Teacher quality and student achievement: Evidence from Brazilian longitudinal data

Larissa da Silva Marioni - PPGEA/UFJF
Ricardo da Silva Freguglia - PPGEA/UFJF
Naércio Aquino Menezes Filho - INSPER and FEA/USP

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Abstract

An important issue in the literature of education economics is identifying inputs that are relevant for students' learning. Particularly, there is no consensus about the influence of teacher's quality on the students' performance. The aim of this paper is to identify the impact of teachers to the learning process, using longitudinal student-level data from Brazil. We estimate an education production function including students, teachers and match fixed effects. The results show that teachers' characteristics, such as education, experience and income impact Portuguese test scores. As for mathematics, these variables were not statistically significant, indicating that only unobserved teachers characteristics explain differences in students' scores. The results also show that a student changing from the worst to the best Portuguese teacher would move from the first to the third learning level, according to the proficiency scale. As for mathematics, a student moving from the worst to the best teacher would move from the first to the fifth level.

KEYWORDS: educational production function, teacher quality, match fixed effects, GERES.

JEL: I21, I24

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1 Introduction

Attention to basic education is not paid only in developed countries but also in developing countries[1], as education exerts strong influence on economic development and reduction of social inequalities. A country having individuals with a higher education level also exhibits higher rates of growth and innovation. Several studies have revealed that proficiency tests carried out with children and youngsters bear strong relation with success in the labor market (Leibowitz, 1974; Murnane, Willett, and Levy, 1995; Neal and Johnson, 1996; Keane and Wolpin, 1997; Cameron and Heckman, 1998; Hanushek and Rivkin, 2006a; Cunha and Heckman, 2007, 2009).

Economists have long been concerned with poor educational results occurring persistently in mainly developing countries. These countries show poor quality levels of education where most students in elementary and secondary schools show lower learning performance as compared to their counterparts in developed countries. Such a gap may be explained by differences in family characteristics together with low school quality in developing countries.

The school would be able to make the difference if it could count on better school inputs, management and teaching practices so as to improve its educational performance. Therefore, what really matters is to find which inputs affect the children's learning performance in order to put effective public policy into action.

According to Glewwe and Kremer (2006), many changes in education policy have been recently made in developing countries. Additionally, differently from developed countries, many education policy assessments have been carried out in developing countries that made education studies a promising research effort in the last years.

Teachers play a crucial role in education policy discussions. Parents, teachers and educational managers emphasize the teacher's central role in determining the school quality, mainly in the elementary school together with the relevant role played by school infrastructure in the learning process. Teachers' motivation and ability in transmitting knowledge are characteristics that could make the difference in a classroom. However, in spite of being relevant, teacher quality is difficult to observe in this process, since many times evidence can only be indirectly inferred. Among the observable characteristics, experience, background and salary are mostly used in the literature, despite their controversial effects found in the production function models (Hanushek and Rivkin, 2006b)

[1] According to the Organization for Economic Cooperation and Development (OECD, 2013), projects using student tests to assess teachers' performance can be found in countries like Mexico and Chile.

According to Cunha and Heckman (2007), school background and abilities occur by means of a dynamic process, in which acquired ability level in one stage acts as a basis for ability formation in the following stage. Such ability inequalities among individuals appear in the initial life span and persist all through time. Therefore, children counting on lower investment in their initial educational cycle present major weaknesses in subsequent cycles.

According to Barros and Mendonça (1997), access to a good educational system is crucial for individual and national development. The existing poor educational level and lagging expansion of the educational system in Brazil are a characteristic of the country's national development, which makes it different from those of other Latin American and Asian countries with similar income levels.

One of the hypotheses justifying such a difference in the student proficiency levels is related to the teacher quality. The teacher maintains direct relationship to students and mainly aims to encourage their learning process. Additionally, it is the teacher who makes some choices as to the subject to be taught in the classroom, for instance, classroom drills, student performance assessment and ways of encouraging students' class participation.

However, the greatest challenge facing economists is to develop models that are able to estimate the significance of the teachers' effect on their students learning. One cause of difficulty to find a strong relation between school inputs and students' performance derives from the weakness of the variables used to measure quality. Menezes-Filho (2007) found that salary had an impact on students' performance only in the private school network, while school background showed a little impact and the teachers' qualification had no impact at all. Nevertheless, this does not mean that teachers are not important, only that unobserved features such as motivation and didactic are what really matter.

The hypothesis to be tested in the present study is whether a good quality teacher positively influences the students' performance through time.The aim of this study is to identify the impact of the teachers' contribution on the students' learning process by knowing their proficiency in Portuguese language and mathematics in public and private schools. This study estimates an education production function (EPF) including fixed effects of students, teachers and the match between them, in order to detect the observed effect of teacher quality, as well as attempting to avoid the bias arising from disregard of the variables of students' performance. Such a specification has not been used yet in the empirical literature on the teacher's effect on the students' performance. According to Tiebout (1956), students may not be randomly distributed among schools, due to the fact that families' choices concerning the school and its neighborhood depend on their preferences and resources and this nonrandom choice may contaminate estimates of teacher or school effects.

The longitudinal microdata of the GERES project - *Estudo Longitudinal da Geração Escolar 2005* (longitudinal study of learning generation) allows treating heterogeneity by means of a fixed-effect model, which controls fixed effects of students, teachers and the match between them and relate the remaining differences in proficiency to differences in teachers' characteristics.

This paper is organized as follows: in addition to this introduction, section 2 constitutes an approach of the theoretical framework. Section 3 presents the database used together with the descriptive statistics. The study methodology is presented in section 4, while section 5 deals with the analysis of the results obtained. Finally, section 6 presents final considerations, while bibliographic references are described in section 7.

2 Literature review

Coleman et al. (1966) reflected on the relevance of analyzing factors affecting students' learning. Since then the literature concerning this subject attempts to relate factors associated with students' school performance with school factors and those factors that are external to school. And as far as economists started discussing on this subject, a new concept of analysis was introduced with the aim to investigate educational relationships, which was based on the production function used for analyzing firms in general.

For Hanushek (1979), production function and theoretical issues, which are related to ideal choices made by firms, provide a structural base for describing an efficient production, as well as their adequate response to technology advances or cost entries, etc. Changes in familiar characteristics are difficult to be handled by means of public policies as they are hardly sensitive to these policies, in addition to be expensive in the short run. School inputs should be analyzed with care, as they are frequently subject to public policies viewing school performance, such as aggregate financial resources and premises like laboratories and libraries. Other inputs may be classified as community and classroom factors. Community factors are relevant for the analysis because federal states have different policies as well as different educational expenditures and different labor laws. However, variables of community may be correlated with location policies, since decisions may be affected by household choices (omitted variable bias) or government choices (endogenous decisions).

The education production function approach associates knowledge attainment to a productive process. In order to study and understand the school proficiency determinants, the relation between inputs and proficiency may be described as[2]

$$A_t = f(F_t, S_t, O_t, \epsilon_t) \tag{1}$$

where A_t is the educational result, F_t the vector of family characteristics and background accumulated through time t; S_t is the vector for school inputs and teachers accumulated in time t; O_t the vector for other relevant inputs, such as classroom factors, accumulated in time t, and ϵ_t is the random error term in time t.

[2]Krueger (1999); Lee and Barro (2001); Hanushek (2003); Todd and Wolpin (2007).

Education is strongly associated with success in the labor market[3]. Standardized proficiency tests tend to become a common assessment tool of students' results.Therefore, measuring the education result given by A_t is the central issue. However, the difficulty in acquiring longitudinal data is the major problem to be tackled.

EPF does not count on a particular specification, as it depends on functional form, aggregation level, relevant control variables and identification. Omitted variables are possible, not only for students, but also for classroom, teachers and schools. Unobserved characteristics of individuals, such as natural abilities, can also give birth to bias, since they may be correlated with observable characteristics as familiar characteristics, for example.

According to Belfield (2000), schools can be viewed as educational firms using their resources to produce knowledge. Therefore, microeconomic theory could be used to analyze schools. The literature on this matter points to that school production function is concave in relation to school production factors. Therefore, students in poorer schools would benefit more from improved inputs up to the point that the impact of such improvements would not be significant any more. However, most studies of production function have been carried out in developed countries, which can usually count on more available resources. Results in developing countries could be different (Felício and Fernandes, 2005).

The test of academic performance is an important predictor of wages, participation in crime, health and success in other economic and social aspects of life. Recent studies show that non-cognitive skills have proved important in predicting such outcomes. Non-cognitive traits capture the concept of Marshall character and include perseverance, motivation, self-esteem, self-awareness and future behavior. There is substantial heterogeneity in cognitive and non-cognitive skills (Cunha and Heckman, 2007).

According to Cunha and Heckman (2007, 2009) the family environment and investments affect children's performance in school. The optimal time of investment depends on the outcome that is desired. The substantial heterogeneity that affects the intervention at different ages suggests that a universal policy to combat the adverse effects of early disadvantage is not appropriate. The optimal investment must be adapted to the specificities that create hardships that can cause further disadvantages in children. The authors also claim that parental income is not an adequate measure of resources available to the child, even if the income is used to measure child poverty. The good family relationships is more important than income, because parents with higher education may be available to the children, even if the family is going through an adverse financial situation. Thus, even children raised in poor cultural backgrounds can receive encouragement from parents and succeed.

Britto (2012) points out that the Educational Production Function does not believe that the school

[3]Brazilian authors study labor market issues related to the educational level status. For further information, see: Menezes-Filho, Fernandes, and Picchetti (2006); Freguglia, Menezes-Filho, and Souza (2007); Fernandes and Menezes-Filho (2012).

has other objectives as developing non-cognitive characteristics of students, such as live and conduct group activities and have self-confidence and motivation to study. These skills in addition to school goals are key to the development of the student, not just for schooling but also in their personal lives.

Another issue that the author points out concerns socioeconomic factors, e.g., parental education, family economic status, conditions of where the student lives, influencing student performance in school and are often difficult to be observed, in addition it is hard to separate the effect of these from other features.

According to Tavares (2012), assuming that the school industry is relatively rigid compared to the production technology, a hypothesis to explain the difference in results between schools with the same inputs is given by the differences in the forms of management. The empirical investigation of EPF presented evidence that the elements of school management are linked to educational outcomes.

According to Britto (2012) in the setting of Educational Production Function is not possible to identify which technology that optimizes the function, if it were possible schools wish to achieve feasible combinations that generate the desired level of education. Identify the optimum conditions for each institution, however, is very complicated because some schools may be more efficient and/or best management practices that generate differences in efficiency of the school

Parents, teachers and managers emphasize repeatedly the crucial role played by teachers in determining the school quality. Studying the teacher quality is essential as it is directly related to educational policy-making. Identifying the teacher quality and helping make policies that enhance school quality is thus the challenge to be tackled with.

As for Barros, Mendonça, Santos, and Quintaes (2001), the teachers' characteristics that are used as their quality measure show a lower-than-expected relation to students' performance. Therefore, schooling level, professional background and experience, although used for determining teachers' selection and salaries, may not explain differences in results of their students. The teachers' quality may depend on their unobserved characteristics, such as motivation and leadership.

According to Hanushek (2003), one of the main restraints to educational production function studies is the use of a small number of observed characteristics designed to detect the school quality and teacher quality. Although parametric approach is based on standard regression techniques, information on the teacher quality variation is limited, as data used are frequently administrative or obtained from surveys using a restricted set of characteristics. Among available characteristics, the teacher's education background and experience are very important characteristics under consideration as they affect teachers' pay both in the USA and other countries. This author found evidence that the master's degree did not have a systematic relation to teacher quality if it were to be measured via student performance. On the other hand, the teacher's experience would keep a positive and significant relation to student performance.

To say the school makes a difference for students is greatly controversial. According to Rivkin,

Hanushek, and Kain (2005), this can be just put into simple questions of whether there are or not systematic and significant differences between school and teacher abilities to improve students' learning and how relevant are such teacher quality differences in determining student proficiency. Finally, in case these quality differences can be captured through observable characteristics of teachers and schools as, for example, class size, teachers' experience and background, the reach of such effects should be checked.

