Poverty trends in Europe: A multivariate dependence analysis based on copulas

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\textbf{Abstract}

There is a widespread agreement that poverty is more than just low incomes, as it also involves deprivations in other dimensions such as education, health or labour. In this multidimensional setting, analysing the dependence between dimensions becomes an important issue since higher dependence means higher concentration of deprivations and this could make overall poverty worse. In this paper we look at poverty in Europe by focusing on the AROPE rate, which is the headline indicator to monitor and implement effective poverty-reduction policies in the framework of the Europe 2020 strategy. We propose measuring the multivariate dependence among the three dimensions included in the AROPE rate (income, material needs and work intensity) using copula-based methods. The copula approach focuses on the positions of the individuals across dimensions, rather than on the specific values that those dimensions attain for such individuals, and so it allows for other types of dependence beyond linear correlation. In particular, we analyse how the concordance among the poverty dimensions has evolved in EU countries

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from 2008 to 2014 by applying four multivariate copula-based extensions of Spearman’s rank coefficient. The results show a general increase in the dependence between poverty dimensions, with average lower orthant dependence being, in general, higher than average upper orthant dependence. Moreover, countries with higher AROPE rates also tend to experiment more dependence between poverty dimensions.

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1 Introduction

There is a widespread agreement that poverty is a multimimensional phenomenon involving not only low incomes, but also deprivations in other dimensions such as education, health or labour; see, for instance, Sen (1985, 1987). Because of that, attention has been increasingly focused on multi-dimensional approaches to the analysis of poverty, to the point where the European Union (EU) has adopted a multidimensional poverty and social exclusion indicator as a tool to monitor and implement effective poverty-reduction policies in the framework of the Europe 2020 Strategy. The indicator at hand, namely the AROPE (At Risk Of Poverty or social Exclusion) rate, is based on three measures: relative income poverty; material deprivation; and work intensity. Also, the United Nation Development Program (UNDP) adopted, in 2010, the Multidimensional Poverty Index (MPI) which is based on the Alkire and Foster (2011) proposal and also considers three dimensions: education, health and standard of living. Based on these indices (or any other multivariate indicator), several authors examined the incidence and intensity of multidimensional poverty in developed and non-developed countries; see, for instance, the contributions of Nolan and Whelan (2011), Whelan et al. (2014), Alkire and Apablaza (2016), White (2017) and Atkinson et al. (2017), in the European context.
Nevertheless, in a multidimensional setting, focusing solely on multidimensional poverty indices misses an important part of the picture, which is the possible interactions between the dimensions of poverty. In fact, this interrelation issue is the main feature that distinguishes unidimensional and multidimensional analyses. In this context, several authors argue that incorporating those relationships can be relevant, since higher dependence means higher concentration of deprivations and this could make overall poverty worse; see, for instance, Atkinson and Bourguignon (1982), Duclos et al. (2006) and Ferreira and Lugo (2013). In spite of its relevance, the problem of measuring the dependence between dimensions of poverty has been scarcely addressed in the literature and this is the scope of this paper. In particular, we propose complementing the analysis based on poverty indices by measuring the multivariate dependence among poverty dimensions using copula-based methods. To illustrate our results we focus on the relationship between the indicators of the AROPE rate (income, material needs and work intensity).

The copula approach focuses on the positions of the individuals across dimensions, rather than on the specific values that those dimensions attain for such individuals. The advantage of this approach is that it enables the decomposition of the joint distribution function of all dimensions into its univariate marginals and the dependence structure captured by the copula. Moreover, copulas allow building scaled-free measures of dependence that capture other types of dependence beyond linear correlation. Actually, the well-known Spearman’s rho and other related measures of bivariate association can be expressed in terms of copulas; see Nelsen (2006) and Joe (1990) for a comprehensive review of copulas. Furthermore, in a multivariate setting, neither the concept of concordance nor the generalization of the bivariate coefficients of concordance is unique. For instance, in the trivariate case, there are more than eight copula-based generalizations of the bivariate Spearman’s rho coefficient. In this paper, we apply four of these coefficients, namely those proposed by Nelsen (1996, 2002), which are based on average orthant dependence concepts, as well as the coefficient proposed by García et al. (2013) for the
Aplications of bivariate copula-based methods in welfare context date back to Dardanoni and Lambert (2001), Quinn (2007b,a) and Bø et al. (2012); see also the recent contribution of Aaberge et al. (2018). In a multi-dimensional framework, the first contribution employing copula-based methods in welfare economics is Decancq (2014). He analysed temporal evolution of well-being in Russia by means of a multivariate Kendall’s tau and a multivariate version of Spearman’s rho applied to the dimensions included in the Human Development Index (HDI). Later, Pérez and Prieto-Alaiz (2015) extend Decancq’s results by considering several multivariate versions of Spearman’s rho and apply them to study how the dependence between the dimensions included in the AROPE rate has evolved in Spain over the period 2009-2013. Also, Pérez and Prieto-Alaiz (2016) analysed the multivariate dependence between the dimensions of the HDI using data from 187 countries and three copula-based measures of multivariate association: Spearman’s footrule, Gini’s gamma and Spearman’s rho.

In this paper, we first review the definitions and main properties of the multivariate generalizations of the Spearman’s coefficient we use. Then, we discuss the empirical versions of these coefficients and show a theoretical result which proves that the relationship between three of these coefficients remains the same between their estimators. In the empirical application, we apply these coefficients to perform geographical and temporal comparisons of the dependence between the three dimensions of the AROPE rate in the 28 EU countries over the period 2008-2014. The data we use comes from the EU-Statistics on Income and Living Conditions (EU-SILC) survey, which is the EU reference source for comparative statistics on income distribution and social inclusion at the European level. Our analysis complement the information given by the AROPE rate on the incidence of multidimensional poverty with information on the degree of multivariate dependence between its dimensions. As far as we know, this is the first time that multivariate copula-based methods are applied in the European context.

Our results show variations between EU countries but, in most of them we observe that, re-
gardless of the coefficient considered, there has been a general increase in the multivariate
dependence between poverty dimensions over the period analysed. Noticeably, the highest in-
crease corresponds to Greece and Spain, two of the countries most severely hit by the last
economic crisis. Moreover, in general, the maximal dependence of poverty is found in the lower
orthant over all the years considered. These results suggest that small (high) values of the
three poverty dimensions tend to occur together, and this simultaneous concentration of small
(large) values of income, no material privations and work intensity, is more likely to occur in
2014 than in 2008. Therefore, it seems that, after the crisis, most EU countries have become
more polarized.

