Macroeconomic Policies in an Open Economy

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September 2017
Job Market Paper

Abstract

I build up a two period partially open economy model with domestic and foreign investors and financial intermediaries facing a foreign debt constraint. The key frictions of the model are the limited liability faced by banks and that the individual bank cannot internalize the effect of their decisions on the interest rates. Risky bank loans are funded by domestic and foreign bank debts. Due to these two frictions, banks overestimate the expected present value of future dividends. Thus, capital in equilibrium is inefficiently high and bank risk-taking is excessively high. The novelty and less intuitive result is that under realistic calibration a lower foreign interest rate or a higher access to international credit market decreases the excessive level of capital or the excessive bank risk-taking since the marginal bank debt is domestic and hence the foreign rate affects the marginal cost of credit indirectly through the inefficient additional marginal benefit created by the limited limited. In addition, the limited liability distorts bank liability composition preferences which creates an inefficient low foreign debt share. In this simple model, a macro-prudential policy oriented to reduce the level of risky investment simultaneously restore the credit efficient level and the bank liability composition.

Keywords: Limited Liability, Commitment Problem, Bank risk-taking and welfare analysis.

JEL Classification: F41, G01, G21, G28.

1 Introduction

In the international credit market emerging economies are showing a more active participation and it is expected that the still undeveloped and small banking system in these emerging economies becomes more sophisticated and important. Hence, it is crucial to monitor the impact of higher exposure to the international credit market on

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the domestic financial stability and soundness. In particular, it becomes more important to monitor the effects of foreign shocks as foreign interest rates shocks and access to the international credit market shocks on the excessive bank risk-taking. The goal of this paper is to contribute to the study of the effects of theses shocks and to provide macroprudential policy recommendations.

There is a lot of literature that explain the risk-taking channel of domestic monetary policy and less attention has been put on the effects of the bank risk-taking of foreign monetary policy and the effects of the access to cheap foreign funds from small open economies on the excessive bank risk-taking and on the liability composition of banks. In an open economy framework, to my knowledge there are few studies that address these issues formally. Hence, the contribution of this research is to propose a framework to study the interaction of the excessive bank risk-taking behavior and the openness of a small open economy, and to study the role of macro-prudential policy in restoring the social efficient allocation. In order to do this I develop a two period partially open economy model.

This paper develops a two-period small open economy model with financial intermediaries and domestic and foreign depositors. Then, financial intermediaries fund their risky lending activities with domestic and foreign deposits. The model studies the relation between the excessive risk-taking by banks, the access to foreign funds and the bank liability composition in emerging economies. Mainly, it try to assess if the foreign interest rate or a higher access to foreign funds might affect the excessive risk-taking of domestic banks and if the excessive risk-taking behavior of banks might affect the liability composition of banks in emerging economies. This paper might shed light on how the prudential policy should be implemented in a small open economy.

An important assumption of the model is that financial intermediaries faces a collateral constraint for foreign borrowing, however, do not face any constraint for domestic borrowing. To motivate this asymmetry in the collateral constraint for domestic and foreign debt I am assuming that foreign investors have less available information about the bank risk assets and hence the agency problem that might exist between the foreign investors and banks is stronger than the one between the banks and the domestic investors.

The key two frictions of the model are the limited liability of banks and the fact that the individual bank cannot internalizes the effects of their decisions on the interest rates. This latter is because I assume that bankers cannot commit. Under unlimited liability there are not inefficiencies, so the allocations are the same to a social planner aimed to maximize the domestic welfare. Under binding limited liability, the interaction of these key two frictions results in banks overestimating the expected net present value of

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1The foreign borrowing constraint follows the spirit of Kiyotaki and Moore (1997).
future dividends. In particular, they underestimate the marginal funding cost of domestic debt. In equilibrium, due to the decreasing marginal capital productivity assumption, the aggregate capital and debt are going to be inefficiently high. In addition, since the foreign collateral constraint is binding by assumption and the foreign collateral is independent of the excessive level of lending, the excessive level of credit is funded purely with domestic debt and hence there there is an inefficiently low foreign debt participation.

