Banks’ balance sheets, uncertainty and macroeconomy

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Abstract: Motivated by data on weak credit growth despite highly accommodative monetary policy in many advanced economies and by ample evidence on important effects of uncertainty in the course of the financial crisis of 2007-2009, this paper studies relationships between economic uncertainty and asset portfolio allocation of the banking sector. The theoretical model here is guided by an empirical finding showing that an increase in uncertainty leads to reallocation of portfolio of assets by commercial banks: banks reduce issuance of loans, in particular, of commercial and industrial loans, while increasing the stock of safe assets - cash and Treasury and agency securities. To account for this evidence, I build a DSGE model that incorporates a portfolio-optimizing banking sector facing non-diversifiable credit risk, where banks’ attitude to risk and expected profitability help to explain the endogenous movements of the risk premium. Precautionary mechanism is in play: in addition to remunerating for risk of defaults, the premium charged by risk-averse banks provides self-insurance from profitability reduction brought about by heightened uncertainty about entrepreneurial productivity. Banks reduce their exposure to credit risk by cutting down the share of risky lending in their asset portfolios and increasing the share of risk-free assets. Financial accelerator mechanism amplifies the portfolio reallocation effect of uncertainty shock, as increased external finance premium reduces entrepreneurial demand for capital, putting downward pressure on real price of capital and on borrowers’ net worth, what depresses demand for capital further.

Keywords: Uncertainty shocks, DSGE model, Stochastic volatility, Financial accelerator, Bank Portfolio.

JEL Classification Numbers: E44, E51, G21.

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1 Introduction

Being an important factor of slow economic recovery, weak growth of credit in the aftermath of the financial crisis of 2007-2009 has been a serious concern of policymakers. Despite highly accommodative stance of monetary policy and various policies to enhance credit supply and to support credit demand, near-zero or negative growth of bank lending has been experienced by many advanced economies for a number of years\(^1\) (see Figures A.1 and A.2 in Appendix A). Given that efficient credit allocation is one of the pillars of growth\(^2\), its weakness hinders the full and sustained economic recovery.

A number of recent studies demonstrate that uncertainty has played a prominent role in shaping credit market developments during the financial crisis of 2007-2009\(^3\). I make this result specific for the banking sector in this work. In particular, I estimate a set of vector autoregression models and show that commercial banks reallocate their portfolios of assets following uncertainty shocks by reducing the issuance of business loans and increasing their holdings of safe assets - cash and Treasury and agency securities\(^4\). This result is robust to the type of uncertainty measure used - macro- and microeconomic uncertainty, - and to representation of an asset variable in the VAR - either in levels or as a share of total portfolio.

Based on this finding and responding to calls for more sophisticated modeling of financial intermediaries due to their nontrivial role in the recent financial crisis, I build a general equilibrium model, introducing two features into the banking sector modelling. First, I use a firm-theoretical model of bank behaviour and model banks as optimizing their balance sheet structure by solving the portfolio problem, where banks choose to allocate their funds between risky lending to entrepreneurs and risk-free government bonds. Second, motivated

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\(^1\)For details see IMF Global Financial Stability Report, October 2013).


\(^3\)See, for example, Stock and Watson (2012), Balke and Zeng (2013), Caldara et al. (2016), Baum et al. (2008) and Quagliariello (2008).

\(^4\)Another study demonstrating these effects is Pirozhkova (2016).
by empirical evidence and along the lines of theoretical literature, I model banks as risk-averse agents\(^5\). In practice banks should hold a sufficient level of capital to protect themselves from risk of insolvency, what underpins their negative attitude to uncertainty about future profitability. Aksoy and Basso (2014) and Danielsson et al. (2011) show that in presence of a Value-at-Risk constraint banks behave like risk-averse agents. Assuming concave preferences allows uncertainty to play a non-trivial role in banks’ decision how to allocate assets in their portfolios.

In theoretical model suggested here, banking sector faces non-diversifiable credit risk. This risk emerges, because loans are subject to default and because lending rate on non-defaulted loans specified in the loan contract is non state-contingent on future outcomes. I modify the optimal debt contract structure proposed originally in Bernanke et al. (1999) to allow lending rates be non-contingent on realization of shocks, such that banks could obtain non-zero profits. Heightened idiosyncratic uncertainty - a greater cross-sectional dispersion of productivity, - increases the rate of entrepreneurial defaults, and banks respond by increasing risk premium. Importantly, precautionary mechanism is in play: risk-averse banks charge lending rate, which in addition to remunerating for the increased expected defaults, provides self-insurance from profitability reduction. Thereby banks’ expectations about their future profitability play key role in driving the endogenous movements of credit spread. In this respect the amplification mechanism in my model is in line with the one in Aksoy and Basso (2014), where term spreads’ variations are brought about by expectations of banks of their future profitability\(^6\). Due to increasing external finance premium, the demand for loans falls. The asset portfolios of banks are reallocated: the share of risky lending goes down, while the share of risk-free assets (government bonds), acting as a buffer stock, goes up.

The simulations of the general equilibrium model reproduce a pattern, specified by a key

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\(^6\)Aksoy and Basso (2014) also provide empirical evidence that corroborates the link between expected bank profitability and term spreads movements.
postulate of the modern portfolio theory - that choices of an agent with concave preferences are characterized by a positive premium to the amount that she is willing to pay to avoid a fair gamble. Importantly, there are significant differences in the ways how this result is obtained in the modern portfolio theory and in the model suggested here. First, in my case risk is not measured by the variance of distribution of returns, as it is done commonly in the modern portfolio approach\(^7\). Instead, in the model suggested here credit risk is a downside measure, specifically, it is a probability that idiosyncratic productivity of a borrower is lower than the one that allows her to pay back the loan (a borrower who cannot pay back the loan declares default). The size of credit risk, that the bank is subject to, is determined endogenously and follows from the structure of the optimal debt contract between banks and entrepreneurs. Second, I don’t employ a quadratic utility function, which is conventionally used in the modern portfolio theory to analyze the problem of portfolio allocation of the risk-averse investor and to demonstrate how concavity of investor’s preferences affects the optimal choice of the fraction of portfolio invested in the risky asset. I assume a constant relative risk aversion type of utility function for banks. Third, risk-free rate in the suggested model is unknown ex-ante, in contrast to it being known in advance in the modern portfolio theory; additionally, it is determined endogenously, responding to the movements of output and inflation according to the Taylor rule.

Uncertainty has received a substantial attention as a factor that exerts an important impact on economic developments during the Great Recession. Stock and Watson (2013) argue that the decline of output and employment in the Great Recession was mainly due to financial and uncertainty shocks. Recent empirical macro- and microeconomic research documents strong negative relationship between uncertainty and growth. This is demonstrated, for example, in cross-country studies of Ramey and Ramey (1995) and Engle and Rangel (2008). A VAR approach is used by Bloom (2009) to show that there is a drop and rebound of industrial production following the impact of uncertainty shock. By estimating a

\(^7\)There is a growing amount of works, however, where risk is characterized by a downside measure. See, for example, Chaigneau and Eeckhoudt (2016).
fully fledged DSGE model, Justiniano and Primiceri (2008) demonstrate that the decline in volatility of output in the mid-1980s happened due to change in volatility of various types of technology shocks\textsuperscript{8}. Additionally, Aastveit et al. (2013) and Bloom et al. (2012) show that increased uncertainty weakens the effectiveness of monetary policy. As for microeconomic evidence, significant negative effect of uncertainty on investment in the firm-level panel data is shown by Leahy and Whited (1996). Guiso and Parigi (1999) document the negative impact of uncertainty on firms’ expectations of demand. Bloom, Bond and van Reenen (2007) demonstrate that uncertainty gives rise to the ”caution effect”, while Panousi and Papanikolaou (2012) show that negative effect of uncertainty appears to be management risk-aversion. To sum up, uncertainty is demonstrated to be an important factor that drives the dynamics of economy at both macro and micro levels.

In existing literature some papers analyze the effects of heightened uncertainty about total factor productivity\textsuperscript{9}, while other works investigate the impact of shocks to idiosyncratic productivity of firms\textsuperscript{10}. I contribute to this literature by evaluating the impact that idiosyncratic uncertainty makes on the portfolio reallocation of the banking sector and resulting general equilibrium effects. Uncertainty is modelled as a time-varying volatility of idiosyncratic productivity component of entrepreneurs, so that in times of heightened uncertainty the probability of the events on the tails of the distribution of entrepreneurial productivity is higher. This implies not only increased credit risk, what induces risk-neutral banks to charge a higher risk premium to compensate for the greater possible losses, but also greater uncertainty about the future bank profitability, what makes risk-averse banks increase risk premium further to self-insure against profitability reduction. I find that the proposed mechanism of precautionary motive of risk-averse banks produces significant portfolio reallocation

\textsuperscript{8}Among other papers, showing that uncertainty shocks produce economic contractions are Bachmann et al. (2013), Alexopoulos and Cohen (2009), Bachmann and Bayer (2011) and Knotek and Khan (2011).

\textsuperscript{9}See, for example, Fernandez-Villaverde and Rubio-Ramirez (2007), Justiniano and Primiceri (2008), Fernandez-Villaverde et al. (2011), Basu and Bundick (2012), Bloom et al. (2012), among others.

\textsuperscript{10}Bloom et al. (2012), Christiano et al. (2013) and Bachmann and Bayer (2011) study the effect of changing volatility of cross-sectional dispersion of firm-level productivity. The relative importance of aggregate and idiosyncratic uncertainty, which are sometimes referred to as macro and micro uncertainty, is studied in Balke et al. (2012) and Cesa-Bianchi and Fernandez-Corugedo (2014).
effects, it helps to explain an additional portion of the risky lending reduction and business cycle movements.

Another stream of literature, related to my work, explores the role of credit frictions as a factor that contributes to business cycle fluctuations. Despite employing different workhorse models with various types of frictions, the key studies in this literature - Bernanke et al. (1999), Holmstrom and Tirole (1997), Kiyotaki and Moore (1997) and Carlstrom and Fuerst (1997), - agree that financial frictions have significant effects on movement of aggregates. They don’t only make an impact as amplifying the effect of exogenous shocks\(^{11}\), but also act as a source of disturbancies that play an important role for business cycles\(^{12}\). The recent and growing literature anlyses the role of credit market imperfections in amplifying the effect of uncertainty. Among those, Arellano et al. (2012) demonstrate that higher uncertainty is a factor that reduces factor inputs of firms and their output, when firms are subject to costly default. Christiano et al. (2013) and Gilchrist et al. (2013) show that idiosyncratic uncertainty shocks increase the external financial premium in presence of asymmetric information in lending relationships. Benes and Kumhof (2015) also analyze the general equilibrium model with financial accelerator and endogenous risky lending; they focus on bank capital adequacy requirements and demonstrate that countercyclical capital buffers increase welfare. Bonciani and van Roye (2013) show that stickiness of banking retail interest rates amplifies the effect of TFP uncertainty on economy. Balke and Zeng (2013) show that the financial crisis of 2007-2009 was mostly due to decline in financial intermediation that originated from output and uncertainty shocks.