The rest of the paper is organized as follows. Section 2 first introduces the copula, summarizes
its basic properties and describes the positive orthant dependence concepts. Then, it introduces
several copula-based trivariate generalizations of the bivariate Spearman’s rho and discusses
their main properties and how to estimate them using the empirical copula. Section 3 illustrates
the use of these tools to measure how the dependence between the three indicators of the
AROPE rate (income, material needs and work intensity) has evolved in the EU-28 countries
over the period 2008-2014. Finally, Section 4 concludes the paper with a summary of the main
results.

2 Methodology

2.1 Copulas and orthant dependence

Copulas are multivariate distribution functions whose one-dimensional margins are uniform on
the interval (0,1). More precisely, a $d$–dimensional copula $C$ is a multivariate distribution
function $C : I^d \rightarrow I$, with $I = [0,1]$, defined for every $\mathbf{u} = (u_1, \ldots, u_d) \in I^d$ as $C(\mathbf{u}) = p(\mathbf{U} \leq \mathbf{u}) = p(U_1 \leq u_1, \ldots, U_d \leq u_d)$, where $U_i$ is $U(0,1)$, for $i = 1, \ldots, d$. The importance
of copulas in statistics relies on the Sklar’s theorem (Sklar, 1959). This theorem establishes that, if \( X = (X_1, \ldots, X_d) \) is a \( d \)-dimensional random vector with joint distribution function

\[
F(x) = F(x_1, \ldots, x_d) = p(X_1 \leq x_1, \ldots, X_d \leq x_d)
\]

and univariate marginal distribution functions \( F_i(x_i) = p(X_i \leq x_i) \), for \( i = 1, \ldots, d \), then there exists a copula \( C : \mathbb{I}^d \rightarrow \mathbb{I} \) such that, for all \((x_1, \ldots, x_d) \in \mathbb{R}^d\), \( F \) can be represented as

\[
F(x_1, \ldots, x_d) = C(F_1(x_1), \ldots, F_d(x_d)).
\] (1)

Hence, copulas are functions that join or “couple” multivariate distribution functions to their one-dimensional marginal distribution functions. If the margins \( F_1, \ldots, F_d \) are all continuous, the copula \( C \) in (1) is unique; otherwise \( C \) is uniquely determined on \( \text{Ran}F_1 \times \ldots \times \text{Ran}F_d \).

Conversely, if \( C \) is a \( d \)-copula and \( F_1, \ldots, F_d \) are univariate distribution functions, the function \( F \) defined in (1) is a joint distribution function with margins \( F_1, \ldots, F_d \). Throughout this section, we assume that \( F_1, \ldots, F_d \) are all continuous.

In a multidimensional poverty setting, the random vector \( X = (X_1, \ldots, X_d) \) represents the relevant \( d \) dimensions of poverty for a population and the transformed variables \( U_i = F_i(X_i) \), with \( i = 1, \ldots, d \), attach to each individual in the population its position in all dimensions.

For instance, an individual with position vector \((1, \ldots, 1)\) will be top-ranked in all dimensions. Each random variable \( U_i \) is \( U(0, 1) \) and the joint distribution of the vector \( U = (U_1, \ldots, U_d) \) is the copula \( C \) defined above. Therefore, for a given real vector \( u = (u_1, \ldots, u_d) \in \mathbb{I}^d \), the value \( C(u) \) represents the proportion of individuals in the population with positions outranked by \( u \), i.e. with a lower or equal position than \( u \) in all dimensions. For instance, \( C(0.25, \ldots, 0.25) \) will represent the probability that a randomly selected individual is simultaneously in the 1st quartile (“low ranked”) in all dimensions, i.e., the probability that he/she is simultaneously “poor” in all dimensions.
Any copula $C$ satisfies the Fréchet-Hoeffding bounds inequality

$$W(u) \leq C(u) \leq M(u),$$

for every $u = (u_1, \ldots, u_d) \in I^d$, where $W(u) = \max(u_1 + \cdots + u_d - d + 1, 0)$ and $M(u) = \min(u_1, \ldots, u_d)$. $M$ is always a copula and represents maximal dependence, i.e. the case when each of the random variables $X_1, \ldots, X_d$ is almost surely a strictly increasing function of any of the others (the outcomes in all dimensions are ordered in the same way). $W$ is only a copula if $d = 2$, in which case it represents perfect negative dependence. Another important copula is the independent copula, defined as $\Pi(u) = u_1 \times \cdots \times u_d$, which accounts for the case where the variables $X_1, \ldots, X_d$ are independent. Noticeably, the volumes between the independent copula $\Pi$ and the copulas $M$ and $W$, respectively, are:

$$a_d = \int_{I^d} [M(u) - \Pi(u)]d\mathbf{u} = \frac{1}{(d + 1)} - \frac{1}{2^d},$$

$$b_d = \int_{I^d} [\Pi(u) - W(u)]d\mathbf{u} = \frac{1}{2^d} - \frac{1}{(d + 1)!}.$$

If $d = 2$ (bidimensional case), we have $a_2 = b_2 = 1/12$, but if $d > 2$, the independent copula $\Pi$ is closer to $W$ than to $M$. Actually, it can be shown that $\lim_{d \to \infty} \frac{b_d}{a_d} = 0$.

Finally, if $\mathbf{U} = (U_1, \ldots, U_d)$ is a random vector of variables $U(0, 1)$ whose joint distribution function is the copula $C$, the survival function $\overline{C} : I^d \to I$ is defined as:

$$\overline{C}(\mathbf{u}) = p(\mathbf{U} > \mathbf{u}) = p(U_1 > u_1, \ldots, U_d > u_d).$$

In general, $\overline{C}$ is not a copula. Moreover, if $U_1, \ldots, U_d$ are independent random variables, then its survival function is $\overline{\Pi}(\mathbf{u}) = (1 - u_1) \times \cdots \times (1 - u_d)$. For a complete survey on the copula theory, see Nelsen (2006).

