Entrepreneurial Production Function and Firm Dynamics

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May 2017

Abstract

The objective of this paper is to understand the role of business owners’ working hours in the production of their firms. We use a novel dataset on US start-ups to show that their owners work non-negligible hours, which correlate strongly with the firm-level output and employment. In order to interpret better the empirical evidence, we study an entrepreneurial production function that parametrizes the role of owners’ hours into the degree of substitutability/complementarity they have with the hired labor. The calibration of the model to match important empirical features suggests large substitutabilities between owners’ hours and their hired labor. We show that hours adjustments of owners can be crucial in coping with the financial constraints, and omitting these can lead to a substantial overstatement of the impact of financial frictions on firms’ life-cycle profiles.


Keywords: Entrepreneurship, labor supply, production, firm dynamics.

1 Introduction

One of the most staggering facts about US business dynamics is the large heterogeneity in the life-cycle profiles of new start-ups. Hopenhayn (2013) finds that close to 50% of jobs created

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by an entering cohort are lost due to exit within the first 5 years. Among the ones that survive, many remain small, but a significant portion of start-ups sustain rapid employment growth to become large corporations and contribute to the growth and levels of aggregate output in the economy.

Typically, discrepancies between firms’ life-cycle performances are attributed to factors that entrepreneurs take as given, such as the quality of the business idea and exogenous profitability shocks. Along this line, Hopenhayn (1992), Hopenhayn and Rogerson (1993), and Cabral and Mata (2003) study the growth of business start-ups with heterogeneous quality of ideas and exogenous idiosyncratic profitability shocks. A key question in this respect is whether markets are able to allocate resources to those firms with the most promising business outlook. Imperfect borrowing markets shape life-cycle employment growth patterns and hinder this efficient allocation of resources, as shown theoretically and quantitatively by more recent influential papers such as Cagetti and De Nardi (2006), Buera and Shin (2013) and Midrigan and Xu (2014).

Meanwhile, the role of the entrepreneurial hours can be key in determining their firms’ performance. Correctly identifying this role is crucial in studying firm dynamics for three main reasons. First, to the extent that entrepreneurial hours substitute the hired labor, entrepreneurs can exert their own effort to overcome financial constraints when these make it hard to hire external labor. On the other hand, if owner hours are rather complementary to the hired labor, natural limit of their total time puts an endogenous limit to the span-of-control of their firms.

Second, some part of the variation in total factor productivity among firms might be related to their entrepreneurs’ effort. Ignoring the role of the variation in the latter on the firm-level variation in productivity can lead to overestimating the role of uncertainty that entrepreneurs’ take as given. To the extent that they can foresee and choose the quantity of their managerial input, the uncertainty that they consider would be milder than the one estimated by a researcher that does not take into account entrepreneurs’ function in their business.

In a similar vein, the final motive to better capture entrepreneurs’ role in their firms is to improve our estimates of misallocation of resources. In particular, variation in the
entrepreneurial input might naturally lead to variation in the marginal products of labor, which the literature often takes as evidence of misallocation of resources.

In this paper, we first use data from the US to highlight new facts on owner hours and firm performance of the firms. Our main data source is the Kauffman Firm Survey (KFS), a panel dataset on US start-ups provided by the Kauffman Foundation. Meanwhile, we also complement our empirical analysis using, Survey of Business Owners, a cross-sectional data provided by the US Census. We show that owners’ hour form an non-negligible part of the total labor supply within firms. Owner’s contribution as an input to the firm is sizable not only for firms without employees. The share of owners in their firms’ total working hours is around 80 percent during the first year of operation. As firms age, they get bigger and the role of entrepreneurs in their firms diminishes. The median share of owners’ hours in the firm almost halves by the eighth year of operation. Importantly, this decrease is almost exclusively driven by an increase in hired labor, not a decrease in owners’ working hours. Even in year eight of operation at firms with more than 32 employees, the average weekly working hours of the owner is around 40. Such a high work load in a large firm would be difficult to rationalize, if the owner’s hours could be easily substituted by outside labor.

Moreover, we show that owner hours correlate positively with alternative measures of firm performance such as output, profits, and employment. This pattern is particularly strong for small firms, though across age, the strength of this relationship is rather stable. We also document that firms whose owners work particularly long and short hours in the initiation of the business, have better life-cycle outcomes in the following years.

The empirical patterns we highlight suggest there might be gains from understanding better the role of owner hours in their production units. In order to do so, we build a dynamic model of entrepreneurship with two important features. First, entrepreneurs face borrowing constraints and have to decide how much labor and capital to hire. Second, and the non-standard feature of our model is that entrepreneurs’ working hours matter for their firms’ production. Here we use a general functional form to model the complementarity/substitutability of entrepreneurial input to the hired labor, and later let the data dictate us the parameter value regarding the nature of this relationship.

We then calibrate our model parameters to match important features of our data. We
discuss the implications of different degrees of the aforementioned substitutability on firm-level outcomes, and use these to identify the correct parameter for this feature. We show that the calibrated model performs well in many untargeted aspects of the data, such as the relationship between owner hours and output.

Using our calibrated model, we highlight the role of the variation in owner hours with three sets of exercises. First, we show that owners’ initially long (short) hours persistently brings the output and employment profiles of firms to a higher (lower) level. Second, we use a counterfactual exercise to argue that shutting down the hours adjustments in our model would lead to significant reductions in total output, and many owners would end up exiting the market if they are not allowed adjust their hours. Finally, we show that a recalibration of the model without allowing adjustments in owner hours would overstate the effects of financial frictions, since their own effort provide an important cushion to firm owners in coping with the borrowing constraints.