I contribute to the literature on credit frictions by showing that in the financial accelerator framework a la Bernanke et al. (1999), where lenders are risk-averse and choose their balance sheets volumes before observing shock values, heightened idiosyncratic uncertainty leads,

\(^{11}\)See, for example, Gertler and Karadi (2011) and Balke (2000) on this.

\(^{12}\)For example, Christiano et al. (2010) distinguishes between a banking technology shock and a bank reserve demand shock, which have consequential effects on movements of total output. Hafstead and Smith (2012) examine the role of a shock to bank-specific loan productivity or a shock to the cost of bank intermediation.
first, to widening of credit spread, and second, to lowering of the volume of bank credit. I demonstrate that allowing for concave preferences of banks gives rise to the precautionary savings motive, such that assets portfolios of banks are reallocated, what is consistent with the empirical results presented in section 2. I show that financial accelerator mechanism works to amplify these effects, as the reduced demand for capital from entrepreneurs induces price of capital to go down, such that entrepreneurial net worth decreases, implying even higher risk premium charged by banks and the further reduction of bank credit. In section 2 I present a partial equilibrium model of bank portfolio choice. Econometric evidence suggesting the portfolio reallocation effects of uncertainty shocks is presented in section 3. Section 4 describes the general equilibrium model and calibration strategy. Section 5 discusses the results of the model simulations and the effects of uncertainty shocks on model economy.

2 Uncertainty and banks’ portfolio reallocation: Empirical evidence

In this section I analyse the effects of uncertainty shock on reallocation of banks’ portfolios of assets empirically. Following Bachmann et al. (2013), I use the forecasters’ disagreement about future inflation as a benchmark measure of uncertainty. I also consider other measures of uncertainty for robustness check of results: the news-based uncertainty index, economic policy uncertainty index, a commonly used in the literature VIX/VXO index\(^\text{13}\), realised unconditional volatility of GDP growth, conditional volatility of GDP growth, cross-sectional standard deviation of firms’ profit growth and cross-firm stock return variation. Two latter proxies measure microeconomic uncertainty, while all the other measure macroeconomic uncertainty; both volatility and ‘vagueness’ uncertainty measures are used.

\(^{13}\)Instead of VIX, I use the VXO index - a CBOE S&P 100 Volatility Index that features a longer data series, starting from 1986Q1 instead of 1990Q1, as in the case of VIX.
The forecasters’ disagreement about future inflation measures the dispersion between individual forecasters’ predictions about future levels of the Consumer Price Index and is used with data coming from the Federal Reserve Bank of Philadelphia’s Survey of Professional Forecasters. News-based uncertainty index quantifies newspaper coverage of economic uncertainty, related to policy. In particular, it is the index of search results from 10 large newspapers, from which a normalized index of the volume of news articles discussing economic policy uncertainty is constructed\textsuperscript{14}. The composite economic policy uncertainty index developed in Baker, Bloom and Davis (2016) captures the compound effect on policy uncertainty of several factors, including, first, the news-based uncertainty, second, uncertainty about the future path of the federal tax code, and third, disagreement of professional forecasters about government spending and inflation. The VXO index is a market estimate of implied volatility, calculated using quotes on S&P 100 Index option, and is often used as a measure of short-term uncertainty. This stock market volatility has also been previously used as a proxy for uncertainty at the firm level (see, for example, Leahy and Whited (1996) and Bloom et al. (2007)). A realized unconditional volatility of GDP growth is calculated as a rolling sample standard deviation over a 5 years window, whereas conditional volatility of GDP growth is estimated as heteroscedasticity of real GDP growth with GARCH (1,1)\textsuperscript{15}. In a latter case, the volatility is estimated as a conditional variance from GARCH model.

The measures of microeconomic uncertainty used in the current analysis are cross-sectional standard deviation of firms’ profit growth and cross-firm stock return variation. The former one measures the within-quarter cross-sectional spread of pretax profit growth rates normalized by average sales. As suggested by Bloom (2009), profit growth has a close fit to productivity and demand growth in homogenous revenue functions, and hence, its standard deviation across firms could be used as a pertinent proxy for idiosyncratic or microeconomic uncertainty. The latter microeconomic uncertainty measure, suggested in Bloom et

\textsuperscript{14}For details, see www.policyuncertainty.com

\textsuperscript{15}A similar measure of macroeconomic uncertainty is constructed in Cesa-Bianchi and Fernandez-Corugedo (2014) on TFP data.
Table 1: Pairwise correlation coefficients between uncertainty measures

<table>
<thead>
<tr>
<th></th>
<th>UV GDPg</th>
<th>CV GDPg</th>
<th>VXO</th>
<th>NB UI</th>
<th>P UI</th>
<th>FD</th>
<th>SD pr g</th>
<th>CF SRV</th>
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</thead>
<tbody>
<tr>
<td><strong>Unconditional volatility of GDP growth</strong></td>
<td>1***</td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
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<tr>
<td><strong>Conditional volatility of GDP growth</strong></td>
<td>0.75***</td>
<td>1***</td>
<td></td>
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<tr>
<td>VXO</td>
<td>0.15*</td>
<td>0.41***</td>
<td>1***</td>
<td></td>
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<td></td>
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<tr>
<td>News-based uncertainty index</td>
<td>0.40***</td>
<td>0.38***</td>
<td>0.53***</td>
<td>1***</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Policy uncertainty index</td>
<td>0.63***</td>
<td>0.47***</td>
<td>0.41***</td>
<td>0.88***</td>
<td>1***</td>
<td></td>
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</tr>
<tr>
<td>Forecasters’ disagreement</td>
<td>0.53***</td>
<td>0.31**</td>
<td>0.24***</td>
<td>0.14*</td>
<td>0.49***</td>
<td>1***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Standard deviation of firms’ profit growth</td>
<td>0.13</td>
<td>0.33**</td>
<td>0.41**</td>
<td>0.18**</td>
<td>0.09</td>
<td>-0.02</td>
<td>1***</td>
<td></td>
</tr>
<tr>
<td>Cross-firm stock return variation</td>
<td>0.11</td>
<td>0.51***</td>
<td>0.75***</td>
<td>0.53***</td>
<td>0.43**</td>
<td>0.14*</td>
<td>0.29**</td>
<td>1***</td>
</tr>
</tbody>
</table>

Note. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. The following abbreviations are used: UV GDPs - unconditional volatility of GDP growth, CV GDPg - conditional volatility of GDP growth, VXO - VXO index, NB UI - News-based uncertainty index, P UI - Policy uncertainty index, FD - forecasters’ disagreement about future inflation, SD pr g - standard deviation of firms’ pretax profit growth, CF SRV - cross-firm stock return variation. The sample is 1954Q4-2015Q4 or the longest one over this period, for which data is available.

al. (2016), is an interquartile range of firms’ monthly stock returns. This uncertainty measures volatility of the perceptions of the stock market participants about firms’ performance. Campbell et al. (2001) demonstrate that in booms cross-sectional spread of stock returns is about 50% lower than in recessions; Bloom et al. (2016) also show that this uncertainty measure is countercyclical.

Table 1 shows that pairwise correlations between various uncertainty measures range from very low and insignificant (for example, between cross-firm stock return variation and unconditional volatility of GDP growth) to high and significant (for example, between VXO index and cross-firm stock return variation), Figure B.1 plots series for the uncertainty measures discussed here. Composite policy uncertainty index and news-based uncertainty index co-move together (correlation coefficient 0.88), because the latter one is one of the
components of the former. High correlation is also observed between conditional and unconditional volatility of GDP growth (0.75). Generally, microeconomic uncertainty measures tend to be correlated with macroeconomic ones to the less extent than macroeconomic uncertainty proxies between each other. In particular, this refers to standard deviation of pretax profit growth that shows only week or moderate correlation with other uncertainty measures. Hence, there are significant differences between dynamic properties of distinct measures of uncertainty. However, despite these differences, I find that the effects of uncertainty shocks on banks’ assets portfolio reallocation in the estimated VAR model are remarkably similar across alternative uncertainty measures.

I estimate a structural orthogonalized VAR on the U.S. 1985-2015 quarterly data to analyze the impact of uncertainty shock on the components of bank assets: safe assets (which include cash and Treasury and agency securities, i.e. assets with low/minimal level of risk) and loans issued for non-financial corporations (commercial and industrial loans). The baseline VAR model includes, in the following order, eight variables: an uncertainty measure, the real GDP, the GDP deflator, the indebtedness of corporate sector measured by its leverage, simple capital ratio of banks, a component of banks’ assets (safe assets or commercial and industrial loans), portfolio credit risk, measured by charge-off rate on loans, and the Federal Funds rate. Thus, I include in a model a standard set of variables that comprise a small-scale monetary policy VAR, an uncertainty measure and a set of financial and credit variables, which are potentially important as determinants of banks’ balance sheet structure, and thus, should be controlled for. The sample covers the period from 1985Q1 to 2015Q3.

I place the uncertainty measure as the first variable in the VAR model. This Cholesky ordering implies that uncertainty shocks influence all other variables contemporaneously,

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16 The shock in the Federal Funds rate equation is considered the monetary policy shock; I go along McCallum (1983), Bernanke and Blinder (1992), Bernanke and Mihov (1998), Sims (1986) and Sims (1992) in that.

17 The model with VXO index as uncertainty measure ranges from 1986Q1 to 2015Q3 - from the earliest date that VXO index series is available for.
such that uncertainty is an underlying characteristic of the state of economy being unaffected by other shocks within the same period (i.e. within a quarter)

In subsequent periods uncertainty responds to all shocks through its relation to the lags of other variables as specified in the VAR model.

Federal funds rate is placed the last in the VAR. The assumption is that monetary authority observes and responds to contemporaneous information on all other variables. I consider this is a plausible assumption given that data on prices, industrial output, aggregate employment and other indicators of aggregate real economic activity are available to the FED on monthly basis. Leverage of non-financial corporates is placed after real activity and inflation measures based on the assumption that companies observe contemporaneous values of uncertainty, real activity and inflation, when making decision about how much debt to incur, whereas all credit variables are not observed by them. Capital ratio of banks is placed before loans. Capital adequacy requirements affect the amount of risky assets banks can have on their balance sheets, and that is the reason why I assume that banks see and take into account the level of their capital ratio when making decisions about risky loans issuance. Assets’ components variable (safe assets or loans) is placed after the capital ratio. The assumption is that banks observe contemporaneous information on uncertainty, real activity, inflation, indebtedness of corporates and capital ratio, when deciding on loans issuance and how much safe assets to hold. Charge-off rate on loans is placed after loan volumes. I assume that the value of loans removed from the books and charged against loss reserves is affected by the volume of loans issued by banks to firms contemporaneously.

Estimation results are shown in Appendix B as impulse responses to one standard deviation innovation in uncertainty with 90% bias-corrected bootstrap confidence bands calculated as in Kilian (1998). I report here the impulse responses for the benchmark uncertainty measure - the forecasters’ disagreement about future inflation, the result of robustness checks,

\footnote{The recursively identified vector autoregressions with uncertainty measures to trace the dynamic responses of measures of economic activity to surprise increases in uncertainty are also used in Bloom (2009), Baker, Bloom and Davis (2016) and Bachmann et al. (2013), where in the latter one the uncertainty measure is ordered first in the VAR.}
where alternative measures of uncertainty are used, are available from the author upon request.

