Long-Term Care Needs: Implication for Savings, Welfare and Public Policy

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Abstract

Contrary to the predictions of standard life cycle models, individuals dissave slowly during retirement. I investigate the role of long-term care (LTC) needs in the saving decisions of the elderly, those above age 70, and quantify its importance relative to bequests and medical expenses. For this purpose, I construct a model of savings for old people where LTC needs are stochastic, individuals are heterogeneous in their access to informal care, and there is an optimal choice of care hours bought in the market. Health is estimated using a new dynamic latent variable model that summarizes the rich information contained in health surveys into 4 parsimonious health groups. This allows me to account for heterogeneity in both LTC needs and survival probabilities. Results imply that motives associated with LTC needs are significantly more important than medical expenses or bequest motives as drivers of savings for higher-income elderly. On the other hand, the middle class saves little for LTC because their children provide substantial informal care and show strong bequest motives. Medical expenditures matter little because the health states in which they are high are also short-lived due to higher mortality. Equivalent variation calculations indicate that a 4.5% subsidy in the price of care is revenue neutral, since it reduces government transfers to the poor from means-tested programs. On the other hand, it is welfare improving for both the poor and the rich.

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1 Introduction

More than a third of the US total wealth is in hands of individuals aged 65 or older while they represent only 12% of the total population (Wolff, 2004). Moreover, individuals above age 65 are projected to represent one fifth of the total population by 2050. Most OECD countries face similar demographic trends. Therefore, understanding how the elderly manage their finances is becoming increasingly important.

Contrary to the predictions of a standard life-cycle model (Huggett, 1996), many elderly dissave slowly during retirement. Identifying the reasons why the simple model fails is crucial for the optimal design of policies related to health care, Social Security, and insurance markets. This paper has three goals: i) to analyze to what extent long-term care (LTC), defined as assistance performing basic tasks of everyday life, drives the savings decision of individuals late in life; ii) to compare its relative importance over bequest motives and medical expenses; and finally iii) to evaluate the effect of different LTC policies on welfare and redistribution.

LTC is likely very important driver of savings since it is insured to a lesser extent than medical expenses. In the US, Medicare is the main source of medical insurance for individuals aged over 65 but it does not cover most LTC expenses. Medicare only pays for nursing homes stays following a prior hospitalization of at least three days and for the first 20 days. Moreover, home health aide services are uncovered. On top of that, LTC is expensive when paid out of pocket; the average price of home care aide is around $21 per hour\(^1\) and the price of a private room at a nursing home is $48,000 per year (Stewart, Grabowski, & Lakdawalla, 2009).

In order to understand the financial risk associated to LTC, it is important to determine a metric for LTC needs. For this purpose, using the Health and Retirement Study (HRS) and the methodology in Amengual, Bueren, and Crego (2017), I document that LTC needs can be parsimoniously represented by four different health status. The four health status summarize the information contained on reported difficulties with Activities of Daily Living (ADLs) and

\(^1\)LongTermCare.gov
Instrumental Activities of Daily Living (IADL). First, healthy individuals do not need help with daily self-care activities. Then, physically and mentally frail individuals are in need of assistance with activities related to mobility and cognition, respectively. Finally, impaired individuals are in need of assistance with both physical and cognitive tasks. Estimated transitions probabilities uncover a large heterogeneity in LTC risk across gender and income: females and the poor facing significantly larger LTC risks.

Based on the previous health classification, I present new empirical facts consistent with the idea that LTC expenses are endogenous. As such, individuals adjust LTC expenditures based on LTC needs, financial resources and access to care from relatives. First, individuals spend increasing resources on formal care (FC) hours provided by paid helpers as health deteriorates. For example, physically frail individuals consume on average 0.7 hours of FC per day or $5,365 per year. When impaired, the average consumption of FC rises to 2.8 hours per day or $21,460 per year. Given that the median income of a single retired individual is around $16,840, LTC constitutes a significant financial risk faced by the elderly. Second, for a given health status, richer individuals spend significantly more on care. For example, an individual in the top quartile of the income distribution consumes 1.2 hour more of care per day than an individual in the bottom quartile when impaired. Finally, I find that access to informal care (IC) provided by relatives, lowers the demand of FC.

Motivated by these facts, I develop and estimate a model of single retired individuals including heterogeneity in both LTC needs and family types. Family types differ in the hours of IC provided by children when the elderly is in need of LTC. Besides, agents in the model derive utility from regular consumption and care hours. Families provide IC is for free but agents can also decide to buy extra care at a market price ($21 per hour). The marginal utility of care hours is allowed to differ across agents’ LTC needs so that individuals in the model, as in the data, are able to adjust their LTC expenses. Family type affects the agents’ savings decisions in two important ways. First, agents belonging to families that provide more IC need to save less for LTC expenses, thus reducing the need to accumulate precautionary savings related to LTC. Meanwhile, I allow the intensity of the bequest motive to vary across family types. Furthermore, common to previous literature,
agents are heterogeneous in their wealth, permanent income, gender, and transitory and persistent medical shocks. Finally, agents have the option to access a government means-tested program that provides a consumption floor and LTC services if necessary. This is the first paper to introduce LTC needs heterogeneity in a setting where LTC expenses are endogenous.

Family types are modeled as follows. I summarize the distribution in IC provided by relatives in the data by considering that each individual in the model belongs to one out of three possible family types. First, in close families, relatives provide intense IC when the elderly is in need. Second, in distant families, relatives provide moderate care. Thirdly, individuals on their own do not receive care from their relatives.

I use the Method of Simulated Moments (MSM) to estimate the parameters in the model. I minimize the distance between simulated life-cycle profiles of assets across different cohorts and permanent income (PI) groups and their empirical counterparts. Identification of health dependent preferences parameters is achieved by matching reported FC hours and Medicaid recipiency rates by health status, PI, and by family type.

The estimated model is able to match the pattern of the targeted features of the data. The estimated preferences parameters imply that as health deteriorates, agents optimally decide to increase the share of consumption devoted to FC. For example, an individual in the top quartile of the PI distribution spends around 16%, 33%, and 53% of his consumption in FC if physically frail, mentally frail and impaired respectively. Besides, bequest parameters show that individuals in close families show much larger bequest intensity than individuals on their own who hold negligible bequest motives.

Counterfactual simulations show that, first, LTC is a crucial driver of savings for the elderly rich. Individuals in the top quartile of the income distribution would dissave much faster in a world without LTC needs. For example, by age 90, the median level of assets would be 1/3 or $50,000 lower than in the benchmark model. Second, differences in access to IC show that individuals with relatively little access to IC have a much larger exposure to LTC financial risk. Indeed, the median level of assets of individuals without access to IC would be 50% lower at age 90 in a world without LTC while the median level of assets after
retirement would be unchanged for individuals in close families.

Then, I compare the relative importance of LTC to bequest motives and medical expenses. Bequests have a small effect on the savings behavior of the elderly rich. On the other hand, bequests motives seem an important driver of savings for individuals with families that provide IC. For example, the median wealth of individuals in close families and distant families would be 50% and 20% lower by age 90 in an economy without bequest motives, respectively.

As opposed to previous studies, agents in the proposed model self-insure little for medical expenses. This is because the estimated medical expenses’ process implies that, as in the data, people who suffer from large medical cost face larger chances of dying (Pauly, 1990). More precisely, I document that, the two-year probability of dying for individuals aged 70 and over is 9.3%. While individuals in the top 90, 95, and 99 percentiles of the medical expenses distribution, mortality rate rises to 15.9, 18.2, and 23.2, respectively. Using the proposed model, mortality is smaller at top quantiles (12.3, 13.2, and 16.3, respectively) but much closer to the data than using two health groups based on self-reported health as in De Nardi, French, and Jones (2010) (10.3, 10.8, and 11.3, respectively) or no health covariate as in Kopecky and Koreshkova (2014) (9.3 for all quantiles). Thus, the positive correlation between medical expenses and mortality rates limits the underlying financial risk that agents face.

Finally, I use the estimated model to evaluate the welfare gains from government subsidies to the price of FC. Interestingly, I find that small subsidies to FC decrease the enrollment rate in government means-tested program. As a results government transfers to the poor are reduced. More precisely, a 4.5% subsidy in the price of formal care is revenue neutral, reduces the progressivity of government transfers, but increases welfare for both the rich and the poor. Furthermore, I find that expansions of LTC hours provided by means-tested program are valued at more than its cost by both the rich and the poor. Subsidies to formal care, in spite of being regressive, are however, more efficient at increasing welfare for every dollar spent by the government.

The rest of the paper is organized as follows. In section 2, I review the most closely
related literature. In section 3, I explain the health classification used in the paper and present empirical facts on the adjustment of FC consumption. In section 4, I present the model. In section 5, I discuss parameter estimates and model fit. Section 6 present some counterfactuals that quantify the forces affecting the saving behavior. Section 7 presents a welfare and cost analysis of different LTC policies. Section 8 concludes.

2 Related Literature

This paper is related to the literature on the importance of health expenses (medical and LTC) on the savings behavior of the elderly. It is the first study to quantify how much the financial risk implied by LTC affects the savings pattern of the elderly with heterogeneous LTC needs and where LTC expenses are endogenous. Furthermore, I provide a new set of estimates on the relative importance of bequest, LTC, and medical expenses as drivers of late-in-life dissaving.