Our paper contributes to the literature by identifying the role of owners’ hours in the production of their firms. Accordingly, we take part in the recently developing line of research studying firm dynamics by introducing the managerial input as a factor in the production function. Bhattacharya, Guner, and Ventura (2013) studies the role of distortions in firms’ life-cycle profiles using a production function with the total factor productivity depending on the endogenous managerial ability. In particular, the evolution of the managerial ability depends on the investment made in such skills. In Lee (2012), managerial ability enters the production function in a similar way, and this ability depends on whether the manager is also the owner of the firm or not. The present paper differs from this line of research by restricting the sample to firm owners, and specifying the managerial input to their hours. This earns us ease of identification, as allowed by our dataset, at the expense of generality.

Labor supply of entrepreneurs to their businesses so far has not received much attention from the macroeconomic literature. Two contemporaneous exceptions are Yurdagul (2017) and Allub and Erosa (2014). The former paper takes a simple approach to owners’ hours than the present paper, and assumes a particular degree of complementarity between entrepreneurs’ hours and her labor by introducing owner hours as shifters of Hicks neutral productivity as in the aforementioned models with managerial input. It then studies the
role of differences in working hours flexibilities between entrepreneurs and workers in explaining the entrepreneurial decisions in the US. Meanwhile, Allub and Erosa (2014) uses a production function to which entrepreneurs can contribute both with their managerial hours and blue-collar hours. The focus is the distinction between employer and non-employer firms, and it uses the production function parametrizations in the literature with managerial input as a separate factor, to calibrate the weight of managerial hours separately. In contrast, our objective is to understand the reduced form role of owners’ hours in their businesses without distinguishing explicitly the allocation of their time into managerial and blue-collar tasks. For identification purposes, this serves well as our data does not provide information of owner’s hours supplied separately to these routines. Instead, we are able to identify one parameter that is informative on the overall substitutability between hired hours and owners’ hours using the information on the total of each item.

The paper is organized as follows. Section 2 provides evidence using the KFS to emphasize the motivation of the paper. Section 3 describes the model. Section 4 explains the calibration strategy of the paper and Section 5 documents the main results from this exercise, and discusses the model’s predictions for relevant empirical features. Section 6 shows the role of the adjustments in owner hours in shaping their life-cycle patterns, and it studies the implications of omitting these adjustments in the model.

2 Evidence

In this section, we document our motivating facts, which can shed some light on the selection and growth mechanisms underlying firms’ life-cycles. For this purpose, we use the Kauffman Firm Survey, a panel data that follows 4929 US firms starting their business in 2004, for the first eight years of their operations. Importantly, the universe of firms contains all those who either have an employer identification number, pay federal unemployment insurance, or pay income taxes according to Schedule C. The latter assures, differently from the datasets commonly used in literature (such as Business Dynamics Statistics), coverage of firms without

\[^{1}\text{See Robb and Farhat (2013) and DesRoches, Barton, Ballou, Potter, Zhao, Santos, and Sebastian (2007) for detailed descriptions of the data.}\]
any employees. This novel dataset is particularly useful for our purposes, as it provides information on characteristics and working hours of entrepreneurs, firms’ external inputs such as labor and capital and their revenues.

In order to complement our empirical analysis from the KFS, we also use a cross-sectional dataset provided by the US Census, namely the Survey of Business Owners (SBO) that is publicly available for year 2007. As KFS, SBO also covers entrepreneurs without any hired employees. The main advantage of the SBO data over the KFS is that it has a larger sample size (in the sense of the number of firms) and that it provides information from a wide range of cohorts (rather than one cohort as in KFS). Its limitation is the lack of panel structure and less detailed information on hours and the firms’ balance sheets.

In our KFS sample, as is the case for many panel data on firms, there is a large failure rate. In particular, 51 percent of the firms cannot survive to their eighth year of operations. Meanwhile, among the firms that survive all the eight years in the panel, the growth patterns in employment are stark. Among these businesses, an average two-year-old firm is more than twice as large as an average firm in the beginning. Average employment levels for different age groups in the SBO are also in line with the panel dynamics in the KFS, in that older firms tend to have strikingly more employment. In principle, it is not straightforward to argue a selection mechanism lying behind these patterns. However, it is clear that the lifecycle profiles of firms exhibit a vast heterogeneity, ranging from early exits to significant employment growth.

One persistent factor leading to contrasting lifecycle profiles can be the level of entrepreneurial effort in addition to the quality of the business idea. Panel A of Figure 2 shows that owners of firms in the KFS work for their firms for non-trivial amount of hours. Even more striking, owners’ hours remain high as the firm ages and they are higher for larger firms. This is also true in the SBO, in that owners work significant hours for their firms, and particularly so for those with large firms.

Similarly, Panel A of Figure 3 shows that the hours worked by owners form a significant part of the total labor input in firms’ production. In particular, the entrepreneurial hours in the KFS is about 80 percent of the total hours of the hired workers in the first year of operations for the median firm in the sample, and gradually declines to 70 percent of the
hired hours as firms age to seven years. Across size groups we also see consistent patterns, with firms employing 1 to 3 workers having the corresponding ratio around 1. As we look at larger firms, the owners’ hours relative to hired hours decline in a monotone way and become negligible as a share of total hours by the size group 64 to 127. Panel B shows that the patterns are qualitatively the same, and quantitatively very similar in the SBO.

