Unemployment Risks and Intra-Household Insurance *
(Very preliminary draft)

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April 11, 2017

Abstract

We consider an economy with incomplete markets and intra-household risk sharing, where households are formed by a job-seeker and an employed spouse and differ by the productivity of the spouse. We study the constrained efficient private provision of insurance within the household through the labor supply of the spouse, and what unemployment risks should be publicly insured away. Unlike the spouse’s total income, neither productivity nor labor supply is observed. We characterize the directed search equilibrium, and show that the spouse’s labor supply is negatively affected by unemployment benefits regardless of the search outcome of the worker in line with the empirical evidence. We also show that the optimal unemployment benefits are contingent on the household’s total income as it affects the trade-off between consumption-smoothing and job search incentives. Moreover, we numerically explore the welfare gains of implementing a household-income-based unemployment insurance.

Keywords: Unemployment Risks, Intra-Household Risk-sharing, Directed Search, Efficient Private Insurance

JEL Codes: J08, J22, J64, J65

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*We thank Nezih Guner, Jean-Baptiste Michau, Claudio Michelacci, and Aysegul Sahin for their feedback. We are also grateful to numerous participants at the SEA (2015), ENTER Jamboree (2016), Istanbul SaM Workshop (2016), the Annual SaM Conference (2016) and the ESEM (2016) as well as seminar participants at CREST and Universitat de Girona. We acknowledge financial support from the Spanish Ministry of Science and Technology under Grant No. ECO 2013-46395, ECO2015-67602-P from MINECO/FEDER, UE, and from the Spanish Ministry of Economy and Competitiveness through the Severo Ochoa Programme for Centres of Excellence in R&D (SEV-2015-0563).

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1 Introduction

A consistent finding in public economics is that welfare gains can be obtained from making policy instruments contingent on observable characteristics. For example, Weinzierl (2011) and Farhi and Werning (2013) find that optimal income taxation varies over the life cycle, and Michelacci and Ruffo (2015) extend this result to unemployment insurance (UI) benefits. Likewise, Kleven, Kreiner, and Saez (2009) find that optimal tax rates on an individual’s labor income differ by the earnings of the spouse.

The primary goal of this paper is to study the constrained efficient private intra-household insurance against unemployment risks and, hence, what risks should be publicly insured away. Intuitively, UI benefits should be contingent on the spouse’s earnings to the extent that these help reduce consumption risks and are sensitive to both income shocks and policy.

Significant consumption smoothing at the household level has been estimated in the literature. For example, Heathcote, Storesletten, and Violante (2014) find that only does 40% of individual permanent wage fluctuations pass through to household consumption.\(^1\) Blundell, Pistaferri, and Saporta-Eksten (2016) study the main sources of consumption insurance against a permanent fall in the husband’s earnings, and estimate that, on average, adjusting the wife’s labor supply accounts for approximately 50%, with assets and transfers accounting for 20% each.\(^2\) The analysis of the effects of the husband’s income on his wife’s labor supply goes back at least to Mincer (1962), who showed informal evidence that wives work more if their husbands are unemployed. Blau and Kahn (2007) estimate the average cross-wage elasticity of wife’s hours worked at -0.2.\(^3\) Cullen and Gruber (1996) estimate a 6% increase in the wife’ hours worked in the short run after the displacement of her husband.

Likewise, the crowding-out effects of publicly-provided insurance on private provision appear to be sizable. Two estimates of this distortion are provided by Gruber (1997) and Cullen and Gruber (2000).\(^4\) The former finds that a 10 percent increase in the replacement

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\(^1\) Dynarski and Gruber (1997) estimate the average elasticity of total consumption with respect to male’s earnings at 0.24, which implies that 76 cents are smoothed away for each dollar in male’s earnings.

\(^2\) In contrast, and because of the gender gap in labor supply elasticities, permanent shocks on wives’ wages are primarily smoothed away through savings and government transfers.

\(^3\) The evidence on cross-wage elasticity is quite limited. Using March CPS data, Blau and Kahn (2007) find that almost 80% of married women worked in 2000, and that wage-cross elasticity is negative and significant at both the extensive and intensive margins, increases with education and is twice as high for married women with children under 6. Furthermore, there is no significant difference when including likely cohabitation. Likewise, Hyslop (2001) estimates that a $1 increase in the husband’s hourly wages reduces wife’s annual earnings by $300 and her labor supply by 35 annual hours. We provide more references to the empirical evidence on this source of private insurance in the literature section. Devereux (2004) estimate the cross-wage elasticity to -0.4.

\(^4\) As summarized by Heathcote, Storesletten, and Violante (2009), there is a number of private insurance
rate is associated with a reduction in the consumption drop of 2.8 percentage points. The latter find that hours worked of wives increase by 30% during an unemployment spell of their husbands in the absence of UI, but each dollar of UI reduces the wife’s earnings by 36-73 cents.\(^5\)

Despite the evidence, the literature that addresses unemployment risks has forgotten to draw the labor-market-based intra-household insurance into the picture. We investigate the optimal allocation of unemployment risks in a static economy with directed search and intra-household risk sharing. Households are formed by a job-seeker and an employed spouse, and differ by the productivity of the spouse. Partial insurance against unemployment (or consumption) risks is privately arranged by pooling income and adjusting the spouse’s labor supply. In line with Chetty and Saez (2010), we assume no moral hazard is generated as a result of such an arrangement. The optimal design of the public unemployment insurance scheme factors in the intra-household risk-sharing mechanism, and, hence, takes into account the household’s total income. Therefore, there is a trade-off between the distortions on job creation generated by the taxes necessary to finance the public provision and the forgone leisure stemming from the private arrangements of insurance. Moreover, the optimal design of the public provision of insurance is limited by the heterogeneity across households and the informational frictions: unlike the spouse’s income, neither her working time nor her productivity is observed.

We first characterize the \textit{laissez-faire} equilibrium. We show that the labor supply of the spouses married to unemployed workers is larger than the one of those married to employed workers, which is in good accordance with the empirical evidence referred to above.\(^6\) We also find that an increase in unemployment benefits reduces the labor supply of spouses of both unemployed and employed workers in equilibrium. The former refers to the crowd-out effects

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\(^{5}\)The precise estimate is much larger for instrumented than for potential UI. Cullen and Gruber (1996) also find that the effects of UI on the hours conditional on working, but not their employment likelihood, are large and significant for wives of employed husbands with high unemployment risk, but not for the unemployed group. This suggests that households anticipate those risks. The responsiveness to UI also varies over the life cycle. For example, the crowd-out effects are much larger in household with small children and for young couples.

\(^{6}\)We consider spouses in a frictionless labor market, and, hence, we make no distinction between the extensive and intensive margins.

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\(^5\)Engen and Gruber (2001) estimate that the negative percentage effect of UI on asset holdings is twice as large for singles as for married unemployed workers. However, as pointed out by Chetty and Finkelstein (2012), the magnitude of the effect of UI on private savings is modest, and the median wealth holdings are very low as reported by Engen and Gruber (2001) and Chetty (2008). See also Kolsrud, Landais, Nilsson, and Spinnnewijn (2015) for Sweden. Kaplan (2012) documents that a large number of low-skilled youth males move back to their parents’ place after job loss. Moreover, using Canadian survey data, Browning and Crossley (2001) estimate a much smaller effect of the replacement rate on household total expenditure, and a significant effect on married workers whose spouse was unemployed.

\(^6\)The precise estimate is much larger for instrumented than for potential UI. Cullen and Gruber (1996) also find that the effects of UI on the hours conditional on working, but not their employment likelihood, are large and significant for wives of employed husbands with high unemployment risk, but not for the unemployed group. This suggests that households anticipate those risks. The responsiveness to UI also varies over the life cycle. For example, the crowd-out effects are much larger in household with small children and for young couples.
of the public provision of insurance, whereas the general equilibrium (income) effects through wages explain the latter, which is also consistent with the evidence reported by Cullen and Gruber (2000).

We then address the normative questions: What is the constrained efficient allocation of risks in this economy? Can it be decentralized in the market economy? We first show that the insurance level in the equilibrium allocation falls short of the optimal level as welfare gains are obtained from redistributing resources from households with the two members employed to households with one unemployed worker. Because of the lack of redistributive instruments, two sources of private insurance are excessively at work: the intensive margin of the labor supply of the spouse and the job creation margin in the labor markets.

In the case of ex-ante homogeneous households, we show that the planner’s allocation can be decentralized in the market economy. This implementation requires three fiscal instruments: unemployment benefits financed by lump sum and a proportional income tax on newly employed workers.

In Section 6, we quantitatively illustrate the welfare gains of moving from the current system to the planner’s solution. Relative to the present-day system, two outcomes are of interest. First, the welfare gains differ significantly over the distribution of households. Second, productivity gains also take place since the labor supply increases (reduces) for more (less) productive spouses. Finally, we explore how far away it is from the planner’s solution a simple policy consisting of a replacement rate and a dependency allowance contingent on the spouse being unemployed. As of 2015, nine states of the U.S. provide such an allowance, and its amount varies across states.  

In our analysis of the optimal unemployment insurance, we abstract from the persistent effects of unemployment on earnings by focusing on unemployment risks and the short run, thereby lessening the effects on the spouse’s labor supply. Consistent to the short run approach, no frictions to adjustments in the labor supply of the spouse have been modeled.

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8Stevens (1997) finds that earnings remain approximately 9% below expected levels 6 years after job loss. Using PSID data, Stephens (2002) documents that husbands’ earnings remain about 20% lower 3-4 years after displacement, and wives’ working hours keep increasing during this period. He estimates a 11% increase on average in annual working hours of wives, which includes both the intensive and extensive margins and offsets over 25% of their husbands’ lost earnings, which is in line with the figure estimated by Morissette and Ostrovsky (2008) for Canada in the 1990s. Dynarski and Gruber (1997) also find a large response in wives’ earnings following their husbands’ job loss using CEX data for high-school and college graduates, but not significant using PSID data. Using CPS data, Mankart and Oikonmou (2014) estimate that wives are 7.7% more likely to participate in the labor market in the month in which their husband becomes unemployed, which is almost as high as the overall participation probability of wives.
We also abstract from moral hazard problems and monitoring as well as administrative costs, features that would augment the relative costs of public insurance.

The paper proceeds as follows. After a brief summary of the related literature, Section 2 shows our data work. Section 3 describes the economy. In Section 3, we study the market equilibrium. Section 4 analyzes the planner’s solution. In Section 5, we undertake a numerical exercise, and Section 5 concludes. All proofs are relegated to the Appendix.

1.1 Related Literature

This paper contributes to several branches of the labor literature. First, in the search literature, several attempts have been undertaken to examine the optimal level of unemployment benefits under various sources of private insurance. For example, in a random search model, Krusell, Mukoyama, and Şahin (2010) find that the sizable negative effects on job creation limit significantly the generosity of the optimal public provision of insurance in an economy where workers can insure themselves through savings. Although not focused on the optimal unemployment insurance, Acemoglu and Shimer (1999) show that private markets offer insurance against unemployment risks to job-seekers who can direct their search. In the search literature, Burdett and Mortensen (1978) were the first in stating that the participation decision of a household member depends on the employment state of the other members. To the best of our knowledge, no attempt has been done to introduce households in this framework to study the optimal insurance.9

Second, Ortigueira and Siassi (2013) quantitatively assess unemployment insurance with couples in the Aiyagari-Hugget framework, in which households are hit by exogenous employment shocks. Instead, in our setting, the public insurance scheme as well as the private provision of insurance affect the search decisions and job opportunities of the unemployed. Attanasio, Low, and Sánchez-Marcos (2005) estimate larger welfare costs of uncertainty in the husband’s earnings in the absence of the ability of their wives to adjust the labor supply.