Kotlikoff et al. (1989) underline the importance of health expenditures for understanding the lack of dissaving of the elderly. However, Hubbard, Skinner, and Zeldes (1994) and Palumbo (1999) find such expenses to have a small effect. Using more recent data, De Nardi et al. (2010) for the US and Dobrescu (2015) for Europe find health related expenses to be crucial drivers of savings. My results are consistent with theirs in the sense that health expenditures (medical and LTC) are an important driver of savings. On top, the proposed model allows me to identify the independent contribution played by medical and LTC. My estimates show that LTC needs are much more important than medical expenses as drivers of the elderly rich savings which is key for addressing effective policies aiming at increasing the welfare of the old.

Kopecky and Koreshkova (2014) estimate the independent contribution of medical and nursing home expenses on aggregate wealth. They find that savings for out-of-pocket health expenses account for 13.5 percent of the aggregate wealth, half of which is due to nursing home expenses. In their model nursing home is an exogenous state where individuals are forced to spend around $35,000 (94% of average earnings) for basic care services plus the
level of consumption they choose. In their model, middle-income individuals save more for nursing home expenses than those at the top of the permanent earnings distribution because they face higher expenses relative to their income. Besides, Lockwood (2014), using a model where LTC choices are exogenous, finds bequest motives to be significantly more important than LTC as a driver of savings. Through the lens of his model, in order to match the dissaving pattern of the upper-middle class, a strong bequest motive is necessary. On the other hand, in this paper, I find that the richer spend significantly more on FC when in need of LTC than the poor (in line with the data) which increases late in life risk faced by the elderly upper-middle class and the rich. As a result, the elderly rich save more than the middle class for LTC. Thus, the proposed model estimates a weaker bequest motive than previous literature.

In a recent work that uses high-quality data on survey answers to hypothetical scenarios, Ameriks, Briggs, Caplin, Shapiro, and Tonetti (2015) show that the marginal utility of consumption increases strongly when in need of LTC; its associated impact on precautionary savings remains, however, unclear. In contrast to their work I allow for heterogeneity in LTC needs and the role of family as important source of care. Firstly, in Ameriks et al. (2015) the authors only consider one LTC status. Therefore, two individuals that in the data show very different LTC needs (an individual with a broken arm facing difficulties bathing and an Alzheimer’s patient who requires constant care) will derive the same utility from care in their model. Secondly, I include help provided by the family. Even though the authors focus on a sample of very rich individuals who have lower access to IC, 27% of individuals in the HRS in the top quartile of the income distribution belongs to a close family. Besides, related to the role of family, answers given to hypothetical scenarios in their data, reveal a large heterogeneity in the bequest motives which is absent in their modeling strategy. The proposed model, on the other hand, is able to predict the large fraction of individuals (those on their own) who are not willing to leave any bequest even at relatively large wealth levels.

My paper is also linked to a recent strand of the literature that analyzes family care arrangements (Fahle 2015; Ko 2017; Mommaerts 2015). The lead paper in this field is Barczyk and Kredler (2017) where the authors propose a model where IC is the outcome
of an intergenerational bargaining process played between elderly parents and their adult child. In the model, children have an incentive to provide IC to protect future bequests. On the other hand, the elderly parents want to accumulate wealth up to a threshold value that makes children provide care. Compared to the data, the model overestimates the dissaving pattern of the elderly rich. This is because wealthier individuals in the model derive low utility from holding assets above the threshold that induce their children to provide care. Their calibrated economy implies that individuals in the top quintile of PI distribution are twice as likely to receive IC than individuals in the bottom quintile which is at odds with the data as individuals in lower PI groups have higher chances of belonging to closer families. The authors are well aware of this by writing “This divergence might be driven by the fact that in reality, family values run so strong for some that economic incentives matter less”. The proposed model ignores children economic incentives and focus on the parent’s savings decisions taking as given the “values” of their children.

Finally, this paper is complementary to empirical studies on bequest behavior in response to children’s attention and caregiving. M. Brown (2006) and Groneck (2016) find that end-of-life transfers favor both current and expected caregiver. In line with their results, estimated preferences parameters show that individuals in close families hold stronger bequest motives.

3 LTC Needs and LTC Choices

The data for this paper comes from the HRS, which is a representative sample of individuals aged over 50 for the United States. The HRS is a biennial panel conducted by the University of Michigan since 1992. The survey includes detailed information on assets, income, health and LTC services utilization. I use nine interviews from the HRS 1998-2014 for individuals aged over 70.


3.1 LTC in the US

In 2013, formal LTC costs in the US added up to $310 billion which corresponds to around 1.5 percent of GDP or 9.4% of all U.S health care spending. These are financed through two main sources: public through Medicaid or private through out-of-pocket spending. Insurance plays a small role as only about 10% of retirees buy LTC insurance and only 7% of LTC expenditures are paid by private insurance policies\(^2\). Furthermore, Medicare only covers stays up to 20 days following a hospital stay and does not cover health aide services.

**Medicaid.** — Public expenditures on LTC are almost entirely paid by Medicaid. Medicaid is, however, means-tested and therefore only available to impoverished elderly. To become eligible individual assets must be below $2,000. Medicaid offers coverage of both home based services as well as institutional care. Medicaid only covers basic care needs so that individuals can be reluctant to access it (Ameriks, Caplin, Laufer, & Van Nieuwerburgh, 2011). In the community, Medicaid offers homemaker (including light housekeeping, grocery shopping or laundry), personal care (assistance with daily routines) and meals (provides meals at their homes or in senior centers) services. In nursing homes, Medicaid covers for room, board, and care. By 2014, Medicaid spending on home services was $80.6 billion or 53% of total Medicaid spending on LTC versus 47% on nursing homes.

**Private LTC.** — Since only the impoverished have access to Medicaid, most individuals pay LTC out-of-pocket. On top, LTC in the US is expensive. The average annual rate for a semi-private nursing home is around $48,000 per year (Stewart et al., 2009) and the hour of FC is estimated to be around $21 per hour (LongTermCare.gov).

3.2 LTC needs

In order to analyze how LTC expenses affect the savings behavior of the elderly, I need first to define a metric for LTC needs. The HRS contains a wide array of variables about different aspects of elderly’s health. In Amengual et al. (2017), we propose a latent variable

\(^{2}\text{J. R. Brown and Finkelstein (2007) Finkelstein (2008), Lockwood (2014), Mommaerts (2015), and Ko (2017) analyze the reasons why the insurance market is so small} \)
Econometric Model.— The HRS is an unbalanced panel of individuals $i = 1, \ldots, N$ followed for $t_i = 1, \ldots, T_i$ periods which correspond from ages $a^i_1$ to age $a^i_{T_i}$. We consider that an individual $i$ at time $t$ belongs to a latent health group $h_{i,t}$ out of $H$ possible ones. If the individual belonged to group $g$, the probability of reporting difficulties with the $k$'th I-ADL, say $x_{i,k,t} = 1$, is $\iota_{k,g}$. Under the assumption that I-ADLs are independently distributed conditional on the health status, the joint distribution of $\mathbf{x}_{i,t} = (x_{1,i,t}, x_{2,i,t}, \ldots, x_{K,i,t})'$ is characterized by

$$p(\mathbf{x}_{i,t}|h_{i,t} = g) = \prod_{k=1}^{K} \iota_{k,g}^{x_{k,i,t}} (1 - \iota_{k,g})^{1-x_{k,i,t}},$$

where $t_g = (t_{1,g}, t_{2,g}, \ldots, t_{K,g})'$. We take into account health dynamics by explicitly modeling the transition probabilities across groups. In particular, an individual $i$ at time $t$, with gender $s$ and in PI quantile $b$ who belongs to group $g$ transits to group $c$ with probability

$$\pi_{g,c}(a_{it}, s_i, b_i) = \frac{\exp[f_{g,c}(a_{it}, s_i, b_i)]}{1 + \sum_{c \in \mathcal{H}} \exp[f_{g,c}(a_{it}, s_i, b_i)]},$$

where $\mathcal{H}$ is the set that contains the $H$ health groups. The remaining possible event is

3ADLs: Some difficulty with dressing (DRESS), using the toilet (TOILET), bathing (BATH), getting in or out of bed (BED), to walk across a room (WALK) and eating (EAT). IADLs: Some difficulty with preparing a hot meal (MEALS), shopping for groceries (SHOP), managing money (MONEY), taking medications (MEDS), using a phone (PHONE), and using a map (MAP).

4Along the paper I use I-ADLs to denote the set of both ADLs and IADLs.
that the individual dies, which is an observable state that occurs with probability

$$\pi_{g,D}(a_{it}, s_i, b_i) = \frac{1}{1 + \sum_{c \in H} \exp[f_{g,c}(a_{it}, s_i, b_i)]]}.$$ 

This specification allows health groups to own distinct dynamics as parameters differ according to the current health group. Moreover, to capture within-group heterogeneity, transition probabilities can depend on age, gender, PI ranking (I split PI distribution in quartiles, $Q = 4$) and interaction terms through the function $f_{g,c}(a, s, b)$:

$$f_{g,c}(a, s, b) = \beta_{1,g,c} + \beta_{2,g,c}a + \beta_{3,g,c}s + \sum_{q=2}^{Q} \beta_{4,g,c,q}1_{q=b} + \beta_{5,g,c}(a \times s) + \sum_{q=2}^{Q} \beta_{6,g,c,q}(1_{q=b} \times a) \quad (3)$$

**Estimation and Results.—** In practice, I set the number of latent health groups $H = 4^5$. Estimation of the econometric model delivers two sets of parameters: $[\hat{\beta}, \hat{i}]$. $i$ shows that individuals are classified as physically frail, mentally frail, impaired or healthy, represent individuals’ LTC needs suitably. Figure 1 shows the probability of reporting difficulty with I-ADLs in each LTC need group. The impaired have physical and cognitive limitations while the healthy have no or light difficulties with I-ADLs. In turn, the physically frail have limited mobility, while the mentally frail have difficulties with more cognitive tasks such as managing money.

