Fair Innings? The Utilitarian and Prioritarian Value of Risk Reduction over a Whole Lifetime

Matthew D. Adler,[1] Maddalena Ferranna,[2] James K. Hammitt,[3] Nicolas Treich[4]

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Abstract

The social value of risk reduction (SVRR$_i$) is the marginal social value of reducing an individual's fatality risk, as measured by some social welfare function (SWF). This Article investigates SVRR, using a lifetime utility model in which individuals are differentiated by age, lifetime income profile, and lifetime risk profile. We consider both the utilitarian SWF and a "prioritarian" SWF, which applies a strictly increasing and concave transformation to individual utility.

We show that the prioritarian SVRR provides a rigorous basis in economic theory for the "fair innings" concept, proposed in the public health literature: as between an older individual and a similarly situated younger individual (one with the same income and risk profile), a risk reduction for the younger individual is accorded greater social weight even if the gains to expected lifetime utility are equal. The comparative statics of prioritarian and utilitarian SVRRs with respect to age, and to (past, present, and future) income and baseline survival probability, are significantly different from the conventional value per statistical life (VSL). Our empirical simulation based upon the U.S. population survival curve and income distribution shows that prioritarian SVRRs with a moderate degree of concavity in the transformation function conform to lay moral judgments regarding lifesaving policies: the young should take priority but income should make no difference.

Key Words

Social welfare function (SWF), benefit-cost analysis (BCA), value of statistical life (VSL), fair innings, social value of risk reduction (SVRR), utilitarian, prioritarian, risk regulation

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[1] Richard A. Horvitz Professor of Law and Professor of Economics, Philosophy, and Public Policy, Duke University. adler@law.duke.edu. Corresponding author.
[2] Research Associate, Harvard T.H. Chan School of Public Health, Harvard University. mferranna@hsph.harvard.edu.
[3] Professor of Economics and Decision Sciences, Harvard University. jkh@hsph.harvard.edu.
[4] Research Faculty, INRA; Toulouse School of Economics, University Toulouse Capitole. nicolas.treich@inra.fr.
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Introduction

Is it socially more important to save the lives of younger individuals, than to save the lives of the old? It seems hard to dispute that younger individuals should take priority with respect to lifesaving measures to the extent that age inversely correlates with life expectancy remaining, at least if the younger and older individuals are similarly situated with respect to the other determinants of well-being (health, income, etc.). If Anne is similarly situated to Bob, except for being younger, and a given reduction in Anne's current mortality risk produces a larger increase in her life expectancy than the same reduction in Bob's, the risk reduction for Anne seems socially more valuable.

But some have argued that the young should take priority with respect to lifesaving measures, and health policy more generally, on fairness grounds—not merely on the utilitarian basis that lifesaving measures directed at the young tend to yield a greater increase in life expectancy and expected lifetime well-being. Harris (1985, p. 91) introduced the idea of "fair innings" into the public health literature. "The fair innings argument requires that everyone be given an equal chance to have a fair innings, to reach the appropriate threshold but, having reached it, they have received their entitlement. The rest of their life is the sort of bonus which may be canceled when this is necessary to help others reach the threshold." Others who have endorsed some version of the fair innings concept include Williams (1997); Daniels (1988); Lockwood (1988); Nord (2005); Bognar (2008, 2015); Ottersen (2013). The notion that the young should receive priority with respect to lifesaving measures is reflected, not merely in the academic literature on fair innings, but also in surveys of citizen preferences regarding health policy (see Bognar [2008] for references; Dolan and Tsuchiya [2012]).

Bognar (2015, p. 254) uses the following thought experiment to crystallize the fair innings concept.

> [Y]ou have only one drug and there are two patients who need it. The only difference between the two patients is their age. . . . You have to choose between saving: (C) a 20-year old patient who will live for 10 more years if she gets the drug; or (D) a 70-year old patient who will live for 10 more years if she gets the drug.
>
> Both patients would spend the remaining ten years of their life in good health. So there is no difference in expected benefit. The only difference is how much they have already lived when they receive the benefit.
>
> … [According to] the fairness-based argument for the fair innings view, you should … prefer C to D.

We'll build on the suggestion of Bognar (2015) in using the term "fair innings" to mean the following: as between a policy that produces a given gain in expected lifetime well-being for a younger person, and an otherwise-identical policy that produces the same gain in expected lifetime well-being for an older person, it is ethically better for society to undertake the first policy.

While fair innings in this sense is an intuitively appealing idea, it is *not* supported by the current economic literature regarding the valuation of lifesaving. That literature generally focuses on benefit-cost analysis (BCA), which is the dominant tool in governmental practice for assessing fatality risk-reduction policies. The methodology of BCA does *not* support the idea that gains to the young are socially more valuable than equal gains for the old.[5]

In this Article, we examine the fair innings concept as part of a broader analysis of the use of social welfare functions (SWFs) to value risk reduction, and a comparison of the SWF framework to BCA. We show, in particular, that "prioritarian" SWFs place greater weight on gains to expected lifetime well-being accruing to younger rather than older individuals. We thus demonstrate, for the first time, that the fair innings concept has a rigorous basis in welfare economics—specifically in the SWF framework, not BCA.

BCA appraises government policies by summing individuals' monetary equivalents—an individual's monetary equivalent for a policy being the amount of money she is willing to pay or accept for it, relative to the status quo. In turn, the value per statistical life (VSL) is the concept that captures how BCA values fatality risk reduction. VSL is the marginal rate of substitution between an individual's survival probability in a period, and her income. Put differently, VSL is the coefficient that translates a change in someone's survival probability into a monetary equivalent. Individual i's willingness to pay or accept for a small change Δp in survival is approximately $(\Delta p)\mathrm{VSL}_i$.

BCA, although now widespread, is controversial. A different framework for evaluating policy—one that has strong roots in economic theory and plays a major role in various bodies of scholarship within economics—is the social welfare function (SWF). The SWF framework measures policy impacts in terms of interpersonally comparable *utilities*, not monetary equivalents. Each possible outcome is a vector of individual utilities, and a given policy is a probability distribution over such vectors. The SWF, abbreviated $W(\cdot)$, assigns a social value to a policy P, $W(P)$, in light of the probability distribution over outcomes and, thus, utility vectors that P corresponds to. On the SWF framework, see generally Adler (2012, 2019); Blackorby, Bossert and Donaldson (2005, chs. 2-4); Bossert and Weymark (2004); Weymark (2016).

In previous work (Adler, Hammitt and Treich [2014]), we analyzed the application of the SWF framework to risk policies and compared how it values risk reduction to VSL. The key construct in our analysis was the social value of risk reduction (SVRR). The SVRR for individual i is the social value per unit of risk reduction to individual i—social value as captured by the SWF $W(\cdot)$. SVRR_i is just $\frac{\partial W}{\partial p_i}$, and the change in the SWF that occurs with a change Δp in individual i's survival probability p_i is approximately $(\Delta p)\mathrm{SVRR}_i$.

[5] See below, Part II, explaining why BCA does not support fair innings.

Using the simple, one-period model that is often employed in the literature on VSL, Adler, Hammitt and Treich (2014) calculated $SVRR_i$ for different types of SWFs: the utilitarian, "ex ante prioritarian," and "ex post prioritarian" SWFs. (Utilitarianism ranks outcomes by summing utility numbers, while prioritarianism does so by summing a strictly increasing and strictly concave transformation of utility numbers, thereby giving priority to those at lower utility levels. The idea of utilitarianism dates back hundreds of years to the writings of Jeremy Bentham; prioritarianism is a more recent concept, pioneered by the moral philosopher Derek Parfit [2000]. The ex ante and ex post prioritarian SWFs are two distinct specifications of prioritarianism for the case of uncertainty.) We analyzed the comparative statics of $SVRR_i$ and VSL_i with respect to individual wealth and baseline risk.

The current Article significantly expands the analysis of Adler, Hammitt and Treich (2014). We use a much richer model of individual resources and survival. An individual's life has multiple periods, up to a maximum T (e.g., 100 years). Each individual is characterized by a lifetime risk profile (a probability of surviving to the end of each period, conditional on her being alive at its beginning); a lifetime income profile (an income amount which she earns in each period if she survives to its end); and a current age. This multi-period setup permits a more nuanced analysis of $SVRR_i$ and VSL_i. In particular, we can now examine the comparative statics of $SVRR_i$ and VSL_i with respect to an individual's *age* as well as with respect to (past, present and future) income and baseline fatality risk.

Although the SWF framework is widely used in some areas of economics, such as optimal tax theory (see Tuomala [2016] for an overview) and climate economics (Botzen and van den Bergh [2014]), little research has been undertaken applying this framework to the policy domain of fatality risk reduction—a major arena of governmental policymaking (Graham [2008]). We aim to make headway in exploring this important and understudied topic, and to raise its profile in the research community.

Part I sets forth the model and the SWFs we will consider. Part II analyzes the comparative statics of $SVRR_i$ and VSL_i with respect to age. We provide a formal statement of the fair innings concept, via properties which we term "Priority for the Young" and "Ratio Priority for the Young." We show that the ex ante prioritarian $SVRR_i$ and ex post prioritarian $SVRR_i$ both display Priority for the Young and indeed the logically stronger property of "Ratio Priority for the Young." By contrast, VSL_i does not have either property. [6]

Part III analyzes the comparative statics of $SVRR_i$ and VSL_i with respect to income and baseline risk. Part IV undertakes an empirical exercise, based on the U.S. population survival curve and income distribution, to illustrate the $SVRR_i$ concept and to estimate the impact of age and (within each age cohort) income on $SVRR_i$ and VSL_i.

[6] As explained in Part II, Priority for the Young and Ratio Priority for the Young are defined relative to a utilitarian baseline, and so it's trivial that the utilitarian $SVRR_i$ does not display these properties.

Parts I through IV focus on the case of "myopic" consumption: each individual consumes in a given period what she earns then. Part V extends the analysis to the case of "endogenous" consumption—where individuals maximize expected lifetime well-being by saving income for future consumption or financing current consumption by borrowing against anticipated earnings.

Our headline results are as follows. First, we demonstrate that the SWF framework—by contrast with BCA—provides a rigorous basis for the "fair innings" concept. The social value of risk reduction (SVRR), as calculated using an ex ante or ex post prioritarian SWF, gives extra social weight to risk reduction among younger individuals above and beyond the additional weight they receive in virtue of greater life expectancy remaining. Part II demonstrates this for the case of myopic consumption; Part V extends our fair-innings results to endogenous consumption.[7]

Second, we show that the manner in which BCA values risk reduction is significantly different from the SWF framework, *regardless* of which SWF is used (utilitarian, ex ante prioritarian, ex post prioritarian). These differences are multifold. The prioritarian SVRRs display Priority for the Young and Ratio Priority for the Young, while VSL does not. Further, as established in Part III, the comparative statics of VSL with respect to income and baseline risk are different not only from the ex ante and ex post prioritarian SVRRs, but also from the utilitarian SVRR. Finally, Part IV demonstrates that these analytic differences may be empirically quite significant. In particular, VSL increases much more steeply with income in each age group than the utilitarian SVRR, while the prioritarian SVRRs are flat or decrease with income.[8]

I. **The Model**

There is a population of N individuals. The life of a given individual i is divided into periods 1, 2, …, t, … T, with T the maximum number of periods that any individual can live.

Death and survival are conceptualized as follows: An individual who is alive at the beginning of a given period may either die before the period ends, or survive to the end of the period (equivalently, be alive at the beginning of the following period). Let $p_i(t)$ denote individual i's probability of surviving to the end of period t, given that she is alive at the beginning of period t. We'll generally refer to $p_i(t)$ as a "survival probability." Individual i is

[7] As explained in Part V, this part does not exhaustively consider the different possible variation of endogenous consumption. Rather, we focus on the annuities/surprise case (see Part V) and show that our fair innings results extend to this case. Examining further variations of endogenous consumption is a topic for future research.
[8] The properties of the ex ante prioritarian and ex post prioritarian SVRR depend, to some extent, on which concave transformation function is used—embodying the degree of priority for the worse off. Thus, in Part III, the prioritarian SVRRs with a moderately concave transformation function are flat with income, while the prioritarian SVRRs with a more concave transformation decrease with income.

characterized by a vector of such probabilities, one for each period up to T—for short, her "risk profile." (We assume $0 < p_i(t) < 1$ for all $t > 1$ and that $p_i(1) = 1$.[9])

Government makes a policy choice (see below for more detail) at a point in calendar time, denoted "the present." The present is the *beginning* of the "current period."

For any individual now alive, the current period is some period in her life. (For example, if Betty has already lived 4 periods, the current period is number 5 in Betty's life.) Let A_i denote the number of the current period for individual i. We will also refer to this as the "age" of individual i (but please note that the present time is at the beginning of the current period, so that an individual "age 1" is at the beginning of period 1 of her life, an individual "age 2" at the beginning of period 2, and so forth).

Let $\pi_i(t; A_i)$ denote individual i's probability of surviving to the end of period t of her life, given that she is currently alive at the beginning of period A_i, with $t \geq A_i$. $\pi_i(t; 1)$ or, for short, $\pi_i(t)$, is just the individual's probability at birth of surviving until the end of period t. Then:

$$\pi_i(t; A_i) = \frac{\pi_i(t)}{\pi_i(A_i - 1)} = \prod_{s=A_i}^{t} p_i(s) .$$

Finally, let $\mu_i(t; A_i)$ denote individual i's current probability of surviving to the end of period t and then dying during the next period $(t + 1)$. In other words, $\mu_i(t; A_i)$ is the individual's current probability of living exactly t periods. In the case of $t = (A_i - 1)$, this is the probability of dying before the end of the current period (A_i), i.e., $\mu_i(t; A_i) = 1 - p_i(A_i)$. For $t \geq A_i$, we have that:

$$\mu_i(t; A_i) = (1 - p_i(t+1))\pi_i(t; A_i) .$$

The earnings process is as follows: if an individual survives to the end of period t, she earns an income amount $y_i(t) \geq 0$. Individual i, thus, is characterized by a vector of incomes, $(y_i(1), \ldots, y_i(T))$—her "income profile."

Period consumption, like period income, is modelled as occurring only if the individual survives to the end of the period. An individual's consumption during period t, if she survives to the end of period t, is denoted $c_i(t)$. Until Part V of the Article, we assume "myopic" consumption: $c_i(t) = y_i(t)$. The individual consumes in each period whatever she earns then, rather than saving earnings for future consumption or financing consumption by borrowing against future earnings. Part V considers endogenous rather than myopic consumption.

We assume that an individual's lifetime well-being is the discounted sum of period well-being, where $u(\cdot)$ is the period utility function and $\beta = 1/(1 + \varphi)$, $\varphi \geq 0$ the constant utility discount rate. $U_i(s)$ denotes the individual's lifetime well-being if she lives exactly s periods.