The novelty and less intuitive result is that under realistic calibration the lower foreign interest rate or the higher access to international credit market reduces the excessive bank risk-taking and hence the excessive level of capital. Since the foreign collateral constraint binds, the marginal debt is domestic, and changes of the foreign rate does not affect directly the marginal cost of capital, it only affects the cost of capital indirectly by affecting the inefficient additional marginal benefit created by the interaction of the two frictions. In particular, a lower foreign rate reduces the probability of default for a given capital and hence reduces the number of times that banks will avoid to fully honor their obligations since it faces limited liability, and hence the additional marginal benefits are lower. It results in higher marginal costs of capital and lower capital. Finally, in this simple model a macro-prudential policy oriented to reduce the level of risky investment automatically restores the efficient liability composition.

This paper is organized as follow. Section 2 presents the literature review. Section 3 describe the model. Section 4 shows the closed economy version of the model. In section 5 I move to the open version of the model. Section 6 presents the model with all the key ingredients and the parameter values. Section 7 shows the numerical results. Finally, section 8 concludes.

2 Literature Review

The literature review focuses on theoretical and empirical research related to the work presented here. In particular, this paper is related to the bank-risk taking in open economy models facing some type of constraint and feature that might create inefficiencies in the banking sector. I also review the role of macroprudential policies in these models and how my research my complement or contribute to the current state of art.

Within the closed economy models, there are some papers oriented to show that capital requirements as a measure of control for the excessive bank risk taking is a good instrument in an economy featuring some type of realistic feature. As in this paper Collard et al. (2015) shows that under limited liability there is a socially excessive risk from banks. This motivates the use of capital requirements to mitigate this excessive

\[ \text{lower capital produce a second round effect by reducing even more the probability of default and then increases by more the marginal cost of capital.} \]
risk. Begenau (2015) shows in a DSGE model how capital requirements can reduce bank funding costs and increase lending when households have preferences for safe and liquid assets. Christiano and Ikeda (2013) also show that binding capital requirements increases welfare by reducing the leverage which in turn reduces the risk to the creditors who can not overstate banker effort. The role of capital requirements in mitigating the inefficiencies created by the bailouts is study in Nguyen (2014). In general, I contribute to this literature presenting an open framework to study the role of capital requirements in the presence of foreign markets This paper is similar to Collard et al. (2015) but the main difference in addition to the open credit market is that the probability of default of bank is endogenous and with this I am more able to study the effects of foreign interest rate on the excessive bank risk taking.

Regarding open model that study the role of macro-prudential policies as stabilizers of both the real and financial sectors,\textsuperscript{3} A model with a domestic and international collateral constraints is developed in Caballero and Krishnamurthy (2001). A policy oriented to reduce the ex-ante amount of foreign debt reduce the distortions that might create a binding internal foreign constraint for instance a fire sales of local assets and avoid the contagion on the domestic collateral constraint. This is in line with the external over-borrowing literature of Bianchi (2011), Bianchi and Mendoza (2011) and Korinek (2011). For instance, in Bianchi (2011) private agents do not take into account the negative effects on the foreign collateral value through a reduction next period non-tradable good prices of a higher current level of foreign debt. I depart from this literature first with the source of the inefficiency in the model I develop, the inefficiency is not due to a borrowing constraint, it is due to the limited liability of banks and the fact that banks do not internalizes the effects of their decisions on the interest rate. Hence, it is not the result of this paper that foreign debt is excessively high, but that the level of capital is excessively high. I want to contribute to this literature by modeling a banking that decides for an inefficiently high level of risk taking.