Given $\hat{\beta}$ and $\hat{i}$, I can compute $p(h_{it}|x_{it}^{T_i}, \hat{\beta}, \hat{i})$, a $4 \times 1$ vector which contains the smoothed probability that each individual belongs to each health group at any point in time where $x_{it}^{T_i}$ represents individual’s past, current, and future information on I-ADLs and potential death events. The first, the second, the third, and the fourth element of $p(h_{it}|x_{it}^{T_i}, \hat{\beta}, \hat{i})$ correspond to the probability of belonging to the healthy, the physically frail, the mentally frail and the impaired group, respectively. Therefore, LTC needs become more acute as the probability of belonging to the fourth group increases. $p(h_{it}|x_{it}^{T_i}, \hat{\beta}, \hat{i})$ represents the LTC measure I use throughout the paper.

$^5$For details on the estimation procedure and how we select the optimal number of health groups, the reader is referred to the original paper.
The proposed measure of LTC has three main advantages with respect to others used in the previous literature. First, the measure relies on LTC needs indicators and not on observed LTC choices. De Nardi et al. (2010), Kopecky and Koreshkova (2014), Lockwood (2014) or Barczyk and Kredler (2017) rely on endogenous LTC measures (for example, nursing home utilization or hours of care). These assumptions might imply large discrepancies between actual and modeled LTC risks. For example, if richer individuals consumed more care hours than the poor as a pure income effect, using FC hours as a measure for LTC needs would overestimate the LTC risk faced by the rich. To focus on reported difficulties allow us to eliminate the bias. Secondly, the classification maximizes the representativeness of the observed difficulties with ADLs and IADLS observed in the population. Ameriks et al. (2015) consider individuals as in need of LTC if they report one or more difficulties with ADLs. Ko (2017) classifies individuals as healthy if they report none or one difficulty with ADLs, in light need if has two or three and as impaired in case she reports difficulty with more than three ADLs. In contrast, our estimation results show that not all I-ADLs are as informative for predicting need of care and that there is large heterogeneity in the implied needs for
individuals reporting the same number of I-ADLs. For example, if a person has difficulties with getting in or out of bed, she belongs to the physically frail group with a probability higher than one-third and with less than 5% to the mentally frail. On the other hand, an individual incapable of eating is much more likely to belong to the impaired rather than the physically frail group. Finally, by modeling health groups and health dynamics jointly allows us to exploit future and past information including potential death events. Moreover, potential misreporting is smoothed out by the algorithm.

3.3 Heterogeneous LTC Risks

Using transition parameters of the econometric model, Figure 2 plots the share of individuals by health status for females at the bottom PI (left panel) and at the top (right panel) PI. The figure shows that there are large differences across PI groups already present at age 70 as the probability of being healthy is around 25 p.p. larger for the rich. In their 70’s, most individuals are healthy, however, health deteriorates fast with age for both groups. At age 82 for the poor females and age 88 for the rich females, most individuals are in some need of LTC.
Table 1 summarizes LTC expected risk across groups by displaying average duration in each health status at age 70 by PI and gender. The last column is the sum the average duration of all possible status or individual’s life expectancy. The table shows large differences in life expectancy. A female in the top of the income distribution expects to live 7 more years than a man at the bottom of the income distribution. Furthermore, there are large differences in healthy life expectancy across PI groups. Richer individuals in spite of living longer; they spend shorter periods of time in need of LTC. Females live on average 3.3 more years than men and of these, 2 years are spent in need of LTC.

Table 1: Expected life expectancy of each health state at age 70 by permanent income group (Gradient: Top-Bottom)

<table>
<thead>
<tr>
<th>PI</th>
<th>Life Expectancy</th>
<th>Physically frail</th>
<th>Mentally frail</th>
<th>Impaired</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Healthy</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Males</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bottom</td>
<td>10.0</td>
<td>6.4</td>
<td>1.9</td>
<td>0.9</td>
</tr>
<tr>
<td>Lower-middle</td>
<td>11.6</td>
<td>8.1</td>
<td>1.9</td>
<td>0.9</td>
</tr>
<tr>
<td>Upper-middle</td>
<td>12.4</td>
<td>9.2</td>
<td>1.8</td>
<td>0.8</td>
</tr>
<tr>
<td>Top</td>
<td>13.9</td>
<td>11.3</td>
<td>1.4</td>
<td>0.6</td>
</tr>
<tr>
<td>Gradient</td>
<td>3.9</td>
<td>4.9</td>
<td>-0.5</td>
<td>-0.3</td>
</tr>
<tr>
<td>Females</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bottom</td>
<td>13.4</td>
<td>7.7</td>
<td>3.2</td>
<td>1.2</td>
</tr>
<tr>
<td>Lower-middle</td>
<td>14.9</td>
<td>9.4</td>
<td>3.1</td>
<td>1.2</td>
</tr>
<tr>
<td>Upper-middle</td>
<td>15.9</td>
<td>10.7</td>
<td>2.9</td>
<td>1.1</td>
</tr>
<tr>
<td>Top</td>
<td>17.0</td>
<td>12.6</td>
<td>2.3</td>
<td>1.0</td>
</tr>
<tr>
<td>Gradient</td>
<td>3.6</td>
<td>4.9</td>
<td>-0.9</td>
<td>-0.2</td>
</tr>
</tbody>
</table>

Females live longer because conditional on health status, they face larger chances of surviving. Figure 3 shows two-year mortality rates for females at the top PI and for males at the bottom PI. There are large differences in the two-year probability of survival across health status. To be impaired is an important predictor of mortality as the two year-mortality rate is around 2.5 times larger for the impaired than for the healthy group. Differences in life expectancy across PI groups are mainly driven by:

- Differences in health already present at age 70.
- Conditioning on surviving, poor individuals show higher probabilities of moving to a
health group with higher mortality during first years after retirement.

- Richer individuals have larger probabilities of recovery from a LTC need status.

### 3.4 LTC as a Choice

In this section, I provide evidence that individuals adjust their LTC expenses depending on their needs, financial resources and help provided by relatives. On top, I show that individuals increasingly decide to use LTC services provided by Medicaid as health deteriorates. For this purpose, I use the HRS helpers files which contain information about hours of care and the identity of the caregiver. I restrict the analysis to singles. I classify care as IC if the caregiver is a relative or a friend and as FC in case she is a paid helper or a professional. I compute statistics of care hours by health status based on the previous health classification. To do so, I construct 10,000 bootstrap samples using $p(h_{it} | x_{it}^T, \beta, \iota)$, compute the statistic for each sample and report the average across samples. Thus, a given individual in a given wave might have different health status across samples. For example, if an individual has 90% probabilities of belonging to the healthy group, 10% probability of belonging to the
<table>
<thead>
<tr>
<th></th>
<th>Physically frail</th>
<th>Mentally frail</th>
<th>Impaired</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bottom</td>
<td>0.8</td>
<td>1.0</td>
<td>2.3</td>
</tr>
<tr>
<td>L-M</td>
<td>0.6</td>
<td>0.9</td>
<td>2.9</td>
</tr>
<tr>
<td>U-M</td>
<td>0.5</td>
<td>1.2</td>
<td>2.7</td>
</tr>
<tr>
<td>Top</td>
<td>0.9</td>
<td>1.3</td>
<td>3.5</td>
</tr>
<tr>
<td>Total</td>
<td>0.7</td>
<td>1.0</td>
<td>2.8</td>
</tr>
</tbody>
</table>

**Table 2:** Average formal care hours by permanent income quartile and health status

physically frail group and 0% of belonging to the mentally frail and impaired group, she will be sampled on average 9,000 times as healthy and 1,000 as physically frail.

Firstly, the last row of table 2 shows that care consumption increases as health deteriorates. The physically frail show an average consumption of care of 0.7 hours per day. Then, the mentally frail seem to be in slightly higher need since they consume 1 hour per day. Finally, the impaired are the ones showing the largest consumption of care with 2.8 hours. These correspond to around $5,365 per year (or 24% of the observed average income), $7,665 (or 35% of the observed average income) and $21,462 (or 100% of the observed average income) when physically frail, mentally frail and impaired respectively.

Secondly, table 2 also shows that individuals adjust their level of LTC spending to their available financial resources as the rich consume more care than the poor. Besides, differences across income groups widen as health deteriorates. Physically frail individuals in the top PI consume 0.1 more hours of FC than those in the bottom. If impaired, individuals in the top consume 1.3 more hours of care.