In this paper, we use copulas to study measures of multivariate association derived from multi-
variate dependence concepts. The notions of dependence in the multivariate case can be defined in different ways. The one we handle in this paper is positive orthant dependence and it is defined as follows (Nelsen, 2006):

- **X** is positively lower orthant dependent (PLOD) if \( C(u) \geq \Pi(u) \), for each \( u \in I^d \), that is, if the probability that the variables \( X_1, \ldots, X_d \) are simultaneously small is at least as great as it would be were they independent.

- **X** is positively upper orthant dependent (PUOD) if \( C(u) \geq \Pi(u) \), for each \( u \in I^d \), that is, if the probability that the variables \( X_1, \ldots, X_d \) are simultaneously large is at least as great as it would be were they independent.

- **X** is positively orthant dependent (POD) if both inequalities hold.

The corresponding negative concepts are defined by reversing the sense of the inequalities above. For \( d = 2 \), PLOD and PUOD are the same and reduce to POD. Obviously, the same reduction occurs with the analogous negative concepts.

In this framework, the differences \( C(u) - \Pi(u) \) and \( \overline{C}(u) - \overline{\Pi}(u) \) can be regarded as measures of “local” lower and upper orthant dependence, respectively; see Nelsen (1996). Accordingly, the copula-based measures of multivariate association to be introduced in next Section, are based on these differences.

### 2.2 Copula-based multivariate extensions of Spearman’s rho

The first copula-based generalization of bivariate Spearman’s rho, due to Wolff (1980) and Nelsen (1996), is focused on the difference \( C(u) - \Pi(u) \) and it is defined as:

\[
\rho_d = \frac{1}{a_d} \int_{I^d} [C(u) - \Pi(u)]d\Pi(u) = \frac{(d + 1)}{2^d - (d + 1)} \left[ 2^d \int_{I^d} C(u)d\Pi(u) - 1 \right].
\]
Following Nelsen (1996), $\rho_{d}^{-}$ can be regarded as a multivariate measure of average lower orthant dependence. In fact, $\rho_{d}^{-}$ measures, to some extent, “how far” from the independence (copula $\Pi$) our multivariate data (represented by its copula $C$) are in the lower orthant. Moreover, using Lemma 3.1 in Dolati and ´Ubeda-Flores (2006), $\rho_{d}^{-}$ can be alternative written as:

$$\rho_{d}^{-} = \frac{(d+1)}{2^d - (d+1)} \left[ 2^d \int_{\mathbb{I}^d} \Pi(u) dC(u) - 1 \right].$$

(3)

The definition of $\rho_{d}^{-}$ in (3) as a rescaled expectation was already proposed by Joe (1990).

In a similar fashion, Nelsen (1996) defined a second generalization of Spearman’s rho, derived from average of upper orthant dependence, which is given by:

$$\rho_{d}^{+} = \frac{(d+1)}{\alpha_d} \int_{\mathbb{I}^d} [C(u) - \Pi(u)] d\Pi(u) = \frac{(d+1)}{2^d - (d+1)} \left[ 2^d \int_{\mathbb{I}^d} \Pi(u) dC(u) - 1 \right].$$

(4)

From this expression, $\rho_{d}^{+}$ could be thought of as a rescaled “distance” between $C$ – representing the behaviour of our data in the upper orthant – and $\Pi$ – representing independence in such orthant. Alternatively, expression (4) can be written as a rescaled expectation, as proposed by Joe (1990), as follows:

$$\rho_{d}^{+} = \frac{(d+1)}{2^d - (d+1)} \left[ 2^d E \left( \prod_{i=1}^{d} (1 - U_i) \right) - 1 \right].$$

The third copula-based multivariate version of Spearman’s rho, due to Nelsen (2002), is the average of the two generalizations described above, namely:

$$\rho_{d} = \frac{\rho_{d}^{-} + \rho_{d}^{+}}{2} = \frac{(d+1)}{2^d - (d+1)} \left[ 2^{d-1} \left( \int_{\mathbb{I}^d} C(u) d\Pi(u) + \int_{\mathbb{I}^d} \Pi(u) dC(u) \right) - 1 \right].$$

(5)

This coefficient $\rho_{d}$ is further discussed in Dolati and ´Ubeda-Flores (2006) as an example of

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1 Joe (1990) denote $\rho_{d}^{-}$ and $\rho_{d}^{+}$ as $\omega(F)$ and $\pi(F)$, respectively.
Average Orthant Dependence (AOD) measure of multivariate concordance. See also the discussion in Taylor (2007) showing that $\rho_d$ is a measure of multivariate concordance according to its axiomatic definition. Hence, as Decancq (2014) points out, $\rho_d$ can be interpreted as a “normalised” probability of concordance between the distribution of $X$, as represented by their copula $C$, and independence, as represented by the copula $\Pi$. Thus, the more concordance among the dimensions of $X$ the higher the value of $\rho_d$.

When the copula of $X$ is the upper bound $M$, the three measures defined above attain their maximum value, 1, and they all become zero when the components of $X$ are independent ($C = \Pi$). A lower bound for the three of them is $\lfloor 2^d - (d + 1)! \rfloor / \{d! [2^d - (d + 1)] \}$; see Nelsen (1996). Noticeably, for $d = 2$, the three coefficients above, $\rho_2^-, \rho_2^+$ and $\rho_2$, reduce to bivariate Spearman’s rho. Moreover, in the trivariate case ($d = 3$), $\rho_3$ becomes the average of the three pairwise Spearman’s rho coefficients, that is:

$$\rho_3 = \frac{\rho_{12} + \rho_{13} + \rho_{23}}{3},$$

where $\rho_{ik}$ denotes the pairwise Spearman’s rho coefficient for the bivariate random variable $(X_i, X_k)$, with $1 \leq i < k \leq 3$; see Nelsen (1996).

The advantage of $\rho_d^-$ and $\rho_d^+$ is that they are capable of revealing some forms of dependences that $\rho_d$ fails to detect. See, for instance, a trivariate example in Nelsen (1996) where $\rho_{12} = \rho_{13} = \rho_{23} = \rho_3 = 0$, presumably indicating no dependence at all, but $\rho_3^+ = 2/15$ and $\rho_3^- = -2/15$, indicating some degree of positive upper and negative lower orthant dependence, respectively, that is missed by $\rho_3$. In spite of this advantage, there are still some forms of multivariate dependence that the coefficients $\rho_d^+$ and $\rho_d^-$ may fail to detect when they take values near zero.