There are at least three potential mechanisms shaping the role of entrepreneurial hours in production of firms. First, and the more direct one is the limited span-of-control of individuals on their enterprises. The total time and effort that a person, or a few partners, can devote to their firm is limited by nature. The right panel of Figure 2 illustrates this “natural limit” through the flattening pattern of entrepreneurial hours’ at the upper end of the size distribution of firms. Accordingly, as firms age and employ more workers, it is almost mechanical that the functions of entrepreneurs appear to decrease relative to the size of their operation. The other two mechanisms underlying the labor supply of entrepreneurs are how complementary/substitutable their hours are with respect to hired workers, and financial constraints on firms’ input decisions. To the extent that firms are financially constrained and owners’ hours can substitute external labor, entrepreneurs might prefer to work longer hours, especially in the initiation of their operations. This would be consistent with the decreasing pattern in owner hours as firms age illustrated in the left panel of Figure 2. Meanwhile, managerial tasks, which form an important part of an owners’ workload naturally
Panel A: KFS
(a) Over age
(b) Over size

Panel B: SBO
(a) Over age
(b) Over size

Figure 2: Owner hours across ages (left) and size

complement the hired workers’ jobs. This can create a higher opportunity cost of not working for owners of larger firms, and can explain the increasing pattern of owner hours for larger firms.

Importantly, there is a positive relationship between owner hours and output of firms. Figure 4 shows that owner hours (in logs) are positively correlated with various output measures and profits in both of our datasets. This relationship is particularly strong for small firms, and it gets weaker as we look at the larger ones. Notice that the magnitudes are similar in the two datasets, as both exhibit a correlation around 0.4 for the smallest firms which diminishes to 0 for firms larger than 15-20 employees. The corresponding correlation
Figure 3: Owner hours relative to hired labor across ages (left) and size

Patters are significantly more flat across age. In particular, for the firms in the KFS, output-owner hours correlations are stable around 0.4 for all ages and this is not different for our findings from the SBO dataset. Clearly, these findings do not indicate a causal relationship between owner hours and firm performance. However, it is a consistent pattern between the two datasets that longer owner hours are typically associated with more output and profit, especially for smaller firms, in which owner hours form a particularly high share of total labor.

One aspect for which we can exploit the panel structure of the KFS is to see how do initially long and short hours of owners relate to the following life-cycle patterns of firms.
Figure 4: Correlation between owner hours and output across size (left) and ages
Figure 5 shows the starkly different output and employment profiles of firms whose owners work more than 60 and less than 20 hours a week in the initial year of operations. In the former group of firms, the output is about 2 log points larger for the latter group for the initial years, and the differences do not seem to diminish over time. In terms of the hired labor, the group of firms with longer initial owner hours hire about 50 hours of work more on average in the initial year, which increases to about 150 hours by the second year and the differences remain stable from there on.

![Figure 5: Initial owner hours and the following life-cycle](image)

Overall, our findings first suggest that owners work non-negligible hours for their firms, and even those that operate medium size units shoulder a significant fraction of the total labor supply of their firms. In addition to being far away from 0, another property that we show from the owner hours is that they are typically associated with better output and employment outcomes over the life-cycle. Whether such correlations arise from causal relationships from owner hours to output and employment outcomes or not, it is evident that there can be potential gains from understanding better, “What is the role of owner hours in their production units?”

In the next section, we introduce our theoretical framework to identify the nature of this role, particularly the extend to which entrepreneurs complement or substitute the hired labor in their firms.
3 Model

The economy is populated by a unit mass of potential entrepreneurs who are infinitely lived. Time is discrete, and entrepreneurs discount the future at rate $\beta$. They possess a production technology where the labor supply of the owner is an input into the firm’s production function:

$$Y_{it} = z_{it} \left( K_{it}^{\alpha} \left( \kappa h_{it}^\lambda + L_{it}^\lambda \right)^{\frac{1-\alpha}{\lambda}} \right)^{\eta},$$

where $Y_{it}$ is value added of firm $i$ in period $t$, $K_{it}$ is the employed capital, $h_{it}$ is the hours of the owner, and $L_{it}$ is the hours of hired workers. $z_{it}$ is the stochastic idiosyncratic firm productivity and $\kappa$ determines the weight of entrepreneurial hours in the firm. $\lambda$ governs the substitutability of owners’ hours and hired labor.

At the beginning of each production period, the entrepreneur decides to continue her business, or exit the market. In case she continues, she decides on the next period capital stock, her own hours decision, and how much outside labor to hire at market prices $w$. Hiring a worker has a fixed per period cost $\xi$. In addition to endogenous exit decisions, entrepreneurs face exogenous exit shocks, which arrive with probability $\chi$ at the end of each period.

Together with the productivity shocks, the second source of exogenous variation is the disutility of working. We include this feature in the model to generate the observed variation in owners’ labor supply, consistently with the previous literature pointing at the role of the preference shocks in matching the variation in the working hours for workers and entrepreneurs. (See for instance Erosa, Fuster, and Kambourov (2016) and Yurdagul (2017).)

We abstract from the wealth effects in the labor supply by assuming GHH preferences.\(^2\) This specification proves useful for our purposes, as it simplifies the solution of the optimal labor supply of firm owners, hence help illustrate the model mechanisms and identify the model parameters.

