The Role of Commitment in the Financing of Status Goods Production∗

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April 10, 2017

Abstract

We develop a dynamic model of a horizontally differentiated product duopoly with the purpose to investigate the relationship between the market niche each firm targets, the social status of its brand, and the technology each firm adopts. We provide conditions under which a firm is able to credibly offer a high status goods. We extend the model to include a firm’s financing decisions. The presence of financial markets per se does not fundamentally alter the equilibrium of the original game. We further extend the model to include the possibility of commitment to private financing. A firm that cannot credibly supply status goods in the original game might be able to do it in the game with commitment, because the discount factor required to implement the equilibrium with commitment is smaller than in the game without commitment. The present paper provides a novel link between financial and capital markets: concerns with the social status of a firm’s products.

Keywords: brand, competition, commitment, status, capital structure.

JEL classification: G32, C78, L13, L15.

∗We thank the suggestions and helpful comments of Costas Azariadis, Patrick Beissner, Ben Chen, Yijuan Chen, Rafael da Matta, Simona Fabrizi, Sander Heinsalu, Marc Hofstetter, George Mailath, Wilfredo Maldonado, Stephen Martin, Miguel Martínez, Mieszko Mazur, Lars Norden, Jaime Orrillo, Jorge Streb, James Taylor, participants at the 16th SAET Conference on Current Trends in Economics, the 2016 Meeting of the Association for Public Economic Theory, the 2016 LACEA-LAMES, the First Asia-Pacific Industrial Organization Conference, the 2016 Australasian Economic Theory Workshop and the 2016 Luso-Brazilian Finance Network annual meeting, and seminar participants at the Australian National University, University of Brasília, and Catholic University of Brasília. All remaining errors are our own responsibility. Rogério Mazali would like to thank CNPq and the Research School of Economics at ANU for partial financial support.

†This paper has previously circulated under the title “Social Status, Reputation, Financing, and Commitment”.

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1 Introduction

In the last decades, many economists have dedicated their efforts to understand the market for status or luxury goods. Unlike regular goods, these goods might have little or no intrinsic value, in the sense that consuming them provides little or no direct utility. In the terminology of Pollak (1967, 1976), Koopmans (1960), among others, these goods provide relatively low “cardinal utility”. The main component of the value these goods provide come from their “ordinal utility”, that is, the utility given by the position the consumer of a status good occupies in a rank. Because of this feature, Frank (1985, 2005) refers to these goods as positional goods.

Most firms that produce high-end status goods are private. Why is it the case? By retaining private status, these firms choose to give up valuable financing sources for their projects. Hence, these firms could be forced to pass on positive net present value (NPV) projects due to financial constraints. We argue that producers of high status goods might have something to gain out of retaining private status, namely, credibility for the social status their products. The perceived status level of a good is closely related to the exclusivity of its consumption. A high status good producer has to commit to producing few units of its good in order to charge a high price for the good’s status value. Once a particular good has acquired high-status with its customers, its producer has incentives to deceive consumers and sell additional units to consumers of lower social strata. The producer might have to switch technologies and acquire industrial machinery in order to mass-produce its good. If high social strata consumers anticipate that firms will do that, they will never pay such a high price for the firm’s good in the first place. As a result, there will be no market for status goods.

We develop a dynamic stylized model of a horizontally differentiated product duopoly. The model investigates the relationship between the market niche each firm targets and the social status of its brand. In the basic model, two duopolistic firms offer a horizontally differentiated product that can yield their consumers extra utility from social status. Consumers use firms’ technology decisions as a signal of the social status that the good each firm produces will have. In our model, firms produce a pure status good that gives no direct utility to its consumers. Facing monopolistic competition, firms choose the price of their goods. Prices can include a status premium for the status utility they give to their consumers, if these consumers attribute status value to the good. Firms choose the technology with which they produce their goods (handicraft or industrial manufacturing). We provide conditions under which there is an equilibrium of the dynamic game in which one firm produces high status goods using a handicraft technology while the other adopts an industrial manufacturing technology. Such strategy profile is implementable as a Subgame Perfect Equilibrium if the fixed costs of the acquisition of industrial machinery are sufficiently large, if the marginal costs of producing with the handicraft technology are sufficiently
small, and if the common discount factor is sufficiently large. Under suitable assumptions, the firm adopting the industrial technology obtains a larger market share than its rival, which adopts the handicraft technology and sells its good to a smaller market niche at a higher price.

Whenever the high status producer deviates from its prescribed actions, its product loses its perceived status. This loss of social status works as a punishment carried out by consumers. If this punishment is able to deter a deviation by the status good producer, then the producer of the “no status” good can increase its profit by “softening punishment”. In this case, the pivotal condition to implement the equilibrium turns out to be incentive conditions for a firm deviating from handicraft production of a status good to return to the equilibrium path. We extend the model to include financing decisions and show that the presence of financial markets per se does not fundamentally alter the equilibrium of the original game. As a matter of fact, we show that the classical Modigliani and Miller (1958) result holds for all public firms in the game with financial markets.

One important feature of the equilibrium we present is that, despite the fact that the loss of status imposes a penalty on the firm producing the status good in case of deviation, we are not making use of the Folk Theorem in any of its forms. In our model, a stage game action has two dimensions: price and technology choice. When current period technology choices induce a change in consumers’ perceptions of status, the following period stage game is fundamentally different from its predecessor. Because of this feature, we cannot apply the classical Folk Theorem result that implies that, with simple punishment strategies such as those described in Abreu (1986, 1988) or in Lambson (1987, 1994, 1995), any payoff above minmax payoffs is attainable in a Subgame Perfect Equilibrium. The equilibrium we describe make use of strategies such that, for a fixed pair of technology choices by the two firms, each firm chooses a price that is a one-period best response to the price chosen by the other firm.

We further extend the model to allow firms to commit to a particular financing policy. More precisely, in the case that the above strategy profile is not a Subgame Perfect Equilibrium, firms can commit to producing a high status product by restricting its ability to adopt an industrial, mass-production technology. We show that in many cases firms that produce high status goods will purposely adopt more restrictive financing policies, such as retaining private status. Retaining private status works as a commitment device that provides the high status product firm the credibility it needs to convince customers and rivals that it will keep producing high status products and not deviate to obtain short-term gains by “invading” its rival’s market niche. Although for a firm that seeks to produce high status goods, its technology decision affects its financing, for a firm producing using the industrial technology, financing decisions are irrelevant, and the classical Modigliani and Miller (1958) result holds. Following a stream of literature that started with Titman (1984) and Brander and Lewis (1986), our paper provides
a reason why product and capital markets are linked and affect one another through concerns about a product’s perceived status social.

We assume the social status component of the utility of consumers is given by the exclusivity of its consumption. In particular, we assume the status value of the good is inversely proportional to the number of individuals consuming it, and that only the most exclusive good has this extra status utility component. Even if consumers do not care directly about the number of individuals consuming a particular good, matching concerns might give rise to behavior that looks “as if” consumers cared about how exclusive their goods are. For example, Mazali and Rodrigues-Neto (2013) and Koford and Tschoegl (1998) develop models that generate equilibria in which consumers care *ex post* about the rarity or exclusivity of its consumption. One way or another, this status component is only learned through experience. We make assumptions regarding consumers’ beliefs regarding the social status of a particular brand that are, in essence, equivalent to assuming: (i) that the status component of consumption is a pure experience good, as defined in Nelson (1970), and analyzed by Shapiro (1983a) and Villas-Boas (2004, 2006), among others; (ii) that consumers update their beliefs about the true status value of the good through social learning instead of individual learning, as in Bergemann and Välimäki (2006).

Most firms that produce high-end status goods are private. For example, most watchmakers of high status, such as Cartier, and Rolex, are private companies, as well as fashion giants such as Armani, Versace, and jewelers H. Stern and Swarovski. Of the world’s top ten jeweler companies according to Luxe High Life’s catalogue, only one (Tiffany & Co.) is publicly traded. Another is a subsidiary in a public conglomerate (Harry Winston, part of the Swatch Group), and the other eight are all privately held companies (Cartier, Van Cleef & Arpels, Piaget, Bvlgari, Mikimoto, Graff, Buccelati and Chopard).¹ Most of the supersport car manufacturers are completely private, 100% independent endeavors, or a part of a conglomerate with significant restrictions on access to funds from other companies in the conglomerate. McLaren, Aston Martin, W Motors and Koenigsegg are in the first group while Bugatti and Lamborghini are in the second. Out of the top 10 world’s most expensive cars of 2015, according to Digital Trends², six are manufactured by private independent companies (Zenvo ST1, Pagani Huayra, Aston Martin One-77, Koenigsegg One:1, W Motors Lycan Hypersport, and Koenigsegg CCXR Trevita), while three are manufactured by private firms within public conglomerates (Mansory Vivere, Bugatti Veyron, and Lamborghini Veneno). Only two, both manufactured by Ferrari N.V., are manufactured by a public company (Ferrari LaFerrari and Ferrari F60 America). Nevertheless, Ferrari has traded as a private company for most of its life, only announcing its intention to go public in 2015.³ The

decision of Ferrari going public is part of a spin-off operation that dismembered Ferrari from the Fiat-Chrysler Automobiles conglomerate. An article published in Fortune magazine in January 2016 seems to imply that being part of the conglomerate was not beneficial for Ferrari in any way and that Ferrari was not able to obtain financing from the conglomerate, given the high leverage of the conglomerate.\(^4\) Other high-end status goods that are part of conglomerates have restrictions on obtaining financing from parent companies. Lamborghini S.p.A, for example, was acquired by the Volkswagen group in 2011. Instead of maintaining it under direct control of Volkswagen, the group’s management decided to keep it under control of one of its higher luxury subsidiaries, Audi A.G.. This ownership structure made it very difficult for Lamborghini to obtain financing from Volkswagen, because it would need approval from the Audi management as well. These restrictions in internal financing make many of these high-status car makers that are part of large conglomerates behave as if they are completely independent private companies with respect with financing.

The production of status goods also seems to be related to the technology used to produce them. Indeed, many of the high-end status goods are almost completely handmade. Watches, haute couture, jewelry, oriental tapestry are examples of industries that are notoriously well known for the handicraft of the goods they produce. Even the high-tech supercar factories have many “hands on the job” procedures in assembling one of their expensive car models. Television documentaries produced by the National Geographic (“Ultimate Factories: Car Collection” and “Mega Factories”) and TLC (“Rides”) show extensive footage from inside factory plants, documenting many of these procedures.

In order to acquire the trust of high-end consumers, a firm producing high status goods would have to “burn the bridges” and restrict its ability to acquire the funds necessary to obtain industrial machinery. By doing so, the firm makes it virtually impossible for it to produce “cheap goods” and sell them as if they were of high status. Customers can then trust that the firm will not sell additional units and thus they attribute high status to the good the firm produces.

The market for positional or status goods has been extensively studied by the economic literature. Hopkins and Kornienko (2004, 2009, 2010) showed how the market for status goods could be modeled in a game theoretic fashion, on which status goods are used as signaling devices of hidden abilities, and the resulting market will have similar characteristics as those of ordinal utility goods. Pesendorfer (1995) studied the market for fashion, showing that it the has characteristics of status goods, and that its dynamics implies new designs will appear whenever the current design is copied by sufficiently many people in the lower strata of the society, so that the current design loses its status value. Rayo (2013) provided conditions under which a monopolist facing no fixed costs would choose to offer the same variety (or brand, or model)

of status good to a positive measure of consumers instead of fully customizing status goods in that vicinity. Mazali and Rodrigues-Neto (2013) showed that the presence of fixed costs in creating new varieties implies full customization will never occur in equilibrium, and the market equilibrium will be stratified in a similar fashion as in Burdett and Coles (1996). A summary of theoretical studies on social status in economics can be found in Truyts (2010). For empirical studies related to social status in economics, refer to Heffetz and Frank (2010).

The present study is related to a stream of literature that started with Titman (1984) and Brander and Lewis (1986), relating a firm’s financial decisions to its product market decisions. Brander and Lewis show that firms can adopt a high debt financing policy to commit themselves to an aggressive strategy in a duopolistic product markets. The resulting equilibrium had lower prices and higher quantities than those obtained in the traditional Cournot model of quantity competition with no debt. Titman (1984) showed that customer warranty concerns can influence a firm’s capital structure decisions because when a firm that produces a durable good goes into financial distress, customers stop purchasing from the firm, which increases the probability of asset liquidation. Firms that want to avoid this warranty concerns trap have to limit the amount of debt they issue. Many other studies show a relationship between a firm’s capital structure and product or input markets. Taken as a whole, we can refer as these models as the “stakeholder theory of capital structure” (Titman, 1984; Bronars and Deere, 1994; Klein and Leffler, 1981; Maksimovic and Titman, 1991). These studies consider the fact that the firm’s non-financial stakeholders, such as customers, workers, suppliers, etc., affect and are affected by the debts of a firm they have a stake in. More recently, researchers published studies linking a firm’s capital structure to product development and product differentiation. Holmström (1989) argues that, as firms grow and evolve into complex conglomerates whose shares are publicly traded, they become more intolerant to failure and therefore tend to shy away from risky innovative R&D projects. Manso (2011) shows that managerial compensation schemes can be set to give managers the appropriate incentives to innovate. Those incentives include great tolerance for failure and long-term stability, and can be obtained with the appropriate combination of stock options with long vesting periods, option repricing, golden parachutes, and managerial entrenchment. Ferreira, Manso and Silva (2014) show that uncertainties regarding how innovative a project is compounded with the presence of liquidity traders in the market cause firms with operational profitable innovative projects to remain private, while firms with conservative projects go public. Our study shows that, even if a project is not that innovative, if its intended market niche is most upscale, firms will also have incentives to remain private.

Our study is also closely related to a stream of literature that relates reputation with incentives

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5 For a broad review of this literature, see Istaitieh and Rodríguez-Fernández (2006).
to produce high quality products. Kreps and Wilson (1982) and Milgrom and Roberts (1982) were among the first to point out that reputational concerns could motivate many economic phenomena. Shapiro (1982, 1983, 1983a) was one of the first to apply the concept of reputation to analyze a firm’s incentive to provide high quality goods. Holmström (1999) applies it to managerial incentives to show that the implicit contract defined by career concerns might not be sufficient to overturn short-turn incentives to shirk. Mailath and Samuelson (2000) build a model in which firms have a short-term incentive to exert low effort, but could obtain higher operational profits if they could commit to exerting high effort in every period. Competent firms exert high effort while incompetent firms shirk. As there is occasional exit in the market, incompetent firms may acquire good reputation firms and then deplete it, while competent firms tend to acquire firms with average reputation and build it up. Tadelis (1999) shows that, in a dynamic adverse selection model in which firms can trade their brandnames, firms with bad reputations are able to trade their brandnames in equilibrium. Tadelis (2002) finds conditions under which finitely lived firms have incentives to exert high effort until the end. Atkeson, Hellwig and Ordoñez (2015) show that barriers to entry created by the government have an adverse selection effect on the market that can only be overcome if consumers slowly learn about the products true quality, so that reputation incentives are sufficient for a fraction of entrants to pay the entry costs and invest in quality. Maksimovic and Titman (1991) show that high debt can compromise a firm’s ability to commit to producing high quality products and thus reduce firm value, unless the firm has assets with high salvage value in liquidation. In this case, debt may increase the firm’s ability to commit to producing high quality products and increase firm value. They also show that appropriate dividend restrictions can eliminate this incentive to produce low quality goods. Klein and Leffler (1981) also evaluate a firm’s incentives to honor contracts (including implicit contracts firmed with consumers) as a function of its salvage value. They show that the condition for a firm to continuously honor contracts with consumers is that the price they obtain is significantly above salvageable production costs, and thus firms would make positive operational profits at every period.

The results shown here are closely related to Maksimovic and Titman (1991). However, there are important differences. In our case, the access to public financial markets destroys the firm’s ability to commit to producing high status goods, regardless of the nature of the firm’s assets (industrial machinery). If firms cannot commit to selling high status goods, the only alternative left is to limit their own access to funds so that it becomes impossible for the firm to obtain access to industrial machinery. Unlike in Maksimovic and Titman (1991), debt does not compromise the firm’s ability to commit to producing high-end goods any more than equity does. What matters to compromise the firm’s credibility is the firm’s ability to raise enough funds to finance
the acquisition of industrial machinery, and any publicly traded asset has the same effect on the firm’s credibility.

The remainder of the paper is organized as follows. Section 2 describes the basic setup of the model and the definition of equilibrium used throughout the paper. Section 3 describes many stage game outcomes that are used in the analysis of the dynamic game. Section 4 characterizes the equilibrium of the dynamic game and proves that the profile described is indeed a Subgame Perfect Equilibrium. Section 5 describes optimal punishment strategies used in equilibrium. Section 6 discusses commitment devices and their relation to financial constraints purposely set up by firms. Section 7 discusses some of the economic implications of the results developed in the current study. Appendix A has all the proofs. Appendix B has a discussion on the equilibrium in the financial markets.

