Measuring Long Run Risks for Brazil

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Abstract

We study the temporal structure of risk prices, risk exposures and expected market returns for Brazil assuming the economy follows a long run risk model. Hansen, Heaton, and Li (2008) show how to approximate the stochastic discount factor associated with the model around a log-linear process, where the term structure of expected returns can be obtained by vector autoregression methods. We apply these methods using aggregate consumption, dividends and the gross domestic product as endogenous variables. Consumption and dividends are decomposed using Beveridge and Nelson (1981) methods, where martingale components that dominates asymptotic risk premia are extracted. We also identify temporary and long run shocks to consumption using Blanchard and Quah (1989) identification scheme. Results indicate that consumption and dividends respond in opposite directions to temporary shocks in the short run, generating negative risk premium for temporary shocks. When the investment horizon is larger, risk prices for temporary shocks are essentially zero and the equity premium is dominated by risk exposure to the long run shock.

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1 Introduction

In recent years, a growing literature analyzes the temporal properties of the interaction between macroeconomic risk exposures and asset pricing, see e.g. Parker and Julliard (2005), Lettau and Wachter (2007) and Hansen and Scheinkman (2009). At the same time, long run risks models, as in Bansal and Yaron (2004), have been successful as a consumption based explanation for the high equity premium found in US data. In this paper, we combine these literatures and analyze the temporal asset pricing behavior of a long run risks model applied to Brazilian data. The model consists on an endowment economy where aggregate consumption and dividend growth contain predictable components and there is a representative agent with Epstein-Zin preferences. We follow Hansen, Heaton, and Li (2008) and approximate the stochastic discount factor around a log-linear process that can be recovered by preference parameters and a log-linear specification for consumption growth.

Although previous studies show that the standard CCAPM model is capable of explaining the equity premium in Brazil with reasonable relative risk aversion, see e.g. Bonomo (2002), it does not generate interesting results regarding risk exposure and price dynamics. The standard consumption model is also incapable of differentiating between short and long term macroeconomic shocks in the determination of the equity premium. Under Epstein-Zin preferences, if consumption growth is predictable then risk premium will depend on the temporal behavior of the responses of aggregate consumption and dividends to different shocks.

We estimate the long run risks model solution proposed by Hansen, Heaton, and Li (2008) using Brazilian data and a VAR composed by real per capita aggregate consumption, gross domestic product and aggregate dividends. Consumption variation is the main source of aggregate risks in this economy, determining, along the utility parameters, risk prices. We use the approximate dynamics of the stochastic discount factor to determine a temporal structure for risk prices that, combined with risk exposures, dictate expected returns.
To decompose the series into temporary and long run components, we use Beveridge and Nelson (1981) decompositions to isolate martingale components of consumption and dividends. The asymptotic excess return will depend only on the size of the martingale component in both series. The GDP series is used as an additional source of predictability for consumption and is necessary to identify long run shocks. The identification scheme for long run shocks follow Blanchard and Quah (1989), where one of the shocks is assumed to have no long run impact on the level of consumption. Exposures to this shock therefore have no pricing implication in the long run. The remaining shock is denominated long run risk shock, capturing innovations that affects permanently consumption and GDP. Results indicate that the response of dividends to both the long run and temporary shocks are stronger than the responses of consumption. Estimations also suggest that consumption and dividends respond in opposite directions to temporary shocks in the short run, decreasing the expected excess return obtained by the model. In our sample, the long run shock is the dominant force behind risk premia for dividend flows.

2 The Model

The model follows Hansen, Heaton, and Li (2008), a log-linear long run risk model where shocks are restricted to have deterministic volatility. Consumption growth has a predictable component, so current shocks to the state variable contains new information about future levels of consumption. The representative agent has Epstein-Zin recursive utility, so the stochastic discount factor will depend not only on innovations in consumption growth, but also on innovations regarding his entire future utility path, summarized by his continuation value, so the volatility of shocks affecting consumption in the long run is a major component of risk premium.

Estimating the stochastic discount factor associated with Epstein-Zin preferences is difficult because it depends on the realization of a value function not observed in data. The solution adopted by Hansen, Heaton, and Li (2008) is to approximate the value
function around a log-linear case, where it can be fully described by parameters of the utility function and the dynamics of the state variables. This log-linear framework will allow us to use VAR methods to estimate the model and identify long run shocks.