Akinci and Queralto (2014) in an open-economy version of Gertler and Kiyotaki (2010) shows that level of bank leverage is excessively high and that a subsidy on equity issuance reduces the probability of a financial crisis improving welfare.\textsuperscript{4} To do this they make the incentive constraint occasionally binding to capture infrequent financial crisis. Hence, the inefficiency comes as from the fact that banks do not internalize the benefits that comes from a lower probability that the constraint binds in the future of issuing equity in ”normal” times.\textsuperscript{5} Then, a subsidy on equity issuance reduces the probability of a

\textsuperscript{3} These models typically assume borrowing constraints which is key to explain the inefficiencies or an exogenous structure for the spread of the domestic and foreign interest rate called country risk premium, see Schmitt-Grohe and Uribe (2003). They justify the presence of these constraints due to some micro-funded moral hazard problems which are not modeled in these.

\textsuperscript{4} This subsidy as it is mentioned in the paper has a flavor of capital requirements.

\textsuperscript{5} In other words, banks do not internalize the consequences of asset price movements on their individual balance sheet positions.
financial crisis improving welfare.

Regarding the limited liability of banks there is some closed economy literature as Sinn 2003, Agur et al. 2013, Gete and Tiernan 2013 that introduce the limited liability as the key market failure in their models which creates over-lending and excessive risk-taking. Sinn 2003 suggests banks should increase the equity levels to make the bankruptcy probability to zero, but since he develops a partial equilibrium model it does not account for the aggregate effects of requiring more equity. I follow this literature in the sense that limited liability creates an excessive risk-taking. In contrast to Sinn 2003, for instance, I allow for a foreign borrowing constraint between foreign creditors and financial intermediaries which defines an external risk-premium and for simplicity I abstract from allowing the bank to choose between different risky projects.

Regarding the bank risk-taking and interest rate literature there is a bunch of empirical and theoretical papers. Most of them suggest that bank risk-taking increase after a reduction of the policy rate, and with increase of the bank risk-taking the social welfare decreases. The last financial crisis showed that after very low Fed rates there was an excessive risk-taking behavior accompanied with less transparency of information and other market failures. In Jiménez, Ongena, Peydró and Saurina (2014) using data from Spain it is suggested that a lower short-term interest rate increases the level of risk of the loans. In the same way Maddaloni and Peydró (2011) shows that lending standards deteriorate after a reduction of the short-term interest rate. Dell’Ariccia, Laeven and Suarez, 2016 states “ex-ante risk taking by banks is negatively associated with increases of short-term interest rates”.

Here I find similar results if I consider the effects of the domestic interest rate, but I find that a expansionary foreign monetary policy leads to a reduction of the excessive bank risk-taking when the marginal funds for credit are obtained from domestic depositors, then in this case a reduction of the foreign rate just reduce the default probability of bank and hence the size of the inefficiency. In general, my contribution is to provide a formal framework to study the effects of changes of interest rate on the bank risk-taking that reduce welfare, in other words, the excessive bank risk-taking. Hence, building an open model allows to explain why is that a domestic and foreign monetary policy might have different effects on the excessive bank risk taking.

Finally, the excessive bank risk-taking is cleared reviewed in Mathias Dewatripont and Xavier Freixas (2012) assessing four reasons for this studied in the literature: 1. The multiplicity of stakeholders and the moral hazard problem of too big to fail encourage more risk-taking, 2. The anticyclical capital to loan rule that depends wrongly on credit to GDP deviations from its trend, 3. Market imperfections produce incentives to hide information and 4. The bailout policies. The model developed here studies the points 1, 3 and 4 and provides similar conclusions.
3 Description of the Model

I develop a two-period model where there are infinite numbers of identical domestic financial intermediaries (banks), domestic investors (domestic households) and foreign investors. Domestic households own banks. Domestic and foreign investors make domestic and foreign deposits respectively on banks. Banks uses an exogenous initial equity and deposits to fund their risky investments. I model a borrowing constraint for banks to motivate a restriction on the bank’s ability to obtain funds. In order to motivate for an asymmetric restriction on the bank’s ability to obtain fund I assume that there is a borrowing constraint on foreign debt only and there is not constraint for domestic borrowing.