Thirdly, I present evidence that care provided by relatives alleviates significantly LTC expenses. Before doing so, I summarize the observed heterogeneity in IC hours provided by relatives in three types of families in the population. For this purpose, I split the distribution of IC hours into three equally likely groups by health status:

- Individuals belonging to close families receive the average of the top tercile of IC hours distribution.
- Individuals belonging to distant families receive the average of the middle tercile of IC
Table 3: Average informal care hours by family type

<table>
<thead>
<tr>
<th>Family</th>
<th>Physically frail</th>
<th>Mentally frail</th>
<th>Impaired</th>
</tr>
</thead>
<tbody>
<tr>
<td>On your own</td>
<td>0.0</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>Distant</td>
<td>0.7</td>
<td>1.3</td>
<td>1.3</td>
</tr>
<tr>
<td>Close</td>
<td>4.7</td>
<td>7.4</td>
<td>9.5</td>
</tr>
<tr>
<td>Total</td>
<td>1.7</td>
<td>2.9</td>
<td>3.6</td>
</tr>
</tbody>
</table>

Table 4: Average formal care hours by family type

<table>
<thead>
<tr>
<th>Family</th>
<th>Physically frail</th>
<th>Mentally frail</th>
<th>Impaired</th>
</tr>
</thead>
<tbody>
<tr>
<td>On your own</td>
<td>1.4</td>
<td>2.1</td>
<td>4.9</td>
</tr>
<tr>
<td>Distant</td>
<td>0.3</td>
<td>0.9</td>
<td>3.5</td>
</tr>
<tr>
<td>Close</td>
<td>0.3</td>
<td>0.5</td>
<td>1.7</td>
</tr>
<tr>
<td>Total</td>
<td>0.7</td>
<td>1.0</td>
<td>2.8</td>
</tr>
</tbody>
</table>

- Individuals *on their own* receive the average of the bottom tercile of IC hours distribution.

Table 3 reports average hours of IC in each family type and table 4 shows average FC hours by family type. Controlling by health status, individuals with closer relatives, significantly reduce their consumption of FC hours. The presence of close relatives therefore, reduce spending when in need of LTC. Individuals on their own purchase between 3 and 4 times more care than those who belong to close families.

Finally, table 5 and 6 report average Medicaid recipiency rates across income and families. As LTC needs become acuter, individuals increasingly rely on LTC services provided by the government. Not surprisingly, Medicaid recipiency rates are inversely related to income. Furthermore, differences across family types are small. When healthy, individuals in closer families seem to receive Medicaid more frequently but the opposite is to true when individuals are in need of LTC.
Physically Mentally

<table>
<thead>
<tr>
<th>PI</th>
<th>Healthy</th>
<th>frail</th>
<th>frail</th>
<th>Impaired</th>
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</thead>
<tbody>
<tr>
<td>Bottom</td>
<td>39</td>
<td>56</td>
<td>53</td>
<td>74</td>
</tr>
<tr>
<td>L-M</td>
<td>13</td>
<td>20</td>
<td>27</td>
<td>45</td>
</tr>
<tr>
<td>U-M</td>
<td>3</td>
<td>8</td>
<td>13</td>
<td>32</td>
</tr>
<tr>
<td>Top</td>
<td>2</td>
<td>3</td>
<td>5</td>
<td>12</td>
</tr>
<tr>
<td>Total</td>
<td>10</td>
<td>23</td>
<td>26</td>
<td>44</td>
</tr>
</tbody>
</table>

**Table 5:** Medicaid recipiency rates (%) by permanent income quartile

<table>
<thead>
<tr>
<th>Family</th>
<th>Physically</th>
<th>Mentally</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Healthy</td>
<td>frail</td>
</tr>
<tr>
<td>On your own</td>
<td>9</td>
<td>23</td>
</tr>
<tr>
<td>Distant</td>
<td>9</td>
<td>19</td>
</tr>
<tr>
<td>Close</td>
<td>12</td>
<td>25</td>
</tr>
<tr>
<td>Total</td>
<td>10</td>
<td>23</td>
</tr>
</tbody>
</table>

**Table 6:** Medicaid recipiency rates (%) by family type

4 Model

Motivated by the previous empirical exercise, I build a structural model that can reproduce the main features of the data: (i) as health deteriorates, individuals increase their consumption of FC hours, (ii) richer individuals consume more care hours, (iii) Medicaid constitutes an important source of insurance as health deteriorates for all income levels, and (iv) increasing access to IC lowers the demand for FC.

The model follows closely De Nardi et al. (2010) but at the same time incorporates new ingredients to match LTC choices observed in the data. First, agents in the model suffer health shocks that affect individuals marginal utility of care consumption. Second, there are two types of care: FC is provided by paid helpers while IC is provided by relatives for free. Agents are heterogeneous over IC and decide FC consumption taking as given the decision of their relatives. Thus, IC and FC can take place simultaneously. Thirdly, the marginal utility of leaving a bequest is affected by the amount of IC received if in need of LTC, agents can, therefore compensate their relatives for the help provided.

**Preferences.** — Agents start their life at age $a = 70$ and live at most 100 years old.
Every period lasts for two years of time: \( a \in \{70, 72, \ldots, 100\} \). Individuals derive utility from regular consumption and care hours. The individual’s utility depend on health status \( h \) which can take five values: healthy \((h = 1)\), physically frail \((h = 2)\), mentally frail \((h = 3)\), impaired \((h = 4)\) and dead \((h = 5)\). The marginal utility of consuming care hours is allowed to vary depending on health. Furthermore, individuals when in need of LTC \((h > 1 \text{ and } h \neq 5)\), receive IC hours \((l_{ic})\) depending on their family type \((F)\) and their health status.

There are three types of families in the model:

1. When the elderly are on their own \((F = 0)\), children provide little/no care.
2. In distant families \((F = 1)\), children provide moderate IC when their parents are in need of LTC.
3. In close families \((F = 2)\), children provide intense IC when their parents are in need of LTC.

Each period their utility flow is given by,

\[
 u(c, l_{fc}; h, F) = \frac{c^{1-\sigma}}{1-\sigma} + \mu(h)^{\nu} \left[ l_{fc} + \omega l_{ic}(F, h) \right]^{1-\nu} \tag{4}
\]

where, \( c \) is regular consumption expressed in dollar values, \( l_{fc} \) is FC hours. \( \mu \) is the LTC needs shifter, which affects the marginal utility of consuming care hours when individuals have difficulties with I-ADLs. \( \omega \) is an equivalence scale that maps IC into FC hours. \( \sigma \) and \( \nu \) are the risk aversion parameters of regular consumption and care hours respectively.

When the person dies, individuals derive utility from leaving bequest following:

\[
 \phi(k; F) = \lambda(F)^{\delta} \frac{(k + \delta)^{1-\sigma}}{1-\sigma}, \tag{5}
\]

where \( \delta \) captures the extent to which bequests is a luxury good or a necessity. \( \lambda(F) \) captures the intensity of the bequest motive. I allow \( \lambda \) to vary across family types since individuals in close or distant families might have stronger bequest motives to compensate their children for the help provided when in need of LTC.
Health and Medical expenses uncertainty. — Health and survival probabilities are modeled as in section 3.2. Moreover, individuals face uncertainty in out-of-pocket medical expenses \((m)\). I follow French and Jones (2004) and model log health costs as the sum of a white noise process and a persistent AR(1). I allow the variance of the transitory component to be dependent on health status to capture the large dispersion in data when individuals are in need of LTC.

\[
\ln m_t = m(h, b, a, s) + \psi_t(h)
\]

\[
\psi_t(h) = \xi_t + \zeta_t(h), \quad \zeta_t(h) \sim N(0, \sigma^2_\zeta(h))
\]

\[
\xi_t = \rho \xi_{t-1} + \epsilon_t, \quad \epsilon_t \sim N(0, \sigma^2_\epsilon)
\]

Therefore in the model, health and medical expenses are considered as shocks. A different approach, based on Grossman (1972), is to consider medical expenses as investment in health (Ozkan et al. 2011; Yogo 2016). However, many studies in the empirical literature have found such effects to be small: Brook et al. (1983), Fisher et al. (2003) or Finkelstein and McKnight (2008).

Government Insurance. — Agents have the option of using a government program which is means-tested. The cost of using it is that consumer’s wealth is set to zero \(^6\). The government provides a consumption floor \((\underline{c})\) and care hours \((l(h))\) if the agent is in need of LTC \((h \in \{2, 3, 4\})\). \(G = 1\) if the consumer chooses to use the program and \(G = 0\), otherwise. Government transfers are given by:

\[
\max\{0, \underline{c} + pf c_l(h) + m - b - (1 + r)k\}
\]

Timing, budget constraints. — I assume that PI is a constant function of sex and PI ranking. At the beginning of the period, the individual has wealth \((1 + r)k_t \geq 0\). \(r\) is the risk-free interest rate on savings. The individual receives non-asset income \(b(PI, s)\), and medical expenses \(m\) are realized. Children provide for free IC to their parents. Then, the

---

\(^6\)In reality, Medicaid has an asset disregard threshold which modal value across states is $2,000. For simplicity, I set this threshold to zero.
individual decides how much to consume $c$, how many hours of FC to purchase $l_{fc}$ and how much to save or to access government insurance $G$. Finally, the health shock hits and individuals who die leave all their assets as bequests. Therefore next period’s assets are given by:

$$k_{t+1} = (1 + r)k_t + b - m - p_{fc}l_{fc},$$

(10)

where $p_{fc}$ denotes the price of an hour of FC. Individuals face a borrowing constraint such that $a_{t+1} \geq 0$.