2.1 Stochastic Processes

Assume there’s a Markov process $x_t$ following a vector autoregression of the form

$$x_{t+1} = \mu_x + Ax_t + Bw_{t+1} \tag{1}$$

where $\{w_t\}$ is a sequence of independent and identically distributed normal variables such that $w_t \sim N(0,I)$. We assume that all eigenvalues of $A$ lie inside the unit circle to guarantee the stationarity of $x_t$. We can write $x_t$ as a first order autoregression without loss of generality, as any VAR(p) process can be transformed into a VAR(1) by appropriately redefining variables.

As will become clear, investor’s preferences in this economy generates concern about consumption risks in the short and long run, so dynamics of consumption is a central element for asset pricing. Consumption growth is modeled in a log-linear framework and assumed to be predictable by the state variable $x_t$. The logarithm of consumption growth $\Delta c_{t+1}$ satisfy

$$\Delta c_{t+1} = \mu_c + \Phi_c x_t + \Gamma_c w_{t+1} \tag{2}$$

The matrix $\Phi_c$ determines patterns of predictability and persistence of consumption, directly impacting investor’s long run risks concerns. To highlight the long run components of consumption, we use Beveridge and Nelson (1981) decomposition to write

$$c_t = \mu_c t + \pi_c \sum_{j=1}^{t} w_j + \Phi^*_c x_t \tag{3}$$

where $\mu_c$ is a time trend, $\pi_c \sum_{j=1}^{t} w_j$ is a martingale component and $\Phi^*_c x_t$ captures transi-
tory movements in consumption. Initial conditions for \( c_t \) does not have any implications in our results, so we set \( c_0 = 0 \) for simplicity. Parameters of this decomposition can be inferred from the dynamics in 2:

\[
\pi_c = \Gamma_c + \Phi_c (I - A)^{-1} B \\
\Phi^*_c = -\Phi_c (I - A)^{-1}
\]

When \( \Phi_c = 0 \), consumption follows a pure random walk process and shocks to consumption growth have only permanent effects. On the other side, the consumption process will just have transitory components if the long run effects of shocks, measured by \( \pi_c = \Gamma_c + \sum_{j=1}^{\infty} \Phi_c A^j B \), are zero. This condition will later be explored to identify shocks by assuming that one component of \( w_{t+1} \) has no long term effects on the level of consumption and income.

The logarithm of aggregate dividend growth \( \Delta d_{t+1} \) follows the same structure of consumption, satisfying

\[
\Delta d_{t+1} = \mu_d + \Phi_d x_t + \Gamma_d w_{t+1}
\]

We also use Beveridge and Nelson’s decomposition to write the dividend process as

\[
d_t = \mu_d t + \pi_d \sum_{j=1}^{t} w_j + \Phi^*_d x_t
\]

where parameters are defined analogously as their consumption counterparts.

### 2.2 Preferences

There is a representative agent with recursive preferences as in Epstein and Zin (1989) satisfying the constant elasticity of substitution (CES) recursion
\[ V_t = \left( (1 - \beta)C_t^{1-\rho} + \beta R_t (V_{t+1})^{1-\rho} \right)^{1/\rho} \]  

(8)

where \( \beta \) is the subjective discount factor and \( \rho \) is the reciprocal of the intertemporal elasticity of substitution (IES). The function \( R_t(\cdot) \) applies a risk correction to the continuation value \( V_{t+1} \) and also has a CES functional form

\[ R_t(V_{t+1}) = \mathbb{E} \left[ V_{t+1}^{1-\gamma} | \mathcal{F}_t \right]^{1/\gamma} \]  

(9)

where \( \mathcal{F}_t \) represents information available in period \( t \) and \( \gamma > 0 \) controls the risk aversion of the representative agent. The constant relative risk aversion (CRRA) family of utilities is the solution to a special case of equation 8 where \( \rho = \gamma \). The more general Epstein-Zin preferences implies that intertemporal elasticity of substitution and risk aversion can be determined separately, so amplifying the price of risk with a larger \( \gamma \) does not have the negative side effect of simultaneously raising the risk free rate.

We are interested in the asset pricing implications of the stochastic discount factor associated with these preferences, either in the short and long run. The stochastic discount factor is the ratio of marginal utility of consumption between time \( t + 1 \) and \( t \) and links the current price of an asset with its future dividend stream. Under the CES formulation of Epstein-Zin preferences, the stochastic discount factor \( S_{t,t+1} \) can be written as

\[ S_{t,t+1} = \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\rho} \left( \frac{V_{t+1}}{R_t(V_{t+1})} \right)^{\rho-\gamma} \]  

(10)

As we can see, not only consumption growth generates variations in the stochastic discount factor, but also variations of the value function with respect to its certainty equivalent, that is, innovations to all future stream of utilities impact \( S_{t,t+1} \). A persistent positive shock alters the prospect of future consumption, and consequently the stream of future utilities, so assets having a positive covariance with this shock will demand higher expected return when compared to the CRRA case.