[9] As discussed below, we assume that $A_i(t) \geq 2$ for all i—that is, that every individual has survived to the end of the first period of her life—and to ensure this we assume $p_i(1) = 1$.

$U_i(s)=\sum_{t=1}^{s}\beta^t u(y_i(t))$.[10] We assume that $u(\cdot)$ is continuously differentiable and that $u'(\cdot) > 0$, $u''(\cdot) < 0$.

Note that the above formula for lifetime well-being includes a term for a given period t iff the individual survives to the end of the period. If she doesn't survive to the end of a given period, her period utility is zero. Further, our analysis assumes that, if i does survive to the end of period t, with consumption $c_i(t)$ in that period, $u(c_i(t)) > 0$. Note that if $u(c_i(t)) < 0$, increasing $p_i(t)$ may have the effect of *lowering* i's expected lifetime well-being. We wish to focus here on the case in which risk reduction is beneficial to individuals—not the unusual case in which it may be harmful—and so assume that $u(c) > 0$ for all $c \geq 0$. We maintain this assumption both for the analysis of myopic consumption, $c_i(t) = y_i(t)$; and, in Part V, for the analysis of endogenous consumption.[11]

We consider three different social welfare functions (SWFs): the utilitarian SWF, denoted W^U; the ex ante prioritarian SWF, denoted W^{EAP}; and the ex post prioritarian SWF, denoted W^{EPP}.

1. **Utilitarianism**. $W^U = \sum_{i=1}^{N} V_i$, where V_i is the expected lifetime well-being of individual i, given that his current age is A_i. $V_i = \sum_{t=1}^{A_i-1}\beta^t u(y_i(t)) + \sum_{t=A_i}^{T}\pi_i(t;A_i)\beta^t u(y_i(t))$

2. **Ex ante prioritarianism**. Let $g(\cdot)$ be a strictly increasing and strictly concave transformation function. Then $W^{EAP} = \sum_{i=1}^{N} g(V_i)$.

3. **Ex post prioritarianism**. Given that individual i is currently age A_i, the probability of attaining lifetime well-being $U_i(t)$ with $t \geq A_i$ is given by $\mu_i(t; A_i)$, as previously defined. Thus

$$W^{EPP} = \sum_{i=1}^{N}\left[(1-p_i(A_i))g(U_i(A_i-1)) + \sum_{t=A_i}^{T}\mu_i(t;A_i)g(U_i(t))\right]$$

The application of SWFs to lifetime well-being has a strong ethical justification. (Adler 2012, ch. 6). While much of the SWF literature uses one-period models for reasons of tractability, there is also a significant body of work using multiperiod or lifetime numbers as the

[10] Since consumption occurs at the end of each period, the discount factor β is raised to the power t rather than $(t-1)$.

[11] One way to understand why it would be true that $u(c) > 0$ for $c \geq 0$ is to assume that government provides individuals a subsistence level of consumption sufficient to ensure period utility better than premature death. c then denotes consumption above this subsistence level.

input to an SWF.[12] (For discussion of this literature, see Adler [2012, p. 245]; Boadway [2012, pp. 86-106]; Tuomala [2016, pp. 360-64].)

Note that prioritarianism (in both ex ante and ex post versions) is a family of SWFs, corresponding to all the possible strictly increasing and strictly concave transformation functions $g(\cdot)$. The choice of $g(\cdot)$ defines a specific W^{EAP} and W^{EPP}. However, our analysis will be generic, holding true for any $g(\cdot)$. We do assume that $g(\cdot)$ is continuously differentiable, so that $g'(\cdot) > 0$ and $g''(\cdot) < 0$.

The "Atkinson" family of $g(\cdot)$ functions[13], which have attractive axiomatic properties and are regularly used in the economic literature on prioritarianism,[14] are such that $g(0)$ may be undefined. In order for our analysis to accommodate the possibility that $g(0)$ is undefined, we assume that $A_i \geq 2$ for all i.[15] Because the period length can be arbitrarily short, this is not a significantly restrictive assumption.

Government's policy: As mentioned, government enacts a policy at the beginning of the current period. The policy changes individuals' current-period survival probabilities. That probability, for individual i, changes from $p_i(A_i)$ to $p_i(A_i) + \Delta p_i$, with $\Delta p_i > 0$. Her probability of surviving until the end of period $t > A_i$ is now as follows:

$$[p_i(A_i) + \Delta p_i] \prod_{s=A_i+1}^{t} p_i(s) = \pi_i(t; A_i)\left[1 + \frac{\Delta p_i}{p_i(A_i)}\right]$$

We might posit that the policy is a "surprise" (individuals didn't anticipate it). Alternatively, we might posit that the possibility of the policy was anticipated and previously assigned some probability by each individual. This surprise/non-surprise question doesn't matter for the analysis of myopic consumption,[16] but is relevant if consumption is endogenous. See Part V.

The social value of risk reduction ($SVRR_i$) for a given SWF $W(\cdot)$ is just $\left.\frac{\partial W}{\partial p_i(t)}\right|_{t=A_i}$, that is, $\lim_{\Delta p_i \to 0} \frac{\Delta W}{\Delta p_i}$. We denote the utilitarian $SVRR_i$ as S_i^U, the ex ante prioritarian $SVRR_i$ as S_i^{EAP}, and the ex post prioritarian $SVRR_i$ as S_i^{EPP}.

[12] Similarly, while much empirical work on income inequality focuses on annual income, there is also a significant body of work that looks at the inequality of lifetime income. See, for example, Bönke et al. (2015), Bowlus and Robin (2004), Guvenen et al. (2017), Huggett et al. (2011), Nilsen et al. (2012).
[13] $g(u) = (1-\gamma)^{-1}u^{1-\gamma}$, $\gamma > 0$, $\gamma \neq 1$; and $g(u) = \ln u$ if $\gamma = 1$. See Adler (2012, ch. 5).
[14] Indeed, our empirical exercise (see explanation, Part IV) uses an Atkinson $g(\cdot)$ function.
[15] Note that the expression above for the ex post prioritarian SVRR uses the $g(\cdot)$ value of the lifetime well-being of a life with length $(A_i - 1)$. In order to avoid $(A_i - 1) = 0$, we assume $A_i \geq 2$ for all i.
[16] Modelling the policy as anticipated or, instead, a "surprise" will matter only insofar as individuals make choices; but in the myopic-consumption case, individuals make no choices at all—neither to affect their period mortality risks, nor their period consumption.

$SVRR_i$ is the marginal change in social welfare per unit of current risk reduction for individual *i*. To be sure, a governmental policy may well have effects other than changing individuals' current survival probabilities. It may also change their survival probabilities in future periods. And a risk-reduction policy will surely have costs, which will be reflected in a change to individuals' current or future incomes. The *total* effect of a policy on social welfare, ΔW, will be approximately equal to the sum, across individuals, of $(SVRR_i)\Delta p_i$ plus corresponding terms for changes to future survival probabilities and to incomes.[17] $SVRR_i$ captures that *portion* of a policy's total impact on social welfare that result from changes to individual *i*'s current survival probability.

Further, by comparing $SVRR_i$ to $SVRR_j$, for two individuals *i* and *j*—as we do below—we can determine the relative social impact of risk reductions for the two. Consider a change Δp to someone's current survival probability. That risk change, if accruing to individual *i*, results in a change of social welfare by approximately $SVRR_i(\Delta p)$. If accruing to individual *j*, it results in a change of social welfare by approximately $SVRR_j(\Delta p)$. Thus (for a small Δp) the first social welfare change is larger than/smaller than/equal to the second iff $SVRR_i$ is larger than/smaller than/equal to $SVRR_j$.

Calculating $\lim_{\Delta p_i \to 0} \frac{\Delta W}{\Delta p_i}$ for W^U, W^{EAP}, and W^{EPP} yields the formulas for $SVRR_i$, which are respectively as follows:

$$S_i^U = \sum_{t=A_i}^{T} \frac{\pi_i(t;A_i)}{p_i(A_i)} \beta^t u(y_i(t))$$

$$S_i^{EAP} = g'(V_i) S_i^U$$

$$S_i^{EPP} = -g(U_i(A_i - 1)) + \sum_{t=A_i}^{T} \frac{\mu_i(t;A_i)}{p_i(A_i)} g(U_i(t))$$

Note that our assumption that $u(y_i(t)) > 0$ for all *i*, *t*—it is always better to survive a period than to die before its end—ensures that S_i^U, S_i^{EAP}, and $S_i^{EPP} > 0$ for all *i*. Risk reduction is always a social benefit.

Throughout this Article, as in Adler, Hammitt and Treich (2014), we assume that individuals have common preferences, represented by a common period utility function. This

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[17] Assume that a policy changes individual *i*'s survival probability in period *t* by Δp_i^t, with $t > A_i$; and her income (and consumption, given myopic consumption) by Δy_i^t, with $t \geq A_i$. As in the text, let Δp_i denote the policy change to the individual's current survival probability. Then, by the total-differential approximation from calculus, ΔW is approximately equal to: $\sum_i \left(SVRR_i \Delta p_i + \sum_{s=A_i+1}^{T} \frac{\partial W}{\partial p_i(t)}\Big|_{t=s} \Delta p_i^t + \sum_{s=A_i}^{T} \frac{\partial W}{\partial y_i(t)}\Big|_{t=s} \Delta y_i^t \right)$.

common function $u(\cdot)$ is, at the same time, the basis for calculating individuals' lifetime well-being values, $U_i(t)$, for purposes of the various SWFs. In the standard analysis, a particular person i's VSL_i is the change in her expected utility, per unit of survival probability, divided by the change in her expected utility, per unit of income (or wealth or consumption). In our model, given the above assumptions about $u(\cdot)$ and $U_i(t)$, VSL_i can be defined more specifically as follows:

$$VSL_i = \frac{S_i^U}{p_i(A_i)u'(y_i(A_i))\beta^{A_i}} \ .$$

That is, VSL_i is the utilitarian SVRR divided by the expected marginal utility of i's current consumption.

In the text below, to avoid clutter, we will often drop the individual subscript and use the terms "SVRR" and "VSL" to mean, respectively, SVRR_i and VSL_i.

II. Age Effects and "Priority for the Young"

The effect of age on the SVRR has never been addressed by the academic literature.

Here, we analyze what our model implies with respect to age effects on SVRR as well as VSL by considering two individuals i and j, with identical risk profiles and income profiles, but the first older than the second ($A_i > A_j$). In what follows, we often drop subscripts on incomes or probabilities where these are the same for i and j, e.g., $y(t)$ indicates $y_i(t) = y_j(t)$.

The Utilitarian SVRR: Recall that $S_i^U = \sum_{t=A_i}^{T} \frac{\pi_i(t;A_i)}{p_i(A_i)} \beta^t u(y_i(t))$. Observe that S_i^U is equal to the difference between (1) individual i's expected lifetime well-being conditional on surviving the current period and (2) her realized lifetime well-being if she dies during the current period (does not survive it). The intuition for this result is straightforward. Consider the simple case in which individual i would die for certain during the current period, absent governmental intervention, and intervention ensures that she survives the period. In this case, clearly, the change in utilitarian social welfare (ΔW^U) that results from the intervention is just the difference between individual i's expected lifetime conditional on surviving the current period, and her realized lifetime well-being if she dies during the current period. For short, let's term this difference the "utilitarian gain from saving individual i."

More generally, consider a policy which increases individual i's current survival probability by Δp_i. The change in utilitarian social welfare that results from the Δp_i increase is just $\Delta p_i \left(\sum_{t=A_i}^{T} \frac{\pi_i(t;A_i)}{p_i(A_i)} \beta^t u(y_i(t)) \right)$. Thus S_i^U, the change in utilitarian social welfare per unit of current-period risk reduction for individual *i*, is nothing other than $\sum_{t=A_i}^{T} \frac{\pi_i(t;A_i)}{p_i(A_i)} \beta^t u(y_i(t))$: the utilitarian gain from saving individual *i*.

What, then, are the relative magnitudes of S_i^U and S_j^U, for two individuals of different ages ($A_i > A_j$) but with identical risk and income profiles? In other words, how does the utilitarian gain from saving an individual depend upon her age?

It can be shown that $S_j^U - S_i^U$ equals:

$$\sum_{t=A_j}^{A_i-1} \pi(t;A_j+1)\beta^t u(y(t)) + \left(\pi(A_i;A_j+1)-1\right)\sum_{t=A_i}^{T} \pi(t;A_i+1)\beta^t u(y(t)) .$$

Thus the utilitarian SVRR decreases/is unchanged/increases with age iff the value of this formula is positive/zero/negative.

The first term in this formula (for short, the "duration term") is positive. By increasing the *younger* individual's current survival probability, we increase her chance of surviving the periods A_j, A_j+1, …, $A_i - 1$ in her life, and that probability change for each such period yields an increment in expected lifetime well-being (by increasing her chance of accruing consumption utility with respect to that period). This increment to expected lifetime well-being with respect to periods A_j, A_j+1, …, $A_i - 1$ does not occur if we increase the *older* individual's survival probability, since he has already survived those periods.

The second term in the formula above (for short, the "risk term") is negative. By increasing either individual's current survival probability, we increase that individual's chance of surviving periods A_i, $A_i + 1$, …, T in his or her life, and thereby increase his or her chance of accruing consumption utility with respect to those periods. The risk term captures the *difference* between the magnitude of this benefit for the younger individual and its magnitude for the older one. Since the older individual is sure to be alive at the beginning of period A_i, while the younger individual is not, this difference is negative.

Clearly, if income can increase with age, the magnitude of the risk term may exceed that of the duration term, and thus the utilitarian gain from saving the older individual *i* may be greater than that of saving the younger one. What if constant income is assumed? With a constant income profile and a constant risk profile, the duration term predominates and the utilitarian SVRR decreases with age. More generally, it can be shown that if income is constant and the risk profile is such that survival probabilities do not increase with age, the utilitarian SVRR decreases with age. (See Appendix.)

Ex ante Prioritarian SVRR: A simple manipulation shows that $S_j^{EAP} - S_i^{EAP} = g'(V_j)\left(S_j^U - S_i^U\right) + S_i^U\left(g'(V_j) - g'(V_i)\right)$. We noted immediately above in discussing the utilitarian SVRR that the quantity $(S_j^U - S_i^U)$ equals a positive "duration" term plus a negative "risk" term. The first part of the formula here, namely $g'(V_j)\left(S_j^U - S_i^U\right)$, incorporates those terms. This part is positive iff $(S_j^U - S_i^U)$ is positive. The second part of the formula here, $S_i^U\left(g'(V_j) - g'(V_i)\right)$, is a third term ("priority for the young"), which is always positive. Because $V_i > V_j$ (the older individual has greater expected lifetime well-being) and $g(\cdot)$ is strictly concave, $g'(V_i) < g'(V_j)$.