There is no consensus in the literature on the relevance of specific effects coming from the teacher behavior nor any empirical conclusion as an evidence of teachers' role in determining proficiency and success of individuals in the labor market. The teacher role can be overestimated by parents and students or - a different explanation - measurable characteristics, as teacher experience, could scarcely explain the real teacher quality variation.

The heterogeneity in the school background and school quality make it difficult to define an appropriate curriculum for all students. Often the development of the curriculum is geared to the needs of the elite, leaving other students behind. This makes national tests are characterized by low performance and causes a higher rate of repetition and drop out. The quality of schools and spending on students are still low when compared with learning in developed countries and teachers have little incentive. In recent years, however, the educational systems of developing countries have expanded very fast (Glewwe and Kremer, 2006).

Rivkin et al. (2005) developed an estimator of the variance of teacher quality in order to avoid the problem of selection of students that could cause bias in the estimates. This estimator is based on patterns of differences in performance gains within schools. The authors do not focus only on the measurable characteristics of teachers and schools as is done in the literature but also rely on student outcomes to evaluate the overall magnitude of the effect of the teacher, regardless of the identification and measurement of specific effects. This semi parametric approach generates an estimate of the role of teacher quality in academic development and information on the degree to which specific factors explain the differences in the efficiency of the teacher.

Academic performance at some point is a cumulative function of family experiences before and at the time of analysis, experience of community and school. However, it is difficult to get all available information and studies are subject to bias caused by the presence of omitted variables.

An alternative approach focuses on determining the rate of learning over specific periods of time; the advantage of this analysis is that it eliminates unobserved influences. This model is known as value-added model that explicitly controls for variations in the initial conditions when it is observed, for example, as school influences student performance during a given year. Although this model eliminates the potential bias specification, the inclusion of initial notes as a means of considering the last result reduces the likelihood that historical factors omitted introduce significant bias.

The most significant problems in estimating these matters come from the match of students with

teachers and schools. This is because, in addition to the families' choice as to city sections and schools for their children,students' classes school are determined by directors and other managers. Additionally, estimates are subject to bias, due to the fact that decision-makers have access to data on students, teachers and schools which are not entirely available to a research effort or are measured with errors (Rivkin et al., 2005).

The wage of teachers over time in relation to other workers is an initial measure of teacher quality. If the correlation between alternative employment opportunities and quality of education is low and school districts do not systematically hire the best teachers available any change in supply tends to move the average quality in the same direction. The ratio of wages to the studies show that these are more likely to be positively related to student achievement than negatively related. And the problems are related to lack of historical data and missing data. Another issue in relation to earnings is about the stock of teachers. Wages affect the choice of the first job and the change between jobs. Hanushek and Rivkin (2006b) argue that as all reviews of the effects of wages compare salaries gifts with the effectiveness of the existing stock of teachers, this stock raises doubts about the results.

Another crucial issue for the quality of the teacher on salary is the responsiveness of current teachers to possible wage changes. The nature of employment of teachers (schedules and holiday coincident with the children) and the intrinsic rewards of teaching may have limited substitutes thus making decisions to enter or stay in school are less sensitive to earnings.

One way to check the implications of aggregate wage movements on quality has been to identify the impacts on the observable characteristics of teachers as proxies for quality. Should examine the effects of wages and non-pecuniary factors on flows in and out of education and implicitly the supply of teachers with particular characteristics. Four characteristics are given special attention in the literature: experience, measured result or skill, expertise and credentials of teacher (Hanushek and Rivkin, 2006b).

Many studies on teacher quality focus on specific features of the teachers' proficiency, controlling for differences in students. The number of current and past factors affecting proficiency at some point of time make it difficult to estimate effects of specific features. In fact, the most important is to check whether any observed association between schools or teachers with student outcomes capture a causal relationship (Hanushek and Rivkin, 2006a).

Studies in the literature - Croninger, Rice, Rathbun, and Nishio (2007), Buddin and Zamarro (2009) - show that after the first two years of experience the additional years have little impact on student learning. And this relationship is not linear. More experienced teachers are not necessarily the best performing in the classroom, they may reduce their efforts to be better remunerated for service time and not quality.

Therefore, it is important for schools to seek and hold the best teachers, based on their performance in classroom, since teaching quality positively affects students' learning. For parents, it is

important to know about the teaching staff of the school before enrolling their children in a school. They attempt to enroll their children in classes with top teachers seeking the best learning process possible for them. The best teacher is not necessarily the one showing the highest educational background. Choosing a good teacher mainly depends on his/her performance in classroom. Additionally, the teacher's work should not be restrained to classroom. The teacher must be able to motivate all agents in the educational process: students, parents and the community (Barros et al., 2001).

3 Data and descriptive statistics

The microdata used in this study were taken from the *Projeto* GERES - *Geração Escolar*. The GERES Project follows students' performance in a sample of public and private schools during the first four years in an elementary school. The project goals are described as follows: i) identifying the school characteristics that maximize students' learning and minimize the social origin impact on their learning process; ii) identifying the school characteristics that make the probability of grade repetition to decrease; iii) identifying the school characteristics that make the probability of absenteeism to decrease. Therefore, it is a longitudinal study that allows following students' learning through time.

GERES followed up first grade students in the elementary school in 2005 and then through 2008 by means of an annual test designed to estimate proficiency levels in mathematics and Portuguese language. Questionnaires for teachers, directors, parents and students were applied with the aim to assess family and school factors influencing the learning process. The project evaluated 303 public and private schools in 5 big Brazilian cities[4] - Belo Horizonte (MG), Rio de Janeiro (RJ), Campo Grande (MS), Salvador (BA)[5] e Campinas (SP).

Each city was considered a stratum, and a complex probabilistic sample of students, classes and schools was selected based on the 2003 School Census. Schools having only multigrade classes in the first grade were excluded from the sample[6]. Private schools having less than 10 students enrolled in the second grade of the elementary school were also excluded, in addition to other schools with less than 20 students. Furthermore, schools in rural zones and those having first grade students only during nighttime classes were also excluded[7]. Cognitive tools, tests of Portuguese language and mathematics, contextual tools and questionnaires for students, parents, teachers and directors were then applied in the selected schools (Brooke and Bonamino, 2011).

Before tests were applied to the sample of GERES, a pre-test was taken. A pre-test was conducted among students in public and private schools in Rio de Janeiro and Juiz de Fora. The analysis

[4]The selected municipalities were those presenting the most adequate conditions for developing the analysis, including localization of universities that took part in the study Brooke and Bonamino (2011).

[5]The city of Salvador was removed a year before the end of the research. Therefore, the last panel year did not included this city.

[6]Despite the fact that 47.5% of total primary schools were excluded, only 12.5% of total students were disregarded.

[7]Class activities beginning at 4pm.

of pre-tested items was taken from percentile of (parameter for difficulty) and Item Response Theory (IRT). The most suitable items, with greater technical and pedagogical quality, were included to tests administered to students.

In each wave, students present in class were submitted to reading tests in Portuguese and math tests[8]. Two test versions were constructed for each wave - an easier test and a more difficult one - in this way minimizing the possibility of measuring error estimated for students with different profiles. Proficiency variables were estimated based on the item response theory (IRT) together with items from SAEB during the fourth stage, which permitted that GERES students' marks could be compared to those estimated in the *Sistema de Avaliação da Educação Básica* - SAEB (basic education assessment system).

The first wave, which was carried out for obtaining a diagnostic assessment, was accomplished in March 2005 and served as adjustment for subsequent tests. The second wave was carried out in November 2005. The remaining waves,were fulfilled in November 2006, 2007 and 2008. From the second wave on, the tests fulfilled were defined based on previous waves. It is worth mentioning that all registered students could not be followed up due to change of schools. This research carefully distinguished between "school aggregate value" and "selection effect", i.e., unsuccessful students were still included in the sample provided that they remained in the same school or changed to another school present in the sample. However, students who have changed to another school not pertaining to the sample were excluded and this could represent a selection bias; this school would show a higher performance not because it has reached a better level, but because poor performance students have been excluded from the sample.

The scale of proficiency GERES was built considering the assumptions of IRT from the clustering process and taking into account the notion of proximity of items to represent measures of difficulty of the items and proficiencies through the same size scale skills (Brooke and Bonamino, 2011). Measures of proficiency must be interpreted in a qualitative way. The tables below show a snapshot of the interpretation of the proficiency levels of the lower grades of elementary school. The levels are in ascending order and are cumulative, i.e., students at the highest levels dominate not only the skills that are level, but also the skills of lower levels.

[8]Tests were applied to students present in class. Therefore, differences as for Portuguese and mathematics may arise. Part of the loss of students may be attributed to school transfers, as well as transfers from different nets and cities.

Table 1: Interpretation of GERES scale for Portuguese

Level	Zone	Interpretation
1	<129	• Recognize the letters of the alphabet also the writing direction. Are able to read sentences with either the Portuguese canonical structure (subject + verb + object) as phrases the escape this pattern, linking them to an image. • finds an information on a label. • These are considered basic skills and their consolidation is essential for students to progress in their development as readers, as they are placed as conditions essential to read with understanding and autonomously.
2	≥129 <143	• Demonstrate recognize syllables of a word. • Find information in a short text (up to 10 lines) with little information and familiar language and a median length of text (up to 15 lines). • Identify the subject indicated by the title, a simple informative text also a report from the verbal and non-verbal cues of the cover of a magazine. • They infer an information in a serial and identify the purpose of a ticket.
3	≥143 <166	• Find information in texts of various genres (median length story, short informational texts and median length text and time maps). • Identify the subject indicated by the title of short informative text and a leaflet campaigning efforts, through the association of verbal and non-verbal. • They infer an information in a less familiar genre text, relying on nonverbal cues, also the meaning of a word in a poem with simple language. • Establish logical-discursive relations between parts of a text through verbal forms, identifying a distant referent, as well as cause-consequence relationship without explicit marking, in a tale with a family theme.
4	≥166 <182	• Extend the skills related to conducting inference. • Recognize, on a strip, mood effects, the effect of the use of ellipsis direction and onomatopoeia. • Establish relationships between texts of the same genre, recognizing differences in information. Establish relations between parts of a text through lexical substitution and cause-consequence, in a short poem and median length text. • Identify partners in a fable and makes explicit that show them in a speech.
5	≥182 <201	• Infer information in text with mixed language, in median length text (up to 15 lines) with simple language and short informative text, which uses graphical elements (bar code), apart from recognizing the effect of humor in a joke. • Establish logical-discursive relations of time in a poem marked by adverbial and cause-consequence in a legend. • Recognize the opinion of a character, this in direct discourse, and the narrator. Also identify the order in which events occur in a literary narrative, featuring a character, the interlocutors, through explicit markings identifying them as well as the narrator of a longer story (up to 30 lines).
6	≥201	• Find information in short informative text with more sophisticated vocabulary and expository-argumentative text. • Identify subject of an informative text and a song lyric. • Infer, in a poem, the meaning of a word as well as brands recognize the irony in text with simple language and familiar themes of meaning and effect of the use of a diminutive. • Establish logical-discursive relations of cause-consequence in informational text with more sophisticated vocabulary and in a tale. They can also recognize the point of view of enunciating through the use of an adjective.

Source: Brooke and Aguiar (2010).