Figure B.2 shows that a positive shock to uncertainty induces a significant reduction of output (by 0.09% on impact and by 0.25% after 3 quarters), inflation (by 0.03% on impact and by 0.05% after 5 quarters) and federal funds rate (by 0.09 annual % after 9 quarters from the shock impact), while capital ratio and charge off rate on business loans go up (the former - by 0.43% and the latter - by 4.31% after 3 quarters from the shock impact). Controlling for aggregate demand, inflation, corporate sector indebtedness and capital ratio of banks, commercial and industrial loans go down following an uncertainty shock, by 0.48%. Figure B.3 demonstrates that there is a significant increase of the safe assets holdings by banks after an exogenous spike in uncertainty - by 0.5% after 4 quarters and by 1.12% after 8 quarters from the impact of uncertainty shock. Figure B.4 shows that the result of the business loans reduction following an exogenous increase of uncertainty holds not only for the volume of loans issued, but also for the share of commercial and industrial loans in portfolios of assets of banks. The share of business loans goes down by 0.05 pp and stays reduced for a period up to 11 quarters after a positive shock to uncertainty. Figure B.5 demonstrates that the result of the safe assets increase after an uncertainty shock holds for the share of safe assets in the portfolios of banks: this share goes up by 0.22 pp with this increase being significantly positive for after 15 quarters after the impact of the shock. Finally, I look at the response of the share of total loans in banks’ portfolios to a positive uncertainty shock. The share of total loans in total banks’ assets is essentially the share of risky loans that banks choose to issue. Apart from commercial and industrial loans, total loans include consumer loans and real estate loans. Figure B.6 shows that the share of risky lending in banks’ assets goes down after an uncertainty shock; the decrease is 0.1 pp and it stays significantly negative for 15 quarters after the shock impact. The differences between the dynamics of the share of business loans and the total share of risky lending in the assets of banks are explained by the fact that these are substantially different factors, which determine the dynamics
of consumer and real estate loans' issuance as compared with commercial and industrial loans (see Pirozhkova (2016) for details). As a robustness check, I use other measures of macroeconomic and microeconomic uncertainty, and the result of the reduction of business loans (and the share of risky loans in total assets of banks) and the increase of safe assets (and the share of safe assets in the portfolios of banks) after a positive shock to uncertainty continue to hold\textsuperscript{19}.

As additional robustness checks, I use an alternative estimation period, which excludes the time interval following the financial crisis: the estimation period is 1985-2007. I also use an alternative identification scheme, which assumes that uncertainty reacts to movements in all the other model variables within a quarter, and place an uncertainty measure the last in the VAR. These robustness checks confirm my findings.

My empirical study of the impacts of uncertainty shocks in the vector autoregression framework reveals that there are significant portfolio reallocation effects in bank assets following surprise increase in uncertainty. Specifically, banks increase their holding of riskless asset, while the risky lending issuance is reduced.

3 Uncertainty and risk-averse banking sector

This section provides intuition for the dynamics of bank portfolio obtained in the general equilibrium setup of the model in the following section. It introduces the idea of bank’s precautionary motive when there is uncertainty about bank future profitability.

Consider a case with a representative risk-averse bank in economy. The bank funds its activity by issuing deposits for households $D_t$ and allocates its funds to corporate loans $L_t$ and riskless government bonds $B_t$. Issuing loans is risky and yields higher return than the return on government bonds: risk premium compensates for risk of issuing loans. Bank’s

\textsuperscript{19}The results of all the robustness checks are available from the author upon request.
stylized balance sheet constraint is:

\[ D_t = L_t + B_t, \]  

(1)

and the expected profits of the bank are:

\[ E_t(\pi_{t+1}) = (r^L_t L_t) \nu_t + r^g_t B_t - r^d_t D_t, \]  

(2)

where \( r^L_t \) is the bank’s lending rate, \( r^g_t \) is risk-free rate on government bonds, \( r^d_t \) is rate on deposits and \( \nu_t \) is the share of non-defaulted loans\(^{20}\).

Under the assumption of bank being risk-neutral the no-arbitrage condition implies that deposit rate coincides with risk-free rate, and risk spread is accounted by default rate on loans, while bank gets no profit:

\[ r^d_t = r^g_t = r^L_t \ast \nu_t. \]  

(3)

In this case bank would satisfy all the demand for loans and deposits adjusting the volume of government bonds held on its balance sheet.

Instead of risk-neutrality let’s assume that bank’s preferences are represented with utility function \( u(\cdot) \) featuring risk-prudence, following the definition of prudence from Kimball (1990): \( u'(\cdot) > 0, u''(\cdot) < 0 \) and \( u'''(\cdot) > 0 \)\(^{21}\). Then the portfolio problem of the bank is:

\[ \max_{L_t, B_t, D_t} E_t(u(\pi_{t+1})) \quad s.t.(1) \]  

(4)

The optimizing behaviour of a risk-averse bank is different from that of a risk-neutral one. In maximizing its expected utility, risk-averse bank takes into account the stochastic nature of the share of non-defaulted firms \( \nu_t \)\(^{22}\). Ultimately, \( \nu_t \) could depend on stochastic

\[^{20}\]Throughout the paper all the interest rates are gross rates. Lower case letters denote nominal interest rates and upper case letters denote real interest rates.

\[^{21}\]The terms risk-averse and risk-prudent are used interchangeably here.

\[^{22}\]In the general equilibrium setup of the model this share is endogenous: it is a function of the equilibrium threshold level of idiosyncratic productivity shock that separates bankrupt and non-bankrupt entrepreneurs.
properties of idiosyncratic return to capital (including its time-varying volatility that I refer to as idiosyncratic uncertainty) or/and on stochastic properties of the aggregate productivity process.

To examine the impact of uncertainty on bank’s choice of risky and safe assets share in its portfolio, consider that increased uncertainty induces banks to take into account a wider range of possible values of future profits around $E(\pi_{t+1})$: from the lowest possible $\pi_{t+1}^L$ to the highest possible $\pi_{t+1}^H$ (see Figure 1). Because the function of marginal profit $u'(\pi_{t+1})$ is convex ($u''(\cdot) > 0$), Jensen inequality implies $E(u'(\pi_{t+1})) > u'(E(\pi_{t+1}))$. Thus, when there is uncertainty about future returns/profitability, expected marginal utility is higher than in the case of no uncertainty. Higher expected marginal utility of profit in period $t+1$ requires lower level of expected profit $E_t(\pi_{t+1})$, what is attained by changing the structure of its portfolio: issuing less risky loans $L_t$, which pay higher return, and increasing the holdings of safe low-yield government bonds $B_t$. Hence, the effect of heightened uncertainty on the portfolio of risk-averse bank is a greater share of safe assets; risk-averse banks smooth out their profit across states of nature by increasing their riskless bonds holding motivated by

![Figure 1: Impact of uncertainty on expected profit of a bank](image)

This cut-off value is a solution of the entrepreneurial maximization problem. See the Optimal Debt Contract section for details.
precautionary considerations. Specifically, they choose to ensure the relative stability of the profit aiming at avoiding the possible realisation of critically high level of defaults.

4 The general equilibrium model

The theoretical model suggested in this paper is a general equilibrium model based on the costly state verification setup and the financial accelerator mechanism drawn from Bernanke et al. (1999) with risk-averse portfolio optimizing banking sector and non-diversifiable credit risk. I endogenize bank credit spread and analyze the impact of idiosyncratic uncertainty shocks on portfolio reallocation of the banking sector and macroeconomic outcomes. The model economy is populated with households, entrepreneurs, banks, capital goods producers, final good producers, the government and a monetary policy authority. Households consume, supply labour and save via bank deposits. Entrepreneurs produce intermediate goods using capital, financed either internally from the net worth or externally by borrowing funds from the banks. The banking sector allocates deposits raised from households to risky loans and risk-free bonds. Capital goods producers sell capital, that they create, to entrepreneurs. Final good producers resell intermediate goods, produced by entrepreneurs, with a markup. The government issues riskless bonds and buys the final good. The central bank implements monetary policy.

4.1 Banking sector

There is a representative bank in economy owned by households, that provides loans to entrepreneurs and funds its investments by households’ deposits. The bank also buys government bonds that pay risk-free rate. At time $t$ the balance sheet of the bank is:

$$D_t = L_t + B_t,$$  \hspace{1cm} (5)
where current period deposits $D_t$ constitute liabilities, and loans $L_t$ issued for entrepreneurs and risk-free government bonds $B_t$ purchased today comprise asset side of the balance sheet. At the end of period $t$ the bank chooses how to allocate its funds in portfolio of assets, which will generate the return in period $t + 1$: $L_t$ and $B_t$ are chosen at time $t$, i.e. the timing of bank assets corresponds to the time, when the loans are issued and bonds are purchased and not when the payoff occurs. A specialized loan branch within the bank issues loans for entrepreneurs and performs the monitoring of loans. It receives $L_t$ from the parent branch at period $t$ and commits to pay back non-state contingent nominal interest rate $r^e_t$, set by the parent branch at time $t$. The allocation of assets between lending and safe assets holding is decided by the parent branch.

The expected profit of the banking sector $\Pi_{t+1}$ is the difference between its expected income and expenses, where incomes comprise the principal and interest on non-defaulted loans, the assets of defaulted entrepreneurs less the costs of monitoring them and the principal and interest on government bonds, while expenses paid include the principal and interest on deposits issued to households in period $t$:

$$E_t \Pi_{t+1} = E_t[(1 - F(\tilde{\omega}_{t+1}))r^L_t L_t + (1 - \mu)V^d_{t+1} + r^G_t B_t - r^D_t D_t]$$

where $(1 - F(\tilde{\omega}_{t+1}))$ is the ex-post share of non-defaulted borrowers, $r^L_t$ is lending rate for loans issued at time $t$, $(1 - \mu)V^d_{t+1}$ is the value of assets of defaulted firms took over by the bank after paying the monitoring costs $\mu$, $r^D_t D_t$ is bank’s payment for households’ deposits issued at $t$ and $r^G_t B_t$ is the return on government bonds purchased at $t$. Bank profit $\Pi_{t+1}$ is transferred lump-sum to households at the end of period $t + 1$.

I allow for concave preferences of the banking sector. First, there is multiple evidence demonstrating that banks act as risk-averse agents in empirical literature. This is shown, for example, in Ratti (1980), Bhaumik and Piesse (2001), Nishiyama (2007) and Raju (2014). Additionally, some papers demonstrate that the choices of bank managers reveal their risk-
averse type of preferences. Second, the assumption of concavity of bank preferences is a common one in theoretical literature on financial intermediation. Third, the use of this assumption could be justified by the fact that in practice banks should hold a sufficient level of capital to protect themselves from the risk of insolvency. As a result, banks are non-neutral to uncertainty about future payoffs on loans and profitability, instead, they have negative attitude to this uncertainty. Aksoy and Basso (2014) and Danielsson et al. (2011) show that in presence of the Value-at-Risk constraint banks behave like risk-averse agents. In particular, they show that Lagrange muplier associated with the capital constraint enters into the banks’ lending decision problem just like a risk-aversion parameter, and as a result, adopting the risk-averseness assumption is isomorphic to modelling risk-neutrality of risk-constrained banks. The aforementioned considerations support the use of assumption of concavity of bank preferences that I employ here.