In the log-linear environment we are using, it is easier to work with the logarithm of
the SDF, which we represent by $s_{t,t+1}$:

$$s_{t,t+1} = \log(\beta) - \gamma g_{t+1} + (\rho - \gamma) \left[ v_{t+1} - \frac{1}{1 - \gamma} \log \mathbb{E}_{t} \left[ \exp \{(1 - \gamma)(v_{t+1} + g_{t+1})\} \right] \right]$$ (11)

The usual problem with this stochastic discount factor is that we do not observe $V_{t+1}$, so it can be hard to estimate. One solution, explored by Epstein and Zin (1991), is to perform a transformation of the stochastic discount factor so that it depends on returns of the consumption claim asset, proxied by the market return. A caveat of this approach is that aggregate dividends and consumption do not behave similarly in data, being poorly correlated.

The solution used in Hansen, Heaton, and Li (2008) is to approximate the general stochastic discount factor around a simpler, log-linear one. If $\rho = 1$, the value function recursion 8 has the functional form

$$V_{t} = C_{t}^{1-\beta} \mathcal{R}_{t}(V_{t+1})^{\beta}$$ (12)

Defining $v_{t} = \log \left( \frac{V_{t}}{C_{t}} \right)$, we can write

$$v_{t} = \frac{\beta}{1 - \gamma} \log \mathbb{E}_{t} \left[ \exp \{(1 - \gamma)(v_{t+1} + g_{t+1})\} | \mathcal{F}_{t} \right]$$ (13)

The solution to 13 when $\rho = 1$ has the log-linear structure $v_{t+1} = \mu_{v} + U_{v} x_{t}$, where

$$\mu_{v} = \frac{\beta}{1 - \beta} \left( \mu_{c} + \frac{1 - \gamma}{2} |\Gamma_{c} + \Phi_{c} H|^{2} \right)$$ (14)

$$U_{v} = \beta U_{c}(I - \beta A)^{-1}$$ (15)

The cumulated response of a shock to consumption, discounted by the subjective discount factor $\beta$, is given by $U_{v}$, controlling the impact of innovations from the state variable $x_{t}$ to the value function. Plugging $v_{t+1}^{1}$ to equation 11, we find the solution
$s_{t,t+1}^{1} = \mu_{s} + \Phi_{s}x_{t} + \Gamma_{s}w_{t+1}$ to the stochastic discount factor in the special case when $\rho = 1$, where

$$
\begin{align}
\mu_{s} &= \log \beta - \mu_{c} - \frac{(1 - \gamma)^2}{2}|\Gamma_{c} + U_{v}B|^2 \\
\Phi_{s} &= -\Phi_{c} \\
\Gamma_{s} &= -\gamma\Gamma_{c} + (1 - \gamma)U_{v}B
\end{align}
$$

The solution proposed in Hansen, Heaton, Lee, and Roussanov (2007) is to approximate the logarithm of the stochastic discount factor $s_{t,t+1}$ around $\rho = 1$. We can write

$$s_{t,t+1} \approx s_{1,t,t+1}^{1} + (\rho - 1)Ds_{1,t,t+1}^{1}$$

where $Ds_{1,t,t+1}^{1}$ is the derivative of the stochastic discount factor process with respect to $\rho$ evaluated at 1. We can compute $s_{1,t,t+1}^{1}$ and $Ds_{1,t,t+1}^{1}$ using parameters of the consumption process and the state variable. $Ds_{1,t,t+1}^{1}$ is a complicated quadratic function of $w_{t+1}$ and $x_{t}$ so we refer to Hansen, Heaton, Lee, and Roussanov (2007) for computations and formulas for the derivative process.

