets.

The endowment that every bank $B_i \in B$ receives during the interaction process with other banks is not enough to cover the withdrawals by early depositors, i.e., $e_i < \omega_i$. In this way, banks in the model can be said to be wealth constrained, which implies that early depositors can be served only if banks prematurely sell a fraction of their investments at $t = 1$. This early sell of assets is assumed to occur at a fire-sale price, which in turn is assumed to depend on the size of the investment at hand that a fraction of which is to be prematurely sold, according to the following:

(i) Large projects have a discount factor of $\rho^*$: one unit of investment in a large project paying $r_i^*$ at $t = 2$ is priced at $\rho^*r_i^*$ at $t = 1$, with $0 < \rho^* < 1$;

(ii) Small projects have a discount factor of $\rho$: one unit of investment in a small project paying $r_i$ at $t = 2$ is priced at $\rho r_i$ at $t = 1$, with $0 < \rho < 1$. 

Figure 7: Portfolio decision of banks at a particular round of interaction.
Since loans are always the size of an investment in a small project, i.e., $1, the fire-sale cost associated with them is taken to be the same as that for small projects, $\rho$. Another important assumption is that the fire-sale cost of large projects is higher than that of small projects and loans:

$$0 < \rho^* < \rho < 1.$$  \hfill (17)

Government or central bank intervention, as discussed next, is a way of alleviating the costs imposed on banks due to the premature sell of assets, by means of enhancing the liquidity of the market.

### 2.4 Government Intervention

The fire-sale cost that banks face due to their maturity mismatch problem might prevent them from investing in large projects, due to the fact that, compared to small projects, they are more costly to be negotiated before maturity. Large projects, however, are precisely the ones offering a higher payoff, and the assumption that they do not embed any extra risk would make them preferable from the point of view of aggregate output.

In this context, therefore, it is natural that policies set by the government or the central bank should aim at promoting investments in large projects, and the only way considered by the model to achieve this goal is through interventions in the market that reduce the fire-sale cost of projects, in particular the large ones. Accordingly, liquidity-enhancing interventions cause the discount factor, or the price of projects being negotiated before maturity, to be altered in the following way:

(i) Large projects: with intervention, one unit of investment in a project paying $r^*_i$ at $t = 2$ is priced at $[\rho^* + \gamma^* (1 - \rho^*)] r^*_i$ at $t = 1$;

(ii) Small projects: with intervention, one unit of investment in a project paying $r_i$ at $t = 2$ is priced at $[\rho + \gamma (1 - \rho)] r_i$ at $t = 1$.

Thus, with $\gamma$ and $\gamma^*$ defined as the intervention parameters for small and large projects, respectively, under no intervention, i.e., $\gamma^* = \gamma = 0$, the original discount factors apply, i.e., $\rho$ for small projects and $\rho^*$ for large ones. On the other hand, with full intervention, i.e., $\gamma^* = \gamma = 1$, there is no fire-sale cost to be incurred when projects are negotiated.

In order to capture the effects of what could be thought as a too-big-to-fail policy, it is assumed that large projects command more support from the government than
small ones, i.e., $\gamma^* > \gamma$. Despite such a policy, however, large investments are still assumed to be more costly to be sold than small ones, i.e.:

$$\rho + \gamma(1 - \rho) > \rho^* + \gamma^*(1 - \rho^*).$$

(18)

2.5 Timeline of Events

With the ingredients of the model in hand, the timeline of events is the following:

- **$t = 0$**:
  1. Banks meet in a pairwise fashion, which will give rise to a network structure after the interaction process is finished. At each round of meetings banks decide:
     (i) Whether or not to form a link (make a loan or borrow);
     (ii) How much to invest in the short-term bond;
     (iii) How much of the long-term asset (project or loan) to sell in order to meet the needs of early depositors.

- **$t = 1$**:
  1. Banks execute the selling strategy of assets;
  2. The proceeds from the sell of assets, together with the payoff from short-term bonds, are used to pay early depositors.

- **$t = 2$**:
  1. Payoffs from long-term assets (projects and loans) are realized;
  2. Banks pay late depositors and clear positions with other banks, consuming the remains as profits.

The next section details the decision making process carried out by banks at the interaction stage, which leads them to choose whether or not to create links with one another at each of their meetings.
3 Link Formation

The network structure that emerges at the end of the interaction process is the result of banks’ investment decisions, made at each of the pairwise meetings they participate in. Every time a loan is made, a link in the network is formed, and the celebration of a loan contract - as well as any other type of investment - will be based solely on the profitability that it entails. Therefore, it is necessary to consider the benefits and costs underlying all the investment opportunities that banks face at each meeting.

In what follows, it is considered a meeting of two arbitrary banks, $i$ and $j$, and how they evaluate each option from the menu of investments that arise once they meet. The profitability of each investment opportunity is determined by the budget constraints (BCs) that must be satisfied, which are intrinsically associated to each type of investment available. The profit entailed by each type of investment is discussed next, and a comparison of them - which is the basis for the banks to decided whether or not to form links - is detailed subsequently.

3.1 Investment in a Small Project

If bank $i$, upon a meeting with bank $j$, is to invest in a small project, the budget constraints (BCs) needed to be satisfied at each date are:

$$1 + b_i \leq 1 + e_i \quad \text{(BC at } t = 0)$$

$$b_i + \alpha_i^t r_i [\rho + \gamma (1 - \rho)] \geq \omega_i \quad \text{(BC at } t = 1)$$

$$(1 - \alpha_i^t) r_i = (1 - \omega_i) + e_i + \pi_i \quad \text{(BC at } t = 2)$$

where:

- $\pi_i$: profit of bank $i$ with an investment in a small project;
- $b_i$: investment in the short-term bond;
- $\alpha_i^t$: fraction of the small project to be liquidated at $t = 1$.

The budget constraint at $t = 0$ expresses that total expenses, i.e., investment in the small project, $\$1$, plus investment in the short-term bond, $b_i$, cannot exhaust the amount of total resources available, namely deposits, $\$1$, and the endowment, $e_i$. At $t = 1$, the revenue from the short-term bond, $b_i$, plus the proceeds from the
liquidation of a fraction of the small project, \( \alpha_i r_i [\rho + \gamma (1 - \rho)] \), should suffice to service early depositors, \( \omega_i \). Finally, at \( t = 2 \), the fraction not liquidated of the small project, \( (1 - \alpha_i) r_i \), must allow the bank to meet the demand from late depositors, \( 1 - \omega_i \), plus the amount owed to equity holders, \( e_i \). What is left from the payoff of the small project after paying late depositors and equity holders constitutes the profit of the bank, \( \pi_i \).

Since bank \( i \) does not want to (i) waste resources and (ii) sell a higher fraction of the small project than what is necessary, the budget constraints at \( t = 0 \) and \( t = 1 \) will be binding, allowing one to solve for \( b_i \) and \( \alpha_i^* \):

\[
\begin{align*}
    b_i &= e_i, \\
    \alpha_i^* &= \frac{\omega_i - e_i}{r_i [\rho + \gamma (1 - \rho)]}.
\end{align*}
\]

Substituting into the expression for bank \( i \)'s profit, \( \pi_i \), becomes:

\[
\begin{align*}
    \pi_i &= r_i - \left\{ (1 - \omega_i) + e_i + \frac{\omega_i - e_i}{[\rho + \gamma (1 - \rho)]} \right\} \\
    \Leftrightarrow \pi_i &= r_i - r_i^*,
\end{align*}
\]

where:

\[
    r_i^* := (1 - \omega_i) + e_i + \frac{\omega_i - e_i}{[\rho + \gamma (1 - \rho)]}.
\]

Obviously, bank \( i \) would be willing to accept deposits that it could channel to a small project as long as \( \pi_i \geq 0 \), i.e., \( r_i \geq r_i^* \), which is an assumption maintained for every \( i \in N \).

3.2 Investment in a Large Project

If bank \( i \) is to invest in a large project, a loan agreement has to be established so that $1 is borrowed from bank \( j \). The budget constraints are then modified in the following way:
\[ 2 + b_i \leq 2 + e_i \quad \text{(BC at } t = 0) \]

\[ b_i + \alpha_{i^*,r_i^*} [\rho^* + \gamma^* (1 - \rho^*)] \geq \omega_i \quad \text{(BC at } t = 1) \]

\[ (1 - \alpha_{i^*,r_i^*}) r_i^* = (1 - \omega_i) + e_i + y_{ij} + \pi_{ij}^* \quad \text{(BC at } t = 2) \]

\[ y_{ij} \geq r_j \quad \text{(IR of the Lender)} \]

Differently from an investment in a small project, the budget constraint at \( t = 0 \) shows an extra $1, representing the loan that needs to be taken to finance the investment in a large project. As a result, at \( t = 2 \) there is also an extra expense representing the repayment of the loan, i.e., principal plus interest, denoted by \( y_{ij} \), that bank \( i \) owes to bank \( j \). The cost of the loan will depend on how much bank \( j \) will charge for it, which makes the profit of the bank in a large project to depend on the characteristics of the lender. This justifies the choice of \( \pi_{ij}^* \) to denote the profit of bank \( i \) with an investment in a large project that is partially financed by a loan taken from bank \( j \).

The individual rationality constraint in bank \( i \)'s problem, IR, represents the opportunity cost of bank \( j \), who could use the 1 it obtains from depositors to invest in a small project, rather than to lend it. Therefore, the compensation for the loan that bank \( i \) needs to offer to bank \( j \) should be as large as the later payoff with a small project, i.e., the repayment of the loan must be such that \( y_{ij} \geq r_j \). Without loss of generality, the borrower is assumed to have all the bargaining power and, as such, offers the minimum compensation for the loan, at which the lender is indifferent between lending or not, resulting in \( y_{ij} = r_j^6 \).

