Government Financing with Taxes or Inflation*

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Abstract

We calculate the effects of an increase in government spending under two financing alternatives: labor income taxes or inflation. A standard cash-in-advance model implies that it is optimal to finance the increase in spending with inflation rather than with taxes. However, when we allow agents to select the moment in which they rebalance their portfolio, this conclusion is reversed. The welfare cost of financing the government with inflation becomes higher. The robustness of this result is studied by considering government spending in the form of transfers or consumption expenditures and alternative definitions of seigniorage.

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1. Introduction

We calculate the effects of an increase in government spending in a model in which agents choose the frequency of trades of bonds for money, as in Baumol (1952) and Tobin (1956). The increase in expenditures can be either in the form of transfers or government consumption. The increase in expenditures can be financed either by an increase in inflation or by an increase in distortionary taxes in the form of labor income taxes. Cash-in-advance models have a given fixed time period between trades of bonds for money, usually a month or a quarter. We, instead, let agents decide the length of the time periods. This change implies a more elastic demand for money, a better fit to the data, and different predictions on the effects of financing the government with inflation.\(^1\)

When agents are allowed to choose the time period between trades of assets, the welfare cost of financing the government with inflation is substantially higher than in the model with fixed periods. An analyst may conclude that it is optimal to finance an increase in government spending with inflation when the model has fixed periods. Having endogenous periods reverts this conclusion. It becomes optimal to finance an increase in government spending with distortionary taxes. It is crucial to consider changes in the frequency of trades to calculate the effects of policies that involve changes in inflation.

We consider an increase in government spending of 5 percent in the form of an increase in transfers and take the initial government-output ratio to be 20 percent.\(^2\) If the increase in expenditures is financed with an increase in labor income taxes, then having fixed or endogenous periods implies similar predictions for the welfare cost: 0.95 percent in terms of income. If the increase in expenditures is financed

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\(^1\) Alvarez et al. (2009) show that a model with exogenous timing for the trades in the financial markets is able to generate short-run variation in the velocity of money. An alternative to imply long run changes in velocity is the introduction of cash and credit goods. But this still implies small variation in velocity, as shown in Hodrick, Kocherlakota, and Lucas (1991). Alvarez et al (2002) have endogenous market segmentation of risky assets but agents use all money holdings in every period, which implies a constant velocity. Williamson (2008, 2009) introduces a model with goods market segmentation.

\(^2\) In our simulations, this increase in expenditures implies an increase in the government-output ratio from 20 to about 21 percent.
with an increase in inflation, the predicted welfare cost with fixed periods is 0.49 percent in terms of income. With endogenous periods, the predicted welfare cost is 1.45 percent. A difference of 0.96 percentage points.\(^3\)

One of the reasons for the difference in estimates is the larger decrease in the demand for money with endogenous periods when inflation increases. For the same rate of inflation, the smaller demand for money implies a decrease in seigniorage as compared with a model with fixed periods. With fixed periods, the inflation rate necessary to cover the 5 percent increase in government spending is equal to 5.5 percent per year. With endogenous periods, this inflation rate is equal to 12.7 percent per year. A model with fixed periods predicts that a smaller inflation rate generates the same seigniorage as a higher inflation rate in a model with endogenous periods.

The values for inflation and seigniorage implied by this example are within realistic estimates. According to our simulations, seigniorage revenues as a percentage of output are 2.19 percent of output with fixed periods and 1.93 percent of output with endogenous periods. Sargent et al. (2009) estimate that seigniorage exceeded 10 percent of output for Brazil in 1993 and for Argentina several times during 1970-1990. Click (1998) calculates seigniorage to be 2.5 percent of output on average for a database of 90 countries during 1971-1990 and finds that seigniorage exceeded 10 percent of output in some cases.\(^4\) Kimbrough (2006) cites evidence that seigniorage may reach 17 percent of GDP in some countries but that usually it does not exceed 10 percent. Kimbrough considers seigniorage from 5 to 15 percent of output. Our simulations imply an increase in seigniorage revenues of 1 percentage point, and total seigniorage revenues around 2 percent of output, which is much below the admitted maximum value of 10 percent.

\(^3\)This difference is substantial. One percent of income is equivalent to more than 130 billion dollars every year or more than one thousand dollars distributed to every household in the United States every year (2010 dollars, data from the BEA and from the US Census Bureau).

\(^4\)Click (1998) calculates seigniorage to be 10.5 percent of government spending on average but to exceed 100 percent of government spending for various countries. In our case, seigniorage is 10.3 percent of government consumption with fixed periods and 9.2 percent with endogenous periods. The database of Click includes countries from all continents. In this database, U.S. has a seigniorage of 0.43 percent of output, the lowest seigniorage is 0.38 percent of output for New Zealand and the highest seigniorage is 14.8 percent of output for Israel.
Having endogenous or fixed intervals is not relevant when the increase in spending is financed by an increase in labor income taxes. When financing is made through labor income taxes, inflation is about the same before and after the increase in spending. Changes in the demand for money are then insignificant. As as result, it is not important to take into account the timing of the trades between bonds and money. The changes in labor taxes are approximately the same with endogenous or fixed intervals; labor taxes increase to about 32 percent. Other effects, such as those on output and consumption, are also similar when financing is made through taxes. Endogenous trading frequency is relevant when the change in policy implies a change in inflation and, consequently, a significant change in the demand for money.

Agents in the model pay a fee in goods to have access to financial markets, where they trade bonds for money. The payment of a fee implies staggered visits to the asset market. As inflation increases, it is optimal to increase the frequency of the trades in the financial markets to decrease the real money holdings. The increase the frequency of the trades increases the frequency of fee payments.

We interpret the aggregate fee payments in the model as the size of the financial sector. Therefore, we can use our model to estimate the increase in the financial sector implied by the increase in inflation. In our simulations, an increase in inflation from 0 to 10 percent per year implies an increase in financial services of 1 percentage point. Our estimates are in accordance with the evidence in English (1999). English (1999) finds a positive relation between inflation and the size of the financial sector and estimates that a 10 percent increase in inflation in U.S. implies an increase in the financial sector of 1.3 percentage points.

Cooley and Hansen (1991, 1992) study the effects of inflation in a cash-in-advance economy with distortionary taxation. Their economy is similar to the economy that we have here, except that we allow agents to choose optimally the timing of their transactions in the asset market. In Cooley and Hansen (1991), decreasing inflation from 10 to zero percent per year, with seigniorage revenues replaced with labor income taxes, implies a welfare loss of 1.018 percent of output. In Cooley and Hansen (1992), financing the government with distortionary taxes implies a welfare cost of 13.30 percent of GDP when compared with lump sum taxation. If financing is

\footnote{About this unexpected result, see also Benabou (1991) and Wright (1991).}
done with inflation, it implies a welfare cost of 12.43 percent. The difference of 0.87 percent between financing with inflation and distortionary taxes is the loss associated with decreasing inflation to zero and increasing taxes to keep government revenues constant. Here, we confirm that replacing inflation with labor income taxes implies a welfare loss when agents cannot choose the timing of transactions in the asset market. However, once we lift the restriction on the timing of asset transactions, the loss of decreasing inflation is reversed to a gain.

This paper is related to Silva (2012). Silva (2012) finds that the inclusion of the trading frequency decision increases the estimates of the welfare cost of inflation. This effect is surprising as the ability to change the trading frequency gives more flexibility to the agents and could make the welfare cost of inflation decrease. To the contrary, the welfare cost of 10 percent instead of zero inflation is four times larger in an endogenous frequencies framework than in a fixed frequencies framework (1.3 vs. 0.3 percent in terms of output). As we discuss below, the reason for the small welfare cost of inflation in cash-in-advance models is the small interest-elasticity of the demand for money when the frequency of trades is fixed. Here, we extend this framework to study the effects of government spending financed with distortionary taxation or inflation. We find that taking into account the decision on the frequency of trades substantially changes the estimates of the welfare cost of inflation. For our surprise, this effect is so large in certain cases that it reverses a prescription of increasing inflation to a prescription of keeping inflation low.