The intuition behind the formula is as follows. Ex ante prioritarian social welfare, W^{EAP}, is the sum of individuals' transformed expected lifetime well-beings— transformed by a strictly increasing and strictly concave $g(\cdot)$ function. The effect of this transformation is to give greater social weight to changes in expected lifetime well-being that accrue to individuals at a lower level of expected lifetime well-being. The differential ex-ante-prioritarian benefit of saving a younger rather than older individual reflects the differential gains to expected lifetime well-being of saving the younger one $(S_j^U - S_i^U)$. But it also reflects the fact that the younger individual has a lower level of expected lifetime well-being and thus takes priority ($g'(V_j) > g'(V_i)$).

We now define "Priority for the Young" more formally. In defining this property, we incorporate a utilitarian baseline. The utilitarian social evaluation of risk reduction for a younger versus an older individual depends on a comparison of the gains to expected lifetime well-being from saving one or the other. Utilitarianism prefers to save the young only to the extent that doing so produces a larger increment in expected lifetime well-being. The ex ante prioritarian SWF reflects "Priority for the Young," relative to the utilitarian baseline, defined as follows:

Proposition I-A : *The Ex Ante Prioritarian SWF displays Priority for the Young*

$$S_j^U - S_i^U > 0 \Rightarrow S_j^{EAP} - S_i^{EAP} > 0 \text{ and } S_j^U - S_i^U = 0 \Rightarrow S_j^{EAP} - S_i^{EAP} > 0$$

Priority for the Young is a precise expression, using the SVRR formalism, of the fair innings concept. Ex ante prioritarianism never assigns a smaller or equal per-unit value to risk reduction for the younger individual if the utilitarian per-unit value is larger than for the older individual. ($S_j^U - S_i^U > 0 \Rightarrow S_j^{EAP} - S_i^{EAP} > 0$.) Further, if the utilitarian per-unit values are equal, ex ante prioritarianism places a larger per-unit value on risk reduction for the younger individual.

($S_j^U - S_i^U = 0 \Rightarrow S_j^{EAP} - S_i^{EAP} > 0$.).[18] Recall, finally, that the utilitarian per-unit value of risk reduction for any person, young or old, is just the gain to expected lifetime from saving her.

We can illustrate the Priority-for-the-Young property using the Bognar (2015) thought experiment presented in the Introduction. Consider two patients, a young patient *j* and an older patient *i*, who are respectively at the beginning of periods two and three of their lives. The maximum lifespan is 3 periods. The patients have a common risk profile, with p_2 the common survival probability for period two and p_3 the common survival probability for period three. Assume also that the patients are equally well off, materially. Each faces the same, constant, income profile, with period utility normalized to 1.

Finally, assume that p_3 is close to zero, so that $S_j^U \approx S_i^U = 1$. Thus, if we have one dose of a drug that can increase the patient's current survival probability, utilitarianism is indifferent between giving the drug to the younger or the older patient; the utilitarian SVRRs are approximately equal. However, we easily obtain that $S_j^{EAP} \approx g'(1+p_2) > S_i^{EAP} \approx g'(2)$, by the concavity of $g(\cdot)$.[19] Ex ante prioritarianism tells us to give the drug to the younger individual, who has a lower expected lifetime well-being (it is uncertain whether she will survive the second period, while the older patient will definitely live at least two periods). Ex ante prioritarianism gives greater weight to a given increase in expected lifetime well-being if it accrues to an individual at a lower level of expected lifetime well-being, and so the younger patient takes priority.

Not only does ex ante prioritarianism satisfy Priority for the Young. We can prove a logically stronger result, namely that the relative social value of risk reduction for young versus old individuals is always greater with ex ante prioritarianism than with utilitarianism.

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[18] Priority for the Young is stated as a conjunction of two properties, namely $S_j^U - S_i^U > 0 \Rightarrow S_j^{EAP} - S_i^{EAP} > 0$ *and* $S_j^U - S_i^U = 0 \Rightarrow S_j^{EAP} - S_i^{EAP} > 0$. Note that this conjunction is logically stronger than the property $S_j^U - S_i^U \geq 0 \Rightarrow S_j^{EAP} - S_i^{EAP} \geq 0$. This latter property allows for the ex ante prioritarian SVRRs to be equal even though the utilitarian gain from saving the younger individual is greater ($S_j^U - S_i^U > 0$ and $S_j^{EAP} - S_i^{EAP} = 0$), while Priority for the Young defined conjunctively as we have done does not allow this.

Priority for the Young *can* be expressed as a single property, namely: $S_j^U - S_i^U \geq 0 \Rightarrow S_j^{EAP} - S_i^{EAP} > 0$. This property is logically equivalent to the conjunctive formulation; we use the latter in our exposition because we believe it is easier to immediately grasp.

[19] V_j, the expected lifetime well-being of the younger individual, is $(1 - p_2)$ (1) + $p_2(1 - p_3)$ (2) + $p_2p_3(3) = 1 + p_2 + p_2p_3$; while V_i, the expected lifetime well-being of the older individual, is $(1 - p_3)$ (2) + $p_3(3) = 2 + p_3$. Thus, $S_j^U = \frac{\partial V_j}{\partial p_2} = 1 + p_3$, while $S_i^U = \frac{\partial V_i}{\partial p_3} = 1$. $S_j^{EAP} = g'(1 + p_2 + p_2 p_3)(1 + p_3)$ and $S_i^{EAP} = g'(2 + p_3)$.

$(S_j^{EAP} / S_i^{EAP}) > (S_j^U / S_i^U)$.[20] If utilitarianism prefers to reduce the younger individual's risk (the utilitarian gain from saving her is greater), ex ante prioritarianism has a yet greater degree of priority for the young. If utilitarianism is indifferent (the utilitarian gains are equal), ex ante prioritarianism gives priority to the young. Finally, although ex ante prioritarianism may prefer to reduce the risk of the older individual (if the utilitarian gain from saving her is sufficiently greater), in this case it always give less priority to the older individual than utilitarianism does.

Proposition I-B: *The Ex Ante Prioritarian SWF displays Ratio Priority for the Young*

$$(S_j^{EAP} / S_i^{EAP}) > (S_j^U / S_i^U)$$

Note that Ratio Priority for the Young implies Priority for the Young (Proposition I-B implies Proposition I-A), but not vice versa.

Ex post prioritarian SVRR: It can be shown that $S_j^{EPP} - S_i^{EPP}$ equals:

$$\sum_{t=A_j}^{A_i-1} \mu(t;A_j+1)g(U(t)) + (\pi(A_i;A_j+1)-1)\sum_{t=A_i}^{T} \mu(t;A_i+1)g(U(t)) + \left(g(U(A_i-1)) - g(U(A_j-1))\right)$$

Although this formula is different from $S_j^{EAP} - S_i^{EAP}$, it nonetheless reflects the same three factors. The first term of the formula is a positive "duration term," reflecting the increased chance for the younger individual of surviving periods A_j through $A_i - 1$; the second term is a negative "risk term," reflecting the chance she will not survive to period A_i; and the third term is a positive "priority for the young" term.

We saw above that the ex ante prioritarian SWF displays "Priority for the Young": it prefers to reduce the younger individual's risk even when utilitarianism is indifferent, and prefers to do so whenever utilitarianism does. The same is true for the ex post prioritarian SWF.

Proposition II-A : *The Ex Post Prioritarian SWF displays Priority for the Young*

$$S_j^U - S_i^U > 0 \Rightarrow S_j^{EPP} - S_i^{EPP} > 0 \text{ and } S_j^U - S_i^U = 0 \Rightarrow S_j^{EPP} - S_i^{EPP} > 0$$

See Appendix for proof.

The intuition for this result is as follows. Ex post prioritarian social welfare, W^{EPP}, is the sum of individuals' expected transformed lifetime well-beings—transformed by a strictly increasing and strictly concave $g(\cdot)$ function.[21] The ex post prioritarian SVRR,

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[20] $S_i^{EAP} = g'(V_i)S_i^U$ and $S_j^{EAP} = g'(V_j)S_j^U$. Thus we have that $\frac{S_j^{EAP}}{S_i^{EAP}} = \frac{g'(V_j)}{g'(V_i)}\frac{S_j^U}{S_i^U} > \frac{S_j^U}{S_i^U}$.

[21] While ex ante prioritarianism applies this function to expected lifetime well-being, ex post prioritarianism applies it to realized lifetime well-being.

$S_i^{EPP} = -g(U_i(A_i - 1)) + \sum_{t=A_i}^{T} \frac{\mu_i(t;A_i)}{p_i(A_i)} g(U_i(t))$, is the difference between individual *i*'s expected transformed lifetime well-being conditional on surviving the current period, and her transformed lifetime well-being if she does not survive. Equivalently, it is the *expected difference* between her transformed lifetime well-being conditional on surviving the current period (given her possible lifespans if she does survive and their probabilities), and her transformed lifetime well-being if she does not survive.

Consider now two individuals, one (*j*) younger than the second (*i*), with a common income and risk profile. The ex post prioritarian SWF places less value on a risk reduction for *i* than for *j* because *i*'s lifetime well-being if she dies during the current period, $U(A_i - 1)$, is greater than *j*'s if she dies during the current period, $U(A_j - 1)$—and thus the very same increase in lifetime well-being for the two individuals translates into a smaller change in transformed lifetime well-being for *i*. Assume that *i*, if she survives the period, has probability μ of realizing a level of lifetime well-being which is ΔU greater than her level of lifetime well-being if she dies now. And assume that the same is true for *j*. The utilitarian value of a chance μ of increment ΔU is the same for both individuals, namely $\mu(\Delta U)$. The ex post prioritarian value of a chance μ for individual *j* of increment ΔU is $\mu\left(g(U(A_j - 1) + \Delta U) - g(U(A_j - 1))\right)$, while for *i* it is $\mu\left(g(U(A_i - 1) + \Delta U) - g(U(A_i - 1))\right)$. The first value is greater than the second by virtue of the strict concavity of $g(\cdot)$, because $U(A_j - 1) < U(A_i - 1)$.

We can again use the Bognar (2015) thought experiment, now to illustrate why ex post prioritarianism satisfies Priority for the Young. Following the example above, we have that utilitarianism is (approximately) indifferent between giving the drug to the younger patient and giving it to the older one, if p_3 is small. $S_j^U \approx S_i^U = 1$. However, $S_i^{EPP} = g(3) - g(2)$, while $S_j^{EPP} \approx g(2) - g(1)$ if p_3 is small.[22] (If the older individual survives the period, her expected transformed lifetime well-being is $g(3)$; her transformed lifetime well-being if she does not is $g(2)$. If the younger individual survives the period, her expected transformed lifetime well-being, with p_3 small, is approximately $g(2)$; her transformed lifetime well-being if she does not is $g(1)$.). By the concavity of $g(\cdot)$, $g(3) - g(2) < g(2) - g(1)$.

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[22] Let G_j and G_i denote each individual's expected transformed lifetime well-being. $G_j = (1-p_2)g(1) + p_2(1-p_3)g(2) + p_2p_3(g(3)$. $G_i = (1 - p_3)g(2) + p_3g(3)$. Then $S_j^{EPP} = \frac{\partial G_j}{\partial p_2} = [g(2) - g(1)] + p_3[g(3) - g(2)]$, while $S_i^{EPP} = \frac{\partial G_i}{\partial p_3} = g(3) - g(2)$.

We saw above that ex ante prioritarianism satisfies not merely Priority for the Young but also the (logically stronger) Ratio Priority for the Young. The same is true for ex post prioritarianism.

Proposition II-B: *The Ex Post Prioritarian SWF displays Ratio Priority for the Young*

$$(S_j^{EPP} / S_i^{EPP}) > (S_j^U / S_i^U)$$

See Appendix for Proof.

VSL. As is well known, the effect of age on VSL is ambiguous. (Aldy and Viscusi, 2007; Hammitt, 2007). VSL reflects the influence of age on the utilitarian SVRR (the numerator of VSL), plus an additional effect: the change in expected marginal utility of consumption (the denominator of VSL) with age.

Let $C_i = p(A_i)\beta^{A_i}u'(y(A_i))$ and similarly for C_j. Then $\text{VSL}_j - \text{VSL}_i$ equals:

$$\frac{1}{C_j}\left(S_j^U - S_i^U\right) + S_i^U\left(\frac{1}{C_j} - \frac{1}{C_i}\right).$$

Note that the expected marginal utility of consumption for the younger individual (C_j) may be larger than for the older individual (C_i)—which can occur if the younger individual has less consumption and/or a greater current survival probability. If $C_j > C_i$, the second term in the above formula for $\text{VSL}_j - \text{VSL}_i$ will be negative even if $S_j^U = S_i^U$. Further, if $S_j^U > S_i^U$, the second term will again be negative if $C_j > C_i$, and its magnitude may exceed that of the first term.

In short, VSL does not satisfy either component of Priority for the Young.

Proposition III-A: VSL does not display Priority for the Young (*either component*)

It is not the case that $S_j^U - S_i^U = 0 \Rightarrow VSL_j - VSL_i > 0$; and it is not the case that $S_j^U - S_i^U > 0 \Rightarrow VSL_j - VSL_i > 0$

In other words: BCA may prefer a risk reduction for the older individual even if the utilitarian gains are equal, indeed even if the utilitarian gain to saving the younger one is larger.

Because Ratio Priority for the Young implies Priority for the Young, the proposition that VSL fails to satisfy Priority for the Young implies (by contraposition) that it fails to satisfy Ratio Priority for the Young.

Proposition III-B VSL does not display Ratio Priority for the Young

It is not the case that $(VSL_j / VSL_i) > (S_j^U / S_i^U)$

A Summary: Table 1 summarizes the results of our analysis of age effects on the utilitarian, ex ante prioritarian, and ex post prioritarian social value of risk reduction (SVRR), and on VSL.

Table 1: SVRRs, VSL and Priority for the Young

	Priority for the Young	Ratio Priority for the Young
Utilitarian SVRR[23]	No	No
Ex Ante Prioritarian SVRR	Yes	Yes
Ex Post Prioritarian SVRR	Yes	Yes
VSL	No	No

One "takeaway" from our analysis is that the concept of prioritarianism, in both its ex ante and ex post variants, provides a rigorous basis for the fair innings concept—as precisely expressed by the properties Priority for the Young and Ratio Priority for the Young. Ex ante prioritarian social welfare, W^{EAP}, is the sum of a strictly increasing and concave transformation function, $g(\cdot)$, applied to each individual's expected lifetime well-being. The ex ante prioritarian SVRR has the priority-for-the-young properties because a given increment in expected lifetime well-being is accorded greater social weight when provided to an individual at a lower level of expected lifetime well-being. The ex post prioritarian SVRR has the priority-for-the-young properties for a different reason. The ex post prioritarian SWF, W^{EPP}, applies the $g(\cdot)$ function to individuals' *realized* (not expected) lifetime well-being levels. As compared to older persons with the same lifetime risk and income profile, younger persons face a lottery over possible realized lifetime well-being levels with a greater chance of lower realized levels, and a smaller chance of higher realized levels. A given increment in realized lifetime well-being is upweighted by W^{EPP}, if it befalls someone at a lower level of realized lifetime well-being.