Third, a number of papers have emphasized that optimal policies are contingent on observable characteristics. Michelacci and Ruffo (2015) show that welfare gains would be obtained if a revenue-neutral reform would imply larger UI benefits for young workers.

Our work is also closely related to the optimal income taxation literature, starting from Mirrlees (1971). Boone and Bovenberg (2004) and Hungerbühler, Lehmann, Parmentier, and

9 As Guler, Guvenen, and Violante (2012) point out, there has been no continuation of their work until very recently. They analyze the case of couples jointly searching for jobs in different locations. As in our setting, wage dispersion arises in equilibrium in those models as the spouse’s income affects the reservation value of the job-seekers when there is perfect income pooling. They do not pay attention, however, to the efficient distribution of unemployment risks.
Van der Linden (2006) analyze optimal taxation in a frictional labor market. While in the latter, the demand side and wages are exogenous, workers are risk neutral and unemployment benefits are constant in the latter. None of these deals with two-member households. The optimal income taxation with couples is the focus of Kleven, Kreiner, and Saez (2009). In an economy with heterogeneity in spouse’s wage rate and worker’s participation costs, they find that optimal tax rates on an individual’s income differ by the earnings of the spouse. In contrast to our setting, in their economy, unemployment is voluntary, wages are exogenous and constant across types, and the effects on the demand side of the market are overlooked, and the utility function is quasi-linear in consumption, which eliminates the income effects on labor supply of the spouse. Chetty and Saez (2010) also examine the optimal design of (income and health) insurance programs in the presence of private insurance.

The crowding-out effects of public intervention have also been analyzed in different settings. Krueger and Perri (2011) investigate the optimal degree of progressivity in the income tax as a public risk-sharing device in the presence of limited private insurance markets. They and Attanasio and Ríos-Rull (2000) show that the introduction of mandatory public insurance may indeed backfire and reduce total insurance. As these two papers show, the specifics of the private insurance scheme matter for the net gains of the public provision. For example, the crowd-out effects of the public UI would have been larger if we had modeled potential inabilities of the households to smooth consumption in the short run due to e.g. habit formation or consumption commitments.

Finally, we model heterogeneity across households in the spouse’s income, but other dimensions of heterogeneity are also potentially key for the design of the public provision of insurance. For example, differences across workers result from their unemployment duration and over the lifecycle as studied by Hopenhayn and Nicolini (1997) and Michelacci and Ruffo (2015), respectively.

[To be completed]
Table 1: Summary Statistics

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Std. Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age</td>
<td>38.358</td>
<td>8.803</td>
</tr>
<tr>
<td>Female</td>
<td>.516</td>
<td>.450</td>
</tr>
<tr>
<td>Married</td>
<td>.567</td>
<td>.496</td>
</tr>
<tr>
<td>Children under 18</td>
<td>1.057</td>
<td>1.261</td>
</tr>
<tr>
<td>Spouse employed</td>
<td>.762</td>
<td>.426</td>
</tr>
<tr>
<td>if men</td>
<td>.640</td>
<td>.479</td>
</tr>
<tr>
<td>if women</td>
<td>.881</td>
<td>.324</td>
</tr>
<tr>
<td>Spouse NILF</td>
<td></td>
<td></td>
</tr>
<tr>
<td>if men</td>
<td>.276</td>
<td>.447</td>
</tr>
<tr>
<td>if women</td>
<td>.056</td>
<td>.229</td>
</tr>
<tr>
<td>Spouse’s income (if &gt; 0)</td>
<td>3788.111</td>
<td>3855.923</td>
</tr>
<tr>
<td>if men</td>
<td>2696.291</td>
<td>2650.780</td>
</tr>
<tr>
<td>if women</td>
<td>4598.887</td>
<td>4376.034</td>
</tr>
<tr>
<td>HH net liquid wealth</td>
<td>52361.66</td>
<td>856131.4</td>
</tr>
<tr>
<td></td>
<td>(median 6.74)</td>
<td></td>
</tr>
<tr>
<td>Nonemployment duration</td>
<td>21.007</td>
<td>23.748</td>
</tr>
</tbody>
</table>

Note.- Net liquid wealth is defined as total wealth minus home, vehicles and business equity and also net of unsecured debt.

2 Data

We use data from the Survey of Income and Program Participation (SIPP) for the U.S. for the 1996, 2001, 2004 and 2008 panels, covering from 1996:6 to 2013:6. Surveyed individuals are interviewed every four months and report for the previous four-month period a number of demographic and economic variables, in particular their labor market status, income and hours worked. We consider two labor market status: employment, $E$, and nonemployment, $E'$.

We restrict our dataset to individuals aged 25-55, who are quite attached to the labor market, and for which we have precise information of the duration of their unemployment. An observation is a $E'E'$ spell. We eliminate observations with a $E'$ spell shorter than 3 weeks. We end up with 42,334 spells. Table 2 shows the summary statistics for the whole sample. Since only 45.60% of the individuals in our sample have a single spell, dealing with all spells would overweight short spells. Therefore, we further restrict our dataset to the very first spell of all individuals in our sample, which accounts for 73.74% of the observations.\footnote{We largely follow Cullen and Gruber (2000) and Chetty (2008).}

\footnote{We conduct robustness checks by extending the dataset to the first two observations of all workers.}
<table>
<thead>
<tr>
<th></th>
<th>times=1 (31,216 observations)</th>
</tr>
</thead>
<tbody>
<tr>
<td>HH wealth Q1</td>
<td>-1.132*** (.029)</td>
</tr>
<tr>
<td>HH wealth Q2</td>
<td>-1.183*** (.028)</td>
</tr>
<tr>
<td>HH wealth Q3</td>
<td>-1.169*** (.035)</td>
</tr>
<tr>
<td>HH wealth Q4</td>
<td>-1.106*** (.029)</td>
</tr>
<tr>
<td>HH wealth Q5</td>
<td>-1.121*** (.029)</td>
</tr>
<tr>
<td>Previous log hourly wage</td>
<td>.021*** (.005)</td>
</tr>
<tr>
<td>Married</td>
<td>.037 (.029)</td>
</tr>
<tr>
<td>Married × Spouse has no wage</td>
<td>.005 (.033)</td>
</tr>
<tr>
<td>Married × Spouse’s wage Q1</td>
<td>.068** (.035)</td>
</tr>
<tr>
<td>Married × Spouse’s wage Q2</td>
<td>-.040 (.035)</td>
</tr>
<tr>
<td>Married × Spouse’s wage Q3</td>
<td>-.073** (.035)</td>
</tr>
<tr>
<td>Married × Spouse’s wage Q4</td>
<td>-.083** (.035)</td>
</tr>
<tr>
<td>Married × Spouse’s wage Q5</td>
<td>-.115*** (.036)</td>
</tr>
<tr>
<td>Female</td>
<td>-.165*** (.014)</td>
</tr>
<tr>
<td>Children under 18</td>
<td>-.027*** (.006)</td>
</tr>
</tbody>
</table>

Table 2: Cox Hazard Model Estimates

Note.- Coefficients reported can be interpreted as the percentage change in hazard rate associated with the quintile of the spouse’s average wage of the previous three months prior to re-employment. Only first spell of any individual. The model includes the unemployment rate, a year line, a quadratic polynomial of age, and dummies for seam effects, white and black, high-school, college degree and postcollege degree, homeownership, state, occupation and industry. The net liquid wealth dummies are indicator variables for whether the household wealth falls into the corresponding quintile.
We estimate a Cox proportional hazard model, which is reported in Table 2. Both columns show a negative relationship between the hazard rate of a worker and his or her spouse’s wage. Notice that these estimates do not vary significantly when including information on net liquid wealth. To the extent that there is no moral hazard problems within the household, the higher exit rates from nonemployment for workers whose spouse’s wage is below the median points to the liquidity effects of the private intrahousehold insurance. This suggests that large consumption-related welfare gains might be expected from UI benefits if contingent on spouse’s earnings. However, it may be the case that workers in the bottom part of the spouse’s wage distribution are intrinsically different from their counterparts above the median, and, hence, the differences in exit rates be not explained by the intrahousehold insurance.

[To be completed]

3 Benchmark Model

Consider an economy populated by a measure one of two-member households and a large continuum of risk-neutral firms. The mass of active firms is pinned down by free entry. Households are formed by an unemployed worker and her spouse endowed with market productivity \( x \in [\underline{x}, \overline{x}] \), with \( \underline{x} > 0 \).\(^{12}\) The former searches for a job, whereas the spouse chooses her labor supply \( \ell \in [0, 1] \). Let \( F(x) \) denote the measure of households with type below level \( x \), and it is assumed to be a differentiable cdf. Following the optimal taxation literature since Mirrlees (1971), productivity and labor supply (either hours worked or effort) are private information, whereas an individual’s total earnings are observable by the government and the planner.\(^{13}\) It is convenient to think in terms of observable variables, and, hence, we model the spouse as deciding total earnings \( y \) instead of labor supply \( \ell = y/x \).\(^{14}\)

To begin with, we follow Guler, Guvenen, and Violante (2012) and assume that households accounting for almost 93% of the observations. Cullen and Gruber (2000) deals with this issue by equally weighting all the observations of a given individual so that her total weight is one.

\(^{12}\)Alternatively, \( x \) can be interpreted as ability or hourly wage. This latter interpretation neglects general equilibrium effects. The assumption of a positive lower bound is made for expositional reasons. Single-earner households can be thought of as the case with the spouse’s productivity \( x \) being arbitrarily small.

\(^{13}\)As pointed out by Salanie (2011), if labor supply were interpreted as hours worked, the government could force employers to report them.

\(^{14}\)For simplicity, we make the assumption that spouses work in a frictionless labor market and decide their labor supply at a given productivity. According to CPS data, not-in-the-labor-force wives amount to 30 percent of married women. To abstract from the underlying reasons of their non-participation decision—caring of children and elderly, etc.—, we model their market productivity at zero. Similarly, single primary earners are assumed to be married to a zero-productivity spouse. The empirical literature has not unraveled whether the increase in wives’ hours results from increasing working time at the same job or from job-switching.
are the decision-making units, and consumption is a public good within the household.\footnote{In Section 4.3.2, we extend the analysis to a cooperative model of the household.} They derive utility from consumption $c$ and leisure of the spouse.\footnote{We abstract from whether leisure of the household members are substitutes or complements by assuming indivisible labor supply of the unemployed worker.} We impose the following assumptions on the utility function $v(c, y/x)$ that describes the preferences of a couple:

A1. $v$ is thrice continuously differentiable.

A2. $v$ is increasing in consumption and leisure: $v_c > 0, v_\ell < 0$

A3. $v$ is strictly concave: $v_{\ell\ell}, v_{cc} < 0$, and $v_{\ell\ell}v_{cc} - v_{c\ell}^2 > 0$.

A4. Weak complementarity: $v_{c\ell} \leq 0$.

A5. $\lim_{\ell \to 0} v_\ell < \lim_{\ell \to 0} v_c$ and $\lim_{\ell \to 1} v_c < \lim_{\ell \to 1} v_\ell$.

The first three conditions are fairly standard. We also assume that the cross-partial derivative is non-positive, meaning complementarity between consumption and leisure, which includes the case of additive separability between consumption and leisure.\footnote{Complementarity between consumption and leisure is a sufficient, but not necessary condition for the results. Attanasio and Weber (1995) and Meghir and Weber (1996) provide empirical evidence in this regard. Most macro models assume additive separability. See e.g. Heathcote, Storesletten, and Violante (2014) and Kleven, Kreiner, and Saez (2009).} The last condition ensures the existence of an interior solution in the household’s problem.