The response of output depends on the trading frequency considered. With a fixed trading frequency, the inflation rate acts essentially as a labor income tax. As discussed in Cooley and Hansen (1989) and others, an increase in inflation encourages agents to substitute away from labor toward leisure, which decreases output. This effect is also present in the endogenous trading frequency case. However, the need to decrease the demand for money leads to further effects. To decrease the demand for money, it is necessary to increase the financial sector, captured by the increase in the trading frequency. As the financial sector also requires labor and capital, the

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6 It is possible to increase the elasticity of the demand for money with fixed periods, which increases the welfare cost of inflation. However, as shown in Silva (2012), this requires increasing the value for the elasticity of intertemporal substitution from 2 or smaller, which are usually considered plausible values for this parameter, to values such as 50 or larger.
increase in the financial sector implies a positive effect on labor supply and output. This effect on the labor supply is small as the financial sector as a fraction of output increases only one percentage point when the increase in spending is financed through inflation. However, the effect might be enough to change the sign of the output response. Under certain conditions, there is a positive government multiplier. In the case of transfers, the multiplier is \(-1.98\) with fixed timing and \(0.03\) with endogenous timing. If the increase in spending is in government consumption, the multiplier is \(-0.01\) with fixed timing and \(1.09\) with endogenous timing. With taxes, the multiplier is similar with fixed or endogenous timing (about \(-2.2\) in the case of transfers and about \(-0.2\) in the case of government consumption). Therefore, the model implies a government multiplier slightly larger than one if the increase in spending is in the form of government consumption and if it is financed through inflation.\(^7\)

The remainder of the paper is organized as follows. Section 2 describes the model in detail. In section 3, we solve the model for the competitive equilibrium steady state. In section 3, we specify the welfare cost criterion, discuss our simulations, and report the results of an increase in government spending that is financed either by distortionary taxes or inflation. Section 4 concludes.

2. The Model

We extend the general equilibrium Baumol-Tobin model in Silva (2012). Money must be used to purchase goods, only bonds receive interest payments, and there is a cost to transfer money from bond sales to the goods market. Agents exchange bonds for money infrequently. The infrequent sales of bonds for money occur as in the models of Grossman and Weiss (1983), Rotemberg (1984) and Alvarez et al. (2009). The difference from these models is that the timing of bond sales is endogenous. Moreover, we have capital and labor and, in addition, we introduce a government that finances itself with distortionary taxes. The model can be understood as a cash-in-advance model with capital and labor with the decision on the size of the

\(^7\)According to Ramey (2011), the evidence points to a government expenditures multiplier between 0.8 and 1.5. For permanent increases in government expenditures, the multiplier is closer to the one in our experiments. In this case, Ramey points to a multiplier of 1.2 (in Baxter and King 1993). See also the discussion in Hall (2009) and Woodford (2010).
holding periods. Apart from the heterogeneity of agents and the decision on the holding periods, the model is similar to standard cash-in-advance models in Cooley and Hansen (1989) and Cooley (1995).

Time is continuous and denoted by \( t \in [0, +\infty) \). At any moment, there are markets for assets, for the consumption good, and for labor. There are three assets, money, claims to physical capital, and nominal bonds. The markets for assets and the market for the consumption good are physically separated.

There is an unit mass of infinitely lived agents with preferences over consumption and leisure. Agents have two financial accounts: a brokerage account and a bank account. They hold assets in the brokerage account and money in the bank account. We assume that readjustments in the brokerage account and money in the bank account have a fixed cost. As only money can be used to buy goods, agents need to maintain an inventory of money in their bank account large enough to pay for their flow of consumption expenditures until the next transfer of funds.

Let \( M_0 \) denote money in the bank account at time zero. Let \( B_0 \) denote nominal government bonds and \( k_0 \) claims to physical capital, both in the brokerage account at time zero. Index agents by \( s = (M_0, B_0, k_0) \).

The agents pay a cost \( \Gamma \) in goods to transfer resources between the brokerage account and the bank account. \( \Gamma \) represents a fixed cost of portfolio adjustment. Let \( T_j (s) \), \( j = 1, 2, \ldots \), denote the times of the transfers of agent \( s \). Let \( P(t) \) denote the price level. At \( t = T_j (s) \), agent \( s \) pays \( P(T_j (s))\Gamma \) to make a transfer between the brokerage account and the bank account. The agents choose the times \( T_j (s) \) of the transfers.

The consumption good is produced by firms. Firms are perfect competitors. They hire labor and rent capital to produce the good. The production function is given by \( Y(t) = Y_0 K(t)^\theta H(t)^{1-\theta} \), where \( 0 < \theta < 1 \) and \( K(t) \) and \( H(t) \) are the aggregate quantities of capital and hours of work at time \( t \). Capital depreciates at the rate \( \delta \), \( 0 < \delta < 1 \).

The agent is a composition of a shopper, a trader, and a worker, as in Lucas (1990). The shopper uses money in the bank account to buy goods, the trader manages the brokerage account, and the worker supplies labor to the firms. The firms transfer
their sales proceeds to their brokerage accounts and convert them into bonds.\textsuperscript{8}

The firms pay \( w(t)h(t,s) \) and \( r^k(t)k(t,s) \) to the worker for the hours of work \( h(t,s) \) and capital \( k(t,s) \) supplied, \( w(t) \) are real wages and \( r^k(t) \) is the real interest rate on capital. The firms make the payments with a transfer from the brokerage account of the firm to the brokerage account of the agent. When the firms make the payments to the agents, the government collects \( \tau_Lw(t)h(t,s) \) in labor income taxes and sends \( T \) in transfers to the agents. With the payments of the firm, the brokerage account of the worker is credited by \( (1-\tau_L)w(t)h(t,s)+r^k(t)k(t,s)+T \). These credits can be used at the same date for purchases of bonds.\textsuperscript{9}

The government issues bonds that pay a nominal interest rate \( r(t) \). Let the price of a bond at time zero be given by \( Q(t) \), with \( Q(0) = 1 \). The nominal interest rate is \( r(t) = -d\log Q(t)/dt \). Let inflation be denoted by \( \pi(t) \), \( \pi(t) = d\log P(t)/dt \). To avoid the opportunity of arbitrage between bonds and capital, the nominal interest rate and the payment of claims to capital satisfies \( r(t)-\pi(t) = r^k(t)-\delta \). That is, the rate of return on bonds must be equal to the real return on physical capital discounted by depreciation. With this condition satisfied, the agents are indifferent between converting their income into bonds or capital.

Money holdings at time \( t \) of agent \( s \) are denoted by \( M(t,s) \). Money holdings just after a transfer are denoted by \( M^+ (T_j(s),s) \) and they are equal to \( \lim_{t\rightarrow T_j, t>T_j} M(t,s) \).

Analogously, \( M^- (T_j(s),s) = \lim_{t\rightarrow T_j, t<T_j} M(t,s) \) denotes money just before a transfer. The net transfer from the brokerage account to the bank account is given by \( M^+ - M^- \). If \( M^+ < M^- \), the agent makes a negative net transfer, a transfer from the bank account to the brokerage account, which is immediately converted into bonds.

Money holdings in the brokerage account are zero, as bonds receive interest payments and it is not possible to buy goods directly with money in the brokerage account. All money holdings are in the bank account. To have \( M^+ \) just after a transfer at \( T_j(s) \), agent \( s \) needs to transfer \( M^+ - M^- + P(T_j(s))\Gamma \) to the bank account, \( P(T_j(s))\Gamma \) is used to buy goods to pay the transfer cost.

\textsuperscript{8}In Silva (2012), the firms keep a fraction \( a \) of the sales proceeds in money and transfer the remaining fraction \( 1-a \) to their brokerage accounts of the workers, \( 0 \leq a < 1 \). However, the value of \( a \) has little impact on the welfare cost, on the demand for money and on other equilibrium values.

\textsuperscript{9}We also studied a version in which the deposits could only be used in the following period. The results are not affected by this change.
Define a holding period as the interval between two consecutive transfer times, that is \([T_j(s), T_{j+1}(s)]\). The first time agent \(s\) adjusts its portfolio of bonds is \(T_1(s)\) and the first holding period of agent \(s\) is \([0, T_1(s))\). To simplify the exposition, let \(T_0(s) = 0\), but there is not a transfer at \(t = 0\), unless \(T_1(s) = 0\).

Denote \(B^{-}(T_j(s), s), B^{+}(T_j(s), s), k^{-}(T_j(s), s),\) and \(k^{+}(T_j(s), s)\) the quantities of bonds and capital just before and just after a transfer. During a holding period, bond holdings and capital holdings of agent \(s\) follow

\[
\dot{B}(t, s) = r(t) B(t, s) + P(t) (1 - \tau_L) w(t) h(t, s) + T, \tag{1}
\]

\[
\dot{k}(t, s) = (r^k(t) - \delta) k(t, s). \tag{2}
\]

The way in which equations (1) and (2) are written implies that labor income and government transfers are converted into nominal bonds, and that interest payments to capital are converted into new claims to capital. This is done to simplify the expressions of the law of motion of bonds and capital. The agent is indifferent between the asset allocations on bonds and capital, as \(r(t) = r^k(t) - \delta\).