To overcome this drawback, Nelsen and Úbeda-Flores (2012) develop, in the 3-dimensional case, new coefficients of directional dependence which are defined as follows. Let $\alpha = (\alpha_1, \alpha_2, \alpha_3)$, with $\alpha_i \in \{-1, 1\}$, denote the eight vertexes of the cube $I^3$ determining the eight directions
in which we could measure dependence in trivariate distributions. For each direction $\alpha$, a directional $\rho$-coefficient is defined as:

$$\rho_3^\alpha = \frac{\alpha_1 \alpha_2 \rho_{12} + \alpha_1 \alpha_3 \rho_{13} + \alpha_2 \alpha_3 \rho_{23} + \alpha_1 \alpha_2 \alpha_3 \frac{\rho^+ - \rho^-}{2}}{3}.$$  \hspace{1cm} (7)

Noticeably, $\rho_3^{(1,1,1)} = \rho_3^+$ and $\rho_3^{(-1,-1,-1)} = \rho_3^-$. Finally, García et al. (2013) introduced the index of maximal dependence $\rho_3^{\text{max}}$ as the largest of the eight directional $\rho$-coefficients defined in (7).

This index can be alternatively calculated as follows:

$$\rho_3^{\text{max}} = \frac{2}{3} \max\{\rho_{12}, \rho_{13}, \rho_{23}, 3\rho_3\} - \min\{\rho_3^+, \rho_3^-\}. \hspace{1cm} (8)$$

The applications of these coefficients to welfare economics is scarce. Decancq (2014) first employed the multivariate version of Spearman’s rho in (5) to illustrate how the dependence between dimensions of well-being has evolved in Russia between 1995 and 2005. The same coefficient is also used by Pérez and Prieto-Alaiz (2016) to analyse the dependence between the dimensions of well-being from 1980 till 2014 for the countries included in the 2015 Human Development Report. Moreover, a complete application with all the coefficients described above can be found in Pérez and Prieto (2015) who study how the dependence between the dimensions included in the AROPE rate has evolved in Spain over the period 2009-2013.

### 2.3 Non-parametric estimation

In practice, the copula $C$ is unknown and the coefficients described in Section 2.2 must be estimated from the data. Therefore, empirical versions of these coefficients are required. Let $\{ (X_{ij}, \ldots, X_{dj}) \}_{j=1,\ldots,n}$ be a sample of $n$ serially independent random vectors from the $d$-dimensional vector $X = (X_1, \ldots, X_d)$ with associated copula $C$. Let $R_{ij}$ be the rank of $X_{ij}$ among $\{X_{i1}, \ldots, X_{in}\}$, with $i = 1, \ldots, d$ and $j = 1, \ldots, n$. The copula $C$ can be estimated by the
empirical copula defined as:

\[
\widetilde{C}_n(u) = \frac{1}{n} \sum_{j=1}^{n} \prod_{i=1}^{d} 1_{\{\widetilde{U}_{ij} \leq u_i\}}, \quad \text{for } u = (u_1, \ldots, u_d) \in I^d,
\]

where \(1_A\) denotes the indicator function on a set \(A\) and \(\widetilde{U}_{ij} = \frac{R_{ij}}{n}\).

Statistical inference for \(\rho_d^-\) and \(\rho_d^+\) based on the empirical copula is discussed in Schmid and Schmidt (2007) and Schmid et al. (2010). In particular, these authors propose estimating the coefficients \(\rho_d^-\) and \(\rho_d^+\) defined in Section 2.2 by replacing the copula \(C\) in (2) and (4), respectively, with the empirical copula in (9). However, Pérez and Prieto-Alaiz (2016) demonstrate that the resultant statistics are not proper estimators of their population counterparts, since they can take values out of the parameter space. The modifications proposed by Blumentritt and Schmid (2014) and Bedo and Ong (2014), based on using the so-called pseudo-observations, \(U_{ij}^* = R_{ij}/(n + 1)\), instead of \(\widetilde{U}_{ij} = R_{ij}/n\), have still some drawbacks, as they fail to achieve the maximum value 1 for maximal dependence and take a narrower range of values that they should be.

To overcome these problems, Pérez and Prieto-Alaiz (2016) propose alternative feasible non-parametric estimators of \(\rho_d^-\) and \(\rho_d^+\), based on the results in Joe (1990), which are given by the following expressions, respectively:

\[
\hat{\rho}_d^- = \frac{1}{n} \sum_{j=1}^{n} \prod_{i=1}^{d} \widetilde{U}_{ij} - \left(\frac{n+1}{2n}\right)^d = \frac{1}{n} \sum_{j=1}^{n} \prod_{i=1}^{d} R_{ij} - \left(\frac{n+1}{2}\right)^d,
\]

\[
\hat{\rho}_d^+ = \frac{1}{n} \sum_{j=1}^{n} \prod_{i=1}^{d} \widetilde{U}_{ij} - \left(\frac{n+1}{2n}\right)^d = \frac{1}{n} \sum_{j=1}^{n} \prod_{i=1}^{d} R_{ij} - \left(\frac{n+1}{2}\right)^d,
\]

where \(R_{ij} = n + 1 - R_{ij}\) and \(\widetilde{U}_{ij} = R_{ij}/n\). By construction, both \(\hat{\rho}_d^-\) and \(\hat{\rho}_d^+\) achieve their maxi-
mum value 1 for maximal dependence. Moreover, it is worth mentioning that the estimators in (10) and (11) keep the same if they are computed using \( U_{ij}^* = R_{ij}/(n+1) \) and \( \overline{U}_{ij}^* = \overline{R}_{ij}/(n+1) \) instead of \( \tilde{U}_{ij} \) and \( \tilde{\overline{U}}_{ij} \), respectively.

To estimate the coefficient \( \rho_d \) in (5), we propose the following plug-in estimator

\[
\hat{\rho}_d = \frac{\hat{\rho}_d^- + \hat{\rho}_d^+}{2},
\]

where \( \hat{\rho}_d^- \) and \( \hat{\rho}_d^+ \) are the estimators in (10) and (11), respectively. Noticeably, this estimator coincides with the estimator of \( \rho_d \) proposed by Dolati and Úbeda-Flores (2006) in the framework of AOD measures of multivariate concordance; see Appendix.