For those familiar with the literature on prioritarianism under uncertainty, it will be striking that *both* ex ante prioritarianism *and* ex post prioritarianism have the properties Priority for the Young and Ratio Priority for the Young. This literature demonstrates a range of significant axiomatic differences between the two variants of prioritarianism (for example, regarding the ex ante Pareto principles and stochastic dominance axioms) (Adler 2012, 2019).

[23] Because Priority for the Young, and Ratio Priority for the Young are defined as a stronger preference for the young than the utilitarian preference, it is trivial that the utilitarian SVRR doesn't have these properties. By contrast, our results for the ex ante and ex post prioritarian SVRRs and for VSL are not trivial, but require mathematical analysis.

The current analysis shows that, notwithstanding these important differences, the two approaches are alike in supporting the fair innings concept.

Second, the analysis extends an important finding of Adler, Hammitt and Treich (2014). That article, as mentioned, used a single-period model which was not suited to study age effects. What it *did* study was the effect of income and baseline risk on the utilitarian, ex ante prioritarian, and ex post prioritarian SVRRs and on VSL. Here, Adler, Hammitt and Treich (2014) found that BCA and the SWF framework value risk reduction in significantly different ways. The present analysis confirms that finding, now with respect to age effects. By contrast with ex ante and ex post prioritarian SVRRs, VSL does not satisfy Priority for the Young or Ratio Priority for the Young.

III **The Effects of Income and Baseline Risk**

We now consider how SVRR and VSL vary between individuals of the same age, but with different income or risk profiles.

A. *Sensitivity to Income*

We consider first whether SVRR and VSL increase, decrease, or are unchanged by a *single-period* difference in income. Two individuals *i* and *j* are identical in age ($A_i = A_j$), in their risk profiles, and in their income profiles except that $y_j(t) = y_i(t) + \Delta y$, $\Delta y > 0$, for some single period *t*. The period in which the individuals' incomes differ can be the current period, in which case $t = A_i = A_j$, or it can be a past or future period. We determine whether $\text{SVRR}_j > \text{SVRR}_i$, $\text{SVRR}_j = \text{SVRR}_i$, or $\text{SVRR}_j < \text{SVRR}_i$ by examining the sign of $\frac{\partial S_i}{\partial y_i(t)}$.

We find as follows. <u>Utilitarian SVRR</u>. The utilitarian SVRR is independent of a single-period change to past income (since the formula for S_i^U depends only on present and future income), while it is increasing with a single-period change to present or future income. $\frac{\partial S_i^U}{\partial y_i(t)} = \frac{\pi_i(t;A_i)}{p_i(A_i)}\beta^t u'(y_i(t)) > 0 \quad \forall t \geq A_i$. The intuition here is that preventing the current death of an individual with greater present or future income produces a larger gain in expected lifetime well-being.

<u>Ex Ante Prioritarian SVRR.</u> Unlike the utilitarian SVRR, the ex ante prioritarian SVRR is "history dependent"—sensitive to individuals' *past* characteristics. Specifically, the ex ante prioritarian SVRR decreases with a single-period increment to *past* income. $\frac{\partial S_i^{EAP}}{\partial y_i(t)} = g''(V_i) S_i^U \beta^t u'(y_i(t)) < 0 \quad \forall t < A_i$. The intuition is the following: If two individuals are

identical except that the first has lower past income, then preventing either of their deaths in the current period produces the same increment in expected lifetime well-being, but the first individual has a lower baseline level of expected lifetime well-being, thus takes priority under W^{EAP}).

As for a single-period increment to *present* or *future* income: $\frac{\partial S_i^{EAP}}{\partial y_i(t)} = \frac{\pi_i(t;A_i)}{p_i(A_i)} \beta^t u'(y_i(t)) \left(g''(V_i) \sum_{s=A_i}^{T} \pi_i(s;A_i)\beta^s u(y_i(s)) + g'(V_i) \right) \ \forall t \geq A_i$. Thus $\frac{\partial S_i^{EAP}}{\partial y_i(t)}$ is positive/negative/zero iff $-\frac{g''(V_i)}{g'(V_i)}$ is less than/greater than/equal to $\frac{1}{\sum_{s=A_i}^{T} \pi_i(s;A_i)\beta^s u(y_i(s))}$.

Thus we have that the effect of single-period increment to present income or future income on the ex ante prioritarian SVRR is ambiguous.[24] The ex ante prioritarian SVRR may increase, decrease, or even remain constant after that increment.

We can say a bit more about the determinants of the comparative statics. Note that $V_i > \sum_{s=A_i}^{T} \pi_i(s;A_i)\beta^s u(y_i(s))$. Thus, manipulating the above equation, we have the following: if $-\frac{g''(V_i)V_i}{g'(V_i)} \leq 1$, then $\frac{\partial S_i^{EAP}}{\partial y_i(t)} > 0$. In short, if $g(\cdot)$ is such that coefficient of relative risk aversion is always less than or equal to 1, a one-time increase to present or future income will increase the ex ante prioritarian SVRR. The intuition for this result is as follows: The individual with lower present or future income has a lower *level* of expected lifetime well-being, so takes priority under W^{EAP}, but reducing her current risk produces a smaller increase in expected lifetime well-being, and so W^{EAP} prefers reducing the other individual's risk if $g(\cdot)$ is not very concave.

<u>Ex Post Prioritarian SVRR</u>. The ex post prioritarian SVRR, too, is history dependent. Like the ex ante prioritarian SVRR, it decreases with a single-period increment to past income.

$$\frac{\partial S_i^{EPP}}{\partial y_i(t)} = -\beta^t u'(y_i(t)) \left[g'(U_i(A_i - 1)) - \sum_{t=A_i}^{T} \frac{\mu_i(t;A_i)}{p_i(A_i)} g'(U_i(t)) \right] < 0 \quad \forall t < A_i$$

However, unlike its ex ante counterpart, the ex post prioritarian SVRR *always* increases with a change to present or future income.

[24] By "ambiguous" we mean the following. The comparative statics of SVRR or VSL with respect to a parameter of interest (present income, future income, permanent income, age, etc.) are "ambiguous" if (a) we can find some combination of the other parameters and strictly increasing and strictly concave $u(\cdot)$ and $g(\cdot)$ such that SVRR or VSL is increasing in the parameter of interest, and (b) some alternative combination of the other parameters and the same $u(\cdot)$ and $g(\cdot)$ such that SVRR or VSL is decreasing in the parameter of interest.

$$\frac{\partial S_i^{EPP}}{\partial y_i(t)} = \beta^t u'(y_i(t)) \sum_{s=t}^{T} \frac{\mu_i(t;A_i)}{p_i(A_i)} g'(U_i(s)) > 0 \quad \forall t \geq A_i$$

The reason for the divergence between S_i^{EAP} and S_i^{EPP} as regards sensitivity to present or future income is subtle. The social value, as per W^{EPP}, of preventing an individual from dying during the current period is the expected difference between the transformed lifetime well-being of the longer lives she might lead were she to survive the current period, and the transformed lifetime well-being of her life were it to end now. Increasing present or future income *increases* that expected difference in transformed lifetime well-being.

VSL Because VSL_i equals S_i^U divided by the expected marginal utility of *i*'s current income, the comparative statics of VSL with respect to past and future income are the same as for the utilitarian SVRR. Further, because the utilitarian SVRR (the numerator of VSL) is increasing in current income, and the denominator is decreasing, VSL also increases in current income—indeed more quickly than the utilitarian SVRR.

Next, we consider the effect on SVRR and VSL of a change in *permanent income*. Two individuals *i* and *j* are identical except that $y_j(t) = y_i(t) + \Delta y$ *for all periods*. We find as follows; see Appendix for proofs. Utilitarian SVRR. Increasing in permanent income. Ex ante Prioritarian SVRR. Ambiguous. Increasing in permanent income if $g(\cdot)$ is not too concave. Ex post prioritarian SVRR. Ambiguous. VSL. Increasing in permanent income.

B. *Sensitivity to Baseline Risk*

We consider first whether SVRR and VSL increase, decrease, or are unchanged by a *one-period* difference in survival probability. Two individuals *i* and *j* are identical except that $p_j(t) = p_i(t) + \Delta q$, $\Delta q > 0$, for some single period *t*. We determine whether $\text{SVRR}_j > \text{SVRR}_i$, $\text{SVRR}_j = \text{SVRR}_i$, or $\text{SVRR}_j < \text{SVRR}_i$ by examining the sign of $\frac{\partial S_i}{\partial p_i(t)}$.

Neither SVRR nor VSL is sensitive to change in past survival probabilities—as is obvious—and so we discuss only the results for the case of a one-period change to present survival probability ($t = A_i = A_j$) or future survival probability.

We find as follows. Utilitarian SVRR. The utilitarian SVRR is insensitive to a one-period change in present survival probability. (Note that the formula for S_i^U is equivalent to $\beta^{A_i} u(y_i(A_i)) + \sum_{t=A_i+1}^{T} \beta^t u(y_i(t)) \prod_{s=A_i+1}^{t} p_i(s)$, so is not a function of $p_i(A_i)$.) It is increasing in a one-period change in future survival probability. $\frac{\partial S_i^U}{\partial p_i(t)} = \sum_{s=t}^{T} \frac{\pi_i(s;A_i)}{p_i(A_i)} \beta^s u(y_i(s)) \frac{1}{p_i(t)} > 0 \quad \forall t > A_i$.

The intuition is that preventing a current death produces a bigger increase in expected lifetime well-being if the individual has a lower chance of dying in future periods.

<u>Ex Ante Prioritarian SVRR.</u> The ex ante prioritarian SVRR is decreasing in current survival probability. (Note that $\dfrac{\partial S_i^{EAP}}{\partial p_i(t)} = g''(V_i)\left(S_i^U\right)^2 < 0$.) The effect of a one-period change in future survival probability is ambiguous.

$\dfrac{\partial S_i^{EAP}}{\partial p_i(t)} = \sum_{s=t}^{T} \dfrac{\pi_i(s;A_i)}{p_i(A_i)} \beta^s u(y_i(s)) \dfrac{1}{p_i(t)} \left[g'(V_i) + g''(V_i) \sum_{s=A_i}^{T} \pi_i(s;A_i)\beta^s u(y_i(s)) \right]$. This is greater than/equal to/less than 0 iff $\dfrac{-g''(V_i)}{g'(V_i)}$ is less than/equal to/greater than $\dfrac{1}{\sum_{s=A_i}^{T} \pi_i(s;A_i)\beta^s u(y_i(s))}$.

Observing again that $V_i > \sum_{s=A_i}^{T} \pi_i(s;A_i)\beta^s u(y_i(s))$, we have a parallel result here as for the effect of present income, future income, and permanent income on the ex ante prioritarian SVRR (see above): a one-period change to future survival probability will increase the ex ante prioritarian SVRR if the coefficient of relative risk aversion for $g(.\cdot)$ is uniformly less than or equal to one.

The intuitions for these results are that an increase in current survival probability increases the individual's expected lifetime well-being, hence gives her lower priority as per W^{EAP}; while an increase in future survival probability has competing effects, both increasing the change to expected lifetime well-being of preventing the individual's current death, and increasing her level of expected lifetime well-being, with the first effect predominating if $g(\cdot)$ is not too concave.

<u>Ex Post Prioritarian SVRR</u>. The ex post prioritarian SVRR is insensitive to a change to current survival probability. (Note that $\mu_i(t;A_i) = (1 - p_i(t+1)) \prod_{s=A_i}^{t} p_i(s)$ for $t \geq A_i$, hence $p_i(A_i)$ drops out of the formula for S_i^{EPP} .) It is increasing in a one-period change to future survival probability. $\dfrac{\partial S_i^{EPP}}{\partial p_i(t)} = -\dfrac{\pi_i(t-1;A_i)}{p_i(A_i)} g(U_i(t-1)) + \sum_{s=t}^{T} \dfrac{\mu_i(s;A_i)}{p_i(A_i)p_i(t)} g(U_i(s)) > 0 \quad \forall t > A_i$.

<u>VSL.</u> VSL increases with a one-period change to future survival probabilities, and decreases with a change to current survival probability. This follows immediately from the results for the utilitarian SVRR—since VSL is the utilitarian SVRR divided by a denominator term that does not depend upon future survival probability, and increases as current survival probability does.

Next, we consider the effect on SVRR and VSL of a *permanent* difference in survival probability. Two individuals *i* and *j* are identical except that $p_j(t) = p_i(t) + \Delta q$, $\Delta q > 0$, for *every* period *t*. Our results are as follows; see Appendix for proofs. Utilitarian SVRR. Increasing with a permanent increase in survival probability. Ex ante Prioritarian SVRR. Ambiguous. Increasing with a permanent increase in survival probability if $g(\cdot)$ is not too concave. Ex Post Prioritarian SVRR. Increasing with a permanent increase in survival probability. VSL. Ambiguous.

C. *A Summary*

The comparative statics of the SVRRs and VSL with respect to income and survival probability are summarized in Table 2 immediately below:

Table 2: Comparative Statics of SVRRs and VSL with respect to Income and Survival Probability

	Income: Single-period difference	**Income: permanent difference**	**Survival probability: single-period difference**	**Survival probability: permanent difference**
Utilitarian SVRR	Past period: *Independent* Current period: *Increasing* Future period: *Increasing*	*Increasing*	Current period:*Independent* Future period: *Increasing*	*Increasing*
Ex Ante Prioritarian SVRR	Past period: *Decreasing* Current period: *Ambiguous* Future period: *Ambiguous* Note: The SVRR increases with a one-period increment to current or future income if $g(\cdot)$ is not too concave (coefficient of relative risk aversion ≤ 1)	*Ambiguous* Note: The SVRR increases with an increment to permanent income if $g(\cdot)$ is not too concave	Current period: *Decreasing* Future period: *Ambiguous* *Note*: The SVRR increases with a one-period increment to future survival probability if $g(\cdot)$ is not too concave (coefficient of relative risk aversion ≤ 1)	*Ambiguous* *Note*: The SVRR increases with a permanent change to survival probability if $g(\cdot)$ is not too concave
Ex Post Prioritarian SVRR	Past period: *Decreasing* Current period: *Increasing* Future period: *Increasing*	*Ambiguous*	Current period:*Independent* Future period: *Increasing*	*Increasing*
VSL	Past period: *Independent* Current period: *Increasing* Future period: *Increasing*	*Increasing*	Current period: *Decreasing* Future period: *Increasing*	*Ambiguous*

Much about this table is noteworthy. First, timing matters. Whether individuals who differ with respect to income, or with respect to survival probability, have divergent SVRRs or VSLs depends upon whether the income or survival probability difference occurs in the past, the present, or the future. Consider the columns for "income: single-period difference" and "survival probability: single-period difference." The following is true for each of the three

SVRRs and for VSL: (1) its comparative statics (independent, increasing, decreasing, or ambiguous) are *not* the same for past, current, and future-period differences in income, and moreover (2) its comparative statics are *not* the same for current and future-period differences in survival probability.