Two key assumption in the model are that banks face limited liability and banks can not commit. In order to capture the fact that the risk-free interest rate in emerging economies is lower than developed economies, I assume that the opportunity cost of domestic investors is lower that the opportunity cost of the foreign investors. For simplicity, I assume that households have not access to the international credit market and banks can costlessly identify if an investor is domestic or foreign, and that domestic and foreign investors invest in risky assets only through banks.

In addition, I assume that domestic and foreign investors are risk-neutral and there is not deposit insurance for domestic and foreign deposits (debt), banks are not able to issue equity at \( t = 0 \) and there is only one type of risky investment. These simplifying assumptions will not affect the main results of the model.

The timing of the mode is the following: At \( t = 0 \) investors make bank deposits and financial intermediaries fund their risky lending (investment) activities with deposits and an exogenous initial equity. At \( t = 1 \) the outcome of the bank’s investment is realized and banks transfer the dividends to domestic households since they own banks. In this two period model, the dividends are the same of the equity at \( t = 1 \). If the bank has unlimited liability, the bank never defaults, and hence the equity at \( t = 1 \) can be positive or negative, and hence the dividends can be positive or negative depending on the economic conditions at \( t = 1 \). If the bank has limited liability, the value of the final equity can not be negative and hence the final dividends cannot be negative, then each time the size of the obligations is higher that the income from the risky investments, the bank defaults, depositors are not fully honor and dividends are zero.

The individual bank takes the loans to buy capital \( K_0 \) and invests it on a risky project at \( t = 0 \). At period \( t = 1 \) it receives \( Z_1 K_0^a \) plus the undepreciated capital \((1-\delta)K_1\), where \( Z_1 \) is the technology shock. \( Z_1 \) has a lognormal distribution, \( \ln(Z_1) \sim N(\mu_z, \sigma^2_z) \). \( F \) is the CDF and \( f \) is the PDF of \( Z_1 \). I assume for simplicity the capital is fully depreciated, \( \delta = 1 \).
In the following I look at the equilibrium for the closed economy and then I open the economy and within each of them the unlimited and liability cases. This is going to help me see how the inefficiency looks after opening the economy and to explore how the new factors such that the access to foreign credit market and the foreign interest rate might affect the size of the inefficiency. Moving from a closed to an open economy allows me to show more clearly the mechanism of how a lower interest rate might decrease the excessive bank risk-taking.

4 Closed Economy

In a closed economy the individual bank can only fund their loans or business investments, $K_0$, with domestic deposits, $D_0$, since it invests the exogenous initial equity, $N_0$. The balance of the bank is,

$$K_0 = D_0 + N_0.$$ (1)

For illustrative purposes I start presenting the equilibrium when banks have unlimited liability (ULL) and then I add this limited liability (LL) feature in order to explain and measure the inefficiencies and welfare losses, caused by the interaction of the limited liability and the fact that banks can not commit, having into account that the equilibrium under ULL corresponds to the socially efficient allocation.

4.1 Banks with Unlimited Liability

The final net worth of the bank is,

$$N_1 = Z_1 K_0^\alpha - \bar{R}_0 D_0,$$

where $N_0$ is the exogenous initial value of bank’s equity and $\bar{R}_0$ is the promised gross interest rate at $t = 0$ and it is determined in equilibrium and taken as given by the individual bank. Notice that when the bank has unlimited liability, the bank does not default even when $N_1 < 0$, since when the final equity is negative, bank’s owner receives negative dividends, which is in fact not realistic that is why we then move to the limited liability case. Hence, when $N_1 \geq 0$, bank’s transfer positive dividends to bank’s owner, otherwise $N_1 < 0$ and hence bank’s owner receive negative dividends.