I assume constant relative risk aversion (CRRA) type of utility function. The flow of utility is separable across periods and takes the form:

$$u(\Pi_t) = \frac{(\Pi_t)^{1-\kappa}}{1-\kappa}. \quad (7)$$

At time $t$ the management of the parent branch of the bank chooses the share of the portfolio to be invested in risky loans $\alpha_t = \frac{L_t}{D_t}$ to maximize its expected utility, taking into account the balance sheet constraint and taking as given interest rates on deposits, loans and risk-free bonds:

$$\max_{\alpha_t} \mathbb{E}_t \sum_{s=0}^{\infty} S_{t,t+s+1} u(\Pi_{t+s+1}), \quad (8)$$


VaR constraint is a quantile measure of losses distribution, which limits the probability of portfolio losses. It states that losses of bank portfolio should not exceed the value of its net worth $NW_t$, thus, ensuring solvency of the bank with probability $(1-\alpha)$: $VaR_\alpha(Loss_t) \leq NW_t$. Importance of accounting for VaR constraint has been emphasized, for example, by Adrian and Shin (2013).
where $S_{t,t+1} = \beta \frac{C_{t+1}^{\sigma}}{C_t^{\sigma}}$ is the households’ stochastic discount factor. Because both deposit rate and rate on government bonds are riskless, I use a simplifying assumption that $r^G_t = r^D_t$ in each period, and rearrange the expression for bank profit (6) (see Technical appendix C for details) and obtain:

$$
E_t\Pi_{t+1} = E_t\left[ L_t((1 - F(\bar{\omega}_{t+1}))r^L_t - r^G_t) + (1 - \mu)V^d_{t+1} \right].
$$

(9)

The first order condition of the bank problem is (details can be found in the Technical appendix C):

$$
E_t[\Pi^{-\kappa}_{t+1}(r^{L,RA}_t(1 - F(\bar{\omega}_{t+1})) - r^G_t)] = 0,
$$

(10)

where $r^{L,RA}_t$ stands for the lending rate charged by a risk-averse bank.

To emphasize, the decision about the share of risky loans in portfolio $\alpha_t$ is made at time $t$ (accordingly, interest rate on loans $r^L_t$ is set at $t$), while the expected value of future profitability at $t + 1$ is taken into account. In contrast, risk-neutral banks, who are profit maximizers, have linear preferences about their future profit. Hence, the optimality condition assuming banks’ risk-neutrality is:

$$
r^{L,RN}_t E_t[(1 - F(\bar{\omega}_{t+1}))] = r^G_t,
$$

(11)

where $r^{L,RN}_t$ stands for the lending rate charged by a risk-neutral bank.

As seen from (10) and (11), both types of banks charge lending rates, which are set to compensate for the risk of defaults on entrepreneurial loans $F(\bar{\omega}_{t+1})$. The difference is that banks with concave preferences also take into account the expected value of their future profitability, specifically, the marginal utility of future profit. By using the fact that $r^L_t$ becomes known at $t$ and applying the definition of covariance and the linearity property of
expectations to (53), I expand this optimality condition to

$$r_t^{L,RA} \left[ \frac{\text{Cov}(\Pi_{t+1}^\kappa, (1 - F(\bar{\omega}_{t+1}))}{\mathbb{E}_t[\Pi_{t+1}^\kappa]} + \mathbb{E}_t[(1 - F(\bar{\omega}_{t+1}))] \right] = r_t^G. \quad (12)$$

By comparing (11) and (12), one can see that the difference between lending rates $r_t^{L,RN}$ and $r_t^{L,RA}$ depends on the sign of covariance $\text{Cov}(\Pi_{t+1}^\kappa, (1 - F(\bar{\omega}_{t+1}))$ in the left hand side of (12). Given that the share of non-defaulted entrepreneurs is correlated with bank profitability positively, its correlation with the marginal utility of profit is negative due to assumption of bank preferences’ concavity. Hence, $\text{Cov}(\Pi_{t+1}^\kappa, (1 - F(\bar{\omega}_{t+1})) < 0$. The negative covariance in (12) decreases the multiplier of $r_t^{L,RA}$ in the left hand side of (12) comparing to the multiplier of $r_t^{L,RN}$ in the left hand side of (11). Therefore, under equal risk-free rates and under the same expected rate of entrepreneurial defaults, the lending rate charged by risk-averse banks exceeds the one charged by risk-neutral banks $r_t^{L,RA} > r_t^{L,RN}$.

The difference in risk premia charged by risk-neutral and risk-averse banks is explained by the precautionary mechanism: in view of heightened uncertainty risk-averse banks insure themselves from future profitability reduction anticipating increasing defaults. They increase risk premium today to diminish profitability reduction tomorrow that could arise due to elevated uncertainty. Banks’ expectations about their future profitability play a key role in driving the endogenous movements of credit spread in the model. This link between bank spreads and their expected profitability was shown first in Aksoy and Basso (2014) to deliver endogenous movements in term spreads’ variations; they also provide empirical evidence to corroborate it.

To show the effect of default risk and expected profitability on risk premium, I manipulate the optimality condition for risk-averse banks (12) to obtain (see Technical appendix C for details)

$$RP_t^{RA} = \frac{1 - \frac{\text{Cov}(\Pi_{t+1}^\kappa, (1 - F(\bar{\omega}_{t+1}))}{\mathbb{E}_t[\Pi_{t+1}^\kappa]} - \mathbb{E}_t[(1 - F(\bar{\omega}_{t+1}))]}{\frac{\text{Cov}(\Pi_{t+1}^\kappa, (1 - F(\bar{\omega}_{t+1}))}{\mathbb{E}_t[\Pi_{t+1}^\kappa]} + \mathbb{E}_t[(1 - F(\bar{\omega}_{t+1}))]} = r_t^G \cdot (13)$$

20
The risk premium charged by risk-neutral banks is:

\[ RP_{t}^{RN} = \frac{1 - \mathbb{E}_t[(1 - F(\tilde{\omega}_{t+1}))]}{\mathbb{E}_t[(1 - F(\tilde{\omega}_{t+1}))]} r^G_t. \]  \hspace{1cm} (14)

One can see that both risk premia are increasing in the expected risk of default:

\[ \frac{\partial RP_{t}^{RA}}{\partial \mathbb{E}_t[F(\tilde{\omega}_{t+1})]} > 0, \quad \frac{\partial RP_{t}^{RN}}{\partial \mathbb{E}_t[F(\tilde{\omega}_{t+1})]} > 0 \]  \hspace{1cm} (15)

Additionally, risk premium of the risk-averse banking sector \( RP_{t}^{RA} \) features two properties. First, it decreases in expected bank profitability:

\[ \frac{\partial RP_{t}^{RA}}{\partial \mathbb{E}_t[\Pi_{t+1}]} < 0. \]  \hspace{1cm} (16)

Second, it increases in tightness of relationship between profits and the rate of loans repayment:

\[ \frac{\partial RP_{t}^{RA}}{\partial \text{Cov}(\Pi_{t+1}, (1 - F(\tilde{\omega}_{t+1})))} > 0, \]  \hspace{1cm} (17)

i.e. the more risk-averse bank relies on loans’ repayment as a source of profit (in contrast to relying on assets of defaulted entrepreneurs), the higher risk premium it charges. These last two channels constitute the precautionary mechanism that characterizes the choices of risk-averse bank and induces lending rate charged by risk-averse to exceed the rate charged by risk-neutral bank.

Notably, the analytical result of this section is in line with the conclusions of the modern portfolio theory that are derived under different assumptions. First, in the model here risk is not measured by the variance of distribution of returns, as it is done commonly in the modern portfolio approach. Instead, risk is a downside measure - a probability that idiosyncratic productivity of a borrower is lower than the one that allows her to pay back the loan. Second, I don’t employ a quadratic utility function, which is conventionally used.
in the modern portfolio theory, but a constant relative risk aversion type of utility function for banks.

Another consideration worth noting is a distinct nature of the eminent precautionary saving mechanism, which arises in the households’ consumption-saving problem, and the precautionary motive at work suggested here. In particular, the precautionary saving is brought about by higher variance of exogenous shock to household income, which increases the optimal choice of saving, if marginal utility is convex\(^{27}\). Instead here a positive shock to the variance of idiosyncratic entrepreneurial productivity component increases the optimal value of lending rate charged on loans by banks, when their preferences feature concavity.

### 4.2 Households

A representative household chooses consumption \(C_t\), total labour supply \(H^h_t\) and bank deposits \(D_t\) to maximize its expected discounted lifetime utility

\[
\max_{C_t, H^h_t, D_t} \mathbb{E}_t \sum_{k=0}^{\infty} \beta^k [\ln(C_{t+k}) + \xi \ln(1 - H^h_{t+k})]
\]

subject to budget constraint

\[
P_tC_t + D_t \leq W_t H^h_t + \Pi_t + r^P_{t-1} D_{t-1},
\]

where \(P_t\) is aggregate price level, \(W_t\) is nominal wage, \(\Pi_t\) is the profit of banks paid out to households, who own the banks as dividends, and \(r^P_{t-1} D_{t-1}\) is the nominal return on savings deposits issued in \(t - 1\).

The first-order conditions for consumption, labour and deposits are:

\[
C^{-1}_t = \beta \mathbb{E}_t [C^{-1}_{t+1} r^P_{t+1} \pi^{-1}_{t+1}],
\]

\(^{27}\)I follow the formulation of the precautionary saving mechanism along the lines of Rothschild and Stiglitz (1971) and Kimball (1990).
where the first one is Euler equation for real consumption, and the second one is intratemporal condition that determines tradeoff between real consumption and leisure. Stochastic discount factor of a representative household is defined as

\[
SDF_{t,t+1} = \beta \mathbb{E}_t \frac{C_{t+1}^{t-1}}{C_{t-1} \pi_{t+1}}. \tag{22}
\]

### 4.3 Entrepreneurs

Entrepreneurs are producers of the wholesale output \( Y_t \). They live for a finite number of periods and are risk-neutral; in each period a probability of survival \( \gamma \) is constant. Entrepreneurs combine capital \( K_t \), purchased in period \( t - 1 \), with labour \( H_t \) hired in \( t \) to produce wholesale output in period \( t \). Production function is assumed to be constant returns to scale, what enables using it as an aggregate relationship, rather than focusing on production function of each entrepreneur:

\[
Y_t = A_t K_t^\alpha H_t^{1-\alpha}. \tag{23}
\]

where \( A_t \) is an exogenous parameter of aggregate productivity.