Second, the prioritarian SVRRs, ex ante and ex post, are history-dependent—specifically, with respect to income. Each is decreasing with a one-period change to past income—by contrast with the utilitarian SVRR and VSL, which are independent of past income.

Third, this table confirms a key finding of Adler, Hammitt, and Treich (2014), using a simpler single-period model: the manner in which VSL values risk reduction is *not* robust to a change in social evaluation framework. VSL differs, in some significant way, from *each* of the SVRRs. VSL and the utilitarian SVRR have the same comparative statics with respect to income, but not survival probability. VSL and the prioritarian SVRRs have different comparative statics with respect to both income and survival probability.[25]

Fourth, the choice *within* the prioritarian family, between ex ante and ex prioritarian approach, is seen to be significant. The ex ante prioritarian SVRR is decreasing in current survival probability and ambiguous with respect to future survival probability, while the ex post prioritarian SVRR is independent of current survival probability and increasing in future survival probability. Both SVRRs are decreasing in past income, but: the ex ante prioritarian SVRR is ambiguous with respect to current and future income, while the ex post prioritarian SVRR is *increasing* with current and future income.[26]

This table, of course, concerns comparative statics: the direction of impact on VSL and the SVRRs of changes in risk and survival probability. It doesn't show the magnitude of impact—another type of difference between the various approaches. This difference will emerge in the following Part, where we empirically estimate VSL and the SVRRs for the US population.

IV. SVRRs and VSL for the US Population

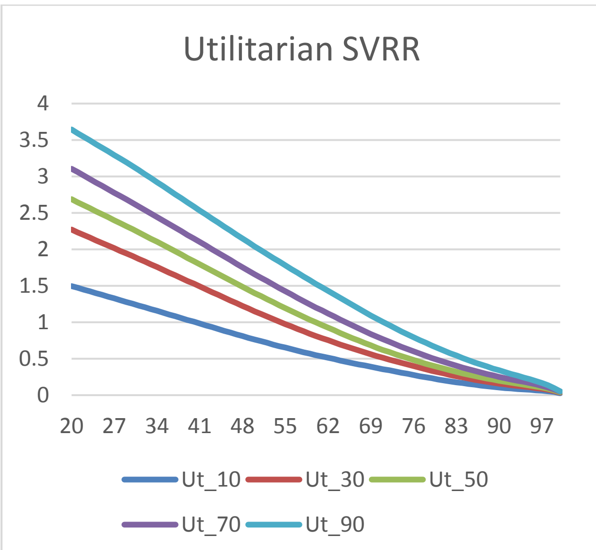
In this Part, we illustrate the SVRR and VSL concepts, and estimate their relative magnitudes, by calculating SVRR and VSL for cohorts of individuals characterized by varying risk profiles, income profiles and ages. The income and survival data for this exercise derive from the actual U.S. population. The U.S. Census Bureau collects data on the income distribution by age range. We used this to estimate the percentiles of the income distribution for each age. Assuming zero mobility (movement across percentiles), we determined a lifetime

[25] Except that, if g(·) has a coefficient of relative risk aversion less than or equal to one, the comparative statics of ex ante prioritarianism with respect to survival probability are the same as BCA.
[26] See Adler and Treich (2017), finding significant differences between ex ante and ex post prioritarianism in a model of intergenerational consumption allocation.

income profile for each percentile. The risk profile was based upon the actual U.S. population survival curve, and then adjusted by income percentile to reflect income differences in life expectancy.[27]

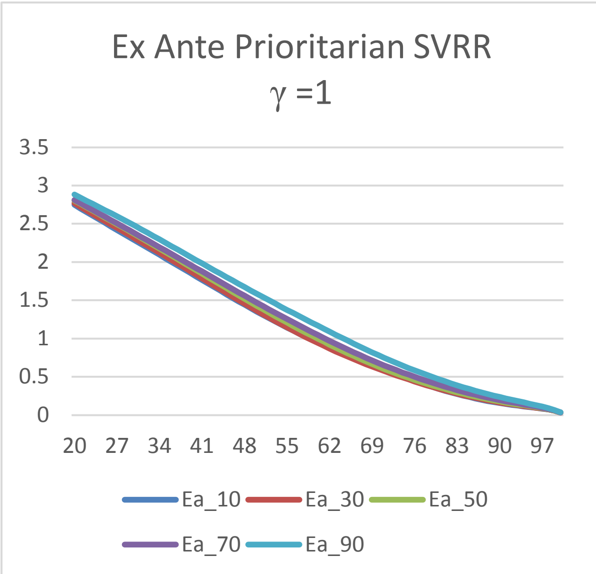
As per the analysis in Parts II and III, we assume that an individual's annual consumption in a given year is just her income. A logarithmic utility function was used. The utility discount rate was set to 0. For the prioritarian SVRRs, we used an "Atkinson" (isoelastic) SWF with both a moderate inequality-aversion parameter (γ =1) and higher such parameter (γ = 2). This yields four different prioritarian SVRRs (namely ex post or ex ante, with γ =1 or 2).

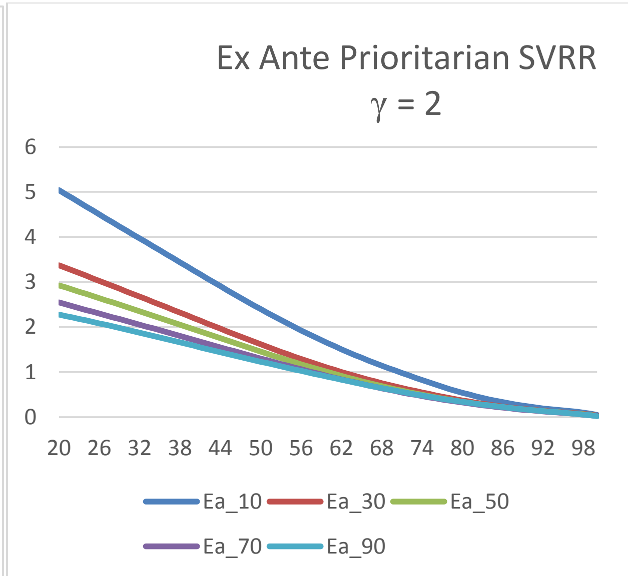
Figure 1: SVRRs and VSL by Age and Income Percentile for the U.S. Population

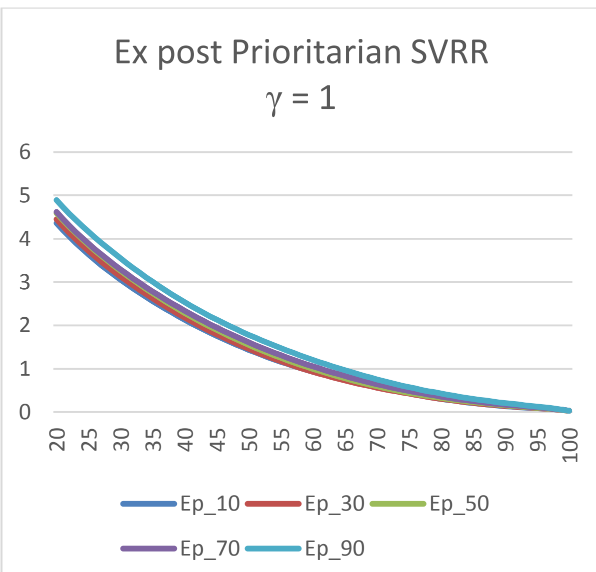


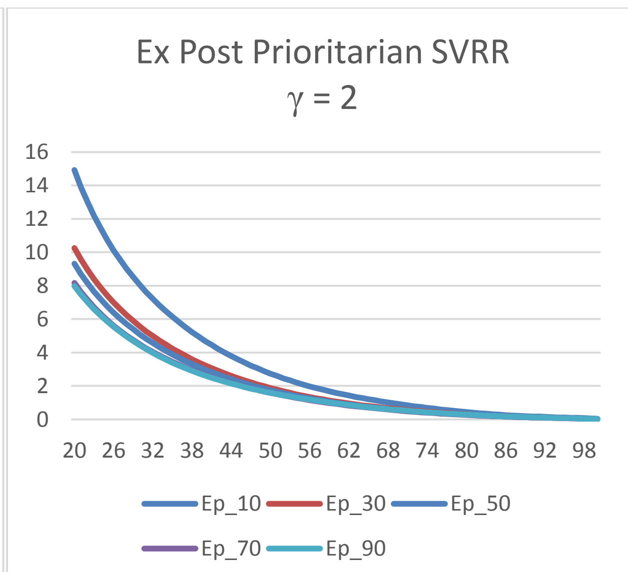
[27] Specifically, data on the U.S. income distribution was taken from the Current Population Survey (CPS) income tables. See https://www.census.gov/data/tables/time-series/demo/income-poverty/cps-pinc.html. We used the table PINC_01_1_1_1 (total work experience, both sexes, all races) for 2016. A fourth degree polynomial in age was fitted to the data. We assume that income for ages 20-25 is the same as for ages 25-30 and that income for ages 75-100 is the same as for ages 70-74. The U.S. population survival curve was taken from the life tables compiled by the Centers for Disease Control and Prevention. See https://www.cdc.gov/nchs/products/life_tables.htm. We used the 2014 life tables (National Vital Statistics Reports, vol. 66, no. 4, August 24, 2017).

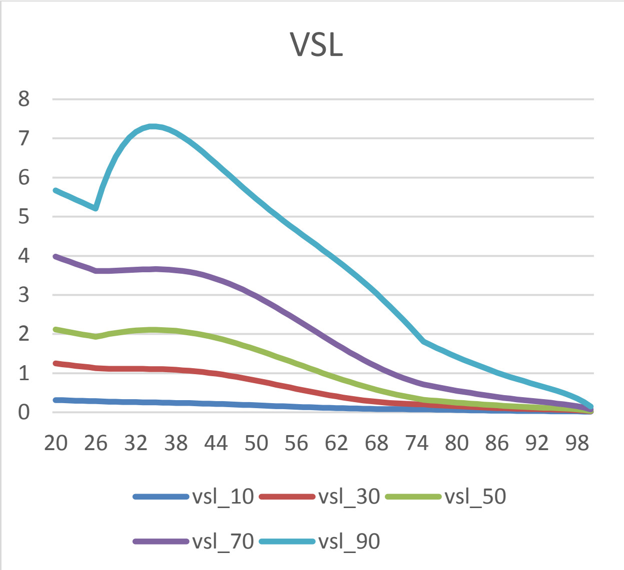
In the simulation exercise, we calculated SVRR and VSL, by age, for the 10^{th}, 30^{th}, 50^{th}, 70^{th}, and 90^{th} percentiles of the U.S. income distribution. In order to do so, we adjusted the U.S. population survival curve, as taken from the life table, by multiplying the annual mortality risk for each age by a scaling factor to reflect the individual's income. These scaling factors were, respectively, 1.5, 1.2, 1, 0.9, and 0.7 for, respectively, the 10^{th}, 30^{th}, 50^{th}, 70^{th}, and 90^{th} percentiles of income. The scaling factors were taken from Adler (2017, appendix C), who in turn estimated them to match findings by Chetty et al. (2016) regarding differences in life expectancy across income classes.


Ex Ante Prioritarian SVRR
γ =1
3.5
3
2.5
2
1.5
1
0.5
0
20 27 34 41 48 55 62 69 76 83 90 97
Ea_10 Ea_30 Ea_50
Ea_70 Ea_90


Ex Ante Prioritarian SVRR
γ = 2
6
5
4
3
2
1
0
20 26 32 38 44 50 56 62 68 74 80 86 92 98
Ea_10 Ea_30 Ea_50
Ea_70 Ea_90


Ex post Prioritarian SVRR
γ = 1
6
5
4
3
2
1
0
20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 95 100
Ep_10 Ep_30 Ep_50
Ep_70 Ep_90


Ex Post Prioritarian SVRR
γ = 2
16
14
12
10
8
6
4
2
0
20 26 32 38 44 50 56 62 68 74 80 86 92 98
Ep_10 Ep_30 Ep_50
Ep_70 Ep_90



The panels in Figure 1 display the utilitarian SVRR, the prioritarian SVRRs, and VSL as a function of age, for various percentiles of the income distribution. The results are normalized so that 1 represents the SVRR or VSL for a 60 year old, median income individual.

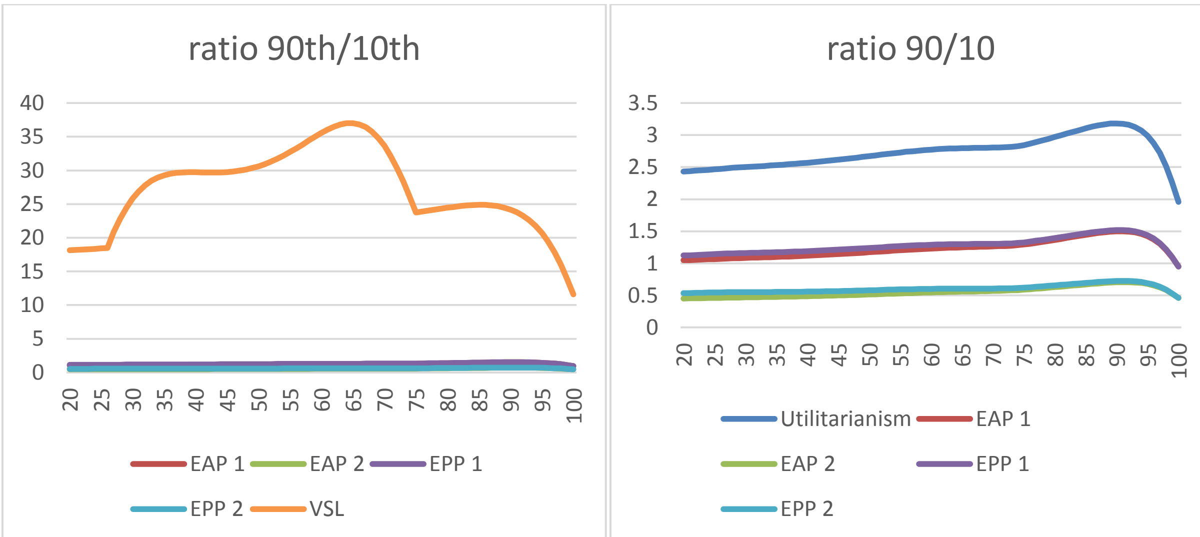
As the panels show, all the SVRRs decrease with age (even though this is not theoretically required—see Part II). The utilitarian SVRR and ex ante prioritarian SVRR have a similar degree of priority for the young at median income. Shifting from ex ante to ex post prioritarianism increases priority for the young.