2 Setup

2.1 Firms and Investors

There are two firms, \( j \in \{0, 1\} \), each living for infinitely many periods, \( t = \{0, 1, 2, \ldots\} \). There is a common discount factor, \( 0 < \delta < 1 \). The firms produce a single physical indivisible good that carries the firm’s brand. This homogeneous physical good is differentiated due to consumer preferences over brands. Let good \( j \) be the good produced by Firm \( j \). This terminology is independent of the technology adopted by Firm \( j \). The two firms can exit and re-enter the market freely.

Let \( \{m, h\} \) be the set of technologies available for production. Denote by \( \tau_j \) the technology adopted by Firm \( j \). By adopting a handicraft technology \( (h) \), each firm may produce its good at zero fixed costs and marginal cost \( c > 0 \). Alternatively, firms may produce using a manufacturing/industrial, mass-producing technology \( (m) \) at zero marginal cost and positive fixed cost \( K > 0 \). Industrial machinery is completely depreciated after one period. After then, if the firm wants to keep mass-producing its product, it needs to purchase the industrial machinery again by paying the same fixed cost \( K \).

2.2 Consumers

Each consumer lives for one period and may purchase good \( j \in \{0, 1\} \) by paying price \( p_j \). Only the first unit of the good provides utility to its consumers. Thus, consumers have no incentives to purchase additional units of good \( j \). If a consumer purchases a unit of good \( j \), he/she obtains no additional utility by purchasing a unit of good \(-j\). Consumers are indexed in the unit interval by \( i \in [0, 1] \). Index \( i \) can represent geographical location, as in the standard Hotelling (1929)
model, endowment, as in Cole, Mailath, and Postlewaite (1992), individual ability, as Mazali and Rodrigues-Neto (2013), individual “pizazz”, as in Burdett and Coles (1997), consumer switching cost, as in Klemperer (1987, 1995), income, as in Hopkins and Kornienko (2004), among others. If \( i \) represents location, for example, then a consumer’s position defines his transportation costs and preferences according to the distance to the different shops.

At the end of each period, another generation of consumers is born and replaces the previous. The previous generation dies and thus leaves the game. The direct utility of consumer \( i \) from consuming good \( j \) in a single period is given by \( B(i, j) \), where:

\[
B(i, j) = \begin{cases} 
  u - i, & \text{if } j = 0; \\
  u - 1 + i, & \text{if } j = 1.
\end{cases}
\]

where \( u > 0 \) is a constant. The function \( B(i, j) \) is the component of the utility indicating intrinsic preferences. In the absence of status concerns, this static model is identical to Hotelling’s (1929) model of spatial competition.

Besides direct utility, a good might also have a status-enhancing component. Let \( S_t(j) \in \{0, 1\} \) be an indicator variable that represents the beliefs of consumers at time \( t \) regarding whether good \( j \) has high status. This variable assumes the value \( S_t(j) = 1 \) if consumers think that good \( j \) provides social status to its consumers at period \( t \). If consumers think that good \( j \) does not provide any social status to its consumers at period \( t \), then \( S_t(j) = 0 \). As in Mazali and Rodrigues-Neto (2013), an exogenous social norm determines which good, if any, provides social status. This social norm assigns \( S_t(j) = 1 \) if and only if Firm \( j \) was the only firm to produce a handcrafted good in period \( t - 1 \). On the other hand, if at time \( t - 1 \) Firm \( j \) uses the industrial technology, then the social norm assigns \( S_t(j) = 0 \). If both firms use the handicraft technology at \( t - 1 \), then \( S_t(j) = 0 \). If we impose the hypothesis that \( c \geq 1/2 \), it can be shown that this hypothesis is equivalent to assuming that the status component of good \( j \) is an experience good, that consumers only attribute status to the most exclusive good in the previous period, and that “word of mouth goes quickly”. Social status being an experience good means that only after its consumption individuals are able to infer its true social status value. “Word of mouth goes quickly”, as in Ahn and Suominen (2001), means that, once a particular consumer experiences a particular status level for this good, it becomes public information. Each good lasts exactly one period, and cannot be passed on from one generation to another.\(^6\)

\(^7\)Alternatively, we could assume that consumers rationally make predictions regarding the status of the goods offered, in a way similar to how consumers predict quality in Mailath and Samuelson (2001), Tadelis (2002), or Atkeson, Hellwig and Ordoñez (2015). The dynamics that arises from this assumption is rather complex and not central to the present study. Therefore, for the sake of simplicity, we simply assume that the consumption of status is an experience good with social learning through a “word of mouth goes quickly” assumption.

\(^7\)For further details on “word of mouth” communication models, check Ellison and Fudenberg (1995) and Banerjee and Fudenberg (2004).
In the Hotelling (1929) model, consumers on the left (sufficiently small index $i$) buy good 0 and consumers on the right consume good 1. Let $i_t^*$ be the consumer that, at time $t$, is indifferent between goods 0 and 1; that is, the consumer on the border of the two market niches. The magnitude of the status component of the utility function depends on the current exclusivity of the status good. Let $D_t(j, i_t^*)$ be the demand for the good produced by Firm $j$ at time $t$ when a mass of $i_t^* \in [0, 1]$ consumers buys from Firm 0. Demands are $D_t(0, i_t^*) = i_t^*$ and $D_t(1, i_t^*) = 1 - i_t^*$.

The status component of the time $t$ utility is $S_t(j)[1 - D_t(j, i_t^*)]$. The more exclusive a good perceived as of high status, the more utility its consumers obtain from it. Let $U_t(i, j; i_t^*)$ denote the time $t$ utility of a consumer with index $i \in [0, 1]$ from buying the good from Firm $j \in \{0, 1\}$ when a mass of $i_t^* \in [0, 1]$ consumers buys from Firm 0. This utility is:

$$U_t(i, j; i_t^*) = B(i, j) + S_t(j)[1 - D_t(j, i_t^*)] - p_j.$$

### 2.3 Stage Game

The game starts at $t = 0$ and is repeated infinitely. At every stage game, events occur in the following order:

1. Firms simultaneously choose $\tau_j$ and $p_j$;

2. Consumers observe prices and simultaneously choose a good to buy;

3. Period payoffs are realized, period $t + 1$ consumers and firms observe status and update their beliefs.

### 2.4 Repeated Game

An action $a_j \in A_j$ for Firm $j$ is a pair $a_j = (p_j, \tau_j)$ defining price $p_j$ and technology $\tau_j$ to be played by each firm in a stage game. The set $A_j$ represents the set of all possible actions available to firm $j$. Let $H_t = (H_0^t, H_1^t)$ be the finite history of the repeated game up to time $t$, where $H_j^t = (a_j^0, \ldots, a_j^t)$, where $a_j^t$ is the action played by Firm $j$ at time $t$. Let $H_t$ be the set of possible histories up to time $t$. Thus, $H_t \in \mathcal{H}_t$. Let $H$ be an infinite-history, an element of $\mathcal{H}$, the set of all possible infinite histories of the game.

Let $\alpha_j^t : \mathcal{H}_t \rightarrow A_j$ be the function that assigns, for each finite history $H_t \in \mathcal{H}_t$, an action $a_j \in A_j$. A strategy $\sigma_j$ for Firm $j$ is an infinite sequence of functions $\sigma_j = (\alpha_j^0, \alpha_j^1, \ldots, \alpha_j^t, \ldots)$ that state, for every period $t$, an action to be taken for every possible finite history of the game up until $t$. 

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After each finite history $H_t$, there is a subgame starting at $t+1$. Let $G(H_t)$ be the continuation subgame starting at period $t + 1$ after the occurrence of history $H_t$. A continuation strategy $\sigma_j(H_t)$ is the restriction of a strategy from $t + 1$ onwards such that the complete history of the game has finite history $H_t$ from periods 0 to $t$. A continuation profile is a pair of continuation strategies $(\sigma_0(H_t), \sigma_1(H_t))$ where each player plays a continuation strategy with the same finite history $H_t$.

Let $V_j(\sigma)$ represent the discounted flow of stage game payoffs. Let $\mathbb{I}_m(\cdot)$ be an industrial technology indicator function. More precisely, $\mathbb{I}_m(m) = 1$, and $\mathbb{I}_m(h) = 0$. Thus, for each $j \in \{0, 1\}$:

$$V_j(\sigma) = \sum_{t=0}^{+\infty} \delta^t [\pi_j(a_t) - K \mathbb{I}_m(\tau_j(t))]$$

A Nash Equilibrium of the continuation subgame is a profile of continuation strategies such that no player has a unilateral profitable deviation. A Subgame Perfect Equilibrium is a strategy profile such that, in every subgame that is reached after finite history $H_t$, the continuation profile induced by $\sigma$ is the Nash Equilibrium of the subgame. The focus of the analysis is on profiles that characterize Subgame Perfect Equilibria.

**Example 1** As an example of a strategy profile that may be implementable in equilibrium is one in which both firms choose the handicraft technology, and play Hotelling prices in every period. Any deviation from this profile is ignored by the players. Subsection 3.7 describes the stage game payoffs that each player obtains if both firms play such a strategy.

Another example of a strategy profile that may be implementable as a Subgame Perfect Equilibrium is one in which each firms choose different technologies along the equilibrium path. The complete strategy profile and its implementability as a Subgame Perfect Equilibrium will be discussed in Section 4.

### 3 Stage Game Analysis

#### 3.1 Consumer Demands

Consider the index $i^*$ such that consumers with this index are indifferent between buying from Firm 0 or from Firm 1. In this case, the demands of the firms are $D(0, i^*) = i^*$ and $D(1, i^*) = 1 - i^*$. The following lemma states consumer demands as functions of prices $p_0$ and $p_1$. 

Lemma 1 If consumers do not attribute status value to good 1 at period $t$, that is, if $S_t(1) = 0$, then the aggregate demands for goods 0 and 1 are given, respectively, by

\[ i^* = \frac{1 + p_1 - p_0}{2}, \]
\[ 1 - i^* = \frac{1 - p_1 + p_0}{2}. \]

If otherwise consumers do attribute status value to good 1 at period $t$, that is, if $S_t(1) = 1$, then the aggregate demands for goods 0 and 1 are given, respectively, by

\[ i^* = \frac{1 + p_1 - p_0}{3}, \]
\[ 1 - i^* = \frac{2 - p_1 + p_0}{3}. \]

The good produced by Firm 1 is the most exclusive when $i^* > 1/2$. This is true if and only if its price is significantly greater than the price of the other good, in the sense that $p_1 - p_0 > 1/2$. In the following subsections, we will consider a series of stage games that are relevant for the equilibrium of the Dynamic Game.

3.2 Stage Game Along the Equilibrium Path

Suppose that Firm 0 adopts the industrial technology, while Firm 1 adopts the handicraft technology. Suppose also that consumers attribute social status utility to good 1 and no social status to good 0. Firms simultaneously decide their prices. The equilibrium outcome of this stage game, proven in Lemma 8 in Appendix A, is given by prices $p_0^* = 4/3 + c/3$, $p_1^* = 5/3 + 2c/3$, demands $i^* = 4/9 + c/9$, $1 - i^* = 5/9 - c/9$, and profits $\pi_0^* = (4 + c)^2/27$, and $\pi_1^* = (5 - c)^2/27$.

Exclusiveness has long been considered a key factor in defining whether a particular product is a status good or not. The marginal cost of production under the handicraft technology being sufficiently high in the sense that $c > 1/2$ exactly when the status good is the most exclusive. Also, the operational profit of Firm 0 is larger than the operational profit of Firm 1 if and only if the marginal cost of production under the handicraft technology is larger than 1/2. Therefore the following three statements are equivalent: (i) $\pi_0^* > \pi_1^*$; (ii) $c > 1/2$; and (iii) $i^* > 1/2$.

3.3 Firm 0’s One-Shot Deviation

Consider the scenario in which consumers attribute status to good 1 only. Suppose that Firm 1 plays the action described in Subsection 3.2. What is the best possible deviation for Firm 0 in such a stage game? The solution to this problem by Firm 0 yields the outcomes of this stage game, with prices $p_0^{D0} = 4/3 + 5c/6$ and $p_1^{D0} = p_1^* = 5/3 + 2c/3$, demands $i^{D0} = 4/9 - c/18$ and $1 - i^{D0} = 5/9 + c/18$, and operational profits $\pi_0^{D0} = (8 - c)^2/108$, and $\pi_1^{D0} = (5/3 - c/3)(5/9 + c/18)$.

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8See, for instance, Silverstein et al (2008), Chapter 3.
3.4 Firm 1’s One-Shot Deviation

Suppose that Firm 1 wants to increase its one-period operational profit above \( \pi^*_1 \), by secretly switching to the industrial technology. Consumers and Firm 0 still believe Firm 1’s good is handmade, and Firm 1 knows its rival and consumers have these beliefs. Firm 0 plays \( \tau^D_0 = m \) and \( p^D_0 = p^*_0 \), because it is unaware of Firm 1’s deviation.

When Firm 1 deviates, it raises \( K \) in order to produce its good at zero marginal cost, and sells it for a price \( p^D_1 \) that maximizes its current operational profit given consumers’ beliefs. Firm 1 sells these goods to the \( 1 - i^D \) consumers on the right end of the unit interval. Lemma 10 in Appendix A computes the outcomes of this stage game. These outcomes are \( p^D_0 = \frac{4}{3} + \frac{c}{3} \) and \( p^D_1 = \frac{5}{3} - \frac{c}{6} \), demands \( i^D = \frac{4}{9} - \frac{c}{18} \) and \( 1 - i^D = \frac{5}{9} + \frac{c}{18} \), and profits \( \pi^D_0 = \frac{(4 + c)(8 - c)}{27} \) and \( \pi^D_1 = \frac{(10 + c)^2}{108} \).

3.5 Returning to the Equilibrium Path

Consumers attribute no status to either good. Firm 1 uses the handicraft technology, while Firm 0 uses the industrial technology. These technology choices are common knowledge. Lemma 11 in Appendix A computes the outcomes of this stage game. These outcomes of this stage game are prices \( p^P_0 = 1 + \frac{c}{3} \) and \( p^P_1 = 1 + \frac{2c}{3} \), demands \( i^P = \frac{1}{2} + \frac{c}{6} \) and \( 1 - i^P = \frac{1}{2} - \frac{c}{6} \), and profits \( \pi^P_0 = \frac{(3 + c)^2}{18} \) and \( \pi^P_1 = \frac{(3 - c)^2}{18} \).

3.6 Alternative to Regaining Status

Suppose that consumers do not assign status to any good. Suppose further that Firm 0 believes that Firm 1 will try to regain good 1’s social status by adopting the handicraft technology and charging price \( p^P_1 = 1 + \frac{2c}{3} \). Then, Firm 0 adopts the industrial technology and charges price \( p^P_0 = 1 + \frac{c}{3} \). In this case, Firm 1 may have a short-run incentive to keep adopting the industrial technology and charge a price that maximizes its one-period profit. In other words, Firm 1 may have an incentive to play a stage game best reply to price \( p^P_0 = 1 + \frac{c}{3} \). In this scenario, the stage game outcome includes prices \( p^{ARR}_0 = p^{ARR}_1 = 1 + \frac{c}{3} \) and \( p^{ARR}_1 = 1 + \frac{c}{6} \), demands \( i^{ARR} = \frac{1}{2} - \frac{c}{12} \) and \( 1 - i^{ARR} = \frac{1}{2} + \frac{c}{12} \), and operational profits \( \pi^{ARR}_0 = \frac{(6 - c)(c + 3)}{36} \) and \( \pi^{ARR}_1 = \frac{(c + 6)^2}{72} \).

3.7 Hotelling With Same Technology and No Status

Suppose consumers attribute no status to either good. The technologies used by each firm are common knowledge. Suppose further that these facts are common knowledge between the two firms. The stage game is reduced to a standard Hotelling competition game. Let \( \Pi_h(\cdot) \) be a
handicraft technology indicator function. More precisely, \( I_h (h) = 1 \), and \( I_h (m) = 0 \). Lemma 13 in Appendix A computes the outcomes of this stage game, namely prices \( p_0^H (\tau) = 1 + c.I_h (\tau) = p_1^H (\tau) \), demands \( i^H = 1 - i^H = 1/2 \), and operational profits \( \pi_0^H = \pi_1^H = 1/2 \).