Following Bernanke et al. (1999) and Carlstrom and Fuerst (1997), it is assumed that entrepreneurs supply labour in the general labour market to supplement their income. This assumption is made for a technical reason in order for new and bankrupt entrepreneurs to have some net worth that allows them to start operations. Total labour input, used in the wholesale good production, is a composite of labour, supplied by entrepreneurs \( H_t^c \), and the
household labour supply $H^h_t$:\(^{28}\)

$$H_t = (H^e_t)^{1-\Omega}(H^h_t)^{\Omega}.$$ \hspace{1cm} (24)

Net worth of entrepreneurs is composed of entrepreneurial equity $V_t$ - the wealth gained by operating the firm, - and of entrepreneurial wage $w^e_t$. At the end of period $t$ aggregate entrepreneurial net worth $NW_t$ is

$$NW_t = \gamma V_t + w^e_t;$$ \hspace{1cm} (25)

where $\gamma V_t$ is time $t$ equity value of entrepreneurs, who survive. Those entrepreneurs, who don’t survive, consume their equity: $C^e_t = (1-\gamma)V_t$.

Entrepreneurs sell their output to final good producers at the wholesale price $P^W_t$, so the gross markup of retail goods over wholesale goods is $X_t = \frac{P_t}{P^W_t}$ and the marginal product of capital is $\alpha\left(\frac{P^W_t}{P_t}\right)\frac{Y_t}{K_t}$. Undepreciated capital is sold back to capital producers at the end of every period, so ex-post aggregate return of holding a unit of capital from $t$ to $t+1$ is the sum of capital gains from reselling the capital and capital rents ($Q_t$ is the real price of capital):

$$R^k_{t+1} = \alpha\left(\frac{P^W_t}{P_t}\right)\frac{Y_{t+1}}{K_{t+1}} + Q_{t+1}(1-\delta)Q_t.$$ \hspace{1cm} (26)

Demand for household and entrepreneurial labour is obtained by setting the respective real wages $w_t$ and $w^e_t$ to marginal products of labour:

$$w_t = \Omega(1-\alpha)\frac{Y_t P^W_t}{H^h_t P_t},$$ \hspace{1cm} (27)

$$w^e_t = (1-\Omega)(1-\alpha)\frac{Y_t P^W_t}{H^e_t P_t}.$$ \hspace{1cm} (28)

\(^{28}\)It is assumed that entrepreneurs supply their labour inelastically (labour does not enter their utility) with total entrepreneurial labour input being normalized to one. The share of income that goes to entrepreneurial labour is set small enough in calibrations, as a result, there is no significant impact of this production function alteration on my results.
4.4 The optimal debt contract

I modify the conventional structure of the optimal debt contract as suggested by Bernanke et al. (1999) to make the bank lending rate be non-contingent on future shocks. In the original formulation of the financial accelerator framework the risk of entrepreneurial default is idiosyncratic and diversifiable, such that lending rate is contingent on future realisation of productivity shocks. In the model here credit risk is non-diversifiable and lending rate in debt contract is not made contingent on future productivity outcomes. Hence, in my formulation zero-profit condition of banks is replaced with incentive compatibility constraint that allows for non-zero profit outcomes of banks.

Entrepreneurs purchase capital, which is going to be used in production in $t+1$, in the end of $t$. This capital acquisition is financed either through entrepreneurial net worth $NW_t$ or by borrowing from banks $L_t$:

$$Q_tK_{t+1} = L_t + NW_t. \quad (29)$$

There is an idiosyncratic disturbance $\omega^j$ to firm $j$’s return on capital, so that ex-post gross return to capital of firm $j$ is $\omega^j R^k_{t+1}$. $\omega^j$ is i.i.d. across entrepreneurs and time and $F(\omega)$ is a continuous cumulative distribution function over a non-negative support $\mathbb{E}(\omega^j) = 1$ with $\mathbb{P}[\omega \leq x] = F(x)$. Information about a realized return to capital of an entrepreneur is private and bank has to pay a monitoring cost to observe it. The monitoring cost is a constant share $\mu$ of the realized gross return on capital of a firm: $\mu \omega^j R^k_{t+1} Q_t K^j_{t+1}$.

$\bar{\omega}$ is as a cutoff value of idiosyncratic shock such that entrepreneurs, who receive any value lower than this threshold, are unable to repay their loans:

$$\bar{\omega}_{t+1} R^k_{t+1} Q_t K_{t+1} = r^L_t \pi_{t+1}^{-1} L_t. \quad (30)$$

Entrepreneurs with $\omega^j \geq \bar{\omega}$ are able to pay back their loans and receive $\omega^j R^k_{t+1} Q_t K^j_{t+1}$.
$r_t^L \pi_{t+1}^{-1} L_t$. Entrepreneurs with draws $\omega^j < \bar{\omega}$ default and receive nothing, leaving the bank with $(1 - \mu) \omega^j R_{t+1}^k Q_t K_{t+1}^j$. Due to constant returns to scale assumption, $\bar{\omega}$ specifies how the expected aggregate gross return of entrepreneur $R_{t+1}^k Q_t K_{t+1}$ is divided between the bank and the entrepreneur. $\Gamma(\bar{\omega})$ is the expected gross share of entrepreneurial return going to the bank

$$\Gamma(\bar{\omega}) \equiv \int_0^{\bar{\omega}} \omega f(\omega) d\omega + \bar{\omega} \int_{\bar{\omega}}^{\infty} f(\omega) d\omega,$$

while $\mu \Xi(\bar{\omega})$ is the expected monitoring costs$^{30}$:

$$\mu \Xi(\bar{\omega}) \equiv \mu \int_0^{\bar{\omega}} \omega f(\omega) d\omega.$$

Hence, the net fraction of entrepreneurial return going to the bank is $\Gamma(\bar{\omega}) - \mu \Xi(\bar{\omega})$, and the share of return that stays with entrepreneur is $1 - \Gamma(\bar{\omega})$.

The optimal loan contract maximizes the gross return on capital of entrepreneur subject to incentive compatibility constraint of the bank:

$$\max_{K_{t+1}, \bar{\omega}_{t+1}} \mathbb{E}_t [(1 - \Gamma(\bar{\omega}_{t+1})) R_{t+1}^k Q_t K_{t+1}]$$

subject to

$$\mathbb{E}_t [(\Gamma(\bar{\omega}_{t+1}) - \mu \Xi(\bar{\omega}_{t+1})) R_{t+1}^k Q_t K_{t+1}] = r_t^c L_t / \mathbb{E}_t \pi_{t+1},$$

where the incentive compatibility constraint formulates that the amount of real receipts of the bank loan branch from issuing loans at time $t$ (left hand side) is equal to repayments to the bank parent branch under the rate of return that ensures the expected utility maximization according to (8) and where $r_t^c$ is defined as:

$$\frac{r_t^c L_t}{\mathbb{E}_t \pi_{t+1}} = \mathbb{E}_t [(1 - F(\bar{\omega}_{t+1})) r_t^L L_t + (1 - \mu) V_{t+1}^d].$$

\footnote{$\Gamma'(\bar{\omega}) = 1 - F(\bar{\omega})$ and $\mu \Xi'(\bar{\omega}) \equiv \mu \bar{\omega} f(\bar{\omega})$.}
Hence, the lending rate \( r^L_t \), the amount of loans \( L_t \) issued and the interest rate \( r^e_t \) that the loan branch must pay back to the parent branch, satisfy the incentive compatibility constraint of the loan branch and maximize the entrepreneurs’ expected return at the moment, when loans mature.

Solving the optimal loan contract problem yields the following optimality condition:

\[
\frac{R^k_{t+1}}{r^e_t \pi^t_{t+1}} = \frac{\Gamma'(\bar{\omega}_{t+1})}{(1 - \Gamma(\bar{\omega}_{t+1}))(\Gamma'(\bar{\omega}_{t+1}) - \mu \Xi'((\bar{\omega}_{t+1}) + \Gamma'(\bar{\omega}_{t+1}))(\Gamma(\bar{\omega}_{t+1}) - \mu \Xi((\bar{\omega}_{t+1}))},
\]  

which, together with the incentive compatibility constraint, pins down the optimal choice of capital \( K^t_{t+1} \) and of the threshold value of idiosyncratic shock to capital return \( \bar{\omega}_{t+1} \). In its turn, \( \bar{\omega}_{t+1} \) and the variance of idiosyncratic shock, being time-varying in the case of idiosyncratic uncertainty, specify the rate of default on loans, which affects changes in risk premium that the banking sector charges. These optimality conditions introduce the financial accelerator mechanism to the model, such that external finance premium increases in the leverage ratio of entrepreneurs.

### 4.5 Idiosyncratic uncertainty

To introduce idiosyncratic uncertainty, I assume that variance of entrepreneurial idiosyncratic productivity shocks \( \omega \) is time-varying\(^{31}\). \( \omega \) is distributed log-normally - \( \omega \sim \log N(1, \sigma^2_\omega) \), hence, the log of \( \omega \) is normally distributed. The mean of \( \omega \) is fixed to one, and the variance of the log-normal distribution is defined as \( (\sigma^{Id}_t)^2 = \log(1 + \sigma^2_\omega) \). This variance \( \sigma^{Id}_t \) is assumed to be time-varying and is affected by shock, which I refer to as the source of idiosyncratic uncertainty\(^{32}\). The log-deviation of \( \sigma^{Id}_t \) from its steady state is modeled as:

\[
\log\left(\frac{\sigma^{Id}_t}{\sigma^{Id}}\right) = \rho_\sigma \log\left(\frac{\sigma^{Id}_t}{\sigma^{Id}}\right) + \sigma_\sigma \epsilon^\sigma_t.
\]  

\(^{31}\)This formulation of idiosyncratic productivity draws from Christiano et al. (2010) and Dorofeenko et al. (2008).

\(^{32}\)Empirical evidence that corroborates dispersion of idiosyncratic entrepreneurial productivity as a source of uncertainty is provided, for example, in Bloom et. al (2012) and Bloom (2014).
\( \sigma^\sigma \) is the standard deviation of innovations to \( \sigma_i^{ld} \) and \( \epsilon^\sigma \) follows standard normal distribution. Positive innovations to idiosyncratic uncertainty shocks increase the dispersion of entrepreneurial return to capital \( \sigma_i^{ld} \). The illustration of this increase is given in Figure 2: higher variance of idiosyncratic shock to entrepreneurs’ productivity changes the shape of the distribution shifting the mass of distribution to the left leaving when the mean of the distribution unaffected. The intuition behind idiosyncratic shock is the following: the higher dispersion of productivity implies higher probability of defaults on loans, and given costly state verification, a higher risk premium and lending rates, what leads to lower entrepreneurial demand for capital.