At each date \(T_j(s), j = 1, 2, \ldots\), agent \(s\) readjusts its portfolio. At the time of a transfer \(T_j(s)\), the quantities of money, bonds, and capital satisfy

\[
M^{+}(T_j) + B^{+}(T_j) + P(T_j) k^{+}(T_j) + P(T_j) \Gamma = M^{-}(T_j) + B^{-}(T_j) + P(T_j) k^{-}(T_j), \tag{3}
\]

\(j = 1, 2, \ldots\) The portfolio of money, bonds and capital chosen, plus the real cost of readjusting, must be equal to the current wealth. With the evolution of bonds and capital in (1)-(2), we can write \(B^{-}(T_j)\) and \(k^{-}(T_j)\) as a function of the interest payments accrued during a holding period \([T_{j-1}, T_j)\). Substituting recursively and using the non-Ponzi conditions \(\lim_{j \to +\infty} Q(T_j) B^{+}(T_j) = 0\) and \(\lim_{j \to +\infty} Q(T_j) P(T_j) k^{+}(T_j) = 0\), we obtain the present value constraint

\[
\sum_{j=1}^{\infty} Q(T_j(s)) [M^{+}(T_j(s), s) + P(T_j) \Gamma] \leq \sum_{j=1}^{\infty} Q(T_j(s)) M^{-}(T_j) + W_0(s), \tag{4}
\]

where \(W_0(s) = B_0 + P_0 k_0 + \int_0^{\infty} Q(t) P(t) (1 - \tau_L) w(t) h(t, s) dt\). The constraint
(4) states that the present value of money transfers and transfer fees is equal to the present value of deposits in the brokerage account, including initial bond and capital holdings.

In addition to the present value budget constraint (4), the agents face a cash-in-advance constraint

\[ \dot{M}(t,s) = -P(t) c(t,s), \quad t \geq 0, \quad t \neq T_1(s), T_2(s), \ldots \] (5)

This constraint emphasizes the transactions role of money, that is, agents need money to buy goods. At \( t = T_1(s), T_2(s), \ldots \), constraint (5) is replaced by \( \dot{M}(T_j(s), s)^+ = -P(T_j(s)) c^+(T_j(s)) \), where \( M(T_j(s), s)^+ \) is the right derivative of \( M(t,s) \) with respect to time at \( t = T_j(s) \) and \( c^+(T_j(s)) \) is consumption just after the transfer.

The agents choose consumption \( c(t,s) \), hours of work \( h(t,s) \), money in the bank account \( M(t,s) \), and the transfer times \( T_j(s), j = 1, 2, \ldots \). They make this decision at time zero given the paths of the interest rate and of the price level. The maximization problem of agent \( s = (M_0, k_0, B_0) \) is then given by

\[
\max_{c,h,T_j,M} \sum_{j=0}^{\infty} \int_{T_j(s)}^{T_{j+1}(s)} e^{-\rho t} u(c(t,s), h(t,s)) \, dt
\] (6)

subject to (4), (5), \( M(t,s) \geq 0 \), and \( T_{j+1}(s) \geq T_j(s) \), given \( M_0 \geq 0 \). The parameter \( \rho > 0 \) is the intertemporal rate of discount. The utility function is \( u(c(t,s)) = \log c(t,s) + \alpha \log (1 - h(t,s)) \). Preferences are a function of consumption and hours of work only, the transfer cost does not enter the utility function. These preferences are derived from the King, Plosser and Rebelo (1988) preferences \( u(c,h) = \left[ \frac{(1-h)^{\eta + 1} - 1/\eta}{1-1/\eta} \right] \), with \( \eta \rightarrow 1 \), which are compatible with a balanced growth path.\(^{10}\)

As bonds receive interest and money does not, the agents transfer the exact amount of money to consume until the next transfer. That is, the agents adjust \( M^+(T_j) \), \( T_j \), and \( T_{j+1} \) to obtain \( M^-(T_{j+1}) = 0, j \geq 1 \). We can still have \( M^-(T_1) > 0 \) as \( M_0 \)

\(^{10}\)We also solved a version of the model with \( \eta \neq 1 \) and a version with Greenwood, Hercowitz, and Huffman (1988) preferences. Moreover, we considered cash and credit goods. Silva (2012) considers indivisible labor, analyzed by Hansen (1985). These changes in the model do not affect our conclusions.
is given rather than being a choice. Using (5) and $M^{-} (T_{j+1}) = 0$ for $j \geq 1$, money just after the transfer at $T_{j}$ is

$$
M^{+} (T_{j} (s), s) = \int_{T_{j}}^{T_{j+1}} P (t) c (t, s) dt, \quad j = 1, 2, \ldots
$$

(7)

The government makes consumption expenditures $G$ and transfers $T$, distributed to agents in lump sum form, taxes labor income at the rate $\tau_{L}$, and issues nominal bonds $B (t)$ and money $M (t)$. The government controls the aggregate money supply at each time $t$ by making exchanges of bonds and money in the asset markets. The financial responsibilities of the government at time $t$ satisfy the period budget constraint

$$
r (t) B (t) + P (t) G + P (t) T = \dot{B} (t) + \tau_{L} P (t) w (t) H (t) + \dot{M} (t).
$$

(8)

That is, the government finances its responsibilities $r (t) B (t) + P (t) G + P (t) T$ by issuing new bonds, using revenues from labor taxes, and by issuing money. With the condition $\lim_{t \to \infty} B (t) e^{-rt} = 0$, the government budget constraint in present value is given by

$$
B_{0} + \int_{0}^{\infty} Q (t) P (t) (G + T) dt = \int_{0}^{\infty} Q (t) \tau_{L} w (t) H (t) dt + \int_{0}^{\infty} Q (t) P (t) \frac{\dot{M} (t)}{P (t)} dt.
$$

(9)

Seigniorage is equal to the real resources obtained by issuing money, $\dot{M} (t) / P (t)$.

The market clearing conditions for money and bonds are $M (t) = \int M (t, s) dF (s)$ and $B_{0} = \int B_{0} (s) dF (s)$, where $F$ is a given distribution of $s$. The market clearing condition for goods takes into account the goods used to pay the transfer cost. Let $A (t, \delta) \equiv \{ s : T_{j} (s) \in [t, t + \delta] \}$ represent the set of agents that make a transfer during $[t, t + \delta]$. The number of goods used on average during $[t, t + \delta]$ to pay the transfer cost is then given by $\int_{A (t, \delta)} \frac{1}{\delta} \Gamma dF (s)$. Taking the limit to obtain the number of goods used at time $t$ yields that the market clearing condition for goods is given by $\int c (t, s) dF (s) + \dot{K} (t) + \delta K (t) + G + \lim_{\delta \to 0} \int_{A (t, \delta)} \frac{1}{\delta} \Gamma dF (s) = Y$. The market clearing for capital and hours of work are $K (t) = \int k (t, s) dF (s)$ and $H (t) = \int h (t, s) dF (s)$.

An equilibrium is defined as prices $P (t), Q (t)$, allocations $c (t, s), M (t, s), B (t, s)$,
$k(t,s)$, transfer times $T_j(s)$, $j = 1, 2, \ldots$, and a distribution of agents $F$ such that (i) $c(t,s)$, $M(t,s)$, $B(t,s)$, $k(t,s)$, and $T_j(s)$ solve the maximization problem (6) given $P(t)$, $r(t)$, and $s^k(t)$ for all $t \geq 0$ and $s$ in the support of $F$; (ii) the government budget constraint holds; and (iii) the market clearing conditions for money, bonds, goods, capital, and hours of work hold.

3. Solving the Model

As we study the long run effects of financing the government with taxes or inflation, we focus on an equilibrium in the steady state. In this equilibrium, the nominal interest rate is constant at $r$ and the inflation rate is constant at $\pi$. Moreover, the aggregate quantities of capital and labor are constant at $K$ and $H$, and output is constant.