The utilitarian SVRR increases with income: at every age, individuals in higher income percentiles have larger SVRRs. This is reversed for the prioritarian SVRRs with $\gamma = 2$; at every age, SVRR decreases with income. $\gamma = 1$ is an intermediate case, in which the utilitarian preference for income is almost neutralized but not reversed. Note here that the lines displaying the ex ante and ex post prioritarian SVRR as a function of age are virtually the same for all income percentiles. Thus the prioritarian SVRRs with moderate inequality aversion conform to lay moral judgments regarding lifesaving policies, namely that the young should take priority but income should make no difference.

VSL decreases with age for individuals above 40. At earlier ages, for some income percentiles, VSL displays the inverted U ("hump") shape often described in the literature.

The most striking difference between VSL and all the SVRRs concerns income effects: VSL increases with income at all ages, and much more steeply than even the utilitarian SVRR. This can be observed in Figure 1, and is displayed very clearly in Figure 2, which shows the ratio between SVRR or VSL at the 90th and 10th income percentiles as a function of age. That ratio is between 0.5 and 3 for all the SVRRs, while generally exceeds 20 for VSL.

Figure 2: Ratio of SVRRs and VSL at 90th Percentile Income to SVRR and VSL at 10th Percentile Income by Age

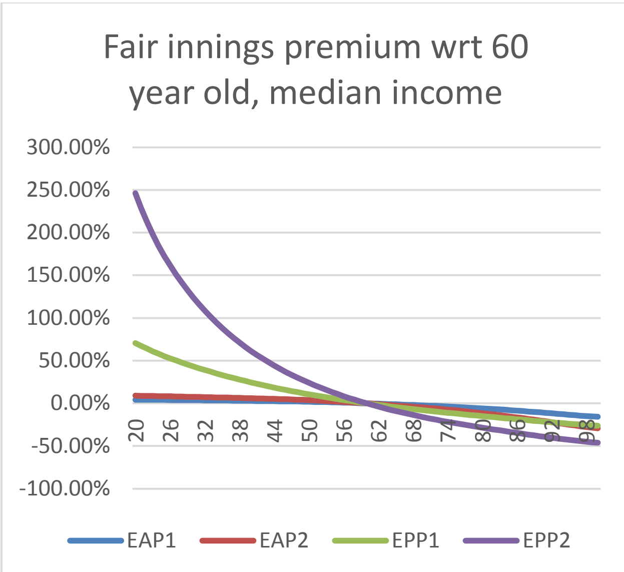


Our exercise here also sheds light on the U.S. government's practice of employing a single, population-average VSL, to value risk reduction (Robinson, 2007). Such an approach is not only inconsistent with the theory of BCA—as Figure 1 shows, VSL varies by age and income—but also with the SWF framework. All of the SVRRs vary, at least, by age, and some by both age and income.

Finally (see Figure 3) we estimate a "fair innings premium." Recall that both ex ante prioritarian and ex post prioritarian SVRRs have the property of Ratio Priority for the Young: the ratio of prioritarian SVRRs, between a younger and older person with the same lifetime income and risk profile, is always larger than the ratio of utilitarian SVRRs. Figure 3 shows the magnitude of this difference in ratios. For individuals of the median income profile and associated risk profile, we calculate the percentage by which the ratio between the ex ante or ex post prioritarian SVRR of an individual of each age and a 60-year-old's ex ante or ex post prioritarian SVRR exceeds the utilitarian young-to-old ratio.[28]

[28] That is, we calculate $[(S_j^{EAP} / S_{60}^{EAP}) / (S_j^{U} / S_{60}^{U})] - 1$ and $[(S_j^{EPP} / S_{60}^{EPP}) / (S_j^{U} / S_{60}^{U})] - 1$ for each age *j*.

Figure 3: Fair Innings Premium



V. Endogenous Consumption

Our analysis of the social value of risk reduction (Parts I-III), and empirical exercise (Part IV), assumed exogenous ("myopic") consumption. An individual's income profile specifies what she earns in each period if she survives to its end. If she does indeed survive a given period, her consumption in that period is equal to her income. (If she doesn't survive, she is not alive to earn income or engage in consumption with respect to that period.)

The natural next step is to consider endogenous consumption. Individuals can shift resources between periods, by saving or borrowing. An individual's consumption during a period is not necessarily equal to her income, but instead is the upshot of her income profile plus savings and borrowing.

Modelling endogenous consumption depends, first, on how capital markets are modelled. See Shepard and Zeckauser (1984); Cropper and Sussman (1990). One possibility is that actuarially fair annuities are available. An individual can invest wealth at a rate of return that reflects both the market rate and her probability of survival. (For example, if individual *i* make a one-period investment of w at the beginning of period t, with r the market rate of interest, she receives $\frac{w(1+r)}{p_i(t)}$ at the end of the period if she survives, and nothing if she does not.) She can also borrow against future income, again at a rate that reflects both the market rate and her survival probability.

A different possibility, so-called "Robinson Crusoe," assumes that the capital markets offer only "plain vanilla" opportunities to save and borrow, rather than contractual terms and prices that reflect individual survival probabilities. An individual who invests w at the beginning of period t receives $w(1+r)$ at the end of the period—specifically, this amount is paid to her individually if she survives the period, and to her estate if she does not (so that the ex ante value of the payment to the individual depends upon her "bequest function"). An individual can borrow against future income only if the income is certain to be earned (and thus an individual who earns income $y_i(t)$ only if she survives to the end of t, and who finds herself at the beginning of period $t^* \leq t$, cannot borrow against that conditional income if $\pi_i(t; t^*) < 1$.

Second, modelling endogenous consumption depends on whether the government's policy intervention is treated as a "surprise" or, instead, as an event whose possible occurrence individuals previously anticipated. Individual i is currently at the beginning of period A_i of her life. Government now enacts a risk-reduction policy such that individual's survival probability for this period changes from $p_i(A_i)$ to $p_i(A_i + \Delta p_i)$. To handle the case of previously anticipated policies, we might imagine that individuals assign subjective probabilities to the various possible government interventions. Specifically, let $\theta_{i,t}^{\Delta p,t^*}$ denote the probability that individual i, at the beginning of period t, assigns to government in period t^* changing her survival probability by Δp. The individual saves and borrows in light of her income profile, risk profile, *and* the probabilities of policy change (the $\theta_{i,t}^{\Delta p,t^*}$ values). The "surprise" case is just the case in which $\theta_{i,t}^{\Delta p,t^*}$ values are uniformly zero.

Note that the surprise/anticipation question is orthogonal to that of fair annuities/Robinson Crusoe. Intersecting these two dimensions yields four different ways to model endogenous consumption. See Table 3.

Table 3: Modelling Endogenous Consumption

Market structure: Fair annuities Policy anticipated? No (a "surprise" policy)	Market structure: Fair annuities Policy anticipated? Yes
Market structure: Robinson Crusoe Policy anticipated? No (a "surprise' policy)	Market structure: Robinson Crusoe Policy anticipated? Yes

A full treatment of the social value of risk reduction with endogenous consumption requires considering each of the four boxes in Table 3 (and perhaps other possibilities as well). Such an analysis is well beyond what can be accomplished in the current Article. Instead, as an initial step in the investigation of endogenous consumptions, and an initial robustness check of our main results, we consider the upper left hand box in Table 3: actuarially fair annuities and a "surprise policy." For short, we'll refer to this case as the "annuities/surprise" case.

We conceptualize it as follows. At birth (the beginning of period 1), individual *i* contracts with a risk-neutral investor. Individual *i* assigns the investor rights to the stream of future conditional income amounts $y_i(1), \ldots, y_i(T)$ that *i* will receive at the end of each period if she survives that period. With a market interest rate of r, the investor is willing to pay $w_i = \sum_{t=1}^{T} \frac{\pi_i(t) y_i(t)}{(1+r)^t}$. w_i is individual *i*'s wealth at birth. The individual can then invest this wealth, so that she receives consumption $c_i(t)$ at the end of period t if she survives to the end of t (otherwise nothing). A risk-neutral investor who agrees to supply this conditional consumption will require a payment at birth from individual *i* in the amount of $\frac{\pi_i(t) c_i(t)}{(1+r)^t}$.

We assume that individual *i*, at birth, invests her wealth w_i so as to maximize expected lifetime well-being. In other words, she chooses a lifetime consumption plan $(c_i(1), \ldots c_i(T))$, so as to maximize expected lifetime well-being $\sum_{t=1}^{T} \pi_i(t) \beta^t u(c_i(t))$, under the constraint that $\sum_{t=1}^{T} \frac{\pi_i(t) c_i(t)}{(1+r)^t} = \sum_{t=1}^{T} \frac{\pi_i(t) y_i(t)}{(1+r)^t}$. Individual *i*'s consumption at the end of t, if she indeed survives to this point, is equal to her planned-at-birth consumption $c_i(t)$, which the investor pays out according to the terms of the at-birth contract.

Because of the availability of actuarially fair annuities, the individual's consumption is "smoothed" relative to the myopic case: it increases monotonically, decreases monotonically, or remains flat regardless of the shape of the income profile. Let λ_i be the Lagrangian multiplier associated with the constraint immediately above. Recall also that $\beta = 1/(1 + \varphi)$, with $\varphi \geq 0$ the utility discount rate. For each $t \geq 1$, the first order condition for a lifetime consumption plan that maximizes expected lifetime well-being is as follows: $\frac{1}{(1+\varphi)^t} u'(c_i(t)) = \lambda_i \frac{1}{(1+r)^t}$. From this we derive the Euler condition: $\frac{u'(c_i(t))}{u'(c_i(t+1))} = \frac{(1+r)}{(1+\varphi)}$. Consumption increases/decreases/remains constant with age iff r, the market interest rate, is greater than/less than/equal to the utility discount rate.

We now consider the social value of risk reduction and VSL. The utilitarian SVRR, ex ante prioritarian SVRR, ex post prioritarian SVRR and VSL for the annuities/surprise case are each calculated using the same formula as for the myopic case (as given in Part I), except that period consumption in these formulas is equal to planned consumption as per the lifetime consumption plan (denoted as $c_i(t)$), rather than period income.

Annuities/surprise SVRRs

$$V_i = \sum_{t=1}^{A_i-1} \beta^t u(c_i(t)) + \sum_{t=A_i}^{T} \pi_i(t;A_i)\beta^t u(c_i(t))$$

$$U_i(s) = \sum_{t=1}^{s} \beta^t u(c_i(t))$$

$$S_i^U = \sum_{t=A_i}^{T} \frac{\pi_i(t;A_i)}{p_i(A_i)} \beta^t u(c_i(t))$$

$$S_i^{EAP} = g'(V_i) S_i^U$$

$$S_i^{EPP} = -g(U_i(A_i-1)) + \sum_{t=A_i}^{T} \frac{\mu_i(t;A_i)}{p_i(A_i)} g(U_i(t))$$

$$VSL_i = \frac{S_i^U}{p_i(A_i)u'(c_i(A_i))\beta^{A_i}}$$

Note that the introduction of the government policy, changing *i*'s current survival probability from $p_i(A_i)$ to $p_i(A_i + \Delta p_i)$, does not lead to a change in her lifetime consumption plan. Since *i* at birth does not anticipate this policy (or other changes in her risk profile), nor do other participants in the capital market, she and her counterparties are willing to sign a binding at-birth contract, based on initial (pre-policy) survival probabilities, in which she promises to trade future income earned conditional on survival for wealth up-front, and wealth up-front for future consumption conditional on survival.

It follows immediately that the ex ante prioritarian and ex post prioritarian SWFs display priority for the young and indeed ratio priority for the young in the annuities/surprise case—just as they do in the myopic case. Recall (see Part II) that the analysis of age effects involves two individuals *i* and *j* with identical risk and income profiles, but the first older than the second ($A_i > A_j$). In the myopic case, we show that priority for the young and ratio priority for the young hold true *for every* such A_i and A_j—for any common risk profile of the two individuals and any common income profile (i.e., any common consumption profile, since income = consumption in this case.) Consider now that, in the annuities/surprise case, *i* and *j* formulate the same lifetime consumption plan at birth. Thus the annuities/surprise $SVRR_i$ and $SVRR_j$ are equal, respectively, to the myopic $SVRR_i$ and $SVRR_j$ for individuals with an income profile equal to that lifetime consumption plan. But since myopic priority for the young and myopic ratio priority for the young hold true *for any* income profile, they hold true in this case. By extension, then, annuities/surprise priority for the young and ratio priority for the young hold true by virtue of the fact that *i* and *j* have a common risk profile and a common lifetime consumption plan, *whatever that plan may happen to be*.

Conversely, the annuities/surprise VSL does not display priority for the young or ratio priority for the young. In Part II, we showed that myopic VSL does not display these properties because expected marginal utility of consumption can decrease with age. Clearly, the expected marginal utility of consumption can decrease with age in the annuities/surprise case.[29]

In short, all of our results regarding priority for the young in the myopic case, summarized in Table 1, carry over to the annuities/surprise case. See Table 4.

Table 4: Annuities/Surprise and Priority for the Young

	Priority for the Young	Ratio Priority for the Young
Utilitarian SVRR	No	No
Ex Ante Prioritarian SVRR	Yes	Yes
Ex Post Prioritarian SVRR	Yes	Yes
VSL	No	No

Next, we consider the effect of income and baseline risk on the annuities/surprise SVRRs and VSL. Recall that our analysis for the myopic case considered both single-period differences in income (with the single period being past, present or future), and a permanent difference in income. In the annuities/surprise case, there is no need to differentiate these situations. If *i* and *j* are the same age and have the same risk profile, and also have the same income profile except that *j* has more income in one or more periods, then *j* has greater wealth at birth ($w_j > w_i$), regardless of whether the periods in which income differs are past, present and future. In turn, it is wealth at birth and the risk profile that determines the individual's consumption plan, not the precise timing of incomes.

In short, the comparative statics of SVRR and VSL with respect to current income, past income, future income, and permanent income should be the same. We therefore consider only permanent income. Our results are displayed in Table 5; see Appendix for derivations.