![Figure 2: The effect of an idiosyncratic uncertainty shock.](image)

### 4.6 Capital goods producers

A competitive sector of capital goods producers buys final goods from retailers as investment goods and existing undepreciated capital \((1 - \delta)K_t\) from entrepreneurs and combines

28
them to create capital for the next period $K_{t+1}$, which is then sold to entrepreneurs\footnote{I assume that capital producers rent the capital stock from entrepreneurs and use it to produce new capital; since this rent and subsequent return of capital happen within one period, the rental price is supposed to be zero.}:

$$K_{t+1} = I_t + (1 - \delta)K_t.$$  \hspace{1cm} (38)

Capital adjustment costs are introduced to allow for the price of capital to vary, following Kiyotaki and Moore (1997). Drawing from Christensen and Dib (2008), I adopt a quadratic capital adjustment costs function, specified as $\frac{\chi}{2}(\frac{I_t}{K_t} - \delta)^2K_t$. The optimization problem of the capital goods producers is to choose the value of investment $I_t$ that maximizes their profits:

$$\max_{I_t} [Q_tI_t - I_t - \frac{\chi}{2}(\frac{I_t}{K_t} - \delta)^2K_t].$$  \hspace{1cm} (39)

The first-order condition is

$$Q_t = 1 + \chi(\frac{I_t}{K_t} - \delta).$$  \hspace{1cm} (40)

This standard condition of Tobin’s Q relates the real price of capital to the marginal adjustment costs. Due to capital adjustment costs, the response of investment to various shocks slows down, which it its turn has an impact on the real price of capital. Variations in the price of capital, absent if there are no capital adjustment costs, contribute to volatility of net worth of entrepreneurs, what has a direct impact on workings of the financial accelerator mechanism. Hence, (40) represents the supply of capital and the demand for capital from entrepreneurs is formulated by (36) together with (34).

4.7 Retailers

The sector of retailers is introduced into the model to incorporate price rigidity. Specifically, I adopt Calvo price setting framework. There is a unit mass of monopolistically competitive retailers. They purchase wholesale goods from entrepreneurs at the nominal
wholesale price $P_t^w$ and resell them at their own retail price. Let $Y_t(i)$ denote the quantity of output resold by retailer $i$ and let $P_t(i)$ denote the nominal price the retailer receives. Total final goods are a composite of individual retail goods

$$Y_t = \left[ \int_0^1 Y_t(i)^{(\eta-1)/\eta} di \right]^\eta, \quad (41)$$

and the corresponding aggregate price index is given by

$$P_t = \left[ \int_0^1 Y_t(i)^{1-\eta} di \right]^{1/\eta}, \quad (42)$$

where $\eta > 1$ is elasticity of substitution in the retail market.

The standard monopolistic competition demand curve for individual retailers is

$$Y_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\eta} Y_t. \quad (43)$$

The retailer chooses its sale price $P_t(i)$ to maximize its profits taking as given the aggregate demand, price level and wholesale good price.

To incorporate price stickiness, I introduce Calvo pricing such that retailers are free to change their price each period with probability $1 - \theta$. Let $P_t^*(i)$ denote the price chosen by retailers who are able to change their price. The aggregate price evolves according to (see Technical appendix C for details)

$$P_t = \left[ \theta P_{t-1}^{1-\eta} + (1 - \theta)(P_t^*)^{1-\eta} \right]^{1/\eta}. \quad (44)$$

### 4.8 Monetary authority

I assume that monetary policy instrument is the nominal risk-free interest rate $r_t^G$ that the central bank is able to set following the standard Taylor rule with interest rate smoothing,
responding to deviations of inflation and output from their steady state values:

\[
\log\left(\frac{r_G}{\bar{r}_G}\right) = \rho_r \log\left(\frac{r_{G-1}}{r_G}\right) + \psi_x \log\left(\frac{\pi_t}{\pi}\right) + \psi_y \log\left(\frac{y_t}{y}\right) - \epsilon^m_t, \tag{45}
\]

where \(\pi\) and \(y\) are the steady state levels of inflation and output and \(\epsilon^m_t\) in an i.i.d. white noise process denoting monetary policy shock. It is assumed that the supply of risk-free government bonds is perfectly elastic, such that any volume of demand for bonds from the banking sector is met.

4.9 Market clearing

The final good market clearing condition states that total output equals the sum of households’ and entrepreneurial consumption, government spending, total resources used to create new capital goods and the monitoring costs of banks:

\[
Y_t = C_t + C_e^e + G_t + I_t + \mu V^d_t. \tag{46}
\]

4.10 Calibration

I calibrate the model at a quarterly frequency, setting fairly standard values for the DSGE model parameters. The discount rate \(\beta\) is set to 0.99 to target an annualized average risk-free interest rate of 3.8%, what is the average rate of three-month Treasury bills in 1983Q1-2016Q3. The capital share \(\alpha\) is set to 1/3, the share of entrepreneurial labour income \((1 - \Omega)\) is set to 0.01, such that the labour share of households \((1 - \alpha)(1 - \Omega)\) is 0.66. The parameter of leisure utility, \(\xi\) (1.87) is set to target the time that households spend working to 1/3. The elasticity of substitution between final goods \(\eta\) is set to 5, and \(\theta\), the Calvo parameter, is set to 0.75; these values imply that firms are able to adjust their prices on average once in four quarters and that a steady state value of markup of 25%.

I choose the quadratic adjustment costs functional form of the capital goods production
function drawing from Christensen and Dib (2008), because quadratic adjustment costs, being a type of smooth "convex" adjustment costs that increase in the squared rate of investment, do not generate real options effects of uncertainty on economy. I want to analyze banks’ precautionary mechanism of uncertainty as the main channel of influence of uncertainty on variables, that is why I aim at leaving aside other potential channels of impact. I set the marginal adjustment cost parameter $\chi$ equal to 0.5882 - the parameter estimate from Christensen and Dib (2008). Depreciation rate $\delta$ is set to its standard value of 0.025. The steady state share of government expenditures in total output is taken to be 0.2.

The values for monitoring costs $\mu$, for the steady state of the variance of idiosyncratic productivity shock $\sigma^{Id}$ and for the survival rate of entrepreneurs $\gamma$ are set jointly to match the data on entrepreneurial defaults (the average number of non-performing loans in 1988Q1-2016Q1 equal to 2.23%), the leverage ratio of entrepreneurs of 1.84 (measured as the average value of total assets to net worth of nonfinancial corporate business in 1983Q1-2016Q2) and the real rate of return on capital expenditures of 15.4% in the steady state (estimate of Poterba (1998)). In that way, the fraction of realized payoffs lost in bankruptcy, $\mu$, is set to 0.21 (a number inbetween of that from Carlstrom and Fuerst (1997) - 0.25, and Bernanke et al. (1999) - 0.12), the survival rate of entrepreneurs $\gamma$ is set to 0.95 (the value in Carlstrom and Fuerst (1997) is 0.947, and the one in Bernanke et al. (1999) is 0.97), while steady state standard deviation of idiosyncratic productivity shock is set to 0.364 (somewhat higher than in Carlstrom and Fuerst (1997) - 0.207, and in Bernanke et al. (1999) - 0.28). The steady state annualized value of lending rate $r^L$ is set to 6.86% to match the average of prime loan rate on historical data from 1983Q1 to 2016Q3. I draw the parameters of the Taylor rule followed by monetary authority from the estimated values in Christiano et al. (2010): $\psi_\pi$ is set to 2.39, $\psi_y$ - to 0.36 and $\rho_r$ - to 0.85. From Christiano et al. (2010) I also use the estimated parameter values for autoregressive process of the government expenditure and variance of idiosyncratic productivity component of entrepreneurs$^{34}$: $\rho_g = 0.938, \sigma^g = 0.021,$

$^{34}$Christiano et al. (2010) refer to idiosyncratic uncertainty shock as to 'risk shock'.
\[ \rho_\sigma = 0.79 \text{ and } \sigma_\sigma = 0.05. \]

4.11 Solution method

The traditional linear approximation of the model solution implies that uncertainty shocks do not play a role due to certainty equivalence. For the variability of the second moment to enter the decision rules of economic agents, a third-order approximation is used. As discussed in Fernandez-Villaverde et al. (2010), the third-order Taylor expansion allows to simulate and to evaluate the effect of an uncertainty shock. I use the perturbation method to solve the model.

Dynare 4.4 is used to compute the third-order approximation around the non-stochastic steady state. As Fernandez-Villaverde et al. (2010) note, the third-order approximation moves the simulated paths of states and controls away from their steady state values, because the expected value of the variables depends on the variance of shocks\(^{35}\). Hence, I compute impulse responses as deviations from the mean of ergodic distributions of the data generated by the model, rather than deviations from the steady states, as this allows to take into account the second-order effects in a more comprehensive way. This approach is proposed by Fernandez-Villaverde et al. (2010) and is used in other studies investigating the effects of uncertainty shocks (see, for example, Born and Pfeifer (2011) and Cesa-Bianchi and Fernandez-Corugedo (2014)). The details of the computation of impulse responses functions are given in Appendix D.

I employ the pruning procedure proposed by Kim et al. (2008) to deal with the problem of explosive behaviour of the simulated time series when high-order perturbations are used to approximate the solution of the model.

\(^{35}\)As has been shown by Schmitt-Grohe and Uribe (2004), the expected value of any variable in high order approximations differs from its deterministic steady-state value.
5 The effect of uncertainty shocks

In this section I analyse the effect of an exogenous increase in idiosyncratic uncertainty on the model economy, and specifically, on banks’ portfolio allocation. I consider two versions of the model - with risk-averse banking sector and with risk-neutral banks - to compare the features of portfolio reallocation following a positive uncertainty shock for two specifications of banking sector preferences. I find that, first, idiosyncratic uncertainty shock makes a significant impact of macroeconomic and credit variables; second, that banks’ attitude to uncertainty makes a difference not only for portfolio allocation, but also for the dynamics of aggregate investment and output, and third, that financial accelerator mechanism works to amplify the effect of uncertainty shock. I analyse impulse responses to one standard deviation increase of idiosyncratic uncertainty innovation ($\epsilon^g_t$). Figure 3 plots impulse responses of the model variables to an idiosyncratic uncertainty shock.

Figure 3: Impulse response functions to idiosyncratic uncertainty shock

![Graph showing impulse responses](image-url)
When a positive idiosyncratic uncertainty shock hits, it means that the dispersion of idiosyncratic productivity of entrepreneurs increases. This implies that some entrepreneurs earn higher returns, while others bear greater losses. Hence, a greater fraction of entrepreneurs are further on the left tail of the productivity probability distribution function, such that more entrepreneurs are unable to repay their loans. As a result, the default rate on loans goes up. This can be seen on the figure 3, Defaults section. Defaults increase by 15% for both risk-neutral and risk-averse banks responding to a spike of idiosyncratic uncertainty. Due to costly state verification framework adopted in this model, higher defaults imply that banks’ expected costs associated with bankruptcies go up, so banks of both types of specifications increase their lending rates to compensate for it. Lending rate of the risk-neutral banking sector goes up by 0.12 annual percent, while the one of the risk-averse bank increases it by 0.38 annual percent, and 0.26 pp of difference between them is explained by precautionary mechanism. The higher cost of external debt induces entrepreneurs to reduce their demand for capital: firms borrow less and reduce their investment expenditures. Specifically, loans issuance goes down by 1% under risk-averse banks specification, and the issuance of loans of risk-neutral banks is 0.8%; 0.2% of the loans reduction is explained by the banks’ precautionary motive. Notably, banks reallocate their portfolios following a positive idiosyncratic uncertainty shock: the share of risky loans goes down by 0.8% in case of risk-averse banking sector, and by 0.48% in case of risk-neutral banks with 0.32% of reduction of the risky loans share being explained by the precautionary motive of banks. The share of riskless bonds increases in portfolios of both types of banks: risk-neutral banks increase the share of risk-free bonds by 0.43%, and risk-averse banks increase it by 0.64%. Again, there’s a role for the banks’ precautionary motive, as there is a difference between the sizes of the increase. Crucially, the deleveraging of entrepreneurs is observed, and under the risk-averse type of banks preferences specification the reduction of the leverage is greater, than under the risk-neutral type.