4 Equilibrium of the Repeated Game with Status

4.1 Equilibrium with Product Differentiation on Status

This subsection describes the dynamic strategy profile \( \sigma^* \) in which the status good is produced at every period along the equilibrium path, for sufficiently patient agents. Along the equilibrium path, Firm 1 chooses to use the handicraft technology and charges price \( p_1^* \) according to equation (30). Along the equilibrium path, Firm 0 uses the industrial technology and charges price \( p_0^* \) according to equation (29). Consumers attribute status value for good 1, no status value to good 0, and demand these goods according to equations (31) and (32). Consumers attribute no status to either good in case Firm 1 deviates by changing its production technology to industrial.

Call \( a_j = (p_j, \tau_j) \) the action taken by Firm \( j \) in a given stage game. Define strategy profile \( \sigma^* = (\sigma_0^*, \sigma_1^*) \). Firm 0 plays according to the following strategy \( \sigma_0^* \):

1. At \( t = 0 \), Firm 0 plays \( a_0^* \);
2. At each \( t > 0 \), if Firm 0 observes Firm 1’s period \( t - 1 \) action with the handicraft technology, then Firm 0 plays action \( a_0^* \) in period \( t \);
3. At each \( t > 0 \), if Firm 0 observes Firm 1’s period \( t - 1 \) action with the industrial technology, then Firm 0 plays action \( a_0^P = (p_0^P, m) \) in period \( t \);

Firm 1 plays according to the following strategy \( \sigma_1^* \):

1. At \( t = 0 \), Firm 1 plays \( a_1^* \);
2. At each \( t > 0 \), if Firm 1 played an action with the handicraft technology in period \( t - 1 \), then Firm 1 plays action \( a_1^* \) in period \( t \);
3. At each \( t > 0 \), if Firm 1 played an action with the industrial technology in period \( t - 1 \), then Firm 1 plays action \( a_1^P = (p_1^P, h) \) in period \( t \).

Strategy profile \( \sigma^* = (\sigma_0^*, \sigma_1^*) \) is the candidate equilibrium of the dynamic game. Figure 1 depicts strategy profile \( \sigma^* \). The next few subsections show that \( \sigma^* \) is indeed a Subgame Perfect Equilibrium if Firm 1 is sufficiently patient.
Two states can occur if firms play according to $\sigma^*$. In one state, consumers attribute status to good 1, but not to good 0. In the other, consumers attribute no status to either good. In either state, strategy $\sigma_0^*$ prescribes that Firm 0 adopts the industrial technology. Strategy $\sigma_1^*$ prescribes Firm 1 to adopt the handicraft technology at either state. In the first state, the one describing the equilibrium path, the prescribed prices according to $\sigma^*$ are $p_0^*$ and $p_1^*$. In the state with no status, prescribed prices according to $\sigma^*$ are $p_0^P$ and $p_1^P$. The repeated game starts at the state with status attributed to Firm 1, and stays there as long as Firm 1 adopts the handicraft technology. Deviations by Firm 0 are ignored. Deviations in price by Firm 1 are also ignored. If Firm 1 uses the industrial technology, then in the following period the game moves to the state in which firms have no status. The game remains in the no status state if Firm 1 keeps adopting the industrial technology. The game returns to the state in which consumers assign status to good 1 as soon as Firm 1 returns to adopting the handicraft technology.

Moving from one state to the other in case Firm 1 changes its technology from handicraft to industrial implies a loss of status to its good in the following period. This “consumer punishment” may provide sufficient incentives to deter Firm 1 from ever deviating from the prescribed actions.

Let $V_0^*$ and $V_1^*$ be the equilibrium payoffs of the repeated game $G$. These payoffs correspond to the value of the firms’ total assets along the equilibrium path.

4.2 Short-Term Incentives to Deviate

In principle, either firm may have incentives to deviate to obtain larger short-term profits. Consider Firm 0’s incentives to deviate from the prescribed action $a_0^*$ on the equilibrium path by switching to the handicraft technology. If Firm 0 plays $a_0^*$, it obtains $\pi_0^{D_0} = (8 - c)^2 / 108$. Firm 0 obtains one-period gains from deviating if
\[ \pi_{0}^{D0} > \pi_{0}^{*} - K, \text{ that is, if:} \]
\[ \frac{(8 - c)^2}{108} > \frac{(4 + c)^2}{27} - K. \]  
Inequality (5) is equivalent to 36K > 16c + c^2.

Now consider Firm 1’s short-term incentives. Firm 1 obtains higher short-term gains if it deviates whenever \( \pi_{1}^{D} - K > \pi_{1}^{*} \). Assume that this inequality holds, and suppose that Firm 1 deviates from its prescribed actions by switching to the industrial technology and charging price \( p_{1}^{D} \) as defined in equation (44). Substituting (34) and (48) into the condition \( \pi_{1}^{D} - K > \pi_{1}^{*} \) yields:
\[ \frac{(10 + c)^2}{108} - K > \frac{(5 - c)^2}{27}. \]  
Inequality (6) is equivalent to 36K < 20c - c^2.

Lemma 2 compares conditions establishing short-term incentives to deviate for each firm. Firm 0 is able to obtain short-term gains by deviating when \( K \) is sufficiently large in comparison to \( c \). In this case, Firm 1 is not able to obtain short-term gains by deviating. Similarly, Firm 1 has short-term incentives to deviate when \( K \) is sufficiently small in comparison to \( c \). In this case, Firm 0 has no short-term incentives to deviate.

**Lemma 2** There is no combination of parameters in which both firms have short-term incentives to deviate. If Firm 0 cannot obtain one-period gains by deviating from the equilibrium path, that is, if 36K \( \leq \) 16c + c^2, then Firm 1 can obtain one-period gains by deviating from the equilibrium path, that is, 36K < 20c - c^2.

Likewise, if Firm 1 cannot obtain one-period gains by deviating from the equilibrium path, that is, if 36K \( \geq \) 20c - c^2, then Firm 0 can obtain one-period gains by deviating from the equilibrium path, that is, 36K > 16c + c^2.

Henceforth, let us assume that Firm 0 cannot obtain short-term profits by deviating, that is, 36K \( \leq \) 16c + c^2. In this case, Firm 1 always has short-term incentives to deviate, and thus 36K < 20c - c^2.

**4.3 Deviations from the Equilibrium Path**

If Firm 1 deviates, by playing \( (p_{1}^{D}, m) \), it makes a higher operational profit in the current period. This is true because consumers still believe they are purchasing a status good. Firm 0 still charges \( p_{0}^{*} \) and uses the industrial technology. If Firms adopt these actions, Firm 1 is able to catch a much larger market share, because part of Firm 0’s former market niche is now served by Firm 1. Firm 0’s operational profit is reduced from \( \pi_{0}^{*} \) to \( \pi_{0}^{D} \). Not only Firm 1 reduces Firm 0’s current operational profits by occupying part of its market niche, but also reduces Firm 0’s
future prospects. This occurs because, once Firm 1 adopts the industrial technology, the game moves to the no status state. The fact that no good has any status value reduces the aggregate profit.

Consider the strategy profile \( \sigma^* \). In order to prove that \( \sigma^* \) is indeed a Subgame Perfect Equilibrium, we need to verify whether: (i) it is credible for Firm 1 not to deviate from the equilibrium path; and (ii) once good 1 has lost its status, we need to check if Firm 1 has incentives to regain status and return to the equilibrium path.

Consider first one-shot deviations in which Firm 1 only switches its price, but maintain its use of the handicraft technology. If Firm 1 deviates in prices only, it cannot obtain a higher current period profit, because \( p_1^* \) is already a best response to \( p_0^* \) once technologies are fixed. Also, not switching technologies does not switch the state of the game in the following period. Thus, future payoffs are not changed when Firm 1 does not change its technology and changes its price only.

Now consider a one-shot deviation in which Firm 1 plays \( (p_1^D, m) \) in the current period. Firm 1 does not have incentives to deviate from the equilibrium path if:

\[
V_1^* \geq \pi_1^D - K + \delta \pi_1^P + \delta^2 V_1^*. \tag{7}
\]

If Firm 1 deviates, consumers attribute no status value to Firm 1’s product in the following period, which reduces Firm 1’s profits, working as a “consumer punishment” for deviation. It might be the case that this “non-strategic punishment” by consumers is sufficient to deter deviation, and no active punishment from Firm 0 is necessary. This deviation is not profitable for Firm 1 if:

\[
\frac{\pi_1^D}{1 - \delta} \geq \pi_1^D - K + \delta \pi_1^P + \delta^2 \frac{\pi_1^D}{1 - \delta},
\]

where \( \pi_1^D \) is given by equation (48), and \( \pi_1^P \) is given by (56). This inequality is equivalent to \((1 + \delta)\pi_1^D \geq \pi_1^D - K + \delta \pi_1^P \), or \( \delta \geq \hat{\delta}_1 \), where \( \hat{\delta}_1 \) is defined as:

\[
\hat{\delta}_1 = \frac{\pi_1^D - K - \pi_1^*}{\pi_1^* - \pi_1^P}. \tag{8}
\]

**Lemma 3** A one-shot deviation in which Firm 1 plays \( (p_1^D, m) \) in the current period is not profitable for Firm 1 for sufficiently large values of \( \delta \) and \( K \), and sufficiently small values of \( c \). More precisely, deviation is not profitable if and only if \( \delta \geq \hat{\delta}_1 \), where \( \hat{\delta}_1 \) is given by equation (8) or, alternatively, by:

\[
\hat{\delta}_1 = \frac{-6c^2 + 120c - 216K}{-4c^2 - 8c + 92}. \tag{9}
\]
For all values of \( c \) such that \( c \leq 5 - \sqrt{27/2} \), the denominator of equation (9) is positive. For relatively small \( K \) or small \( \delta \) and for relatively large \( c \), the inequality \( \delta \geq \tilde{\delta}_1 \) may not hold. Thus, for sufficiently low values of \( K \) (or sufficiently low values of \( \delta \), or sufficiently large values of \( c \)), Firm 0 needs to actively punish deviations to prevent Firm 1 from doing it. To see this, consider the following example.

**Example 2** Assume \( c = 1 \). Condition \( \delta \geq \tilde{\delta}_1 \) holds if and only if \( K \geq (57 - 40\delta) / 108 \). If \( K < 17/108 \), then, \( \delta < 1 < \hat{\delta}_1 \), and the loss of status assigned by consumers is not sufficient to deter deviation from Firm 1.

### 4.4 Incentives For Firm 1 to Return to the Equilibrium Path

The strategy \( \sigma_1^* \) prescribes that Firm 1 uses the handicraft technology in the no status state. Firm 1, however, may choose to continue adopting the industrial technology. In the absence of status, Firm 1 has incentives to return to using the handicraft technology if and only if:

\[
\pi_1^P + \delta \pi_1^* + \delta^2 \pi_1^* \frac{\pi_1^{ARR}}{1 - \delta} \geq \pi_1^{ARR} - K + \delta \pi_1^P + \delta^2 \frac{\pi_1^*}{1 - \delta}. \tag{10}
\]

If Firm 1 uses the handicraft technology to regain its status, it obtains current payoffs of \( \pi_1^P \), and a continuation payoff of \( \delta \pi_1^* + \delta^2 \pi_1^* / (1 - \delta) \). If Firm 1 delays its return to the equilibrium path for one period, insisting in using the industrial technology for another period, then it obtains current payoffs of \( \pi_1^{ARR} - K \), and a continuation payoff of \( \delta \pi_1^P + \delta^2 \pi_1^* / (1 - \delta) \). Solving inequality (10) for \( \delta \) yields \( \delta \geq \tilde{\delta}_1 \), where:

\[
\tilde{\delta}_1 = \frac{\pi_1^{ARR} - K - \pi_1^P}{\pi_1^* - \pi_1^P}. \tag{11}
\]

**Lemma 4** Suppose Firm 1 deviates and good 1 suffers a social status loss. Then, Firm 1 prefers to return to the equilibrium path (regaining its status) for sufficiently large values of \( \delta \) and \( K \), and sufficiently small values of \( c \). More precisely, returning to the equilibrium path is more profitable if and only if \( \delta \geq \tilde{\delta}_1 \), where \( \tilde{\delta}_1 \) is given by equation (11) or, alternatively, by:

\[
\tilde{\delta}_1 = \frac{-9c^2 + 108c - 216K}{-4c^2 - 8c + 92}. \tag{12}
\]

The lemma above implies that Firm 1 has incentives to return to the equilibrium path once it has deviated and lost its status if it is sufficiently patient. Also, Firm 1 has incentives to return to the equilibrium path if the fixed cost associated with the industrial technology is sufficiently large, or if the marginal cost associated with the handicraft technology is sufficiently low.

The next result shows that the binding constraint is that \( \delta \geq \tilde{\delta}_1 \), that is, that Firm 1 does not have an incentive to deviate from the equilibrium path. As a consequence of Proposition 1, the necessary and sufficient condition for \( \sigma^* \) to be a Subgame Perfect Equilibrium of the repeated game is that Firm 1 is sufficiently patient in the sense that \( \delta \geq \tilde{\delta}_1 \).
Proposition 1 If Firm 1 is sufficiently patient so that it has incentives not to deviate from the equilibrium path, then it also has incentives to return to the equilibrium path once it finds itself in the punishment stage. The reciprocal result does not hold, in general. Mathematically, \( \tilde{\delta}_1 \geq \tilde{\delta}_1 \). Therefore, the strategy profile \( \sigma^* \) is a Subgame Perfect Equilibrium if and only if Firm 1 is sufficiently patient in the sense that \( \delta \geq \tilde{\delta}_1 \).

5 Ownership Choice and Capital Structure

This section analyzes a modified version of the repeated game described in the previous sections. In this modified game, besides choosing their prices and production technologies, firms also choose their ownership and capital structures. It also shows that the value of the firm does not change with the firms’ financial choices and, therefore, that product market decisions can be analyzed independently of the financial decisions of the firms.

5.1 Basic Setup for the Modified Game

Let \( o_j \in \{ \text{public, private} \} \) denote the ownership structure adopted by Firm \( j \). If Firm \( j \) adopts the industrial technology firm, it needs to finance the acquisition of industrial machinery. Implementing mass production in assembly lines with computerized procedures and robotics is usually an expensive project. A firm that makes this move usually has to look for external funds to finance it. Hence, from now on we always assume that firms are financially constrained, so that they do not have internal funds to acquire the industrial technology. Assume that public firms have access to financial capital markets, and are able to issue publicly traded stocks and bonds. Public firms may finance the cost \( K \) of industrial machinery through the issuance of stocks or debt. Therefore, if Firm \( j \) chooses \( o_j(t) = \text{public} \), it has access to both the industrial and handicraft technologies, and thus \( \tau_j(t) \in \{ h, m \} \).

If a public firm issues debt to finance part or all of the fixed cost \( K \), the firm repays its debt in the same period, after operational profits are realized. Assume that there is no intra-period cost of capital; the firm repays the same amount when paying off its debt. Assume that investors are numerous, homogeneous and rational, so that they pay the fair price for the firms’ shares. Also, assume investors have no market power and invest in the firm only if the firm repays them accordingly, that is, if the firm is able to generate non-negative value. Firms’ managers act in the interest of a firm’s current shareholders. Managers maximize the present value \( V_j \) of the discounted cash flows of the share of current shareholders on future profits.

Let \( \phi_j \) be the fraction of firm \( j \) that current shareholders give up to new shareholders that occurs if the firm issues stock to finance a fraction of \( K \). If a public firm \( j \) issues bonds, assume that in each period it borrows \( K \psi_j \), for some \( 0 \leq \psi_j \leq 1 \), to finance the fraction \( \psi_j \) of the
cost of machinery. If Firm $j$ chooses $o_j = \text{public}$ and $\tau_j = m$, then the coordinates $\phi_j$ and $\psi_j$ must be such that the total amount raised with assets issuance is at least $K$. If Firm $j$ chooses $o_j = \text{public}$ and $\tau_j = h$, then it may raise as much capital as their managers decide. So $\phi_j$ may take any value between 0 and 1, and $\psi_j$ may take any non-negative value.

Private firms do not have access to financial markets, and cannot issue any publicly traded securities: stocks or debt. If Firm $j$ chooses $o_j(t) = \text{private}$, the only technology available to it is the handicraft technology, and we must have $\tau_j(t) = h$. If Firm $j$ chooses $o_j(t) = \text{private}$, then it must also choose $\phi_j = \psi_j = 0$, and $\tau_j(t) = h$.

Let $a_j = (p_j, \tau_j, \phi_j, \psi_j, o_j)$ be a typical action taken by Firm $j$ in a given stage game. In each period of the repeated game, firms $j = 0, 1$ choose their actions $a_j$ simultaneously. As before, the repeated game starts at $t = 0$ and continues at $t = 1, 2, 3, \ldots$. As with the original game, firms have a common discount factor $\delta$. One might consider the hypothesis of choosing an ownership structure at every period a strong assumption. However, because each period in our model represents a whole generation, and therefore very long periods of time, choosing a possibly different ownership structure is actually a more realistic assumption than assuming that the ownership structure is chosen at the beginning of the game and fixed afterwards.