For the bidimensional case \((d = 2)\), all the estimators above, namely \( \hat{\rho}_2^- \), \( \hat{\rho}_2^+ \) and \( \hat{\rho}_2 \), collapse to the well-known sample version of bivariate Spearman’s rho defined as

\[
\rho_S = \frac{12}{n(n^2 - 1)} \sum_{j=1}^{n} R_{1j}R_{2j} - \frac{3(n+1)^2}{n - 1}.
\]

In the trivariate case \((d = 3)\), the estimators \( \hat{\rho}_3^- \) and \( \hat{\rho}_3 \) reduce to:

\[
\hat{\rho}_3^- = \frac{8}{n(n-1)(n+1)^2} \sum_{j=1}^{n} R_{1j}R_{2j}R_{3j} - \frac{n+1}{n-1},
\]

\[
\hat{\rho}_3 = \frac{8}{n(n-1)(n+1)^2} \sum_{j=1}^{n} \overline{R}_{1j}\overline{R}_{2j}\overline{R}_{3j} - \frac{n+1}{n-1}.
\]

Moreover, it can be shown (see Appendix) that property (6) continues to hold for the corresponding empirical coefficients, that is

\[
\hat{\rho}_3 = \frac{\hat{\rho}_3^- + \hat{\rho}_3^+}{2} = \frac{\hat{\rho}_{12} + \hat{\rho}_{13} + \hat{\rho}_{23}}{3},
\]

where \( \hat{\rho}_{ik} \) denotes the bivariate sample Spearman’s rho for the pair \((X_i, X_k)\). Hence, in the trivariate case, the sample version of the coefficient \( \rho_3 \) can be computed as the average of their
corresponding pairwise sample coefficients.

García et al. (2013) construct plug-in estimators for the coefficients $\rho_3^\alpha$ and $\rho_{3}^\text{max}$ by replacing in (7) and (8), respectively, the population bivariate and trivariate Spearman’s coefficients by their empirical counterparts in equations (13)-(16). As expected, the estimators $\hat{\rho}_3^+\alpha$ and $\hat{\rho}_3^-\alpha$ in (14)-(15) come up as particular cases of the estimated directional coefficient $\hat{\rho}_3^\alpha$ for $\alpha = (1, 1, 1)$ and $\alpha = (-1, -1, -1)$, respectively. Moreover, García et al. (2013) derive the asymptotic distribution of the estimators at hand and show that they are asymptotically unbiased, consistent and asymptotically normally distributed.

3 Empirical application

Several recent papers point out that, over the years of the Great Recession, there has been a general increase in the incidence of multidimensional poverty in the European Union; see, for example Duiella and Turrini (2014) and Atkinson et al. (2017), among others. Moreover, this increase is explained mostly by the rise in severe material deprivation and low work intensity rates and has been particularly important in those countries most affected by the economic crisis such as Greece, Spain or Ireland.

As we said before, multidimensional poverty depends not only on the proportion of individuals deprived in each dimension but also on the degree of interdependence among the poverty dimensions. In this context, we propose complementing (not replacing) the information given by the incidence of multidimensional poverty with measures of multivariate dependence among poverty dimensions. In particular, we apply the copula-based methods described in Section 2 to measure the evolution of the dependence among the poverty dimensions in the EU28 countries over the period 2008-2014. This analysis will give us a general picture of how the degree of poverty has evolved in Europe during and after the Great Recession.
3.1 Data and variables

For the empirical application we have selected the indicators of poverty that are included in the AROPE rate, namely income, material needs and work intensity. The selection of these dimensions is based on the relevance of the AROPE rate in the European context, as it is the headline indicator to monitor the progress in reducing poverty, which is one of the headline targets of the Europe 2020 strategy.

The data comes from the EU-Statistics on Income and Living Conditions (EU-SILC) survey. In particular, we use the cross-sectional surveys of all years of the period 2008-2014.

The measure of income is the equivalised disposable income, which is calculated as the total income of the household, after tax and other deductions, divided by the equivalised household size\(^2\).

The work intensity of a household is the ratio of the total number of months that all working-age household members\(^3\) have worked during the income reference year and the total number of months they could have theoretically worked during the same period.

With respect to material deprivation, it is originally defined as the enforced lack in a number of essential items, namely: 1) the capacity of facing unexpected expenses; 2) one-week annual holiday away from home; 3) a meal involving meat, chicken or fish every second day; 4) an adequately warm dwelling; 5) a washing machine; 6) a colour television; 7) a telephone; 8) a car; 9) the capacity to pay their rent, mortgage or utility bills. We use a variable that indicates the number of no-privations out of the nine possible, taking the values: 0 (having all the 9 possible deprivations), 1 (having eight out of the nine aforementioned deprivations), \ldots, 9 (having no deprivations).

\(^2\)The equivalised household size is defined according to the modified OECD scale, which gives a weight of 1 to the first adult, 0.5 to other household members aged 14 or over and 0.3 to household members aged less than 14.

\(^3\)Eurostat considers that a working-age person is a person aged 18-59 years, excluding also the students aged 18-24 years.
Thus, the three variables considered (equivalised disposable income, work intensity, and number of no deprivations) keep the same relationship with the phenomenon being measured, poverty. That is, high values of each variable convey lower chance to be poor, while low values of each variable convey higher chance to be poor.

The unit of analysis is the household. We only work with subsamples of households for which we have complete information for all the three ranking variables. In particular, in these subsamples, households composed only of children, of students aged 18-24 and/or people aged 60 or more are excluded, due to their missing values in the variable work intensity.

In the copula-based framework we deal with how the individuals are ranked in the dimensions considered, rather than with the exact level attained by the individuals in these dimensions. Hence, in each dimension, households are ranked according to the variable defining it. Following the strategy of Decancq (2014), if a tie occurs, the households are untied using information from other variables. If a tie still occurs for some households after ranking them according to both the primary and secondary variables, they are ranked randomly.