Table 2: Interpretation of GERES scale for mathematics

Level	Zone	Interpretation
1	<89	• Small quantities are compared with the use of graphics, highlighting what has the greatest amount also the height of objects, indicating the lowest and highest. • Identify the numberic symbol (numbers with 2 digits) and compare two-digit natural numbers, with and without graphic support. • They perform selective counting, managing associate quantities to numbers. • Coordinate actions count and join amounts to solve simple problem situations to determine the total up to 20. • Solve problems involving the ideas of counting and removing a quantity of another (minuend up to 10), from graphic support.
2	≥89 <139	• Localize an object between two others indicate their sizes, pointing out which one is the smallest, highest or average. • Identify the ascending order of groups of objects randomly arranged. • They pool in small quantities with units and tens graph or using the Brazilian monetary system support. • They solve addition problems (action of joining) and subtraction (removing action and complete) without graphic support. • Identify the subtraction operation as the solution of a given situation, also read and interpret information in tables or simple tables.
3	≥139 <186	• Show a greater expansion of the numeric field that reaches the set of numbers represented for three and four digits. • Identify those numbers associating written out to the numeric symbol • Identify the predecessor of a number and perform the decomposition. • Resolve complex problems involving subtraction with idea of complementation, comparison and equalization, multiplication involving the multiplicative principle and division with meaning that are spread in more advanced stage of construction and are resolved when inserted into contexts, which indicates that the child's understanding of action operative principle.
4	≥186 <222	• Solve problems involving the composition and decomposition of monetary values this ability due to other previously sedimented. • They compare units of measure applied to measurable quantities present in the daily context, such as mass, capacity and time measurements in situations involving month and year. • Understand the Decimal Numbering System and significance of the fundamental operations in problem solving. • Solve problems involving subtraction of natural numbers with unknown initial state, as well as problems involving multiplication of natural numbers and exact division of a natural number by another two digits, with rest.
5	≥222	• Progression in the ability to solve numerical problems involving the different meanings of operations, such as equalization with the change of a quantity, to compare, in subtracting rational in decimal form, proportionality, as well as those involving the rectangular configuration of the multiplication. • Skills such as problem solving involving the fundamental operations are broadened and go for consolidation. • Solve problems that insert operative actions with natural, rational numbers, in the form of decimal and percent, measures of length, mass and capacity and its applications, such as calculating the perimeter and area, and also measures time and value.

Source: Brooke and Aguiar (2010).

As for the present study, unbalanced panels for Portuguese and mathematics were used. Salvador was excluded from the analysis, as the research in this city was started when students were attending the second grade and, for this reason, the last observation was held in 2007 when these students concluded the fourth grade. Table 3 shows the number of students and schools present in the sample in each stage.Total observations for proficiency tests amounted to 54,591 for the Portuguese language

and 54,533 for mathematics all through the five stages. Table 4 presents the basic statistics to the defined variables to be used in this study.

Table 3: GERES sample

		Wave 1	**Wave 2**	**Wave 3**	**Wave 4**	**Wave 5**
Portuguese	Students	9,447	10,07	12,607	10,263	12,204
	Schools	199	200	240	204	213
	Teachers	427	428	812	401	474
Mathematics	Students	9,464	10,064	12,623	10,237	12,145
	Schools	199	200	240	204	213
	Teachers	427	428	813	401	474

Source: GERES database (2005-2008).

Note: The number of schools varies over the waves because some of them were closed or incorporated.

The pair represented by (i, j) - student-teacher - determines the match between them. Therefore, in case the students' teacher has been changed, this combination was taken as a new match. It is worth noting that, if a student was matched with the same teacher for two distinct periods only one match was considered with this same teacher, no matter two distinct time periods have occurred. This is because the quality fixed effect of this match was constant over time. For this reason, it should taken as a single combination, tough distinct in time. The same occurred with unsuccessful students. In case a student has failed and repeated the grade with the same teacher, such a tie in different years was the taken as the same match. However, in case a student has failed and repeated the grade with a different teacher, then a new match was then registered.

Table 4: Descriptive statistics

Variable	Continuous/ Categorical	Categories	Mean/ Percentage	Standard error	Mean/ Percentage	Standard error
			Portuguese		Math	
Dependent Variable						
Portuguese Proficiency	Continuous		141,75	32,63		
Mathematics Proficiency	Continuous				172,11	69,37
Teacher Variables						
Education	Categorical	High School	10,66		10,58	
		Higher Education - Pedagogy	57,03		57,12	
		Higher Education - Others	32,31		32,3	
Qualification	Categorical	Do not made/complete postgraduate	52,2		52,18	
		Update	8,57		8,54	
		Specialization	37,34		37,38	
		Master's degree	1,72		1,72	
		Doctorate	0,18		0,18	
Experience	Categorical	Until 4 years	7,28		7,3	
		5 to 10 years	14,89		14,91	
		11 to 15 years	18,96		18,9	
		More than 15 years	58,87		58,89	
Other job	Categorical	Yes	45,19		45,26	
		No	54,81		54,74	

Source: GERES database (2005-2008).

Note: The Portuguese and math proficiencies percentages are different due to a different number of students in each proficiency.

Table 4: Descriptive statistics (conclusion)

Variable	Continuous/ Categorical	Categories	Mean/ Percentage	Standard error	Mean/ Percentage	Standard error
			Portuguese		Math	
Teacher Variables						
Tenure	Categorical	Until 1 year	19,66		19,7	
		1 to 2 years	14,04		14,04	
		3 to 4 years	16,74		16,76	
		5 to 10 years	22,28		22,34	
		11 to 15 years	15,06		14,96	
		More than 15 years	12,33		12,21	
Number of worked hours at school	Categorical	Until 20h/week	22,74		22,81	
		21h to 25h/week	30,61		30,64	
		26h to 30h/week	11,48		11,46	
		31h to 40h/week	20,84		20,76	
		More than 40h/week	14,33		14,33	
Teachers family earnings	Categorical	Until R$1900	33,03		33,07	
		R$1901 to R$3100	35,17		35,11	
		More than R$3100	31,8		31,83	

Source: GERES database (2005-2008).

Note: The Portuguese and math proficiencies percentages are different due to a different number of students in each proficiency.

4 Empirical strategy

The advancement of this study in relation to other works present in the literature is due to the use of a theoretical framework that attempts to explain the teacher quality by using the Educational Production Function (EPF) and controlling the unobserved heterogeneity of students, teachers and the match between them. In case the sources of unobserved heterogeneity are not controlled, estimates may be biased if the teacher quality is correlated with any of such unobserved characteristics.

The microeconometric specification is based on the Woodcock (2007) model. Although this author had proposed a model for firms, the present study applies EPF in accordance with the linear specification proposed by Todd and Wolpin (2003, 2007). According to the literature on the subject, such an approach has not been applied to this area yet.

As for Todd and Wolpin (2007),the fixed effects specification may be implemented for both a contemporary model and value-added model. Estimators within fixed effects are feasible in the presence of several observations concerning an individual for different years. If a model that does not control any kind of fixed effect is used, then two problems of omitted variable bias would appear, i.e., that of student fixed effect and that of teacher fixed effect. Having in mind that the database is an unbalanced panel showing n students, J teachers, T time periods and P explaining variables in the sample and changing along, then the adequate model controlling these two fixed effects[9] may be represented by equation (2):

$$Y_{ijt} = \boldsymbol{x}_{\boldsymbol{ijt}}^{'}\boldsymbol{\beta} + \gamma_i + \theta_j + \epsilon_{ijt} \tag{2}$$

Where Y_ijt is the student grade i, with teacher j, in time t; $x_{ijt}^{'}$ is the vector $(1 \times P)$ of observable control variables, which vary along time both for the student i and teacher j, also during time t; γ_i is the fixed effect of student i and θ_j, the fixed effect of teacher j; and finally ϵ_{ijt} is the random error term.

Lets consider that the estimated model is:

$$\boldsymbol{Y} = \boldsymbol{X\beta} + \boldsymbol{\vartheta} \tag{3}$$

in this case, with no controlled fixed effects:

$$\begin{aligned} \hat{\boldsymbol{\beta}} = (\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}'\boldsymbol{Y} = (\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}'\left[\boldsymbol{X\beta} + \boldsymbol{A\gamma} + \boldsymbol{P\theta} + \boldsymbol{\epsilon}\right] = \\ = \boldsymbol{\beta} + (\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}'\boldsymbol{A\gamma} + (\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}'\boldsymbol{P\theta} + (\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}'\boldsymbol{\epsilon} \end{aligned} \tag{4}$$

[9]Fixed effects are taken as specific abilities of both students and teachers. Intelligence is an example of unobserved ability.

because of the hypothesis $E[\boldsymbol{\epsilon}|\boldsymbol{X}, \boldsymbol{A}, \boldsymbol{P}]= \mathbf{0}$:

$$E\left(\hat{\boldsymbol{\beta}}\right) = \boldsymbol{\beta} + E\left[(\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}'\boldsymbol{A}\boldsymbol{\gamma}\right] + E\left[(\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}'\boldsymbol{P}\boldsymbol{\theta}\right] \tag{5}$$

$\hat{\boldsymbol{\beta}}$ is thus a biased estimator of $\boldsymbol{\beta}$, in which part of the bias is assigned to the correlation between the control variables **X** and the fixed effect of students; and the other bias part is assigned to the correlation between the control variables **X** and the teacher fixed effect (Wooldridge, 2002). Therefore, estimation should be made according to equation (2), which takes the two fixed effects into account.

By estimating the mean of equation(2) and subtracting the original equation:

$$Y_{ijt} - \bar{Y}_{ij} = (\boldsymbol{x}'_{\boldsymbol{ijt}} - \bar{\boldsymbol{x}}'_{\boldsymbol{ij}})\boldsymbol{\beta} + (\epsilon_{ijt} - \bar{\epsilon_{ij}}) \tag{6}$$

The estimation (6) by ordinary least squares does not show bias of the omitted variable and then the estimator β is consistent. Although consistent, interpreting the estimator in this model would not be correct, provided that - in addition to individual fixed effects - the fixed effects of the interaction between them, i.e., the match between student-teacher, described as δ_{ij} here, be considered.

By adapting Casalecchi (n.d.) model for workers and firms to students and teachers, the student's ability may be divided into two, i.e., a general portion and a specific portion. The general portion is given by the student fixed effect and it may be viewed as an ability that may be used in any class the student might belong to. On the other hand, the specific portion comprises specific abilities the student not always uses, but that could be used depending on the teacher's abilities. The impact of such specific abilities - of both students and teachers - is set by the match between them.

Therefore, by also controlling the student-teacher match, it is possible to capture not only the student individual ability, which affects his/her grade whoever the teacher is, but also the combination of the abilities of student *i* with that of a given teacher *j*. Despite the fact that abilities are constant during the period considered, the amount of student ability each teacher makes use of may change in case the teacher is substituted.

Therefore, this study adopts the linear model of Todd and Wolpin (2003, 2007) for specifying the fixed effects of students, teachers and the match between them. Take the following model to be tested into account:

$$Y_{ijt} = \boldsymbol{x}'_{\boldsymbol{ijt}}\boldsymbol{\beta} + \gamma_i + \theta_j + \delta_{ij} + \epsilon_{ijt} \tag{7}$$

where Y_{ijt} is the grade of student *i*, with teacher *j*, in time *t*; $\boldsymbol{x}'_{\boldsymbol{ijt}}$ is the vector $(1 \times P)$ that represents the remaining observable control variables that vary through time of both student *i* and teacher *j*

also in time *t*; γ_i is the fixed effect of student *i*, which in addition to the unobserved component aggregates the variables of student *i* that are constant through time; θ_j is the teacher fixed effect, which (in the same way as that of the student) is comprised of the unobserved component and those variables constant in time associated to teacher *j*; δ_{ij} is the effect of the match between student *i* and teacher *j* and; ϵ_{ijt} is the random error term.