There are two counteracting forces that have a potential to drive the entrepreneurial de-
mand for capital. The partial equilibrium effect implies that the increased cost of borrowing induces entrepreneurs to reduce their optimal choice of leverage, such that capital demand goes down. The general equilibrium effect takes into account that as a result of capital reduction, following a spike in uncertainty, the rental rate of capital goes up. Hence, entrepreneurs have an incentive to increase their leverage to benefit from this high value of capital return, which would give rise to an upward pressure on the demand for capital. The model simulations show that the partial equilibrium effect dominates the general equilibrium one: the forces that bring entrepreneurs’ demand for capital down due to the increase of the interest rate on loans outweigh the general equilibrium incentives to build up leverage because the return on capital has gone up. As a result, entrepreneurial leverage and investment fall.

The reduction of the demand for capital means that the price of capital goes down; the resulting net worth of entrepreneurs follows the dynamics of the price of capital closely: in both cases the negative impact of uncertainty with risk-averse banks exceeds the one with risk-neutral banks, meaning that the precautionary motive of banks is at work, however, its impact is rather small comparing to the impact on credit variables. Importantly, the rebound of the price of capital and of net worth occurs after 4 quarters from the impact of uncertainty shock. This rebound means that the demand for capital starts reviving after 3-4 quarters from the uncertainty shock impact. However, banks continue keeping their lending rates high - lending rates are still increased after 3-4 quarters from the shock impact, implying that the supply of credit stays depressed longer than the demand for credit following idiosyncratic uncertainty shock. Notably, the deleveraging process of the entrepreneurial sector continues to occur beyond 4 quarters after the uncertainty shock hits.

The reduction of investment after the positive idiosyncratic uncertainty shock is sizeable: it drops by 11% in the economy with risk-averse banks, and by 7.6% in the economy with risk-neutral banks. 3.4% - a substantial difference - can be attributed to the precautionary motive of banks. Notably, the dynamics of investment following uncertainty shock - the drop and the following rebound in 4 quarters, - is in line with the result in Bloom (2009). The
decrease of aggregate output follows investment: it goes down by 0.8% under risk-averse banking sector and by 0.6% under risk-neutral one, with these results being roughly in line with the existing evidence on macroeconomic effects of uncertainty shocks.

The presence of financial accelerator mechanism amplifies the effect of uncertainty shock. In particular, due to reduced capital demand, the price of capital falls, making entrepreneurial net worth go down, what reduces the demand for capital further. Lower investment forces consumption to go up on impact of uncertainty shock. A similar result is documented by Bloom et al. (2012) and by Christiano et al. (2014). In the latter, they draw an analogy of the effect of uncertainty shock to the increase of tax rate on capital return, which hinders saving and investment, while boosting consumption, in a way similar to a spike in the tax rate. However, the increase of consumption following uncertainty shock is clearly inconsistent with data. A possible way to fix the positive response of consumption to uncertainty shock in the model might be introducing the shock to the first moment - to the level of entrepreneurial productivity - simultaneously with the uncertainty shock. The reason to do that is that in practice the cases of elevated uncertainty often go along with the negative first moment shock, as suggested by Bloom (2014)\textsuperscript{36}. Monetary authority responds to depressed output by easing monetary conditions: nominal interest rate is reduced, what together with heightened inflation implies decreasing real saving rate, such that households are encouraged to consume more and work less. After 2 quarters the negative wealth effect builds up, weaker capital demand acts to reduce output further, so the consumption also goes down. It features the additional 0.05% reduction after 6 quarters from the shock impact under the risk-averse bank specification comparing to the risk-neutral case.

It is important to notice, that this model is aimed to study the workings of the risk premia and risk aversion transmission mechanism of uncertainty with a particular focus on the precautionary motive of the banking sector. I reduced the amount of transmission mechanisms via which uncertainty might have an impact on variables in the suggested model

\textsuperscript{36}I refer to Figure 5 of Bloom (2014) here.
deliberately. One has to consider that this analysis is not supposed to be a comprehensive study of the effects of uncertainty shocks and should take it into account while making interpretations of the results obtained here.

To summarize, the suggested model produces significant negative effects of idiosyncratic uncertainty shock on macroeconomic variables - especially, on investment. Considering two versions of the model - with risk-neutral and risk-averse banks - allows to see that precautionary mechanism explains a substantial additional share of dynamics of credit variables: of the lending rate increase, of the decrease of the volume of loans issued, of the increase of the share of portfolio invested in risk-free bonds and of the leverage reduction. Financial accelerator plays a role and amplifies the impact of uncertainty shock.

6 Conclusion

In this paper I propose a DSGE model with a portfolio-optimizing banking sector to account for the empirical evidence of uncertainty shocks inducing asset portfolio reallocation of commercial banks. I model uncertainty as a time-varying cross-sectional dispersion of entrepreneurial productivity. This modelling conforms with the evidence of portfolio reallocation that I obtain as a response of banks to uncertainty shock, when microeconomic measures of uncertainty is used, in particular, cross-section standard deviation of firms profit growth and cross-firm stock return variation. I modify the standard financial accelerator framework of Bernanke, Gertler and Gilchrist (1999) to allow bank lending rates be non-contingent on aggregate shocks. Risk-averse banks face non-diversifiable credit risk and, by invoking a precautionary mechanism, increase risk premium following a spike in uncertainty by more than risk-neutral banks do. My result is in line with the conclusions of the modern portfolio theory, but the approach used here is advantageous comparing to that of the modern portfolio theory in several aspects. First, the risk of investment is not measured by variance of returns, but is a downside measure, what is a more adequate approach for
the case of default risk. Second, I don’t employ a quadratic utility function, but assume a more plausible representation of preferences characterized by constant relative risk aversion. Third, I adopt the general equilibrium approach, such that default risk is endogenous and also time-varying, as well as risk-free rate not being known in advance and being determined endogenously responding to movements in output gap and inflation. The suggested model allows to replicate the effects of uncertainty on the dynamics of banks’ balance sheet items, observed in the data.
References


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Appendix A  Credit market conditions

Figure A.1: Bank credit growth

Note. The plotted numbers are year over year real quarterly credit growth figures. Source: BIS.

Figure A.2: Bank credit growth, selected countries

Note. The plotted numbers are year over year real quarterly credit growth figures. Source: BIS.
Figure A.3: Uncertainty and banks’ business loans growth

Note. Shaded areas are NBER recessions. Sources: Federal Reserve Board H.8 Release, NBER.

Figure A.4: Uncertainty and banks’ safe assets growth

Note. Shaded areas are NBER recessions. Sources: Federal Reserve Board H.8 Release, NBER.
Appendix B  VAR evidence on effects of uncertainty shocks

Figure B.1: Measures of macroeconomic uncertainty
Figure B.2: Impulse responses to an uncertainty shock of a model with commercial and industrial loans: 1985-2015.

Note. The responses of the model’s variables to one-standard deviation shock to uncertainty. The solid black lines represent median responses, the solid blue lines represent the 90% bias-corrected bootstrap confidence bands, which are calculated as in Kilian (1998). The units of the horizontal axes are quarters. The units of the vertical axis are percents. According to Akaike information criterion, the VAR order includes four lags.
Figure B.3: Impulse responses to an uncertainty shock of a model with safe assets: 1985-2015.

Note. The responses of the model’s variables to one-standard deviation shock to uncertainty. Safe assets include cash and Treasury and agency securities. The solid black lines represent median responses, the solid blue lines represent the 90% bias-corrected bootstrap confidence bands, which are calculated as in Kilian (1998). The units of the horizontal axes are quarters. The units of the vertical axis are percents. According to Akaike information criterion, the VAR order includes four lags.
Figure B.4: Impulse responses to an uncertainty shock of a model with the share of commercial and industrial loans: 1985-2015.

Note. The responses of the model’s variables to one-standard deviation shock to uncertainty. The share of commercial and industrial loans in banks’ portfolios is calculated as the percentage in the total banks’ assets. The solid black lines represent median responses, the solid blue lines represent the 90% bias-corrected bootstrap confidence bands, which are calculated as in Kilian (1998). The units of the horizontal axes are quarters. The units of the vertical axis are percents. According to Akaike information criterion, the VAR order includes four lags.
Figure B.5: Impulse responses to an uncertainty shock of a model with the share of safe assets: 1985-2015.

Note. The responses of the model’s variables to one-standard deviation shock to uncertainty. The share of commercial and industrial loans in banks’ portfolios is calculated as the percentage in the total banks’ assets. The solid black lines represent median responses, the solid blue lines represent the 90% bias-corrected bootstrap confidence bands, which are calculated as in Kilian (1998). The units of the horizontal axes are quarters. The units of the vertical axis are percents. According to Akaike information criterion, the VAR order includes four lags.
Figure B.6: Impulse responses to an uncertainty shock of a model with the share of total loans: 1985-2015.

Note. The responses of the model’s variables to one-standard deviation shock to uncertainty. The share of total loans in banks’ portfolios is calculated as the percentage of the sum of commercial and industrial loans, real estate loans and consumer loans in the total banks’ assets. The solid black lines represent median responses, the solid blue lines represent the 90% bias-corrected bootstrap confidence bands, which are calculated as in Kilian (1998). The units of the horizontal axes are quarters. The units of the vertical axis are percents. According to Akaike information criterion, the VAR order includes four lags.
Appendix C  Technical appendix

Appendix C.1 Banking sector

At time $t$ the balance sheet of the bank is:

$$D_t = L_t + B_t. \quad (47)$$

The profit of the bank is the difference between its income and expenses:

$$\mathbb{E}_t \Pi_{t+1} = (1 - F(\bar{\omega}_{t+1})) r_t^L L_t + (1 - \mu) V_{t+1}^d + r_t^G B_t - r_t^D D_t. \quad (48)$$

Using the simplifying assumption that $r_t^G = r_t^D$ in each period $t$, I rearrange the expression for bank profit $(\mathbb{E}_t \Pi_{t+1} = (1 - F(\bar{\omega}_{t+1})) r_t^L L_t + (1 - \mu) V_{t+1}^d + r_t^G B_t - r_t^D D_t)$, between the third and the forth lines below I use the balance sheet identity holding at time $t$:

$$\Pi_{t+1} = (1 - F(\bar{\omega}_{t+1})) r_t^L L_t + (1 - \mu) V_{t+1}^d + r_t^G B_t - r_t^D D_t =$$

$$= (1 - F(\bar{\omega}_{t+1})) r_t^L L_t + r_t^G B_t - r_t^G D_t + (1 - \mu) V_{t+1}^d =$$

$$= (1 - F(\bar{\omega}_{t+1})) r_t^L L_t + r_t^G (B_t - D_t) + (1 - \mu) V_{t+1}^d =$$

$$= L_t (1 - F(\bar{\omega}_{t+1}))(r_t^L - r_t^G) + (1 - \mu) V_{t+1}^d. \quad (51)$$