### 2.3 Pricing

We want to study the temporal pricing properties of the stochastic discount factor derived from the long run risk model, where parameters of the underlying stochastic processes are estimated using Brazilian data. To understand the implications of the model for asset pricing, define the process $D_{t}^{*}$ representing the long term component of dividends in 7:

$$D_{t}^{*} = \exp \left( \mu_{d}t + \pi_{d} \sum_{j=1}^{t} w_{j} \right)$$

It will be useful, to construct a recursive formulation for pricing, to work with dividends $D_{t}$ composed as the product of the long run component $D_{t}^{*}$ and a transitory term.
exp (\(\phi_0 + \phi_1 x_t\)), forming a stochastic process similar to 7:

\[ D_t = D_t^* \exp(\phi_0 + \phi_1 x_t) \tag{21} \]

To compute asset pricing behavior through time, we follow Hansen, Heaton, and Li (2008) and construct the auxiliary operator \(P(\cdot)\) to price dividend streams. This operator is determined by the stochastic discount factor \(s_{1,t,t+1}^1\) and the long run terms of the dividend process 20. For a given transitory component \(\exp(\phi_0 + \phi_1 x)\), the operator yields another function \(P(\exp(\phi_0 + \phi_1 x))\) such that

\[
P(\exp(\phi_0 + \phi_1 x)) = \mathbb{E}[\exp(s_{t,t+1} + \mu_d + \pi_d w_{t+1} + \phi_0 + \phi_1 x_{t+1}) | x_t = x] \tag{22}
\]

\[
= \exp \left( \mu_s + \mu_d + \phi_0 + \phi_1 \mu_x + \frac{|\Gamma_s + \pi + \phi_1 B|^2}{2} + (\Phi_s + \phi_1 A)x \right) \tag{23}
\]

\[
= \exp (\phi_0^* + \phi_1^* x) \tag{24}
\]

where

\[
\phi_0^* = \mu_s + \mu_d + \phi_0 + \phi_1 \mu_x + \frac{|\Gamma_s + \pi + \phi_1 B|^2}{2} \tag{25}
\]

\[
\phi_1^* = \Phi_s + \phi_1 A \tag{26}
\]

We can compute the price in \(t\) of a dividend paid in \(t+1\) satisfying 21 using the formula

\[
\mathbb{E}[\exp(s_{t,t+1})D_{t+1} | F_t] = D_t^* P(\exp(\phi_0 + \phi_1 x_t)) \tag{27}
\]

The pricing operator \(P(\cdot)\) offers an easy way to calculate time \(t\) prices of a dividend paid in any subsequent period \(t+j\) by iteration. To see how the iteration can be computed,
define $S_{t,t+j} = \prod_{i=1}^{j} S_{t+i-1,t+i}$ and write

$$
\mathbb{E}[S_{t,t+j}D_{t+j}|F_t] = D_t^* \mathbb{E} \left[ \exp \left( \sum_{i=1}^{j} (s_{t+i-1,t+i} + \pi_d w_{t+i}) + \mu_d j + \phi_0 + \phi_1 x_{t+j} \right) \bigg| x_t = x \right]
$$

$$
= D_t^* \mathcal{P}^j(\exp(\phi_0 + \phi_1 x_t))
$$

$$
= D_t^* \exp(\phi_0^{(j)} + \phi_1^{(j)})
$$

where $(\phi_0^{(j)}, \phi_1^{(j)})$ can be computed iterating $j$ times equations 25 and 26. The coefficient $\phi_1^{(j)}$ converges to

$$
\bar{\phi}_1 = \Phi_s (I - A)^{-1}
$$

The term $\phi_0^{(j)}$ does not converge in general, it will diverge to $-\infty$ using our estimated parameters, but the difference $\phi_0^{(j)} - \phi_0^{(j-1)}$ converges to

$$
-\nu = \mu_s + \mu_d + \bar{\phi}_1 \mu_x + \frac{|\Gamma_s + \pi + \bar{\phi}_1 B|^2}{2}
$$

Before computing the temporal behavior of returns, let $\mathcal{G}(\cdot)$ represent the cash flow growth operator, a variation of 22 where

$$
\mathcal{G}(\exp(\phi_0 + \phi_1 x)) = \mathbb{E} \left[ \exp (\mu_d + \pi_d w_{t+1} + \phi_0 + \phi_1 x_{t+1}) \big| x_t = x \right]
$$