The model identification hypothesis described in equation (7) requires that the correlation between explaining variables and the error term be captured by a variable that remains stable through the years.If the estimated model is that of equation(7), then:

$$E(\hat{\boldsymbol{\beta}}) = \boldsymbol{\beta} \tag{8}$$

The analysis of this work is to be made by using the fixed effects model. This method assures robustness to the task in the face of the elements that are constant over those years. Such elements are to be estimated with the aim to compare the Pooled OLS (POLS) model and those of fixed effects for students (EF_i), teachers (EF_j) and the match between them (EF_{ij}).

$$POLS : Y_{ijt} = \boldsymbol{x}_{\boldsymbol{ijt}}^{'}\boldsymbol{\beta} + \epsilon_{ijt} \tag{9}$$

$$EF_i : Y_{ijt} = \boldsymbol{x}_{\boldsymbol{ijt}}^{'}\boldsymbol{\beta} + \gamma_i + \epsilon_{ijt} \tag{10}$$

$$EF_j : Y_{ijt} = \boldsymbol{x}_{\boldsymbol{ijt}}^{'}\boldsymbol{\beta} + \theta_j + \epsilon_{ijt} \tag{11}$$

$$EF_{ij} : Y_{ijt} = \boldsymbol{x}_{\boldsymbol{ijt}}^{'}\boldsymbol{\beta} + \gamma_i + \theta_j + \delta_{ij} + \epsilon_{ijt} \tag{12}$$

After the change occurred within them, the fixed-effects models EFi, EFj and $EFij$ will be estimated by ordinary least squares with the aid of STATA software 12.0, respectively[10]

$$Y_{ijt} - \bar{Y}_i = (\boldsymbol{x}_{\boldsymbol{ijt}}^{'} - \bar{\boldsymbol{x}}_{\boldsymbol{i}}^{'})\boldsymbol{\beta} + (\epsilon_{ijt} - \bar{\epsilon}_i) \tag{13}$$

$$Y_{ijt} - \bar{Y}_j = (\boldsymbol{x}_{\boldsymbol{ijt}}^{'} - \bar{\boldsymbol{x}}_{\boldsymbol{j}}^{'})\boldsymbol{\beta} + (\epsilon_{ijt} - \bar{\epsilon}_j) \tag{14}$$

$$Y_{ijt} - \bar{Y}_{ij} = (\boldsymbol{x}_{\boldsymbol{ijt}}^{'} - \bar{\boldsymbol{x}}_{\boldsymbol{ij}}^{'})\boldsymbol{\beta} + (\epsilon_{ijt} - \bar{\epsilon}_{ij}) \tag{15}$$

In equation (13),the mean is estimated for the same student *i*, while *j* and *t* vary; in equation (14) the mean is estimated for the same teacher *j*, while *i* and *t* vary and; in equation (15) the mean is estimated for the match (*i, j*), while *t* varies.

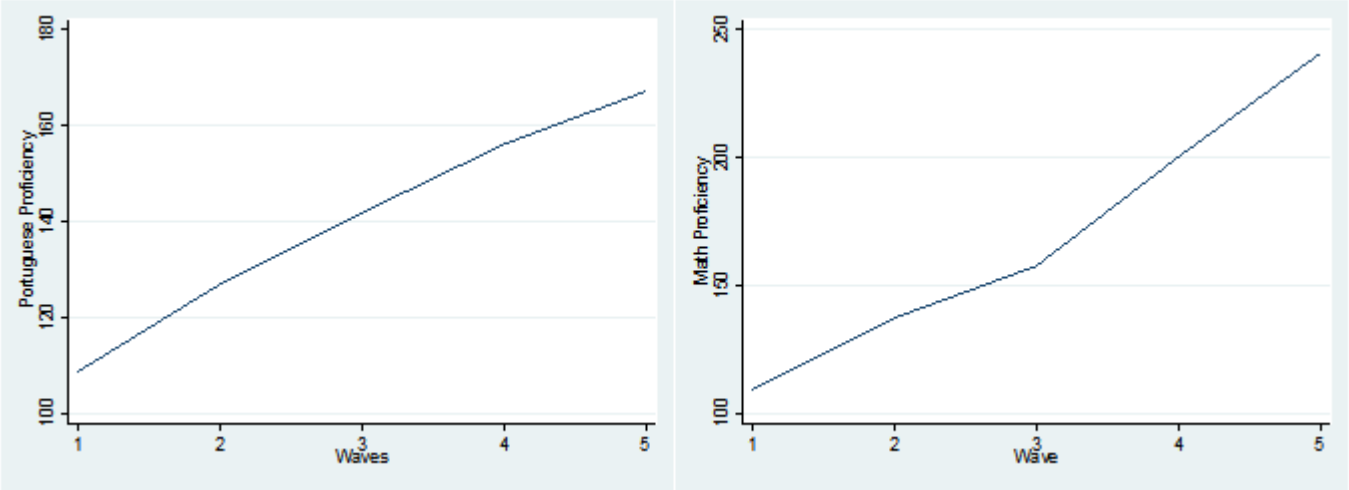
[10]See Casalecchi (n.d.) for details of derivations.

5 Results

5.1 Descriptive Analysis

Figure 1 below shows the evolution of the average proficiency of students and Figure 2, the density of Portuguese language and mathematics, respectively.

Figure 1: Average student proficiency by waves

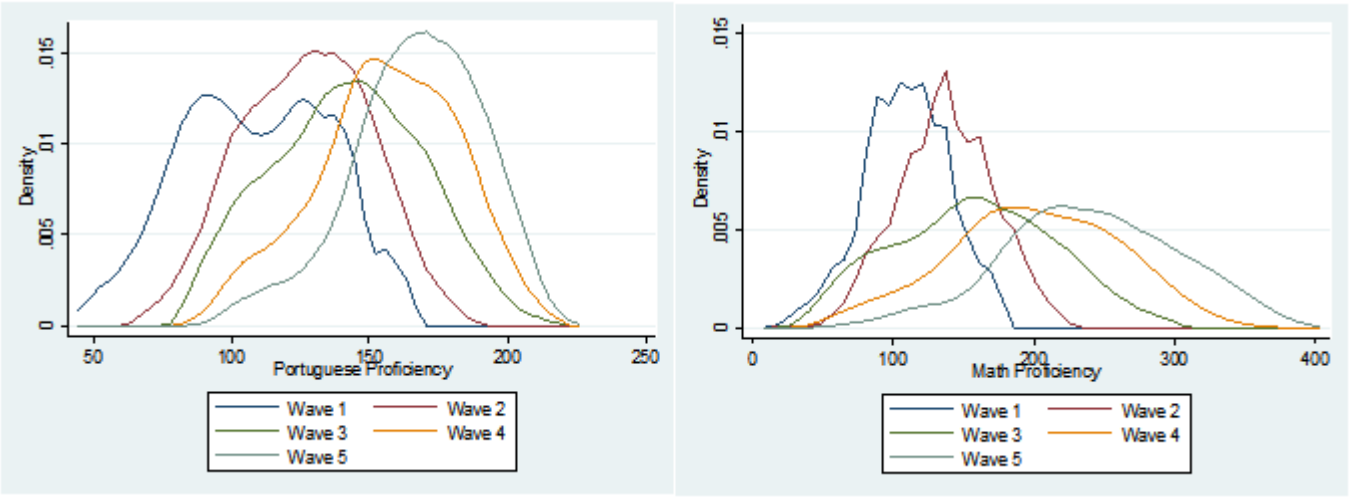


Source: GERES (2005-2008).

As can be seen in Figure 1, students have entered with the same proficiency level in both disciplines. However, the average proficiency in math has exceeded that of Portuguese over the years.Such a difference in the evolution of both disciplines suggests that they should be analyzed separately. Indeed, as Brooke and Bonamino (2011) emphasized, the proficiency scales of Portuguese and mathematics are different in GERES, which justifies a distinct analysis[11]

[11] See Brooke and Bonamino (2011) for the GERES scale construction.

Figure 2: Proficiency density



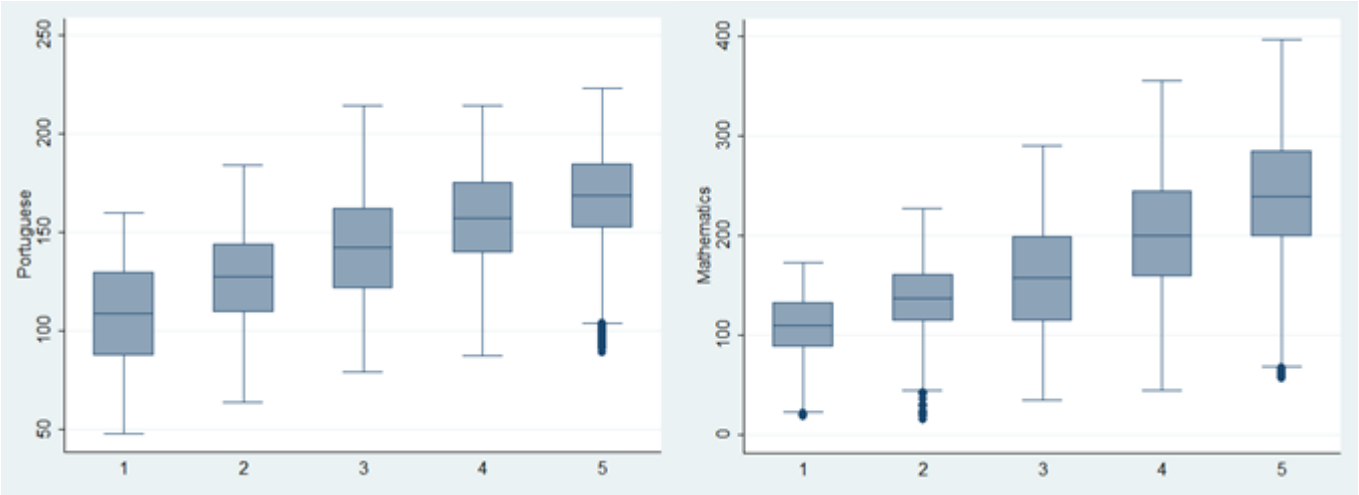
Source: GERES (2005-2008).

Figure 2 shows the proficiency densities for Portuguese and mathematics[12]. It can be seen that the Portuguese proficiency density has increased over the years, but it is not so scattered. As for the mathematics proficiency, density is more concentrated in years 1 and 2. The observed change in mathematics proficiency in year 3 can be explained by the fact that schools put more focus on children's reading and writing in years 1 and 2, and only after then more emphasis on children's abilities in mathematics should be laid. According to Brooke and Aguiar (2010), the teachers' attention is more focused on literacy goals as for Portuguese in the first grade of the elementary school. And intensification of math studies is postponed until the following year.

In order to investigate if the dispersion of the math scores were derived from upper outliers in the sample,it was built a boxplot of proficiencies along the waves (FIGURE 3). In Portuguese only on occasion 5 shows the presence of outliers below. For mathematics, instead, there is less outliers on occasions 1, 2 and 5. Absence of upper outliers strengthens the argument that the variation in density of math scores can be derived from the focus given to the skills of the Portuguese in the first grade of elementary school.

[12]The kernel specification function used is that of Epanechnikov's.

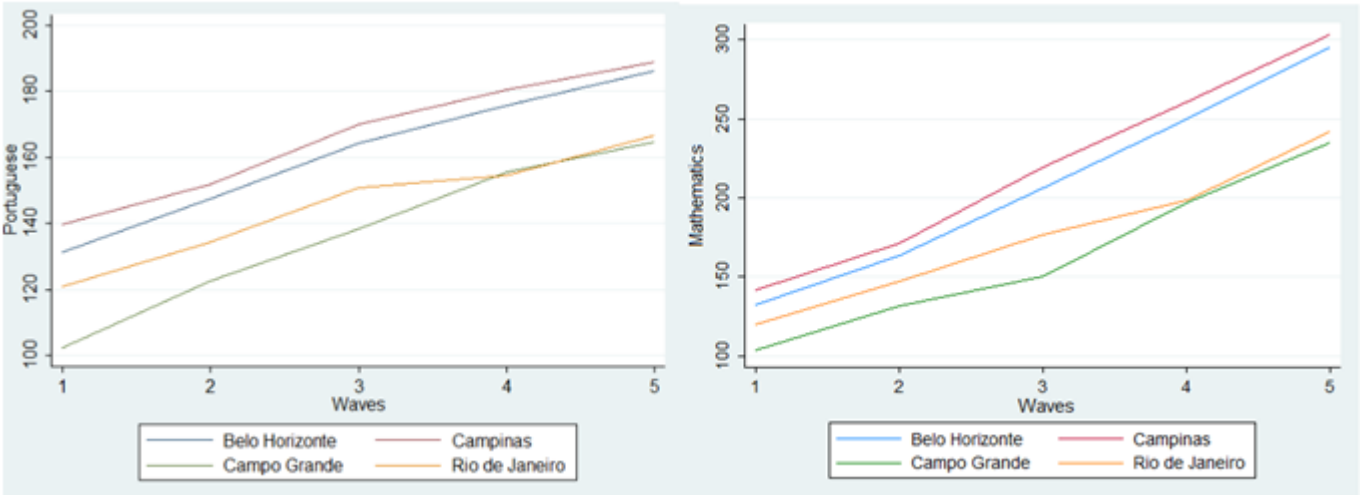
Figure 3: Boxplot of proficiencies along the occasions



Source: GERES (2005-2008).