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\(^2\)See Greenwood, Hercowitz, and Huffman (1988).
Formally, the value of being an entrepreneur is:

\[
V(z, a, v) = \max_{a', h, K, L} \left( \frac{c - v h^{1+\phi}}{1+\phi} \right)^{1-\gamma} - \psi + \beta(1 - \chi) E_{z', v'|z, v} \max\{V(z', a', v'), 0\} \tag{1}
\]

\[
c = \tilde{\Pi} + (1 + r)a - a' \tag{2}
\]

\[
\tilde{\Pi} = z \left( K^\alpha \left( \kappa h^\lambda + L^\lambda \right)^{\frac{1-\alpha}{\lambda}} \right)^\eta - wL - (r + \delta)K - I_{L>0}\xi \tag{3}
\]

\[
I_{L>0}(wL + \xi) + (r + \delta)K \leq \theta a \tag{4}
\]

\[
h \geq 0, \quad L \geq 0 \tag{5}
\]

where $\delta$ represents the capital depreciation rate, $\gamma$ is the coefficient of relative risk aversion, $\phi$ governs the individual labor supply elasticity, $v$ is the disutility of work, and $\psi$ is a shifter in the utility function. In case of exit, the individual receives a stock value of 0.

We assume that profitability of the idea, $z_{it}$, follows an AR(1) process:

\[
\log z_{it} = (1 - \rho_z) \log z_0 + \rho_z \log z_{i,t-1} + \epsilon_{it}, \quad \epsilon_{it} \sim N(0, \sigma_\epsilon).
\]

At birth, entrepreneurs draw a disutility of work parameter $\log v_{i0} \sim N(\log v_0, \sigma_v^2)$. Afterwards, the log disutility of working follows a highly persistent AR(1) process:

\[
\log v_{it} = (1 - \rho_v) \log v_0 + \rho_v \log v_{i,t-1} + u_{it}, \quad u_{it} \sim N(0, \sigma_v).
\]

We can isolate the static problem of an entrepreneur, taking as given the productivity level $z$, hours devoted to working $h$, and wealth $a$:

\[
\Pi(z, h, a) = \max_{K, L \geq 0} z \left( K^\alpha \left( \kappa h^\lambda + L^\lambda \right)^{\frac{1-\alpha}{\lambda}} \right)^\eta - wL - (r + \delta)K - I_{L>0}\xi
\]

s.t.

\[
I_{L>0}(wL + \xi) + (r + \delta)K \leq \theta a.
\]

The following section describes the calibration of the model parameters. Then the description of the main results from the benchmark model will follow.
4 Calibration

The model period is one year, in line with the interval between the waves of our panel data. Our calibration strategy splits the parameters into two groups. First group is the standard parameters that are not directly related to our model features, and we take them from the literature. In particular, we follow Cagetti and De Nardi (2006) in setting our capital share at 0.33, depreciation rate at 0.06, and risk aversion at 1.5. We set the discount factor $\beta$ following Midrigan and Xu (2014). We assume a risk free interest rate of 0.05 and normalize the wage to 1. For the parameter $\phi$, we use value of 3.33 which corresponds to an intermediate degree of labor supply elasticity.

We calibrate our second group of parameters to match certain targeted moments that seem natural choices for each parameter. Importantly, most of our calibration targets are statistics on age 8 of our sample firms, the last year of our panel in which we expect firms to be less subject to the frictions we do and do not model explicitly.

Regarding the stochastic process of productivity, our structural model features only persistent shocks. We assume that log productivity in the data follows a mixture of a persistent $AR(1)$ process and transitory shocks:

$$
\ln(z_{it}) = \nu_{it} + \tau_{it}
$$

$$
\nu_{it} = (1 - \rho_z)z_0 + \rho_z \nu_{it-1} + \epsilon_{it}
$$

$$
\tau_{it} = MA(q).
$$

We identify the autocorrelation of persistent shocks by minimum distant estimation of the autocovariance function of output growth. This leads to $\rho_z = 0.92$. We use the mean of the productivity process, $z_0$, to match an average hired labor of 104 hours at age 8. We target cross-sectional output dispersion with the standard deviation of productivity shocks, $\sigma_\epsilon$. In the data, a significant fraction of firms does not produce output (their intermediate inputs are larger than their final output). To avoid distortions from the lower tail, we match the 90/50 ratio of output in the data.

We want to assure that the largest firms can grow as much as in the data. A crucial
parameter determining the optimal size and the profitability of firms is the span-of-control parameter $\eta$. We calibrate this parameter to match the upper tail of the size distribution in our sample, namely the ratio of total labor hired by the 95th to 90th percentile in the size distribution. The resulting value we obtain for $\eta$ is 0.89. This is within the range of the literature, which typically varies from 0.79 to 0.95.\footnote{See for instance, Buera, Kaboski, and Shin (2011) which assumes a value of 0.79, Midrigan and Xu (2014) which uses a value of 0.85, and Cagetti and De Nardi (2006) which uses a value of 0.88.}

We calibrate the parameters of the disutility of work process to replicate key relevant moments of entrepreneurial hours. We set the mean disutility, $v_0$, to match that entrepreneurs work on average 38 hours in year 8. We set the standard deviation of initial value, $\sigma_{v0}$, and that in the ergodic distribution, $\sigma_v$, to match the 75/25 ratio of owner hour dispersion at ages 0 and 8 in the data, respectively. Finally, we match with the persistence of the disutility, $\rho_v$, the autocorrelation in owner hours.