Analogously to an investment in a small project, the budget constraints at \( t = 0 \) and at \( t = 1 \) will bind, for there is no reason why bank \( i \) would either waste resources or liquidate more of the project than necessary. Adding to that the fact that bank \( i \) pays the minimum interest to the lender, the following holds:

\[ y_{ij} = r_j, \quad b_i = e_i, \quad (25) \]

\[ \alpha_{i^*,r_i^*} = \frac{\omega_i - e_i}{r_i^* [\rho^* + \gamma^* (1 - \rho^*)]} \quad (27) \]

\(^6\)One way of breaking the indifference point towards bank \( j \) extending a loan would be to impose a cost due to asymmetric information with investments in projects. Since, presumably, market forces lead banks to be more scrutinized than projects, a loan to another bank would be preferable to an equivalent investment in a small project, ceteris paribus.
Bank $i$’s profit with a large project when borrowing from bank $j$, $\pi^*_{ij}$, is then:

$$\pi^*_{ij} = r^*_i - \left\{ (1 - \omega_i) + e_i + r_j + \frac{\omega_i - e_i}{\rho^* + \gamma^*(1 - \rho^*)} \right\}.$$  \hfill (28)

### 3.3 Investment in Loans

From the assumptions that (i) the discount factor for loans is the same as for small projects and (ii) the repayment on a loan is the same as the one in a small project - due to the rationality constraint discussed previously - it results that loans and small projects are perfect substitutes. If bank $i$ decides to extend a loan, therefore, the investment in the short-term bond and the fraction of the loan that will need to be sold before maturity will be the same as those that apply in the analysis of an investment in a small project, being omitted here. For practical purposes, therefore, investments in loans and small projects are indistinguishable.

### 3.4 The Decision to Form Links

By comparing the profits associated to each investment opportunity that arise following their meeting, banks $i$ and $j$ will decide whether to (i) stay in autarky, i.e., use the funds available for that particular round of interaction to invest separately in their respective small projects, or (ii) engage in an interbank loan, leading the borrower to invest in a large project and the lender in a loan. Following the pairwise meeting of banks $i$ and $j$, these possible outcomes and their associated parameter conditions are summarized below:

1. **Bank $i$ wants to borrow from bank $j$, but not vice versa:** bank $i$ is better off investing in a large project, $\pi^*_{ij} > \pi_i$, whereas the opposite is true for bank $j$, $\pi_j > \pi^*_{ji}$. From expressions (22) and (28), that is the case if the following holds:

   $$\frac{r^*_i - (r_i + r_j)}{\omega_i - e_i} > \frac{[\rho + \gamma (1 - \rho)] - [\rho^* + \gamma^* (1 - \rho^*)]}{\rho + \gamma (1 - \rho)} \frac{r^*_j - (r_j + r_i)}{\omega_j - e_j},$$  \hfill (29)

   where the first inequality represents the fact that $\pi^*_{ij} > \pi_i$, whereas the second that $\pi_j > \pi^*_{ji}$. 

   26
2. **Bank \( j \) wants to borrow from bank \( i \), but not vice versa:** Analogous to the previous condition, but now with bank \( j \) being the one interested in borrowing, \( \pi_{ji}^* > \pi_j \), and bank \( i \) in lending, \( \pi_i > \pi_{ij}^* \):

\[
\frac{r_j^* - (r_j + r_i)}{\omega_j - e_j} > \frac{[\rho + \gamma (1 - \rho)] - [\rho^* + \gamma^* (1 - \rho^*)]}{[\rho + \gamma (1 - \rho)] [\rho^* + \gamma^* (1 - \rho^*)]} \geq \frac{r_i^* - (r_i + r_j)}{\omega_i - e_i},
\]

(30)

where the first inequality represents the fact that \( \pi_{ji}^* > \pi_j \), whereas the second that \( \pi_i > \pi_{ij}^* \).

3. **Both banks \( i \) and \( j \) want to borrow:** In this case, it holds that \( \pi_{ij}^* > \pi_i \) and \( \pi_{ji}^* > \pi_j \), equivalent to:

\[
\min \left\{ \frac{r_i^* - (r_i + r_j)}{\omega_i - e_i}, \frac{r_j^* - (r_j + r_i)}{\omega_j - e_j} \right\} > \frac{[\rho + \gamma (1 - \rho)] - [\rho^* + \gamma^* (1 - \rho^*)]}{[\rho + \gamma (1 - \rho)] [\rho^* + \gamma^* (1 - \rho^*)]}.
\]

(31)

In this scenario, it is assumed that the bank ending up being the borrower is the one to make the largest profit out of the loan. From (28), if \( \pi_{ij}^* > \pi_{ji}^* \), i.e.:

\[
[r_i^* - (1 - \omega_i) - e_i - r_j] - [r_j^* - (1 - \omega_j) - e_j - r_i] > \frac{(\omega_i - e_i) - (\omega_j - e_j)}{\rho^* + \gamma^* (1 - \rho^*)},
\]

(32)

then bank \( i \) ends up borrowing from bank \( j \). On the other hand, if \( \pi_{ji}^* > \pi_{ij}^* \), i.e.:

\[
[r_j^* - (1 - \omega_j) - e_j - r_i] - [r_i^* - (1 - \omega_i) - e_i - r_j] > \frac{(\omega_j - e_j) - (\omega_i - e_i)}{\rho^* + \gamma^* (1 - \rho^*)},
\]

(33)

then it is bank \( j \) who borrows from bank \( i \).

4. **Neither bank \( i \) nor bank \( j \) wants to borrow:** in this scenario, it follows that both banks are better off investing in their respective small projects, with \( \pi_i > \pi_{ij}^* \) for bank \( i \) and \( \pi_j > \pi_{ji}^* \) for bank \( j \) both holding, which is equivalent to:

...
\[
\frac{[\rho + \gamma (1 - \rho)] - [\rho^* + \gamma^* (1 - \rho^*)]}{[\rho + \gamma (1 - \rho)] [\rho^* + \gamma^* (1 - \rho^*)]} > \max \left\{ \frac{r_i^* - (r_i + r_j)}{\omega_i - e_i}, \frac{r_j^* - (r_j + r_i)}{\omega_j - e_j} \right\}.
\]

(34)

In this scenario, therefore, banks remain in autarky.

4 Intervention and Network Structure

The previous conditions driving the formation of links between banks hold for any set of parameters. The question that arises is how government and central bank policies that enhance market liquidity - denominated government intervention for short - affect the structure of the network being formed. The effects of government intervention on the banking network structure can be captured by comparing the network that would emerge under no government intervention, i.e., when \(\gamma^* = \gamma = 0\), with the network that would emerge when government and central bank policies that enhance market liquidity are in place, i.e., when \(\gamma^* \geq \gamma > 0\). The crudest way of comparing the networks under these two intervention scenarios is by counting the number of links present in each of the networks, for which the following result holds:

**Proposition 1.** For any fixed set of parameters, the network that emerges under government intervention has at least the same number of links (interbank loans) as the analogous network that would emerge under no government intervention.

**Proof:** See Appendix. 

Proposition 1 establishes that government intervention does not brake links between banks that would exist otherwise, and it eventually leads banks to engage in new interbank loans. In other words, intervention makes the network of banks to be more connected. This is a straightforward result since intervention policies that enhance market liquidity make large projects less costly to be prematurely liquidate, and therefore the incentives for banks to engage in interbank loans increase.

There is, however, a less intuitive way by which the network structure can be affected by government intervention. As the following proposition shows, intervention might reverse the direction of the flow of money between two banks that, under no intervention, would be engaging in a loan agreement. In other words, a loan agreement between a borrower bank \(i\) and a lender bank \(j\), prevailing under no intervention, might be switched to one where bank \(i\) becomes the lender and bank \(j\) the borrower, upon government intervention:
Proposition 2. Consider a network of banks formed without government intervention. If two arbitrary banks engage in a loan agreement, the identity of the borrower and the lender might change if one considers the network that would prevail under government intervention.

Proof: See Appendix.

To understand the above result, consider a meeting between bank $i$ and bank $j$. Under no intervention, suppose that this meeting results in a celebration of a loan agreement where bank $i$ borrows 1 from bank $j$. The loan agreement does not imply that bank $i$’s profit with the large project, which is being partially financed by the loan taken, is higher than bank $j$’s profit with the loan being extended. Rather, it implies that bank $i$’s profit investing in a large project is higher than the profit it could obtain with a small project, and the opposite for bank $j$. Under government intervention, this scenario will remain the same for bank $i$, but it might change for bank $j$. In this case, both banks would be willing to borrow to finance an investment in a large project, and the tie-breaking rule might now favor bank $j$, leading to a change in the identity of the borrower and the lender in regards to the interbank loan.

Government intervention, therefore, has a pivotal role in determining the shape of the network structure that emerges from the interaction among banks. It also follows, under a so called too-big-to-fail policy (TBTF) whereby government intervention enhances the market liquidity of large projects in a more pronounced way than it does for small projects, i.e., $\gamma^* > \gamma > 0$, that the incentives for the creation of links are even stronger when compared to the case where intervention is the same irrespective of the size of the project, i.e., $\gamma^* = \gamma > 0$:

Proposition 3. Under a TBTF intervention policy, i.e., $\gamma^* > \gamma > 0$, the conditions for banks to create links are easier to be satisfied than those under an intervention policy that takes place irrespective of the size of projects, i.e., $\gamma^* = \gamma > 0$.

Proof: See Appendix.