We can use the operator 33 to write the conditional expected value dividend $\mathbb{E}[D_{t+j}|F_t]$ as the product of $D_t^*$ and the growth operator iterated $j$ times:

$$
\mathbb{E}[D_{t+j}|F_t] = D_t^* \mathcal{P}^j(\exp(\phi_0 + \phi_1 x_t))
$$
The limiting growth in $G(\cdot)$ is given by

$$
\eta = \mu_d + \frac{\pi_d \cdot \pi_d}{2} \tag{35}
$$

The new notation can be used to write the temporal behavior of returns in a simple way. Let $R_{t,t+j}$ represent the return of holding a claim for $D_{t+j}$ from $t$ to $t+j$, whose expected value satisfies

$$
\mathbb{E}[R_{t,t+j}] = \frac{\mathbb{E}[D_{t+j}]}{\mathbb{E}[S_{t,t+j}D_{t+j}]} \tag{36}
$$

$$
= \frac{G^j[\exp(\phi_0 + \phi_1 x_t)]}{P^j[\exp(\phi_0 + \phi_1 x_t)]} \tag{37}
$$

It is useful to scale by the horizon $j$ and take logarithms so we can compare the temporal structure of expected returns:

$$
\log \left( \mathbb{E}[R_{t,t+j}]^{1/j} \right) = \frac{\log G^j[\exp(\phi_0 + \phi_1 x_t)] - \log P^j[\exp(\phi_0 + \phi_1 x_t)]}{j} \tag{38}
$$

By taking limits in 38, we can compute the long run expected excess return $\varrho$ for holding an asset paying $D_{t+j}$. The long run return is the difference between two components: the asymptotic cash flow growth $\eta$ and the asymptotic decay of pricing $\nu$:

$$
\varrho = \varsigma^* + \pi^* \cdot \pi_d \tag{39}
$$

where

$$
\varsigma^* = -\xi_0 - \Phi_s(I - A)^{-1}B \tag{40}
$$

$$
\pi^* = -\mu_s - \frac{|\pi^* \cdot \pi^*|^2}{2} \tag{41}
$$

The coefficient $\varsigma^*$ is the long run asymptotic risk free rate, as it does not depend on
the cash flow exposure to risks. The term $\pi^* \cdot \pi_d$ measures the expected long run returns, where $\pi_d$ is the exposure to long run risks and $\pi^*$ its corresponding price.

3 Data Description

We use IBGE data to obtain quarterly and annual series for Brazilian aggregate consumption and gross domestic product (GDP) from 1987 to 2015. GDP is used as a proxy for income as the disposable income series is not available after 2010. All series were deflated using IPCA, a price index consisting mainly of consumption goods in Brazil. Annual population data from 1987 to 2013 is also taken from IBGE, and linear methods were used to extrapolate the series to 2014-2015 since spline procedures generate unreliable near zero population growth for these years. To obtain quarterly data for population, we interpolated the annual series using a cubic spline.

Financial data was obtained using the Economatica software, taking real dividends from 1988 to 2015 of all firms listed in IBOVESPA at least once during the sample period. In each year, firms that were not traded at the end of the period were excluded from the sample. We generate a series for aggregate dividends by summing all dividends paid by firms belonging to the sample in each year. We chose to work with annual data, despite the limited size of the sample, because quarterly dividend payments in Brazil do not have a clear seasonal pattern through the years, as shown in figure 1. Consumption and dividend growth have low correlation in the quarterly sample even after controlling for seasonality, in fact they have a small negative correlation.

Descriptive statistics for each series are presented in table 1. We annualized the statistics to make comparison easier. As we can see, consumption and GDP behave similarly between the two frequencies, in part due to the seasonal treatment by IBGE that attempts to connect quarterly and annual growth. Dividend growth, though, differ specially in standard deviations, as firms usually pay dividends only twice a year, and zero in the remaining quarters, generating high volatility. As we have argued, this pattern
cannot be changed by seasonal treatment of the series.

Table 1: Sample Moments

<table>
<thead>
<tr>
<th>Series</th>
<th>Mean</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Quarterly data</td>
</tr>
<tr>
<td>Consumption growth</td>
<td>2.79%</td>
<td>2.71%</td>
</tr>
<tr>
<td>GDP growth</td>
<td>2.47%</td>
<td>2.56%</td>
</tr>
<tr>
<td>Dividend growth</td>
<td>8.79%</td>
<td>77.97%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Annual data</td>
</tr>
<tr>
<td>Consumption growth</td>
<td>2.78%</td>
<td>2.24%</td>
</tr>
<tr>
<td>GDP growth</td>
<td>2.5%</td>
<td>2.52%</td>
</tr>
<tr>
<td>Dividend growth</td>
<td>8.4%</td>
<td>22.49%</td>
</tr>
</tbody>
</table>

We first investigate some of the statistical properties of the series of consumption, GDP and dividend before specifying a VAR model for them. Figure 2 plots log real consumption and GDP per capita from 1996 to 2015, suggesting that none of the series is stationary but they may share a common stochastic trend, being cointegrated. To investigate those conjectures, we first test if each series has a unit root, and then test the hypothesis of cointegration for consumption and GDP. The permanent income hypothesis, see e.g. Campbell (1987), implies that consumption and income should be cointegrated with $(1, -1)$ as the cointegrating vector, so we test if this is true in the data we collected,
using GDP as a proxy for income.