**Solution method.**—To save on state variables, I redefine the problem in terms of cash in hand, $x$:

$$x = (1 + r)k + b(s, PI) - m.$$

(11)

Given a set of parameter values, I can solve the model numerically by backward induction starting at age $T$. We can write the model in recursive form in terms of cash in hand. $\beta$ represents the discount factor. The value function is given by:

$$V_t(s, x, b, h, \zeta, F) = \max_{c, l_{fc}, G} \left\{ u(c, l_{fc}; h, F) + \beta \pi_{a,b,s,h,h' \neq 5} E_t [V_{t+1}(s, x', b, h', \zeta', F)] + \beta \pi_{a,b,s,h,h'=5} \phi(k'; F) \right\}$$

(12)

subject to

$$x' = (1 - G) \left[ (1 + r)(x - c - p_{fc}l_{fc}) - m' \right]$$

(13)

$$G = 1 \iff \begin{cases} 
    c = \bar{c} \\
    l_{fc} = l(h)
  \end{cases}$$

(14)

By first-order conditions, the optimal intra-temporal allocation expresses hours of FC in terms of consumption and IC received:

$$l_{fc} = \max \left\{ \mu(h) p_{fc}^{-1/\nu} c^{\sigma/\nu} - \omega l_i(h, F), 0 \right\}$$

(15)
5 Estimation

I estimate the model with a two step Method of Simulated Moment (MSM) estimator following Gourinchas and Parker (2002) and Cagetti (2003). In the first step, I estimate all the parameters that can be identified out of the model. In the second stage, the remaining parameters are estimated using the model and taking as given the parameters from the first step.

First stage parameters include PI, health transitions, hours of care received in each type of family, hours of care provided by Medicaid LTC services and medical expenses’ process. In the second stage, I estimate the set of parameters $\theta = (\beta, \sigma, \nu, \delta, \omega, \mu, \xi)$ that minimize the distance between simulated wealth, Medicaid recipiency rates and care hours moments with their empirical counterparts.

5.1 First Stage Parameters

Permanent income. — PI includes Social Security benefits, defined benefit pension benefits, and annuities. Since in the model agents have access to social insurance, I do not include means-tested government transfers such as Supplemental Security Income or food stamps. For each individual, I compute average PI across waves in which she is observed and I split the PI distribution into four quartiles. Each individual in the simulation is given the median non-asset income by gender. Table 7 shows that median annual income ranges from around $11,000 per year in the bottom PI to $33,000 per year in the top PI. The representative female earns on average between 2% and 10% less than the median male by PI quartile.

<table>
<thead>
<tr>
<th></th>
<th>Females</th>
<th>Males</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bottom</td>
<td>13,516</td>
<td>9,350</td>
</tr>
<tr>
<td>L-M</td>
<td>14,247</td>
<td>14,550</td>
</tr>
<tr>
<td>U-M</td>
<td>19,731</td>
<td>20,524</td>
</tr>
<tr>
<td>Top</td>
<td>31,533</td>
<td>35,266</td>
</tr>
</tbody>
</table>

Table 7: Median Income by permanent income quartile

Medical expenses. — The health expenditure model is estimated using HRS data 1996-
Figure 4: Two Year Average Medical Expenses in Thousands by permanent income quartile. Left panel: Males; right panel: Females

2014. Details on the estimation procedure can be found in the appendix. I drop from the sample individuals living in nursing homes or in Medicaid since LTC expenditures and government transfers are modeled explicitly in the model. I estimate jointly the mean and variance of log-medical expenditures. The mean is modeled as a age, age square, sex, PI ranking dummies, health dummies and PI interacted with health.

To better interpret the estimates, I simulate medical expenses histories based on artificial individuals. Figure 4 shows average medical spending as individuals age for different PI. Medical expenditures increase fast as health deteriorates from around $4,500 every two years at age 70 to around $7,500 at age 96. Relatively poorer individuals, in spite of being in worse health, tend to spend less on medical expenditures with differences widening as individuals age. Females spend similar amounts in their 70’s and around 30% more in their 90’s. Figure 5 shows average medical expenditures for healthy and impaired individuals. We observe that conditional on health status, age has a very small effect on medical spending. Therefore, the secular increase in medical expenditures over age is driven by worsening health of survivors.

Table 8 shows estimates of the persistent component and the transitory component. The variance of the transitory component increases as individual’s health deteriorates. Further-
more, the persistent component has an autocorrelation coefficient of 0.93 so that innovations to the persistent component have long lived effects.

*Hours of care provided by the family and by Medicaid.*— IC hours by family type are set at their average in the HRS data as defined in section 3.4. These correspond to care hours in table 3. Table 9 shows average FC hours by health status for individuals in Medicaid in the HRS, these are the ones I use in the estimation.

### 5.2 Second Stage Moments

*Empirical wealth moments.*— I select single retired individuals who were interviewed in 1998. Wealth moments track the evolution of wealth over time as members of the sample age. I split the sample into 4 6-year birth cohort by PI and by family type. The age ranges of these cohorts are 70-75, 76-81, 82-87 and 88-92. For each cohort and PI/family type, I compute the median wealth in each wave from 1998 to 2014. Thus there are 128 wealth moments by PI (4 PI × 8 waves × 4 cohorts) and 96 wealth moments by family type (3 family types × 8 waves × 4 cohorts) \(^7\). Each cohort’s wealth moments trace the evolution over time of the

---

\(^7\)I drop cells with less than thirty observations
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Variable</th>
<th>Median (Standard deviation)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$</td>
<td>Autocorrelation, persistent part</td>
<td>0.93 (0.01)</td>
</tr>
<tr>
<td>$\sigma^2_\epsilon$</td>
<td>Innovation variance, persistent part</td>
<td>0.07 (0.01)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma^2_\zeta(h = 1)$</td>
<td>Healthy</td>
<td>0.98 (0.01)</td>
</tr>
<tr>
<td>$\sigma^2_\zeta(h = 2)$</td>
<td>Physically frail</td>
<td>1.09 (0.03)</td>
</tr>
<tr>
<td>$\sigma^2_\zeta(h = 3)$</td>
<td>Congitively frail</td>
<td>1.32 (0.05)</td>
</tr>
<tr>
<td>$\sigma^2_\zeta(h = 4)$</td>
<td>Impaired</td>
<td>1.52 (0.07)</td>
</tr>
</tbody>
</table>

**Table 8:** Persistence and variance of innovations to medical expenses

<table>
<thead>
<tr>
<th></th>
<th>Physically frail</th>
<th>Mentally frail</th>
<th>Impaired</th>
</tr>
</thead>
<tbody>
<tr>
<td>$l(h)$</td>
<td>1.0</td>
<td>1.2</td>
<td>2.6</td>
</tr>
</tbody>
</table>

**Table 9:** Average formal care hours in Medicaid
median wealth of its surviving members. Later waves contain fewer people due to death. Of the 2,958 individuals in the sample in 1998, 375 are still alive in 2014.

*Medicaid and hours of care moments.*— The empirical LTC moments are hours of care and Medicaid recipiency rates by PI and family type described in section 2. These correspond to tables 2, 5, 4 and 6. Thus there are 21 moments for FC hours and 28 moment for Medicaid recipiency rates.

### 5.3 Simulation Procedure

I simulate a large number of artificial individuals. Each of these individuals is endowed with a value of the state vector \((a, s, k, b, h, \zeta, \xi, F)\). \((a, s, k, b)\) are drawn from the data distribution in 1998. \(\zeta\) and \(\xi\) are Monte Carlo draws from discretized versions of the estimated shock processes.

*Sampling family types.*— Following the empirical section, individuals in close families, distant families or on their own belong to the top, middle or bottom tercile of the IC distribution, respectively. However, I face two difficulties when assigning families to individuals: first, individuals who are always health do not report IC and second individuals might report a different amount of IC such that they might change from one tercile to another across waves. To deal with these issues, I proceed as follows:

1. For individuals who report only one type of family whether because their family type is only observed only once (50% of individuals with at least one observed IC report) or because the individual reports the same family type in different waves (25% of individuals with at least one observed IC report) are given their observed family type. For example, if I observe that an individual is healthy from 1998 to 2004, then in 2006 he is impaired and he reports belonging to a close family (top tercile of the IC distribution for impaired individuals) and dies in 2008, I consider this individual belonging to a close family with probability 1.

2. For individuals who report more than one family type (25% of individuals with observed family type), I take a random draw based on the observed frequency. For example, if I
observe that an individual belongs to a distant family in one wave and to a close family in the following wave, I assign this individual to a close family with 50% probability and to a distant family with 50% probability.

3. For individuals for which I do not observe the family type whether because they are always healthy or because IC hours is missing, \( p(F) \) is estimated based on individuals specific covariates. Using individuals in cases 1 and 2, I run an ordered logit model of the family type against time invariant covariates that have been identified on the literature as determinants of IC (estimated effect in parentheses):

- Females (+)
- PI (=)
- Whether has children (+)
- Race (hispanic and african +)
- Religion (postestants more than jewish)
- Marital status (widowed more than never married and divorced)
- Education (-)
- Child went to college (-)
- Females child (+)

I then compute the predicted probability of family type for individuals for which I do not observe it. To simulate the family type I take random draws from these probabilities.

Figure 6 plots the histogram for the predicted probability of belonging to a close family. Individuals in poorer families have larger probabilities of belonging to a close family. 12% of individuals in the bottom PI have more than one-half chances of belonging to a close family while only 1.5% in the top PI do so.

**Sampling health status.** First, following De Nardi et al. (2010), the simulation uses each individual’s survival history in 2000-2014 to ensure that individuals contribute to the
same wealth moments in the simulation as in the data. This protects against the mortality bias arising from the fact that richer individuals face larger chances of survival than in the parsimonious model. Second, in order to ensure that the simulated health draws have the same persistence as the estimated health process, I use the Kim smoother proposed by Kim (1994) (see Appendix for further details on how the smoother works for sampling health status for each individual).