We assume that firms start operating with some initial assets. These assets are drawn randomly from a normal distribution (truncated at zero). We calibrate the mean initial asset holdings, $\mu_a$, targeting the mean size of firms during their first year of operation. This implies a value of initial mean assets equivalent to a payroll of 1.1 workers for a year. We set the standard deviation, $\sigma_a$, to 2.3 which corresponds to the dispersion of initial equity in the SBO data.

In the data, many firms (almost) do not grow over subsequent years but some firms grow very rapidly. We consider this rapid growth as informative on how binding credit constraints are, as with time, owners of productive firms can accumulate wealth to cope with the constraints and grow closer to their desired size. In fact, in our model, more severe debt constraints (lower $\theta$) imply steeper growth profiles of the average firm. Accordingly, we calibrate $\theta = 4.1$ to match the mean employment growth between years one and eight.

At the same time, as shown above, a key fact from the data is that many firms do not hire an employee at any stage of their first eight years. We calibrate the fixed costs of hiring labor, $\xi$, to match the amount of firms with zero employees at age 8 in the SBO data. This implies a fixed cost of 12% of one yearly worker.

We want to assure that, as in the data, 49 percent of firms survive until age eight. To
this end, we set the shifter in the utility function to $\psi = -3.35$. In our model, endogenous exit rate becomes negligible for older firms and the exogenous exit rate $\chi$ determines the rate of exit. Assuming constant entry, this implies that the number of firms with ages between $j$ and $j + T$, relative to the number of firms with ages between $h$ and $h + T$ for $h > t$, $T > 0$ and $t$ large enough is given by $(1 - \chi)^{h-t}$. In SBO, the number of firms of age 18 to 27 is 59 percent of that of firms from 8 to 17. If there is no endogenous exit beyond age 8, and there is constant entry -as is the case in our model, this requires a $\chi$ of 5.1 percent.

**Identification of complementarity between owners’ hours and hired labor.** An important parameter for our purposes is the substitutability parameter, $\lambda$, in the labor aggregation. The identification of this parameter comes from the relationship between firm size and owner hours.

The model does not allow for full characterization of the solution, mostly due the non-convexities associated to the financial constraints of entrepreneurs and the exit choice which is discrete in nature. However, the role of the substitutability between owners’ hours and hired labor, $\lambda$, can still be highlighted using some particular cases. Suppose we consider the problem of an entrepreneur that is not restricted by the borrowing constraint and optimally hires external labor. The first order conditions for this entrepreneur implies that the own hours and the hired labor satisfy:

$$vh^{\phi+1-\lambda} = \kappa wL^{1-\lambda}. \quad (6)$$

Holding the disutility of working constant and assuming imperfect substitution (i.e. $\lambda < 1$), nonconstrained entrepreneurs hiring workers would work less as their firms get bigger. As the complementarity between own and hired hours increases (i.e. lower $\lambda$) increases in size would translate more into increases in owner hours, since then the marginal productivity of owner hours depend more positively on the size of the firm. Needless to say, taking into account the endogeneity of the labor choice, fixed costs of hiring, and the financial constraints, this mechanism would translate only indirectly into the relationship between $\lambda$ and observed hours-size relationship. Meanwhile, the aforementioned intuition highlights an important
channel that is key in our identification of $\lambda$. In this vein, we calibrate $\lambda$ to match the fact that an entrepreneur spends on average 54 hours in a firm with $16 - 31$ employees.

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<th>Value</th>
<th>Basis</th>
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<td>$\alpha$</td>
<td>0.33</td>
<td>Cagetti and De Nardi (2006)</td>
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<td>$\delta$</td>
<td>0.06</td>
<td>$\gamma$</td>
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<td>$\gamma$</td>
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<td>$\beta$</td>
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<td>Midrigan and Xu (2014)</td>
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<td>Inverse of labor supply elasticity</td>
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<td>$r$</td>
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<td>$\psi$</td>
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Table 1: Parameters

Note: Unless noted otherwise, the target statistics come from the KFS data that we use in Section 2; and the statistics are from firms of age 8.

5 Results

5.1 Moments

The model matches the targeted moments well. The overall average of owner hours at year 8, the target in setting the average disutility of working, is at 38 hours per week for all three alternative span-of-control choices of the model, and it is 37 hours in the data. Similarly, owners of businesses with 15 to 31 employees work around 54 hours per week in the model and
in the data, which is targeted with the choice of the substitutability parameter $\lambda$. The model also matches the ratio of 75th to 25th percentiles of owner hours at years 0 and 8, the targets for the dispersion in disutility of working in the beginning and in the ergodic distribution. Importantly, the average firm size in year 8 and in the initiation in the model and in the data are also very close, around 105 hired hours for year 8, and around 57 hired hours in the beginning of operations. These have been the targeted moments for the tightness of financial constraints and the initial level of assets of firm owners. Our productivity process also replicates well the output inequality and the persistence observed in the data. The ratio of 90th to 50th output percentiles are around 12 for the data and for the model, and persistence is around 0.90. Finally, we target the survival rate to age 8 with the preference parameter for entrepreneurship, $\Psi$, and the fraction of non-employer firms at age 8 with the fixed costs of employing $\xi$, and our models match both moments very tightly, with around 50 percent rates for both moments.

The model performance in the related untargeted moments are also assuring. In particular, the model fits well the untargeted distributional moments of owner hours, hired labor, output and the ratio of hired labor to owner hours, observed in the data. The model fits also the average hired labor to owner hours around 2.7 observed in the data. This tells us that our simplified approach of setting the parameter of relative weight of owner hours in the production, $\kappa$, equal to unity is rather innocuous as this would have been a natural target for this parameter.