![Figure 2: Consumption and GDP](image)

We apply the Augmented Dickey Fuller (ADF) test for unit root in the processes of consumption, GDP and aggregate dividends separately. Given a series \( \{x_t\} \), the test consist in regressing

\[
\Delta x_t = \alpha + \delta t + \rho x_{t-1} + \beta_1 \Delta x_{t-1} + \ldots + \beta_p \Delta x_{t-p} + v_t
\]  

(42)

and testing the null hypothesis of \( \rho = 0 \). If we cannot reject the null, then \( \{x_t\} \) has a unit root. We choose the number of lags in each series using Schwarz information criterion. Table 2 presents results for the ADF test under two specifications of 42, one including the time trend, and the other with the restriction \( \delta = 0 \). The results show that we cannot reject at 5\% level that consumption, GDP and aggregate dividends have a unit root, either for the quarterly and the annual series.

To investigate if consumption and GDP are cointegrated, we use the Johansen procedure, Johansen (1991), to test an unrestricted specification for cointegration and also if consumption and GDP have the cointegration vector \((1, -1)\). Results in table 3 indicate that we cannot reject the hypothesis that GDP and consumption are cointegrated at 5\% of significance, for both frequencies. When we restrict the cointegrating vector to be \((1, -1)\), we also cannot reject that the two series are cointegrated at 5\% using quarterly
Table 2: Testing for Unit Riots

<table>
<thead>
<tr>
<th>Series</th>
<th>ADF test without time trend</th>
<th></th>
<th>ADF test with a time trend</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Quarterly data</td>
<td>Annual data</td>
<td>Quarterly data</td>
<td>Annual data</td>
</tr>
<tr>
<td></td>
<td>ADF Statistics</td>
<td>p-value</td>
<td>ADF Statistics</td>
<td>p-value</td>
</tr>
<tr>
<td>Consumption</td>
<td>-0.29</td>
<td>0.92</td>
<td>-0.87</td>
<td>0.78</td>
</tr>
<tr>
<td>GDP</td>
<td>-0.91</td>
<td>0.77</td>
<td>-0.83</td>
<td>0.79</td>
</tr>
<tr>
<td>Dividend</td>
<td>-2.03</td>
<td>0.27</td>
<td>-0.92</td>
<td>0.76</td>
</tr>
</tbody>
</table>

Table 3: Cointegration Tests

<table>
<thead>
<tr>
<th>H₀</th>
<th>Unrestricted Cointegration Test</th>
<th></th>
<th>Restricted Cointegration Test</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Quarterly data</td>
<td>Annual data</td>
<td>Quarterly data</td>
<td>Annual data</td>
</tr>
<tr>
<td></td>
<td>Trace Statistics</td>
<td>p-value</td>
<td>Trace Statistics</td>
<td>p-value</td>
</tr>
<tr>
<td>At most one</td>
<td>13.30</td>
<td>0.11</td>
<td>3.45</td>
<td>0.06</td>
</tr>
<tr>
<td>cointegrating relation</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Cointegration vector (1, -1)</td>
<td>1.44</td>
<td>4.72</td>
<td>0.03</td>
</tr>
</tbody>
</table>

frequency and 1% for annual data.

4 Empirical Results

Having analyzed the stochastic relations between consumption, GDP and dividends, we specify dynamic models to identify shocks. We start by analyzing dynamics for \((c_t, y_t)\) and studying its implications for the long run risks model. Aggregate dividends does not help to predict consumption and GDP in our samples, so we can exclude the dividends series for now, simplifying the analysis without significant impact on estimated parameters. To
estimate the dynamics of consumption and GDP, we explore the cointegration between
the two series and use the triangular representation to specify a VAR on \( z_t = (\Delta c_t, c_t - y_t) \):

\[
z_t = \mu + \Phi_1 z_{t-1} + \ldots + \Phi_p z_{t-p} + u_t
\]

(43)

where the matrices \( \Pi \) and \( \Phi_i \) are \( 2 \times 2 \), for \( i = 1, \ldots, p \), \( \mu \) is a 2-dimensional vector and \( u_{t+1} \)
has zero mean and covariance matrix \( \Omega \). The number of lags is once again chosen according
to Schwarz and Akaike information criteria, where preference is given for specifications
with higher number of lags, so the size of shocks are not unnecessarily enlarged and we
can avoid autocorrelation of residuals.