5.2 Financial Markets Equilibrium

To raise the capital amount $K$, at every period, Firm 0’s shareholders must issue shares and bonds. The total proceeds of this issuance must yield $K$. Old shareholders must give away a fraction $\phi$ to new shareholders. The remaining $\psi K$ are financed through the issuance of debt. Proposition 2 shows that, regardless of the choices of $\phi$ and $\psi$, the value of Firm 0 is the same. Appendix B contains a complete discussion of this proposition, alongside other issues regarding the equilibrium in financial markets.

**Proposition 2** (Modigliani and Miller, 1958) The value of the industrial technology firm is independent on how it finances its capital investment $K$. Thus, the firm’s investment decisions in product markets are independent of the firm’s financial decisions.

This result occurs because none of the basic assumptions of the original Modigliani-Miller model is broken. The fact that Firm 0’s value to old shareholders is equal to the present value of the company’s cash flows comes as a no arbitrage condition. Despite the dilution process that takes place every time the firm has to raise capital for new machinery, old shareholders are not expropriated in any way, because they receive their “fair value” for the shares they give away. Because Firm 0’s value is independent of financing, we can ignore financing strategies in calculating the firm’s values.
The validity of Modigliani and Miller’s theorem is another point in which the present study differs from Maksimovic and Titman (1991). In their study, firms are never indifferent between the two sources of financing. Increasing debt always change incentives for firms to offer high quality products, because it increases the probability of financial distress. In our study, public companies are indifferent between debt and equity financing. The Modigliani-Miller result holds in our model because the form of financing for a public company does not interfere with the Firm 1’s incentives to offer high status goods.

5.3 Ownership Structure and Product Market Equilibrium

Because firms can change their ownership structure at every period, both technologies are always available to both firms. As a result, the strategic aspects of the modified game incorporating ownership and financial decisions are intrinsically the same as in the game analyzed in Section 4. Because Modigliani and Miller’s theorem holds (Proposition 2), then the inclusion of either financial or ownership structure decisions does not fundamentally change the repeated game. The equilibrium payoff of each firm is the same as the one obtained in Section 4.

It is possible to write the firms’ values as discounted sums of one-period profits. Along the equilibrium path, firm values are:

\[
V_0^* = \frac{\pi_0^* - K}{1 - \delta}, \tag{13}
\]

\[
V_1^* = \frac{\pi_1^*}{1 - \delta}, \tag{14}
\]

Remark 1 The equilibrium value of Firm 0 is at least as large as the value of Firm 1 when \(V_0^* \geq V_1^*\). This inequality holds if and only if \(\pi_0^* - K \geq \pi_1^*\). This is true when the fixed cost is sufficiently small in the sense that \(K \leq \pi_0^* - \pi_1^*\). Because \(\pi_0^* - \pi_1^* = (2c - 1)/3\), then:

\[V_0^* \geq V_1^* \text{ if and only if } 3K \leq 2c - 1.\]

6 Ownership Choice with Commitment

There are combinations of parameter values for \(\delta, K\) and \(c\) such that \(\delta < \hat{\delta}_1\). In this case, the strategy profile \(\sigma^*\) described in Section 4 is not a Subgame Perfect Equilibrium. However, there may exist alternative setups of the repeated game in which there is a Subgame Perfect Equilibrium such that, along the equilibrium path, firms produce using different technologies and consumers attribute status to the handmade good.
6.1 Basic Setup of the Game with Commitment

Consider a modified version of the game described in Section 5 in which Firm 1 has a “private financing only” provision on its constitution. Hence, Firm 1 cannot finance the purchase of $K$ and therefore does not have access to the industrial technology. Firm 1 is only able to produce handmade goods. If Firm 0 always produces using the industrial technology, then a status good is always available in the market.

The sequence of events at every stage game are the same as in the game of Section 4. Let $O_j$ represent the set of feasible ownership structures available to Firm $j$. Then, $O_0 = \{\text{private, public}\}$ and $O_1 = \{\text{private}\}$. As a consequence, this commitment setup implies $\tau_1 = h$ at every period. For Firm 0, choosing to be public is always possible, and thus the industrial technology is always available.

Let $G_M$ denote the modified game. Let $\sigma_{M,j}$ be a typical strategy for Firm $j$, and $\sigma_M = (\sigma_{M,0}, \sigma_{M,1})$ be a strategy profile of $G_M$. An action $a_j \in A_j$ for Firm $j$ is defined by a vector $a_j = (p_j, \tau_j, \phi_j, \psi_j, o_j)$ that lists price $p_j$, technology $\tau_j$, stock issuance $\phi_j$, debt issuance $\psi_j$, and ownership structure $o_j$, to be played by each firm in a stage game. The set $A_j$ represents the set of all possible actions available to firm $j$. Let $H_t = (H_{0,t}, H_{1,t})$ be the finite history of the repeated game up to time $t$, where $H_{j,t} = (a_{j,1}, \ldots, a_{j,t})$, where $a_{j,t}$ is the action played by Firm $j$ at time $t$. Let $\mathcal{H}_t$ be the set of possible histories up to time $t$. Thus, $H_t \in \mathcal{H}_t$. Let $H$ be an infinite-history of $G_M$. Infinite history $H$ is an element of $\mathcal{H}$, the set of all possible infinite histories of the game.

Let $\alpha_j^t : \mathcal{H}_t \rightarrow A_j$ be the function that assigns, for each finite history $H_t \in \mathcal{H}_t$, an action $a_j \in A_j$. A strategy $\sigma_j$ for Firm $j$ is an infinite sequence of functions $\alpha_j^t : \mathcal{H}_t \rightarrow A$, that is, $\sigma_j = (\alpha_j^0, \alpha_j^1, \ldots, \alpha_j^t, \ldots)$.

Consider the following strategy $\sigma_{M,0}^*$ for Firm 0:

1. If Firm 0 has played an action with $\tau_0 = m$ at $t - 1$, then, at period $t$, Firm 0 plays $a_{0,M}^* = (p_0^*, m, \phi_0^*, \psi_0^*, \text{public})$, where $\phi_0^*$ and $\psi_0^*$ are any pair of values of $\phi_0$ and $\psi_0$ that raise the amount $K$;

2. If Firm 0 has played an action with $\tau_0 = h$ at $t - 1$, then, at period $t$, Firm 0 plays $a_{0,M}^* = (p_0^*, m, \phi_0^*, \psi_0^*, \text{public})$, where $\phi_0^*$ and $\psi_0^*$ are any pair of values of $\phi_0$ and $\psi_0$ that raise the amount $K$.

Consider the following strategy $\sigma_{1,M}^*$ for Firm 1:

1. If Firm 0 has played an action with $\tau_0 = m$ at $t - 1$, then, at period $t$, Firm 1 plays $a_{1,M}^* = (p_1^*, h, 0, 0, \text{private})$;
2. If Firm 0 has played an action with $\tau_0 = h$ at $t - 1$, then, at period $t$, Firm 1 plays $a_{1,M}^P = (p_1^P, h, 0, 0, \text{private})$.

We claim that the strategy profile $\sigma_M^* = (\sigma_{M,0}^*, \sigma_{M,1}^*)$ constitutes a Subgame Perfect Nash Equilibrium of the modified game with commitment for sufficiently patient agents. The next few subsections show that $\sigma_M^*$ is indeed a Subgame Perfect Equilibrium. They also show that the commitment path allows for the status good to be provided in equilibrium for a much wider set of parameter values $\delta$ than would be possible in the original game. In fact, for many values of $K$ and $c$, commitment would enlarge the set of values of $\delta$ for which status goods are provided in subgame perfect equilibria.

6.2 Analysis of the Game with Commitment

Suppose both firms play according to strategy profile $\sigma_M^*$. Along the equilibrium path, Firm 0 and Firm 1 play $a_{0,M}^*$ and $a_{1,M}^*$, in every period. These actions yield profits $\pi_0^* - K = (4 + c)^2 / 27 - K$ and $\pi_1^* = (5 - c)^2 / 27$. Consumers assign status to good 1 at every period. This means that good 1 is more valuable than it would be in the absence of status. Intuitively speaking, “there is more money on the table” to be shared by the two firms. The following lemmata show that Firm 0 does not have a profitable deviation from $\sigma_M^*$.

Suppose that, in every period leading to period $t - 1$, firms 0 and 1 have been playing actions $a_{0,M}^*$ and $a_{1,M}^*$. Suppose that Firm 0 deviates in period $t$, by using the handicraft technology, $\tau_0(t) = h$, and charging a best response price to $a_{1,M}^*$, namely $p_0^D = 4/3 + 5c/6$. Firm 0 obtains profit $\pi_0^D = (8 - c)^2 / 108$. In the following period, it returns to adopting the industrial technology. However, good 1 does not have status at period $t + 1$, and thus Firm 1 plays $p_1^I = 1 + 2c/3$ and $\tau_1 = h$, while Firm 0 plays $\tau_0 = m$ and $p_0^P = 1 + c/3$, obtaining $\pi_0^P = (3 + c)^2 / 18$.

At period $t + 2$, Firm 1 regains its status and the game returns to the equilibrium path, with firms playing $(a_{M,0}^*, a_{M,1}^*)$ in every period. The following lemma shows that this set of actions does not constitute a profitable deviation for Firm 0.

**Lemma 5** Suppose that $c < 5 - \sqrt{27}/2$ and Firm 0 does not have a short-term incentive to deviate from the equilibrium path, that is, $\pi_0^D < \pi_0^* - K$. Furthermore, suppose that, until period $t - 1$, Firm 0 and Firm 1 have always been playing actions $a_{M,0}^*$ and $a_{M,1}^*$. Consider the deviation at period $t$ in which Firm 0 uses $\tau_0(t) = h$, and charges price $p_0^D = 4/3 + 5c/6$. After period $t$, Firm 0 returns to using the industrial technology, $\tau_0 = m$, every period. In period $t + 1$, Firm 0 plays $\tau_0 = m$ and $p_0^P = 1 + c/3$, while Firm 1 plays $p_1^I = 1 + 2c/3$ and $\tau_1 = h$. At period $t + 2$ and in every period after $t + 2$, both firms play $a_{M,0}^*$ and $a_{M,1}^*$, as in the equilibrium path. This deviation is not profitable for Firm 0.
Could there be another profitable deviation for Firm 0? Once Firm 0 changes its technology, the stage game after the deviation changes in \( t + 1 \), because Firm 1 loses its status. Thus, Firm 0 can potentially obtain a profitable deviation from the prescribed action in \( t + 1 \) after deviating at \( t \). In this case, at \( t + 1 \), instead of playing the prescribed action \( a^P_0 \), Firm 0 can keep using the handicraft technology for yet another period, charging a price best-response to \( \tau_1 = h \) and \( p^P_1 \). The outcomes of this scenario are shown in Appendix Section A.3.11. Firm 0 charges price \( p^{D02}_0 = 1 + 5c/6 \) and obtains profits \( \pi^{D02}_0 = (6 - c)^2/72 \). Lemma 6 below shows that this is not a profitable deviation for Firm 0.

**Lemma 6** Suppose that \( c < 5 - \sqrt{27/2} \) and Firm 0 does not have a short-term incentive to deviate from the equilibrium path, that is, \( \pi^{D0}_0 < \pi^*_0 - K \). Furthermore, suppose that, until period \( t-1 \), firms 0 and 1 had been playing actions \( a^*_M,0 \) and \( a^*_M,1 \) in every period. Consider the deviation by Firm 0 in periods \( t \) and \( t+1 \) as follows: at period \( t \), Firm 0 uses \( \tau_0(t) = h \), and charges price \( p^{D0}_0 = 4/3 + 5c/6 \); at period \( t+1 \), Firm 0 uses \( \tau_0(t+1) = h \), and charges price \( p^{D02}_0 = 1 + 5c/6 \). Firm 1 plays \( \tau_1 = h \) and \( p^P_1 \) in periods \( t+1 \) and \( t+2 \). This is not a profitable deviation for Firm 0.

Firm 0 does not have a profitable deviation from \( \sigma^*_M,0 \). Given the strategy Firm 0 plays, Firm 1 is constrained to use the handicraft technology and already charges a best response price to \( p^*_0 \). Therefore, Firm 1 does not have a profitable deviation either. Hence, \( \sigma^*_M \) is a Nash Equilibrium of the continuation game starting at \( t \).

**Proposition 3** *(Equilibrium of the Game with Commitment)* Suppose that \( c < 5 - \sqrt{27/2} \) and Firm 0 does not have a short-term incentive to deviate from the equilibrium path; that is, \( \pi^{D0}_0 < \pi^*_0 - K \). Then, the strategy profile \( \sigma^*_M \) is a Subgame Perfect Equilibrium of the modified game with commitment \( G_M \).

### 6.3 Comparison to Hotelling Payoffs

First, we would like to compare the equilibrium described in Subsection 6.2 with an alternative equilibrium in which both firms adopt the handicraft technology and pay a standard Hotelling game in every period. Firm 0 would prefer this outcome if it attains a higher value in this equilibrium. The relevant technology pair for our benchmark Hotelling equilibrium is then \((h,h)\).

Suppose that the firms play a standard Hotelling game with both firms adopting the handicraft technology at every period. In period \( t = 0 \), consumers assign status to good 1, and for every period \( t \geq 1 \), consumers assign status to neither firm. Firm 0 obtains \( \pi^{HS}_0 = 16/27 \) at \( t = 0 \) and \( \pi^H_0 = 1/2 \) at every period \( t \geq 1 \). Then, Firm 0’s value is:

\[
V^H_0(h,h) = \pi^{HS}_0 + \frac{\delta}{1-\delta} \pi^H_0.
\]
Substituting the profit expressions for $\pi^H_0$ and $\pi^{HS}_0$ yields:

$$V^H_0(h,h) = \frac{16}{27} + \frac{\delta}{1-\delta} \frac{1}{2} = \frac{32 - 5\delta}{54 (1-\delta)}.$$ 

If Firm 0 follows $\sigma^*_{M,0}$, it obtains payoffs of $\pi^*_0 - K = (4 + c)^2 / 27 - K$ at every period. Firm 0’s value becomes:

$$V^*_0 = \frac{\pi^*_0 - K}{1-\delta} = \frac{1}{1-\delta} \left[ \frac{(4 + c)^2}{27} - K \right] = \frac{(4 + c)^2}{27} - \frac{27K}{27 (1-\delta)}.$$ 

Firm 0 has a higher value in the equilibrium $\sigma^*_M$ of the game with commitment than in the equilibrium of the standard Hotelling game in which both firms use the handicraft technology if $V^*_0 > V^H_0(h,h)$, that is, if:

$$\frac{\pi^*_0 - K}{1-\delta} \geq \pi^{HS}_0 + \frac{\delta}{1-\delta} \pi^H_0.$$ 

Substituting the profit expressions for $\pi^H_0$, $\pi^{HS}_0$, and $\pi^*_0$ yields:

$$\frac{(4 + c)^2}{27} - K \geq \frac{32 - 5\delta}{54}.$$ (15)

This inequality is equivalent to $\delta \geq \delta^C_0$, where $\delta^C_0$ is:

$$\delta^C_0 = \frac{\pi^{HS}_0 - (\pi^*_0 - K)}{\pi^{HS}_0 - \pi^H_0} = \frac{54K - 16c - 2c^2}{5}.$$ 

We have that $\delta^C_0 < 1$. To see that, note that the case in which $\delta = 1$ corresponds to the situation in which firms do not discount the future, and firm value is simply the sum of stage game payoffs. In this case, inequality $\delta < \delta^C_0$ reduces to $\pi^*_0 - K > \pi^H_0$, which holds by assumption.

We would also like to compare the equilibrium described in Subsection 6.2 with an alternative equilibrium in which both firms adopt the industrial technology and pay a standard Hotelling game in every period. Firm 1 would prefer this outcome if it attains a higher value in this equilibrium. The relevant technology pair for our benchmark Hotelling equilibrium is then $(m,m)$.