Combined with the other primitives of the model, government’s intervention policy will determine the structure of the network that emerges following banks’ interaction process. The network structure that will take place can equivalently be expressed through a matrix representing how banks are connected, and based on that the balance sheet of every bank can be characterized in terms of assets and liabilities. This in turn allows for the analysis of the fragility of the banking system in the presence of shocks that affect banks’ assets, i.e., the payoff of projects and other losses due to contagion. This is the subject of the next section.
5 Financial Networks and Shocks

The question of how government and central bank policies that enhance market liquidity affect financial fragility is intrinsically connected to not only the structure of banks’ connectivity in the network formed, but also to the characterization of their balance sheets and the type of shocks they face. The balance sheet characterization will provide the networth that banks have available to absorb shocks, and the shocks themselves will represent the losses on which the degree of fragility of a financial network depends.

5.1 Characterization of the Balance Sheet of Banks

The interaction process of banks will lead them to decide whether or not to form links, i.e., to engage in interbank loans that allow the borrowers to make investments in large projects. The connectivity structure of the network can then be described by the following matrix:

\[
X = \begin{bmatrix}
0 & \chi_{12} & \cdots & \chi_{1N} \\
\chi_{21} & 0 & \cdots & \chi_{2N} \\
\vdots & \vdots & \ddots & \vdots \\
\chi_{N1} & \chi_{N2} & \cdots & 0
\end{bmatrix},
\]

(35)

In the matrix, \(\chi_{ij}\) is an indicator function such that \(\chi_{ij} = 1\) if row-bank \(i\) lends to column-bank \(j\), and \(0\) otherwise. Matrix \(X\) will be crucial for the understanding of how shocks to projects’ payoffs propagate in the network, since it represents how banks are connected. The propagation of shocks, i.e., contagion, will depend on the networth of banks, which in turn is defined by their balance sheets. Together with the fire-sale parameters for small and large projects, \(\rho\) and \(\rho^*\), and the government intervention ones, \(\gamma\) and \(\gamma^*\), the following vectors that represent the other primitives of the model\(^7\) will allow the financial network to be completely characterized:

\[
r^* = \begin{bmatrix} r^*_1 \\ r^*_2 \\ \vdots \\ r^*_N \end{bmatrix}, r = \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_N \end{bmatrix}, \omega = \begin{bmatrix} \omega_1 \\ \omega_2 \\ \vdots \\ \omega_N \end{bmatrix}, e = \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_N \end{bmatrix},
\]

(36)

\(^7\)The vectors represent the payoff of a large project, \(r^*_i\), payoff of a small project, \(r_i\), fraction of early depositors, \(\omega_i\), and endowment, \(e_i\), that are available to bank \(i\) in every one of the meetings it has with the other banks in the network.
Before characterizing the balance sheet of banks, it is useful to note that, for an arbitrary bank \( i \in N \), the row-sum in matrix \( X \) gives its number of debtors, i.e., the number of loans made, \( n_L^i = \sum_{j \in N} \chi_{ij} \), whereas the column-sum gives its number of creditors or, equivalently, the number of large projects undertaken, \( n^i_{L,\star} = \sum_{j \in N} \chi_{ji} \). Since in each of the \( N - 1 \) rounds of interaction banks will necessarily be making an investment, the number of small projects in a bank’s portfolio is given by \( n_L^i = (N - 1) - n_{L,\star}^i - n_{L,\star}^i \).

Recall that, by assumption, banks are short of capital to serve early depositors, and this forces banks to liquidate a fraction of their assets before maturity, at date \( t = 1 \). That fraction is \( \alpha^i \) for investments in either loans or small projects, as in (54), and \( \alpha^i_{\star} \) for investments in large projects, as in (27). Hence, on the balance sheet of bank \( i \) at \( t = 2 \), there will be \( a^i_L = r_i (1 - \alpha^i) n_L^i \) worth of loans, \( a^i_{L,\star} = r^i_{\star} (1 - \alpha^i_{\star}) n^i_{L,\star} \) of large projects, and \( a^i_e = r_i (1 - \alpha^i_{\star}) n^i_{\star} \) of small projects. The size of the asset side of bank \( i \)'s balance sheet is, therefore, given by \( a^i = a^i_L + a^i_{L,\star} + a^i_e \).

The liability side of the balance sheet is composed of the amount owed to late depositors, loans taken from other banks (debt) and networth (equity endowment plus profits). Recall that $1 is collected from depositors in each of the \( N \) rounds of interaction, and the fraction of these deposits that remains to be claimed at \( t = 2 \) is \( 1 - \omega_i \). The amount owed to late depositors is, therefore, \( l^i_\omega = (N - 1) (1 - \omega_i) \).

Debt owed to other banks comes from the loans taken in order to finance investments in large projects. Moreover, banks’ loan repayments should cover the opportunity cost of lenders, which is the payoff they would get by investing in a small project rather than extending a loan, as in the discussion of the individual rationality constraint that is present when a bank considers to invest in a large project. This results in the debt owed to other banks by an arbitrary bank \( i \) being given by \( l^i_d = \sum_{j \in N} \chi_{ji} r_j \).

As for the networth, \( W^i \), bank \( i \)'s equity is given by \( l^i_e = (N - 1) e_i \), as in every round of interaction bank \( i \) receives an equity endowment of \( e_i \). Profits come from investments, i.e., loans and/or projects. Loans and small projects yield the same profit, \( \pi_i \), as in (22), and since there are \( n^i_L \) loans and \( n^i_{\star} \) small projects, the total profit from investments in these two assets is \( (n^i_L + n^i_{\star}) \pi_i \). Bank \( i \)'s profit with an investment in a large project financed by a loan taken from bank \( j \) is \( \pi^i_{\star j} \), as in (28), and hence bank \( i \)'s total profit from investments in large projects is \( \sum_{j \in N} \chi_{ji} \pi^i_{\star j} \). Total profits are then \( l^i_\pi = (n^i_L + n^i_{\star}) \pi_i + \sum_{j \in N} \chi_{ji} \pi^i_{\star j} \). With networth being \( W^i = l^i_e + l^i_\pi \), the size of the liability side of bank \( i \)'s balance sheet is given by \( l = l^i_\omega + l^i_d + W^i \).

In this way, for any network formed, with or without government intervention, the balance sheet of every bank is characterized, with assets (investments in loans, small and large projects) and liabilities (debt to other banks and networth) being
readily determined. Knowledge of the balance sheet of banks makes it possible to
determine the impact on the financial network following negative shocks to projects’
payoffs, which is done in the next section.

5.2 Negative Shocks to Assets’ Payoffs

The shocks to be introduced affect the payoffs that banks are entitled to receive from
their investments in projects. In this way, one can interpret that the shocks in the
model originate from the real side of the economy. It is important to highlight that
the shocks are to be understood as probability zero events, or perturbations of the
network, as in Allen and Gale (2000). If bank \( i \) has made an investment in a large
project with a payoff of \( r_i^* \), a shock of \( \delta_i^* \) implies that it receives only \( r_i^* (1 - \delta_i^*) \).
Analogously, if the investment was in a small project, upon a shock the project’s
payoff would be \( r_i^s (1 - \delta_i^s) \) rather than \( r_i^s \), for a shock of magnitude \( \delta_i^s \). The vectors
of shocks to the payoffs of small and large projects are given, respectively, by:

\[
\delta = \begin{bmatrix}
\delta_1 \\
\delta_2 \\
\vdots \\
\delta_N 
\end{bmatrix}, \quad \delta^* = \begin{bmatrix}
\delta_1^* \\
\delta_2^* \\
\vdots \\
\delta_N^* 
\end{bmatrix}.
\] (37)

Shocks cause banks to face a loss in the asset side of their balance sheets, whereas
their liabilities remain the same. The total loss a bank \( i \) will face is written as
\( \Delta_i = \Delta_{iL}^i + \Delta_{i^*}^i + \Delta_i^r \), where \( \Delta_{iL}^i = a_{ir} \delta_i \) corresponds to the loss on small projects,
\( \Delta_{i^*}^i = a_{i^*} \delta_i^* \) to the loss on large projects, and \( \Delta_i^r \) to the loss on loans made to other
banks, i.e., due to contagion.

Bank \( i \)'s loss on loans made to other banks, \( \Delta_{iL}^i \), depends on the bankruptcy
status of its debtors and, therefore, is endogenously determined. A bank \( j \) is deemed
bankrupt if it cannot fulfill entirely its obligations with debtors and households, i.e.,
if its total losses are greater than its networth, \( \Delta_j > W_j \).