There are four stages. In stage one, unemployed workers direct their search. That is, they choose a submarket and place an application at cost $\kappa$.\footnote{Labor market participation is costly to make the social planner’s problem analyzed in Section 5 non-trivial.} A submarket or location is defined by a set of job characteristics. In stage two, firms decide on the submarket to place their vacancies, and incur cost $k$ when posting a vacancy. Market productivity of newly employed workers is normalized to 1. As usual in the search literature, each recruiting firm holds a single vacancy. Meetings take place in stage three as described below. Some workers become employed, whereas some other workers remain unemployed and produce $z$ at home. In stage four, spouses decide their total earnings $y$, and both production and consumption take place.

To ensure existence of equilibrium and that all jobless workers search for a job, we make the following two assumptions. First, there is a gap between net market productivity and vacancy creation costs, $1 - z > k$, to ensure that vacancy creation is a profitable activity. Second, cost $\kappa$ is sufficiently small so that $\max_y v(y + 1, y/x) - \max_y v(y + z, y/x) > \kappa$.\footnote{In Section 4.3.2, we extend the analysis to a cooperative model of the household.}
Matching Rates. Meetings are bilateral. Workers find a job at submarket $\omega$ with probability $\nu(q)$, where $q$ denotes the expected queue length or ratio of job-seekers to vacancies, whereas firms fill their vacancies with probability $\eta(q)$. Since the mass of newly employed workers equals the mass of newly filled vacancies in any given submarket $w$, it must be the case that $\nu(q) = \frac{\eta(q)}{q}$. We assume that $\nu$ is a decreasing function to capture the intuition that it is harder to find a job in tighter labor markets, and, hence, $\eta$ is assumed to be increasing. Likewise, the following limit conditions are necessary to ensure existence of equilibrium and planner’s allocations: \[ \lim_{q \to 0} \nu(q) = \lim_{q \to \infty} \eta(q) = 1 \text{ and } \lim_{q \to 0} \nu(q) = \lim_{q \to \infty} \eta(q) = 1. \] Let $\gamma(q) \equiv \frac{q \nu'(q)}{\eta(q)}$ denote the elasticity of the job-filling rate, which is assumed to be a decreasing function.\(^{19}\)

4 Market Economy

In this section, we analyze an economy in which agents make decisions in a decentralized way to maximize their utility. There are potentially infinitely many submarkets. Each submarket is defined by a single-wage offer $w$. Whereas firms decide whether to create a vacancy and what wage to commit to, the household’s decision is twofold. First, it chooses a submarket to submit a job application. Then, after learning the search outcome, it decides the labor supply of the spouse. We start detailing this last stage and, then, proceed backwards.

Stage Four. Let $w$ denote the income of the job-seeker at the end of the period, with $w = z$ if unemployed. We denote the household’s indirect utility function by $V_x$, which is defined as the maximand of the following program

\[ V_x(w) \equiv \max_y \nu(y + w, y/x) \]

The Weierstrass theorem together with Assumption A1 ensures that $V_x$ is well-defined. Notice that the first order necessary condition is also sufficient because of Assumption A3. Moreover, Assumption A5 ensures the existence of an interior solution for the first order condition. Therefore, the following equations uniquely determine the contingent earnings of the spouse, $y^e_x(w)$ and $y^u_x$.

\[ v_c(y + w, y/x)x = -v_c(y + w, y/x) \]

\[ v_c(y + z, y/x)x = -v_c(y + z, y/x) \]

\(^{19}\)These properties are satisfied for the usual matching functions, e.g. the Cobb-Douglas and urn-ball ones.
As can be anticipated from the observation of the household’s problem, the indirect utility function is central in the analysis of the market equilibrium. In the following lemma, we establish properties of function $V_x$ and of the optimal total earnings of the spouse, which are inherited from the assumptions on the utility function $\upsilon$ and follow from using repeatedly the Implicit Function Theorem to the first order conditions. In particular, the labor supply of the spouse is larger if married to an unemployed worker than to an employed one in line with the evidence reported in the Introduction. This is because labor supply is set to equate the marginal utility of leisure and the marginal utility of consumption, which is lower for households with the two members employed, together with the complementarity between consumption and leisure. Furthermore, we conclude that the wage-cross elasticity is negative, which is in line with the evidence reported in the Introduction.

**Lemma 4.1** Function $V_x$ is twice continuously differentiable, strictly increasing and concave. Furthermore, the optimal solution $y^e_x(\cdot)$ is twice continuously differentiable, and strictly decreasing in wages. In particular, $y^e_x(w) < y^u_x$ for all $w > z$.

Moreover, this lemma states that the household utility $V_x(w)$ and its derivative $V'_x(w)$ are increasing and decreasing in the household type $x$, respectively.

**Lemma 4.2** Function $V_x$ increases and its derivative $V'_x$ decreases with the spouse’s wage rate $x$.

**Stage Two.** There is entry of firms in all submarkets as long as expected profits are positive in and out of equilibrium. That is, the following condition must hold both for all $w \in [z, 1]$:

$$\eta(q)(1 - w) \leq k, \text{ and } q \leq \infty, \text{ with complementary slackness.} \quad (4)$$

Intuitively, when the larger the wage, the lower the mass of vacancies posted. In the limit, no positive mass of firms commit to a wage equal to market productivity of workers.

**Stage One.** Job-seekers rationally anticipate the optimal behavior of firms in the second stage, and trade off a higher wage and a higher job-finding probability. The expected utility of a household with the spouse’s productivity $x$ amounts to $(1 - \nu(q(x)))V_x(z) + \nu(q(x))V_x(w)$, where $q(x)$ denotes the expected queue length in submarket $w$.

### 4.1 Equilibrium.

We now turn to the definition of equilibrium.
Definition 1 A directed search equilibrium consists of, for all $x \in [x, \overline{x}]$, household values $U_x$, earnings $y_e^x : [z, 1] \to \mathbb{R}_+$ and $y_u^x \in \mathbb{R}_+$, wages $w_x$, and a queue length function $Q : [z, 1] \to \mathbb{R}_+$ such that:

i) Households optimally direct their search and choose earnings. For all $x \in [x, \overline{x}]$,

(a) $\nu(Q(w))(V_x(w) - V_x(z)) + V_x(z) \leq U_x$, $\forall w \in [z, 1]$, and

(b) For all $w \in [z, 1]$, $y_e^x(w)$ and $y_u^x$ solve the respective household’s problem, and, hence, satisfy conditions (2) and (3), respectively.

ii) Free entry of firms:

$\eta(Q(w))(1 - w) \leq k$, $\forall w \in [z, 1]$, and $Q(w) \leq \infty$, with complementary slackness. In particular, the first inequality is an equality for all $w_x$.

The first equilibrium condition is self-explanatory. The second condition determines the ratio of job-seekers to vacancies both on and off the equilibrium path. Workers form rational expectations about firms’ decisions in stage 2. Specifically, they expect the ratio of job-seekers to firms in any submarket to be determined by the zero-profit condition. Thus, the household’s problem at the beginning of the period is

$$\max_{q \geq 0, w \in [z, 1]} V_x(z) + \nu(q)(V_x(w) - V_x(z))$$

s. to condition (4)

4.2 Equilibrium Characterization

The following proposition states that there exists a unique equilibrium, and characterizes it. Equilibrium condition (6) is the first order condition of the household’s problem. It equates the costs of creating a vacancy to the expected profits, which amount to the probability of filling a vacancy times the share $1 - \gamma(q)$ of the joint value of the firm-worker pair adjusted by the marginal utility of the household. Equilibrium equation (7) is the zero-profit condition expressed in terms of wages. For notational simplicity, we denote hereafter the equilibrium queue length at wage $w_x$ as $q_x \equiv Q(w_x)$.

---

20 For expositional simplicity, we omit the participation decision because of the assumption on sufficiently small search costs.
Proposition 4.3 There exists a unique equilibrium. For any given household type \( x \in [x, \bar{x}] \), the equilibrium pair \((q_x, w_x)\) is characterized by the following system of equations

\[
\begin{align*}
k &= \eta(q)(1 - \gamma(q)) \left( \frac{V_x(w) - V_x(z)}{V_x'(w)} + 1 - w \right) \tag{6} \\
k &= \eta(q)(1 - w) \tag{7}
\end{align*}
\]

The equilibrium is generically separating. Put differently, there is wage dispersion in equilibrium even though workers are equally productive and firms are also homogeneous. This is because the attitudes towards unemployment risks differ across households because of the private insurance arrangement. This poses a source of heterogeneity which has been overlooked when examining wage dispersion using a Mincerian regression. The optimal search strategy depends on the spouse’s wage rate. Similarly to Acemoglu and Shimer (1999), households of higher types apply to higher-wage jobs if the absolute risk aversion of the indirect utility function, \(-\frac{V''_x(w)}{V_x'(w)}\), decreases with the wage rate of the spouse \( x \).\(^{21}\) Consider the case of additively separable, CRRA preferences, \( v(c, \ell) = c^{1-\sigma} + (1-\ell)^{1-\sigma} \). Because absolute risk aversion is decreasing in \( x \), job-seekers married to more productive spouses apply to higher-wage jobs, which are harder to get.

Proposition 4.4 If \(-\frac{V''_x(w)}{V_x'(w)}\) is decreasing in \( x \), then \( w_x \) and \( q_x \) are increasing in \( x \). If it is constant, so are \( w_x \) and \( q_x \).

The following lemma states that the public provision of insurance crowds out private insurance as labor supply of spouses married to unemployed workers decreases with \( z \), the parameter capturing home productivity and unemployment benefits. Furthermore, as is common in search models, wages increase and job-finding rates decrease with \( z \). This result together with the negative relationship between the wage of the primary earner and the labor supply of the secondary earner stated in Lemma 4.1 implies that the labor supply of the spouse married to an employed worker also decreases with \( z \). These direct and indirect negative effects of unemployment benefits on the spouse’s labor supply are consistent with the empirical evidence reported by Cullen and Gruber (2000). They estimate that each $100 in potential benefits lowers the working hours of wives of employed and unemployed workers by 5.2 and 22.7 per month, respectively. The general equilibrium effect through wages of unemployment benefits on the labor supply of the spouse in households with the two members

\(^{21}\)Despite the differences, it is not surprising that we obtain a similar result to theirs as job-seekers can insure themselves through savings in their setting. This condition involves assumptions on the third derivative of the utility function \( v \).
employed has two components. While the positive macro effects of benefits on wages are supported by Hagedorn, Karahan, Manovskii, and Mitman (2015), the second component of the mechanism is in line with the negative cross-elasticities estimated by e.g. Hyslop (2001) and Blau and Kahn (2007) as reported in the Introduction.

**Lemma 4.5 Comparative Statics.**

1. *Wages and queue lengths increase with* \( z \).

2. *The spouse’s labor supply decreases with* \( z \) *regardless of the employment state of the worker.*

### 4.3 Extensions

In this section we examine two particular cases that have been considered in the literature: namely, quasi-linear preferences and a cooperative model of the household.

#### 4.3.1 Quasi-linear Preferences.

Consider first a quasi-linear utility function in consumption, \( v(c, y/x) = c + \phi(y/x) \), where function \( \phi \) is twice continuously differentiable, decreasing and concave. Then, the indirect utility function \( V_x \) is linear in wages. As is well known, labor supply of the spouse is insensitive to the income of the job-seeker and, in particular, to unemployment benefits; hence, household’s consumption increases one-to-one with benefits. The equilibrium conditions are

\[ k = \eta(q_x)(1 - \gamma(q_x))(1 - z), \quad \forall x \in [\underline{x}, \overline{x}] \quad (8) \]

\[ k = \eta(q_x)(1 - w_x), \quad \forall x \in [\underline{x}, \overline{x}] \quad (9) \]

\[ \phi'(y^j_x / x) = -x, \quad \forall x \in [\underline{x}, \overline{x}], j \in \{u, e\} \quad (10) \]

Notice that the first two equations are the counterparts of conditions (6) and (7), whereas the last one is the first order condition of the household’s problem (2). It follows from the first two equilibrium conditions that all workers search in the same market regardless of their spouse’s wage rate.\(^{22}\) The third condition shows no income effects on the labor supply of the spouse, and an increasing income \( y^j_x \) in \( x \).