Average firm growth in hired labor is an important measure of firm success, and is also a good reflection of the financial constraints underlying the firm dynamics. In the data, we find that many surviving firms grow in their size, so that the median growth rate is positive in the sample. By targeting the mean growth rates, the model matches well this feature by generating a positive growth rate for the median surviving firm.
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<tr>
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<tr>
<td>Fraction no workers*</td>
<td>0.52 0.53</td>
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Table 2: Summary of results
An important focus of this paper is the weight of entrepreneurs’ hours in their operations and particularly how they relate to the hired hours for different age and size groups of firms. It is assuring that in this aspect the model matches the observed qualitative and quantitative features of the data. For younger firms, the hours of entrepreneurs are about 80 percent of the hired hours for the average firm and gradually decays down to about 40 percent of the hired hours. Similarly, for the smallest employer firms, the owners’ hours are around 80 percent of the hired hours and for firms of the highest size category in the sample, owners’ hours to hired hours ratio becomes very close to zero.

![Figure 6: Size and age (model)](image)

In the data, we highlighted that the correlations between owner’s hours and the output of their firms are typically decreasing across size categories, and flat across ages. Similarly, the left panel of Figure 7 shows that in our benchmark calibration we find that for larger firms this correlation is clearly smaller, as is the case in the data. The mechanism that the model generates such a pattern is the high degree of substitutability between owners’ hours and hired labor in our calibration. To the extent that owners’ hours and hired labor are substitutes and there is diminishing returns to labor, the marginal productivity of owners’ hours is albeit smaller in larger firms.

The right panel of Figure 7 shows that the correlation between owner’s hours and output implied by the model is around 0.33 for all ages of firms, which also resembles the features in the data both in terms of the levels and in terms of the slope. In our model, there are
different channels that imply different slopes for this relationship. First, as firms age, they are typically less restricted by the financial restrictions and they grow in size. In line with the patterns of this correlation with size (left panel), this gives a decreasing correlation pattern with age. Meanwhile, for the firms that are restricted by the financial constraints, there is a positive correlation between the firms’ exogenous productivity, and the owners’ hours. As we use a sample of survivors of 8 years in these figures across ages, and these firms are more likely to increase their productivity over these years, this gives a positive channel to the relationship between this correlation and age. In result, we have the rather flat profile as shown in the figure. The corresponding correlations with profits, instead of output, are also flat in the model, as is the case in the data.

Figure 7: Correlations between owner hours and output (model)

What is the relationship between the owners’ initial effort in their business and the consecutive life-cycle profiles of their firms? In this aspect there are two different channels that would shape the dynamics in the model. First, owners’ that work long hours in the first years of the business are likely to work more in the following years if the factors that make them work long hours (low disutility of working, high firm productivity) in the beginning persist. The left panel of Figure 9 illustrates this, showing that owners that work particularly long (short) hours in the beginning of the operations typically for long (short) hours in the years to come. This would give a positive affect in keeping output of these firms high (low) persistently. Second, the initial effort of the owners itself can help them build the capital
they need, and increase the following output levels. In fact, the right panel of Figure 9 shows that the firms whose owners work long hours in the beginning maintain a certain output level throughout the 8 years, and do not face a decline in output similar to the decline in their own hours. This is due to the fact that these owners can find resources later on to substitute their hours with those of the hired labor.

![Graphs showing initial hours and the life-cycle (model)](image)

Figure 8: Initial hours and the life-cycle (model)

The correlations depicted in Figure 7 and the initial owner hours and the life-cycle relationship in Figure 9, are important to see the model performance as they replicate closely the patterns highlighted in the data. However, they cannot be looked at through the perspective of causality, as all else equal, output and the owner hours both increase with firm size in our benchmark calibration. Later in Section 6, we study in more detail how the variation in owner hours indeed affects the life-cycle profiles of their firms by exploiting the exogenous variation in the disutility of working in model simulations.
Substitutability between owner hours and hired labor. An important highlight of our calibration is that it dictates a substitutability parameter, $\lambda$, of 0.75 to match the hours of owners of larger businesses. Here, we discuss the role of this substitutability in the model, by running two models with alternative values of $\lambda$, namely setting it equal to 0.3 and 1.

There are many moments that are sensitive to this parameter, but here we focus on two of them. First, is our calibration target for this parameter, which is the hours of owners of large businesses (15 to 31 workers). Consistently with the mechanisms highlighted in Section 4, higher (lower) substitutability indeed implies much longer (shorter) working hours of owners of large firms, as this parameter governs how fast the owners’ role in the business diminishes as firms grow in size. To be specific, Table 3 shows that the case of perfect substitutability implies owners of such firms to work the same as the unconditional mean in the sample. In fact, Figure 9 shows that for any size of firms, owners always work the same amount in case of full substitution. Meanwhile, for a parametrization with less substitutability than the benchmark, as in the case with $\lambda = 0.3$, the model overshoots the hours of owners in big businesses. As the parameterization goes more in the complementarity direction than substitution, marginal productivity of owners’ hours increase more as firms get larger, hence all else equal, imply longer hours for owners of big businesses.