In particular, when a tie occurs in work intensity, households are ranked according to two secondary variables: a variable measuring the intensity in education of the household and a variable measuring the intensity of health of the household. The intensity of education is the sum of the highest ISCED (International Standard Classification of Education) level attained by all members of the household that are not currently in education divided by the highest possible value of this sum. The health intensity indicator is constructed in a similar fashion. In particular, the self-perceived general health of the individuals is used. The health intensity variable is the sum of the values of the self-assesed health indicator of all members of the household divided by the highest possible value of this sum. The relationship between educational attainment and labour market outcomes is well documented in the literature. Educational

\footnote{The representation of each country in the whole cross-country sample does not change when going from the full sample to the restricted one.}
attainment has been found to be negatively associated with the incidence and duration of unemployment; see, for example, Nickell (1979) and Mincer (1991). Furthermore, Farber (2004), Wolbers (2000) and Riddell and Song (2011), among others, have found a positive association between educational attainment and the probability of being re-employed. The literature on the relationship between health and labour outcomes is also extensive; see Chirikos (1993) and Currie and Madrian (1999) for reviews of this issue. In particular, Ettner et al. (1997), Pelkowski and Berger (2004) and García Gómez and López Nicolás (2006), among others, find a positive (negative) relationship between good (bad) health and labour market outcomes.

When there are ties with respect to material deprivation, we rank households with respect to the burden of the housing cost. An overburden of the housing cost can be seen as an indicator of financial stress (Whelan and Maître, 2012; Deidda, 2015) and as an indicator of vulnerability (Brandolini et al., 2013). We use a dummy variable taking the value 1 if the housing cost is a burden for the household as well as the value of the housing cost. Thus, households for which the housing cost is a burden are assigned worse positions than those for which it is not. If a tie still exist for those households for which the housing cost is a burden they are ranked using the value of the housing cost. That is, the higher is the housing cost the worse is the position of the household. If a tie still occurs for households for which the housing cost is not a burden, they are ranked randomly.

Other solutions to the problems of ties can be found in the literature. One alternative is to rank the tied households randomly, without using any additional information. Another solution consists of making the more discrete variables continuous by adding a random perturbation taking values in [0,1]. Denuit and Lambert (2005) show that this procedure preserves the concordance order and so it leaves unchanged the concordance-based dependence measures\(^5\).

\(^5\)We only show the results using information from other variables, since our main results are not altered by the use of these two alternative methods of solving ties. These results are available upon request.
3.2 Estimation results

We next analyse the evolution of the multivariate dependence among income, no material needs and work intensity in the European Union over the period 2008-2014. In order to do that, we compute the estimators of the coefficients introduced in Section 2 for each year of the period considered and for the 28 member states.

Figure 1 shows the evolution $\hat{\rho}_3^-, \hat{\rho}_3^+$ and $\hat{\rho}_3$ in the EU28 countries over the whole period analysed. We observe a general positive trend in the three coefficients, indicating a general increase in the dependence between the dimensions of poverty in the European Union between 2008 and 2014. Moreover, we find that the trends of $\hat{\rho}_3^-$ and $\hat{\rho}_3^+$ are very similar, with a general increase of the dependence both in the lower and in the upper orthant. Thus, not only the dependence among dimensions of poverty has increased over the period of analysis, but also societies have become more polarised, with poverty dimensions being more likely to have simultaneously small (large) values in 2014 than in 2008. Despite the general increase in the dependence between dimensions of poverty in Europe, we can also find different country-specific patterns. In particular, those countries most severely hit by the crisis, namely Spain and Greece, experiment the highest increases both in $\hat{\rho}_3^-$ and $\hat{\rho}_3^+$. The increase in dependence is also important in two countries such as Denmark and The Netherlands, which had, by far, the lowest values of the coefficients in 2008 but have converged to the situation of the rest of Europe. We also observe that Bulgaria, which in 2008 had the highest values of the coefficients, approached the situation of the rest of EU members, experimenting a decrease in the dependence between dimensions of poverty.

Figure 1 also suggests that dependence in the lower orthant is higher than in the upper orthant, as $\hat{\rho}_3^-$ is greater than $\hat{\rho}_3^+$ in the vast majority of countries over the entire period of analysis\(^6\).

\(^6\)In fact, $\hat{\rho}_3^-$ is greater than $\hat{\rho}_3^+$ in all the countries analysed except for Romania, where $\hat{\rho}_3^+$ is greater than
Thus, the dependence between dimensions of poverty is higher in the lower orthant than in the upper orthant.

Moreover, given equation (16) in Section 2, it is possible to decompose the contribution of $\hat{\rho}_-^3$ and $\hat{\rho}_+^3$ to $\hat{\rho}_3$. That is, we can calculate the relative contribution of lower orthant dependence and upper orthant dependence to average orthant dependence. The last two columns of Table 1 in the Appendix show the contribution of $\hat{\rho}_-^3$ and $\hat{\rho}_+^3$ to $\hat{\rho}_3$ in the years 2008 and 2014.

As expected, in all countries except for Romania and for both years the contribution of the dependence in the lower orthant (measured by $\hat{\rho}_-^3$) is greater than the contribution of the dependence in the upper orthant (measured by $\hat{\rho}_+^3$). More importantly, the contribution of both coefficients has remained (in general) rather stable between 2008 and 2014, as the trends observed for $\hat{\rho}_-^3$ and $\hat{\rho}_+^3$ are very similar (see Figure 1 again).

Table 1 in the Appendix also shows the estimated values of $\rho_-^3$, $\rho_+^3$, $\rho_3$ and $\rho_{3}^{\text{max}}$ for the years 2008 and 2014 and for all the EU28 countries. Apart from the general increase in the coefficients between 2008 and 2014 as well as the higher dependence in the lower orthant, we find that in both years and for all countries\(^7\), $\rho_{3}^{\text{max}}$ coincides with $\hat{\rho}_-^3$, indicating that the direction of maximal dependence is $(-1,-1,-1)$. That is, the highest dependence between the dimensions of poverty is found in the lower orthant. This means that small values of the three poverty dimensions (income, no material needs and work intensity) tend to occur together. Furthermore, this simultaneous occurrence is more likely in 2014 than in 2008.