Since banks are owned by households, the objective of the bank is to maximize the expected present value of future dividends. In this two period model, the only future dividend is the one at period $t = 1$ and it is going to be the same that the final equity, $N_1$, since the bank has unlimited liability. Hence, the expected present value of future
bank’s dividends

\[ V_0 = E_0 \{ \beta (Z_1 K_0^\alpha - \bar{R}_D^0 D_0) \} \]  

(2)

where \( \beta \) is the discount factor of domestic households. The representative bank optimally choose the level of domestic deposits, \( D_0 \), to maximize \( V_0 \) subject to (1) taken as given the interest rate \( \bar{R}_D^0 \). The first order condition for \( D_0 \) yields,

\[ \beta (\bar{Z} \alpha K_0^\alpha - \bar{R}_D^0) = 0, \]  

(3)

where \( \bar{Z} = E_0 \{ Z_1 \} \). The condition (3) requires that the marginal product of capital equals the marginal cost of capital represented by the domestic gross interest rate.

The opportunity cost of domestic depositors is going to be the same that gross return of safe government saving bonds. Since banks have unlimited liability, bank’s deposits are safe assets. In equilibrium arbitrage in the market assures that they generate the same return as government bonds which in turn is going to be the inverse of the discount factor of risk-neutral domestic households,

\[ \bar{R}_D^0 = R_B^0 = \frac{1}{\beta}, \]  

(4)

where \( R_B^0 \) is the gross return of safe domestic government saving bonds. In equilibrium the optimal level of capital can be found from (3) and (4),

\[ K_0 = \left( \frac{\bar{Z} \alpha}{R_B^0} \right)^{\frac{1}{1-\alpha}}. \]  

(5)

\[ D_0 = \left( \frac{\bar{Z} \alpha}{R_B^0} \right)^{\frac{1}{1-\alpha}} - N_0. \]

This is not a novelty result, but ti will work as a benchmark for next analysis. It can be seen in (3) or (5) how the domestic risk-free interest rate affects directly the marginal cost of capital in equilibrium.

4.2 Banks with Limited Liability

The unlimited liability assumption is very far from being realistic. Since now bank faces limited liability, the final equity can not take negative values. Hence, the final equity becomes

\[ N_1 = max \{ 0, Z_1 K_0^\alpha - \bar{R}_D^0 D_0 \}. \]
This means that the bank can not transfer negative dividends to households. For a given \( \bar{R}_D^0 \) and \( K_0 \) there is going to be a \( Z^* \) such that

\[
0 = Z^* K_0^\alpha - \bar{R}_D^0 D_0. \tag{6}
\]

It means that if \( Z_1 < Z^* \), the bank is not able to fully honor its obligations and consequently it defaults and \( N_1 = 0 \), otherwise, the bank does not default and \( N_1 \geq 0 \). For convenience, as in Gertler and Kiyotaki (2015) I define the endogenous domestic recovery ratio, \( x_D^1 \), as the fraction of the total claims that it is recovered by the domestic depositors. When the bank does not default, \( x_D^1 = 1 \) since depositors do receive the full amount of claims, however, when the bank defaults, depositors only receive an endogenous fraction, \( x_D^1 < 1 \), of the agreed total payment, \( \bar{R}_D^0 D_0 \), and it hence the recovery ratio must satisfy

\[
0 = Z_1 K_0^\alpha - x_D^1 \bar{R}_D^0 D_0, \tag{7}
\]

which means that the amount recovery by the domestic depositors, \( x_D^1 \bar{R}_D^0 D_0 \), is the bank realized income, \( Z_1 K_0^\alpha \). In general, I can rewrite \( x_D^1 \) as

\[
x_D^1 = \min \left\{ 1, \frac{Z_1 K_0^\alpha}{\bar{R}_D^0 D_0} \right\} \tag{8}
\]

In general equilibrium \( x_D^1 \bar{R}_D^0 \) represents the effective gross return of domestic debt and the probability of default of the bank is given hence by

\[
p_0 \equiv F(Z^*).
\]