The transfer cost $\Gamma$ and the payment of interest on capital and bonds make agents follow $(S, s)$ policies on consumption, money, capital claims, and bonds. For money, agent $s$ makes a transfer at $T_j$ to obtain money $M^+(T_j, s)$ at the beginning of a holding period. The agent then lets money holdings decrease until $M(t, s) = 0$, just before a new transfer at $T_{j+1}$. Symmetrically, the individual bond holdings $B^+(T_j, s)$ is relatively low at $T_j$ and it increases at the rate $r$ until it reaches $B^-(T_{j+1}, s)$, just before $T_{j+1}$. The same applies to the behavior of $k(t,s)$. We assume that all agents behave in the same way in the steady state, in the sense that they follow the same pattern of consumption along holding periods. With constant inflation and interest rates, this implies that the agents start a holding period with a certain value of consumption, $c^+(T_j, s)$, which decreases until the value $c^-(T_{j+1}, s)$, just before the transfer at $T_{j+1}$. The agents look the same along holding periods, although in general they are in different positions of the holding period.\textsuperscript{11}

As the agents follow the same pattern of consumption along holding periods, it can be shown that they also have the same interval between holding periods $N$ (Silva 2011). Let $n \in [0, N)$ denote the position of an agent along a holding period and reindex agents by $n$. Agent $n$ makes the first transfer at $T_1(n) = n$, and then

\textsuperscript{11}For a description of different applications of $(S, s)$ models in economics, see Caplin and Leahy (2010). See Alvarez et al. (2017) for the relation between inventory theoretical models of the demand for money and the welfare cost of inflation.
makes transfers at \( n + N, n + 2N \) and so on. Given that the agents have the same consumption profile across holding periods, the distribution of agents along \([0, N)\) compatible with a steady state equilibrium is a uniform distribution, with density \(1/N\). We can then solve backwards to find the initial values of \( M_0, B_0, \) and \( k_0 \) for each agent \( n \in [0, N) \) that implies that the economy is in the steady state since \( t = 0\).

To characterize the pattern of consumption of each agent, consider the first order conditions of the individual maximization problem (6) with respect to consumption. These first order conditions imply

\[
c(t, n) = \frac{e^{-\rho t}}{P(t) \lambda(n) Q(T_j)], t \in (T_j, T_{j+1}), j = 1, 2, \ldots,}
\]

where \( \lambda(n) \) is the Lagrange multiplier associated to the budget constraint (4). Let \( c_0 \) denote consumption just after a transfer. In the steady state, \( P(t) = P_0 e^{\pi t}, \) for a given initial price level \( P_0, \) and \( Q(T_j) = e^{-rT_j}. \) Therefore, rewriting (10), individual consumption along holding periods is given by

\[
c(t, n) = c_0 e^{(r-\pi-\rho)t} e^{-r(t-T_j)},
\]

taking the largest \( j \) such that \( t \in [T_j(n), T_{j+1}(n)] \). We find aggregate consumption by integrating (11), using the fact that the distribution of agents along \([0, N)\) is uniform. Aggregate consumption is then

\[
C(t) = c_0 e^{(r-\pi-\rho)t} \left( 1 - e^{-rN} \right) / rN.
\]

As aggregate consumption is constant in the steady state, the nominal interest rate and the inflation rate that are compatible with the steady state are such that \( r = \rho + \pi. \)

From (11), (12), and \( r = \rho + \pi, \) individual consumption \( c(t, n) \) decreases during the holding period \( t \in [T_j, T_{j+1}] \) at the rate \( r. \) On the other hand, aggregate consumption

\[\text{See Silva (2011, 2012) for an additional analysis on the distribution of agents in the steady state. Silva (2011) has the characterization of } M_0 \text{ and } B_0 \text{ for each agent } n. \text{ The characterization of } k_0 \text{ is obtained analogously. We obtain government bonds } B_0 \text{ by the government budget constraint.}\]
is constant at $c_0 \frac{1-e^{-rN}}{rN}$. The individual behavior given by $c(t,n)$ is very different from the aggregate behavior, as in other $(S,s)$ models. In particular, the variability of consumption is much larger at the individual level.

A similar situation happens with bonds. During holding periods, individual bond holdings follow equation (1), which implies that $\dot{B}(t,n)/B(t,n) > r$. However, aggregate bond holdings across agents implies that $\dot{B}(t)/B(t) = \pi$. As $r = \rho + \pi$, individual bond holdings grow at a higher rate than aggregate bond holdings. At the transfer dates $T_j$, however, bond holdings decrease sharply as the agent sells bonds for money and transfers money to the bank account at these dates. As $B(t)$ grows at the rate of inflation, the value of aggregate bond holdings is constant in real terms. Figure 1 shows the evolution of individual bond holding for two agents, $n$ and $n'$. Agent $n$ makes the first transfer at time zero and the second transfer at $T_2(n) = N$. Agent $n'$ makes the first transfer at $T_1(n') > 0$ and the second transfer at $T_2(n') = T_1(n') + N$. The fact that $T_1(n) = 0$ implies that agent $n$ starts with zero money holdings and so the agent needs to make a transfer at $t = 0$. Agent $n'$ starts with some money, which delays the first transfer. At time $t > 0$, the agents have different quantities of bonds, $B(t,n)$ and $B(t,n')$.

Given the production function $Y = Y_0K^\theta H^{1-\theta}$, profit maximization implies that $w = (1-\theta)Y_0(K/H)^\theta$ and $r^k = \theta Y_0(K/H)^{-(1-\theta)}$, which are constant in the steady state. With the non-arbitrage condition $r^k - \delta = r - \pi$ and $r = \rho + \pi$, we have $K/H = [\theta Y_0/(\rho + \delta)]^{1/(1-\theta)}$. Therefore, $K/Y = \theta/((\rho + \delta))$. As $K$ is constant in the steady state, the investment output ratio $(K + \delta K)/Y$ is given by $\delta \theta/(\rho + \delta)$.

The first order conditions for $h(t,n)$ imply

$$1 - h(t,n) = \frac{\alpha c_0}{(1 - \tau_L) w}, \quad (13)$$

As wages are constant in the steady state, individual hours of work are constant along the holding periods. As there is a unit mass of agents, $H = h$. With the expression of wages, we obtain the equilibrium value of the hours of work,

$$h = 1 - \frac{\alpha c_0}{(1 - \tau_L)(1-\theta)Y_0(K/H)^\theta}, \quad (14)$$
As $c_0$ depends on $r$ and $N$, equation (14) determines hours of work as a function of $r$ and $N$.

The market clearing condition for goods in the steady state is $C(t) + \delta K + G + \frac{1}{N} \Gamma = Y$. This equation implies

$$c_0 \frac{1 - e^{-\gamma N}}{rN} + \delta K + G + \frac{1}{N} \Gamma = Y.$$  (15)

Dividing by $Y$, we obtain an expression for the consumption-income ratio $\hat{c}_0 \equiv c_0/Y$ in terms of $N$ and the ratio between government spending and output,

$$\hat{c}_0 \frac{1 - e^{-\gamma N}}{rN} + \delta K \frac{G}{Y} + \frac{1}{N} \frac{\Gamma}{Y} = 1,$$  (16)

where $K/Y = \theta/((\rho + \delta)$.

The optimal holding period $N$ is obtained with the first order conditions for $T_j(n)$. 

---

**Fig. 1:** Individual bond and money holdings for two agents, $n$ and $n'$. Agent $n$ makes the first transfer at time zero, $n = 0$; this agent starts with $M_0(n) = 0$. Agent $n'$ starts with $M_0(n') > 0$ and makes the first transfer at $T_1(n') > 0$. At $t$, bond and money holdings are a function of the elapsed time since the last time the portfolio was rebalanced. Money at the beginning of holding periods increase at the rate of inflation, $\pi$. Individual bond holdings grow at the rate $r$ and aggregate bond holdings grow at the rate $\pi$. 

---
As derived in the appendix, \( N \) must satisfy

\[
c_0 r N \left( 1 - \frac{1 - e^{-\rho N}}{\rho N} \right) = \rho \Gamma. \tag{17}
\]

The aggregate demand for money is given by \( M(t) = \frac{1}{N} \int M(t, n) \, dn \). Individual money holdings at \( t \), \( M(t, n) \), are obtained through the cash-in-advance constraint (5), given individual consumption \( c(t, n) \) for an agent that has made a transfer at \( T_j(n) \), \( M(t, n) = \int_{T_j(n)}^{T_{j+1}(n)} P(\tau) c(\tau, n) \, d\tau, \tau \in (T_j, T_{j+1}) \). At any time \( t \), there will be agents in their holding period \( j + 1 \) and others in their holding period \( j \). Taking this fact into account and the behavior of \( c(t, n) \) in (11), it is possible to express real money holdings \( M/P \) in terms of \( r, N, \) and \( c_0 \). As derived in the appendix,

\[
\frac{M}{P}(r) = \frac{c_0(r, N)}{\rho} e^{-r N(r)} \left[ \frac{e^{r N(r)} - 1}{r N(r)} - \frac{e^{(r-\rho)N(r)} - 1}{(r-\rho) N(r)} \right]. \tag{18}
\]

The values of \( \frac{M}{P} \) and \( N \) are written with respect to \( r \) to emphasize their dependency on the nominal interest rate \( r \). The money-income ratio \( m(r) \equiv M/(PY) \) is obtained by dividing (18) by \( Y \). As output is constant in the steady state, the growth rate of the money supply must be equal to the rate of inflation, \( \pi \).\(^{13}\)

The values of \( m(r) \) can be compared with the data on interest rates and money-income ratio. This is done in figure 2. The data in the figure is similar to the data used in Lucas (2000), Lagos and Wright (2005), Ireland (2009), and Silva (2012). We use the same data to facilitate comparison of results. Especially, to facilitate comparison of the welfare cost values. Equation (18) implies an interest-elasticity of the demand for money around \(-1/2\) and semi-elasticity of \(-12.5\). Lucas (2000), Guerron-Quintana (2009), Alvarez and Lippi (2009) and others argue that the evidence on interest rates and money indicate a long-run interest-elasticity of \(-1/2\).