For instance, assume that bank \( i \) has lent to bank \( j \), i.e., \( \chi_{ij} = 1 \), and bank
\( j \) is bankrupt. The losses that will be spread by bank \( j \) are those that cannot be
contained by bank \( j \)'s networth, i.e., those given by \( \Delta_j - W_j > 0 \). The creditors
of bank \( j \) are owed the total amount \( l_{ij}^j \), as in the characterization of banks’ balance
sheets, and their loans will be (negatively) affected in the same proportion: for every
unit borrowed, bank \( j \) will pay only a fraction \( (\Delta_j - W_j) / l_{ij}^j \) of its debt. Hence,
bank \( i \), who expected to receive a payment of \( r_i \) for the fraction \( (1 - \alpha_i^j) \) of the
loan made to bank \( j \) that is still in its balance sheet, will end up receiving only
\( r_i (1 - \alpha_i^j) (\Delta_j - W_j) / l_{ij}^j \). Therefore, the amount:
\[ \Delta^i_L = r_i \left(1 - \alpha^i_r\right) \sum_{j \in N} \chi_{ij} \frac{(\Delta^j - W^j)^+}{l^j_d} \]  

represents the contagion losses suffered by bank \( i \) upon the eventual (partial) default of its debtors, where \((\cdot)^+\) denotes the positive part. The system of equations that determines \( \tilde{\Delta}^i := (\Delta^i - W^i)^+ \) - bank \( i \)'s loss in excess of its networth - is given by:

\[
\tilde{\Delta}^1 = \left[ r_1 \left(1 - \alpha^1_r\right) \sum_{j \in N} \chi_{1j} \frac{\tilde{\Delta}^j}{l^j_d} + \Delta^1_r + \Delta^1_r - W^1 \right]^+ , \\
\tilde{\Delta}^2 = \left[ r_2 \left(1 - \alpha^2_r\right) \sum_{j \in N} \chi_{2j} \frac{\tilde{\Delta}^j}{l^j_d} + \Delta^2_r + \Delta^2_r - W^2 \right]^+ , \\
\vdots \\
\tilde{\Delta}^N = \left[ r_N \left(1 - \alpha^N_r\right) \sum_{j \in N} \chi_{Nj} \frac{\tilde{\Delta}^j}{l^j_d} + \Delta^N_r + \Delta^N_r - W^N \right]^+ .
\]

In matrix form, this system is written as:

\[ \tilde{\Delta} = \left( \tilde{X} \tilde{\Delta} + \Delta_r^* + \Delta_r - W \right)^+ , \]

where \( \tilde{X} \tilde{\Delta} \) represents the vector of contagion losses, \( \Delta_r^* \) and \( \Delta_r \) the vectors of losses on large projects and small projects, respectively, and \( W \) is the networth of individual banks\(^8\). The matrix \( \tilde{X} \) is given by:

\[
\tilde{X} = \begin{bmatrix}
0 & \frac{r_1(1-\alpha^1_r)\chi_{12}}{l^1_d} & \ldots & \frac{r_1(1-\alpha^1_r)\chi_{1N}}{l^1_d} \\
\frac{r_2(1-\alpha^2_r)\chi_{21}}{l^2_d} & 0 & \ldots & \frac{r_2(1-\alpha^2_r)\chi_{2N}}{l^2_d} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{r_N(1-\alpha^N_r)\chi_{N1}}{l^N_d} & \frac{r_N(1-\alpha^N_r)\chi_{N2}}{l^N_d} & \ldots & 0
\end{bmatrix} ,
\]

which can equivalently be expressed as:

---

\(^8\)The dimension of each of these vectors is \( N \times 1 \).
\[ \tilde{X} = \text{diag} \begin{bmatrix} r_1 (1 - \alpha_r^1) \\ r_2 (1 - \alpha_r^2) \\ \vdots \\ r_N (1 - \alpha_r^N) \end{bmatrix} \begin{bmatrix} 0 & \chi_{12} & \cdots & \chi_{1N} \\ \chi_{21} & 0 & \cdots & \chi_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \chi_{N1} & \chi_{N2} & \cdots & 0 \end{bmatrix} \text{diag} \begin{bmatrix} \frac{1}{l_1^d} \\ \frac{1}{l_2^d} \\ \vdots \\ \frac{1}{l_N^d} \end{bmatrix}, \] (44)

where \( \text{diag} \) represents the diagonalization operator of a vector. Matrix \( \tilde{X} \), therefore, is simply a weighted version of \( X \), the matrix representing the network structure. The row-weights are the expected payoff on loans made that are still on banks’ balance sheets, \( r_i (1 - \alpha_r^i) \), whereas the column-weights represent each bank’s per unit amount owed relative to total debt, \( 1/l_i^d \). From (38), an element \( i, j \) of \( \tilde{X} \) represents the per unit loss that bank \( j \) imposes on bank \( i \), conditional on \( i \) having lent to \( j \), i.e., \( \chi_{ij} = 1 \).

Later on, expression (42) will be used to show that government and central bank policies that enhance market liquidity create a trade-off between increasing the networth of banks and exposing them to financial fragility. For, government intervention increases the networth of banks, \( W \), but at the same time it also increases the incentives for them to engage in interbank lending, creating more channels for contagion.

The max operator in expression (42) precludes a closed-form characterization of banks’ total losses from being directly obtained, which would allow one to understand the impact on them from government intervention. However, if shocks are such that banks’ networth become negligible, or almost depleted - a stress scenario - the analysis is simplified, and the role of the government in banks’ losses can be more clearly seen, as shown in the next section.

### 5.2.1 Spread of Shocks in a Stress Scenario

To understand how shocks are transmitted in a stress scenario, where banks have their networth being negligible, or almost depleted, assume that the shocks hitting projects lead to \( W^i \approx 0 \), for every \( i \in N \). Expression (42) is then simplified to:

\[ \tilde{\Delta} = \tilde{X} \tilde{\Delta} + \Delta^r + \Delta_r, \] (45)

and, upon some conditions being satisfied:

\[ \tilde{\Delta} = \left( I - \tilde{X} \right)^{-1} \left( \Delta^r + \Delta_r \right). \] (46)

The necessary conditions for expression (46) to be valid are the existence and uniqueness of the inverse matrix on its right-hand side. The following lemma is used
to address these conditions. Before proceeding to it, recall that a \( n \times n \) matrix \( B \) has a dominant diagonal if there are positive numbers \( d_1, d_2, \ldots, d_n \) such that \( d_j |b_{jj}| > \sum_{i \neq j} d_i |b_{ij}| \), for \( j = 1, \ldots, n \), where \( b_{ij} \) is the \( i \)th row, \( j \)th column element of matrix \( B \).

**Lemma 4.** The \( N \times N \) matrix \( B := (I - \tilde{X}) \) has a dominant diagonal.

**Proof:** See Appendix. \( \square \)

From Theorems 4.C.3, 4.C.4 and 4.C.6 in Takayama (1985), \( (I - \tilde{X}) \) having a dominant diagonal implies that its inverse exists and is unique, and hence that (46) is a valid expression. Not only that, \( (I - \tilde{X})^{-1} \) can be equivalently written as:

\[
(I - \tilde{X})^{-1} = \sum_{k=0}^{\infty} (\tilde{X})^k = I + \tilde{X} + \tilde{X}^2 + \tilde{X}^3 + \ldots. \tag{47}
\]

Combining (46) and (47), it follows then:

\[
\tilde{\Delta} = (\Delta_{r^*} + \Delta_r) + \tilde{X}(\Delta_{r^*} + \Delta_r) + \tilde{X}^2(\Delta_{r^*} + \Delta_r) + \tilde{X}^3(\Delta_{r^*} + \Delta_r) + \ldots. \tag{48}
\]

The terms in the series (48) can be interpreted in the following way: \( (\Delta_{r^*} + \Delta_r) \) corresponds to the first wave of shocks that hits banks from investments in small and large projects. The assumption that banks’ networth is negligible implies that those who are borrowers (partially) default on loans taken. Defaults on loans cause a second wave of shocks, \( \tilde{X}(\Delta_{r^*} + \Delta_r) \), in addition to the first one due to direct losses. The matrix \( X \) provides the factors according to which a unitary loss faced by debtor banks - the column-header banks - is spread among their creditors - the row-header banks. Multiplying \( \tilde{X} \) by \( (\Delta_{r^*} + \Delta_r) \), therefore, transforms per unit losses in total losses.

The second wave of shocks might lead to a third one and so forth, where other banks become bankrupt due not to direct losses in projects, but rather due to contagion. The high order terms in (48) capture precisely this effect and, in case the majority of the loans is concentrated in a few banks, these terms are expected to be negligible.

It is straightforward to make an analogy with input-output analysis. For, the elements of \( \tilde{X} \) can be viewed as the inputs necessary to produce the loss generated by banks, \( \tilde{\Delta} \). In particular, from the analysis preceding (38), one knows that \( r_j (1 - \alpha_j r_i) (\tilde{\Delta}^j) / l_{ij}^j \) represents the unitary loss produced by bank \( j \) and imposed on bank \( i \) - conditional on \( \chi_{ij} = 1 \), i.e., bank \( i \) having lent to bank \( j \). Therefore, the total loss produced by bank \( j \) is given by:
\[ \Delta^j \left[ \frac{r_1 (1 - \alpha_1^j) \chi_{1j}}{l_d^j} + \ldots + \frac{r_N (1 - \alpha_N^j) \chi_{Nj}}{l_d^j} \right] \]

Splitting the total loss into its individual pieces, i.e.:

\[ \tilde{\Delta}^j \left[ \frac{r_1 (1 - \alpha_1^j) \chi_{1j}}{l_d^j} \right] + \ldots + \tilde{\Delta}^j \left[ \frac{r_N (1 - \alpha_N^j) \chi_{Nj}}{l_d^j} \right] \]

one can see that, for any \( i \), \( r_i (1 - \alpha_i^j) \chi_{ij} / l_d^j \) corresponds to the “input” of bank \( i \) in the making of the total loss, or “output”, produced by bank \( j \). These input factors of “production” are elements of the \( j \)'s column in \( \tilde{X} \), as one would expect in a standard input-output matrix. This is where the analogy comes from.

In fact, conditions are satisfied allowing (48) to be approximated by a simpler expression. As it will be argued in the sequence, (48) can be approximated by:

\[ \tilde{\Delta} \cong (\Delta_{r^*} + \Delta_r) + \tilde{X} (\Delta_{r^*} + \Delta_r) + \tilde{X}^2 (\Delta_{r^*} + \Delta_r). \]

The approximation given in expression (51) follows from the fact that the matrix \((I - \tilde{X})\) has not only a dominant diagonal, but a positive one, as shown in the proof of Lemma 4. Using this result, Theorems 4.C.7 and 4.B.2 in Takayama (1985) are combined in the following way. Theorem 4.C.7 states that \( 1 > \tilde{\lambda}_{\tilde{X}} \), where \( \tilde{\lambda}_{\tilde{X}} \) is the Frobenius root of \( \tilde{X} \). On the other hand, from Theorem 4.B.2, if \( w \) is any eigenvalue of \( \tilde{X} \), then \( |w| \leq \tilde{\lambda}_{\tilde{X}} \). Therefore, it can be asserted that all the eigenvalues of \( \tilde{X} \) are smaller than 1 in absolute value. Following Petersen and Pedersen (2012), it then holds that \( \tilde{X}^n \to 0 \) as \( n \to \infty \), and that (48) can be approximated by (51).