The covariance matrix of residuals \( \Omega \) in (43) does not have economic meaning, specially
since residuals are correlated in our sample. We are interested in identifying shocks that
have long run effects on consumption, and consequently on GDP since we assume coin-te-
gration, so we apply Blanchard and Quah (1989) decomposition to \( \Omega \). The identification
consists in assuming that structural shocks \( w_{t+1} = (w_{t+1}^L, w_{t+1}^S) \) are uncorrelated and only
one of them, \( w_{t+1}^L \), has permanent effects on the consumption level.

To see how the identification scheme works, note that the structural and reduced form
shocks are related by the equation \( u_{t+1} = \Gamma w_{t+1} \), so it must be true that

\[
\Omega = \Gamma \Gamma'
\]

(44)

The structural VAR, which we assume to be stationary, can be written as

\[
z_t = \mu_z + \Psi(L) w_t
\]

(45)

where \( L \) is the usual lag operator, \( \mu_z \) is the unconditional mean of \( z_t \) and the operator
\( \Psi(L) \) satisfies \( \sum_{j=0}^{\infty} \Psi_j L^j = (I - \Phi_1 L - \ldots - \Phi_p L^p)^{-1} \Gamma \). Since the first coordinate of \( z_t \)
is consumption growth, the long run effect of \( w_{t+1} \) to the level of consumption is the
sum of the impulse responses, represented by the first line of the matrix \( \sum_{j=0}^{\infty} \Psi_j =
(I - \Phi_1 - \ldots - \Phi_p)^{-1} \Gamma \). Our assumption that \( w_{t+1}^S \) has no long effects on the consumption
level is equivalent to

$$\sum_{j=0}^{\infty} \Psi_{12}^{j} = 0$$  \hspace{1cm} (46)$$

Restriction 44 imposes three restrictions on $\Gamma$ and condition 46 imposes a fourth one, so $\Gamma$ is fully identified. Although both shocks affect consumption in the short run, the effects of $w_{t+1}^{S}$ dissipates as the time length becomes larger.

Figure 3 shows the impulse response function on the level of consumption for both short and long run shocks, where the response for the temporary shock was multiplied by -1 to facilitate comparison. In the short run, $w_{t+1}^{S}$ has a larger effect on consumption, where $w_{t+1}^{L}$ has an almost negligible effect. The short term effect of $w_{t+1}^{L}$ is actually smaller than zero, but this may be the result of uncertainties related to estimation. If the representative agent model with standard utility function is used, one period returns will be driven mostly by the exposure of payoffs to $w_{t+1}^{S}$. Under Epstein-Zin preferences, though, both shocks affects pricing as the full effect of shocks to consumption, discounted by $\beta$, affects risk prices.

Long run consumption growth, given by $\mu_{c} + \pi_{c}^{'}/2$, is 2.62%, a value close to its short run counterpart. This fact can be explained by the small long run impact of shocks. One
Table 4: Long Run Consumption Claim Prices

<table>
<thead>
<tr>
<th>Relative Risk Aversion</th>
<th>$\pi^*$</th>
<th>Excess Return</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Quarterly data</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.0983</td>
<td>0.21%</td>
</tr>
<tr>
<td>10</td>
<td>0.1943</td>
<td>0.42%</td>
</tr>
<tr>
<td>20</td>
<td>0.3862</td>
<td>0.83%</td>
</tr>
<tr>
<td><strong>Annual data</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.1724</td>
<td>0.6%</td>
</tr>
<tr>
<td>10</td>
<td>0.3442</td>
<td>1.21%</td>
</tr>
<tr>
<td>20</td>
<td>0.6877</td>
<td>2.41%</td>
</tr>
</tbody>
</table>

shock has no long run effects on consumption by definition, but $w_{t+1}^L$ is not large even in
the long run, around 2.1%, so $\frac{\pi^* c}{2} \approx 0.00022$.