The Lagrangian of the problem is given by

$$\mathcal{L}_t = \mathbb{E}_t \sum_{s=0}^{\infty} S_{t,t+s+1} u(\Pi_{t+s+1}). \quad (49)$$

The first order condition is:

$$\frac{\partial \mathcal{L}_t}{\partial \alpha_t} = \mathbb{E}_t [u'(\Pi_{t+1}) \frac{\partial \Pi_{t+1}}{\partial \alpha_t} \frac{\partial L_t}{\partial \alpha_t}] = 0, \quad (50)$$

implying that

$$\mathbb{E}_t [\Pi_{t+1}^{-\kappa}(r_t^L (1 - F(\bar{\omega}_{t+1})) - r_t^G) D_t] = 0, \quad (51)$$
where I used the fact that \( \alpha_t = \frac{D_t}{L_t} \). Because \( D_t \) is chosen, and therefore, known at time \( t \) and is a non-zero value, (51) means that the first order condition is actually

\[
E_t[\Pi_{t+1}^{-\kappa}(r_t^L (1 - F(\bar{\omega}_{t+1})) - r_t^G)] = 0,
\]

or

\[
r_t^L E_t[\Pi_{t+1}^{-\kappa}(1 - F(\bar{\omega}_{t+1}))] = r_t^G E_t[\Pi_{t+1}^{-\kappa}].
\]

By using the fact that \( r_t^L \) becomes known at \( t \) and applying the definition of covariance and the linearity property of expectations to (53), I expand this optimality condition to

\[
r_t^L, R_A^t [\text{Cov}(\Pi_{t+1}^{-\kappa}, (1 - F(\bar{\omega}_{t+1}))) + E_t[\Pi_{t+1}^{-\kappa}]E_t[(1 - F(\bar{\omega}_{t+1}))]] = r_t^G E_t[\Pi_{t+1}^{-\kappa}]
\]

hence,

\[
r_t^L, R_A^t [\text{Cov}(\Pi_{t+1}^{-\kappa}, (1 - F(\bar{\omega}_{t+1}))) + E_t[\Pi_{t+1}^{-\kappa}]E_t[(1 - F(\bar{\omega}_{t+1}))]] = r_t^G.
\]

To derive expression for the risk premium, I perform manipulations with (53):

\[
\frac{r_t^L}{r_t^G} = \frac{E_t[\Pi_{t+1}^{-\kappa}]}{E_t[\Pi_{t+1}^{-\kappa} (1 - F(\bar{\omega}_{t+1}))]},
\]

then

\[
\frac{r_t^L - r_t^G}{r_t^G} = \frac{E_t[\Pi_{t+1}^{-\kappa}] - E_t[\Pi_{t+1}^{-\kappa} (1 - F(\bar{\omega}_{t+1}))]}{E_t[\Pi_{t+1}^{-\kappa} (1 - F(\bar{\omega}_{t+1}))]
\]

\[
= \frac{E_t[\Pi_{t+1}^{-\kappa}] - \text{Cov}(\Pi_{t+1}^{-\kappa}, (1 - F(\bar{\omega}_{t+1}))) - E_t[\Pi_{t+1}^{-\kappa}]E_t[(1 - F(\bar{\omega}_{t+1}))]}{\text{Cov}(\Pi_{t+1}^{-\kappa}, (1 - F(\bar{\omega}_{t+1}))) + E_t[\Pi_{t+1}^{-\kappa}]E_t[(1 - F(\bar{\omega}_{t+1}))]
\]

\[
= \frac{1 - \text{Cov}(\Pi_{t+1}^{-\kappa}, (1 - F(\bar{\omega}_{t+1}))) - E_t[(1 - F(\bar{\omega}_{t+1}))]}{E_t[\Pi_{t+1}^{-\kappa} (1 - F(\bar{\omega}_{t+1}))] + E_t[(1 - F(\bar{\omega}_{t+1}))].
\]

**Appendix C.2  Households**

Households maximize their expected discounted lifetime utility

\[
\max_{C_t, H_t, D_t} E_t \sum_{k=0}^{\infty} \beta^k [\ln(C_{t+k}) + \xi \ln(1 - H_{t+k}^h)]
\]

subject to budget constraint

\[
P_t C_t + D_t \leq W_t H_t^h + \Pi_t + r_{t-1}^D D_{t-1}.
\]
The first-order conditions for consumption, labour and deposits are:

\[ \beta^k C_{t+k}^{-1} P_{t+k}^{-1} = \lambda_{t+k} \]  

(59)

\[ \beta^k \xi \frac{1}{1 - H_{t+k}^k} = \lambda_{t+k} W_{t+k} \]  

(60)

\[ \lambda_{t+k} = E_{t+k} [\lambda_{t+k+1} D_{t+k}] \]  

(61)

where \( \lambda_t \) is Lagrange multiplier. These first-order conditions could be written as

\[ C_{t+1} = \beta E_{t} [C_{t+1} R_{t+1} \sigma_{t+1}^{-1}] \]  

(62)

\[ \xi \frac{1}{1 - H_t^k} = C_t^{-1} W_t / P_t. \]  

(63)

### Appendix C.3 Optimal debt contract

\( \omega^j \) is assumed to be i.i.d. across entrepreneurs and time and follow log-normal distribution: \( \omega \sim log \mathcal{N}(1, \sigma^2_\omega) \). Thus, \( \log(\omega_{t+1}) \sim N(-0.5(\sigma_{t}^{Id})^2, (\sigma_{t}^{Id})^2) \). Given this assumption, Bernanke et al. (1999) formulate the following distributions for the debt contract:

\[ z_{t+1} = \ln(\omega_{t+1}) + 0.5(\sigma_{t}^{Id})^2 \]  

(64)

\[ \Xi(\omega_{t+1}) = \Phi^N(z_{t+1} - \sigma_t^{Id}) \]  

(65)

\[ \Gamma(\omega_{t+1}) = \Phi^N(z_{t+1} - \sigma_t^{Id}) + \omega_{t+1}(1 - \Phi^N(z_{t+1})) \]  

(66)

\[ \Xi'(\omega_{t+1}) = \frac{1}{\sigma_{t}^{Id} \sqrt{2\pi}} \exp(-\frac{(\ln(\omega_{t+1}) + 0.5(\sigma_{t}^{Id})^2)^2}{2(\sigma_{t}^{Id})^2}) \]  

(67)

\[ \Gamma'(\omega_{t+1}) = 1 - \Phi^N(z_{t+1}) \]  

(68)

where \( \Phi^N(\cdot) \) is the standard normal c.d.f.

### Appendix C.4 Retailers

Retailers choose \( P_t^*(i) \) to maximize expected profits given by

\[ \mathbb{E}_t \left[ \sum_{k=0}^{\infty} \theta^k SD F_{t+k} \left( \frac{P_t^* - P_{t+k}^w}{P_{t+k}} \right) Y_{t+k}^*(i) \right], \]  

(69)
where $Y^*_{t+k}(i)$ is the demand in period $t+k$ given price $P^*_t$. The first-order conditions from maximizing expected profits can be written as

$$P^*_t = \frac{\eta}{\eta - 1} \frac{\mathbb{E}_t \sum_{k=0}^{\infty} \theta^k SD_{t+k}X_{t+k}^{-1}Y_{t+k}P^\eta_{t+k}}{E_t \sum_{k=0}^{\infty} \theta^k SD_{t+k}Y_{t+k}P^\eta_{t+k}}.$$ 

(70)

where $X_t$ is the optimal price markup such that $P_t = X_t P^w$.

To implement Calvo pricing equations without log-linearization, I summarize the optimal pricing equation with two recursive equations linked by the optimal pricing equation (70). The numerator $n_t$ and denominator $d_t$ in equation (70) can be written recursively as

$$n_t = P^\eta_t Y_t X_t^{-1} + \theta \mathbb{E}_t [SDF_{t+1} n_{t+1}]$$

(71)

$$d_t = P^{\eta-1}_t Y_t + \theta \mathbb{E}_t [SDF_{t+1} d_{t+1}].$$

(72)

Thus, the optimal pricing rule can be written as

$$P^*_t = \frac{\eta}{\eta - 1} \frac{n_t}{d_t}$$

(73)

Let $\hat{P}_t = P^*_t / P_t$ and $F_{1,t} = P_t^{-\eta} n_t$. From equation (71) $F_{1,t}$ is written recursively as

$$F_{1,t} = Y_t X_t^{-1} + \theta \mathbb{E}_t [SDF_{t+1} \pi^\eta_{t+1} F_{1,t+1}].$$

(74)

Substituting $F_{1,t}$ into the rewritten optimal pricing rule (73) yields

$$P^*_t P^{-\eta}_t = \frac{\eta}{\eta - 1} \frac{P_t^{-\eta} n_t}{d_t}.$$ 

(75)

Let $F_{2,t} = P^*_t P^{-\eta} d_t = \hat{P}_t P_t^{1-\eta} d_t$. From equation (72) $F_{2,t}$ is written recursively as

$$F_{2,t} = Y_t \hat{P}_t + \theta \mathbb{E}_t [SDF_{t+1} (\hat{P}_t / P_t^{\pi\eta_{t+1}} F_{2,t+1}].$$

(76)

Using variables $F_{1,t}$ and $F_{2,t}$ the optimal pricing rule is

$$F_{2,t} = \frac{\eta}{\eta - 1} F_{1,t}.$$ 

(77)

$P^*_t$ is the same for all the retailers in each period. Therefore, in each period $1 - \theta$ retailers reset their price
to $P_t^*$ and the aggregate price evolves according to

$$P_t = [\theta P_{t-1}^{1-\eta} + (1 - \theta)(P_t^*)^{1-\eta}]^{\frac{1}{1-\eta}}. \quad (78)$$
Appendix D  Impulse Response Functions Computation

In line with Fernandez-Villaverde et al. (2011) and the approach of Dynare to calculate impulse responses, the following procedure was used to compute impulse responses:

1. I draw a series of random shocks $\epsilon_t = (\epsilon^A_t, \epsilon^W_t, \epsilon^n_t, \epsilon^g_t, \epsilon^m_t)$ for 2096 periods. Starting from the steady state, I simulate the model using $\epsilon_t$.

2. I disregard the first 2000 periods as a burn-in. Based on the last 96 periods, I compute the mean of the ergodic distribution for each variable in the model.

3. Starting from the ergodic mean, I hit the model with a series of random shocks $\epsilon^W_t$ for 96 periods. Simulation $Y^1_t$ is obtained.

4. Obtain $\tilde{\epsilon}^W_t$ by adding one standard deviation to $\epsilon^W_t$ in period 1 and simulate the model starting from ergodic mean and hitting it with $\tilde{\epsilon}^W_t$ to get $Y^2_t$.

5. I obtain IRFs as $Y^2_t - Y^1_t$. 

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