Letting \( \tilde{\Delta} \) denote the approximated version of \( \Delta \) as in (51), the vector of total losses of each bank is:

\[ \tilde{\Delta} = \begin{bmatrix} \tilde{\Delta}^1 \\ \tilde{\Delta}^2 \\ \vdots \\ \tilde{\Delta}^N \end{bmatrix}, \]

with each entry \( \tilde{\Delta}^i \) given by:
\[
\Delta_i = r_i^* \left( 1 - \alpha_i^* \right) \left( \sum_{k \in N} \chi_{ki} \right) \delta_i^* + r_i \left( 1 - \alpha_i^* \right) \left( N - 1 - \sum_{k \in N} \chi_{ik} - \sum_{k \in N} \chi_{ki} \right) \delta_i + \left[ r_i \left( 1 - \alpha_i^* \right) \right] \times \left\{ \sum_{j \in \mathcal{N}} \frac{1}{\theta_d} \left\{ \chi_{ij} + \sum_{k \in \mathcal{N}} \frac{r_k \chi_{ik} \left( 1 - \alpha_k^* \right) \chi_{kj}}{r_d} \right\} \left\{ r_j^* \left( 1 - \alpha_j^* \right) \left( \sum_{k \in \mathcal{N}} \chi_{kj} \right) \delta_j^* + r_j \left( 1 - \alpha_j^* \right) \left[ (N - 1) - \sum_{k \in \mathcal{N}} \chi_{jk} - \sum_{k \in \mathcal{N}} \chi_{kj} \right] \delta_j \right\} \right\}.
\]

As before, the total losses each bank faces can be seen to be composed of losses on large and small projects - the first two terms of expression (53), respectively - and of losses on loans made to other banks - the contagion losses, the last term of the expression.

As far as the effects of government intervention on total losses is concerned, it is readily seen from (53) that the losses due to contagion faced by bank \( i \) will depend on \( \alpha_i^* \), which corresponds to the fraction of loans bank \( i \) needs to sell before maturity in order to meet the demands from early depositors. From the expression for \( \alpha_i^* \):

\[
\alpha_i^* = \frac{\omega_i - e_i}{r_i \left( \rho + \gamma (1 - \rho) \right)}.
\]

it follows that \( \partial \alpha_i^* / \partial \gamma < 0 \), i.e., an increase in the liquidity provided by the government leads to a decrease in the fraction of loans that bank \( i \) has to sell before maturity. This in turn implies that bank \( i \) will have more loans on its balance sheet when unexpected shocks arrive, and so greater will be the losses it will suffer due to contagion.

Any factor that allows banks to hold a higher fraction of loans until maturity will have the same effect as an increase in government support. If bank \( i \) has a smaller fraction of early depositors, \( \omega_i \), a larger endowment of equity, \( e_i \), higher receivables from loans made, \( r_i \), or smaller fire-sale costs, \( \rho \), then the fraction of its loans’ portfolio that it will have to sell before maturity will be smaller. Accordingly, larger will be the amount of loans sitting on bank \( i \)’s balance sheet when unexpected shocks arrive. Exposure to contagion, therefore, will be higher.

6 The Trade-Off Between Networth and Fragility

Government and central bank policies that enhance market liquidity, by decreasing fire-sale costs, directly affect the price of assets and, in turn, have an impact on the networth of banks. On the other hand, market liquidity increases the incentives for banks to engage in interbank loans, allowing them to finance investments that
otherwise would not be feasible. These very same loans bring more channels for contagion among banks, in case negative shocks take place in the economy.

The question that arises is: can government intervention safeguard the health of the financial system, by concomitantly increasing the networth of banks and decreasing their exposure to fragility, through policies affecting the market liquidity of assets? The answer to this question rests on the analysis of the matrix equation (42), replicated below:

$$\tilde{\Delta} = (\tilde{X}\tilde{\Delta} + \Delta_r^* + \Delta_r - W)^+,$$

(55)

The left-hand side of (55), \(\tilde{\Delta}\), represents the total losses that banks face following shocks to their assets, and the right-hand side shows that these losses will depend on the networth \(W\) that banks have available on their balance sheets. Differently from the analysis of the spread of shocks in a stress scenario, where shocks were assumed to be such that banks had their networth depleted, the impact of government intervention on fragility is not straightforward when banks have enough networth to prevent contagion from ensuing.

In order to address the question of whether government policies that enhance market liquidity can concomitantly increase the networth of banks and make the financial network less fragile, it is necessary to understand the conditions under which government intervention undoubtedly leads to an increase in the networth of banks. Proposition 2 implies that, by changing the mix of projects chosen by banks, government intervention does not necessarily increase the total networth of the banking system. However, if the conditions stated in the following corollary to Propositions 1 and 2 are satisfied, the impact on banks networth from government intervention is clear:

**Corollary 5.** Let \(\bar{\pi}^G := \min \{\pi_1^G, \ldots, \pi_N^G\}\) and \(\bar{\pi} := \max \{\pi_1, \ldots, \pi_N\}\) denote the minimum profit with a small project among all the banks, with and without intervention, respectively. If \(\bar{\pi}^G > \bar{\pi}\), then government intervention leads to an increase in the networth of the network of banks, \(\sum_{i \in N} W_i\). Otherwise, that is not necessarily the case.

**Proof:** See Appendix.

Corollary 5 is crucial for the study of the effects of government intervention on the capacity of networks to absorb negative shocks, since it provides the conditions upon which government intervention undoubtedly leads to an increase in the wealth of the banking system. The wealth of the banking system, or the networth of the
financial network, provides the cushion that banks have available to absorb negative shocks.

On the other hand, Proposition 1 establishes that, compared to an analogous network formed under no government intervention, a network where government and central bank policies that enhance market liquidity are present has at least the same number of links. In other words, government intervention leads to an increase in the channels necessary for the spread of negative shocks, or contagion.

Fragility is understood as the exposure of a financial network to the possibility of contagion. One approach to measure fragility is to consider, for a given network, the number of banks that would become bankrupt after the assets of another bank are hit by negative shocks. This in practice constitutes a stress test of the network, in order to see how robust the overall structure is to problems that its individual members might face. In this way, consider a network with \( N \) banks, and take an arbitrary bank \( i \) facing shocks in its projects. The set:

\[
D^i := \{ j \neq i \mid \Delta^j > W^j \} \tag{56}
\]

contains all the banks that become bankrupt as a result of contagion, since the only bank facing direct shocks is bank \( i \). The cardinality of this set, \( |D^i| \), thus, gives the total number of bank failures due only to contagion. For contagion to ensue, however, a necessary condition is that bank \( i \) is bankrupt itself and, therefore, if \( |D^i| \) is positive, the total number of bank failures is \( 1 + |D^i| \). The index:

\[
f^i := \begin{cases} 
1 + |D^i| & \text{if } D^i \neq \emptyset \\
0 & \text{otherwise} 
\end{cases} \tag{57}
\]

gives a measure of the relative fragility of the network to bank \( i \), for a particular realization of shocks that hit bank \( i \)'s projects. Repeating and combining the same exercise for every bank \( j \neq i \) in the network leads to another index:

\[
f := \sum_{i \in N} f^i \tag{58}
\]

which represents a measure of the overall fragility of the network relative to the individual failure of its members.

Based on these measures of fragility, the main result of the paper is enunciated in the following proposition:

**Proposition 6.** Consider a set of parameters according to which a network obtained under government intervention has a total networth higher than that associated to the analogous network that is obtained under no government intervention. In this
context, even though increasing the total wealth of banks, government intervention might lead to a more fragile network.

**Proof:** The proof of the proposition is based on a counter-example. Consider a network composed of 6 banks, generated under $\rho^* = 0.05\rho$, $\gamma^* = 0.8$ and $\gamma = 0.3$, and its non-government counterpart, i.e., the analogous network obtained under $\gamma^* = \gamma = 0$ instead. The other parameters of the economy are:

<table>
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<th>$r^*$</th>
<th>$r$</th>
<th>$\omega$</th>
<th>$\epsilon$</th>
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<td>0.05</td>
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<td>1.08</td>
<td>0.09</td>
<td>0.07</td>
</tr>
<tr>
<td>Bank 5</td>
<td>2.97</td>
<td>1.00</td>
<td>0.15</td>
<td>0.12</td>
</tr>
<tr>
<td>Bank 6</td>
<td>2.71</td>
<td>1.21</td>
<td>0.14</td>
<td>0.10</td>
</tr>
</tbody>
</table>

Table 1: Parameters used for the generation of the networks in Figure 8.

These parameters satisfy the conditions of Corollary 5 and, as such, those of the proposition. Accordingly, government intervention in this economy leads to an increase in the total networth of banks. Under this set of parameters, the networks obtained with and without government intervention are displayed in Figure 8.

Figure 8: Networks generated under the parameters of Table 1.
The effects of government intervention on financial fragility rely on the shocks that are imposed to projects’ payoffs. To avoid relying on a single realization of shocks to draw conclusions regarding the fragility of the networks with and without government intervention, it is necessary to perform simulations varying the size of the shocks.