Figure 4 shows the average proficiency by city analyzed in this work. For the proficiency of Portuguese, the city of Campinas begins the research with the highest marks, followed by Belo Horizonte, Rio de Janeiro and Campo Grande. At the end of the research there is an inversion between Rio de Janeiro and Campo Grande. For math, respecting the differences of scales, the relative behavior of proficiency between the cities is quite similar. This difference in behavior between the municipalities of proficiencies can occur because states have different policies related to education

Figure 4: Evolution of proficiency by city

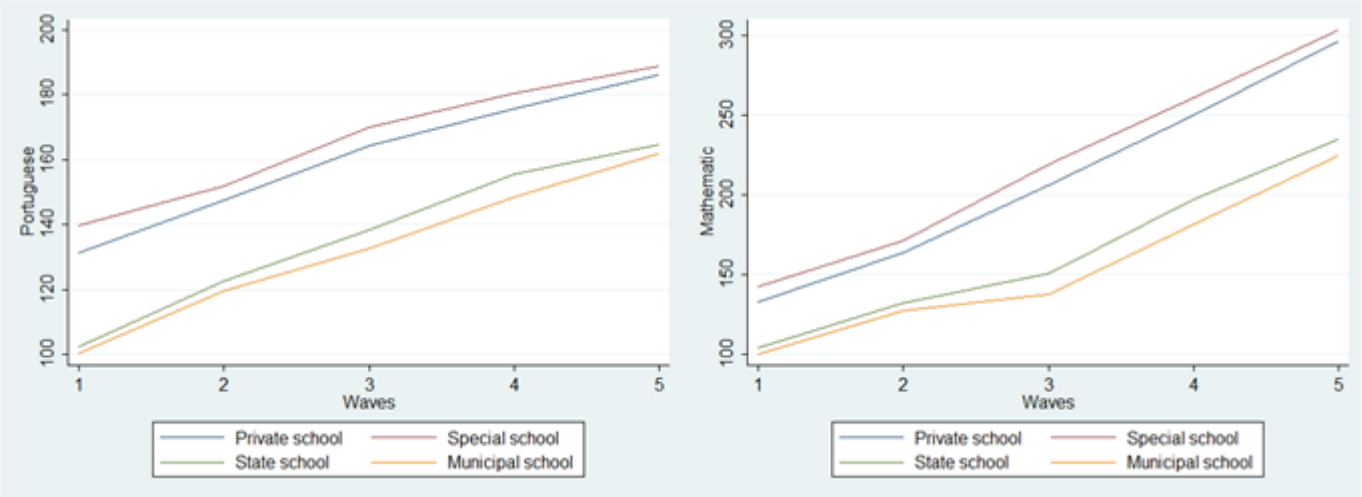


Source: GERES (2005-2008).

According to the literature, private schools have better grades than public schools (Angrist, Bettinger, and Kremer, 2004; França and Gonçalves, 2010). As figure 5, in both proficiencies Portuguese language and mathematics - students from public schools begin primary education at lower levels of

learning than students from private schools. One explanation for this is the difference in social background between these students. The notes of the private schools are superior to the state and municipal schools, the latter showed the worst performance. The exception occurs because of special schools, which notes are higher than the other.

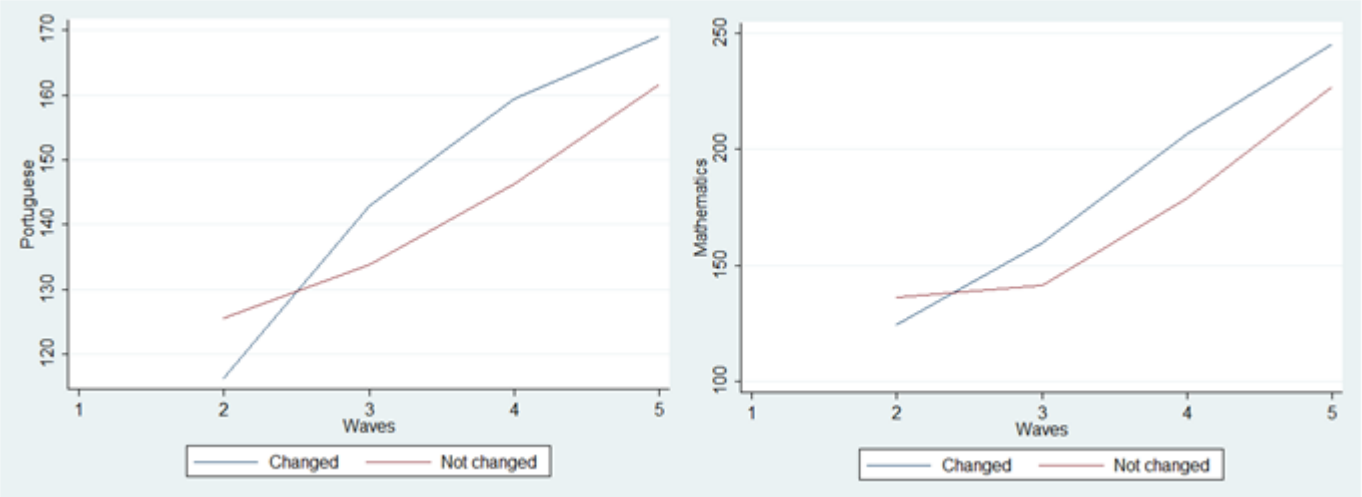
Figure 5: Evolution of proficiency according to the administrative network



Source: GERES (2005-2008).

An important issue to be investigated on the behavior of student learning is related to the change of teacher. As figure 6 it can be seen that the average student proficiency is higher for those who changed their teacher, except for the wave 2. This is because at the time there is no grade promotion of students, which indicates that the student who changes teacher during the school year is less than the remaining school year with the same average grade teacher.

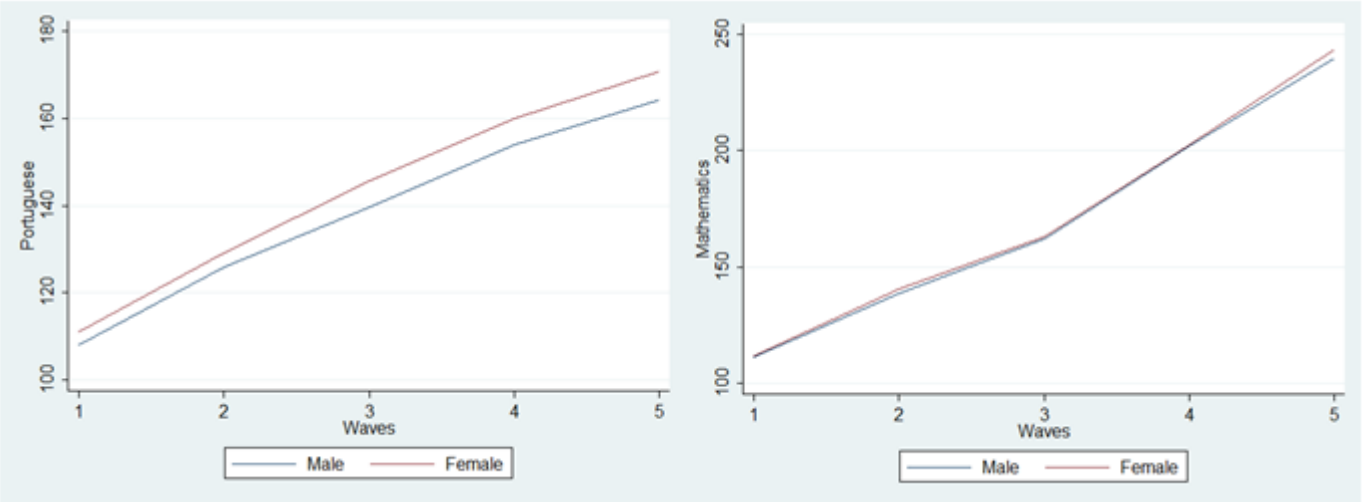
Figure 6: Evolution of proficiency according to the change of teacher



Source: GERES (2005-2008).

The following charts monitoring the evolution of Portuguese language and mathematics proficiencies from some characteristics of students and teachers. By Figure 7 we can see that there are differences between male and female students in both proficiencies. However, the difference is more pronounced in Portuguese than in mathematics. In the latter, both start at the same level and distance themselves from the end of the first stage of basic education (wave 5).

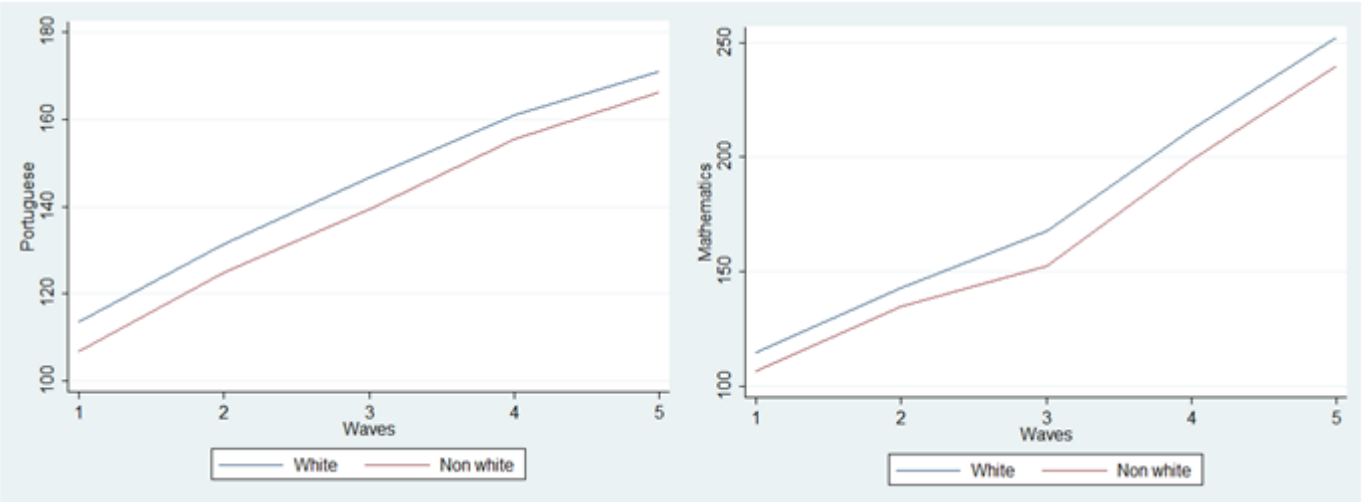
Figure 7: Evolution of student proficiency by gender



Source: GERES (2005-2008).

Analyzing the evolution of proficiencies from the race of students, is that students who have declared whites have higher average grade over the years compared with others, in both proficiencies (FIGURE 8).

Figure 8: Evolution of student proficiency by race

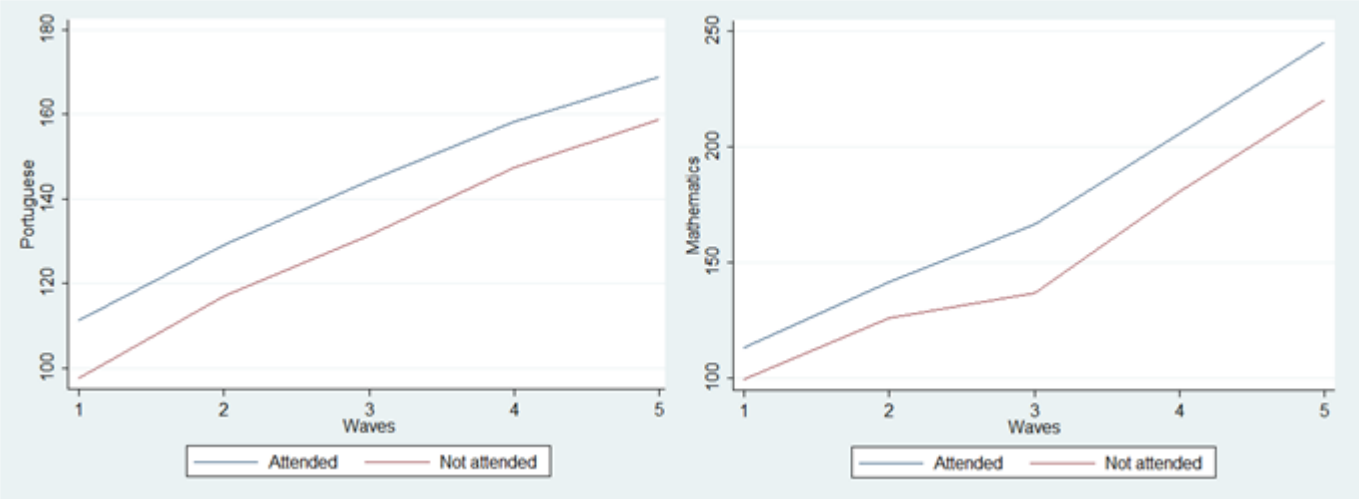


Source: GERES (2005-2008).