The expected present value of the future terminal dividend or final net worth under limited lability is,

\[
V_0 = E_0 \left\{ \beta \left( \max \{0, Z_1 K_0^\alpha - \bar{R}_D^0 D_0\} \right) \right\}. \tag{9}
\]

Hence, when banks have limited liability, the bank cares only on the state of nature when the revenues are higher that all its obligations. The individual bank seeks to maximize the expected present value of future dividends, (9) and to the bank balance sheet, (1), taken as given the interest rate \( \bar{R}_D^0 \). To understand the bank’s incentives when it has limited liability I rewrite (9) as

\[
V_0 = E_0 \left\{ \beta (Z_1 K_0^\alpha - \bar{R}_D^0 D_0) \right\} - \int_{Z_1}^{Z^*} \beta (Z_1 K_0^\alpha - \bar{R}_D^0 D_0) dF(Z_1). \tag{10}
\]

where recall \( Z^* = \frac{\bar{R}_D^0 D_0}{K_0^\alpha} \). The first term of (10) is the discounted expected final equity given that the bank services its deposits under all circumstances. The second term appears by the presence of the limited liability. This measures the advantage resulting from the fact that the bank does not fully service its bonds under all circumstances but
only in the case of non-default. Each time the bank defaults, it can avoid to pay back
that part of the promised deposit repayment that exceeds its revenues, $\bar{R}_0^D D_0 - Z_1 K_0^\alpha$, and this advantage contributes to the final equity to the extent of the probability that it happens, $f(Z_1)$, for each $Z_1 < Z^*$.

The gross interest rate, $\bar{R}_0^D$, is determined in equilibrium and the individual bank cannot affect it. Hence, the first term of (10) delivers the same trade off discussed in the unlimited liability case and the second term motivates the bank for a higher $D_0$ since it produces a positive marginal benefit. The first order condition for $D_0$ yields

$$\beta(\alpha \bar{Z} K_0^{\alpha-1} - \bar{R}_0^D) - \int_0^{Z^*} \beta(\alpha Z_1 K_0^{\alpha-1} - \bar{R}_0^D) dF(Z_1) + \beta(Z^* K_0^\alpha - \bar{R}_0^D D_0) f(Z^*) \frac{\partial Z^*}{\partial D_0} = 0.$$  

Since $Z^* K_0^\alpha - \bar{R}_0^D D_0 = 0$, the foc becomes

$$\beta(\alpha \bar{Z} K_0^{\alpha-1} - \bar{R}_0^D) - \int_0^{Z^*} \beta(\alpha Z_1 K_0^{\alpha-1} - \bar{R}_0^D) dF(Z_1) = 0.$$  

A comparison between the optimality conditions (11) and (3) shows that the bank’s choices are indeed distorted. The first term of (11) in the ULL case gives zero which yields to the optimal decision of domestic debt. When there is limited liability, this decision is not longer optimal since increasing the domestic deposits has an additional advantage represented by the second term of (11).

Domestic investors will know from the general market observation that the the repayment promise of the banks cannot be taken as granted. Hence, the promised interest rate, $\bar{R}_0^D$, has to be enough high to compensate the reduced payment when the bank defaults. Since households are risk neutral, in equilibrium the expected repayment of the bank deposits has to be equal to the repayment of a domestic safe asset which corresponds the alternative investment for the domestic investor, i.e.

$$R_0^B D_0 = E_0\{x_1^D \bar{R}_0^D\} D_0$$  

Hence, the bank pays the agreed amount, $\bar{R}_0^D D_0$, in the case it does not defaults and it pays only a fraction $x_1^D$ of it in the case of failure. The equilibrium values for $D_0$ and $\bar{R}_0^D$ are obtained from (11) and (12). It is not possible o provide a form closed solutions. Hence, for illustrative purposes I rewrite (11) using (12) as

$$\beta(\bar{Z} \alpha K_0^{\alpha-1} - R_0^B) - \int_0^{Z^*} \beta(\alpha Z_1 K_0^{\alpha-1} - x_1^D \bar{R}_0^D) dF(Z_1) = 0.$$  