[29] For example, assume that survival probabilities are non-increasing with age and that the market rate of interest exceeds the utility discount rate, so that planned consumption increases with age.

Table 5: Comparative Statics of Annuities/Surprise SVRRs and VSL with respect to Income and Survival Probability

	Income: permanent difference	**Survival probability: single-period difference**	**Survival probability: permanent difference**
Utilitarian SVRR	*Increasing*	Current period: *Ambiguous* Future period: *Ambiguous*	*Ambiguous*
Ex Ante Prioritarian SVRR	*Ambiguous* Note: The SVRR increases with an increment to permanent income if $g(\cdot)$ is not too concave	Current period: *Ambiguous* Future period: *Ambiguous*	*Ambiguous*
Ex Post Prioritarian SVRR	*Ambiguous*	Current period: *Ambiguous* Future period: *Ambiguous*	*Ambiguous*
VSL	*Increasing*	Current period: *Ambiguous* Future period: *Ambiguous*	*Ambiguous*

Comparing Table 5 with Table 2, it can be seen that each of the annuities/surprise SVRRs has the same comparative statics with respect to permanent income as in the myopic case, as does VSL. This can be explained as follows. In the myopic case, if *i* and *j* have the same risk profile but *j*'s income is greater than *i*'s in every period, then *j*'s consumption is greater in every period. In the annuities/surprise case, the same occurs: if *i* and *j* have the same risk profile but *j*'s income is greater than *i*'s in every period, then $w_j > w_i$, and so in turn *j*'s lifetime consumption plan will have greater consumption in every period than *i*'s.

Notably, however, comparative statics with respect to survival probability do *not* carry over from the myopic to the annuities/surprise case. As summarized in Table 2, the effect of a single-period or permanent change in survival probability on myopic SVRR or VSL is generally *not* ambiguous. Most of the entries in the right two columns of table 2 are "decreasing," "increasing," or "independent." By contrast, the effect of a single-period or permanent change in survival probability on annuities/surprise SVRR or VSL is pervasively ambiguous. "Ambiguous" is the entry in every cell in the right two columns of Table 5.

What explains this difference? In the myopic case, two individuals of the same age with different risk profiles and the same income profile have the same consumption profile. In the annuities/surprise case, two individuals of the same age with different risk profiles and the same income profile will generally have *different* consumption profiles (since their lifetime consumption plans will differ). This extra dimension of difference explains why annuities/surprise comparative statics with respect to survival probability are pervasively ambiguous even though myopic comparative statics are not.

In summary: this Part has considered one variant of endogenous consumption, in which (1) individuals have access to actuarially fair annuities, and (2) do not anticipate government's risk-reduction policy. A central result of our earlier analysis—namely that the myopic ex ante prioritarian and ex post prioritarian SVRR display priority for the young and ratio priority for the young—carries over to this case. At least on this modelling of endogenous consumption, the proposition that prioritarianism always accords more priority to the young than utilitarianism is robust to whether consumption is exogenous or endogenous. Comparative statics with respect to permanent income are also robust. Whether these robustness findings extend to different modellings of endogenous consumption—Robinson Crusoe and/or a policy anticipated in advance—is a matter for future research.

VI. **Conclusion**

This Article has undertaken an extensive analysis of the social value of risk reduction (SVRR). SVRR is the linchpin concept for applying a social welfare function (SWF) to one major policy domain: fatality risk regulation. SVRR$_i$ is defined as $\frac{\partial W}{\partial p_i}$. It is the marginal change in social value, as determined by SWF W, per unit of risk reduction for individual i. We investigate SVRR for three major SWFs (utilitarian, ex ante prioritarian, and ex post prioritarian), using a lifetime model that allows us to differentiate individuals by age, lifetime risk profile, and lifetime income profile.

Economists have intensively investigated the SWF framework in certain policy arenas, such as taxation and climate policy. However, the application of SWFs to the domain of risk regulation has been little studied. Our analysis demonstrates, in detail, how the social weight given to a reduction in a given individual's fatality risk depends upon the functional form of the

SWF. In their comparative statics with respect to income and baseline risk, the three SVRRs differ significantly from each other. At the same time, each of the SVRRs deviates substantially from VSL—the valuation concept for risk reduction that is used by benefit-cost analysis (BCA), currently the dominant methodology in governmental practice and in applied economics. In a nutshell, then, our analysis shows that a rigorous intellectual apparatus with deep roots in welfare economics—the SWF framework—values individual risk reduction in a manner quite different from BCA. In an empirical exercise, we confirm this finding.

Moreover, we show that the "fair innings" concept, popular in the public health literature, has a firm basis *within* welfare economics. Specifically, both the ex ante prioritarian and ex post prioritarian SVRRs satisfy axioms of Priority for the Young and Ratio Priority for the Young. In effect, a young person takes priority over an older person with respect to risk reduction even when the gains in expected lifetime well-being are equal. (By contrast, BCA does not support the fair innings concept; a younger individual may have a *lower* VSL even when the gains to expected lifetime well-being are equal.) As far as we're aware, this Article is the first to provide a rigorous interpretation of "fair innings" using the tools of economics.

Appendix

This Appendix derives formulas/statements presented in the text, where that derivation is not mathematically or logically obvious. The last appendix section, Section D., derives the comparative statics results for the annuities/surprise case, as presented in Part V. Sections A, B and C, which provide backup for earlier parts of the Article, all assume myopic consumption.

A. *SVRR, VSL and Age* (*Myopic Case*)

In what follows, j is the younger person ($A_j < A_i$). Because the two individuals have common risk and income profiles, individual subscripts are dropped where possible (e.g., $p(t) = p_i(t) = p_j(t)$).

Utilitarian SVRR.

(a) The derivation of the formula stated in the text for $S_j^U - S_i^U$ is as follows.

$$S_j^U - S_i^U = \sum_{t=A_j}^{A_i-1} \frac{\pi(t;A_j)}{p(A_j)} \beta^t u(y(t)) + \sum_{t=A_i}^{T} \left(\frac{\pi(t;A_j)}{p(A_j)} - \frac{\pi(t;A_i)}{p(A_i)} \right) \beta^t u(y(t))$$

The first term on the RHS is $\sum_{t=A_j}^{A_i-1} \pi(t;A_j+1)\beta^t u(y(t))$, while the second term is equal to

$$\sum_{t=A_i}^{T}\left(\prod_{s=A_j+1}^{t} p(s)-\prod_{s=A_i+1}^{t} p(s)\right)\beta^t u(y(t))=\sum_{t=A_i}^{T}\left(\pi(A_i;A_j+1)\pi(t;A_i+1)-\pi(t;A_i+1)\right)\beta^t u(y(t))=$$

$$\left(\pi(A_i;A_j+1)-1\right)\sum_{t=A_i}^{T}\pi(t;A_i+1)\beta^t u(y(t))\ .$$[30]

(b) We note in the text that if income is constant and survival probabilities do not increase with age, the utilitarian SVRR decreases with age. Let u be the (positive) utility of the constant income. Note that $S_i^U = u\beta^{A_i} + u\left(\sum_{t=A_i+1}^{T}\beta^t\prod_{s=A_i+1}^{t}p(s)\right)$.

$S_j^U = u\beta^{A_j} + u\left(\sum_{t=A_j+1}^{T-(A_i-A_j)}\beta^t\prod_{s=A_j+1}^{t}p(s)\right) + u\left(\sum_{t=T-(A_i-A_j)+1}^{T}\beta^t\prod_{s=A_j+1}^{t}p(s)\right)$. Note that $\left(\sum_{t=A_i+1}^{T}\beta^t\prod_{s=A_i+1}^{t}p(s)\right)$ and $\left(\sum_{t=A_j+1}^{T-(A_i-A_j)}\beta^t\prod_{s=A_j+1}^{t}p(s)\right)$ each have $(T-A_i)$ terms and that, if $p(t)$ does not increase with time and $0<\beta\leq 1$, it must be the case that $\left(\sum_{t=A_j+1}^{T-(A_i-A_j)}\beta^t\prod_{s=A_j+1}^{t}p(s)\right)\geq\left(\sum_{t=A_i+1}^{T}\beta^t\prod_{s=A_i+1}^{t}p(s)\right)$. Further, $u\beta^{A_j}\geq u\beta^{A_i}$.

Because $u\left(\sum_{t=T-(A_i-A_j)+1}^{T}\beta^t\prod_{s=A_j+1}^{t}p(s)\right)>0$, we have that $S_j^U > S_i^U$.

<u>Ex Post Prioritarian SVRR</u>.

(a) The derivation of the formula stated in the text for $S_j^{EPP}-S_i^{EPP}$ is as follows.

$$S_j^{EPP}-S_i^{EPP}=\sum_{t=A_j}^{A_i-1}\frac{\mu(t;A_j)}{p(A_j)}g(U(t))+\sum_{t=A_i}^{T}\left(\frac{\mu(t;A_j)}{p(A_j)}-\frac{\mu(t;A_i)}{p(A_i)}\right)g(U(t))+\left(g(U(A_i-1))-g(U(A_j-1))\right)$$

[30] In this formula and below, $\prod_{s=A_i+1}^{t}p(s)$ with $t = A_i$ is set equal to 1.

The first term on the RHS is $\sum_{t=A_j}^{A_i-1}\mu(t;A_j+1)g(U(t))$. The second term is equal to:

$$\sum_{t=A_i}^{T}\left((1-p(t+1))\prod_{s=A_j+1}^{t}p(s)-(1-p(t+1))\prod_{s=A_i+1}^{t}p(s)\right)g(U(t))=\sum_{t=A_i}^{T}\left(\pi(A_i;A_j+1)\mu(t;A_i+1)-\mu(t;A_i+1)\right)g(U(t))$$
$$=(\pi(A_i;A_j+1)-1)\sum_{t=A_i}^{T}\mu(t;A_i+1)g(U(t))$$

(b) *Ratio Priority for the Young*. We prove that the ex post prioritarian SVRR displays Ratio Priority for the Young: $(S_j^{EPP}/S_i^{EPP})>(S_j^U/S_i^U)$. It follows that the ex post prioritarian SVRR displays Priority for the Young, and we don't demonstrate that directly. In what follows, we'll abbreviate $g(U(A_j-1))$ and $g(U(A_i-1))$ as $g_{(A_j-1)}$ and $g_{(A_i-1)}$, respectively; and $U(A_j-1)$ and $U(A_i-1)$ as $U_{(A_j-1)}$ and $U_{(A_i-1)}$, respectively.

$S_j^{EPP}=-g_{(A_j-1)}+\sum_{t=A_j}^{A_i-1}\frac{\mu(t;A_j)}{p(A_j)}g(U(t))+\sum_{t=A_i}^{T}\frac{\mu(t;A_j)}{p(A_j)}g(U(t))$. Note now that

$\sum_{t=A_i}^{T}\frac{\mu(t;A_j)}{p(A_j)}=\frac{\pi(A_i)}{\pi(A_j)}$ and $\sum_{t=A_j}^{A_i-1}\frac{\mu(t;A_j)}{p(A_j)}=1-\frac{\pi(A_i)}{\pi(A_j)}$. Thus we have that

$$S_j^{EPP}=\frac{\pi(A_i)}{\pi(A_j)}(g_{(A_i-1)}-g_{(A_j-1)})+\sum_{t=A_j}^{A_i-1}\frac{\mu(t;A_j)}{p(A_j)}\left(g(U(t))-g_{(A_j-1)}\right)+\sum_{t=A_i}^{T}\frac{\mu(t;A_j)}{p(A_j)}\left(g(U(t))-g_{(A_i-1)}\right).$$

$$S_i^{EPP}=-g_{(A_i-1)}+\sum_{t=A_i}^{T}\frac{\mu(t;A_i)}{p(A_i)}g(U(t))=\frac{\pi(A_j)}{\pi(A_i)}\sum_{t=A_i}^{T}\frac{\mu(t;A_j)}{p(A_j)}\left(g(U(t))-g_{(A_i-1)}\right).$$

Turning to the utilitarian SVRR: while the text provides the formula $S_k^U=\sum_{t=A_k}^{T}\frac{\pi_k(t;A_k)}{p_k(A_k)}\beta^t u(y_k(t))$ for $k = i, j$, it's straightforward to arrive at an alternative formula, parallel to that for the ex post prioritarian SVRR, namely: $S_k^U=-U_k(A_k-1)+\sum_{t=A_k}^{T}\frac{\mu_k(t;A_k)}{p_k(A_k)}U_k(t)$.

Thus, we can proceed by steps parallel to those immediately above to derive the following expressions for S_i^U and S_j^U.

$$S_j^U=\frac{\pi(A_i)}{\pi(A_j)}(U_{(A_i-1)}-U_{(A_j-1)})+\sum_{t=A_j}^{A_i-1}\frac{\mu(t;A_j)}{p(A_j)}\left(U(t)-U_{(A_j-1)}\right)+\sum_{t=A_i}^{T}\frac{\mu(t;A_j)}{p(A_j)}\left(U(t)-U_{(A_i-1)}\right).$$

$$S_i^U=\frac{\pi(A_j)}{\pi(A_i)}\sum_{t=A_i}^{T}\frac{\mu(t;A_j)}{p(A_j)}\left(U(t)-U_{(A_i-1)}\right).$$

Observe that $\frac{S_j^{EPP}}{S_i^{EPP}} = \frac{\frac{\pi(A_i)}{\pi(A_j)}(g_{(A_i-1)} - g_{(A_j-1)}) + \sum_{t=A_j}^{A_i-1} \frac{\mu(t;A_j)}{p(A_j)}\left(g(U(t)) - g_{(A_j-1)}\right)}{\frac{\pi(A_j)}{\pi(A_i)} \sum_{t=A_i}^{T} \frac{\mu(t;A_j)}{p(A_j)}\left(g(U(t)) - g_{(A_i-1)}\right)} + \frac{\pi(A_i)}{\pi(A_j)}$, and

that $\frac{S_j^{U}}{S_i^{U}} = \frac{\frac{\pi(A_i)}{\pi(A_j)}(U_{(A_i-1)} - U_{(A_j-1)}) + \sum_{t=A_j}^{A_i-1} \frac{\mu(t;A_j)}{p(A_j)}\left(U(t) - U_{(A_j-1)}\right)}{\frac{\pi(A_j)}{\pi(A_i)} \sum_{t=A_i}^{T} \frac{\mu(t;A_j)}{p(A_j)}\left(U(t) - U_{(A_i-1)}\right)} + \frac{\pi(A_i)}{\pi(A_j)}$. Thus $\frac{S_j^{EPP}}{S_i^{EPP}} > \frac{S_j^{U}}{S_i^{U}}$ iff

$$\frac{\frac{\pi(A_i)}{\pi(A_j)}(g_{(A_i-1)} - g_{(A_j-1)}) + \sum_{t=A_j}^{A_i-1} \frac{\mu(t;A_j)}{p(A_j)}\left(g(U(t)) - g_{(A_j-1)}\right)}{\frac{\pi(A_j)}{\pi(A_i)} \sum_{t=A_i}^{T} \frac{\mu(t;A_j)}{p(A_j)}\left(g(U(t)) - g_{(A_i-1)}\right)} > \frac{\frac{\pi(A_i)}{\pi(A_j)}(U_{(A_i-1)} - U_{(A_j-1)}) + \sum_{t=A_j}^{A_i-1} \frac{\mu(t;A_j)}{p(A_j)}\left(U(t) - U_{(A_j-1)}\right)}{\frac{\pi(A_j)}{\pi(A_i)} \sum_{t=A_i}^{T} \frac{\mu(t;A_j)}{p(A_j)}\left(U(t) - U_{(A_i-1)}\right)}$$