Accordingly, for each bank \( i \in N \) in the network, 1,000 simulations is performed, each of them with shocks hitting large and small projects, \( \delta \) and \( \delta^* \), respectively. The shocks are drawn from independent \( U [0, 1] \) uniform distributions, and are imposed on banks one at a time. The results of the simulations are displayed in Table 2.

<table>
<thead>
<tr>
<th>Bank</th>
<th>LR</th>
<th>Links</th>
<th>Ind Fails</th>
<th>Dir Fails</th>
<th>Networth</th>
<th>Bank</th>
<th>LR</th>
<th>Links</th>
<th>Ind Fails</th>
<th>Dir Fails</th>
<th>Networth</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.00</td>
<td>0.69</td>
<td>4.00</td>
<td>492.00</td>
<td>660.00</td>
<td>4.08</td>
<td>6.00</td>
<td>0.82</td>
<td>3.00</td>
<td>703.00</td>
<td>895.00</td>
<td>1.82</td>
</tr>
<tr>
<td>3.00</td>
<td>0.79</td>
<td>2.00</td>
<td>474.00</td>
<td>644.00</td>
<td>1.86</td>
<td>5.00</td>
<td>0.68</td>
<td>3.00</td>
<td>450.00</td>
<td>624.00</td>
<td>3.67</td>
</tr>
<tr>
<td>2.00</td>
<td>0.63</td>
<td>5.00</td>
<td>475.00</td>
<td>633.00</td>
<td>6.04</td>
<td>2.00</td>
<td>0.62</td>
<td>4.00</td>
<td>273.00</td>
<td>472.00</td>
<td>5.32</td>
</tr>
<tr>
<td>1.00</td>
<td>0.67</td>
<td>3.00</td>
<td>454.00</td>
<td>609.00</td>
<td>3.79</td>
<td>4.00</td>
<td>0.91</td>
<td>0.00</td>
<td>0.00</td>
<td>512.00</td>
<td>0.50</td>
</tr>
<tr>
<td>6.00</td>
<td>0.81</td>
<td>1.00</td>
<td>249.00</td>
<td>449.00</td>
<td>1.42</td>
<td>1.00</td>
<td>0.70</td>
<td>2.00</td>
<td>0.00</td>
<td>203.00</td>
<td>2.81</td>
</tr>
<tr>
<td>4.00</td>
<td>0.91</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.50</td>
<td>3.00</td>
<td>0.83</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.89</td>
</tr>
</tbody>
</table>

Table 2: Ranking of banks according to the number of failures induced.

For each of the networks, i.e., with and without government intervention, the index of fragility \( f \) given by (58) is calculated. The index is obtained by the sum, for each bank, of the number of times it fails plus the number of other banks that fail due to contagion, in each of the simulations performed. The number of times a bank has failed is given by the column “Dir Fails” (direct failures), whereas the number of failures it has caused, i.e., due to contagion, appear on the column “Ind Fails” (indirect failures). For the network with government intervention, \( f = 5,159 \), whereas for the one without intervention, \( f = 4,192 \). Therefore, according to the measure \( f \), government intervention brings more financial fragility, in the economy considered under the parameters of Table 1.

On the other hand, Table 2 shows that the total wealth of the network under government intervention is $17.69, greater than that associated to the analogous network without government intervention, which is $15.01. This comes as no surprise, since the parameters used satisfy the conditions of Corollary 5, and government intervention always lead to an increase in the wealth of the banking system in this case.

The economy just presented, therefore, is an example that shows that, even though increasing the total networth of banks, i.e., the cushion they have available to absorb shocks, government intervention leads to more fragility.
6.1 Analysis

Proposition 6 establishes that government intervention might bring more fragility to the financial network of an economy, despite improving banks’ total networth, i.e., the capacity of the financial sector as a whole to absorb shocks. The main reason behind this result is that government intervention leads to more interbank loans, or links, which at the same time that allow banks to invest in more profitable assets, creates new channels for contagion in the financial network.

An important question that follows regards the identification of the banks that contribute more to the fragility of the financial system, and how these key players are affected by government intervention. For, according to Propositions 1 and 2, government and central bank policies that enhance market liquidity have the potential to alter the configuration of the network, which might change the identity of systemically important banks.

In the present framework, the identification of the banks that contribute the most to financial fragility is based on the index \( f_i \) given by expression (57). For an arbitrary bank \( i \), this index represents the number of bank failures that results from the potential failure of bank \( i \) itself and, in order to avoid it being a measure specific to an arbitrary realization of shocks to projects’ payoffs, it must be simulation-based.

In the example from the proof of Proposition 6, Table 2 presents the banks ranked according to the index \( f_i \), from the most to the least critical, for the networks obtained with and without government intervention. It also presents some typical financial indicators that, when taken together, are often used as a proxy for the systemic importance of a bank. These indicators are, for an arbitrary bank \( i \): the networth, given by \( W_i \) as in the characterization of the balance sheet of banks; the number of (inward) links; and the leverage ratio, LR, calculated as:

\[
LR_i := 1 - \frac{W_i}{a_i} = \frac{l_i^w + l_i^d}{a_i}.
\]  

(59)

The most critical bank in the network under government intervention is bank 5, which is (i) one of the banks with highest networth, (ii) relatively well connected, and (iii) relatively low leveraged. In the network without government intervention, bank 2 is the most critical one, and it is (i) one of the banks with lowest networth, (ii) relatively well connected, and (iii) relatively high leveraged. This analysis points out that, under the present framework, there is not a direct association between the systemic importance of a bank based on the index \( f_i \) and that based on traditional financial indicators.
In this way, the results from Table 2 can be interpreted as saying that an analysis of the systemic importance of banks based solely on traditional financial indicators might be very misleading. Instead, it shows the importance of not only considering the structure of the financial network, but also the need for stress tests that show how individual banks pose a threat to the stability of the financial system.

As previously mentioned, government intervention has the potential to alter the configuration of the financial network and, as a result, the identity of the banks deemed as the most systemically important. Table 2 clearly points out to this effect: not only the most critical banks in the networks with and without government intervention do not coincide, there are also some stark differences in the ranking of the banks when the two networks are compared. For instance, the most critical bank in the network without intervention, bank 6, occupies only the penultimate position in the ranking of banks once government intervention is considered. On the other hand, bank 3, the last one in the ranking without intervention, becomes the second most critical one in the network with intervention.

There is, therefore, a non-linear effect resulting from government intervention, affecting not only the fragility of the financial network as whole, but also the systemic importance of its individual members. Even though a proper analysis of the design of intervention policies is not pursued by the paper, it should be clear that, as far as financial fragility is concerned, government and central bank policies that affect market liquidity must consider the potential impacts on the topology of the financial system, i.e., the network structure.

7 Concluding Remarks

This paper proposes a model of the endogenous formation of financial networks, where government and central bank policies that improve market liquidity - government intervention for short - play a key role in determining the structure of the network. Government intervention leads to the formation of different network structures, each of them with a distinct degree of financial fragility, i.e., the possibility of contagion after an individual bank experiences negative shocks to its assets. This allows one to study the relationship between policies that affect market liquidity and financial fragility.

The main impact of government intervention is on (i) banks’ networth and (ii) number of loans in the interbank market. By improving market liquidity, government intervention gives more incentives for banks to engage in large investments, which are more profitable but at the same time more costly. To finance large investments, banks need to resort to the interbank market, and the consequences are twofold: the
higher profitability of large investments increase the networth of banks, but the loans that need to be taken to finance them create new channels of contagion.

The main result of the paper is precisely to show that, by increasing the networth of banks through policies that enhance market liquidity, government intervention does not necessarily cause the financial network to be less exposed to the possibility of bank failures. This trade-off between networth and financial fragility arises because government intervention has the potential to alter the structure of the financial network, eventually leading to a situation where banks are more connected and exposed to direct and indirect shocks.

The model studied highlights the importance that knowledge of the structure of the network of banks might represent in the design of government and central bank policies. It comes as no surprise that, following the global financial crisis of 2007-2009, many central banks around the world started to (i) collect data of the bilateral exposure of financial institutions, allowing a more detailed identification of financial networks, and (ii) perform stress tests that explicitly take into account the structure of financial networks, which provides a more precise measure of how the financial system is exposed to shocks and the contribution of each financial institution to it, as in the example analyzed in the previous section.

It is the opinion of the author that these efforts, which are more related to the macro-prudential approach to the regulation of banks’ activities, tend to improve the stability of the financial system, as opposed to a micro-prudential approach that tends to focus solely on the balance sheet of individual banks.
Not intended for publication

Appendix

A.1 Proof of Proposition 1

The following lemma is used in the proof of the proposition:

**Lemma 7.** For any $1 > \rho > \rho^* > 0$ and $1 > \gamma^* \geq \gamma \geq 0$, the following is satisfied:

$$\frac{\rho - \rho^*}{\rho \rho^*} \geq \frac{[\rho + \gamma (1 - \rho)] - [\rho^* + \gamma^* (1 - \rho^*)]}{[\rho + \gamma (1 - \rho)] [\rho^* + \gamma^* (1 - \rho^*)]}.$$  \hfill (60)

**Proof:** Consider the function $h(\gamma, \gamma^*)$ defined by

$$h(\gamma, \gamma^*) \equiv \frac{[\rho + \gamma (1 - \rho)] - [\rho^* + \gamma^* (1 - \rho^*)]}{[\rho + \gamma (1 - \rho)] [\rho^* + \gamma^* (1 - \rho^*)]}.$$  \hfill (61)

For $\gamma^* = \gamma$ it follows that

$$h(\gamma, \gamma) = \frac{[\rho + \gamma (1 - \rho)] - [\rho^* + \gamma (1 - \rho^*)]}{[\rho + \gamma (1 - \rho)] [\rho^* + \gamma (1 - \rho^*)]} := H(\gamma).$$  \hfill (62)