It is also possible to decompose $\hat{\rho}_3$ to obtain the relative contributions of each of the pairwise coefficients to overall multivariate dependence; see equation (16). The last two columns of Table 2 in the Appendix display these relative contributions for years 2008 and 2014. Table 2 also shows the estimated values of the three pairwise Spearman’s rho correlation coefficients, namely, the correlation between: income and work intensity ($\hat{\rho}_{12}$), income and material deprivation $\hat{\rho}_-^3$.\(^7\) Except for Romania, where $\rho_{3}^{\text{max}}$ coincides with $\hat{\rho}_+^3$.
(\hat{\rho}_{13}) and work intensity and material deprivation (\hat{\rho}_{23}). As expected, most coefficients have increased its magnitude in 2014 as compared to 2008. With regard to the contribution of each pairwise coefficient to \hat{\rho}_3, we observe that, in general, the highest contribution is given by the dependence between income and work intensity. The lowest contribution is in all cases given by the dependence between work intensity and material deprivation. These relative contributions remain rather stable between 2008 and 2014, although we observe some countries, such as Spain, Cyprus or The Netherlands, for which the contribution of the dependence between work intensity and material deprivation markedly increased between those two years, with a decrease in the relative contribution of the other two pairwise coefficients.

As we have stated before in this paper, the aim of the analysis of the dependence between the dimensions of the AROPE rate is to complement the information given by this measure. In this context, it seems interesting to analyse the relationship between these two aspects, namely the incidence of multidimensional poverty and the dependence between its dimensions. Figure 2 depicts this relationship. In particular, scatter plots of the AROPE rates\(^8\) against \(\hat{\rho}_3\) are presented for years 2008 and 2014\(^9\). We observe a positive relationship between rates of multidimensional poverty and \(\hat{\rho}_3\). That is, countries with high incidence of multidimensional poverty tend to present a high degree of dependence between its dimensions. We can also observe the general increase in the dependence between dimensions of poverty between 2008 and 2014 and the convergence of those countries which had extreme values of \(\hat{\rho}_3\) in 2008 to the situation of the majority of EU28 members.

\(<\text{insert Figure 2 here}>\)

\(^8\)The AROPE rate is calculated as the proportion of households in our sample that are poor in at least one of the three dimensions considered.

\(^9\)We focus on the dependence in the lower orthant because, as we have shown in Figure 1, the patterns followed by the three coefficients are very similar.
4 Conclusions

There is a widespread agreement on the multidimensional nature of poverty. That is, the poverty status of individuals or households is not uniquely determined by their income, but also by other factors such as health, education or labor. In this context, the interrelation between dimensions is a key aspect of multidimensional poverty, as it is the main feature that differentiates it from the unidimensional analysis. Despite its relevance, most of the multidimensional poverty indices proposed so far do not take sufficiently into account this aspect.

In this paper, we propose to measure the dependence between dimensions of poverty using copula-based methods. The copula approach, which focuses on the positions of the individuals across the different dimensions, enables the estimation of scaled-free measures of dependence that capture other types of dependence beyond linear correlation. We propose the use of multivariate generalisations of Spearman’s rho, which measure the dependence between the dimensions both in the lower and in the upper orthant. We have reviewed the definitions and main properties of these coefficients, along with their empirical versions.

In our empirical application, we have used multivariate generalisations of Spearman’s rho to analyse how the dependence among the dimensions of poverty has evolved in the European Union over the period 2008-2014. The dimensions considered are those included in the AROPE rate: income, material needs and work intensity. The aim is to complement the information given by the AROPE rate, that is, the information about the incidence of multidimensional poverty, with measures of the dependence between its dimensions.

Our main conclusion is that the degree of dependence among income, material needs and work intensity has noticeably increased in Europe between 2008 and 2014. This rise is particularly important in those countries most hardly hit by the economic crisis, namely Spain and Greece. Moreover, the increase in the dependence is found both in the lower and in the
upper orthant. This means that not only the dependence among dimensions of poverty has increased significantly from 2008 to 2014, but also the society has become more polarised, with poverty dimensions (income, no material needs and work intensity) being more likely to have simultaneously small (large) values in 2014 than in 2008.

We also find that, in the vast majority of European countries, the dependence in the lower orthant is higher than in the upper orthant and thus contributes more to average orthant dependence. Moreover, the maximal dependence is found in the lower orthant.

With regard to the relationship between the incidence of multidimensional poverty and the dependence between its dimensions, we find it to be positive. That is, countries with high values of the AROPE rate tend to experiment also a higher degree of dependence between the dimensions of poverty.

Acknowledgments

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References


5 Appendix

Proposition 1. The plug-in estimator of $\rho_d$ defined in (12) in Section 2.3, namely

$$\hat{\rho}_d = \frac{\hat{\rho}_d^- + \hat{\rho}_d^+}{2},$$  \hspace{1cm} (A1)

where $\hat{\rho}_d^-$ and $\hat{\rho}_d^+$ are the estimators in (10) and (11), respectively, coincides with the estimator of $\rho_d$ proposed by Dolati and Úbeda-Flores (2006) in the framework of AOD measures of multivariate concordance.

Proof. The estimator of $\rho_d$ proposed by Dolati and Úbeda-Flores (2006), that will be denoted as $\hat{\rho}_d^{DUF}$, is as follows (see Example 5.1 in that paper):

$$\hat{\rho}_d^{DUF} = \frac{1}{n} \frac{n d \sum_{j=1}^{n} \left[ C'(R_{ij}/n + 1, \ldots, R_{dj}/n + 1) + \overline{C}'(R_{ij}/n + 1, \ldots, R_{dj}/n + 1) \right] - a_{d,n}}{b_{d,n} - a_{d,n}},$$  \hspace{1cm} (A2)

where

$$C'(R_{ij}/n + 1, \ldots, R_{dj}/n + 1) = \prod_{i=1}^{d} R_{ij}/n + 1, \quad \overline{C}'(R_{ij}/n + 1, \ldots, R_{dj}/n + 1) = \prod_{i=1}^{d} (1 - R_{ij}/n + 1),$$  \hspace{1cm} (A3)

and

$$a_{d,n} = \frac{1}{2^{d-1}}, \quad b_{d,n} = \frac{1}{n} \sum_{j=1}^{n} \left( \frac{j}{n + 1} \right)^d + \frac{1}{n} \sum_{j=1}^{n} \left( 1 - \frac{j}{n + 1} \right)^d.$$  \hspace{1cm} (A4)