Suppose then that both firms use the industrial technology and play standard Hotelling game at every period. In period $t = 0$, consumers assign status to good 1, and for every period $t \geq 1$, consumers assign status to neither firm. Firm 1 obtains $\pi^{HS}_1 - K = 25/27 - K$ at $t = 0$ and $\pi^H_0 - K = 1/2 - K$ at every period $t \geq 1$. Firm 1’s value is, then:

$$V^H_1(m,m) = \pi^{HS}_1 - K + \frac{\delta}{1-\delta} \left[ \pi^H_1 - K \right].$$
Substituting the profit expressions for $\pi_H^1$, $\pi_H^{HS}$, and $\pi_1^*$ yields:

$$V_H^1(m,m) = \frac{25}{27} - K + \frac{\delta}{1 - \delta} \left[ \frac{1}{2} - K \right] = \frac{50 - 23\delta - 54K}{54(1 - \delta)}.$$  

If Firm 1 follows $\sigma_{M,1}^*$, it obtains payoffs of $\pi_1^* = (5 - c)^2/27$ at every period. Firm 1’s value is, then:

$$V_1^* = \frac{\pi_1^*}{1 - \delta} = \frac{(5 - c)^2}{27(1 - \delta)}.$$  

Firm 1 has a higher value under equilibrium $\sigma_M^*$ in the game with commitment than in the equilibrium of the standard Hotelling game in which both firms use the industrial technology if $V_1^* > V_H^1(m,m)$, that is, if:

$$\frac{\pi_1^*}{1 - \delta} \geq \frac{\pi_H^{HS} - K}{1 - \delta} \left[ \frac{\pi_H^1 - K}{\pi_H^{HS} - \pi_H^1} \right].$$  

Substituting the profit expressions for $\pi_H^1$, $\pi_H^{HS}$, and $\pi_1^*$ yields:

$$\frac{(5 - c)^2}{27(1 - \delta)} \geq \frac{50 - 23\delta - 54K}{54(1 - \delta)}.$$  

This inequality is equivalent to $\delta \geq \delta_1^C$, where:

$$\delta_1^C = \frac{\pi_H^{HS} - K - \pi_1^*}{\pi_H^{HS} - \pi_H^1} = \frac{20c - 2c^2 - 54K}{23}.$$  

Given the assumptions $c < 5 - \sqrt{27}/2$ and $\pi_0^* - K > \pi_0^H$, it is possible to prove that $\delta_0^C < 1$. Hence, $54K < 2c^2 + 16c + 5$.

The lemma below shows that $\delta_1^C > \delta_0^C$. Thus, Firm 1 has a higher value under standard Hotelling competition for a larger set of values of $\delta$ than Firm 0. In other words, whenever Firm 1 has a higher value in the equilibrium $\sigma_M^*$ than in the repetition of standard Hotelling equilibrium at every period, so does Firm 0. The converse, however, is not true. For $\delta_0^C < \delta < \delta_1^C$, Firm 0 has a higher value under $\sigma_M^*$ than under standard Hotelling with handicraft technology, that is, $V_0^* > V_0^H(h,h)$. However, Firm 1 has a higher value under standard Hotelling with the industrial technology than under commitment.

**Lemma 7** If Firm 1 has a higher value $V_1^*$ under $\sigma_M^*$ than in the equilibrium of a standard Hotelling game $V_1^H(m,m)$, then Firm 0 also has a higher value $V_0^*$ under $\sigma_M^*$ than in the equilibrium of a standard Hotelling game $V_0^H(h,h)$, but the converse is not true. Mathematically, $\delta_1^C > \delta_0^C$.  

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The results of Lemma 7 imply that $\delta \geq \delta_C^1$ is the sufficient condition for the equilibrium under commitment to yield an outcome that both firms would consider preferable than what they would obtain in the corresponding standard Hotelling game.

The equilibrium $\sigma^*$ of the original game is qualitatively similar to the equilibrium $\sigma_M^*$ of the game with commitment, yielding the same payoffs $V_0^*$ and $V_1^*$ to each firm. The requirement for $\sigma^*$ to be a Subgame Perfect Equilibrium in the original game is that $\delta \geq \hat{\delta}_1$. The requirement for the equilibrium in the game with commitment to yield an outcome that Pareto dominates (considering payoffs of firms alone) standard Hotelling is that $\delta \geq \delta_C^1$. We can ask then if the option of commitment helps implement the Pareto dominant outcome (from the point of view of firms alone) yielding $V_0^*$ and $V_1^*$ to each firm as a Subgame Perfect Equilibrium. The next proposition proves that $\delta_C^1 < \hat{\delta}_1$. Thus, the measure of the set of discount factors yielding $V_0^*$ and $V_1^*$ as Subgame Perfect Equilibrium payoffs is larger under commitment.

**Proposition 4** If Firm 1 has a short-term incentive to deviate from the equilibrium path with product differentiation on status, then the set of discount factor values such that there is an equilibrium with product differentiation on status in the game without commitment is strictly contained in the set of discount factor values such that there is an equilibrium with product differentiation on status in the game with commitment. Formally, if (6) holds, then $\delta_C^1 < \hat{\delta}_1$.

Proposition 4 states that, if $\delta_C^1 \leq \delta < \hat{\delta}_1$, then: (i) Firm 1 is able to credibly offer status goods to consumers in the game with commitment, but not in the game without them; (ii) Firm 1 obtains a higher equilibrium payoff in the repeated game with commitment than in the standard Hotelling game in which both firms use the industrial technology.

### 6.4 Alternative Commitment Devices

It is not common for stockholders of a private firm to completely forbid their managers from ever issuing publicly traded securities. In practice, not all decisions regarding the firm’s actions are made by the same agents. Shareholders design the firm’s constitution, but have little saying in the many decisions that shape the firm’s daily operations. Managers, on the other hand, cannot change the firm’s constitution, but make all daily operational decisions. Stockholders of a private firm can establish procedures to make the issuance of new shares and new bonds costly, virtually eliminating the managers’ option of accessing public capital markets. These procedures impose extra costs on the issuance of publicly traded securities.

Examples of such procedures include debt covenants between firms and banks that might provide a company with short-term loans for working capital, and supermajority vote requirements for the approval of issuance of new shares. Breaking debt covenants might mean that the firm
would have to pay contractual fees and fines. In order to be able to obtain an IPO/SEO approval from the Board of Directors and/or the General Shareholders' Meetings, managers might have to engage in costly proxy fights to obtain the necessary votes. One way or another, obtaining resources from public capital markets becomes a more expensive endeavor. Many studies have theorized that debt covenants exist to solve agency problems. More specifically, the literature has theorized that debt covenants exist to mediate conflicts of interest between debtholders and managers acting in the name of shareholders that occur in the presence of asymmetric information.\(^9\) This section presents a novel function of such covenants: by indirectly increasing the issuance cost to firms, if the increase they induce in these costs is sufficiently high, they act as a commitment device that prevents the firm from having access to the industrial technology.

Why would a firm go to such great lengths to purposely increase its own cost of issuance of publicly traded securities? We argue that, by doing so, a firm can make access to industrial technology through public financing so expensive that it would never actually do it.

This behavior may be beneficial in the context of the model analyzed in Section 4. Suppose that Firm 1 is not sufficiently patient, in the precise sense that \(\delta < \hat{\delta}_1\). Hence, the strategy profile \(\sigma^*\) is not a Subgame Perfect Equilibrium of the game \(G\). Including these costly issuance procedures is equivalent to raising the public financing cost \(K\).

Let us reconsider the game described in Subsection 6.2. Let \(F \in [0, +\infty]\) represent the additional costs Firm 1 incurs each time it issues publicly traded securities. At the beginning of the game, Firm 1’s shareholders make a continuous choice of \(F\). At every stage game, the total cost of issuing publicly traded securities are given by \(K_F = K + F\). Is it possible to choose \(F\) such that a Subgame Perfect Equilibrium of this newly modified game yields the same payoffs as \(\sigma^*\)?

Recall from equations (8) and (11) that \(\hat{\delta}_1\) and \(\tilde{\delta}_1\) are decreasing in \(K\). In this new game with endogenous issuance costs, \(\hat{\delta}_1\) and \(\tilde{\delta}_1\) are now functions of \(K_F\). Substitute \(K_F = K + F\) into the definition of \(\hat{\delta}_1\). Then, inequality \(\delta \geq \hat{\delta}_1\) becomes \(\delta \geq \hat{\delta}_1(F)\), where:

\[
\hat{\delta}_1(F) = \frac{\pi_1^D - K - F - \pi_1^s}{\pi_1^s - \pi_1^P}.
\] (19)

If \(\delta > \delta^C_1\), Firm 1 wants to set \(F\) such that it has the incentives to remain private, use the handicraft technology and set price \(p_1 = p_1^*\) at every period. For this behavior to be in Firm 1’s best interest, \(F\) has to be set such that both of Firm 1’s incentive constraints hold, namely, Firm 1 does not have incentives to deviate from the equilibrium path \((\delta \geq \hat{\delta}_1)\), and Firm 1 has incentives to comply with the prescribed action once it lost its status and return to the equilibrium path once it has deviated \((\delta \geq \tilde{\delta}_1)\). Proposition 1 shows that \(\hat{\delta}_1 \geq \tilde{\delta}_1\) and, thus,

\(^9\)See, for instance, Jensen and Meckling (1976), Galai and Masulis (1976), and Myers (1977).
\( \delta \geq \hat{\delta}_1 \) is the stricter of the two constraints. Therefore, it suffices for us to focus on the inequality \( \delta \geq \hat{\delta}_1(F) \). Solving this inequality for \( F \) results in \( F \geq F^C \), where:

\[
F^C = \pi_1^D - \pi_1^* - K - \delta \left( \pi_1^* - \pi_1^P \right). \tag{20}
\]

When \( F = 0 \), \( \hat{\delta}_1(0) \) equals \( \hat{\delta}_1 \), the threshold discount rate for the original game \( G \). By equation (19), we have that \( \pi_1^D - K - F = \hat{\delta}_1(0) \left( \pi_1^* - \pi_1^P \right) \). Substituting this expression into equation (20) and simplifying yields:

\[
F^C = \left( \hat{\delta}_1(0) - \delta \right) \left( \pi_1^* - \pi_1^P \right). \tag{21}
\]

Substitute the profit expressions for \( \hat{\delta}_1(0), \pi_1^P \), and \( \pi_1^* \) into \( \hat{\delta}_1(F) \) to obtain:

\[
F^C = \frac{c (20 - c)}{36} - \frac{\delta (23 - 2c - c^2)}{54} - K. \tag{22}
\]

Therefore, we can enunciate the following proposition:

**Proposition 5** Suppose that \( c \leq 5 - \sqrt{27}/2 \), and that before \( t = 0 \), Firm 1 can choose procedures that lead to any increase \( F \) in the costs of issuance of stocks and bonds. Then, there exists a Subgame Perfect Equilibrium of the dynamic game with costly issuance procedures in which the private firm (Firm 1) produces a status good while the public firm (Firm 0) produces a good with no status. In this equilibrium, Firm 1’s shareholders establish procedures to issue new stocks and bonds leading to the procedural costs \( F^C \) defined in equation (22). The overall payoffs of the firms are \( V_0^* \) and \( V_1^* \), as stated by equations (23) and (24). Along the equilibrium path, Firm 0 uses the industrial technology and Firm 1 uses the handicraft technology. Firms sell their goods at prices \( p_0^* \) and \( p_1^* \), given by (29) and (30), and sell their goods to market niches \( i^* \) and \( 1 - i^* \), given by (31) and (32). Firm 0 can sustain its production through public financing by issuing shares and/or debt.

The following remark shows that the break-even point for Firm 1’s short-term incentives occurs for \( \delta \leq \delta_1^C \) and, therefore, \( F^C \) is decreasing in \( \delta \) in the relevant region, that is, for \( \delta \) in the interval \([\delta_1^C, \hat{\delta}_1]\).

**Remark 2** Suppose \( c < 5 - \sqrt{27}/2 \) and \( \delta_1^C \leq \delta \leq \hat{\delta}_1 \). Suppose further that the value of \( F^C \) is as defined in (22). Then, Firm 1 still has short-term incentives to deviate, that is, \( 108 \left( K + F^C \right) < 20c - 3c^2 \), as in the dynamic game of Section 4.

From equation (22), it is straightforward to verify that \( F^C \) is linearly decreasing in \( K \), with \( dF^C/dK = 1 \). The intuition for this feature is straightforward. Because both \( K \) and \( F \) increase the overall costs of external financing, they are one-to-one perfect strategic substitutes.
An increase of one dollar in financing cost $K$ reduces in one dollar the necessary amount of endogenously driven issuance costs $F$ to sustain the equilibrium.

Because $c < 5 - \sqrt{27}/2$, we have that $c < 2\sqrt{6} - 1$, and therefore $\pi_1^* - \pi_1^P > 0$. Hence, the term $F^C$ is decreasing in $\delta$, unless Firm 1’s short-term deviation constraint does not hold. Start with $\delta < \hat{\delta}_1(0)$. As the discount factor increases, the gap between the actual $\delta$ and the necessary discount factor to implement $\sigma^*$ as a Subgame Perfect Equilibrium of the original game $G$ is reduced. Hence, the necessary increase in issuance costs $F$ to implement the same payoffs in the equilibrium of the game with endogenous issuance costs decreases.

As the marginal cost $c$ of producing with the handicraft technology increases, deviating from the equilibrium path becomes more tempting to Firm 1. This increased temptation occurs as a result of the fact that, as Firm 1 deviates and adopts the industrial technology, it no longer has to pay this marginal cost. To prevent a more tempting deviation requires a higher self-imposed issuance cost for publicly traded securities, that is, a larger $F^C$. This fact can be easily verified by taking the derivative of $F^C$ with respect to $c$ in equation (22). The next remark summarizes the comparative statics results for $F^C$.

**Remark 3** The necessary amount $F^C$ of endogenously driven issuance costs $F$ to sustain the equilibrium is linearly decreasing in $K$, with $dF^C/dK = 1$, linearly increasing in $c$, that is, $dF^C/dc > 0$, and decreasing in $\delta$.

Assume shareholders control the firm’s constitution, and are able to pick the value of the issuance procedures cost $F$ before $t = 0$. Afterwards, all decisions regarding production technologies and prices are made by Firm 1’s manager at every period. These procedures are chosen so that Firm 1’s managers never choose to go public, and thus Firm 1 credibly offers the status good.

### 7 Conclusion

The current study provides a model for the financing of status goods production. Two firms compete in a Hotelling market in which a product may yield status utility to their consumers if this product is the only one produced with a handicraft technology in the previous period (or, alternatively, a product yields status utility to its consumers if it is the most exclusive good, status is an experience good and information regarding status is learned socially through a “word of mouth goes quickly” process). Firms can attain equilibria in which they both use the same technology and charge prices symmetrically in a standard Hotelling model. In this case, neither good yields status utility to consumers. Another equilibrium has one firm producing with the handicraft technology while the other produces with the industrial technology. The handmade
good has status value to its consumers. The status good firm has a short-term incentive to switch to the industrial technology, mass-produce its product and obtain high profits for one period, while its product still enjoys status value, before it loses it. If the discount factor is sufficiently high, then this industrial configuration can be sustained as a Subgame Perfect Equilibrium, regardless of the firms’ ownership and capital structures. Merely adding financial markets to the model does not change this result.

For values of the discount rate that are not sufficiently high, the industry configuration described above cannot be sustained as a Subgame Perfect Equilibrium. A status good producing firm can then commit to not issuing public securities, thus remaining private. This restrictive financing policy eliminates the status good firm’s access to the industrial technology. This commitment policy can be implemented through provisions in the firm’s constitution preventing the firm from issuing public securities, or through adding costly procedures that would purposely drive issuance costs up to the point that the firm’s managers would never issue securities. The status good producing firm can produce using the handicraft technology only. In this setup, an industry configuration in which the private firm produces using the handicraft technology while the other firm goes public and produces using the industrial technology can be attained at a Subgame Perfect Equilibrium. The private firm’s product has status value, while the public firm’s product does not. For intermediate values of the discount rate, both firms attain a higher value than in a standard Hotelling equilibrium in which both firms adopt the same technology.

If the discount rate is reflected in the equilibrium interest rates and firms’ stock and bond returns, and cost of capital in general, the model just presented has the following empirical implication: in countries with high discount factors, interest rates are low and, thus, firms’ costs of capital are low. In these countries, we should see more firms that offer status goods having publicly traded securities than in countries with high interest rates and firms’ costs of capital (low discount rates). In countries with moderate interest rates and costs of capital, it would be difficult to sustain the equilibrium described in Section 4, but the outcome with commitment described in Section 6 can be attained at a Subgame Perfect Equilibrium. It follows that in countries with moderate interest rates, we should observe a tendency of status goods producing firms to be private. In countries with high interest rates, firms would have a higher value if they do not produce a status good and instead adopt the same technology of their rivals and compete in a standard Hotelling game producing a good that yields no status value to consumers. As a result, in such countries, we would observe very few status goods firms operating, and these firms would tend to be private. Also, changes in legal and institutional environments that reduce firms’ costs of capital would cause some status goods producing firms to go public, so we should observe an increase in the number of IPOs from firms in this industry following the legal or institutional change. Legal and institutional changes that increase firms’ costs of capital would
have the opposite effect.

A Appendix: Proofs

A.1 Introduction
No proofs.

A.2 Setup
No proofs.