Procedure.—Given a guess for my parameter vector \( \theta \), I solve the model using value function iteration. This yields a set of policy function which allows me to simulate for each artificial individual her savings decision, consumption, FC hours consumed and access to Medicaid. I look in the parameter space the values that minimize the distance between the empirical and the simulated moments. The estimation of \( \theta \) is based on 197 moment conditions. The weighting matrix used in the estimation is the inverse of the diagonal of the estimated variance-covariance matrix of the moment conditions. More precisely estimated moments receive greater weight in the estimation.
5.4 Parameter Identification

Before I present the estimation results, I explain how FC and Medicaid moments allows for the identification of parameters.

First, the high dissaving rate for the relatively poor can be matched with increasing generosity of government insurance or bequest motives that become active at relatively high levels of wealth. By requiring the model to match Medicaid recipiency rates by health status, I am restricting the size of consumption floor. Furthermore, as individuals with worse health status are closer to death, differences of the Medicaid recipiency rates allows to better identify the strength of the bequest motive across individuals with different probabilities of death.

Secondly, coefficients of relative risk aversion for regular consumption and care hours have to reproduce dissavings rates in line with the data. The gradient in FC hours allows me to get further identification. Dividing both sides of equation 15 by regular consumption and multiplying by $p_{fc}$, I get the optimal relative share of FC consumption to regular consumption.

$$\frac{p_{fc}^{1/\nu}}{c} = \mu(h)p_{fc}^{1-1/\nu}c^{\sigma/\nu-1}$$

If $\sigma$ is larger than $\nu$ relatively wealthier individuals (large $c$) will spend a larger share of their resources on care. Values of $\sigma$ and $\nu$ will be therefore identified by the observed differences on FC hours between the rich and the poor.

[To be completed]

5.5 Estimated Parameter values

Table 10 shows the estimated preference parameters. The estimate of the discount factor $\beta$ is 0.97 is in line with the previous literature. The estimate for $\sigma$, the coefficient of risk aversion for regular consumption is 4.2. This value is close to typically used in the literature. (De Nardi et al., 2010) estimated $\sigma = 3.8$ while Ameriks et al. (2015) estimated $\sigma = 5.6$. My estimate of $\nu$, the coefficient of risk aversion for care hours, is 3.0. These estimates imply that relatively rich individuals spend a larger share of their resources on FC than relatively...
Table 10: Estimated Preference Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>discount factor</td>
<td>0.97 (0.01)</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>RRA, consumption</td>
<td>4.24 (0.02)</td>
</tr>
<tr>
<td>$\nu$</td>
<td>RRA, care hours</td>
<td>3.03 (0.01)</td>
</tr>
<tr>
<td>$\delta$</td>
<td>bequest curvature $\times 10^3$</td>
<td>13.1 (1.30)</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>consumption floor $\times 10^3$</td>
<td>10.9 (0.77)</td>
</tr>
<tr>
<td>$\omega$</td>
<td>IC-FC equivalence</td>
<td>0.87 (0.06)</td>
</tr>
</tbody>
</table>

Bequest utility multiplier
- $\lambda(F = 0)$: On your own 0.06 (0.01)
- $\lambda(F = 1)$: Distant 0.62 (0.06)
- $\lambda(F = 2)$: Close 0.83 (0.05)

Care utility multiplier
- $\mu(h = 2)$: Physically frail 1.01 (0.05)
- $\mu(h = 3)$: Mentally frail 2.71 (0.11)
- $\mu(h = 4)$: Impaired 6.55 (0.20)

Notes: Standard errors in parentheses.

the poor. This result is in line with De Nardi et al. (2010) where the authors find that health related expenses (medical and nursing home expenses) to be luxury goods as they are much higher for individuals with higher PI.

The consumption floor is estimated at $10,916 every two years, similar to the value in De Nardi, French, and Jones (2016) ($6,670 per year) and relatively lower to the SSI benefit for single of about $7,800 per year.

My estimates over the care multiplier across different health status, imply that as health deteriorates, individuals marginal utility of consuming care increases.

Strength of the bequest motive.— Following Ameriks et al. (2015), I compare the intensity of the bequest motive implied by my estimated preference parameter to those estimated in lead papers. In figure 7, I present the optimal bequest allocation that a healthy individual would choose one period before death when calibrated according to each paper’s baseline estimate of risk aversion and bequest parameters. The individual would be solving the following maximization problem:
Figure 7: Bequest allocation across studies when healthy: De Nardi et al. (2010): DFJ, Ameriks et al. (2015): ABCST and Lockwood (2014) (left panel) vs proposed model (right panel)

\[
\max_{c,k} \frac{c^{1-\sigma}}{1-\sigma} + \lambda \frac{(k+\delta)^{1-\sigma}}{1-\sigma}
\]
\[
s.t. W = c + k
\]

The figure shows that the bequest motive in the model becomes active around the same level as in De Nardi et al. (2016) and Lockwood (2014) for individuals in close and distant families. Individuals on their own, on the contrary, do not show any bequest motive. De Nardi et al. (2016) and Lockwood (2014) also use the HRS data for estimating their models. In the HRS, 70% of the sample owns less than $100,000 in assets the period before dying. It is therefore important to notice that all these studies point towards the idea that the intensity of bequest motives is moderate for asset poor individuals.

On the other hand, my results are relatively closer to Ameriks et al. (2015) for relatively larger wealth levels. In Ameriks et al. (2015), authors estimate preference parameters in their model using strategic survey questions (SSQ) that use choices made in hypothetical scenarios to estimate preference parameters and wealth data for a richer population than the one in HRS. Therefore my estimated bequest intensity at higher wealth level lines up well with
preferences parameters estimated using a sample of relatively rich individuals.

Figure 8 show the optimal bequest allocation when individuals are in bad health across different models. Lockwood (2014) has no health dependent preferences so the bequest motive is unchanged. In De Nardi et al. (2010) differences across health status are very modest. In contrast, in Ameriks et al. (2015) and in the proposed model, the bequest motive becomes active at much larger wealth levels (around $50,000). For relatively higher wealth levels, individuals in the model show less intense bequest motives.

Finally, Ameriks et al. (2015) document in based on their SSQ a large fraction of individuals that are not willing to leave any bequest even at large levels of wealth ($100,000-$200,000) even if their model is not able to match it. The proposed model, on the other hand, predicts that individuals on their own have no bequest motives. Allowing for heterogeneity in bequest motives, therefore, seems to be important for matching the data.

5.6 Fit of the Model to Targeted Moments

Figure 9 and 10 show median assets in the data (solid line) and in the model (dashed line) across PI groups and family types, respectively. The model matches well wealth trajectories
Figure 9: Median Net Worth by permanent income quartile: Data versus Model

in both cases and is able to reproduce the fact that the top PI group dissaves at a lower rate than poorer income groups.

Table 11 shows that the model reproduces the observed increase in care hours as health deteriorates. In spite of matching difference across income groups, the model overestimates hours of care consumed by the top PI when mentally frail and impaired.

Table 12 shows that the model is also able to reproduce the patterns of Medicaid usage across health status and PI.

Table 13 shows how the model fits FC hours across family types. As in the data, individuals with more access to IC, consume less FC. In general, the model fits the data also quantitatively except individuals on their own where the model slightly under predicts FC hours consumed. Finally, table 14 shows that the model is able to reproduce the fact that when healthy, the share of individuals in Medicaid is larger for individuals in close families than for individuals on their own. The opposite is true when individuals are impaired. The model tends to exaggerate the differences across family types when individuals are impaired.
Figure 10: Median Net Worth by Family Type: Data versus Model

Table 11: Average formal care hours by permanent income quartile and health status: model vs data
### Table 12: Medicaid recipiency rate by permanent income quartile and health status: model vs data

<table>
<thead>
<tr>
<th>PI</th>
<th>Healthy Model</th>
<th>Healthy Data</th>
<th>Physically frail Model</th>
<th>Physically frail Data</th>
<th>Mentally frail Model</th>
<th>Mentally frail Data</th>
<th>Impaired Model</th>
<th>Impaired Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bottom</td>
<td>42</td>
<td>39</td>
<td>55</td>
<td>56</td>
<td>64</td>
<td>53</td>
<td>79</td>
<td>74</td>
</tr>
<tr>
<td>L-M</td>
<td>7</td>
<td>13</td>
<td>10</td>
<td>20</td>
<td>16</td>
<td>27</td>
<td>42</td>
<td>45</td>
</tr>
<tr>
<td>U-M</td>
<td>7</td>
<td>3</td>
<td>8</td>
<td>8</td>
<td>9</td>
<td>13</td>
<td>17</td>
<td>32</td>
</tr>
<tr>
<td>Top</td>
<td>5</td>
<td>2</td>
<td>8</td>
<td>3</td>
<td>7</td>
<td>5</td>
<td>12</td>
<td>12</td>
</tr>
</tbody>
</table>

### Table 13: Average formal care hours by family type and health status: model vs data

<table>
<thead>
<tr>
<th>Family</th>
<th>Physically frail Model</th>
<th>Physically frail Data</th>
<th>Mentally frail Model</th>
<th>Mentally frail Data</th>
<th>Impaired Model</th>
<th>Impaired Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>On your own</td>
<td>0.9</td>
<td>1.4</td>
<td>1.7</td>
<td>2.0</td>
<td>3.7</td>
<td>4.9</td>
</tr>
<tr>
<td>Distant</td>
<td>0.5</td>
<td>0.3</td>
<td>1.2</td>
<td>0.9</td>
<td>3.3</td>
<td>3.4</td>
</tr>
<tr>
<td>Close</td>
<td>0.2</td>
<td>0.3</td>
<td>0.5</td>
<td>0.5</td>
<td>1.4</td>
<td>1.7</td>
</tr>
</tbody>
</table>