\(^{22}\)Notice that the absolute risk aversion of the indirect utility function \( V_x \) is zero.
Consider next quasi-linear preferences in leisure: \( v(c, y/x) = \psi(c) - y/x \), where \( \psi \) is a twice continuously differentiable, increasing and concave function. The equilibrium conditions are the same as before, except for the last one, which is replaced by

\[
\psi'(w_x + y^e_x), \psi'(z + y^u_x) = 1/x
\]  

(11)

There is full insurance because the labor supply of the spouse adjusts to make consumption invariant to the search outcome. This intra-household way of completing markets intuitively eliminates all consumption risks, and, as a result, job-seekers no longer trade off job-finding rates and wages, and all search in the same market regardless of productivity \( x \).\(^{23}\) Needless to say, consumption increases with the spouse’s productivity because so does the spouse’s income, and the difference \( y^u_x - y^e_x \) is also constant in \( x \).

### 4.3.2 Cooperative Model

The benchmark economy hosts a unitary model of the household, in which households are the decision-making units that maximize a utility function subject to a budget constraint. This modeling has been questioned on empirical and theoretical grounds. See Chiappori and Donni (2009) for a survey. Therefore, it is worth checking the robustness of our results in a cooperative model of the household. In such models, each member of the household has their own preferences, and the decision-making process is usually not made explicit. Consumption is no longer a public good. Instead, the two members of the household pool income and decide on their individual consumption and labor supply. Importantly for our analysis, cooperative models ensure Pareto efficient intra-household outcomes.

For notational simplicity, let \( m \) and \( f \) denote the index of the two members of the household. Likewise, \( \alpha \) stands for the Pareto weight on the utility of the first member, and captures the \( m \)’s relative power within the household. To abstract from the interaction between policy and intra-household power distribution, we assume that the Pareto weights do not depend on income, and in particular, they are insensitive to wage \( w \) and productivity \( x \) as well as unemployment benefits \( z \).\(^{24}\) The indirect utility function of a household of type

---

\(^{23}\)Formally, productivity \( x \) determines consumption, and, hence, the objective function in problem (5) becomes \( \nu(q)(w - z) \). Both the objective function and the constraint are independent of household type, and we are then solving the standard program in the basic directed search model with risk-neutral workers.

\(^{24}\)In a collective model of the household instead, the Pareto weights may depend on the relative earnings, total income and other called distribution factors out of the model.
\( x \) is

\[
V_x(w) = \max_{c^f, c^m, y} \alpha v^m(c^m, \ell) + (1 - \alpha)v^f(c^f, y/x)
\]

s. t. \( c^f + c^m = y + I_e w + (1 - I_e)z \)

where \( v^f \) and \( v^m \) satisfy properties A1-A5, labor supply \( \ell \) is exogenous, and \( I_e \) is an indicator function that values one if the worker is employed and zero otherwise.

Notice that the first order conditions establish the following risk-sharing policy, which depends on the Pareto weights, \( \alpha v^m_c = (1 - \alpha)v^f_c \). That is, the marginal utility must be equal across members after adjusting for the weight distribution within the household. Likewise, the household marginal gains of an increase in wages must be equal to the marginal gains from an equivalent increase in the spouse’s leisure, \( V'_x(w) = -(1-\alpha)\frac{v^f_\ell}{x} \).

The following lemma states that we obtain the same results as in the benchmark case.

**Lemma 4.6** Function \( V_x \) is twice continuously differentiable, strictly increasing and concave. The optimal solution \( y_x^e \) is twice continuously differentiable, and strictly decreasing in the wage. In particular, \( y_x^e(w) < y_x^u \) for all \( w > z \) and \( x \). Furthermore, the spouse’s hours worked decrease with \( z \) regardless of the employment status of the worker. The equilibrium allocation is determined by equations (6) and (7).

## 5 Constrained Efficiency

The main result of this section is that constrained efficiency cannot be attained in the market economy because private provision of insurance against consumption risks is inefficiently limited, and the insurance mechanisms, in the labor market through vacancy creation and within the household through the spouse’s labor supply, are excessively used. We first characterize the constrained efficient allocation.

### 5.1 The Planner’s problem

As usually assumed in the search literature, the social planner maximizes a utilitarian welfare function. It sets a mass of vacancies, assigns search strategies and transfers to workers, and faces the same coordination frictions as agents encounter in the market economy. Moreover, the planner cannot observe the type of the households and the spouse’s labor supply; however, both the spouse’s income \( y \) and the worker’s employment state are observable.
More specifically, the planner designs a symmetric incentive compatible revelation mechanism that consists of a menu of contracts \(\{(q_x, c^e_x, c^u_x, y^e_x, y^u_x) | x \in [\underline{x}, \overline{x}]\}\) indexed by the household’s announcement of its type. The mechanism is symmetric in the sense that all households reporting a given type are treated identically. For any reported type \(x\), the mechanism specifies a location where to submit an application and the associated job-finding probability, \(\nu(q_x)\), consumption as well as the spouse’s income contingent on the search outcome, \((c^e_x, c^u_x)\) and \((y^e_x, y^u_x)\).

We say that a mechanism is feasible if total consumption promises do not exceed total output net of vacancy creation costs, i.e. if the following resource constraint holds

\[
\int_{\underline{x}}^{\overline{x}} \frac{k}{q_x} dF(x) = \int_{\underline{x}}^{\overline{x}} (\nu(q_x)(1 + y^e_x - c^e_x) + (1 - \nu(q_x))(z + y^u_x - c^u_x)) dF(x)
\]  

(13)

To simplify notation, let us denote \(U_x \equiv U_x(x)\). The mechanism must be compatible with agents’ incentives. This implies that job-seekers must truthfully reveal their types. That is, the following incentive compatibility constraints must hold.

\[
U_x(\hat{x}) \geq U_x(x'), \quad \forall x, x' \in [\underline{x}, \overline{x}]
\]  

(ICC\(_x\))

Furthermore, the value of job-search must exceed the application cost to ensure that participating in the market is desirable. That is, the following set of participation conditions must also hold.

\[
U_x \geq \kappa + \nu(c^u_x, y^u_x/x)
\]  

(PC\(_x\))

We will refer as the constrained efficient allocation to the feasible incentive-compatible mechanism that solves the planner’s problem, which can be written as

\[
\text{Planner’s problem:} \quad \max \int_{\underline{x}}^{\overline{x}} U_x dF(x) \\
\text{s. to (PC}_x\text{), (ICC}_x\text{) and (RC) for all } x
\]

To understand the importance of each one of these constraints, let us consider what allocation would be obtained if either one were subtracted. Obviously, resources are limited.
If the participation conditions were eliminated, the planner would promise equal bundles regardless of the search outcome, and this allocation could trivially not be decentralized. If the incentive compatibility constraint were eliminated instead because types were observable and preferences were additively separable, then the planner’s allocation could be decentralized in the market economy as we will comment later. However, consumption would be constant across types and higher types would produce more, yielding a declining expected utility over productivity levels. This result is well-known in the optimal taxation literature, see e.g. Mankiw, Weinzierl, and Yagan (2009). To see that incentive compatibility ensures that households with more productive spouses obtain higher values notice that

$$\mathcal{U}_x \geq \mathcal{U}_x(x') = \nu(q_{x'}) v(c_{x'}, y_{x'}/x) + (1 - \nu(q_{x'})) v(c_{x'}, y_{x'}/x) > \mathcal{U}_{x'}, \text{ for } x' < x$$

where the first inequality is condition (ICC$_x$), and the second inequality results from function $v$ being strictly increasing in the household’s type. The intuition underlying this result is that if the sign of the inequality were reversed, higher-type households would have incentives to misreport their type. The following lemma states existence of the planner’s solution as a straightforward result of Weierstrass Theorem.

**Lemma 5.1** There exists a solution to the planner’s problem. Furthermore, the expected utility in the constrained efficient allocation is monotonic over household types.

To characterize the planner’s solution, it is generally convenient to reduce the dimensionality of the problem by replacing the incentive-compatibility condition (ICC$_x$) by a first- and a second-order condition\(^\text{25}\)

\[
\begin{align*}
\dot{U}_x &= -\nu(q_x) u\frac{y_u^x}{x^2} - (1 - \nu(q_x)) v_u^x y_u^x, \quad (\text{FOC - ICC}_x) \\
-\nu'(q_x) q_x u\frac{y_u^x - y_u^x}{x^2} - \nu(q_x) &\left(\frac{\partial c_x}{\partial x} y_u^x + \nu_x y_x^x + \nu_y y_x^x/x + v^c_x \right) \\
-\frac{1}{x^2} \left(1 - \nu(q_x)\right) &\left(\frac{\partial u}{\partial c_x} y_u^x + \frac{\partial u}{\partial y_x} y_y^x/x + v_u^x\right) \geq 0 \quad (\text{SOC - ICC}_x)
\end{align*}
\]

where, for notational simplicity, $\nu^j \equiv v(c^j_x, y^j_x/x)$ for $j \in \{u, c\}$ and for all $x$, and $\dot{n} \equiv \frac{dn}{dx}$

\(^\text{25}\)The FOC of the (ICC$_x$) indeed says that the total differential with respect to $\dot{x}$ is zero at $\dot{x} = x$,

$$\frac{d\mathcal{U}_x(\dot{x})}{d\dot{x}} \bigg|_{\dot{x}=x} = \frac{\partial \mathcal{U}_x(\dot{x})}{\partial q_x} \dot{q}_x + \frac{\partial \mathcal{U}_x(\dot{x})}{\partial c_x} \dot{c}_x + \frac{\partial \mathcal{U}_x(\dot{x})}{\partial y_x} \dot{y}_x + \frac{\partial \mathcal{U}_x(\dot{x})}{\partial u} \dot{u} + \frac{\partial \mathcal{U}_x(\dot{x})}{\partial y} \dot{y},$$

which is equivalent to the expression above. To obtain the second order condition at $\dot{x} = x$, we differentiate again with respect to $\dot{x}$. To simplify the second derivative, we benefit from this condition to hold for all $x$. After some tedious, but straightforward calculations, we obtain expression (SOC-ICC$_x$).
denote the derivative of variable $n$ with respect to $x$. These necessary conditions are local. The following lemma states that they are also sufficient.

**Lemma 5.2** The above necessary conditions are also sufficient.

### 5.2 Efficiency in the market economy

The following proposition states that constrained efficiency is not achieved in the *laissez-faire* equilibrium. It is easy to see that the equilibrium allocation belongs to the feasible set of the planner’s problem. That is, the resource and participation constraints hold, and the equilibrium allocation is also incentive compatible. However, efficiency gains can be obtained by redistributing resources to increase consumption of the households with an unemployed member. Put differently, the private provision of insurance against consumption risks falls short of the constrained efficient level. Moreover, inefficiency may also result from the ex-ante heterogeneity across households and the market’s inability of redistributing resources among them.

**Proposition 5.3** Under Assumptions A1-A5, the equilibrium allocation is not constrained efficient.

Next, we examine the inefficiency result by looking at several particular cases.

### 5.3 Quasi-linear preferences in consumption

We first consider the case of a quasi-linear utility function, $v(c, y/x) = c + \phi(y/x)$, where function $\phi$ is decreasing and concave as assumed in Section 4.3.1. The following proposition states that the equilibrium allocation is constrained efficient. That is, the planner asks all job-seekers to search in the same location and the labor supply of the spouse is not affected by the search outcome. Importantly, worker types are separated costlessly, and the marginal rate of substitution is 1 for all households.