To close the model, we need an equation that links government spending with the

\(^{13}\)When \( Y \) grows at a constant rate, \( c_0 \) grows at the same rate of \( Y \) and the money-income ratio is constant (Silva 2011).
tax revenue. The government budget constraint (8) implies

\[ rb + G + T = \frac{\dot{B}}{B} + \tau_L wH + \frac{M \dot{M}}{P M}, \]  

(19)

where \( b = \frac{B(t)}{P(t)} \). As \( \frac{\dot{B}}{B} = \pi \) and \( \frac{\dot{M}}{M} = \pi \) in the steady state, we obtain that government consumption and transfers must satisfy the constraint

\[ G + T + (r - \pi) b = \tau_L wH + \pi \frac{M}{P}, \]

(20)

where \( \pi = r - \rho \). Equation (20) states that government spending plus interest payments must be financed through revenues from labor income taxes \( \tau_L wH \) or through seigniorage \( \pi \frac{M}{P} \). If we consider that \( b = 0 \), we have

\[ G + T = \tau_L wH + \pi \frac{M}{P}. \]

(21)

In this formulation, seigniorage is defined as \( S = \pi \frac{M}{P} \), as in Sargent and Wallace (1981), Cooley and Hansen (1991, 1992), Aruoba et al. (2011) and others. In this case, the inflation rate is the analogous to a tax rate on real money holdings.

Alternatively, adding \( (r - \pi) \frac{M}{P} \) to both sides of (20) implies

\[ G + T + (r - \pi) d = \tau_L wH + r \frac{M}{P}, \]

(22)

where \( d = b + \frac{M}{P} \). Seigniorage is now defined as \( S = r \frac{M}{P} \), with the nominal interest rate as the analogous to a tax rate on real money holdings. This formulation emphasizes that, if the government finances itself with money, then it does not pay the interest rate on the quantity of money issued.\(^\text{14}\) Chari et al. (1996), De Fiore and Teles (2003) and others set \( d = 0 \), that is, the nominal assets of agents \( b + \frac{M}{P} \) are equal to zero. This implies

\[ G + T = \tau_L wH + r \frac{M}{P}. \]

(23)

\(^\text{14}\)These steps to write seigniorage with two interpretations from the government budget constraints follow Walsh (2010).
Equations (21) or (22) complete the characterization of the equilibrium. As the literature uses $S = \pi \frac{M}{P}$ or $S = r \frac{M}{P}$ as different definitions of seigniorage, we will make separate simulations for both definitions. This formulation implies five equations (equations 14, 15, 17, 18, and either 21 or 23, depending on the definition of seigniorage) and five equilibrium variables ($N$, $h$, $c_0$, the ratio $M/P$, and $\tau_L$ or $r$, depending on the method of financing). The initial equilibrium price is obtained by setting an initial value for the money supply. Equilibrium output is obtained by writing $Y = Y_0(K/H)^{\theta}h$, where $Y_0$ is normalized to 1. We take as given the values for government consumption $G$ and government transfers $T$.

4. An Increase in Government Spending

We calculate the effects of an increase in government spending in the form of transfers $T$ or in the form of government consumption expenditures on goods and services $G$. The economy is initially in a long run equilibrium, following the equilibrium equations described in section 3.\textsuperscript{15} Given the equilibrium for an initial labor tax $\tau_L$ and interest rate $r$, we change the value of $G$ or $T$ and recalculate the values of $\tau_L$ and $r$ so that the system of equations is satisfied for the new value for government spending.

We change $\tau_L$ and $r$ separately. That is, for financing the increase in spending with labor income taxes, we maintain the value of $r$ at its initial value, and find $\tau_L$ so that the government budget constraint (20) and the remaining equations for the equilibrium are satisfied. Analogously, for financing the increase in spending with inflation, we maintain $\tau_L$ and find the interest rate $r$ such that (20) and the remaining equations for the equilibrium are satisfied. The new inflation rate is given by $r - \rho$.

Welfare Cost

The welfare cost of a fiscal policy $A$ with respect to a fiscal policy $B$ is defined as the income compensation $w_A$ that leaves agents indifferent between an economy under $A$ and economy under $B$. A fiscal policy is defined as the values of $G$, $T$, $\tau_L$ and $r$. Let $c_i$ and $h_i$ denote the equilibrium profiles of consumption and hours of

\textsuperscript{15}Equations (14), (15), (17), (18), and either (21) or (23).
work for all agents in the support of $F$ under policy $i = A$ or $B$. The value of $w_A$ is such that $U^T [(1 + w_A) c_A, h_A] = U^T (c_B, h_B)$, where $U^T$ is the aggregate utility with equal weights for all agents.\footnote{$U^T = \int_0^{\infty} e^{-\rho t} u(c(t,n), h(t,n)) dt dF(n)$. This definition of $w_A$ uses the fact that consumption is homogeneous of degree one in income.} The preferences $u(c,h) = \log c + \alpha \log (1 - h)$ imply

$$1 + w_A = \frac{c_{0,B}}{c_{0,A}} \left( \frac{1 - h_B}{1 - h_A} \right)^{\alpha} \exp \left( \frac{r_A N_A}{2} - \frac{r_B N_B}{2} \right).$$

(24)

The values of $c_{0,i}$, $h_i$, $N_i$, and $r_i$, $i = A, B$, are given by the equilibrium conditions in section 3.

Government consumption does not enter the utility function. Therefore, an increase in $G$ always implies a positive welfare cost with respect to the economy with lower $G$. Government consumption enters the market clearing condition and decreases the availability of private consumption for the same output. In any case, we compare an economy with the same value of government consumption, but in which $A$ denotes financing with inflation and $B$ denotes financing with labor income taxes. If $w_A$ is positive for this case, the interpretation is that the agents in the economy would be better off if the government financed government consumption with taxes rather than with inflation.

A way to circumvent the effect of government consumption through the market clearing condition is to consider an increase in government transfers $T$. In this way, the government taxes the economy in a distortionary way and redistributes the tax revenues in lump sum form.

We maintain the two forms of increasing government spending, through government consumption or transfers, because they yield different results and because they allow the analysis of different aspects of fiscal policy. When the government increases $G$, for example, we can study the government consumption multiplier. Also, we can analyze the behavior of the multiplier when the increase in government consumption is financed with inflation or with taxes.

\textit{Calibration}

As in Cooley and Hansen (1989), we set $\theta = 0.36$ for the parameter for capital in the production function and $\delta = 0.10$ for the depreciation. As in Lucas (2000), we
set $\rho = 3$ percent per annum. We maintain these parameters in all simulations.

The parameters $\alpha$ and $\Gamma$ are set so that hours of work are equal to 0.3 and that the money-income ratio $m(r)$ derived from equation (18) passes through the geometric mean of the data. That is, $r_{avg} = 3.64$ percent p.a. and $m_{avg} = 0.257$ year (this value for $m_{avg}$ in the data implies that the average person in the U.S. holds about one quarter of income in money, or that the average velocity is about $1/0.25 = 4$ per year). Figure 2 shows the resulting $m(r)$ together with the data. Similarly, Lucas (2000) determines the parameters for the demand for money such that the theoretical demand for money passes through the geometric average of the data. Also, Alvarez et al. (2009) obtain the holding period (exogenous in their case) such that the theoretical demand for money approximates the average velocity in the data.

To facilitate the comparison of results, we use the same dataset used in Lucas (2000) and Silva (2012). In particular, we use commercial paper rate for the nominal interest rate and M1 for the monetary aggregate. Data are annual from 1900 to 1997 (the last year in which the commercial paper data are available from the same source). M1 and commercial paper rate were also used by Dotsey and Ireland (1996), Lagos and Wright (2005), Craig and Rocheteau (2008), among others.\footnote{We use these variables to facilitate comparison of our quantitative results with previous models. As it is well known, model demand changed substantially at the beginning of 1990s. Teles and Zhou (2005) and Lucas and Nicolini (2015) propose new monetary aggregates to account for the changes in the demand for money. Changing the monetary aggregate does not change our conclusion that the welfare cost of financing the government with inflation with $N$ endogenous increases substantially.}

We set the initial value of government spending such that the initial ratio of government spending to output is equal to 20 percent. Twenty percent is the average value of this ratio for the United States from 1956 to 2012. We also obtained the predictions for different values of the government-to-output ratio such as zero, 5 percent, and 10 percent. The values change, but the qualitative results do not change.