Second important feature that is very sensitive to the substitutability parameter $\lambda$ is the employment behavior of firms. Especially the size of less productive firms reacts strongly to small variations in $\lambda$, as the owners’ labor supply might be enough to absorb the entire workload of these firms in case they can substitute well the external labor. In fact, Table 3 shows that inserting full substitutability to the benchmark calibration, implies that 80 percent of the firms in age 8 do not employ any workers. Meanwhile, higher complementarities than the benchmark quickly makes workers indispensable for the firms’ production, making all the firms hire at least some labor in the economy when $\lambda$ is set at 0.3.

It should be noted that we do not recalibrate the rest of the parameters in the alternatives described above. When we perform a full recalibration for $\lambda$ equal to 1, even without the fixed costs of hiring, only 40 percent of the firms hire workers. We find this counterfactual as hiring a worker naturally brings some fixed costs. Meanwhile, the implications highlighted in this section remain in a comparison of the recalibrated parameterizations of these alternatives.
6 Role of variation in entrepreneurial hours

In this section, our objective is to highlight the role of owners’ effort in the trajectory of their firms. There are three questions we want to answer: (1) How does the variation in owner hours affect the life-cycle patterns of their firms? (2) What is the role of adjustments in owners’ hours? (3) How does the implications of our model differ from those that do not allow for these adjustments? We answer the first question through an ex-post analysis, in which we exploit the exogenous variation in the disutility of working in our model simulations, and relate it to the output and employment profiles that follow. Answering the second question, instead, calls for a counterfactual exercise, keeping the model parameters as in our benchmark calibration. Meanwhile, the last question should be addressed by recalibrating the model with fixed owner hours, as a researcher who thinks this is the correct model would do.
6.1 Ex-post analysis: Effects of variation in owner hours on the firms’ output and employment

In Section 5 we showed that initially high and low owner hours are associated with particularly good and bad performance of firms later on. As informative as this is for the purposes of squaring the model implications with the patterns observed in the data, that exercise was far from showing a causal relationship. Importantly, we can use the variation in initial disutility of working to highlight the causal channel of the aforementioned findings. In the end, this shock is independent from rest of the shocks and states in our model, and the we can interpret the corresponding differences in the life-cycle patterns as results of the high and low effort of the owners in the beginning.

Figure 10 shows that when owners have particularly low (bottom 20 percent) disutility of work initially, their firms prosper very differently from those of owners with high (top 20 percent) disutility initially. The disutility of working is a mean reverting process, and the difference between the hours of the two groups converge to each other with time, but that does not generate converging patterns in their output or employment. Long owner hours can generate enough support for the firms to keep their profiles up permanently.

6.2 Counterfactual: No hours adjustments in the model

Suppose that we take our benchmark model and its corresponding calibration as the true reflection of what entrepreneurs go through. How does having the ability of adjusting their hours affect the path of their firms? To answer this question, we equate the owners’ hours to the mean in our sample at year 8 of operations, which is 38 hours.

The role of hours adjustments is in strong interaction with the effects of financial frictions in the economy. As we highlight in the calibration section, more severe financial frictions make the employment and output profiles of firms more steeply increasing, as firms fight through financial frictions in the ways they can to grow out of the constraints. Since older firms have more time to accumulate assets and grow out of constraints, this gives an increasing profile of employment against age. Meanwhile, adjusting their own hours is an important cushion that the owners have when they are restricted in size and they want to
produce more and accumulate wealth. Accordingly, hours adjustments can help them grow into higher levels of output and size in the presence of financial constraints.

The results from the experiment of forcing all the owners to work equal to the average in the data are consistent with this intuition. The third column of the Table 4 shows that when the adjustments in hours are omitted, a typical firm cannot grow as much as it does in the baseline with flexible hours. The growth of the mean firm shrinks by 50 percent, and that of the median firm reduces to a third of the baseline.

Another related aspect of firm dynamics that changes sharply without hours adjustments is the rate of survival. In particular, all else equal, taking away the tool of hours adjustments from entrepreneurs reduces the survival probably to a third of the baseline.
6.3 Recalibration: Model without hours adjustments and predicted effects of financial frictions

A different question from the one answered above is “What we would miss if we assumed owner’s hours are equal to 38 hours, and perform the calibration exercise accordingly?” Our focus here is the estimated effects of financial frictions, in that we want to compare the gains of sustaining frictionless financial markets in models with and without hours adjustments.

Importantly, our reference model here is not the standard one with no entrepreneurial hours at all. By construction, such a model could not capture the non-employer firms, hence would target a sample different from the one we used in our analysis. By allowing for owner hours in the production function, even if we do not allow for adjustments of it, we preserve the ability to use the same data and same calibration strategy as in the benchmark. This illustrates better the role of allowing for adjustments in hours. Nevertheless, it should be emphasized that such alternative comparison with the standard model would imply even bigger differences with our findings, so the differences we highlight in this section suggest a lower bound of how relevant is to have hours adjustments in an entrepreneurship model.

Here we repeat the benchmark calibration exercise while exogenously setting the hours of owners equal to the data average at year 8 of 38 hours. Notice that this way we cannot calibrate the substitutability parameter $\lambda$ which targeted the owner hours in large firms. Accordingly, we first keep $\lambda$ at its benchmark value of 0.75. In addition, we replicate the same exercise while setting $\lambda$ at 1, as this would correspond to a more standard approach to labor aggregation within firms.