**Proposition 5.4** Constrained efficiency is attained in the market economy.

Why are these preferences of interest? Certainly, these preferences do not satisfy Assumption A3. However, this case constitutes the benchmark for two different literatures. First, in the optimal income taxation literature, it has been common to assume such preferences because it greatly simplifies the problem and allows for a closed-form solution.\(^{26}\)

\(^{26}\)To make the problem nontrivial, a motive for redistribution is assumed for example by either giving a lower weight to higher-earnings agents or assuming a concave transformation of agent’s utility function.
See Salanie (2011) for a summary, and Kleven, Kreiner, and Saez (2009), for the optimal taxation of couples. Furthermore, this assumption is in fair agreement with the evidence on the income elasticity of labor supply for primary earners, mostly men. In contrast, when referring to secondary earners, mostly wives, labor supply elasticity is much larger, and the empirical evidence reported in the Introduction shows significant income effects of earnings and unemployment benefits on wife’s hours of work.

Second, in the standard directed search model, agents are assumed to be risk neutral. Interestingly, the equilibrium allocation is constrained efficient because wages price waiting time. See Moen (1997). Therefore, although the focus of our analysis is on secondary earners and intra-household insurance, the economy with quasi-linear preferences in consumption is also of interest as a benchmark.

The following lemma establishes that constrained efficiency is also attained in equilibrium when household members have their own (quasi-linear) preferences and cooperate when making their decisions.

**Lemma 5.5** Consider a cooperative model of the household as the one described in Section 4.3.2 in which each member has quasi-linear preferences. Then, the equilibrium is constrained efficient.

### 5.4 Economy with ex-ante homogeneous households

We now examine an economy with a degenerate distribution $F$. Recall that, in the market economy, the equilibrium pair $(w, q)$ is determined by conditions (6) and (7), as stated in Proposition 4.3. For expositional purposes, we assume that neither productivity nor labor supply nor output is observable to the planner, and, hence, incentive compatibility must be taken into account.\(^{27}\) The planner’s problem can be written as follows

\[
\begin{align*}
\max & \quad \nu(q) V(m^e) + (1 - \nu(q)) V(m^u) \\
\text{s. to} & \quad \frac{k}{q} = \nu(q) \left(1 - m^e\right) + (1 - \nu(q)) \left(z - m^u\right) \\
& \quad \kappa \leq \nu(q) \left(V(m^e) - V(m^u)\right)
\end{align*}
\]

where function $V$ resembles expression (1). As stated in Proposition 5.3, the equilibrium is not constrained efficient. This is because of the inability of the market economy to efficiently insure the consumption risks away. Indeed, the income of the spouse is above the

\(^{27}\) We obtain the same results if they were observable as would be the case from our set of assumptions.
constrained efficient level as private intra-household insurance tries to compensate for the lack of redistribution across households.

Nonetheless, the planner’s solution can be implemented in the market economy by setting an unemployment insurance system funded by lump sum and proportional income taxes. In order not to distort the labor supply of spouses, which provides intra-household insurance, their income is not taxed. Furthermore, income taxes are necessary to convey the proper search incentives to job-seekers, and this is done through wages in a directed search economy. Notice that this is the case regardless of whether the spouse’s income is observed or not because the planner factors in the spouse’s optimal decision by equating the marginal utility from an additional consumption unit and an additional unit of leisure. It is worth underscoring that the planner’s solution makes the publicly-provided insurance be based on the intra-household insurance and, hence, on household’s total potential income.

We question again whether the implementation of the planner’s solution still holds when considering a cooperative, in lieu of a unitary, model of the household. The answer is yes. Notice that the planner’s problem written as in (14) allows for a straightforward interpretation of function $V$ as in expression (12) in Section 4.3.2. The proof is straightforward; hence, omitted. Importantly, the specific optimal policy is contingent on the Pareto-weights of the household model as so is the planner’s allocation.

**Proposition 5.6** If households are ex-ante identical, then the spouse’s income is excessively large in equilibrium. Constrained efficiency can be attained in the market economy through the implementation of a public unemployment insurance financed by a proportional income tax on newly employed workers and a lump sum tax. Furthermore, this result also holds with a cooperative model of the household.

If workers’ productivity were observable, the result of the decentralization of the planner’s allocation could be extended to an economy with a non-degenerate cdf $F$ by means of productivity-contingent tax rates. Otherwise, as assumed here, an incentive scheme, based on realized spouse’s earnings, must be set to elicit information on actual intra-household insurance. We next analyze scenarios with ex-ante heterogeneous households averse to consumption risks.

---

28 This not taxing on agents with a high income elasticity is along the lines of Alesina, Ichino, and Karabarbounis (2011).
5.5 Quasi-linear preferences in leisure

We start by considering quasi-linear preferences in leisure: $\nu(c, \ell) = \psi(c) - \ell$, where function $\psi$ is increasing and concave. The following proposition characterizes the planner’s solution.

**Proposition 5.7** The planner’s allocation is characterized by the following conditions

$$
\begin{align*}
  k &= \eta(q_x)(1 - \gamma(q_x))(1 - z), \text{ and } c_x^e = c_x^u = c_x \\
  \dot{c}_x &\geq 0, \text{ and } \nu(q)\dot{y}_x^e + (1 - \nu(q))\dot{y}_x^u \geq 0
\end{align*}
$$

(15)

The participation condition ($PC_x$) is redundant. There exists a subset of positive mass in which the above inequalities are strict and $\psi'(c_x) > \frac{1}{x}$. The equilibrium mass of vacancies is constrained efficient, but labor supply is not. Furthermore, there may be bunching: $\dot{c}_x = \nu(q)\dot{y}_x^e + (1 - \nu(q))\dot{y}_x^u = 0$ within a subset of $[x, \bar{x}]$.

Condition (15) states two outcomes: there is full intra-household insurance, and a single labor market is active. Furthermore, the constrained efficient vacancy creation is the one that maximizes output, which is inherently associated with the previous result. As full insurance is provided within the household, job-seekers behave as though they were risk-neutral agents, and, as a result, the planner aims at maximizing output to redistribute across ex-ante different households. Notice that both full private insurance and output maximization also take place in equilibrium, but there is no redistribution across households. Indeed, the equilibrium consumption level exceeds the planner’s consumption, implying that labor supply of the spouse is inefficiently large.

Furthermore, there may exists a subset of individuals applying to contracts which specify the same consumption level and expected income of the spouse. Following the optimal taxation literature, we refer to this result as *bunching*.

The following lemma states that the planner’s allocation can be implemented in the market economy through a system of transfers across households types. There is no redistribution within-group, instead.

**Lemma 5.8** If there is no bunching in the planner’s solution, then it can be decentralized through the implementation of a tax on household’s total income.

[To be completed]
6 Quantitative Exploration

In this section, we quantitatively explore the welfare gains that would obtain from moving from the U.S. economy to another one with the optimal household-income-based insurance system. Consistently with our previous work, our benchmark hosts a unitary model of the household, and we also make the comparison with a cooperative model.

[To be completed]

7 Conclusions

There are many aspects that are left unexamined for the sake of the theoretical analysis. Important points are the extent of leisure complementarity and assortative mating by skill level (see e.g. Boskin and Sheshinski (1983)), the policy implications on family formation (see e.g. Gayle and Shephard (2016)) and inequality within the household (see e.g. the latter and Alesina, Ichino, and Karabarbounis (2011)). We believe that for short average duration and incidence of unemployment like the ones in the U.S. economy, the policy effects on these dimensions is arguably negligible for a large proportion of the labor force. Instead, assortative mating may have a first order effect on redistribution across households in the system-financing burden.

In the trade-off examined between public and private provision of insurance, we have assumed that the only private costs amount to forgone leisure. However, there might be others such as

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8 Appendix

8.1 Appendix. Proofs of Section 4

Proof of Lemma 4.1.

Consider the first order condition (2). Let $f(w,y) \equiv \psi_c(y + w, y/x) x + \psi_t(y + w, y/x)$. Notice that $\frac{\partial f(w,y)}{\partial y} < 0$ due to Assumptions A3 and A4. Therefore, the Implicit Function Theorem ensures that there exists a unique function $y_x(w)$ such that $f(w, y_x(w)) = 0$ in an open neighborhood of $w$. Indeed, $y_x$ is twice continuously differentiable since so is $f$ because of assumption A1.

To show that $y_x$ is a strictly decreasing function, we differentiate equation (2) with respect to $w$, and obtain

$$
\left(v_{cc}x + v_{tc}\right)\left(\frac{dy_x}{dw} + 1\right) + v_{ct}\frac{dy_x}{dw} + v_{tt}\frac{dy_x}{dw}\frac{1}{x} = 0
$$

$$
\Leftrightarrow \frac{dy_x}{dw} = -\frac{v_{cc}x + v_{tc}}{v_{ct} + 2v_{ct} + \frac{v_{tt}}{x}} < 0. \quad (16)
$$

Moreover, if $z < w$, then total earnings are lower if married with an employed worker, $y^e_x(w) < y^u_x$.

We now make use of these results to prove that function $V_x$ is twice continuously differentiable. We can rewrite it as a composite function of twice continuously differentiable functions, $V_x(w) = v(y^e_x(w) + w, y^e_x(w)/x)$, and, hence, so is it.
To show that function $V_x$ is strictly increasing and concave, we compute the first and second derivatives.

\[
V'_x(w) = v_c > 0 \\
V''_x(w) = v_{cc} \left( \frac{dy_x}{dw} + 1 \right) + v_{cl} \frac{dy_x}{dw} \frac{1}{x} = \frac{v_{cc}v_{ll} - v_{cl}^2}{v_{cc}x^2 + 2xv_{cl} + v_{ll}} < 0
\]

The first derivative is determined using the Envelope Theorem. To compute the second derivative, we have used expression (16). Assumptions A3 and A4 ensure that the second derivative is negative. ||

**Proof of Lemma 4.2.**

Let $V_x(w) \equiv \max_y v(y + w, y/x)$. The first order condition of this maximization problem is $v_c x + v_l = 0$. Then,

\[
\frac{\partial V_x}{\partial x} = -v_l \frac{y}{x^2} > 0
\]

Thus, $V_x$ is increasing in $x$.

Moreover, we obtain from the first order condition

\[
\frac{\partial y_x}{\partial x} = -\frac{v_c + v_{cl}y_x + v_{ll}y_x}{x^2v_{cc} + 2xv_{cl} + v_{ll}}
\]

Recall that $V'_x(w) = v_c$. Therefore,

\[
\frac{\partial V'_x}{\partial x} = v_{cc} \frac{\partial y_x}{\partial x} + v_{cl} \left( \frac{\partial y_x}{\partial x} \frac{1}{x} - \frac{y_x}{x^2} \right) = \frac{(v_{cc} + v_{cl}/x) \left( -v_c + v_{cl}y_x + v_{ll}y_x \right) - \frac{y_x}{x^2} v_{cl} \left( x^2v_{cc} + 2xv_{cl} + v_{ll} \right)}{x^2v_{cc} + 2xv_{cl} + v_{ll}} = \frac{(v_{cc}v_{ll} - v_{cl}^2) y_x - v_c(v_{cc} + v_{cl}/x)}{x^2v_{cc} + 2xv_{cl} + v_{ll}} < 0
\]

where the last expression results after some simplifications. ||

**Proof of Proposition 4.3.**

Consider problem (5) of a household of type $x$. Notice that the constraint establishes a positive relationship between $w$ and $q$. Therefore, the household’s problem can be rewritten only in terms of the wage $w$. Since the resulting objective function is continuous in $w$ and the domain $[z, 1]$ is compact, the Weierstrass Theorem ensures the existence of a solution.
The first derivative becomes
\[
-\nu(q) \frac{1 - \gamma(q) V_x(w) - V_x(z)}{\gamma(q)} + \nu(q) V_x'(w)
\]

When, \( w = z \), the first term of the derivative is 0, while the second term is strictly positive. Likewise, the derivative is negative at \( w = 1 \). Furthermore, the objective function is nonnegative and values \( V_x(z) \) at the two extremes of the domain. These two results together imply that the wage solution must be an interior point. The first order condition becomes

\[
(\text{FOC}): \frac{V_x(w) - V_x(z)}{V_x'(w)} = \frac{k}{\eta(q) 1 - \gamma(q)}
\]

The right hand side of this expression is decreasing in \( q \) and, hence, also in \( w \). The derivative of the left hand side is

\[
1 - \frac{(V_x(w) - V_x(z))V_x''(w)}{V_x'(w)^2} > 0
\]

This expression is positive because function \( V_x \) is concave as stated in Lemma 4.1. Therefore, the solution of the first order condition must be unique, and it is also a sufficient condition.