### Table 14: Medicaid recipiency rate by family type and health status: model vs data

<table>
<thead>
<tr>
<th>Family</th>
<th>Healthy Model</th>
<th>Healthy Data</th>
<th>Physically frail Model</th>
<th>Physically frail Data</th>
<th>Mentally frail Model</th>
<th>Mentally frail Data</th>
<th>Impaired Model</th>
<th>Impaired Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>On your own</td>
<td>11</td>
<td>9</td>
<td>24</td>
<td>23</td>
<td>30</td>
<td>28</td>
<td>54</td>
<td>46</td>
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<tr>
<td>Distant</td>
<td>13</td>
<td>9</td>
<td>18</td>
<td>19</td>
<td>24</td>
<td>24</td>
<td>43</td>
<td>42</td>
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<tr>
<td>Close</td>
<td>14</td>
<td>12</td>
<td>25</td>
<td>25</td>
<td>31</td>
<td>27</td>
<td>38</td>
<td>42</td>
</tr>
</tbody>
</table>
5.7 Informal Validation

[Non targeted moments: To be completed]

6 Counterfactuals

In order to identify the key mechanism in the model, I run a set of counterfactual scenarios. For this purpose, following the same procedure as in, I fix the estimated parameters at their benchmark and change one feature of the model at a time. I then compute the optimal savings decisions, simulate the model, and compare the resulting asset accumulation profile to the asset profile generated by the baseline model. I display the asset profiles for individuals who were aged 70-75 in 1998. To focus on the underlying changes in savings, I rule out attrition and simulate health transitions such that individuals live until to age 100. The picture with mortality bias is very similar.

6.1 How much the Elderly Save for LTC?

To determine the importance of LTC, I simulate the model by setting the utility derived for LTC across health status equal to zero ($\mu(.) = 0$). The left panel of figure 11 shows that expenses associated with LTC are a big determinant of the elderly savings for individuals at the top PI. The solid lines represent the simulated benchmark model while the dashed line represents the economy without LTC needs. While being healthy, individuals in the benchmark model reduce consumption from the first years after retirement until late in life (around age 90). Then, they dissave strongly by consuming care hours when in need of LTC. For a given level of initial wealth, with no LTC needs, individuals in the top PI would hold 32% fewer assets by age 90. LTC has also a large impact on the dissaving pattern of individuals in the upper-middle income PI as the median net worth would be around 25% lower by age 90 given the observed asset levels at age 70.

The right panel of figure 11 displays the same simulation across family types. The figure shows that LTC affects very differently the savings decision depending on how much IC an
individual can access. Median net worth for individuals in close families is not affected by LTC needs. Family care constitutes, therefore, a good insurance against LTC shocks. On the contrary, individuals on their own are forced to self-insure until very late in life. The model suggests that at the age of 90, around 50 percent of the assets held by these individuals are LTC related precautionary savings. Individuals in distant families need also to hold large amounts of precautionary savings against LTC as their level of wealth would be around 25% lower by age 90.

6.2 How much the Elderly Save for Medical Expenses?

I now ask whether out-of-pocket medical expenditures are important drivers of old-age savings. To answer this question, I set all out-of-pocket medical expenditures to zero for everyone and look at the corresponding profiles. This could be seen as an extreme form of insurance provided by the Medicare.

Figure 12 shows that medical expenditure plays a minor role in explaining why rich individuals do not dissave until very late in life both across PI groups and family types. At most individuals in the top PI dissave slightly faster such that at age 90 hold 5% more assets
because of medical expenses.

As discussed in French and Jones (2004), the persistence of catastrophic medical shocks constitutes an important reason why individuals might hold a large amount of assets until late in life. Previous studies have underlined the importance of this persistence however little attention has been given to the relation between catastrophic medical expenses and mortality (Pauly, 1990). The relation is important because a positive correlation would bound the persistence of catastrophic expense while a negative one would imply an even larger precautionary motive. The first row in table 15 shows the two-year probability of dying across medical expenses quantiles in the data for individuals aged 70 and over in the HRS. Given that the average two-year probability in the data is 9.3%, the table shows that an individual in the top 5% of the distribution of medical expenses has around 2 times more chances of dying than the average while an individual in the top 1 has around 2.5. Therefore it seems that catastrophic medical expenses are positively associated to death. In order to identify the precautionary saving associated with death, it is therefore important to capture this correlation. The second row of table 15 shows the correlation implied by my estimated health process. The third and fourth row shows the correlation implied when I use the health variable in De Nardi et al. (2010) (DFJ) and Kopecky and Koreskova (2014) (KK),
respectively. In De Nardi et al. (2010) the authors use a health classification based on two levels of self-reported health while in Kopecky and Koreshkova (2014) all individuals might suffer any medical shock independent of their health status. My medical expenses process is able to generate a strong positive correlation between medical expenses and death even if the increase is not as steep as it is in the data. The correlation is much weaker when using the self-reported health classification and zero when not using any health covariate. Therefore individuals in these models will overestimate the risk of persistent and catastrophic medical expenses. Death represents an important hedge instrument against persistently high medical expenses.

<table>
<thead>
<tr>
<th></th>
<th>90th</th>
<th>95th</th>
<th>99th</th>
</tr>
</thead>
<tbody>
<tr>
<td>&gt; $6,300</td>
<td>15.9</td>
<td>18.2</td>
<td>23.2</td>
</tr>
<tr>
<td></td>
<td>(0.5)</td>
<td>(0.7)</td>
<td>(1.9)</td>
</tr>
<tr>
<td>&gt; $9,600</td>
<td>12.3</td>
<td>13.2</td>
<td>16.3</td>
</tr>
<tr>
<td></td>
<td>(0.4)</td>
<td>(0.6)</td>
<td>(1.5)</td>
</tr>
<tr>
<td>&gt; $25,400</td>
<td>10.6</td>
<td>10.8</td>
<td>11.3</td>
</tr>
<tr>
<td></td>
<td>(0.4)</td>
<td>(0.5)</td>
<td>(1.3)</td>
</tr>
<tr>
<td>Data</td>
<td>9.3</td>
<td>9.3</td>
<td>9.3</td>
</tr>
<tr>
<td></td>
<td>(0.1)</td>
<td>(0.1)</td>
<td>(0.1)</td>
</tr>
</tbody>
</table>

Table 15: Two year probability of dying across quantiles of the medical expenses distribution.

### 6.3 On the Strength of the Bequest Motive

I now ask whether the bequests motives that I estimate from the data are important drivers of old-age savings. To answer this question, I set the utility from leaving bequests to zero ($\lambda(.) = 0$) and look at the corresponding profiles.

The left panel of figure 13 shows that across PI groups, bequest motives are relatively more important for poor individuals. In the economy without bequest motives, the top, upper-middle and lower-middle PI individuals dissave relatively faster such that they hold 9%, 20% and 45% lower wealth at age 90. The fact lower PI group show stronger bequest motives in spite of being considered as luxury goods is driven by the shares of individuals by family type.
Figure 13: Median assets by permanent income quartile: benchmark model (solid) and model without bequest motives (dashed)

The right panel of figure 13 shows how strong bequests motives are across different family types. As expected from preferences parameters, the figure shows that the closer the family, the stronger bequest motives are. Therefore the fact that individuals in the top PI hold fewer assets for a bequest is driven by a composition effect: a larger fraction of richer individuals are on their own.

7 Policy Experiments

\[ V_{70}(s, k + TR, b, h, \zeta; current) = V_{70}(s, k, b, h, \zeta, F; experiment) \]

Table 16: The cost and benefits of subsidizing the price of formal care by 4.5%

<table>
<thead>
<tr>
<th>Permanent Income</th>
<th>Government Transfers</th>
<th>Δ Government Transfers</th>
<th>Equivalent Variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bottom</td>
<td>37.5</td>
<td>-1.3</td>
<td>1.5</td>
</tr>
<tr>
<td>L-M</td>
<td>15.1</td>
<td>-0.1</td>
<td>0.5</td>
</tr>
<tr>
<td>U-M</td>
<td>13.0</td>
<td>0.3</td>
<td>0.7</td>
</tr>
<tr>
<td>Top</td>
<td>11.4</td>
<td>0.7</td>
<td>0.9</td>
</tr>
<tr>
<td>Mean</td>
<td>18.0</td>
<td>0.0</td>
<td>0.9</td>
</tr>
</tbody>
</table>
Table 17: The cost and benefits of increasing care hours provided in Medicaid by 5%

<table>
<thead>
<tr>
<th>Permanent income</th>
<th>Government transfers</th>
<th>∆ Government transfers</th>
<th>Equivalent variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bottom</td>
<td>37.5</td>
<td>0.3</td>
<td>1.4</td>
</tr>
<tr>
<td>L-M</td>
<td>15.1</td>
<td>0.1</td>
<td>0.4</td>
</tr>
<tr>
<td>U-M</td>
<td>13.0</td>
<td>0.1</td>
<td>0.3</td>
</tr>
<tr>
<td>Top</td>
<td>11.4</td>
<td>0.0</td>
<td>0.3</td>
</tr>
<tr>
<td>Mean</td>
<td>18.0</td>
<td>0.1</td>
<td>0.6</td>
</tr>
</tbody>
</table>

Table 18: The cost and benefits of subsidizing the price of formal care by 8.5%

<table>
<thead>
<tr>
<th>Permanent income</th>
<th>Government transfers</th>
<th>∆ Government transfers</th>
<th>Equivalent variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bottom</td>
<td>37.5</td>
<td>-1.9</td>
<td>1.5</td>
</tr>
<tr>
<td>L-M</td>
<td>15.1</td>
<td>-0.1</td>
<td>0.7</td>
</tr>
<tr>
<td>U-M</td>
<td>13.0</td>
<td>0.6</td>
<td>1.1</td>
</tr>
<tr>
<td>Top</td>
<td>11.4</td>
<td>1.3</td>
<td>1.6</td>
</tr>
<tr>
<td>Mean</td>
<td>18.0</td>
<td>0.1</td>
<td>1.3</td>
</tr>
</tbody>
</table>

8 Conclusions

In this paper, I estimate a model of savings for retired single individuals with heterogeneous LTC needs and where LTC expenses are endogenous. It is important to do so for several reasons. First, I show that allowing for different levels of health deterioration is important to measure the risk of being unhealthy for long periods of time. As a result, I find that medical expenses are significantly less crucial as a driver of savings than what previous literature has suggested.