A.3 Stage Game Analysis

A.3.1 Consumer Demands

Proof. (of Lemma 1) Consumers indexed at \(i^*\) is indifferent between purchasing good 0 or 1, that is:

\[
(u - 1 + i^*) + S_t(1) [1 - D_t(1, i^*)] - p_1 = (u - i^*) + S_t(0) [1 - D_t(0, i^*)] - p_0.
\]

Cancelling the common term \(u\) on both sides:

\[
-1 + i^* + S_t(1) [1 - D_t(1, i^*)] - p_1 = -i^* + S_t(0) [1 - D_t(0, i^*)] - p_0.
\]

If consumers do not assign status to either good, then \(S_t(0) = S_t(1) = 0\). Hence, \(-1 + i^* - p_1 = -i^* - p_0\). Solving this last equation for \(i^*\) and \(1 - i^*\) yields equations (1) and (2). When only Firm 1’s good is perceived to have status, then \(S_t(0) = 0\) and \(S_t(1) = 1\). In this case, \(-1 + i^* + [1 - D_t(1, i^*)] - p_1 = -i^* - p_0\). Because Firm 1’s demand is \(D_t(1, i^*) = 1 - i^*\), and thus \(1 - D_t(1, i^*) = i^*\). Hence, \(-1 + i^* + i^* - p_1 = -i^* - p_0\). Solving this equation for \(i^*\) and \(1 - i^*\) yields equations (3) and (4). ■

A.3.2 Stage Game Along the Equilibrium Path

Lemma 8 Equilibrium path prices, demands, and operational profits are, respectively, \(p_0^* = 4/3 + c/3\), \(p_1^* = 5/3 + 2c/3\), \(i^* = 4/9 + c/9\), \(1 - i^* = 5/9 - c/9\), \(\pi_0^* = (4 + c)^2/27\), and \((5 - c)^2/27\). Furthermore, the value functions, evaluated at the equilibrium values, are:

\[
V_0^* = \frac{(4 + c)^2}{27(1 - \delta)} - \frac{K}{(1 - \delta)}, \quad (23)
\]

\[
V_1^* = \frac{(5 - c)^2}{27(1 - \delta)}. \quad (24)
\]
Proof. (of Lemma 8) Let us analyze an equilibrium with product differentiation on status in which one firm, say, Firm 1, adopts the handicraft technology to produce its good while Firm 0 adopts the industrial technology. Then, good 1 will be the more expensive good to produce and will be the more exclusive. Consumers attribute high status to this good if both goods are sold at the equilibrium prices (calculated below) $p^*_0$ and $p^*_1$ and low status to good 0. If things remain unchanged in the next period, the learned status level of goods for the next period are the same as their current status levels. The demands for each product in this case were calculated in the proof of Lemma 1, and are given by equations (3) and (4). The operational profit function of the firm producing regular good is $\pi_0 = p_0^i*$. Substituting equation (3) into Firm 0’s operational profit function yields:

$$\pi_0 = p_0 \left(1 + p_1 - p_0\right).$$

(25)

By equation (23), $\arg \max_{p_0} V^*_0 = \arg \max_{p_0} \pi_0$. Substituting the operational profit function (25) into the old shareholder’s value function yields:

$$V^*_0 = \frac{(1 - \phi)p_0[1 + p_1 - p_0]}{3(1 - \delta + \phi\delta)}.$$

which is the objective function to be maximized by Firm 0’s manager. The first order condition for this firm’s value maximization problem is $1 + p_1 - 2p_0 = 0$. Firm 0’s best reply is:

$$p^{BR}_0 = \frac{1}{2} \left[1 + p_1\right].$$

(26)

We can now turn to Firm 1. The operational profit of Firm 1 when producing the status good is:

$$\pi_1 = (1 - i^*) (p_1 - c)$$

$$= \left(\frac{2 - p_1 + p_0}{3}\right) (p_1 - c).$$

(27)

By equation (24), $\arg \max_{p_1} V^*_1 = \arg \max_{p_1} \pi_1$. The first order condition for Firm 1’s operational profit maximization problem is $-2p_1 + p_0 + c + 2 = 0$. Firm 1’s best reply is:

$$p^{BR}_1 = \frac{1}{2} \left[p_0 + c + 2\right].$$

(28)

We can now calculate the market equilibrium prices and market niches of each firm which will constitute the equilibrium in the product market. The system of best replies of each firm is:

$$\begin{cases} 
2p_0 = 1 + p_1 \\
2p_1 = p_0 + c + 2
\end{cases}$$

\(^{10}\)Because the game is symmetric, all results with firm 1 adopting the industrial technology are a mirror image of the results displayed here.
Solving this system leads to equations:

\[ p_0^* = \frac{4}{3} + \frac{1}{3}c, \quad (29) \]
\[ p_1^* = \frac{5}{3} + \frac{2}{3}c. \quad (30) \]

The difference between these equilibrium prices are:

\[ p_1^* - p_0^* = \frac{1 + c}{3}. \]

Substituting these prices into the demand equations yield equations:

\[ i^* = \frac{4 + c}{9}, \quad (31) \]
\[ 1 - i^* = \frac{5 - c}{9}. \quad (32) \]

Substituting these into the operational profit functions yield the results of equations:

\[ \pi_0^* = \frac{(4 + c)^2}{27}, \quad (33) \]
\[ \pi_1^* = \frac{(5 - c)^2}{27}. \quad (34) \]

Substituting these solutions into the value functions yield equations (23) and (24).

If the equilibrium prices are given by equations (29) and (30), consumer \( i^* \)'s utilities become:

\[ U_i^*(p_0^*) = U_i^*(p_1^*) = u - \frac{5 - c}{9} - \frac{4 + c}{3}. \]

For consumers with index \( i^* \), it is individually rational to buy the good if \( U_i^*(p_0^*) = U_i^*(p_1^*) \geq 0 \), or:

\[ u \geq \frac{5 - c}{9} + \frac{4 + c}{3}. \]

At the solution, the operational profit of Firm 0 is larger than the operational profit of Firm 1, or, mathematically, \( \pi_0^* > \pi_1^* \), if and only if \( c > 1/2 \). Firm 1 produces the high status good. In order for this good to have smaller demand than the regular good (produced by Firm 0), it is necessary and sufficient that \( i^* > 1/2 \). This condition is equivalent to \( c > 1/2 \). This implies that Firm 0’s operational profits are larger than Firm 1’s, that is, \( \pi_0^* > \pi_1^* \). The good produced by Firm 1 has positive demand when \( i^* < 1 \). This condition is equivalent to \( c < 5 \).

A.3.3 Firm 0’s One-Shot Deviation

**Lemma 9** The stage game outcome when Firm 1 deviates is such that the prices, demands, and operational profits \( \pi_0^D \) and \( \pi_1^D \) are given by \( i^{D0} = 4/9 - c/18 \), \( 1 - i^{D0} = 5/9 + c/18 \), \( p_0^{D0} = p_0^* = 4/3 + 5c/6 \), \( p_1^{D0} = p_1^* = 5/3 + 2c/3 \), \( \pi_0^{D0} = (8 - c)^2/108 \), and \( \pi_1^{D0} = (5/3 - c/3)(5/9 + c/18) \).
Proof. (of Lemma 9) Firm 0’s operational profits are given by:

\[ \pi_0^{D0} = (p_0^{D0} - c) i^{D0}. \] (35)

Substituting the demand \( i^{D0} \) in equation (3) results in:

\[ \pi_0^{D0} = \left( \frac{1 + p_1 - p_0^{D0}}{3} \right) (p_0^{D0} - c). \]

Firm 0 maximizes this expression to obtain maximum current operational profit. The first-order condition to this problem is \( 1 + p_1 + c - 2p_0 = 0 \). Firm 0’s best reply is:

\[ p_{0, BR}^{D0} = \frac{1 + p_1 + c}{2}. \]

Firm 1 does not observe Firm 0’s action until it is played. Therefore, in this period, it plays \( p_1^{D0} = p_1^*. \) The system of equations that give us the deviation period prices is:

\[
\begin{aligned}
& p_1^{D0} = p_1^*, \\
& 2p_0^{D0} = 1 + p_1^{D0} + c.
\end{aligned}
\]

The prices solving this system are given by equations

\[ p_0^{D0} = p_0^* = \frac{4}{3} + \frac{5}{6} c, \] (36)

\[ p_1^{D0} = p_1^* = \frac{5}{3} + \frac{2}{3} c. \] (37)

Substituting \( p_1^{D0} \) and \( p_0^{D0} \) into (3) and (4) yields demands in equations

\[ i^{D0} = \frac{4}{9} - \frac{1}{18} c, \] (38)

\[ 1 - i^{D0} = \frac{5}{9} + \frac{1}{18} c. \] (39)

Substituting (36), (37), (38), and (39) back into the operational profit functions lead to equations

\[ \pi_0^{D0} = \left( \frac{4}{3} - \frac{1}{6} c \right) \left( \frac{4}{9} - \frac{1}{18} c \right) = \frac{(8 - c)^2}{108}, \] (40)

\[ \pi_1^{D0} = \left( \frac{5}{3} - \frac{1}{3} c \right) \left( \frac{5}{9} + \frac{1}{18} c \right). \] (41)

\[ \blacksquare \]

A.3.4 Firm 1’s One-Shot Deviation

Lemma 10 The stage game outcome when Firm 1 deviates is such that the prices, demands, and operational profits \( \pi_0^D \) and \( \pi_1^D \) are given by \( p_0^D = p_0^* = 4/3 + c/3, \) \( p_1^D = 5/3 - c/6, \) \( i^D = 4/9 + c/18, \)

\( 1 - i^D = 5/9 - c/18, \) \( \pi_0^D = (4 + c)(8 + c)/27, \) and \( \pi_1^D = (10 - c)^2/108. \)
Proof. (of Lemma 10) Firm 1’s operational profits are given by:

\[ \pi^D_1 = (1 - i^D)p^D_1. \]  (42)

Substituting the demand \(1 - i^D\) in equation (4) results in:

\[ \pi^D_1 = \left( \frac{2 - p^D_1 + p_0}{3} \right) p^D_1. \]

Firm 1 maximizes this expression to obtain maximum current operational profit. The first-order condition to this problem is \(2 - p_0 - 2p_1 = 0\). Firm 1’s best reply is:

\[ p^{D, BR}_1 = 1 + \frac{1}{2}p_0. \]

Firm 0 does not observe Firm 1’s action until it is played. Therefore, in this period, it plays \(p^D_0 = p^*_0\). The system of equations that give us the deviation period prices is:

\[
\begin{cases}
 p^D_0 = p^*_0 \\
 2p^D_1 = 2 + p^D_0.
\end{cases}
\]

The prices solving this system are given by equations:

\[ p^D_0 = p^*_0 = \frac{4}{3} + \frac{1}{3}c, \quad (43) \]
\[ p^D_1 = \frac{5}{3} + \frac{1}{6}c. \quad (44) \]

Substituting \(p^D_1\) and \(p^D_0\) into (3) and (4) yields demands in equations:

\[ i^D = \frac{4}{9} - \frac{1}{18}c, \quad (45) \]
\[ 1 - i^D = \frac{5}{9} + \frac{1}{18}c. \quad (46) \]

Substituting (43), (44), (45), and (46) back into the operational profit functions lead to equations:

\[ \pi^D_0 = \left( \frac{4}{3} + \frac{1}{3}c \right) \left( \frac{4}{9} - \frac{1}{18}c \right), \quad (47) \]
\[ \pi^D_1 = \left( \frac{5}{3} + \frac{1}{6}c \right) \left( \frac{5}{9} + \frac{1}{18}c \right) = \frac{(10 + c)^2}{108}. \quad (48) \]

A.3.5 Returning to the Equilibrium Path

Firm 1 uses the handicraft technology, while Firm 0 uses the industrial technology, and the technology choices are common knowledge. In the equilibrium of this stage game, Firm 0 just plays a best reply to Firm 1’s price. Firm 1 plays a best reply to Firm 0’s price, taking into consideration the fact that consumers consider that Firm 1’s product has no status value, which is given by (28). Firm 0 chooses prices according to equation (26).
Lemma 11  The stage game equilibrium occurring in the period immediately after Firm 1 had deviated has prices, demands, and operational profits $\pi_0^P$ and $\pi_1^P$ given by, respectively, $p_0^P = 1 + c/3$, $p_1^P = 1 + 2c/3$, $i^P = 1/2 + c/6$, $1 - i^P = 1/2 - c/6$, $\pi_0^P = (1 + c/3)^2 / 2$, and $\pi_1^P = (1 - c/3)^2 / 2$.

Proof. (of Lemma 11) Firm 0’s operational profit function is:

$$\pi_0^P = p_0i^P = p_0 \left( \frac{1 + p_1 - p_0}{2} \right).$$

Firm 0 maximizes this function to obtain first order conditions:

$$1 + p_1 - 2p_0 = 0.$$ 

Firm 0’s best reply is:

$$p_0^{P,BR} = \frac{1}{2}[1 + p_1]. \quad (49)$$

We can now turn to Firm 1’s operational profit maximization problem. Its operational profit is:

$$\pi_1 = (1 - i^*)(p_1 - c) = \left( \frac{1 - p_1 + p_0}{2} \right)(p_1 - c).$$

The first order condition for Firm 1’s operational profit maximization problem is $-2p_1 + p_0 + c + 1 = 0$. Firm 1’s best reply is:

$$p_1^{P,BR} = \frac{p_0 + c + 1}{2}. \quad (50)$$

Solving the resulting system of best reply equations (49) and (50) yields the equilibrium prices

$$p_0^P = 1 + \frac{1}{3}c, \quad (51)$$

$$p_1^P = 1 + \frac{2}{3}c. \quad (52)$$

Substituting then back into (3) gives us stage game equilibrium values of the demands

$$i^P = \frac{1}{2} + \frac{1}{6}c, \quad (53)$$

$$1 - i^P = \frac{1}{2} - \frac{1}{6}c. \quad (54)$$

Substituting these values back in the profits functions give us the profit values

$$\pi_0^P = \frac{1}{2} \left( 1 + \frac{1}{3}c \right)^2, \quad (55)$$

$$\pi_1^P = \frac{1}{2} \left( 1 - \frac{1}{3}c \right)^2. \quad (56)$$
A.3.6 Alternative to Regaining Status

This section covers the scenario in which consumers do not assign status to any good, Firm 0 believes that Firm 1 will try to regain its status by adopting the handicraft technology and charging price $p_1^P = 1 + 2c/3$, and thus Firm 0 adopts the industrial technology and charges price $p_0^P = 1 + c/3$. Firm 1 may have a short-run incentive to keep adopting the industrial technology and change a price that maximizes its one-period profit. The following lemma computes the stage game outcome when Firm 1 pursues this short-term incentive.

**Lemma 12** Suppose that, at period $t$, Firm 0 adopts the industrial technology, that is, $\tau_0(H_t) = m$, and suppose that they chose the same technologies in period $t-1$, with $\tau_0(H_{t-1}) = \tau_1(H_{t-1}) = m$, and different technologies in $t-2$, with $\tau_0(H_{t-2}) = m$ and $\tau_1(H_{t-2}) = h$. Suppose that both firms are playing a dynamic strategy that requires Firm 1 to return to the equilibrium path by adopting the handicraft technology, that is, $\tau_1(H_t) = h$. Firm 0 plays the prescribed action $p_0^{ARR} = p_0^P = 1 + c/3$. Suppose further that Firm 1 deviates from the dynamic equilibrium prescription by playing a one-period best-response to Firm 0’s prescribed action. The outcome of the stage game, prices, demands, and operational profits are, respectively, $p_1^{ARR} = p_1^P = 1 + c/6$, $i^{ARR} = (6 - c)/12$, $1 - i^{ARR} = (6 + c)/12$, $\pi_0^{ARR} = (6 - c)(c + 3)/36$, and $\pi_1^{ARR} = (6 + c)^2/72$.

**Proof.** (of Lemma 12) Firm 1 maximizes its one-period profits, that is,

\[
p_1^{ARR} = \arg \max_{p_1} (1 - i^* (p_0^P, p_1)) = \arg \max_{p_1} \left( \frac{1 - p_1 + p_0^P}{2} \right).
\]

The first order condition to this problem is:

\[
1 - 2p_1 + p_0^P = 0,
\]

and thus

\[
p_1 = \frac{1 + p_0^P}{2} = \frac{1 + (1 + c/3)}{2} = 1 + \frac{c}{6}.
\] \hfill (57)

Substituting (57) into (3) and (4) yields the demands for goods 0 and 1:

\[
i^{ARR} = \frac{6 - c}{12},
\] \hfill (58)

\[
1 - i^{ARR} = \frac{6 + c}{12}.
\] \hfill (59)
Substituting \( p_0^{ARR} = p_0^P = 1 + c/3 \), and equations (57), (58) and (59) into the operational profit functions yield:

\[
\begin{align*}
\pi_0^{ARR} &= \frac{(6 - c)(c + 3)}{36}, \\
\pi_1^{ARR} &= \frac{(6 + c)^2}{72}.
\end{align*}
\]

A.3.7 Hotelling With the Same Technology and No Status

Suppose that consumers attribute no status to either good. Suppose further that both firms adopt the same technology, and that these facts are common knowledge between the firms. In this case, the model degenerates into a model that is analogous to the standard Hotelling model of spatial competition. The next result describes the outcome of the stage game in this case.