As said in the text, equations (2) and (3) have a unique solution because of the concavity of the utility function. Therefore, there exists an equilibrium, and it is unique. 

**Proof of Proposition 4.4**

We follow closely the proof of Proposition 2 in Acemoglu and Shimer (1999). Consider types \( x \) and \( x' \). Given that \( V_x(w) \) is strictly increasing in \( w \), there exists an inverse function \( V_x^{-1} \), which is also differentiable. Define function \( f(s) \equiv \max_y v(y + V_j^{-1}(s), y/x) \), which is twice continuously differentiable. Notice that \( V_{x'}(w) = f \circ V_x(w) \). By differentiating with respect to \( w \), we obtain

\[
V_{x'}(w) = f'(V_x(w))V_x'(w), \text{ and } V_{x''}(w) = f''(V_x(w))V_x'(w)^2 + f'(V_x(w))V_x''(w)
\]

Using the first equality, we can rewrite the second expression as

\[
f''(V_x(w))V_x'(w)^2 = V_{x''}(w) - V''_x(w) \frac{V_x'(w)}{V_x'(w)}
\]

Therefore, \( f \) is a concave function, and, hence, \( V_{x'}(w) \) is a concave (convex) transformation of \( V_x(w) \) if and only if \( \frac{V''_x(w)}{V_x'(w)} \) is greater (lower) than \( \frac{V''_x(w)}{V_x'(w)} \). Then, to show that sign of the wage difference it suffices to follow the remaining steps in the proof of Proposition 2.
in Acemoglu and Shimer (1999). Finally, the difference in queue lengths is of the same sign as the wage difference since the equilibrium zero-profit condition establishes a positive relationship between $q$ and $w$.

Proof of Lemma 4.5.

1. The proof of wages and queue lengths as increasing functions of $z$ is analogous to its counterpart in Acemoglu and Shimer (1999). Hence, it is omitted.

2. Similarly to the previous case, by differentiating equation (3) with respect to $z$, we obtain

\[
\left( v_{cc}x + v_{cl} \right) \left( \frac{\partial y_x}{\partial z} + 1 \right) + \left( v_{cl} + v_{ll} / x \right) \frac{\partial y_x}{\partial z} = 0
\]

\[
\Leftrightarrow \frac{\partial f_x}{\partial z} = \frac{\partial y_x}{\partial z} / x = \frac{-xv_{cc} - v_{cl}}{v_{cc}x^2 + 2xv_{cl} + v_{ll}},
\]

which is negative.

Note that $\frac{\partial c_f}{\partial z} = \frac{dx}{dw} \frac{\partial w}{\partial z} \leq 0$ because the first factor is negative according to Lemma 4.1, while the second one was shown above to be non-negative.

Proof of Lemma 4.6.

Consider the first order condition of the household problem (12), for a given $x$. After replacing $c^m$ using the budget constraint, the first order conditions with respect to $c^f$ and $y$ are

\[
f_1(w, c^f, y) = -\alpha v^m_c + (1 - \alpha) v^f_c = 0
\]

\[
f_2(w, c^f, y) = \alpha v^m_c x + (1 - \alpha) v^f_c = 0
\]

Let $f(w, c^f, y) = (f_1(w, c^f, y), f_2(w, c^f, y))$. The Jacobian matrix of $f$ is invertible since

\[
|J| = \begin{vmatrix}
\frac{\partial f_1}{\partial c^f} & \frac{\partial f_1}{\partial y} \\
\frac{\partial f_2}{\partial c^f} & \frac{\partial f_2}{\partial y}
\end{vmatrix} = \begin{vmatrix}
\alpha v^m_c + (1 - \alpha) v^f_c & -\alpha v^m_c + (1 - \alpha) v^f_c / x \\
-\alpha v^m_c x + (1 - \alpha) v^f_c & \alpha v^m_c x + (1 - \alpha) v^f_c / x
\end{vmatrix} = \left( (1 - \alpha) \alpha v^m_c \left( v^f_c x^2 + v^f_{ll} + 2xv^f_{cl} \right) + (1 - \alpha)^2 (v^f_c v^f_{ll} - (v^f_{cl})^2) \right) / x > 0
\]

Therefore, the Implicit Function Theorem ensures that, for a given $x$, there exists unique functions $c^f_x(w)$, $c^m_x(w)$ and $y_x(w)$ such that $f(w, c^f_x(w), y_x(w)) = 0$. Indeed, $y_x$ is twice continuously differentiable since so is $f$ because of assumption A1.
To show that $y_x$ is a strictly decreasing function, we differentiate the two first order conditions (17) and (18) with respect to $w$, and obtain a system of two equations with unknowns $dy_x/dw$ and $dc_x^f/dw$. By manipulating them, we obtain

$$\frac{dy_x}{dw} = \frac{-\alpha v_{cc}^m(v_{cl} + xv_{cc}^f)}{\alpha v_{cc}^m(xv_{cc}^f + 2v_{cl}^f + v_{cl}^f/x) + (1 - \alpha)(v_{cc}^f v_{cl} - (v_{cl}^f)^2)/x} < 0$$

Moreover,

$$\frac{dc_x^f}{dw} = -\frac{dy_x}{dw} \frac{v_{cl}^f/x + v_{cl}^f}{v_{cl}^f + xv_{cc}^f} > 0$$ (19)

We now make use of these results to prove that function $V_x$ is twice continuously differentiable. We can rewrite it as a composite function of twice continuously differentiable functions, $V_x(w) = \alpha v^m (y_x(w) + w - c_x^f(w), \ell) + (1 - \alpha) v^f (c_x^f(w), y_x(w)/x)$, and, hence, so is it.

To show that function $V_x$ is strictly increasing and concave, we compute the first and second derivatives.

$$V'_x(w) = \alpha v^m c_x^f (\frac{dy_x}{dw} + 1 - \frac{dc_x^f}{dw}) + (1 - \alpha) v^f c_x^f \left(\frac{dy_x}{dw} + \frac{dc_x^f}{dw} \frac{1}{x} \right) = \alpha v^m c_x^f > 0$$

$$V''_x(w) = \alpha v^m_{cc} \left(\frac{dy_x}{dw} + xv_{cc}^f + 2v_{cl}^f + v_{cl}^f/x + 2v_{cl}^f + \frac{v_{cl}^f}{x} \right) + \frac{(1 - \alpha)(v_{cc}^f v_{cl} - (v_{cl}^f)^2)}{x} < 0$$

Assumptions A3 and A4 ensure that the second derivative is negative.

Let $\ell_x^e$ and $\ell_x^u$ denote hours worked by the spouse of an employed and unemployed worker, respectively. The proof of $\frac{\partial \ell_x^u}{\partial z} < 0$ is analogous to expression (19). Finally, $\frac{\partial \ell_x^u}{\partial z} = \frac{d\ell_x^e}{dw} \frac{\partial w}{\partial z} < 0$ since the second factor is positive as stated in Lemma 4.5.

### 8.2 Appendix. Proofs of Section 5

**Proof of Lemma 5.2.**

The proof is by contradiction.\(^{29}\) Suppose that such conditions are not sufficient, and, hence, there exists $\hat{x}$ and $x$ such that $\mathcal{U}_x(\hat{x}) > \mathcal{U}_x$. Let us suppose without loss of generality

\(^{29}\)We closely follow Fudenberg and Tirole (1991, Ch. 7, p. 261).
that $\hat{x} > x$. That is, $\int_x^{\hat{x}} \frac{\partial U_x(a)}{\partial a} da > 0$. This integral can be developed to obtain

$$\int_x^{\hat{x}} \frac{\partial U_x(a)}{\partial a} da = \int_x^{\hat{x}} \left( \nu'(q_a) \hat{q}_a \left( v(c_e^a, y_e^a/x) - v(c_u^a, y_u^a/x) \right) + \nu(q_a) \left( v(c_e^a, y_e^a/a) \hat{c}_a^e + v(c_u^a, y_u^a/a) \hat{y}_a^u \right) \right) da \leq$$

$$+ (1 - \nu(q_a)) \left( v(c_u^a, y_u^a/a) \hat{c}_a^u + v(c_u^a, y_u^a/x) \hat{y}_a^u \right) \right) da \leq$$

$$\int_x^{\hat{x}} \left( \nu'(q_a) \hat{q}_a \left( v(c_e^a, y_e^a/a) - v(c_u^a, y_u^a/a) \right) + \nu(q_a) \left( v(c_e^a, y_e^a/a) \hat{c}_a^e + v(c_u^a, y_u^a/a) \hat{y}_a^u \right) \right) da = 0,$$

where the inequality results from the integrand being an increasing function in $x$ because of the (local) second order condition, and the last equality is the necessary first order condition. This is a contradiction, and, hence, the local conditions are also sufficient.

\textbf{Proof of Proposition 5.3}

Let $(q_x, w_x)$ denote the unique equilibrium pair of distributions of queue lengths and wages. We first show that it belongs to the feasible set of the planner’s problem. The resource constraint (RC) holds in equilibrium because the zero-profit condition is satisfied in all submarkets. The participation condition for all households holds with strict inequality because cost $\kappa$ is sufficiently small by assumption. Likewise, utility-maximizing households of type $x$ prefer pair $(q_x, w_x)$ than $(q_{x'}, w_{x'})$ for any $x'$ in equilibrium; hence, the equilibrium allocation is incentive compatible.

Consider now the following alternative allocation: $c_e^x = w_x + y_e^x - \epsilon$, $c_u^x = z + y_u^x + \delta$, such that it is resource-neutral, i.e. $\int_{\bar{x}}^{\hat{x}} \left( -\nu(q_x) \epsilon + (1 - \nu(q_x)) \delta \right) dF(x) = 0$. Notice that this allocation yields a strictly higher value than the equilibrium one due to the concavity of the utility function $\upsilon$. It also belongs to the feasible set as the resource constraint holds. It is straightforward to see that the participation condition (PC$_x$) also holds because the participation cost $\kappa$ is sufficiently small. The incentive compatibility conditions (ICC$_x$) also hold for $\epsilon$ arbitrarily small because Proposition 4.3 ensures that the problem of type $x$ households has a unique solution and the pair $(q_x, w_x - \epsilon)$ satisfies its inequality constraint.\footnote{The household’s problem can be equivalently formalized with an inequality instead of an equality.}
Therefore, the equilibrium allocation is not solution of the planner’s problem.||

**Proof of Proposition 5.4**

Consider the following program

\[
\begin{align*}
\max \int \left( \nu(q_x)(1 + y_x^e + \phi(y_x^e/x)) + (1 - \nu(q_x))(z + y_x^u + \phi(y_x^u/x)) - \frac{k}{q_x} \right) dF(x)
\end{align*}
\]

This is the planner’s unconstrained problem after replacing the consumption values using the resource constraint. Then, we are to show that the equilibrium allocation is feasible and incentive compatible, and is a solution of the unconstrained problem, and, hence, must coincide with the planner’s solution.