Suppose we are interested in pricing the consumption claim, a common assumption in
models about the equity premium puzzle. As we know, the estimated long run exposures
to risks are given by $\pi_c = (0.021, 0)$. Since there are no exposures of consumption to $w_{t+1}^S$
in the long run, it has no impact on the asymptotic excess return. The impact of $w_{t+1}^L$ to
the long run return of a consumption claim when $\rho = 1$, shown in table 4, is computed
using the long risk prices vector $\pi^*$. Since the long run exposure to $w_{t+1}^S$ is zero, we only
report results for $w_{t+1}^L$. The result shows that the asymptotic return on the consumption
claim asset is lower than 1%, even for large coefficients of risk aversion.

We also estimated the model using the annual series for consumption and GDP, keeping
in mind that uncertainty is amplified given the limitations of the sample size. The
estimated exposure of consumption to the long run shock is larger, the accumulated effect
of a shock to $w_{t+1}^L$ is 3.5%, so both the quantity and price of the consumption claim in
the long run are amplified when we compare to the quarterly series. Even in this case,
though, the asymptotic excess return is still low. If we set an upper bound of 10 to the
relative risk aversion, the maximum long run excess return for the consumption claim is
1.21%.

To estimate the long run implications of the model for aggregate dividends in Brazil,
we focused on the the annual frequency, as the lack of seasonal patterns for quarterly data,
Table 5: Granger Causality Test

<table>
<thead>
<tr>
<th>$H_0$</th>
<th>Observations</th>
<th>F Statistics</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta d_{t+1}$ does not Granger cause $\Delta c_{t+1}$</td>
<td>25</td>
<td>0.13</td>
<td>0.71</td>
</tr>
<tr>
<td>$\Delta d_{t+1}$ does not Granger cause $y_{t+1} - c_{t+1}$</td>
<td>25</td>
<td>0.65</td>
<td>0.42</td>
</tr>
</tbody>
</table>

set aside the negative correlation between consumption and dividend growth, generates unreliable results. We add consumption growth to the VAR system 43, so now $z_t = (\Delta c_{t+1}, y_{t} - c_{t}, \Delta d_{t+1})$. In table 5 we show results for a test of Granger causality of dividend growth on both $\Delta c_{t+1}$ and $y_{t} - c_{t}$. The results indicates that we cannot reject the hypothesis that $\Delta d_{t+1}$ does not Granger cause $(\Delta c_{t+1}, y_{t} - c_{t})$, so we are able to estimate the VAR system in blocks and the estimated coefficients of 43 for $(\Delta c_{t}, y_{t} - c_{t})$ are still valid. Both Schwarz and Akaike model selection criteria determined one lag for the VAR dynamics.

To identify shocks under the new system, note that the additional shock, which will be called dividend shock $w^D_{t+1}$, have no impact on $(\Delta c_{t}, y_{t} - c_{t})$ if $\Delta d_{t}$ does not Granger cause them, so the correspondent elements in $\Gamma$ are equal to zero. The additional components of the new matrix $\Gamma$ are identified by the condition $\Gamma \Gamma' = \Omega$.

Figure 4 shows the impulse response function for consumption and dividends to the long run and temporary shocks. As we can see, consumption and aggregate dividend respond similarly to a long run shock, but cash flow is more sensitive to shocks, representing about four times the response of consumption in the short and long run. The behavior of responses to the temporary shock, though, is divergent. First, note that we do not make any assumption linking the long run behavior of dividends and consumption, so the accumulated response of cash flows to a temporary shock need not be equal to zero. We can also see that the two series have opposite responses to the temporary shock, so the risk premium for this shock is negative. It does not mean that the model generates negative equity premium when estimated using annual data because consumption is more exposed to $w^L_{t+1}$, so risk prices are higher for the long run shock.

The temporal pattern of risk premia is shown in figure 5. Excess returns are large in
the short run because, although the exposure of dividends to both temporary and long run shocks is high, exposure of consumption to the long run shock is greater than to the temporary shock, that has a negative price. In the long run, risk prices for temporary shocks are zero and the risk premium is dominated by the large long run shock.
5 Conclusion

We estimate a long run risk model for Brazil applying VAR methods to a log-linear approximation to the stochastic discount factor. To perform the estimation, we use aggregate consumption, aggregate dividends and the gross domestic product in quarterly and annual frequencies. After estimating the model, we are able to compute a temporal structure for risk prices and exposures that determines expected excess returns. We also identify long run shocks affecting consumption and GDP that dominates pricing in the long run.

Our results indicate that consumption and dividends respond in opposite directions to temporary shocks in the short run, having a decreasing effect in the overall cash flow risk premium. The temporary shock has a lasting, though small, impact on dividends but does not receive risk premia compensations, so expected excess returns grow with time as exposures to the long run shock dominates dividends.

References