**Lemma 13** In the Nash Equilibrium of the stage game in which both firms use the same technology and consumers attribute no status to either good, both firms charge the same price. Prices are given by

\[ p_0^H(\tau) = 1 + c.I_h(\tau) = p_1^H(\tau), \] where \( I_h(\cdot) \) is the handicraft technology indicator function. In particular, \( p_0^H = 1 + c = p_1^H \), if both firms adopt the handicraft technology, and \( p_0^H = 1 - p_1^H \) if both firms adopt the industrial technology. Each firm sells its good to half of the market, that is, \( i^H = 1 - i^H = 1/2 \). Their operational profits are \( \pi_0^H = \pi_1^H = 1/2 \).

**Proof.** (of Lemma 13) Firms can, in principle, use either technology to produce their goods. To start with, let us analyze the case in which both firms use the handicraft technology. Firm 0 maximizes its operational profits \( \pi_0^H = (p_0 - c)i^H \). Firm 0’s operational profit function is:

\[ \pi_0^H = p_0i^H = (p_0 - c)\left(1 + \frac{p_1 - p_0}{2}\right). \]

Maximizing this function yields first order conditions:

\[ (1 + p_1 - p_0) - (p_0 - c) = 0. \]

Firm 0’s best reply is:

\[ p_0^{H, BR} = \frac{1}{2} \left[ 1 + p_1 + c \right]. \]

We can now turn to Firm 1’s operational profit maximization problem. By producing the status good, its operational profit is:

\[
\begin{align*}
\pi_1 &= (1 - i^*)(p_1 - c) \\
&= \left(1 - \frac{1}{2} + \frac{p_0}{2}\right)(p_1 - c).
\end{align*}
\]
The first order condition for Firm 1’s operational profit maximization problem is \(-2p_1 + p_0 + c + 1 = 0\). Firm 1’s best reply is:

\[ p_{1,BR}^H = \frac{p_0 + c + 1}{2}. \] (63)

Solving the system of best reply equations in \(p_{0,BR}^H\) and \(p_{1,BR}^H\) to find the equilibrium price leads to:

\[ p_0^H = 1 + c, \] (64)
\[ p_1^H = 1 + c. \] (65)

Substituting these solutions back into (1) and (2), we obtain the threshold index and the demands:

\[ i^H = \frac{1}{2} = 1 - i^H. \]

Each firm obtains operational profit equal to:

\[ \pi_0^H = \pi_1^H = \frac{1}{2}. \] (66)

The equilibrium of the Hotelling model with both firms using the industrial technology can be obtained easily by making \(c = 0\) in the equilibrium described above. In this case, the prices, market niche frontiers and operational profits are:

\[ p_0^{IH} = p_1^{IH} = 1, \]
\[ i^{IH} = \frac{1}{2} = 1 - i^{IH}, \]
\[ \pi_0^H = \pi_1^H = \frac{1}{2}. \] (67)

We have thus completely characterized the equilibrium of the Hotelling setup in which firms use the same technology and there is no status assigned to any good.

Firm 1’s operational profits are higher in this equilibrium with product differentiation than in the Hotelling equilibrium or, mathematically, \(\pi_1^* > \pi_1^H\), if and only if \((5 - c)^2/27 > 1/2\). The profit \(\pi_1^*\) of Firm 1 is larger than its Hotelling profit, \(\pi_1^H\), if only if the marginal cost is sufficiently small in the sense that \(c < 5 - \sqrt{27}/2\).

### A.3.8 Hotelling With Different Technologies and No Status

Suppose consumers attribute no status to either good, that Firm 0 adopts the industrial technology and that Firm 1 adopts the handicraft technology. Suppose further that these facts are common knowledge between the two firms. The next lemma describes the outcome of the stage game in this case.
Lemma 14 Consider the following stage game: consumers attribute no status to either good, Firm 1 adopts the handicraft technology, and Firm 0 adopts the handicraft technology. Suppose further that these facts are common knowledge between the two firms. Then, in the Nash Equilibrium of the stage game, prices, demands, and operational profits are, respectively, \( p_{0}^{HDT} = 1 + c/3 \), \( p_{1}^{HDT} = 1 + 2c/3 \), \( i^{HDT} = 1/2 + c/6 \), \( i^{HDT} = 1/2 - c/6 \), \( \pi_{0}^{HDT} = (3 + c)^{2}/18 \) and \( \pi_{1}^{HDT} = (3 - c)^{2}/18 \).

Proof. (of Lemma 14) Firm 0’s operational profit function is:
\[
\pi_{0}^{HDT} = p_{0}i^{HDT} = p_{0}\left(\frac{1 + p_{1} - p_{0}}{2}\right).
\]
Firm 0 maximizes this function to obtain first order conditions:
\[
1 + p_{1} - 2p_{0} = 0.
\]
Firm 0’s best reply is:
\[
p_{0}^{HDT,BR} = \frac{1}{2} [1 + p_{1}]. \tag{68}
\]
We can now turn to Firm 1’s operational profit maximization problem. Its operational profit is:
\[
\pi_{1} = (1 - i^{*})(p_{1} - c) = \left(\frac{1 - p_{1} + p_{0}}{2}\right)(p_{1} - c).
\]
The first order condition for Firm 1’s operational profit maximization problem is \(-2p_{1} + p_{0} + c + 1 = 0\). Firm 1’s best reply is:
\[
p_{1}^{HDT,BR} = \frac{p_{0} + c + 1}{2}. \tag{69}
\]
Solving the system of best reply equations in \( p_{0}^{HDT,BR} \) and \( p_{1}^{HDT,BR} \) to find the equilibrium price leads to:
\[
p_{0}^{HDT} = 1 + \frac{1}{3}c, \tag{70}
\]
\[
p_{1}^{HDT} = 1 + \frac{2}{3}c. \tag{71}
\]
Substituting these solutions back into (1) and (2), we obtain the threshold index and the demands:
\[
i^{HDT} = \frac{3 + c}{6}, \tag{72}
\]
\[
1 - i^{HDT} = \frac{3 - c}{6}. \tag{73}
\]
Substituting (70), (71), (72) and (73) into the profit functions of Firms 0 and 1 gives us the operational profits:

\[
\pi_{HDT}^0 = \frac{(3 + c)^2}{18}, \tag{74}
\]

\[
\pi_{HDT}^1 = \frac{(3 - c)^2}{18}. \tag{75}
\]

Because the game is symmetric in the absence of status effects, in order to obtain the stage game payoffs firms obtain when Firm 1 adopts the industrial technology while Firm 0 adopts the handicraft technology, just switch the subindices 0 and 1 in equations (70) through (75) to obtain the answer.

A.3.9 Hotelling With Same Technology and Status

Suppose that both firms adopt the same technology at time \( t \), that is, \( \tau_0 (H_{t-1}) = \tau_1 (H_{t-1}) \) and that they chose different technologies in period \( t - 1 \), with \( \tau_0 (H_{t-2}) = m \) and \( \tau_1 (H_{t-2}) = h \). In this case, consumers attribute status value to good 1 and no status value to good 0. The next result describes the outcome of the stage game in this case.

**Lemma 15** In the Nash Equilibrium of the stage game in which both firms use the handicraft technology and consumers attribute status value to good 1 only, prices, demands, and operational profits are, respectively,

\[
p_{HS}^0 = \frac{4}{3} + c, \quad p_{HS}^1 = \frac{5}{3} + c, \quad i_{HS} = \frac{4}{9}, \quad 1 - i_{HS} = \frac{5}{9}, \quad \pi_{HS}^0 = \frac{16}{27}, \quad \pi_{HS}^1 = \frac{25}{27},
\]

and, in the Nash Equilibrium of the stage game in which both firms use the industrial technology and consumers attribute status value to good 1 only, prices are, respectively, \( p_{HS}^0 = \frac{4}{3} \), and \( p_{HS}^1 = \frac{5}{3} \), while demands and operational profits are given by the same expressions as in the case where both firms use the handicraft technology.

**Proof.** (of Lemma 15) Firms can, in principle, use either technology to produce their goods. To start with, let us analyze the case in which both firms use the handicraft technology. Firm 0 maximizes its operational profits \( \pi_{HS}^0 = (p_0 - c)i_{HS} \). Firm 0’s operational profit function is:

\[
\pi_{HS}^0 = p_0i_{HS} = (p_0 - c) \left( \frac{1 + p_1 - p_0}{3} \right).
\]

Maximizing this function yields first order conditions:

\[
(1 + p_1 - p_0) - (p_0 - c) = 0.
\]

Firm 0’s best reply is:

\[
p_{HS,BR}^0 = \frac{1}{2} [1 + p_1 + c]. \tag{76}
\]
We can now turn to Firm 1’s operational profit maximization problem. By producing the status good, its operational profit is:

\[
\pi_1 = (1 - \iota^*) (p_1 - c) = \left( \frac{2 - p_1 + p_0}{3} \right) (p_1 - c).
\]

The first order condition for Firm 1’s operational profit maximization problem is \(-2p_1 + p_0 + c + 2 = 0\). Firm 1’s best reply is:

\[
p_{HS,BR}^1 = \frac{p_0 + c + 2}{2}. \tag{77}
\]

Solving the system of best reply equations in \(p_{0,HS,BR}^1\) and \(p_{1,HS,BR}^1\) to find the equilibrium price leads to:

\[
p_{0,HS}^1 = \frac{4}{3} + c, \tag{78}
\]
\[
p_{1,HS}^1 = \frac{5}{3} + c. \tag{79}
\]

Substituting these solutions back into (3) and (4), we obtain the threshold index and the demands

\[
i^{HS} = \frac{4}{9}, \tag{80}
\]
\[
1 - i^{HS} = \frac{5}{9}. \tag{81}
\]

Substituting prices (78) and (79) and demands (80) and (81) into the profit functions of firms 0 and 1 gives us the operational profits

\[
\pi_{0,HS}^1 = \frac{16}{27}, \tag{82}
\]
\[
\pi_{1,HS}^1 = \frac{25}{27}. \tag{83}
\]

The equilibrium of the Hotelling model with both firms using the industrial technology can be obtained easily by making \(c = 0\) in the equilibrium described above, where we obtain:

\[
p_{0,HS}^1 = \frac{4}{3}, \tag{84}
\]
\[
p_{1,HS}^1 = \frac{5}{3}. \tag{85}
\]

A.3.10 Hotelling With Different Technologies and Status

Suppose consumers attribute status to good 1, that Firm 1 adopts the industrial technology and that Firm 0 adopts the handicraft technology. Suppose further that these facts are common knowledge between the two firms. The next lemma describes the outcome of the stage game in this case.
Lemma 16 Consider the following stage game: consumers attribute status to good 1, Firm 0 adopts the handicraft technology, and Firm 1 adopts the handicraft technology. Suppose further that these facts are common knowledge between the two firms. Then, in the Nash Equilibrium of the stage game, prices, demands, and operational profits are, respectively, $p_{HSDT}^0 = 4/3 + 2c/3$, $p_{HSDT}^1 = 5/3 + c/3$, $i_{HSDT} = 4/9 - c/9$, $1 - i_{HSDT} = 5/9 + c/9$, $\pi_{HSDT}^0 = (4 - c)^2/27$, and $\pi_{HSDT}^1 = (5 + c)^2/27$.

Proof. (of Lemma 16) Firm 0’s operational profit function is:

$$\pi_{HSDT}^0 = (p_0 - c) i_{HSDT} = (p_0 - c) \left( \frac{1 + p_1 - p_0}{3} \right).$$

If Firm 0 maximizes this function, obtains the following first order condition:

$$1 + c + p_1 - 2p_0 = 0.$$

Firm 0’s best reply is:

$$p_{HSDT,BR}^0 = \frac{1}{2} \left[ 1 + c + p_1 \right]. \quad (86)$$

We can now turn to Firm 1’s operational profit maximization problem. Its operational profit is:

$$\pi_1 = (1 - i^*)p_1 = (1 - i_{HSDT})p_1 = \left( \frac{2 - p_1 + p_0}{3} \right)p_1.$$

The first order condition for Firm 1’s operational profit maximization problem is $-2p_1 + p_0 + 2 = 0$. Firm 1’s best reply is, then:

$$p_{HSDT,BR}^1 = \frac{p_0 + 2}{2}. \quad (87)$$

Solving the system of best reply equations in $p_{HSDT,BR}^0$ and $p_{HSDT,BR}^1$ to find the equilibrium price leads to:

$$p_{HSDT}^0 = \frac{4}{3} + \frac{2}{3}c, \quad (88)$$

$$p_{HSDT}^1 = \frac{5}{3} + \frac{1}{3}c. \quad (89)$$

Substituting these solutions back into (3) and (4), we obtain the threshold index and the demands

$$i_{HSDT} = \frac{4 - c}{9}, \quad (90)$$

$$1 - i_{HSDT} = \frac{5 + c}{9}. \quad (91)$$
Substituting (88), (89), (90) and (91) into the profit functions of Firms 0 and 1 gives us the operational profits:

\[
\pi_{HSDT}^0 = \frac{(4 - c)^2}{27}, \quad (92)
\]

\[
\pi_{HSDT}^1 = \frac{(5 + c)^2}{27}. \quad (93)
\]

Because the game is symmetric in the absence of status effects, in order to obtain the stage game payoffs firms obtain when Firm 1 adopts the industrial technology while Firm 0 adopts the handicraft technology, just switch the subindices 0 and 1 in equations (88) through (93) to obtain the answer.

**A.3.11 Firm 0 profitable deviation: deviating after its own deviation**

In this scenario, Firm 1 has lost its status after Firm 0’s deviation at time \( t \) and is required to use the industrial technology and charge price \( p_0^P \), while Firm 1 uses the handicraft technology and charges price \( p_1^P \). Firm 0 would then deviate by maintaining the use of the handicraft technology and charging a best response \( p_{0D02}^P \) to price \( p_1^P \). Firm 0 maximizes its one-period profits, that is,

\[
p_{0D02}^P = \arg \max_{p_0} (p_0 - c) i^* (p_0, p_1^P) = \arg \max_{p_1} (p_0 - c) \left( \frac{1 + p_1^P - p_0}{2} \right).
\]

The first order condition to this problem is:

\[
1 - 2p_0 + p_1^P + c = 0,
\]

and thus

\[
p_{0D02}^P = \frac{1 + p_1^P + c}{2} = \frac{1 + c + (1 + 2c/3)}{2} = 1 + \frac{5}{6}c. \quad (94)
\]

Substituting (52) and (94) into (3) and (4) yields the demands for goods 0 and 1:

\[
i_{D02} = \frac{6 - c}{12}, \quad (95)
\]

\[
1 - i_{D02} = \frac{6 + c}{12}. \quad (96)
\]

Substituting equations (52), (94), (95) and (96) into the operational profit functions yield:

\[
\pi_{D02}^0 = \frac{(6 - c)^2}{72}, \quad (97)
\]

\[
\pi_{D02}^1 = \frac{(6 + c) (3 + 2c)}{36}. \quad (98)
\]
A.4 Equilibrium of the Repeated Game with Status

A.4.1 Equilibrium with Product Differentiation on Status

No proofs.

A.4.2 Short-Term Incentives to Deviate

Proof. (of Lemma 2) In the main text. ■

A.4.3 Deviations from the Equilibrium Path

Proof. (of Lemma 3) In the main text. ■

A.4.4 Incentives For Firm 1 to Return to the Equilibrium Path

Proof. (of Lemma 4) Substituting equations (61), (56) and (34) into equation (11) yields equation (9).

Proof. (of Proposition 1) Equations (8) and (11) show that \( \tilde{\delta}_1 \) and \( \tilde{\delta}_1 \) have a common denominator, namely \( \pi_1^* - \pi_1^P \). This denominator is positive because \( \pi_1^* > \pi_1^P \) if and only if \( 23 - 2c - c^2 > 0 \). This last inequality holds for all \( 0 < c < 5 - \sqrt{27}/2 \). By comparing equations (8) and (11), we have that \( \tilde{\delta}_1 > \tilde{\delta}_1 \) if and only if \( \pi_1^D - \pi_1^* > \pi_1^{ARR} - \pi_1^P \). Substituting the values of \( \pi_1^D, \pi_1^*, \pi_1^{ARR} \) and \( \pi_1^P \) in this inequality and simplifying yields \( 3c^2 + 12c > 0 \), which is trivially satisfied for all \( c > 0 \). ■

A.5 Ownership Choice and Capital Structure

A.5.1 Basic Setup of the Modified Game

No proofs.