The first order conditions of the planner’s unconstrained problem are, for all \( x \),

\[
\begin{align*}
\frac{k}{\eta(q)} &= (1 - \gamma(q))(1 - z), \quad \text{where } q \equiv q_x \\
\phi'(y_x/x) &= -x, \quad \text{where } y_x \equiv y_x^e = y_x^u
\end{align*}
\]

Notice that these equations coincide with the equilibrium conditions (8)-(10), and there exists a unique solution to this system of equations. Therefore, the equilibrium allocation is the solution of the planner’s unconstrained problem. We define consumption levels as \( c_x^e \equiv w + y_x^e/x \) and \( c_x^u \equiv z + y_x^u/x \), where \( w = 1 - \frac{k}{\eta(q)} \) is the equilibrium wage. For this allocation to be the planner’s solution, it remains to show that all constraints of the planner’s problem hold. First, the participation rate holds for all types as it does in equilibrium. Second, the equilibrium allocation is obviously incentive compatible. Therefore, the equilibrium allocation is constrained efficient.||

**Proof of Lemma 5.5**

The proof follows very closely the one for Proposition 5.4. First, for notational simplicity, let \( \tilde{\alpha} \equiv \alpha \mathbb{I}_{\alpha > 0.5} + (1 - \alpha)(1 - \mathbb{I}_{\alpha \geq 0.5}) \).

The equilibrium allocation is determined by condition (8) and (9), whereas the equilibrium condition (10) is replaced by \( \phi'(y_x/x) = -x \frac{\tilde{\alpha}}{1 - \tilde{\alpha}} \), which leads to \( y_x^e = y_x^u \).

As for the unitary model of the household, using the resource constraint, the planner’s objective function can be rewritten as

\[
\int \left( \nu(q_x)\left( \tilde{\alpha}(1 + y_x^e) + (1 - \alpha)\phi(y_x^e/x) \right) + (1 - \nu(q_x))\left( \tilde{\alpha}(z + y_x^u) + (1 - \alpha)\phi(y_x^u/x) \right) - \frac{k}{q_x} \right) dF(x)
\]

constraint.
The necessary conditions of the unconstrained problem are, for all \( x \),

\[
\frac{k}{\eta(q)} = (1 - \gamma(q))(1 - z), \quad \text{where } q \equiv q_x
\]

\[
\phi'(y_x / x) = -x \frac{\tilde{\alpha}}{1 - \alpha}, \quad \text{where } y_x \equiv y^e_x = y^u_x
\]

Since the equilibrium and efficiency conditions are the same, and there exists a unique solution to this system of equations, the equilibrium allocation is the solution of the planner’s problem. ||

**Proof of Proposition 5.6.**

The first order conditions of the planner’s problem (14) are

\[
\nu'(q) \left( (V(m^e) - V(m^u))(1 + \xi_2) + \xi_1 (1 - m^e - z + m^u) \right) = -\xi_1 \frac{k}{q^2}
\]

\[
V'(m^e)(1 + \xi_2) \leq \xi_1 \text{ and } m^e \geq z, \quad \text{with comp. slackness}
\]

\[
V'(m^u) \left( 1 - \xi_2 \frac{\nu(q)}{1 - \nu(q)} \right) \leq \xi_1 \text{ and } m^u \geq z, \quad \text{with comp. slackness}
\]

where \( \xi_1 \) and \( \xi_2 \) are the Lagrange multipliers of the first and second constraints, respectively. Since \( m^e > m^u > z \), we can use the last two conditions to replace the multipliers in the first equation to obtain

\[
(1 - \gamma(q)) \left( \frac{V(m^e) - V(m^u)}{V'(m^e)} + 1 - m^e - z + m^u \right) = \frac{k}{\eta(q)} \quad (20)
\]

Notice that \( \xi_2 > 0 \) because \( m^e = m^u \) otherwise, and constraint (PC) would not hold. Therefore, the participation condition is binding. Let \((\hat{m}^e, \hat{m}^u, \hat{q})\) denote the planner’s solution, which satisfies this necessary condition as well as the two constraints of the planner’s problem with equality.

Consider the following set of fiscal instruments \((b, \tau, T)\), where the first element stands for unemployment benefits, the second one is a proportional income tax rate for newly employed workers, and the last instrument is a lump sum tax paid per household. For the tax-distorted
equilibrium \((w, q)\) to be constrained efficient, the following conditions must hold:

\begin{align*}
\hat{m}^u &= z + b - T \quad (21) \\
\hat{m}^e &= w(1 - \tau) - T \\
\tau \frac{k}{\eta(\hat{q})} &= (1 - \gamma(\hat{q}))(\tau + b) \quad (23) \\
T + \nu(\hat{q})\tau w &= (1 - \nu(\hat{q}))b
\end{align*}

The first two conditions ensure that consumption in the decentralized economy equals its level in the planner’s allocation, where the wage \(w\) is determined by equation (7) evaluated at \(q = \hat{q}\). The third condition is necessary and sufficient for the equilibrium and the efficiency first order conditions (6) and (20) to be equal to one another. Finally, the last condition is the government’s budget constraint. Notice that the government’s equation is the same as the planner’s resource constraint (14) after replacing wages using the zero-profit condition (7). Therefore, to show the implementation of the planner’s solution, we need to determine a solution of the system of equations (21)-(23). We can eliminate \(T\) and write \(b\) in terms of \(\tau\) by subtracting (21) from (22) to obtain

\[
b = \left(1 - \frac{k}{\eta(\hat{q})}\right)(1 - \tau) - z \frac{\hat{m}^e - \hat{m}^u}{\hat{m}^e - \hat{m}^u}.
\]

Notice that \(b\) is strictly positive as both the numerator and denominator must be strictly positive. Then, we can obtain \(\tau\) from equation (23) after replacing \(b\) as

\[
\tau = \frac{1 - \frac{k}{\eta(\hat{q})} - z}{\hat{m}^e - \hat{m}^u \left(1 - \frac{k}{\eta(\hat{q})(1 - \gamma(\hat{q}))} - \frac{1 - \frac{k}{\eta(\hat{q})}}{\hat{m}^e - \hat{m}^u}\right)},
\]

and then the values of \(b\) and \(T\) are uniquely determined. Finally, notice that \(\tau \neq 0\) because otherwise \(b = 0\) according to equation (23) and \(T < 0\) due to condition (21), and, as a result, the government’s budget constraint (24) would fail to hold. \(\|

Proof of Proposition 5.7.
We first rewrite the planner’s problem

\[
\begin{align*}
\max \quad & \int_{x}^{x} U_{x} f(x) dx \\
\text{s. to} \quad & U_{x} = \nu(q_{x})\nu(c_{x}^{e}, y_{x}^{e}/x) + (1 - \nu(q_{x}))\nu(c_{x}^{u}, y_{x}^{u}/x) \\
& \dot{A}_{x} = \left(\nu(q_{x})(1 + y_{x}^{e} - c_{x}^{e}) + (1 - \nu(q_{x}))\left(z + y_{x}^{u} - c_{x}^{u}\right) - \frac{k}{q_{x}}\right)f(x) \quad (24) \\
& A_{x}, A_{\tau} = 0 \\
& U_{x} \geq \kappa + \nu(c_{x}^{u}, y_{x}^{u}/x) \quad (25) \\
& \dot{U}_{x} = \nu(q_{x})\frac{y_{x}^{e}}{x^{2}} + (1 - \nu(q_{x}))\frac{y_{x}^{u}}{x^{2}} \quad (26) \\
& 0 \leq -\nu'(q_{x})p_{x}^{q}(y_{x}^{u} - y_{x}^{e}) + \nu(q_{x})p_{x}^{e} + (1 - \nu(q_{x}))p_{x}^{u} \quad (27) \\
& \dot{q}_{x} = p_{x}^{q} \\
& \dot{p}_{x}^{e} = p_{x}^{e} \\
& \dot{y}_{x}^{u} = p_{x}^{u}
\end{align*}
\]

The control variables are \(c_{x}^{e}, c_{x}^{u}, p_{x}^{q}, p_{x}^{e},\) and \(p_{x}^{u}\), whereas \(U, q_{x}, y_{x}^{e}\) and \(y_{x}^{u}\) are the state variables. As usual in optimal control problems, we transform the resource constraint (RC) into differential equation (24) along with two boundary constraints. Inequality (25) is the participation condition (PC\(_x\)). The incentive compatibility conditions (FOC-ICC\(_x\)) and (SOC-ICC\(_x\)) become (26) and (27), respectively. The last three differential equations are state equations.

The Hamiltonian is defined as

\[
\mathcal{H} = U_{x} f(x) + \lambda_{x}^{1}\left(\nu(q_{x})(1 + y_{x}^{e} - c_{x}^{e}) + (1 - \nu(q_{x}))\left(z + y_{x}^{u} - c_{x}^{u}\right) - \frac{k}{q_{x}}\right)f(x)
\]

\[
+ \lambda_{x}^{2}\left(\nu(q_{x})\frac{y_{x}^{e}}{x^{2}} + (1 - \nu(q_{x}))\frac{y_{x}^{u}}{x^{2}}\right) + \lambda_{x}^{3}\left(U_{x} - \kappa - u_{x}^{u}\right) \\
+ \lambda_{x}^{4}\left(U_{x} - \nu(q_{x})v_{x}^{e} - (1 - \nu(q_{x}))u_{x}^{u}\right) \\
+ \lambda_{x}^{5}\left(-\nu'(q_{x})p_{x}^{q}(y_{x}^{u} - y_{x}^{e}) + \nu(q_{x})p_{x}^{e} + (1 - \nu(q_{x}))p_{x}^{u}\right) \\
+ \lambda_{x}^{6}p_{x}^{q} + \lambda_{x}^{7}p_{x}^{e} + \lambda_{x}^{8}p_{x}^{u}
\]

where \(\lambda_{x}^{1}, \lambda_{x}^{2}, \lambda_{x}^{6}, \lambda_{x}^{7}\) and \(\lambda_{x}^{8}\) are the respective co-state variables, and the multipliers \(\lambda_{x}^{3}\), \(\lambda_{x}^{5} \geq 0\). To simplify notation, we denote \(v_{x}^{j} \equiv \nu(c_{x}^{j}, y_{x}^{j}/x),\) for \(j \in \{u,e\}\).
The following necessary conditions must be satisfied:

\[
\frac{\partial H}{\partial q} = -\dot{\lambda}_x^6 \iff \lambda_x^1 f(x) \left( \nu'(q_x) \left( 1 + y_x^e - c_x^e - z - y_x^u + c_x^u + \frac{k}{q_x^2} \right) - \lambda_x^2 \nu'(q_x)(y_x^u - y_x^e)/x^2 - \lambda_x^4 \nu'(q_x)(\nu_x^e - \nu_x^u) + \lambda_x^5 \left( - \nu''(q_x)p_x^u (y_x^u - y_x^e) + \nu'(q_x)(p_x^e - p_x^u) \right) \right) = -\dot{\lambda}_x^6
\]

\[
\frac{\partial H}{\partial c_x^u} = 0 \iff \lambda_x^4 = -\frac{\lambda_x^1 f(x)}{\nu_x^e}
\]

\[
\frac{\partial H}{\partial c_x^u} = 0 \iff \lambda_x^3 \nu_x^u = -(1 - \nu(q_x)) (\lambda_x^4 \nu_x^u + \lambda_x^1 f(x))
\]

\[
\frac{\partial H}{\partial y_x^e} = -\dot{\lambda}_x^7 \iff \lambda_x^1 \nu(q_x) f(x) + \lambda_x^2 \nu(q_x)/x^2 + \lambda_x^4 \nu(q_x)/x + \lambda_x^5 \nu'(q_x)p_x^q = -\dot{\lambda}_x^7
\]