The value of $\tau_L$ is obtained so that equations (21) or (23) are satisfied. We set the initial value for seigniorage using the mean of the nominal interest rate. This procedure yields an initial value for $\tau_L$ equal to 29.79 percent when seigniorage is given by $S = rM/P$ and equal to 30.99 percent when $S = \pi M/P$. Initial seigniorage is equal to 0.94 percent when $S = rM/P$ and to 0.16 percent when $S = \pi M/P$.
Fig. 2: Money-income ratio implied by the model and U.S. annual data. M1 for the monetary aggregate and commercial paper rate for the nominal interest rate.

We depart from other calibrations in the determination of $\tau_L$ as we obtain $\tau_L$ such that it satisfies the government budget constraint. We do not take this value from estimates of marginal tax rates. In particular, there are no lump sum taxes to close the government budget constraint. Nevertheless, the value of $\tau_L$ is close to the ones in other papers. For example, $\tau_L = 23$ percent in Cooley and Hansen (1992) and $\tau_L = 25.11$ percent Aruoba et al. (2011).

As hours of work and the money-income ratio are equilibrium variables, the values of $\alpha$ and $\Gamma$ so that $h = 0.3$ and $m = m_{avg}$ vary if government spending is in the form of transfers or government consumption, and if seigniorage is defined with the inflation rate or with the nominal interest rate. The values, however, do not vary much. The value of $\alpha$ varies from 1.45 for the case of an increase in transfers and $S = rM/P$, the case discussed in the introduction, to 2.00, for the case of an increase in government spending and $S = rM/P$. The value of $\Gamma$ in these two cases is 36.65 and 51.16 respectively. To have an idea of the meaning of these values, consider the ratio $\Gamma/Y$, which yields the cost of a transfer in working days. This ratio is equal to 2.49 with transfers and 3.41 with government consumption.\textsuperscript{18} These values imply

\textsuperscript{18}Silva (2012) uses $\Gamma = \gamma Y$ for the cost parameter and calibrates $\gamma$. Using $\Gamma = \gamma Y$ implies that the demand for money is homogeneous of degree 1 in income. However, as $Y$ is an equilibrium value, $\gamma Y$ varies with the policy. In spite of these differences, the values of $\gamma$ and of $\Gamma/Y$ are approximately equal. Moreover, we recalculated the simulations with $\Gamma$ and with $\Gamma = \gamma Y$ and the
infrequent transfers from the brokerage account to the bank account, as the initial value of $N$ is equal to 367 days with government consumption and 264 days with transfers. When inflation increases to finance the increase in government spending, the values of $N$, when endogenous, decrease to 197 days and 127 days.

A more informative assessment of the transfer cost is the average cost of financial transfers per year as a fraction of income and the minutes per week devoted to financial services. The average cost of financial transfers per year as a fraction of income is given by $\Gamma/Y \times 1/N$. This value implies 0.95 percent of time devoted to financial services, the same measure for the four cases considered in the simulation (the four combinations between government consumption and transfers, and seigniorage defined with inflation or nominal interest rates). According to the OECD, the average weekly hours of U.S. workers from 1957 to 1997 is equal to 36.5 hours per week. The time devoted to financial transfers implied by the model in the initial steady state is then $\Gamma/Y \times 1/N \times 36.5 \times 60 = 21$ minutes per week.

As in other models with market segmentation, the value of $N$ implied by the parameters is large. For comparison, Alvarez et al. (2009) use $N$ equal to 24 months and 36 months.\(^{19}\) Notice that $N$ is the interval between exchanges of high-yielding assets to low-yielding assets; it is not the interval between ATM withdrawals. Christiano et al. (1996), Vissing-Jorgensen (2002), and Alvarez et al. (2009) show evidence that, in fact, firms and households rebalance their portfolios infrequently, in a way that explains the values found for $N$.

Edmond and Weill (2008) argue that the large values for holding periods in market segmentation models are an effect of the aggregation assumptions in these models. Agents in the model encompass firms and households, and firms hold a large portion of money in the economy (Bover and Watson 2005). Moreover, the use of cash by firms has increased across firms (Bates et al. 2009, Adao and Silva 2016). The parameters reflect the large money holdings found in the data (the money-income ratio of 0.25 in the data imply about 10 thousand dollars per person in money in the U.S., or about 30 thousand dollars per household).\(^{20}\)

\(^{19}\)Alvarez et al. use M2 instead of M1, which requires a higher value for $N$. Moreover, they allow for cash inflows. Their calibrations with cash inflows imply the large values of $N$ such as 36 months.

\(^{20}\)Telyukova (2013) reports that agents maintain large money holdings even when they pay high
**An Increase in Government Spending**

We now set the economy with the initial value for the ratio government spending to output equal to 20 percent and increase the value of government spending by 5 percent. That is, we multiply the initial value by 1.05. This increases the ratio of government spending to output by about 1 percentage point, from 20 percent to 21 percent.

We study two ways of financing the increase in government spending: by increasing labor income taxes $\tau_L$ and by increasing the inflation rate $\pi$. We consider seigniorage to be given by $S = rM/P$ or $S = \pi M/P$. The increase in government spending can be in the form of transfers or in the form of government consumption.

Results are in tables 1-3 and in figures 3-6. We study the effects of the change in policy using the model with a fixed $N$ and with endogenous $N$. The value of $N$ when it is fixed is equal to the optimal value of $N$ for the initial situation. All other parameters are the same for the cases with $N$ fixed and $N$ endogenous. The only difference between the two cases is that $N$ is allowed to change optimally in the case of $N$ endogenous. Figure 5 shows the effects when seigniorage is defined with the interest rate and figure 6 with inflation. The definition of seigniorage has a small impact. The form of the increase in government spending, either transfers or government consumption, the method of financing, and having $N$ fixed or endogenous are more important for the results.

The case of fixed $N$ approximates the standard cash-in-advance model with fixed periods. It is not equal to a standard cash-in-advance model because the agents can still smooth consumption during the interval $N$. So, the demand for money varies with $N$ fixed, although it is approximately constant. However, the effects in this model and in a model in which consumption cannot change are similar. What strongly changes the predictions of the model is the case in which $N$ is endogenous. In this case, the demand for money decreases more strongly when inflation increases, a pattern compatible with the data. Table 1 shows the welfare cost calculations of the change in policy.

As shown in table 1, considering fixed periods underestimates the welfare cost of financing the increase in government spending with inflation. Consider the case of interest rates for credit cards.
an increase in government spending in the form of transfers and seigniorage defined
with the nominal interest rate, in the first row of table 1. Financing the increase
in government spending with inflation implies a welfare cost of 1.45 percent using
the model with endogenous periods. The welfare cost is only 0.49 percent with fixed
intervals. The underestimation of the effects is such that a model with fixed \( N \)
implies that there is a gain in financing the increase in government spending with
inflation, shown by the negative value \(-0.46\) in the table. The result is reversed with
endogenous \( N \).\(^{21}\)

<table>
<thead>
<tr>
<th>Values in % of income</th>
<th>Model and Method of Financing</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( N ) Endogenous</td>
</tr>
<tr>
<td></td>
<td>Inflation Labor Tax From Tax to Inflation</td>
</tr>
<tr>
<td>Transfers, Seigniorage ( r \times M/P )</td>
<td>1.45 0.95 0.50</td>
</tr>
<tr>
<td>Government Cons, Seigniorage ( r \times M/P )</td>
<td>3.03 2.11 0.90</td>
</tr>
<tr>
<td>Transfers, Seigniorage ( \pi \times M/P )</td>
<td>0.97 1.01 (-0.04)</td>
</tr>
<tr>
<td>Government Cons, Seigniorage ( \pi \times M/P )</td>
<td>2.41 2.13 0.28</td>
</tr>
</tbody>
</table>

Table 1: Welfare cost of an increase in government spending, \( w \)

Government expenditures increase 5%. This experiment implies an increase in the government-output ratio from 20% to about 21%. Increase in government expenditures either in the form of transfers or government consumption. The calculation of the welfare cost follows equation (24). Seigniorage defined with the nominal interest rate or with the inflation rate. From tax to inflation: welfare cost of changing the method of financing from labor income tax to inflation. A negative sign implies a welfare gain.