A.5.2 Financial Markets Equilibrium

Proof. (of Proposition 2) See Appendix B. ■

A.5.3 Ownership Structure and Product Market Equilibrium

No proofs.

A.6 Ownership Choice With Commitment

A.6.1 Basic Setup of the Game with Commitment

No proofs.
A.6.2 Analysis of the Game with Commitment

**Proof.** (of Lemma 5) Using $\tau_0(t) = m$ and charging price $p_0^{D0} = 4/3 + 5c/6$ constitute a profitable deviation for Firm 0 if $\pi_0^{D0} + \delta [\pi_0^P - K] > \pi_0^* - K + \delta [\pi_0^* - K]$ . Such deviation is *not* profitable for Firm 0 when:

$$\pi_0^{D0} + \delta\pi_0^P < \pi_0^* - K + \delta\pi_0^*.$$  \hfill (99)

Consider the assumptions $\pi_0^{D0} < \pi_0^* - K$ and $\pi_0^P < \pi_0^*$, which implies $\delta\pi_0^P < \delta\pi_0^*$. Then, inequality (99) always holds. Therefore, the proposed deviation is not profitable for Firm 0. ■

**Proof.** (of Lemma 6) The proposed deviation at periods $t$ and $t + 1$ is profitable if $\pi_0^{D0} + \delta\pi_0^{D02} + \delta^2[\pi_0^P - K] > \pi_0^* - K + \delta [\pi_0^* - K] + \delta^2 [\pi_0^* - K]$. It is *not* a profitable deviation for Firm 0 when:

$$\pi_0^{D0} + \delta\pi_0^{D02} + \delta^2\pi_0^P < [\pi_0^* - K] + \delta [\pi_0^* - K] + \delta^2\pi_0^*.$$  \hfill (100)

By assumption, $\pi_0^{D0} < \pi_0^* - K$ and $\pi_0^P < \pi_0^*$, which implies $\delta\pi_0^P < \delta\pi_0^*$. Thus, if $\pi_0^{D02} < \pi_0^* - K$ , then $\delta\pi_0^{D02} < \delta [\pi_0^* - K]$, and inequality (100) would hold. Simple algebra shows that $\pi_0^{D02} < \pi_0^{D0}$. Given the assumption that $\pi_0^{D0} < \pi_0^* - K$, it follows that $\pi_0^{D02} < \pi_0^* - K$. Thus, the aforementioned deviation is not profitable for Firm 0. ■

**Proof.** (of Proposition 3) In the main text. ■

A.6.3 Comparison to Hotelling Payoffs

**Proof.** (of Lemma 7) The inequality $\delta_1^C > \delta_0^C$ is equivalent to:

$$\frac{20c - 2c^2 - 54K}{23} > \frac{54K - 16c - 2c^2}{5}.$$  

Straightforward calculations shows that this last inequality holds if and only if $27K < c^2 + 13c$. Multiplying both sides by 4 leads to:

$$108K < 52c + 4c^2.$$  \hfill (101)

By assumption, Firm 0 has no short-term incentives to deviate from using the handicraft technology in the original game. Then, $36K < 16c + c^2$, which is equivalent to:

$$108K < 48c + 3c^2.$$  \hfill (102)

The right-hand side of (102) is smaller than the right-hand side of (101). Therefore, inequality (102) implies inequality (101). Thus, $\delta_1^C > \delta_0^C$. ■

**Proof.** (of Proposition 4) We need to show that $\delta_1^C < \hat{\delta}_1$. From equations (8) and (9), we have that:

$$\hat{\delta}_1 = \frac{\pi_1^P - K - \pi_1^*}{\pi_1^* - \pi_1^P} = \frac{120c - 6c^2 - 216K}{92 - 4c^2 - 8c}.$$  \hfill (103)
Equations (17) and (18) lead to:

\[ \delta_1^C = \frac{\pi_1^{HS} - K - \pi_1^*}{\pi_1^{HS} - \pi_1^H} \]
\[ = \frac{20c - 2c^2 - 54K}{23} \]
\[ = \frac{80c - 8c^2 - 216K}{92}. \quad (104) \]

When \( c = 0 \), the expressions (103) and (104) coincide, for all values of \( K \). As \( c \) increases, the numerator on (103) grows by a larger sum than the numerator in (104), because \( 25c - 2c^2 > 0 \) for \( c < 25/2 \). The denominator on (103) is reduced as \( c \) grows, because \( 4c^2 + 8c \) is increasing for \( c > 0 \), while the denominator on (104) remains constant. Because \( c \leq 5 - \sqrt{27}/2 \), it follows that \( \hat{\delta}_1 > \delta_1^C \). ■

A.6.4 Alternative Commitment Devices

Proof. (of Proposition 5) In the main text. ■

B Appendix: Financial Markets Equilibrium

B.1 Setup and Analysis

To raise the capital amount \( K \), at every period, Firm 0’s shareholders must issue shares and bonds. The total proceeds of this issuance must yield at least \( K \). Old shareholders must give away a fraction \( \phi \) to new shareholders to finance \((1 - \psi)K\), while the remaining \( \psi K \) are financed through the issuance of debt. Hence, Firm 0’s value function is a fraction \( 1 - \phi \) of the present value of current and future cash flows, that is:

\[ V_0^* = (1 - \phi)[\pi_0^* - K \psi + \delta V_0^*] \]
\[ V_0^* = \frac{(1 - \phi)[\pi_0^* - K \psi]}{1 - \delta + \phi \delta}. \quad (105) \]

Firm 0’s value function represents the value that shares of current shareholders attain in equilibrium, excluding shareholders who purchase shares issued at the current period.

Let \( V_0^{ALL} \) denote the total value generated by Firm 0. Firm 0’s old shareholders take share \( 1 - \phi \) of this value; that is, \( V_0^* = (1 - \phi)V_0^{ALL} \), while surrendering share \( \phi V_0^{ALL} \) of the firm’s value to new shareholders. If Firm 0’s managers act in the interest of old shareholders, then they choose \( \phi \) and \( \psi \) that maximize \( V_0^* \), subject to the constraint that they must raise at least \( K \) with their stock and bond issuance. If the company decides to finance \( \psi K \) by issuing bonds, there remains \((1 - \psi)K\) to be funded by the issuance of new shares.
By non-arbitrage, the value paid by new shareholders for a share $\phi$ of Firm 0 must coincide with their discounted flow of gains. Hence:

$$\phi V_0^{ALL} = (1 - \psi)K.$$  \hspace{1cm} (107)

Using this as well as equations (106) and $V_0 = (1 - \phi)V_0^{ALL}$ leads to:

$$\phi \left[ \pi_0^* - K\psi \right] \frac{1}{1 - \delta + \phi\delta} = (1 - \psi)K.$$  

As it is shown in the proof of Proposition 2, this equation also holds for values of $\pi_0$ outside the equilibrium path. This proof is discussed below.

B.2 Proofs

**Proof.** (of Proposition 2) The proof is carried out in three steps. First, we show that Modigliani and Miller’s result holds at the equilibrium path. We then show that, if in the current period we observe actions that are different from the equilibrium prescribed ones, but we observe only equilibrium path actions thereafter, Modigliani and Miller’s result still holds. Then, we extend this result by finite induction to any finite number of periods of actions played different from the equilibrium prescribed ones.

**Step (i):** We first show that, at the equilibrium path, the objective function depends neither on $\phi$ nor on $\psi$. The new shareholders pay $(1 - \psi)K$ for a fraction $\phi$ of the firm, expecting to obtain $\pi_0 - K\psi$ in the present and a continuation payoff of $V_0$. Making the new shareholders payment equal to their payoff means that:

$$(1 - \psi)K = \phi \left[ \pi_0 - K\psi + \delta V_0 \right].$$

Using this and equation (105) results in the following no arbitrage condition:

$$(1 - \psi)K = \frac{\phi V_0}{1 - \phi}. \hspace{1cm} (108)$$

Substituting equation (106) into this expression leads to:

$$(1 - \psi)K = \frac{\phi (\pi_0 - K\psi)}{1 - \delta + \phi\delta}.$$}

Cross multiplying and solving for $K\psi$:

$$(1 - \delta + \phi\delta)K - (1 - \delta + \phi\delta)\psi K = \phi \pi_0 - \phi K\psi$$

$$\left(1 - \delta + \phi\delta\right)K - \phi \pi_0 = \left(1 - \delta + \phi\delta\right)\psi K - \phi K\psi$$

$$\left(1 - \delta + \phi\delta\right)K - \phi \pi_0 = [1 - \delta + \phi\delta - \phi] K\psi$$

$$K\psi = \frac{(1 - \delta + \phi\delta)K - \phi \pi_0}{(1 - \delta)(1 - \phi)}.$$
Substituting this formula for $K\psi$ into the objective function of Firm 0’s managers yields:

$$\frac{(1 - \phi)[\pi_0 - K\psi]}{1 - \delta + \phi\delta} = \frac{(1 - \phi)}{1 - \delta + \phi\delta} \left[ \pi_0 - \frac{(1 - \delta + \phi\delta)K - \phi\pi_0}{(1 - \delta)(1 - \phi)} \right]$$

$$= \frac{(1 - \phi)[(1 - \delta + \phi\delta - \phi)\pi_0 - (1 - \delta + \phi\delta)K + \phi\pi_0]}{(1 - \delta + \phi\delta)(1 - \delta)(1 - \phi)}$$

$$= \frac{[(1 - \delta + \phi\delta)\pi_0 - (1 - \delta + \phi\delta)K]}{(1 - \delta + \phi\delta)(1 - \delta)}$$

$$= \frac{(1 - \delta + \phi\delta)[\pi_0 - K]}{(1 - \delta + \phi\delta)(1 - \delta)}$$

$$= \frac{\pi_0 - K}{1 - \delta}.$$ 

This proves that equation (13) holds regardless of the choice of $\psi$.

**Step (ii):** This part shows that, if during the first period, profits differ from their equilibrium values, the Modigliani-Miller result still holds. Let $\pi_0(t)$ be Firm 0’s operational profit in period $t$, and that $\pi_0(t)$ may be different from $\pi^*_0$. Suppose that in period $t + 1$ and beyond, Firm 0’s operational profits are equal to $\pi^*_0$. Let $\phi_t$ be the proportion of the firm that old shareholders sell at time $t$. Let $\psi_t$ be the proportion of capital finance through the issuance of debt at time $t$. Let $V_0(t)$ and $V_0(t + 1) = V_0^*$ be Firm 0’s value at times $t$ and $t + 1$. This notation omits the price $p^*_0$ and focuses on the time. Then, by equation (13):

$$V_0(t + 1) = V_0^* = \frac{\pi_0 - K}{1 - \delta}.$$ 

The value of the Firm 0 at time $t$ is:

$$V_0(t) = (1 - \phi_t)[\pi_0(t) - \psi_tK] + \delta(1 - \phi_t)V_0(t + 1)$$

$$= (1 - \phi_t)[\pi_0(t) - \psi_tK] + \delta(1 - \phi_t)V_0^*.$$  \hspace{1cm} (109)

Assuming the constraint (107) holds with equality, we find the following no arbitrage condition that makes the new shareholders (those buying stocks at time $t$) payment equal to their payoff:

$$\frac{\phi_tV_0(t)}{1 - \phi_t} = (1 - \psi_t)K.$$ 

Solving this equation for $\psi_tK$ leads to:

$$\psi_tK = K - \frac{\phi_tV_0(t)}{1 - \phi_t}.$$ 

Substituting this expression for $\psi_tK$ into equation (109) yields:

$$V_0(t) = (1 - \phi_t) \left[ \pi_0(t) - K + \frac{\phi_tV_0(t)}{1 - \phi_t} \right] + \delta(1 - \phi_t)V_0^*$$

$$\left[ 1 - \frac{\phi_t(1 - \phi_t)}{1 - \phi_t} \right] V_0(t) = (1 - \phi_t)[\pi_0(t) - K] + \delta(1 - \phi_t)V_0^*.$$
Solving for $V_0(t)$ and replacing the equilibrium value $V_0^*$ by the expression in equation (13) yields:

$$V_0(t) = (\pi_0(t) - K) + \delta V_0^*$$
$$= \pi_0(t) - K + \delta \frac{\pi_0 - K}{1 - \delta}.$$

This proves that $V_0(t)$ is not a function of either $\psi$, $\phi_t$ or $\phi^*$. Therefore, Firm 0’s value is independent of financing.

**Step (iii):** Suppose now that profits differ from the equilibrium path for the first $T$ periods, starting at $t$. Let $V_0(t + t')$ and $V_0(t + T) = V_0^*$ be Firm 0’s value at times $t + t'$, for $t' \in \{0, 1, \ldots, T - 1\}$, and $t + T$. By equation (13):

$$V_0(t + T) = V_0^* = \frac{\pi_0 - K}{1 - \delta}.$$

Firm 0’s values $V_0(t + t')$, for $t' \in \{0, 1, \ldots, T - 1\}$, are:

$$V_0(t + T - 1) = (1 - \phi_{t+T-1}) [\pi_0(t + T - 1) - \psi_{t+T-1}K] + \delta(1 - \phi_{t+T-1})V_0(t + T)$$
$$= (1 - \phi_{t+T-1}) [\pi_0(t + T - 1) - \psi_{t+T-1}K] + \delta(1 - \phi_{t+T-1})V_0^*$$

$$V_0(t + t') = (1 - \phi_{t+t'}) [\pi_0(t + t') - \psi_{t+t'}K] + \delta(1 - \phi_{t+t'})V_0(t + t' + 1)$$

$$V_0(t) = (1 - \phi_t) [\pi_0(t) - \psi_t K] + \delta(1 - \phi_t)V_0(t + 1).$$

Assuming the constraint (107) holds with equality, the no arbitrage condition at each time $t + t'$, for $t' \in \{0, 1, \ldots, T\}$, is:

$$\psi_{t+t'}K = K - \frac{\phi_{t+t'}V_0(t + t')}{1 - \phi_{t+t'}}.$$

Substituting this into equations (111) to (113) yields:

$$V_0(t + t') = (1 - \phi_{t+t'}) \left[ \pi_0(t + t') - K + \frac{\phi_{t+t'}V_0(t + t')}{1 - \phi_{t+t'}} \right]$$
$$+ \delta(1 - \phi_{t+t'})(1 - \phi_{t+t'+1}) \left[ \pi_0(t + t' + 1) - K + \frac{\phi_{t+t'}V_0(t + t')}{1 - \phi_{t+t'}} \right] +$$
$$+ \ldots + \delta^{t+t'-1} \prod_{j=t+t'}^{t-1} (1 - \phi_{t+t'+j}) \left[ \pi_0(t + T - 1) - K + \frac{\phi_{t+t'}V_0(t + T - 1)}{1 - \phi_{t+T-1}} \right]$$
$$+ \delta^{T-t'} \prod_{j=t+t'}^{T-1} (1 - \phi_{t+t'+j})V_0^*. $$

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This is the same as:
\[ V_0(t + t') = [\pi_0(t + t') - K] + \delta(1 - \phi_{t+t'+1}) \left[ \pi_0(t + t' + 1) - K + \frac{\phi_{t+t'} V_0(t + t')} {1 - \phi_{t+t'}} \right] + (115) \]
\[ + \cdots + \delta^{T-1} \prod_{j=t+t'}^{T-1} (1 - \phi_{t+j}) \left[ \pi_0(t + T - 1) - K + \frac{\phi_{t} V_0(t + T - 1)} {1 - \phi_{t+T-1}} \right] + \delta V_0^* \].

The term in between keys is \( V_0(t + T - 1) \) by making \( t' = T - 1 \) in (114). By step (ii), this is given by
\[ V_0(t + T - 1) = [\pi_0(t + T - 1) - K] + \delta V_0^*. \]

Similarly, for each \( j \in \{1, \ldots, T\} \), we have:
\[ V_0(t + j) = [\pi_0(t + j) - K] + \delta V_0(t + j + 1). \]

Substituting these expressions into (115), we obtain:
\[ V_0(t) = [\pi_0(t) - K] + \delta [\pi_0(t + 1) - K] + \cdots + \delta^{T-1} \{ [\pi_0(t + T - 1) - K] + \delta V_0^* \}. \]

This expression is independent of all \( \phi_t \). So, Firm 0 value is independent of financing.  

**C Appendix: Inequalities Table**

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<td>Condition for ( \pi_0^* &gt; \pi_1^* ) or ( i^* &gt; 1/2 )</td>
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<td>( 3K \leq 2c - 1 )</td>
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**References**