Figure 9 confirms that students attending pre-school has a positive impact on the learning of

elementary school. Both Portuguese and mathematics, students who attended pre-school had higher scores than those who did not attend.

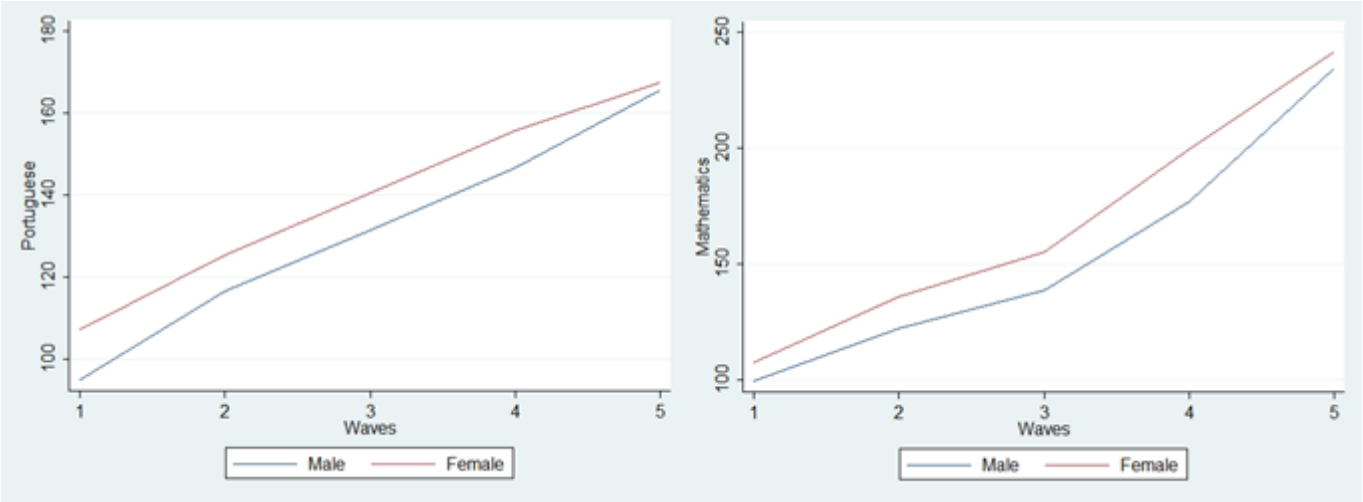
Figure 9: Evolution of proficiency from the pre-school



Source: GERES (2005-2008).

The gender of the teacher, it is possible to observe that the elementary school students who had classes with female teachers had higher scores than those who had classes with male teachers. This occurs in both proficiencies, but according to the evolution shown in Figure 10, this distance because of the gender of the teacher is lower on the last wave.

Figure 10: Evolution of proficiency according to the gender of the teacher

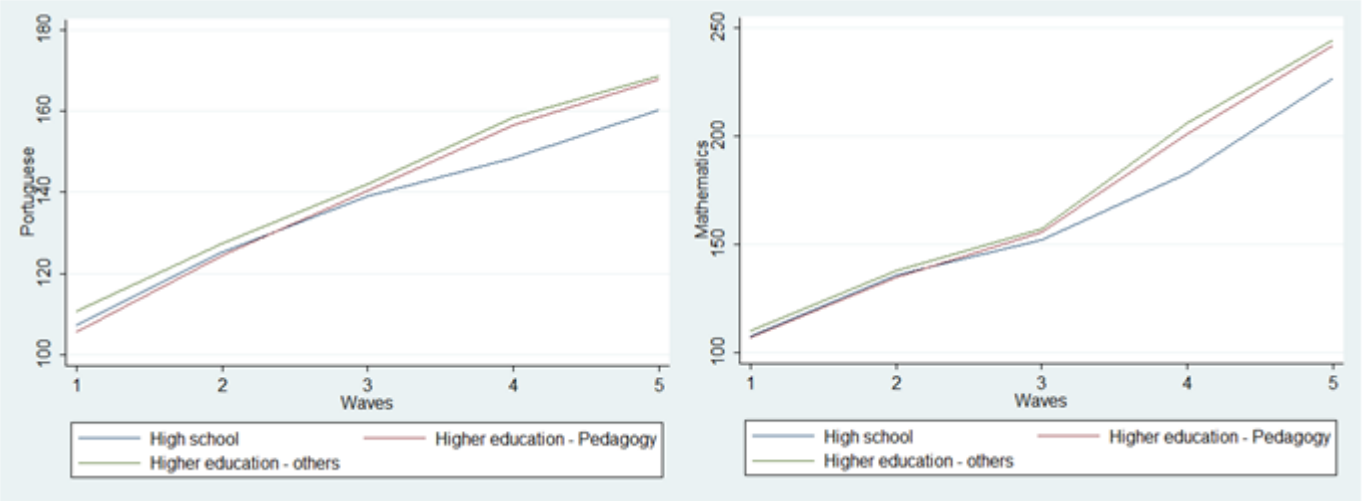


Source: GERES (2005-2008).

As the Figure 11, students have higher grades if the teacher has higher education. However, the teacher has made pedagogy at college has not resulted in higher grades of students in the early

occasions. According Curi (2005), students of pedagogy, who are the future teachers of the lower grades often do not have the professional training required. What happens is that these young teachers sometimes do not have control of the content of the series and/or are insecure and can not relate to the actual content, which has a negative impact on children's learning.

Figure 11: Evolution of proficiency for teacher education



Source: GERES (2005-2008).

5.2 Empirical Analysis

With the aim to compare such different models, regressions were estimated for both discipline proficiencies by means of models of pooled OLS (POLS), student fixed effect (FE_i), teacher fixed effect (FE_j), student fixed effects by controlling with dummies for each teacher in order to control their effects $(FE_i + FE_j)$, and match fixed effects (FE_{ij})[13]. Tables 5 and 6 present the results of such estimates for Portuguese and mathematics, respectively. It should be emphasized that the variables included in the model are categorical[14] and that the highest frequency was the criterion used for defining the reference dummy (omitted category).

As for Portuguese, despite the fact that all categories were significant in all estimates accomplished, schooling was underestimated without controlling the fixed effects, namely: the student fixed effect, the teacher fixed effect and the fixed effect of the match between them (FE_{ij}). The results of column 5 suggest that the teacher's education has a positive effect on the Portuguese grade of students. Therefore, both students whose teachers have high school education level and students having teachers with higher education in other fields show higher grades as compared to students whose teachers have higher education level in pedagogy. Pedagogy students are to be future teachers in the initial school grades and the arguments found in Curi (2005) may explain why sometimes pedagogy

[13]The teacher dummies were not included in the tables of regression results for the sake of simplicity.

[14]The estimated regressions used the variables described in accordance with table 4.

students do not master the content of the discipline and why they lack adequate professional capacity. And all of this brings negative impact on students' proficiency.

As can be seen in column 1, academic degrees without any fixed-effect control (column 1) had a positive and significant impact on all categories, and those students whose teachers had masters or doctorate degrees would show higher grades than those whose teachers had not obtained or completed their postgraduate programs. However, as far as the fixed effects were falling into control, such results started changing, precisely as for their previous uncontrolled characteristics, for example, the teacher didactic. After introducing the match control (column 5), only the category of specialization was significant for the analysis. Nevertheless, it attracted a negative coefficient, which thus suggested that students with teachers having specialization degrees presented lower grades in Portuguese as compared to those whose teachers had not obtained or completed their postgraduate programs.

Experience in turn was significant in all categories containing controls for teacher fixed-effects(columns 3).When the three fixed effects were controlled, those students with less-experienced teachers showed lower grades as compared to those students having more experienced teachers who had more than 15 years of professional experience. Experienced teachers were more liable to be confident of their skills and/or able to better explore the students' learning potential.

The categories analyzed under the teacher's tenure were underestimated when the match fixed effects were not controlled. In accordance with the results obtained when estimating the FE_{ij} regression, , it was observed that students having teachers with less time of experience (categories lower than that of the omitted one) showed higher grades than those having teachers working in the same schools during 5 to 10 years. The remaining categories did not show a significant impact.

Having another job with no control of fixed effects caused a significant and positive impact (column 1). However, the impact was underestimated when fixed effects were controlled (columns 3, 4 and 5). For the match, the impact is not significant, then, students whose teachers have other jobs do not have differences in the notes compared to those students that teachers do not have another job.

The significance of the hourly-load categories analyzed did not keep uniformity for the estimated regressions. When estimating the pooled OLS with no fixed-effect controls (column 1), the category with less worked hours did not show a significant impact, while the remaining categories showing more work hours than those of the omitted category (between 21 and 25 work hours a week) were significant and negative as compared to the omitted category. As for the fixed effects for students, teachers and the match (column 5), the lowest category proved to be significant and negative, suggesting that students whose teachers worked up to 20 week hours had lower grades than those with teachers working between 21 and 25 hours a week. Furthermore, those students with teachers working between 31h and 40h a week had even higher grades as compared to those students with teachers working between 21h and 25 hours a week. One possible explanation is that teachers working fewer hours may have more than one job and your dedication is shared in more than a job, this has a negative impact on their students; Moreover, in general, teachers who work longer hours are teachers in private

schools, which according to the literature, have a positive impact on their students.

The teachers' income was significant in both not omitted categories when it was analyzed disregarding fixed-effect controls. The income bracket lower than that for the omitted category (from R$1,901.00 to R$3,100.00) was negative and significant, while the bracket higher than that of the omitted category was positive and significant. When the fixed effects of students and teachers were taken into account (column 4), the lowest bracket was not significant anymore and the highest bracket (though remaining positive, showed a lower magnitude and became significant at the level of 10%. When the match control was added (column 5), the lowest bracket remained not significant in relation of all usual significance levels. The highest bracket remained positive and significant.

Therefore, the students whose teachers were paid more than R$3,100 had higher grades than those whose teachers received between R$1,901 and R$3,100.