Figure 3 shows the welfare cost of changing the government financing method from labor income taxes to inflation. The values correspond to columns 4 and 7, rows 1 and 2, of table 1, where the table shows the value for the total increase

\(^{21}\)The welfare cost of financing with inflation is larger with endogenous periods. However, the table shows a negative (although small) welfare cost of moving financing from taxes to inflation in row 3. With fixed periods, the gain of inflation is even higher. This happens because there is only one distortionary tax available, because \( S = \pi M/P \), which implies \( b + M/P \neq 0 \), and because of the existence of transfers. See Adao and Silva (2017).
in government spending.\footnote{In the units of the model, the value of $G$ increases from 2.94 to 3.087, shown in figure 3, which correspond to an increase of five percent.} We see from the figure that a model with fixed holding period implies a gain of changing the financing method from taxes to inflation. With endogenous holding periods, there is a positive welfare cost. When inflation increases, the frequency of portfolio rebalancing increases, which increases the costs of cash management. This cost is not taken into account in standard cash-in-advance models. The additional cost of portfolio rebalancing increases substantially the welfare cost of inflation.

![Welfare Cost from Tax to Inflation](image1.png)

(a) Increase in transfers

![Welfare Cost from Tax to Inflation](image2.png)

(b) Increase in government consumption

Fig. 3: Welfare cost of a change the method of financing an increase in government expenditures, $w_A$. The calculation of $w_A$ follows equation 24, where the economy $A$ finances an increase in government expenditures with an increase in inflation and $B$, with an increase in labor income taxes. Seigniorage $r \times M/P$. The total increase in $G$ in the figure reflects an increase from 20\% to 21\% of GDP. Negative values imply a welfare gain of changing policy from taxes to inflation. A standard cash-in-advance model with fixed holding periods implies gains of financing an increase in expenditures with inflation. Here, with endogenous holding periods, there are substantial costs of relying on inflation.

A welfare loss of decreasing inflation in the case of $N$ fixed apparently conflicts with the literature on optimal taxation. Although Phelps (1973) stated that positive inflation can be optimal when only distortionary taxes are available, more recent
results point out that the Friedman rule is optimal in cash-in-advance models with distortionary taxation.  

23 This is the case of Kimbrough (1986), Correia and Teles (1991, 1996), Chari et al. (1996), De Fiore and Teles (2003) and others. However, money is introduced in these contexts with a transactions technology. This is not the case here or in Cooley and Hansen (1991, 1992). Guidotti and Vegh (1993) and Mulligan and Sala-I-Martin (1997) have more restrictive assumptions for the Friedman rule to hold.

Chari et al. (1996) discuss the case of a cash-in-advance economy without a transactions technology, but the optimality of the Friedman rule is obtained when taxes on labor income and consumption are both available. Adao and Silva (2017) show that the Friedman rule is not optimal in the context of Chari et al. (1996), even when both labor and consumption taxes are available, when there are positive transfers. Another aspect, as Lucas (2000) points out, is that these papers assume that the sum of initial money and bonds are equal to zero. This is not the case in Cooley and Hansen. The sum of initial money and bonds equal to zero affects the results of financing with inflation, however, it does not change the conclusion that the welfare cost of inflation is higher with endogenous periods.

Table 2 shows that considering fixed periods underestimates the inflation required to finance the increase in government spending. For the same case as above, the required inflation is 12.72 percent per year with endogenous periods while it is 5.47 percent with fixed periods. With fixed periods, the demand for money does not change much with inflation. This implies that the government is able to finance itself with a smaller rate of inflation. Table 3 shows revenues from seigniorage and from labor taxes in each case. Notice that government spending is still financed mostly with labor income taxes, even when the government uses inflation to finance

\[ \text{Table 2}\]

\[ \text{Table 3}\]

\[ \text{Aruoba et al. (2011) also find a welfare loss of decreasing inflation with distortionary taxation when the initial situation is taken from the data. Da Costa and Werning (2008) have additional results on the Friedman rule. See Kocherlakota (2005) for a survey. Our focus here is on the effects of an increase in government expenditures financed in different ways.}\]
the additional increase in expenditures.\textsuperscript{25}

Table 2: Inflation and labor tax to finance a 5\% increase in government expenditures

<table>
<thead>
<tr>
<th>Model and Method of Financing</th>
<th>$N$ Endogenous</th>
<th>$N$ Fixed</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Inflation, $\pi$</td>
<td>Labor Tax, $\tau_L$</td>
</tr>
<tr>
<td>Transfers, Seigniorage $r \times M/P$</td>
<td>12.72</td>
<td>32.12</td>
</tr>
<tr>
<td>Government Cons, Seigniorage $r \times M/P$</td>
<td>9.68</td>
<td>31.44</td>
</tr>
<tr>
<td>Transfers, Seigniorage $\pi \times M/P$</td>
<td>7.82</td>
<td>33.37</td>
</tr>
<tr>
<td>Government Cons, Seigniorage $\pi \times M/P$</td>
<td>5.79</td>
<td>32.65</td>
</tr>
</tbody>
</table>

Initial inflation rate: 0.64\% p.a. (percent per annum). Initial labor tax: 29.79\% (case $rM/P$) and 30.99\% (case $\pi M/P$). See table 1 for welfare cost calculations and definitions.

The predictions with $N$ fixed or endogenous are different when the increase in government spending is financed with inflation. In this case, the opportunity cost of holding money increases and, therefore, the agents spend resources to decrease their real demand for money. With fixed periods, the agents can decrease slightly the demand for money by making consumption within holding periods steeper. This behavior also shows up with $N$ endogenous, but most of the decrease in the demand for money is obtained by decreasing $N$. The value of $N$ decreases 50\% with the change in inflation. With $N$ fixed, the money-income ratio decreases about 2\% percent. With $N$ endogenous, the money-income ratio decreases about 50\%. An inflation of 12\% p.a. would then make the money-income ratio decrease to the values in the 1980s, corresponding to the data points on the Southeast of figure 2.

We interpret the decrease in the interval $N$ as an increase in the use of financial services. Let $\Gamma \frac{1}{N}$ be the measure of the financial system in the economy. Using this

\textsuperscript{25}A different question is the transition toward the new policy. Studying the equilibrium transition of an aggregated $(S, s)$ model requires techniques that are beyond the objective of this paper. Adao and Silva (2015) study the transition for a simplified version of the model. See also Gertler and Leahy (2008) and Stokey (2008).
measure, the value of financial services as a fraction of output increases from 0.95 percent to 1.97 percent for the case of an increase in transfers financed with inflation and seigniorage defined with the nominal interest rate. With seigniorage defined with inflation, the fraction of financial services to output increases from 0.16 percent to 1.16 percent. For the four cases considered, the fraction of financial services to output increases by about 1 percentage point. English (1999) provides evidence that the share of financial services in the economy increases with inflation. According to English, a 10 percent increase in inflation in U.S. would imply an increase in the financial sector of 1.3 percentage points, which agrees with our results.

Table 3: Revenues from seigniorage and from labor taxes after a 5% increase in G

<table>
<thead>
<tr>
<th>Values in % of output</th>
<th>Model and Method of Financing</th>
<th>N Endogenous</th>
<th>N Fixed</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Inflation</td>
<td>Labor Tax</td>
</tr>
<tr>
<td></td>
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<td>$\tau_LwH$ Seig</td>
<td>$\tau_LwH$ Seig</td>
</tr>
<tr>
<td>Transfers, Seigniorage $r \times M/P$</td>
<td>19.06</td>
<td>1.93</td>
<td>20.56</td>
</tr>
<tr>
<td>Government Cons, Seigniorage $r \times M/P$</td>
<td>19.06</td>
<td>1.71</td>
<td>20.12</td>
</tr>
<tr>
<td>Transfers, Seigniorage $\pi \times M/P$</td>
<td>19.84</td>
<td>1.16</td>
<td>21.36</td>
</tr>
<tr>
<td>Government Cons, Seigniorage $\pi \times M/P$</td>
<td>19.84</td>
<td>0.94</td>
<td>20.89</td>
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</tbody>
</table>

$G$: government expenditures in the form of transfers or government consumption. The initial value of $G$ is such that $G/Y$ is equal to 20%. Initial revenues from labor taxes: 19.06% ($rM/P$) and 19.85% ($\pi M/P$). Initial revenues from seigniorage: 0.94% ($rM/P$) and 0.16% ($\pi M/P$).