Taking the derivative of $H$ yields

$$H'(\gamma) = -\frac{(\rho - \rho^*) \{[\rho^* + \gamma (1 - \rho^*)] + (1 - \gamma) (1 - \rho^*) [\rho + \gamma (1 - \rho)]\}}{[\rho + \gamma (1 - \rho)][\rho^* + \gamma (1 - \rho^*)]^2} < 0,$$  \hfill (63)

the inequality following from $\rho > \rho^*$. For $\gamma \geq 0$, it follows therefore that $H(0) \geq H(\gamma)$, i.e.,

$$\frac{\rho - \rho^*}{\rho \rho^*} \geq \frac{[\rho + \gamma (1 - \rho)] - [\rho^* + \gamma (1 - \rho^*)]}{[\rho + \gamma (1 - \rho)] [\rho^* + \gamma (1 - \rho^*)]}.$$  \hfill (64)

For $\gamma^* \geq \gamma$ one has that

$$\frac{[\rho + \gamma (1 - \rho)] - [\rho^* + \gamma (1 - \rho^*)]}{[\rho + \gamma (1 - \rho)] - [\rho^* + \gamma^* (1 - \rho^*)]} \geq 1,$$  \hfill (65)

and

$$\frac{[\rho + \gamma (1 - \rho)] [\rho^* + \gamma (1 - \rho^*)]}{[\rho + \gamma (1 - \rho)] [\rho^* + \gamma^* (1 - \rho^*)]} \leq 1,$$  \hfill (66)
therefore

\[
\frac{[\rho + \gamma (1 - \rho)] - [\rho^* + \gamma (1 - \rho^*)]}{[\rho + \gamma (1 - \rho)][\rho^* + \gamma (1 - \rho^*)]} \geq \frac{[\rho + \gamma (1 - \rho)] - [\rho^* + \gamma^* (1 - \rho^*)]}{[\rho + \gamma (1 - \rho)][\rho^* + \gamma^* (1 - \rho^*)]}.
\]

(67)

Combining (64) with (67) implies that

\[
\frac{\rho - \rho^*}{\rho \rho^*} \geq \frac{[\rho + \gamma (1 - \rho)] - [\rho^* + \gamma^* (1 - \rho^*)]}{[\rho + \gamma (1 - \rho)][\rho^* + \gamma^* (1 - \rho^*)]},
\]

(68)

which is the desired result.

□

Now one proceeds proving Proposition 1: for an even number of banks, \( N \), consider the original network formed without government intervention, \( \gamma = \gamma^* = 0 \), and take arbitrarily two of them, say banks \( i \) and \( j \). Without loss of generality, assume that

\[
\frac{r_i^* - (r_i + r_j)}{\omega_i - e_i} > \frac{r_j^* - (r_j + r_i)}{\omega_j - e_j}.
\]

(69)

One is interested in what would be the network if, instead, government intervention, \( \gamma^* \geq \gamma > 0 \), was in place. From Lemma 7 it follows that:

\[
\frac{\rho - \rho^*}{\rho\rho^*} > \frac{[\rho + \gamma (1 - \rho)] - [\rho^* + \gamma^* (1 - \rho^*)]}{[\rho + \gamma (1 - \rho)][\rho^* + \gamma^* (1 - \rho^*)]}.
\]

(70)

If in the original network banks \( i \) and \( j \) do not have a link, (34) and (69) imply that

\[
\frac{\rho - \rho^*}{\rho\rho^*} > \frac{r_i^* - (r_i + r_j)}{\omega_i - e_i}.
\]

(71)

Following intervention, however, either

\[
\frac{\rho - \rho^*}{\rho\rho^*} > \frac{[\rho + \gamma (1 - \rho)] - [\rho^* + \gamma^* (1 - \rho^*)]}{[\rho + \gamma (1 - \rho)][\rho^* + \gamma^* (1 - \rho^*)]} > \frac{r_i^* - (r_i + r_j)}{\omega_i - e_i},
\]

(72)

which from (34) implies that banks still do not want to transact, or

\[
\frac{\rho - \rho^*}{\rho\rho^*} > \frac{r_i^* - (r_i + r_j)}{(\omega_i - e_i)} > \frac{[\rho + \gamma (1 - \rho)] - [\rho^* + \gamma^* (1 - \rho^*)]}{[\rho + \gamma (1 - \rho)][\rho^* + \gamma^* (1 - \rho^*)]} > \frac{r_j^* - (r_j + r_i)}{\omega_j - e_j},
\]

(73)
which from (29) implies that bank $i$ borrows from bank $j$ and a link is established, or

\[
\frac{\rho - \rho^*}{\rho \rho^*} > \frac{r_i^* - (r_i + r_j)}{\omega_i - e_i} > \frac{r_j^* - (r_j + r_i)}{\omega_j - e_j} \geq \frac{[\rho + \gamma (1 - \rho)] - [\rho^* + \gamma^* (1 - \rho^*)]}{[\rho + \gamma (1 - \rho)] [\rho^* + \gamma^* (1 - \rho^*)]},
\]

which from (31) implies that both banks want to borrow and that a new link will be created, independently of the identities of the borrower and the lender. Therefore, if in the original network banks $i$ and $j$ do not share a link, with government intervention they will either continue not transacting or instead will create a link.

If in the original network banks $i$ and $j$ do have a loan agreement, then (34) and (69) imply that

\[
\frac{r_i^* - (r_i + r_j)}{\omega_i - e_i} \geq \frac{\rho - \rho^*}{\rho \rho^*}.
\]

Following intervention, either

\[
\frac{r_i^* - (r_i + r_j)}{\omega_i - e_i} \geq \frac{\rho - \rho^*}{\rho \rho^*} > \frac{[\rho + \gamma (1 - \rho)] - [\rho^* + \gamma^* (1 - \rho^*)]}{[\rho + \gamma (1 - \rho)] [\rho^* + \gamma^* (1 - \rho^*)]} \geq \frac{r_j^* - (r_j + r_i)}{\omega_j - e_j},
\]

which from (29) results in bank $i$ borrowing from bank $j$ and a link is kept in place, or

\[
\frac{r_i^* - (r_i + r_j)}{\omega_i - e_i} \geq \frac{\rho - \rho^*}{\rho \rho^*} > \frac{r_j^* - (r_j + r_i)}{\omega_j - e_j} \geq \frac{[\rho + \gamma (1 - \rho)] - [\rho^* + \gamma^* (1 - \rho^*)]}{[\rho + \gamma (1 - \rho)] [\rho^* + \gamma^* (1 - \rho^*)]},
\]

which from (31) means that both banks want to borrow and, no matter what the identities of borrower and lender, they engage in a transaction and a link remains.

Therefore, if in the original network banks $i$ and $j$ do not transact, with intervention they might create a link and, if they do transact, they keep transacting and a link keep existing across the banks. By the arbitrariness of banks $i$ and $j$ the result follows.

\[\square\]

### A.2 Proof of Proposition 2

Consider the original network with an even number $N$ of banks, being formed without government intervention. Take arbitrarily two banks, say $i$ and $j$, and assume without loss of generality that

46
\[
\frac{r^*_i - (r_i + r_j)}{\omega_i - e_i} > \frac{r^*_j - (r_j + r_i)}{\omega_j - e_j}. \tag{78}
\]

For the sake of the argument, assume also that \(i\) borrows from \(j\) in the original network, with \(\gamma^* = \gamma = 0\). That means, from (29) and (31), that either
\[
\frac{r^*_i - (r_i + r_j)}{\omega_i - e_i} \geq \frac{\rho - \rho^*}{\rho \rho^*} > \frac{r^*_j - (r_j + r_i)}{\omega_j - e_j}. \tag{79}
\]

holds or simultaneously that
\[
\frac{r^*_i - (r_i + r_j)}{\omega_i - e_i} > \frac{r^*_j - (r_j + r_i)}{\omega_j - e_j} \geq \frac{\rho - \rho^*}{\rho \rho^*}. \tag{80}
\]

and
\[
[r^*_i - (1 - \omega_i) - e_i - r_j] - [r^*_j - (1 - \omega_j) - e_j - r_i] > \frac{(\omega_i - e_i) - (\omega_j - e_j)}{\rho^*}. \tag{81}
\]

Consider first (79). From Lemma 7, one knows that, with government, \(\gamma^* \geq \gamma > 0\),
\[
\frac{\rho - \rho^*}{\rho \rho^*} > \frac{[\rho + \gamma (1 - \rho)] - [\rho^* + \gamma^* (1 - \rho^*)]}{[\rho + \gamma (1 - \rho)] [\rho^* + \gamma^* (1 - \rho^*)]]. \tag{82}
\]

Therefore, upon intervention it follows that either
\[
\frac{r^*_i - (r_i + r_j)}{\omega_i - e_i} \geq \frac{\rho - \rho^*}{\rho \rho^*} > \frac{[\rho + \gamma (1 - \rho)] - [\rho^* + \gamma^* (1 - \rho^*)]}{[\rho + \gamma (1 - \rho)] [\rho^* + \gamma^* (1 - \rho^*)]} > \frac{r^*_j - (r_j + r_i)}{\omega_j - e_j}. \tag{83}
\]

or
\[
\frac{r^*_i - (r_i + r_j)}{\omega_i - e_i} \geq \frac{\rho - \rho^*}{\rho \rho^*} > \frac{r^*_j - (r_j + r_i)}{\omega_j - e_j} \geq \frac{[\rho + \gamma (1 - \rho)] - [\rho^* + \gamma^* (1 - \rho^*)]}{[\rho + \gamma (1 - \rho)] [\rho^* + \gamma^* (1 - \rho^*)]} \tag{84}
\]

holds. If (83) is the case, from (29) it follows that bank \(i\) continues borrowing from \(j\). If (84), bank \(i\) will keep borrowing from bank \(j\) and not the converse as long as (32) is satisfied, i.e.,
\[
[r^*_i - (1 - \omega_i) - e_i - r_j] - [r^*_j - (1 - \omega_j) - e_j - r_i] > \frac{(\omega_i - e_i) - (\omega_j - e_j)}{\rho^* + \gamma^* (1 - \rho^*)}. \tag{85}
\]
However, one can show that, if

\[
\left(r_i^* - r_j^*\right) - \left(r_j^* - r_i^*\right) \left\{ \frac{\rho^* + \gamma^* (1 - \rho^*)}{1 - [\rho^* + \gamma^* (1 - \rho^*)]} \right\} < (\omega_i - e_i) - (\omega_j - e_j) < \left(r_i^* - r_j^*\right) \left(\frac{\rho \rho^*}{\rho - \rho^*}\right) \tag{86}
\]

then (78) is satisfied whereas (85) is not, i.e., without intervention bank \( i \) borrows from bank \( j \), and with government the opposite is true - bank \( i \) becomes the lender and bank \( j \) the borrower.