Putting back (A3) and (A4) in (A2), the following expression comes up:

$$\hat{\rho}_d^{DUF} = \frac{1}{n (n + 1)^d} \frac{n d \sum_{j=1}^{n} \left[ \prod_{i=1}^{d} R_{ij} + \prod_{i=1}^{d} (n + 1 - R_{ij}) \right] - \frac{1}{2^{d-1}}}{\frac{1}{n (n + 1)^d} \left[ \sum_{j=1}^{n} j^d + \sum_{j=1}^{n} (n + 1 - j)^d \right] - \frac{1}{2^{d-1}}}.$$  \hspace{1cm} (A5)

Now, multiplying both the numerator and the denominator of (A5) by $(n + 1)^d$ and taking into
account that \( \sum_{j=1}^{n} j^d = \sum_{j=1}^{n} (n + 1 - j)^d \), we have:

\[
\hat{\rho}_d^{DUF} = \frac{1}{n} \sum_{j=1}^{n} \left( \prod_{i=1}^{d} R_{ij} + \prod_{i=1}^{d} \overline{R}_{ij} \right) - \frac{(n+1)^d}{2^{d-1}},
\]

(A6)

where \( \overline{R}_{ij} = n + 1 - R_{ij} \). On the other hand, replacing \( \hat{\rho}_d \) and \( \hat{\rho}_d^+ \) in (A1) by their expressions in (10) and (11), respectively, the expression in (A6) comes up. Hence, it turns out that \( \hat{\rho}_d = \hat{\rho}_d^{DUF} \) and the result is proven.

\[\square\]

**Proposition 2.** In the trivariate case \( (d = 3) \), the plug-in estimator of \( \rho_3 \) defined in (12) in Section 2.3 can be computed as the average of their corresponding pairwise sample coefficients, that is,

\[\hat{\rho}_3 = \frac{\hat{\rho}_{12} + \hat{\rho}_{13} + \hat{\rho}_{23}}{3},\]

(A7)

where \( \hat{\rho}_{ik} \) denotes the bivariate sample Spearman’s rho for the pair \((X_i, X_k)\), with \(1 \leq i < k \leq 3\).

**Proof.** First, from equations (13), (14) and (15) in Section 2.3 of the paper, we obtain the following expression for \( \hat{\rho}_3 \).

\[\hat{\rho}_3 = \frac{\hat{\rho}_3 + \hat{\rho}_3^+}{2} = \frac{1}{2} \left[ \frac{8}{n(n-1)(n+1)^2} \sum_{j=1}^{n} (R_{1j} R_{2j} R_{3j} + \overline{R}_{1j} \overline{R}_{2j} \overline{R}_{3j}) - \frac{2(n+1)}{n-1} \right].\]

(A8)

Now, taking into account that \( \overline{R}_{ij} = n + 1 - R_{ij} \), we have

\[\overline{R}_{1j} \overline{R}_{2j} \overline{R}_{3j} = (n + 1)^3 - (n + 1)^2 \sum_{i=1}^{3} R_{ij} + (n + 1)(R_{1j} R_{2j} + R_{1j} R_{3j} + R_{2j} R_{3j}) - R_{1j} R_{2j} R_{3j},\]

and so the summation in (A8) becomes

\[\sum_{j=1}^{n} (R_{1j} R_{2j} R_{3j} + \overline{R}_{1j} \overline{R}_{2j} \overline{R}_{3j}) = n(n+1)^3 - (n+1)^2 \sum_{i=1}^{3} \sum_{j=1}^{n} R_{ij} + (n+1) \sum_{j=1}^{n} (R_{1j} R_{2j} + R_{1j} R_{3j} + R_{2j} R_{3j}).\]
Now, since $\sum_{j=1}^{n} R_{ij} = 1 + 2 + \ldots + n = \frac{n(n+1)}{2}$, the expression above becomes

$$\sum_{j=1}^{n} (R_{1j}R_{2j}R_{3j} + \bar{R}_{1j}\bar{R}_{2j}\bar{R}_{3j}) = -\frac{1}{2}n(n+1)^3 + (n+1)\sum_{j=1}^{n} (R_{1j}R_{2j} + R_{1j}R_{3j} + R_{2j}R_{3j}). \quad (A9)$$

Putting back (A9) in (A8), it turns out that

$$\hat{\rho}_3 = \frac{4}{n(n^2 - 1)} \left( \sum_{j=1}^{n} R_{1j}R_{2j} + \sum_{j=1}^{n} R_{1j}R_{3j} + \sum_{j=1}^{n} R_{2j}R_{3j} \right) - 3\frac{n+1}{n-1}. \quad (A10)$$

On the other hand, the average of the pairwise sample Spearman’s coefficients is

$$\frac{\hat{\rho}_{12} + \hat{\rho}_{13} + \hat{\rho}_{23}}{3} = \frac{1}{3} \left[ \frac{12}{n(n^2 - 1)} \left( \sum_{j=1}^{n} R_{1j}R_{2j} + \sum_{j=1}^{n} R_{1j}R_{3j} + \sum_{j=1}^{n} R_{2j}R_{3j} \right) - \frac{9(n+1)}{n-1} \right] =$$

$$= \frac{4}{n(n^2 - 1)} \left( \sum_{j=1}^{n} R_{1j}R_{2j} + \sum_{j=1}^{n} R_{1j}R_{3j} + \sum_{j=1}^{n} R_{2j}R_{3j} \right) - \frac{3(n+1)}{n-1}. \quad A11 (17)$$

Hence, from (A10) in (17), the result in (A7) comes up. \qed
\[
\hat{\rho}_3^{-3} + 3 \hat{\rho}_3^{+3} + 3 \hat{\rho}_3^{\text{max}}
\]

Table 1: Estimated coefficients of trivariate dependence between the three AROPE dimensions.
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<tr>
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<td>33.46%</td>
<td>33.48%</td>
</tr>
</tbody>
</table>

Table 2: Estimated pairwise Spearman’s rho coefficients between the AROPE dimensions.

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Figure 1: Evolution of $\hat{\rho}$ (solid line), $\hat{\rho}^+$ (dashed line) and $\hat{\rho}$ (dotted line)
Figure 2: Relationship between AROPE rate and the coefficients of multivariate dependence.