Table 5: Portuguese language regressions (continues)

PORTUGUESE					
VARIABLES	**(1) POLS**	**(2)**FE_i	**(3)**FE_j	**(4)**$FE_i + FE_j$	**(5)**FE_{ij}
Education					
Higher Education - Pedagogy (omitted)					
High School	-1.624***	0.669**	-0.164	2.093	8.057**
	(0.376)	(0.304)	-2.476	-2.339	-3.702
Higher Education - Others	2.554***	0.455**	6.277***	3.812**	6.787***
	(0.236)	(0.194)	-1.720	-1.723	-2.562
Qualification					
Do not made/complete postgraduate (omitted)					
Update	0.114	-0.0557	7.107***	3.824***	2.302
	(0.398)	(0.305)	-1.657	-1.367	-1.927
Specialization	1.823***	-0.208	-2.390*	-2.298*	-3.822**
	(0.236)	(0.193)	-1.437	-1.300	-1.622
Master's degree	17.01***	0.316	8.009**	3.559	2.644
	(0.730)	(0.550)	-3.383	-2.994	-4.344
Doctorate	14.60***	-2.538**	-	-8.802	-
	-2.197	-1.118	-	-5.525	-
Experience					
Until 4 years	4.577***	-0.0499	-10.93***	-4.450	-7.659**
	(0.462)	(0.362)	-3.761	-3.047	-3.684
5 to 10 years	0.721**	0.318	-9.770***	-3.791*	-4.580*
	(0.333)	(0.278)	-2.345	-2.142	-2.571
11 to 15 years	-1.071***	-0.258	-4.687***	-1.778	-2.908
	(0.290)	(0.241)	-1.649	-1.450	-1.794
More than 15 years (omitted)					
Tenure					
Until 1 year	-5.715***	0.865***	-3.067	6.837***	12.24***
	(0.343)	(0.284)	-2.067	-2.021	-2.769
1 to 2 years	-4.580***	0.458	-4.072**	2.960*	8.003***
	(0.374)	(0.311)	-1.762	-1.660	-2.305
3 to 4 years	-2.684***	0.230	0.245	3.467***	5.896***
	(0.347)	(0.292)	-1.376	-1.344	-2.021
5 to 10 years (omitted)					
11 to 15 years	-0.213	0.163	4.342**	0.0149	-0.110
	(0.350)	(0.295)	-1.787	-1.838	-2.521
More than 15 years	3.757***	0.382	11.99***	2.734	2.378
	(0.396)	(0.318)	-2.256	-2.059	-2.536
Other job - No (ommited)					
Yes	2.155***	0.173	-0.301	-0.586	-0.957
	(0.283)	(0.204)	(0.923)	(0.765)	(0.957)

Source: GERES database (2005-2008).

Note: Robust standard errors between brackets; *** p<0.01, ** p<0.05, * p<0.1.

Table 5: Portuguese language regressions (conclusion)

PORTUGUESE					
VARIABLES	**(1) POLS**	**(2)**FE_i	**(3)**FE_j	**(4)**$FE_i + FE_j$	**(5)**FE_{ij}
Number of worked hours at school					
Until 20h/week	0.390	-0.114	-5.260***	-6.148***	-7.293***
	(0.296)	(0.271)	-1.478	-1.350	-1.564
21h to 25h/week (omitted)					
26h to 30h/week	-7.212***	0.237	-9.242***	-4.411**	-4.854*
	(0.385)	(0.328)	-2.018	-1.973	-2.822
31h to 40h/week	-3.544***	0.747***	-2.486*	1.366	2.755**
	(0.307)	(0.267)	-1.299	-1.050	-1.241
More than 40h/week	-3.811***	1.062***	1.234	1.317	0.473
	(0.344)	(0.279)	-1.224	(0.960)	-1.140
Teacher's family earnings					
Until R$1900	-1.419***	-0.183	-2.809**	1.655	2.559*
	(0.261)	(0.219)	-1.171	-1.011	-1.385
R$1901 to R$3100 (omitted)					
More than R$3100	3.684***	0.891***	1.376	1.866	2.917*
	(0.266)	(0.211)	-1.216	-1.145	-1.640
Repeating students	-7.312***	-0.860***	-2.550***	-0.290	-0.159
	(0.304)	(0.328)	(0.255)	(0.329)	(0.456)
First grade (omitted)					
Second grade	21.51***	6.775***	19.83***	6.270***	4.866***
	(0.656)	-1.063	-1.425	-1.527	-1.853
Third grade	34.73***	8.627***	41.72***	21.45***	22.31***
	-1.253	-2.034	-4.161	-4.571	-6.984
Fourth grade	44.18***	7.951***	30.58***	24.50**	27.54***
	-1.644	-3.083	-7.768	(11.04)	-7.319
Wave 1 (omitted)					
Wave 2	16.71***	18.38***	20.50***	18.54***	18.55***
	(0.350)	(0.189)	(0.306)	(0.189)	(0.197)
Wave 3	13.08***	27.89***	12.61***	26.99***	28.00***
	(0.712)	-1.048	-1.067	-1.309	-1.571
Wave 4	13.53***	41.03***	4.887	27.60***	26.80***
	-1.295	-2.017	-3.964	-4.285	-6.640
Wave 5	16.87***	53.90***	29.69***	38.39***	35.10***
	-1.669	-3.056	-7.587	(10.88)	-6.825
Constant	108.8***	106.1***	0.00275	106.7***	102.6***
	(0.491)	(0.394)	(0.0839)	-4.122	-2.586
Observations	54,591	54,591	54,591	54,591	54,591
Hausman (χ^2)		2029.15			536.66
R^2	0.430	0.753	0.162	0.784	0.595

Source: GERES database (2005-2008).

Note: Robust standard errors between brackets; *** p<0.01, ** p<0.05, * p<0.1.

The repeating students did not differ in relation to non-repeating notes when it is done the match control. The time dummies and dummies series indicate gain of note associated to the wave in question as compared to the first wave analyzed and on the series compared to the first grade of elementary school, respectively.

For mathematics (table 6) the regression not showing fixed-effect controls (column 1) revealed that all categories were significant. This suggested that students with teachers having high school level had lower grades and students having teachers with higher education in other courses had higher grades both as compared to students having teachers with higher education in pedagogy. After the fixed effects for teachers and students were introduced, the high school category became positive and significant, while the category of higher education in others courses turned out to be not significant. However, when the match effect was introduced (column 5), both categories were not significant, as compared to the omitted category, i.e., higher education - pedagogy. This means that, once the fixed effects were controlled, the teacher school did not impact on students' grades. That is, when the three effects were controlled, the difference between math grades did not depend on the teachers' education degree.

As for teachers' qualification, all categories were positive and significant when there was no control of fixed effects (column 1). However, such results proved to be underestimated when compared to the result with the match control included (column 5). This fact possibly occurred because, in the first case, elements, such as students and teacher's motivation, were not taken into account. As for the last regression, all categories were not significant, except for the master's category that was positive and significant. Therefore, students having teachers with master's degree had higher grades than those whose teachers did not have or completed their post graduation programs.

Experience presented only two significant categories in the POLS and FE_i regressions. The category up to 4 years and that of 5 to 10 years showed a positive sign. However, when the fixed effects of students and teachers were included (column 4), none category was significant. The same was true when the match fixed-effect control was introduced (column 5). Therefore, experience did not have any significant effect on students' math proficiency when fixed-effects control was applied. A possible explanation could be attributed, for example, to the fact that more experienced teachers were more confident to teach. As this was already controlled in the match regression, experience in itself had no significant impact on the students' math grades.

Tenure had a significant effect when analyzing without fixed-effect controls (column 1). The categories lower than the omitted one (from 5 to 10 years) showed a negative impact as compared to that of 5 to 10 years, while higher categories showed a positive impact. When the match fixed effects (column 5) were included - except for the category "more than 15 years" and "3 to 4 years" - the remaining categories were not significant at the usual levels of significance. Therefore, the students having teachers working in the same school for more than 15 years had higher grades as compared to students whose teachers worked in the same school between 5 to 10 years.

Table 6: Math regressions (continues)

MATH					
VARIABLES	**(1) POLS**	**(2)**FE_i	**(3)**FE_j	**(4)**$FE_i + FE_j$	**(5)**FE_{ij}
Education					
Higher Education - Pedagogy (omitted)					
High School	-5.077***	-1.281*	1.953	6.297	-3.886
	(0.751)	(0.760)	-5.144	-5.570	-9.458
Higher Education - Others	3.307***	0.246	-0.649	-5.185	-1.403
	(0.487)	(0.469)	-3.763	-3.898	-5.372
Qualification					
Do not made/complete postgraduate (omitted)					
Update	0.510	1.850**	8.332**	3.637	0.700
	(0.816)	(0.735)	-3.659	-3.384	-4.862
Specialization	3.216***	1.116**	0.0967	1.186	-2.486
	(0.476)	(0.479)	-3.002	-3.117	-3.816
Master's degree	33.95***	1.375	46.77***	43.34***	38.59***
	-1.551	-1.433	-7.426	-7.778	-9.956
Doctorate	36.90***	7.089**	-	0.870	-
	-5.231	-3.038	-	(13.98)	-
Experience					
Until 4 years	9.599***	3.209***	-1.111	5.453	8.779
	(0.922)	(0.871)	-7.653	-8.060	-9.460
5 to 10 years	3.046***	1.372**	-1.895	-0.845	-0.0841
	(0.674)	(0.670)	-5.209	-5.159	-6.281
11 to 15 years	-0.547	0.851	0.295	-1.212	0.932
	(0.575)	(0.586)	-3.739	-3.794	-4.807
More than 15 years (omitted)					
Tenure					
Until 1 year	-10.60***	0.328	-7.144	8.344	9.821
	(0.701)	(0.672)	-4.462	-5.136	-6.850
1 to 2 years	-7.173***	1.364*	-4.136	8.437**	6.312
	(0.760)	(0.772)	-3.826	-4.173	-5.652
3 to 4 years	-3.772***	1.941***	1.882	9.316***	9.218**
	(0.691)	(0.695)	-2.874	-3.046	-4.512
5 to 10 years (omitted)					
11 to 15 years	2.905***	3.188***	14.98***	-0.0443	3.225
	(0.744)	(0.720)	-4.067	-4.509	-6.873
More than 15 years	8.364***	1.892**	32.36***	14.05***	15.06**
	(0.775)	(0.775)	-5.002	-4.943	-6.969
Other job - No (omitted)					
Yes	4.326***	-0.335	0.344	-5.075***	-3.944*
	(0.590)	(0.496)	-2.053	-1.940	-2.334

Source: GERES database (2005-2008).

Note: Robust standard errors between brackets; *** p<0.01, ** p<0.05, * p<0.1.

Table 6: Math regressions (conclusion)

	MATH				
VARIABLES	**(1) POLS**	**(2)**FE_i	**(3)**FE_j	**(4)**$FE_i + FE_j$	**(5)**FE_{ij}
Number of worked hours at school					
Until 20h/week	-0.179	0.654	0.261	-2.071	-6.238
	(0.595)	(0.680)	-3.094	-3.410	-4.074
21h to 25h/week (omitted)					
26h to 30h/week	-9.855***	4.282***	-16.19***	-10.43**	-11.99*
	(0.798)	(0.809)	-4.407	-4.712	-6.491
31h to 40h/week	-8.016***	-1.049	2.347	13.70***	15.14***
	(0.634)	(0.660)	-2.787	-2.899	-3.515
More than 40h/week	-7.475***	2.188***	9.544***	14.99***	12.53***
	(0.683)	(0.688)	-2.776	-2.623	-3.137
Teacher's family earnings					
Until R$1900	-3.215***	1.144**	-4.326*	3.692*	4.144
	(0.525)	(0.534)	-2.408	-2.225	-2.930
R$1901 to R$3100 (omitted)					
More than R$3100	5.304***	-0.700	-1.680	5.199*	1.113
	(0.549)	(0.507)	-2.667	-2.819	-3.875
Repeating students	-15.29***	2.119***	-4.864***	0.729	1.599**
	(0.635)	(0.642)	(0.530)	(0.605)	(0.661)
First grade (omitted)					
Second grade	33.59***	25.60***	39.78***	23.95***	22.87***
	-1.057	-2.163	-2.909	-3.152	-3.830
Third grade	63.96***	51.11***	98.25***	83.08***	88.39***
	-2.459	-4.057	-8.553	-9.896	(17.24)
Fourth grade	87.20***	73.42***	100.9***	94.71***	98.40***
	-3.354	-6.038	(15.76)	(15.69)	(20.31)
Wave 1 (omitted)					
Wave 2	24.76***	29.08***	31.94***	28.54***	28.68***
	(0.460)	(0.293)	(0.522)	(0.288)	(0.284)
Wave 3	17.11***	28.09***	5.536***	22.12***	26.27***
	-1.197	-2.112	-1.997	-2.453	-2.993
Wave 4	28.06***	46.99***	-11.61	6.734	5.803
	-2.559	-4.013	-8.088	-9.195	(16.28)
Wave 5	49.01***	70.02***	31.93**	43.66***	44.69**
	-3.422	-5.968	(15.29)	(14.94)	(18.98)
Constant	109.7***	99.58***	-0.262	96.81***	96.05***
	(0.903)	(0.915)	(0.178)	(10.60)	-5.939
Observations	54,533	54,533	54,533	54,533	54,533
Hausman (χ^2)		1826.89			737.65
R^2	0.480	0.773	0.137	0.824	0.637

Source: GERES database (2005-2008).