\[
\frac{\partial H}{\partial y_x^u} = -\dot{\lambda}_x^8 \iff \lambda_x^1 (1 - \nu(q_x)) f(x) + \lambda_x^2 (1 - \nu(q_x))/x^2 + \lambda_x^3/x + \lambda_x^4 (1 - \nu(q_x))/x
\]

\[
-\lambda_x^5 \nu'(q_x)p_x^q = -\dot{\lambda}_x^8
\]

\[
\frac{\partial H}{\partial A_x} = -\dot{\lambda}_x^1 \iff \lambda_x^1 = \lambda^1, \forall x
\]

\[
\frac{\partial H}{\partial p_x^q} = 0 \iff \lambda_x^6 = \lambda_x^6 \nu'(q_x)(y_x^u - y_x^e)
\]

\[
\frac{\partial H}{\partial p_x^u} = 0 \iff \lambda_x^7 = -\lambda_x^5 \nu(q_x)
\]

\[
\frac{\partial H}{\partial \nu_x^u} = 0 \iff \lambda_x^8 = -\lambda_x^5 (1 - \nu(q_x))
\]

\[
\frac{\partial H}{\partial \nu_x^e} = -\dot{\lambda}_x^2 \iff f(x) + \lambda_x^3 + \lambda_x^4 = -\dot{\lambda}_x^2
\]

\[
\lambda_x^3 \geq 0, \quad \text{and} \quad 0 = \lambda_x^3 \left( U_x - \kappa - \nu_x^u \right)
\]

\[
\lambda_x^5 \geq 0, \quad \text{and} \quad 0 = \lambda_x^5 \left( - \nu'(q_x)p_x^q (y_x^u - y_x^e) + \nu(q_x)p_x^e + (1 - \nu(q_x))p_x^u \right)
\]

and since there are neither initial nor final conditions for $U$, $q_x$, $y_x^e$ and $y_x^u$, the following transversality conditions hold

\[
\lambda_x^2 = \lambda_x^2 = 0, \lambda_x^6 = \lambda_x^6 = 0, \lambda_x^7 = \lambda_x^7 = 0, \lambda_x^8 = \lambda_x^8 = 0
\]

Because the inequality constraints are concave in the control variables, the inequality constraint qualification holds leading to conditions (38) and (39).

We first show that $\lambda^1 > 0$. Using equation (37), we can write $\lambda_x^2 = -\int_x f(t) + \lambda_x^2 + \lambda_x^3 \right) dt$
because of the transversality condition $\lambda_x^2 = 0$. Likewise, the transversality condition implies

$$
\lambda_x^2 = 0 = -1 - \int_{x}^{y} (\lambda_t^3 + \lambda_t^4) \, dt \iff \lambda^1 = \frac{1}{\int_{x}^{y} \frac{1}{v'_e c} \left(1 - \frac{v_u^e}{v'_e} (1 - \nu(q_x)) \right) f(x) \, dx} > 0, \quad (40)
$$

where the sum $\lambda_t^3 + \lambda_t^4$ obtains from equations (29) and (30). The co-state variable is positive as it can easily be shown that $c_x^u \leq c_x^e$ and due to the concavity of function $\nu$.

From equations (35) and (36), we obtain $\lambda_x^7 (1 - \nu(q_x)) = \lambda_x^8 \nu(q_x)$. By differentiating this expression with respect to $x$, we obtain

$$
\dot{\lambda}_x^7 (1 - \nu(q_x)) - \lambda_x^8 \nu(q_x) = -\lambda_x^5 \nu'(q_x) p_x^q
$$

We then subtract equation (32) multiplied by $\nu(q_x)$ from (31) times $1 - \nu(q_x)$ and use this last equality to obtain $\lambda_x^2 = 0$ for all $x$. We obtain that consumption does not vary with the employment state of the worker as it follows from equations (29) and (30) that

$$
\lambda_x^2 v^u_c = -\lambda^1 f(x) (1 - \nu(q_x)) \left(1 - \frac{v^u_c}{v'_e} \right) \lambda^1 = c_x^u = c_x^e, \forall x \implies y_x^e < y_x^u, \forall x
$$

Now, we differentiate equations (34) and (35) with respect to $x$

$$
\begin{align*}
\dot{\lambda}_x^6 &= \dot{\lambda}_x^5 \nu'(q_x) (y_x^u - y_x^e) + \lambda_x^5 \nu''(q_x) p_x^q (y_x^u - y_x^e) + \lambda_x^5 \nu'(q_x) (p_x^u - p_x^e), \\
\dot{\lambda}_x^7 &= -\lambda_x^5 \nu(q_x) - \lambda_x^5 \nu'(q_x) p_x^q,
\end{align*}
$$

and substitute out $\dot{\lambda}_x^6$ and $\dot{\lambda}_x^7$ in equations (28) and (31), respectively, to obtain

$$
\begin{align*}
(\dot{\lambda}_x^5 - \lambda_x^2 x^2) \nu'(q_x) (y_x^u - y_x^e) &= \lambda^1 f(x) \left(\nu'(q_x) \left(\frac{y_x^e - y_x^u}{x v'_e} - 1 - y_x^e + z + y_x^u\right) - \frac{k}{q_x^2} \right), \\
\dot{\lambda}_x^5 - \lambda_x^2 x^2 &= \lambda^1 f(x) \left(1 - \frac{1}{x v'_e} \right) \quad (41)
\end{align*}
$$

By combining these two equations, we obtain expression (15), which implies $p_x^q = \dot{q}_x = 0$.

Next, we show that $\dot{c}_x \geq 0$ and $\nu(q_x) p_x^e + (1 - \nu(q_x)) p_x^u \geq 0$. Notice that the first order
condition of the (ICC) can be stated as follows, using \( \dot{q}_x = 0 \) and \( e^c_x = c^u_x = c_x \):

\[
\frac{dU_x(\hat{x})}{dx} |_{x=x} = 0 \iff \frac{\partial U_x(\hat{x})}{\partial c^c_x} \dot{c}_x + \frac{\partial U_x(\hat{x})}{\partial y^e_x} \dot{y}^e_x + \frac{\partial U_x(\hat{x})}{\partial c^u_x} \dot{c}_x + \frac{\partial U_x(\hat{x})}{\partial y^u_x} \dot{y}^u_x = 0 \iff v^e_c \dot{c}_x = \nu(q) \frac{p^e_x}{x} + (1 - \nu(q)) \frac{p^u_x}{x}
\]

The right hand side is nonnegative because of constraint (27). Therefore, so is \( \dot{c}_x \).

We now prove, by contradiction, that there exists a subset of positive mass within consumption strictly increases with spouse’s productivity. Suppose that there is no subset of positive mass within \((x, \bar{x})\) such that \( \dot{c}_x > 0 \) or equivalently, due to the FOC of the (ICC) together with \( \dot{q}_x = 0, \nu(q_x)p^e_x + (1 - \nu(q_x))p^u_x > 0 \). It follows from expression (40) that \( \lambda^1 = v^e_c \), which is constant in \( x \), and, hence, \( \lambda^4_x = -f(x) \), because of condition (29), and \( \lambda^2_x = 0 \), due to condition (37) and the transversality conditions, for all \( x \in (\bar{x}, \bar{x}) \) except for a zero mass set. Then, equation (41) can be rewritten as \( \hat{\lambda}^5_x = \lambda^1 f(x) \left( 1 - \frac{1}{x x^e} \right) \). Notice that the term in parenthesis is increasing in \( x \) and \( \lambda^5_x = 0 \) and \( \hat{\lambda}^5_x \geq 0 \). It follows that \( \lambda^5_x > 0 \), which is a contradiction because of the transversality conditions. Therefore, there exists a subset \( S \subset [\bar{x}, \bar{x}] \) of positive mass in which both \( c_x \) and \( \nu(q)y^e_x + (1 - \nu(q))y^u_x \) strictly increase with \( x \).

We turn to show that for all \( x \in S \), \( \frac{1}{x x^e} < 1 \). By definition, \( \nu(q)p^e_x + (1 - \nu(q))p^u_x > 0 \) and \( \dot{c}_x > 0 \) for all \( x \in S \). Hence, by continuity, these inequalities hold within an open neighborhood of \( x \in S \). Therefore, \( \lambda^5_x = 0 \) and \( \hat{\lambda}^5_x = 0 \) for all \( x \in S \) because of condition (39).

Equation (41) implies that \( \frac{1}{x x^e} < 1 \) if \( \lambda^5_x < 0 \). We now show that \( \lambda^2_x < 0 \) for all \( x \in (\bar{x}, \bar{x}) \).

We use conditions (29), (37) and (40) to write

\[
\hat{\lambda}^2_x = \lambda^1 f(x) \int_{\bar{x}}^{\bar{x}} \left( \frac{1}{v^e_c(c_x, y^e_x/x)} - \frac{1}{v^e_c(c_x, y^e_x/x)} \right) f(t)dt.
\]

As fraction \( \frac{1}{v^e_c} \) is an nondecreasing function in \( x \) because \( \dot{c}_x \geq 0 \), the derivative of the integral with respect to \( x \) is positive and, hence, \( \hat{\lambda}^2_x \) changes sign at most once and is almost always different from 0. Indeed, the integral is negative at \( x = \bar{x} \) and positive at \( x = \bar{x} \). Since \( \lambda^2_x = \lambda^2_x = 0 \), we have that \( \lambda^2_x \) is negative and convex in \((\bar{x}, \bar{x})\).

Finally, it is immediate to see that the equilibrium allocation is not constrained efficient as equilibrium condition (11) holds neither in S nor out of it where \( \dot{c}_x = 0 \).

**Proof of Lemma 5.8.**

Let \( T \) denote a tax on household’s total income. For the tax-distorted equilibrium to
coincide with the planner’s solution, efficiency condition (15) as well as \( \psi'(c_x) > \frac{1}{x} \) must hold.

The tax-distorted counterparts of equilibrium equations (8), (9) and (11)

\[
(1 - \gamma(q))(\frac{V(w) - V(z)}{\psi(c_x^e)(1 - T'(w + y_x^e))} + 1 - w) = \frac{k}{\eta(q)} \tag{42}
\]

\[
k = \eta(q)(1 - w) \tag{43}
\]

\[
\psi(c_x^e)(1 - T'(w + y_x^e)) = \psi(c_x^u)(1 - T'(z + y_x^u)) = \frac{1}{x} \tag{44}
\]

where \( c_x^e \equiv w + y_x^e - T(w + y_x^e) \) and \( c_x^u \equiv z + y_x^u - T(w + y_x^e) \). Furthermore, the government balances its budget, and, hence,

\[
\int_{\Xi} \left( \nu(q)T(w + y_x^e) + (1 - \nu(q))T(z + y_x^u) \right) f(x)dx = 0
\]

We now compare the planner’s solution with the tax-distorted equilibrium allocation. First, for the consumption levels to be independent of the employment status, it must be the case that \( c_x^e = c_x^u = c_x \). Second, given the planner’s queue length, the tax-distorted equilibrium wage \( w \) is determined by the zero-profit condition (43). Third, by comparing equilibrium condition (44) with its planner’s counterpart, we conclude that the tax function \( T \) must be increasing. Next, by using equation (44) to replace the marginal utility and imposing the equal-consumption condition, we can rewrite equation (42) as

\[
(1 - \gamma(q)) \left( 1 - z - T(w + y_x^e) + T(z + y_x^u) \right) = \frac{k}{\eta(q)}
\]

Therefore, the implementation of the planner’s solution implies \( T(w + y_x^e) = T(z + y_x^u) \), which leads to a pre-tax household’s total income independent of the employment state, \( w + y_x^e = z + y_x^u \). As \( w \) is invariant in \( x \) so is the difference \( y_x^u - y_x^e \).

Finally, it is straightforward to combine the zero-profit condition and the balanced-budget constrained of the government to obtain the resource constraint of the planner. ||