In the second case, i.e., if both (80) and (81) hold simultaneously, the fact that \( \rho^* < \rho^* + \gamma^* (1 - \rho^*) \) implies that

\[
\frac{(\omega_i - e_i) - (\omega_j - e_j)}{\rho^*} > \frac{(\omega_i - e_i) - (\omega_j - e_j)}{\rho^* + \gamma^* (1 - \rho^*)} \tag{87}
\]

and, therefore, (81) being satisfied automatically implies that (32) also is, and hence that with intervention bank \( i \) is kept as the borrower and bank \( j \) the lender.  \( \square \)

A.3 Proof of Corollary 3

As in the proof of Lemma 7, from (67) one has that

\[
\frac{[\rho + \gamma (1 - \rho)] - [\rho^* + \gamma (1 - \rho^*)]}{[\rho + \gamma (1 - \rho)][\rho^* + \gamma (1 - \rho^*)]} > \frac{[\rho + \gamma (1 - \rho)] - [\rho^* + \gamma^* (1 - \rho^*)]}{[\rho + \gamma (1 - \rho)][\rho^* + \gamma^* (1 - \rho^*)]} \tag{88}
\]

For \( \gamma^* > \gamma > 0 \), the right-hand side of the above inequality is the criteria for banks to choose between investing in large or small projects. Running through all possible scenarios after banks’ pairwise meetings - expressions (29), (30), (31) and (34) - one immediately sees that the incentives for both banks to invest in a large project - and hence borrow money - are large with the TBTF policy as opposed to the case where \( \gamma^* = \gamma > 0 \).  \( \square \)

A.4 Proof of Corollary 5

Denote by \( \pi^*_{ij} \) the profits of bank \( i \) with a large project when it borrows from bank \( j \) and by \( \pi_i \) with a small one, and analogously for bank \( j \). By adding a superscript \( G \) one has the same variables but for the case with government intervention. From Proposition 1, the number of links across banks do not decrease when a network is formed with intervention, compared to the one obtained without the government. From Proposition 2, however, if (84) holds,
\[
\frac{r^*_i - (r_i + r_j)}{\omega_i - e_i} \geq \frac{\rho - \rho^*}{\rho \rho^*} > \frac{r^*_j - (r_j + r_i)}{\omega_j - e_j} \geq \left[\frac{\rho + \gamma (1 - \rho)] - [\rho^* + \gamma^* (1 - \rho^*)]}{[\rho + \gamma (1 - \rho)] [\rho^* + \gamma^* (1 - \rho^*)]}\right] \tag{89}
\]

i.e., without the government bank \(i\) borrows from bank \(j\) and the last is actually better-off either lending or investing in a small project - \(\pi^*_ij > \pi_i\) and \(\pi_j > \pi^*_ji\) - but with intervention both banks prefer an investment in a large project - \(\pi^*_ij^G > \pi^*_i\) and \(\pi^*_ji^G > \pi^*_j\) - it might well be that \(\pi^*_ji^G > \pi^*_ij\),

\[
\left[\frac{r^*_i - (1 - \omega_i) - e_i - r_j}{\rho^* + \gamma^* (1 - \rho^*)} < \frac{(\omega_i - e_i) - (\omega_j - e_j)}{\rho^* + \gamma^* (1 - \rho^*)}\right], \tag{90}
\]

which from (33) implies that, with government, the role of banks is switched - bank \(i\) becomes the lender and bank \(j\) the borrower. Given that \(\rho^* + \gamma^* (1 - \rho^*) > \rho^*\), (90) implies that

\[
\left[\frac{r^*_i - (1 - \omega_i) - e_i - r_j}{\rho^* + \gamma^* (1 - \rho^*)} < \frac{(\omega_i - e_i) - (\omega_j - e_j)}{\rho^* + \gamma^* (1 - \rho^*)}\right], \tag{91}
\]

which in turn means that, without the government, the profit of bank \(j\) with a large project was also bigger than that realized by bank \(i\), or that \(\pi^*_ji > \pi^*_ij\). Therefore, without government one has that

\[
\pi_j > \pi^*_ji > \pi^*_ij > \pi_i, \tag{92}
\]

whereas with intervention

\[
\pi^*_ji^G > \max\{\pi^*_j, \pi^*_ij^G\} \quad \text{and} \quad \pi^*_ij^G > \pi^*_i. \tag{93}
\]

Therefore, in the particular cases of banks \(i\) and \(j\), networth increases with government intervention only if

\[
\pi^*_ji^G + \pi^*_i > \pi^*_ij + \pi_j. \tag{94}
\]

From \(\pi^*_ji^G > \pi^*_ij^G\) and \(\pi^*_ij^G > \pi^*_i\) one has that \(\pi^*_ji^G > \pi^*_ij^G\). If \(\pi^*_G > \pi\), then \(\pi^*_ij^G > \pi^*_ij^G > \pi^*_i\) leads to \(\pi^*_i > \pi^*_ij\) and, therefore, (94) is satisfied, i.e., intervention leads to a networth improvement - but not necessarily otherwise. From the arbitrariness of banks \(i\) and \(j\), thus, the result follows. \(\Box\)
A.5 Proof of Lemma 4

The matrix $B$ is given by:

$$B := \left( I - \tilde{X} \right) = \begin{bmatrix}
1 & -r_1(1-\alpha^1_1)\chi_1z & \cdots & -r_1(1-\alpha^1_1)\chi_1n \\
-r_2(1-\alpha^2_1)\chi_2z & 1 & \cdots & -r_2(1-\alpha^2_1)\chi_2n \\
\vdots & \vdots & \ddots & \vdots \\
-r_N(1-\alpha^N_1)\chi_Nz & -r_N(1-\alpha^N_1)\chi_Nn & \cdots & 1
\end{bmatrix}. $$

One needs to show that there are positive numbers $d_1, d_2, \ldots, d_n$ such that $d_j |b_{jj}| > \sum_{i \neq j} d_i |b_{ij}|$, for $j = 1, \ldots, n$, i.e.:

$$d_1 > \sum_{i \neq 1} d_i |b_{i1}| = \sum_{i \neq 1} d_i \frac{r_i (1 - \alpha^i_1) \chi_{i1}}{\sum_{k \in N} \chi_{ki} r_k},$$

$$d_2 > \sum_{i \neq 2} d_i |b_{i2}| = \sum_{i \neq 2} d_i \frac{r_i (1 - \alpha^i_2) \chi_{i2}}{\sum_{k \in N} \chi_{ki} r_k},$$

$$\vdots$$

$$d_N > \sum_{i \neq N} d_i |b_{iN}| = \sum_{i \neq N} d_i \frac{r_i (1 - \alpha^i_N) \chi_{iN}}{\sum_{k \in N} \chi_{ki} r_k}.$$

Suppose that $d_1 = d_2 = \ldots = d_N = d$. The above then becomes:

$$1 > \sum_{i \neq 1} \left(1 - \alpha^i_1\right) \frac{r_i \chi_{i1}}{\sum_{k \in N} \chi_{ki} r_k},$$

$$1 > \sum_{i \neq 2} \left(1 - \alpha^i_2\right) \frac{r_i \chi_{i2}}{\sum_{k \in N} \chi_{ki} r_k},$$

$$\vdots$$

$$1 > \sum_{i \neq N} \left(1 - \alpha^i_N\right) \frac{r_i \chi_{iN}}{\sum_{k \in N} \chi_{ki} r_k}.$$

For any $i$, one knows that $0 < \alpha^i_r < 1$ or, equivalently, $0 < (1 - \alpha^i_r) < 1$, which implies that:
\[
1 = \frac{\sum_{i \neq 1} r_i \chi_{i1}}{\sum_{k \in N} \chi_{k1} r_k} > \sum_{i \neq 1} \left( 1 - \alpha_i \right) \frac{r_i \chi_{i1}}{\sum_{k \in N} \chi_{k1} r_k},
\]
\[
1 = \frac{\sum_{i \neq 2} r_i \chi_{i2}}{\sum_{k \in N} \chi_{k2} r_k} > \sum_{i \neq 2} \left( 1 - \alpha_i \right) \frac{r_i \chi_{i2}}{\sum_{k \in N} \chi_{k2} r_k},
\]
\[
\vdots
\]
\[
1 = \frac{\sum_{i \neq N} r_i \chi_{iN}}{\sum_{k \in N} \chi_{kN} r_k} > \sum_{i \neq N} \left( 1 - \alpha_i \right) \frac{r_i \chi_{iN}}{\sum_{k \in N} \chi_{kN} r_k},
\]
as one wanted to show. □
References