The main difference between these recalibrated models and the benchmark model is the degree of financial frictions required to match the target average growth rate for the firms.
The intuition is simple. For each calibration, we set the initial average assets to match the initial average size. Given this parameter, and given a level of financial frictions, which is the reciprocal of $\theta$ in our model, adjusting their own effort allows firm owners accumulate wealth and push their actual size closer to their desired level. When everyone is forced to work the average hours, then the more productive entrepreneurs lose this channel, and hence cannot grow as fast. Since higher frictions imply steeper growth profiles on average, to generate the same steepness as in the benchmark, the model without hours adjustments demands more severe frictions, i.e. lower $\theta$.

In order to show the impact of financial frictions on output in the benchmark and the two alternatives, we perform the counterfactual of no frictions. In our model, the choice of hours is endogenous and depends on the disutility of working. In general, without frictions, entrepreneurs would readjust their own hours. We avoid this by asking: What amount of labor and capital would entrepreneurs choose when financially unconstrained given their realized hours choice. Then we look at the ratio of the total output for all the firms at a given age in the baseline of each calibration, to that without frictions. Meanwhile, notice that all these three models have a different recalibrated $\theta$ in their baseline. In order to see the robustness of our results, we also perform the above exercise deviating from the $\theta$ value calibrated for the benchmark with hours adjustments.

Figure 11 shows the level of total output compared to the efficient level across each age. The left panel corresponds to the experiment of simply moving from baseline $\theta$ of each recalibration to the no frictions level. The right panel repeats the same experiment, but starting from the $\theta$ of the benchmark calibration. Both figures show that being able to adjust their working hours allows entrepreneurs to partially offset the financial restrictions they are facing, and omitting these adjustments in the production can lead to overestimating the effects of financial frictions.
7 Final remarks

In this paper, we study the role of entrepreneurs’ labor supply in their production units. We first use data on US start-ups to document the empirical regularities suggesting the non-trivial role of owners’ input in the production. We also show systematic patterns between owners’ working hours and the size and output of their firms, which can help us identify various important heterogeneities between firms that are a priori unobservable to researchers.

We build a novel theoretical framework featuring owners’ hours in the production function, which we calibrate to capture various important features of the dynamics in our sample. Importantly, the model allows us to identify the reduced form role of entrepreneurs in their firms, namely the degree of their complementarity and substitutability with their external labor. Our results indicate a significant degree of such substitutability.

An important aspect that entrepreneurial hours can be relevant for the firm dynamics is by mitigating the impact of financial frictions by covering for the external labor they owners cannot hire. To the extent that our model calibration represents the true nature of production, omitting the hours adjustments can lead to a substantial overstatement of the role of financial frictions in determining the aggregate output.
A Appendix: Theoretical results

We can characterize the solution to the firm’s problem within four cases as follows.

**Interiority with employment:** $wL + (r + \delta)K < \theta a$.

$$L = \left[ z \frac{(1 - \alpha)\eta}{w} \left( \frac{\alpha}{1 - \alpha} \frac{w}{r - \delta} \right)^{\alpha\eta} \left[ \kappa \left( \frac{h}{L} \right)^{\lambda} + 1 \right]^\frac{\alpha\eta + (1 - \alpha)\eta}{\lambda - 1} \right]^{\frac{1}{1 - \eta}}$$

$$K = \frac{w}{r - \delta} \frac{\alpha}{1 - \alpha} L \left[ \kappa \left( \frac{h}{L} \right)^{\lambda} + 1 \right]$$

**Binding with employment:** $wL + (r + \delta)K = \theta a$.

$$wL \left( 1 + \frac{\alpha}{1 - \alpha} \left[ \kappa \left( \frac{h}{L} \right)^{\lambda} + 1 \right] \right) = \theta a - \xi$$

$$K = \frac{w}{r - \delta} \frac{\alpha}{1 - \alpha} L \left[ \kappa \left( \frac{h}{L} \right)^{\lambda} + 1 \right]$$

**Interiority without employment:** $(r + \delta)K < \theta a$.

$$K = \frac{\alpha\eta}{r - \delta} \left[ z \left( \frac{\alpha\eta}{r - \delta} \right)^{\alpha\eta} \kappa^{\frac{(1 - \alpha)\eta}{\lambda}} \lambda^{(1 - \alpha)\eta} \right]^{\frac{1}{1 - \alpha\eta}}$$

**Binding without employment:** $(r + \delta)K = \theta a$.

$$K = \frac{\theta a}{r + \delta}.$$

While these necessary conditions are intuitive, the model does not allow further analytical characterization of the solution and we solve the model numerically.
# Appendix: Additional results from the benchmark model

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<tr>
<th>Moment</th>
<th>Data</th>
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<th>Benchmark (Recal, $\lambda = 1$)</th>
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<td>0.53</td>
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Table 5: Detailed results of models without of adjustments in owners’ hours

As the firms survive, they tend to get larger in the model. The left panel of Figure 12 shows that an average firm in the initiation year has about 1 full time worker, which corresponds to a third of an average firm after eight years. In line with this, 70 percent of the firms do not employ any workers in the first year of business in the model, which drops down monotonically to 49 percent after the eighth year. Importantly, both model features highlighted in Figure 12 are in line with those observed in the data.
Hours worked by entrepreneurs exhibit a monotonically increasing pattern with firm age in the model. An entrepreneur works around 33 hours per week in the first year of business and this increases to 38 hours per week by the year eight in business. The relationship between size of the firm and entrepreneurs’ hours are consistent with this, considering that older firms also become larger. In particular, owners of firms employing between 16 and 31 workers work around 53 hours per week, about 80% more than the model implies for owners of firms employing less than a worker.
References