Second, my estimated preferences parameters imply that LTC expenses are luxury goods; that is they are much higher for individuals with more financial resources. Thus, LTC is a more important driver of savings for richer individuals which explains the asymmetry in the dissaving pattern across the income distribution. As a result, I am able to match wealth profiles with weaker bequest motives than previous literature.

Third, I find that access to IC significantly alleviates the financial risk of LTC. How-
ever, given that individuals receiving more IC do not show high dissaving rates, the model estimates a stronger bequest motive for these individuals.

All in all, I find LTC expenses to be the main important channel explaining why the elderly rich do not dissave further. Besides, bequest motives for individuals receiving IC are also identified as an important driver for the middle-class.
References


Appendix A  Smoothed Probabilities

In this appendix I explain the computation of smoothed probabilities. These are used for computing statistics by health status given our estimates of $\hat{\beta}$ and $\hat{\mu}$. The derivation is split in two parts: the filtered probabilities based on Hamilton (1989) and the smoothed probabilities based on Kim (1994).

Filtered probabilities. — For computing the filtered probabilities, I need first to obtain

$$p(x_{i,t+1}, h_{i,t+1}, h_{i,t}|x_i^l) = p(x_{i,t+1}|x_i^l, h_{i,t+1}, h_{i,t}) \cdot p(h_{i,t+1}|h_{i,t}) \cdot p(h_{i,t}|x_i^l)$$

where $p(x_{i,t+1}|h_{i,t+1})$ is given by equation 1, $p(h_{i,t+1}|h_{i,t})$ is given equation 2 and $p(h_{i,t}|x_i^l)$ is available by recursion. Then,

$$p(x_{i,t+1}|x_i^l) = \sum_{k,l} p(x_{i,t+1}, h_{i,t+1}, h_{i,t}=l| h_{i,t}=k)$$

I can thus compute the filtered probabilities as,

$$p(h_{i,t+1}|x_i^{l+1}) = \frac{\sum_{l} p(x_{i,t+1}, h_{i,t+1}, h_{i,t}=l| x_i^l)}{p(x_{i,t+1}|x_i^l)}$$

Smoothed probabilities. — I observe,

$$p(h_{i,t+1}, h_{i,t}|x_i^T) = p(h_{i,t+1}|x_i^T) \cdot p(h_{i,t}|h_{i,t+1}, x_i^T) = p(h_{i,t+1}|x_i^T) \cdot p(h_{i,t}|h_{i,t+1}, x_i)$$

$$= p(h_{i,t+1}|x_i^T) \cdot \frac{p(h_{i,t+1}|h_{i,t}) \cdot p(h_{i,t}|x_i)}{\sum_{l} p(h_{i,t+1}|h_{i,t}=l) \cdot p(h_{i,t}=l|x_i)}$$

Therefore, if we sum over all values of $h_{i,t+1}$, I get my target, $p(h_{i,t}|x_i^T)$.

Sample path for health states, given all the data. — I begin by drawing $h_{i,T}$ from the filtered $p(h_{i,T}|x_i^T)$, I then draw using:

$$p(h_{i,T-1}|h_{i,T}, x_i^T) = \frac{p(h_{i,T}|h_{i,T-1}) \cdot p(h_{i,T-1}|x_i^{T-1})}{\sum_{l} p(h_{i,T}|h_{i,T-1}=l) \cdot p(h_{i,T-1}=l|x_i^{T-1})}$$

(1)
Appendix B  Medical Expenditures Model Estimation

Following French and Jones (2004), I estimate the following model:

\[
\ln m_{it} = X_{it}' \beta + \xi_{it} + \zeta(h)_{it}, \quad \zeta(h)_{it} \sim N(0, \sigma^2_{\zeta}(h_{it})) \tag{2}
\]

\[
\xi_{it} = \rho \xi_{it-1} + \epsilon_{it}, \quad \epsilon_{it} \sim N(0, \sigma^2_{\epsilon}) \tag{3}
\]

\(X_{it}\) consists of a polynomial in age, gender, permanent income dummies, health dummies and permanent income dummies interacted with health dummies. Thus the parameter vector to be estimated is \(\theta = (\beta, \sigma^2_{\epsilon}, \sigma^2_{\xi}(h = 1), \sigma^2_{\xi}(h = 2), \sigma^2_{\xi}(h = 3), \sigma^2_{\xi}(h = 4), \rho)\). I aim at the posterior distribution \(p(\theta | X)\). I specify flat priors for all the parameters in \(\theta\). In order to sample from the joint posterior distribution, I enlarge the parameter space to \((\theta, \xi)\) and apply the following Gibbs algorithm:

- **Block 0**: sample health status.
- **Block 1**: \(p(\xi | \beta, \sigma^2_{\epsilon}, \sigma^2_{\xi}, \rho)\)
- **Block 2**: \(p(\rho | \beta, \sigma^2_{\epsilon}, \sigma^2_{\xi}, \xi)\)
- **Block 3**: \(p(\sigma^2_{\epsilon} | \beta, \sigma^2_{\xi}, \rho, \xi)\)
- **Block 4**: \(p(\sigma^2_{\xi} | \beta, \sigma^2_{\epsilon}, \rho, \xi)\)
- **Block 5**: \(p(\beta | \sigma^2_{\epsilon}, \sigma^2_{\xi}, \rho, \xi)\)

Block 0: Using the filtered and transition probabilities from Amengual et al. (2017), sample health status using the Kim Smoother.

Block 1: Given linearity and normality, the Kalman smoother provides the distribution of each state at each time conditional on all available data. To sample the latent state \(\xi_{it}\), I have to simulate from the smoothed states. Given that the simulation smoother is backwards recursion which requires the Kalman filter output, it is convenient to rewrite the process in
the canonical state-space form as:

\[
\xi_{t+1} = F\xi_t + v_{t+1} \quad (4)
\]
\[
y_t = A'x_t + H'\xi_t + w_t \quad (5)
\]

with

\[
E[v_t v_\tau] = \begin{cases} 
Q, & \text{for } t = \tau \\
0, & \text{otherwise}
\end{cases} \quad (6)
\]
\[
E[w_t w_\tau] = \begin{cases} 
R, & \text{for } t = \tau \\
0, & \text{otherwise}
\end{cases} \quad (7)
\]

where, \( F = \rho, A = \beta, H = 1, Q = \sigma_e^2, R = \sigma_\xi^2(h_{it}). \) The initial condition for the forecast of \( \xi_{1|0} \) based on no observations of \( y \) or \( x \) is set to 0 and the associated mean square error to the unconditional variance: \( P_{1|0} = \frac{\sigma_t^2}{1 - \rho^2}. \) For \( t > 0 \), we have the standard Kalman filter equations that we need to estimate for each individual at a time:

\[
K_t = FP_{t|t-1}H(H'P_{t|t-1}H + R)^{-1} \quad (8)
\]
\[
\xi_{t+1|t} = F\xi_{t|t-1} + (y_t - A'x_t - H'\xi_{t|t-1}) \quad (9)
\]
\[
P_{t|t} = P_{t|t-1} - P_{t|t-1}H(H'P_{t|t-1}H + R)^{-1}H'P_{t|t-1} \quad (10)
\]
\[
P_{t+1|t} = FP_{t|t}F' + Q \quad (11)
\]
\[
\xi_{t|T} = \xi_{t|t-1} + P_{t|t-1}H(H'P_{t|t-1}H + R)^{-1}(y_t - A'x_t - H'\xi_{t|t-1}) \quad (12)
\]

To sample the states, I use Carter and Kohn (1994) algorithm that recursively updates, backwards through time the Kalman smoothed conditional densities of each state given the draw at time \( t + 1 \) following:

\[
\xi_{t|T} = \xi_{t|t} + P_{t|t}F'P_{t+1|t}^{-1}(\xi_{t+1} - F\xi_{t|t}) \quad (13)
\]
\[
P_{t|T} = P_{t|t} - P_{t|t}F'P_{t+1|t}^{-1}FP_{t|t} \quad (14)
\]

then draw \( \xi_t \) from \( N(\xi_{t|T}, P_{t|T}). \)
Block 2 and 5 is identical to the problem of drawing from the posterior coefficients in a linear regression model.

Block 3 and 4 is identical to sample the posterior variance in a linear regression model.