Equivalently, $\frac{S_j^{EPP}}{S_i^{EPP}} > \frac{S_j^{U}}{S_i^{U}}$ iff

$$\frac{\frac{\pi(A_i)}{\pi(A_j)}\left(g_{(A_i-1)} - g_{(A_j-1)}\right) + \sum_{t=A_j}^{A_i-1} \frac{\mu(t;A_j)}{p(A_j)}\left(g(U(t)) - g_{(A_j-1)}\right)}{\frac{\pi(A_i)}{\pi(A_j)}\left(U_{(A_i-1)} - U_{(A_j-1)}\right) + \sum_{t=A_j}^{A_i-1} \frac{\mu(t;A_j)}{p(A_j)}\left(U(t) - U_{(A_j-1)}\right)} > \frac{\sum_{t=A_i}^{T} \frac{\mu(t;A_j)}{p(A_j)}\left(g(U(t)) - g_{(A_i-1)}\right)}{\sum_{t=A_i}^{T} \frac{\mu(t;A_j)}{p(A_j)}\left(U(t) - U_{(A_i-1)}\right)} .$$

Let $\theta = \frac{g_{(A_i)} - g_{(A_i-1)}}{U_{(A_i)} - U_{(A_i-1)}}$. Note that $\frac{\sum_{t=A_i}^{T} \frac{\mu(t;A_j)}{p(A_j)}\left(g(U(t)) - g_{(A_i-1)}\right)}{\sum_{t=A_i}^{T} \frac{\mu(t;A_j)}{p(A_j)}\left(U(t) - U_{(A_i-1)}\right)} \leq \theta$. This is because—by the strict concavity of $g(\cdot)$—each term in the numerator of the preceding fraction is less than or equal to θ times the corresponding term in the denominator. Similarly,

$$\frac{\frac{\pi(A_i)}{\pi(A_j)}\left(g_{(A_i-1)} - g_{(A_j-1)}\right) + \sum_{t=A_j}^{A_i-1} \frac{\mu(t;A_j)}{p(A_j)}\left(g(U(t)) - g_{(A_j-1)}\right)}{\frac{\pi(A_i)}{\pi(A_j)}\left(U_{(A_i-1)} - U_{(A_j-1)}\right) + \sum_{t=A_j}^{A_i-1} \frac{\mu(t;A_j)}{p(A_j)}\left(U(t) - U_{(A_j-1)}\right)} > \theta$$. QED.

B. *SVRR, VSL and Income* (*Myopic Case*)

As discussed in the text, the comparative statics of SVRR with respect to a single-period difference in income are determined by examining the sign of $\frac{\partial S_i}{\partial y_i(t)}$, while the comparative statics of VSL are straightforward in light of the definition of VSL in terms of S_i^U. What follows are derivations of the comparative statics with respect to changes to permanent income.

Utilitarian SVRR. Because the utilitarian SVRR is independent of a single-period change to past income, and increases with a single-period increment to present or future income, it clearly increases with an increment to permanent income.

Ex Ante Prioritarian SVRR. An example suffices to prove that the effect of permanent income is ambiguous. Let $T=2$, $\beta = 1$, $A_i = A_j = 2$, $p_i(2) = p$, and $y_i(1) = y_i(2) = y$. Then $V_i = u(y)(1 + p)$. Denote $(1 + p)$ as q. $S_i^{EAP} = (u(y))g'(u(y)q)$

$\frac{\partial S_i^{EAP}}{\partial y} = u'(y)\left(g''(u(y)q)u(y)q + g'(u(y)q)\right)$. The sign of this expression is positive iff

$g''(u(y)q)u(y)q + g'(u(y)q) > 0$ or, equivalently, $-\frac{g''(u(y)q)}{g'(u(y)q)}u(y)q < 1$.

In this instance, the concavity of $g(\cdot)$ sufficient for the ex ante SVRR to increase with permanent income is, specifically, that the coefficient of relative risk aversion be less than 1. This can be shown to be true in general with a constant income profile ($y_i(t) = y$ for all t), but not when income varies.

Ex Post Prioritarian SVRR. An example suffices to prove that the effect of permanent income is ambiguous. Let $T=2$, $\beta = 1$, $A_i = A_j = 2$, $p_i(2) = p$, and $y_i(1) = y_i(2) = y$. Then $S_i^{EPP} = g(2u(y)) - g(u(y))$. $\frac{\partial S_i^{EPP}}{\partial y} = 2g'(2u(y))u'(y) - g'(u(y))u'(y)$, the sign of which depends upon $2g'(2u(y)) - g'(u(y))$. Depending on $g(\cdot)$, this term can be positive, negative, or zero for $u(y) > 0$.

VSL. Because VSL is independent of a single-period change to past income, and increases with a single-period increment to present or future income, it clearly increases with an increment to permanent income.

C. *SVRR, VSL and Baseline Risk* (*Myopic Case*)

As discussed in the text, the comparative statics of SVRR with respect to a single-period difference in survival probability are determined by examining the sign of $\frac{\partial S_i}{\partial p_i(t)}$, while the comparative statics of VSL are straightforward in light of the definition of VSL in terms of S_i^U.

What follows are derivations of the comparative statics with respect to a permanent (present and future) difference in survival probability.

Utilitarian SVRR. Because the utilitarian SVRR is insensitive to a change in current survival probability, and increases with a one-period increment in future survival probability, it clearly increases with a permanent increment in survival probability.

Ex ante prioritarian SVRR. An example is sufficient to prove that the impact of a permanent increment in survival probability is ambiguous. Let $T = 3$, $\beta = 1$, $A_i = 2$, $p_i(2) = p_i(3) = p$, and $y_i(1) = y_i(2) = y_i(3) = y$. $V_i = u(y)[1 + p + p^2]$, and $S_i^{EAP} = g'(V_i)u(y)(1+p)$. Then the ex ante prioritarian SVRR increases with a permanent increment in survival probability if $\frac{\partial S_i^{EAP}}{\partial p} = g'(V_i)u(y) + u(y)(1+p)g''(V_i)u(y)(1+2p) > 0$, and conversely decreases with a permanent increment in survival probability if $g'(V_i)u(y) + u(y)(1+p)g''(V_i)u(y)(1+2p) < 0$.

Note, in turn, that $g'(V_i)u(y) + u(y)(1+p)g''(V_i)u(y)(1+2p) > 0$ iff $\frac{-g''(V_i)V_i}{g'(V_i)} < \frac{1+p+p^2}{1+3p+2p^2}$.

This last term is bounded below by ½ and above by 1. Thus the ex ante prioritarian SVRR increases with a permanent increment in survival probability if the coefficient of relative risk aversion is sufficiently small, and decreases with a permanent increment in survival probability if the coefficient of relative risk aversion is sufficiently large.

Ex post prioritarian SVRR. Because the ex post prioritarian SVRR is insensitive to a change in current survival probability, and increases with a one-period increment in future survival probability, it clearly increases with a permanent increment in survival probability.

D. *The Annuities/Surprise Case*: *Comparative Statics of SVRR and VSL with respect to Income and Survival Probability* (*Table 5*)

In what follows, "SVRR" and "VSL" means the annuities/surprise SVRR and VSL.

Utilitarian SVRR. The utilitarian SVRR of an individual at age A_i is given by $S_i^U = \sum_{t=A_i}^{T} \frac{\pi_i(t;A_i)}{p_i(A_i)} \beta^t u(c_i(t))$. We analyze the effect of a permanent increment in income on the utilitarian SVRR, by imagining that $y_i(t)$ increases by a constant y in each period, and then considering the sign of $\frac{dS_i^U}{dy}$. $\frac{dS_i^U}{dy} = \sum_{t=A_i}^{T} \frac{\pi_i(t;A_i)}{p_i(A_i)} \beta^t u'(c(t)) \frac{dc_i(t)}{dy}$. By differentiating the first order conditions and the budget constraint with respect to y, we find that $\frac{dc_i(t)}{dy} > 0$ (richer individuals consume more in each period). Thus the utilitarian SVRR increases with a permanent increment in income.

Let us now consider the sensitivity of the utilitarian SVRR to single-period changes in survival probability. The sensitivity with respect to a change in *current* survival probability is given by $\frac{\partial S_i^U}{\partial p_i(A_i)} = \sum_{t=A_i}^{T} \frac{\pi_i(t;A_i)}{p_i(A_i)} \beta^t u'(c_i(t)) \frac{\partial c_i(t)}{\partial p_i(A_i)}$. Differentiating the first order conditions and budget constraint with respect to $p_i(A_i)$ yields:

$\frac{\partial c_i(t)}{\partial p_i(A_i)} = T(c_i(t)) \frac{1}{p_i(A_i)} \frac{\sum_{s=A_i}^{T} \frac{\pi_i(s)}{(1+r)^s}(y_i(s)-c_i(s))}{\sum_{s=1}^{T} \frac{\pi_i(s)}{(1+r)^s} T(c_i(s))}$, with $T(c_i(t)) = -\frac{u'(c_i(t))}{u''(c_i(t))}$. The sign of $\frac{\partial c_i(t)}{\partial p_i(A_i)}$ is ambiguous because the numerator in the preceding formula, $\sum_{s=A_i}^{T} \frac{\pi_i(s)}{(1+r)^s}(y_i(s)-c_i(s))$, can be positive, negative, or zero. It thus follows that the sign of $\frac{\partial S_i^U}{\partial p_i(A_i)}$ is ambiguous.

The sensitivity with respect to a change in *future* survival probability can be determined by examining $\frac{\partial S_i^U}{\partial p_i(F_i)}$, with $F_i > A_i$.

$\frac{\partial S_i^U}{\partial p_i(F_i)} = \sum_{t=A_i}^{T} \frac{\pi_i(t;A_i)}{p_i(A_i)} \beta^t u'(c_i(t)) \frac{\partial c_i(t)}{\partial p_i(F_i)} + \frac{1}{p_i(F_i)} \sum_{t=F_i}^{T} \frac{\pi_i(t;A_i)}{p_i(A_i)} \beta^t u(c_i(t))$. The second term is positive (better future survival rates increase expected lifetime well-being) while the first term is ambiguous since $\frac{\partial c_i(t)}{\partial p_i(F_i)}$ is. The sum of the two terms is ambiguous since it is possible to select parameters such that the second term is arbitrarily small (e.g. if $u(c_i(t)$ is small) and dominated by the first term.

Because the effect of a single-period change in survival probability on the SVRR is ambiguous, so is the effect of a permanent change in survival probability.

<u>Ex ante prioritarian SVRR</u>. The ex ante prioritarian SVRR is related to the utilitarian SVRR as follows: $S_i^{EAP} = g'(V_i) S_i^U$. As with the ex ante utilitarian SVRR above, we analyze the sensitivity of the ex ante prioritarian SVRR to permanent income by determining the sign of $\frac{dS_i^{EAP}}{dy}$, with y a constant increase in income in each period.

$$\frac{dS_i^{EAP}}{dy} = g''(V_i)\frac{dV_i}{dy}S_i^U + g'(V_i)\frac{dS_i^U}{dy} > 0 \text{ iff } -\frac{g''(V_i)}{g'(V_i)} < \left(\frac{dS_i^U}{dy}\bigg/\frac{dV_i}{dy}S_i^U\right) > 0$$, with

$$\frac{dV_i}{dy} = \sum_{t=1}^{A_i-1}\beta^t u'(c_i(t))\frac{dc_i(t)}{dy} + \sum_{t=A_i}^{T}\beta^t \pi_i(t;A_i)u'(c_i(t))\frac{dc_i(t)}{dy} > 0 .$$

The effect of a single-period change to survival probability on the ex ante prioritarian SVRR is ambiguous because $S_i^{EAP} = g'(V_i)S_i^U$ and the effect of a single-period change to survival probability on the utilitarian SVRR is ambiguous.

Ex post prioritarian SVRR. The ex post prioritarian SVRR is given by

$S_i^{EPP} = -g(U_i(A_i - 1)) + \sum_{t=A_i}^{T}\frac{\mu_i(t;A_i)}{p_i(A_i)}g(U_i(t))$, We found in the myopic case that the effect of permanent change in income on the ex post prioritarian SVRR is ambiguous. In the annuities/surprise case, a permanent increase in income leads to an increase in consumption each period (as discussed immediately above in the analysis of the utilitarian SVRR). Thus the ambiguity result for the myopic case carries over to the annuities/surprise case.

The sensitivity of the ex post prioritarian SVRR to a single-period change in survival probability is illustrated by the following example. Assume that $T = 2$, $\beta = 1$, and $A_i = 2$. Denote $c_i(1)$ as c_1 and $c_i(2)$ as c_2. Denote $p_i(2)$ as p. $S_i^{EPP} = g(u(c_2) + u(c_1)) - g(u(c_1))$.

The impact of a change in current survival probability is given by

$\frac{\partial S_i^{EPP}}{\partial p} = g'(u(c_2)+u(c_1))u'(c_2)\frac{\partial c_2}{\partial p} + u'(c_1)\frac{\partial c_1}{\partial p}\left[g'(u(c_1)+u(c_2)) - g'(u(c_1))\right]$. The sign of this expression is ambiguous because $\frac{\partial c_2}{\partial p}$ and $\frac{\partial c_1}{\partial p}$ are. This result generalizes to T periods for $T > 2$, and to changes in future survival probabilities.

VSL. Recall that VSL is related to the utilitarian SVRR as follows.

$VSL_i = \frac{S_i^U}{p_i(A_i)u'(c_i(A_i))\beta^{A_i}}$. In the analysis above regarding the utilitarian SVRR, we showed, first that S_i^U increases with a permanent increase to income; and, second, that consumption in each period also does. Hence the numerator of the preceding fraction increases and the denominator decreases with a permanent increase to income.

As for survival probability: building on the analysis immediately above of the utilitarian SVRR and a single-period change to survival probability, it can be seen that effect of a single-period change on both the numerator of VSL and the denominator is ambiguous.

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