Note: Robust standard errors between brackets; *** p<0.01, ** p<0.05, * p<0.1.

Having another job was significant for all estimated regressions, except for the regression without any fixed effect control (column 1).The coefficient was negative as for the remaining regressions. Therefore, by taking all fixed-effect controls into account (column 5), students and teachers having another job resulted in lower grades than those of students whose teachers did not have a second

job. The possible explanation is that teachers having more than one job would have to divide their dedication into several tasks, a condition that reflected worse students' grades.

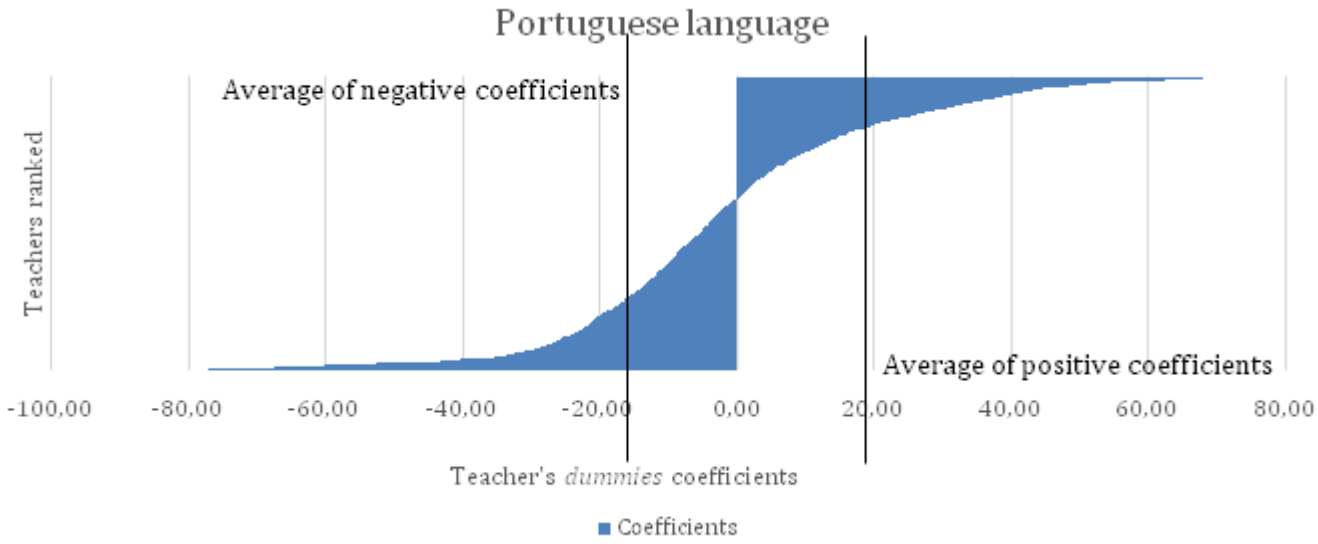
The teachers' hourly load showed that all categories were significant in the regressions with no fixed-effect controls (column 1), as well as with control for the three fixed effects (column 5). Nevertheless, when analyzing the match regression, despite being significant, some categories presented negative signs, but others, positive signs. Similarly to Portuguese, teachers with lower workload may have more than one job and negatively impacting their students, while teachers with higher workload can be teachers of private schools and positively impacting their students.

Finally, the teachers' family earnings was consistent when analyzing the regressions without fixed-effect controls (column 1). The lowest income bracket was negative and significant, while the highest was positive and significant. Therefore, the students having teachers receiving up to R$1,900 had lower grades than those whose teachers received from R$1,900 and R$3,100, while those other students whose teachers received more than R$3,100 had higher grades than those students whose teachers received from R$1,900 and R$3,100. However, when the fixed-effect controls for students and teachers were introduced (column 4), the lowest income bracket (up to R$1,900) was not significant any more, but the highest bracket (over R$3,100) remained positive and significant (now at the 5% significance level and not 1% any more), though with a slightly higher coefficient. However, when the match fixed effect was included (column 5), none category was significant at the usual confidence levels. This suggested that, given the fixed-effect controls of students, teachers and the match between them, the teachers' income did not have any impact on the students' math proficiency.

According Barros et al. (2001), despite characteristics like education and experience are chosen to select teachers from a school it may be that they do not explain the differences in student's proficiency. This is what happened in the case of mathematics. Once controlled the fixed effects of the teacher, the student and match, the observed characteristics of education, teacher education and income did not explain the differences in grades between students for this course. Issues such as motivation and dedication of students and teachers, leadership in the classroom and teaching the teacher's room, the interaction in the classroom between students and teacher characteristics are beyond the variables such as gender and race (included in the controlled fixed effects) are justifying the differences between students.

Two charts constructed with coefficients of teachers' dummies stemming from fixed-effect regressions of students and teachers ($FE_i + FE_j$ - column 4 of tables 5 and 6) are presented below. According to figures 12 and 13, the difference in proficiency between students having the worst teachers (those negatively affecting students' proficiency) and students having the best teachers (those positively affecting students' proficiency) can be verified.

Figure 12: Ranking Portuguese teachers



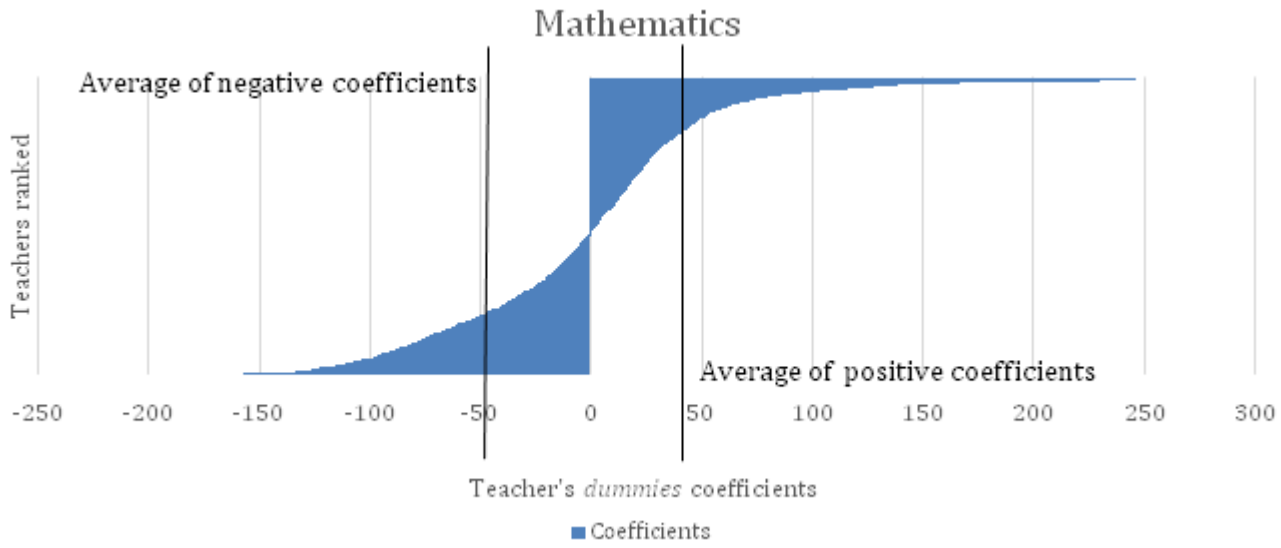
Source: GERES database (2005-2008).

As for Portuguese, the worst teachers had an effect of -16.81 on their students' proficiency points in average while the best teachers had an effect of 18.70 on students' proficiency points in average. The student who goes from worst to best average teacher may have affected their proficiency at 35.51 points. Whereas the initial average score of Portuguese (given by the regression constant, β_0 = 106.7) the student can go to the second level of learning in accordance with Table 2. This means that this student previously only had mastery of basic reading skills shall have a little more complex skills. For example, the student who before recognized letters of the alphabet and located information on a label shall be able to locate information in short texts, recognizing syllables of a word, identify the subject indicated in the title of a simple informational text and to infer some information in a serial and identify the purpose of a ticket.

As for mathematics, the worst teachers had an effect of -48.12 on their students' proficiency points in average while the best teachers had an effect of 40.02 on their students' proficiency points in average. The student who goes from worst to best average teacher can have an effect on the score of 88.14 points. Whereas the initial average grade math (given by the regression constant, β_0 = 96.81) if this student were in the first learning level he/she would be able to reach the third level, in case he/she left the worst teacher to the best teacher according to GERES scale.This student who had only basic math abilities before - would now be able to develop more complex abilities derived from other abilities previously incorporated, such as understanding numbers represented by three or four digits to identify these numbers by linking written language to numerical, identify the predecessor and perform a number decomposition, solving more complex problems involving subtraction with idea of complementation, comparison and equalization, multiplication involving the multiplicative principle and combined meaning of sharing which are more advanced construction process and are resolved

when inserted contexts.

Figure 13: Ranking Math teachers



Source: GERES database (2005-2008).

6 Final comments

The teachers' role in the education system is crucial for students developing their abilities in language and mathematics. Additionally, students' conditions are directly and indirectly involved in public policy discussions. Therefore, student grades results have been increasingly applied in assessing the teachers'. However, there is no consensus on defining the features of a good teacher and his/her relative importance in school performance, as compared to the other factors involved.

The present study investigated the impact of observable variables of teachers' quality on the proficiency level in Portuguese language and mathematics of students in public and private schools that have participated in the GERES project - *Geração Escolar*. Estimations have been made for controlling the characteristics of students, teachers and the match between them, by using the fixed-effect method through an unbalanced longitudinal panel.

The main results have evidenced that - based on the control for the match fixed effect - the Portuguese results suggested that the analyzed characteristics had a significant impact on the students' proficiency. Less-experienced teachers, as compared to those teachers having more than 15 years of experience, revealed a negative impact on their students' grades. Comparatively, teachers having a second job also impacted negatively on their students' performance, as compared to teachers having no other job.

The teachers' income bracket has also showed positive effects on their Portuguese students'

grades. Students whose teachers were paid better showed a higher performance, as compared to those students whose teachers had lower salaries. The seniority variable, however, was not so conclusive because, despite their significance,the analyzed categories did not show uniformity.

As for mathematics proficiency, teachers with master's degree have positively impacted on their students' grades, as compared to those teachers who did not have or completed a post-graduation program. Having a second job, as well as in the case of Portuguese language, negatively impacted math grades. In this case, teachers' seniority suggested that students whose teachers worked in the same school for 15 years or more showed their grades higher than those students whose teachers worked in that school for 5 to 10 years.

On the other hand, variables, such as schooling, experience and income, did not have a significant impact on math grades, when all unobserved characteristics of students, teachers and the match between them were controlled. Quality could be related to unobserved characteristics, as motivation for instance, which - if controlled - were able to explain differences in school grades, as observed in the case of mathematics.

As has also been observed - when the teachers' coefficients were used - when the student changed from the worst teacher of Portuguese to the best one he/she could have a grade improvement of 35.51 points, thus leaping from the first to the third learning level, in accordance with the six-leveled GERES scale for Portuguese proficiency. As for mathematics, students changing from the worst to the best teacher could have a grade improvement of 88.14 points in their proficiency. In this case, according the math GERES scale, students would change from the first learning level to the third learning level (as for mathematics, the scale comprises five levels).

The results found have evidenced that - in addition to the teachers' observable attributes of quality - it was also important to control their fixed effects. Characteristics,such as classroom security, teacher's didactic, dedication and effort employed by students and teachers and their dovetail, were able to influence the students' performance. However, these estimates would be biased without controlling such effects.

Despite the restrained number of focalized municipalities, this study seems to be relevant for assessing the teachers' quality and also as an aid for education policy-making that are crucial for school improvement in developing countries. However, attention should be paid to that assessing the teachers' performance must be carefully carried out as a tool of public policy-making. There are risks associated with individual interests concerning what is to be evaluated, mainly as for the adequacy concerning the research methodology associated with a measure which would affect schools and all agents in question.

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