Figure 4 shows the size of the financial sector, for each value of spending, when the increase in government spending is financed with inflation. In accordance with the empirical evidence, the model with endogenous holding periods implies an increase in the financial sector as a percentage of GDP. With fixed holding period, the size of the financial sector, counterfactually, does not change. As stated above, the total increase in the financial sector implied by the model, of about 1 percentage point,

\footnote{This evidence is also consistent with Aiyagari et al. (1998).}
corresponds to the empirical evidence on the increase in the financial sector when inflation increases 10 percentage points.\textsuperscript{27}

\begin{figure}[h]
\centering
\begin{subfigure}{0.49\textwidth}
\includegraphics[width=\textwidth]{transfers}
\caption{Increase in transfers}
\end{subfigure}
\hfill
\begin{subfigure}{0.49\textwidth}
\includegraphics[width=\textwidth]{government_consumption}
\caption{Increase in government consumption}
\end{subfigure}
\caption{Size of the financial sector when the government finances an increase in expenditures with inflation, $(\Gamma \times 1/N)/Y$. Inflation increases about 10 percentage points in each case (table 2). With fixed periods, the model does not predict an increase in the financial sector. With endogenous periods, the financial sector increases about 1 percentage point, which agrees with the empirical evidence in English (1999) of an increase in inflation of 10 percentage points.}
\end{figure}

An unexpected effect of considering $N$ endogenous is the response of output after the increase in $G$. The response of output implies a government spending multiplier slightly above 1, for $N$ endogenous, when government spending is in the form of government consumption and when it is financed with inflation. When the increase in $G$ is financed with taxes, agents substitute away from labor toward leisure, which decreases output. When $N$ is fixed, inflation and taxes have similar effects. When $N$ is endogenous, the increase in inflation makes agents decrease the demand for money, which requires an increase in the provision of financial services. The increase in financial services implies an increase in output. This effect implies a multiplier above 1 when the increase in government spending is made through consumption and

\textsuperscript{27}The financial sector provides more services than it is implied by the model. In particular, the model abstract from risk sharing, capital allocation, and other financial services. We are concerned here with the increase in the financial sector solely given by the increase in inflation.
financed with inflation. When the increase in spending is in the form of transfers, the multiplier is smaller, but it is still positive. With transfers, the multiplier is equal to $-1.2$ with $N$ fixed and $0.35$ with $N$ endogenous. With government consumption, the multiplier is equal to $-0.01$ with $N$ fixed and $1.09$ with $N$ endogenous.

A multiplier slightly above $1$ is compatible with the values stated in Hall (2009) and Ramey (2011). Woodford (2011) discusses how the multiplier can be above $1$ in models with sticky prices or sticky wages. Here, the multiplier is above $1$ with flexible prices. The only friction in the model are financial frictions. Although output increases, the welfare cost of the economy with high inflation is large. Output increases, but the overall effect is such that welfare decreases.

Having endogenous periods is especially relevant to study policy changes that imply changes in inflation. The effects on the welfare cost, output and consumption are approximately equal with $N$ fixed or endogenous when the increase in government spending is financed with taxes. The predicted increase in $\tau_L$ is also similar in the two cases. The models with $N$ fixed or endogenous yield similar predictions as inflation does not change when taxes are used to cover the increase in government spending.

Endogenous periods matters crucially when the policy change involves an increase in inflation. In this case, the welfare cost of financing with inflation is larger with $N$ endogenous. The inflation rate, output and consumption are also different with $N$ fixed or endogenous.

A model with $N$ endogenous implies a better match on the behavior of the demand for money and on the increase in financial services after an increase in inflation. Given the results obtained here, it is important to consider changes in the frequency of trades to evaluate the effects of inflation. Especially, for the estimates of the welfare cost of different policy changes.

5. Conclusions

We analyze the effects of an increase in government spending. We consider that the increase in government spending can take the form of transfers or government consumption and that it can be financed through taxes or through inflation. The novelty is that the economy reacts to an increase in inflation by increasing the use
of financial services, modeled by the frequency of financial trades. This frequency is fixed in standard cash-in-advance models.

The welfare cost of financing the government with inflation increases substantially when the frequency of financial trades is taken into account. For conventional parameters, a standard cash-in-advance model underestimates the welfare cost of inflation and implies that financing with inflation is optimal. We reverse this result when we take into account the use of financial services. Financing with taxes becomes optimal.

When the government is financed with taxes, the models with fixed or endogenous holding periods imply similar predictions. This is so because the use of financial services and the demand for money do not change, as inflation does not increase, when the government is financed through taxes.

The predictions differ when the change in policy involves an increase in inflation. Higher inflation increases the use of financial services. The increase in the use of financial services appears in the model as an increase in the frequency of payments to transform bonds into money. The welfare cost of inflation increases substantially.

The effects on welfare, output and consumption change with endogenous periods. The match between data and model predictions on the demand for money improves. The increase in the use of financial services implied by the model also agrees with data. We conclude that it is important to consider the decisions on the trading frequency to analyze policies that imply changes in inflation.

References


Appendix

A.1. Optimal Holding Period $N$ (equation 17)

The first order conditions with respect to $T_j (n)$, $j = 2, 3, ...$ imply $e^{-\rho T_j} P(T_j) Q(T_j) \left[ \log c^{-1} (T_j) - \log c^{+1} (T_j) \right] = \lambda [-r(T_j) \int_{T_j}^{T_j+1} \frac{P(t)c(t)}{P(T_j)} dt - c^{+1} (T_j) + \frac{Q(T_{j+1})c^{-1}(T_j)}{Q(T_j)} - \Gamma (r(T_j) - \pi (T_j))].$

The first order conditions with respect to $c(t, n)$ imply $e^{-\rho T_j} c^{+1} (T_j) = \lambda Q (T_j) P (T_j)$ and $e^{-\rho T_j} c^{-1} (T_j) = \lambda Q (T_{j-1}) P (T_j)$. Therefore, $\log \frac{c^{-1} (T_j)}{c^{+1} (T_j)} = -rN_j$. The first order conditions with respect to $T_j (n)$ simplify to $c^{+1} (T_j) rN_j - r \int_{T_j}^{T_j+1} \frac{P(t)c(t)}{P(T_j)} dt = \Gamma (r - \pi)$.

In the steady state, $c(t) = c_0 e^{-r(t-T_j)}$, $c^{+1} (T_j) = c_0$, and $c^{-1} (T_j) = c_0 e^{-r(T_j-T_{j-1})} = c_0 e^{-rN_j}$. These equations imply $c_0 rN_j - c_0 rN_{j+1} \frac{1-e^{-rN_{j+1}}}{\rho N_{j+1}} = (r - \pi) \Gamma$. Setting $N_{j+1} = N_j$, $r - \pi = \rho$, and rearranging yields equation (17) in the text.
A.2. Real Money Holdings (equation 18)

At time \( t > jN \), agents \( n \in [t-jN, N) \) will be in their \( j \)th holding period and agents \( n \in [0, t-jN) \) will be on their \((j+1)\)th holding period. The transfer time \( T_j \) for agent \( n \) is \( T_j \equiv n + (j - 1)N, j = 1, 2, \ldots \). Money demand for agent \( n \in [0, t-jN) \) at time \( t \) is

\[
M_1(t, n) = \int_t^{t+1} e^{-rT_j} c(t) dt,
\]

where \( c(t, n) = c_0 e^{-r(T_j + 1)} \), \( T_j + 1 \leq t < T_j + 2 \). In the steady state, \( P(t) = P_0 e^{\pi t} \), where \( P_0 \) is the initial price level and \( \pi \) is the inflation rate. Therefore, \( M_1(t, n) = \int_t^{t+1} P_0 c_0 e^{\pi t} e^{-r(T_j + 1)} dt \). Analogously, money demand for agent \( n \in [t-jN, N) \) at time \( t \) is

\[
M_2(t, n) = \int_t^{t+1} P_0 c_0 e^{\pi t} e^{-r(T_j + 1)} dt.
\]

Aggregate money demand is

\[
M(t) = \frac{1}{N} \int_0^{t-jN} M_1(t, n) dn + \frac{1}{N} \int_{t-jN}^N M_2(t, n) dn.
\]

This expression yields \( M(t) = \frac{P_0 c_0 e^{\pi t} e^{-r(T_j + 1)}}{1 - e^{r(T_j + 1)}} \). Solving the integral, with \( r - \pi = \rho \), and dividing by \( P(t) \) yields equation (18) in the text.
(a) Effects of an increase in transfers

(b) Effects of an increase in government consumption

Fig. 5: Financing methods and equilibrium variables after an increase in spending. Output and consumption relative to their initial values. Seigniorage $S = rM/P$. 37
Fig. 6: Financing methods and equilibrium variables after an increase in spending. Output and consumption relative to their initial values. Seigniorage $S = \pi M/P$. 

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