Finally,

$$K_0 = \left( \frac{\bar{Z} \alpha}{R_0^B - \theta} \right)^{-\frac{1}{\alpha}}.$$  

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where

\[ \theta_0 \equiv \int_0^{Z^*} (x_1 D_1 \bar{R} - \alpha Z_1 \bar{K}^{-1}) dF(Z_1) = \int_0^{Z^*} Z_1 dF(Z_1) \frac{(1 - \alpha)K_0^\alpha + \alpha K_0^{-1}N_0}{K_0 - N_0} > 0. \]

is positive. And

\[ D_0 = \left( \frac{Z_0}{R_0^\alpha - \theta_0} \right)^\frac{1}{1 - \alpha} - N_0. \]

By comparing (5) and (13) the limited liability generates an inefficiently high level of capital since \( \theta_0 > 0 \). Hence, \( \theta_0 \) represents the inefficient additional marginal benefit of increasing one capital under limited liability. Another way to understand it is that the LL equilibrium is equivalent to a LL equilibrium where the banker makes a mistake and it considers a wrong domestic risk-free interest rate that is \( \theta_0 \) less than the right one.

If I assume that an individual bank can manipulate the interest rates of the deposits, the limited liability has not distortionary effects on the economy. This can be proved by inserting equation (12) into (11) giving again equation (2) when account is taken of (4). As the bank is unable to manipulate risk-free interest rate of domestic deposits, the equilibrium condition is going to be the same than the equilibrium condition for banks with unlimited liability, Thus, in this case the limited liability will not originate any inefficiency in the economy.\(^{6}\)

4.3 Domestic Welfare Losses

Here I am interesting in assessing the effects of the frictions that appears under limited liability on the welfare of the domestic economy. Since households are the owner of banks, I am interested on the welfare of domestic households. Since in equilibrium the risk-neutral domestic households always get in expectation the domestic risk-free interest rate, the welfare losses have to be represented by the lower net present value of terminal dividends generated in the banking activity. Hence, a fair measure of domestic welfare that fully capture the domestic welfare losses if any generated by the inefficiencies that appears under limited liability is given by \( V_0 \). Hence, the domestic welfare losses if there exists under limited liability can be represented as,

\[ WL_0 = V_0^{ULL} - V_0^{LL}. \]  \hfill (14)

And it can be rewritten as\(^{7}\)

\[ WL_0 = \beta \bar{Z} \left[ (K_0^{ULL})^\alpha - (K_0^{LL})^\alpha \right] - (K_0^{ULL} - K_0^{LL}) > 0. \]  \hfill (15)

\(^6\)The formal proof is in the Appendix A.

\(^7\)The proof for the expression of WL is in Appendix D.
The welfare losses are composed by a first negative part, \( (K_0^{ULL})^\alpha - (K_0^{LL})^\alpha \), which represents the fact that under limited liability there is a higher level of capital. The second positive part of the welfare losses, \(-(K_0^{ULL} - K_0^{LL})\), represent the fact that under LL the bank is borrowing more resources and incurs in higher costs. As it is stated next the second positive part dominates the first negative part.

It is possible to rewrite \( WL_0 \) as

\[
WL_0 = \left( \frac{\alpha \bar{Z}}{R_B^0} \right)^{\frac{1}{1-\alpha}} \left\{ \left( \frac{1}{\alpha} - 1 \right) - \left( \frac{1}{\alpha} y^\alpha - y \right) \right\} > 0.
\]

where \( y = \left( \frac{R_B^0}{R_B^0 - \theta_0} \right)^{\frac{1}{1-\alpha}} \). It is proved in Appendix D that welfare losses, \( WL_0 \), are positive, which means that the allocation under limited liability is inefficient. In other words, the allocation under limited liability leads to a lower welfare losses compared to the unlimited liability case which is the efficient one.

4.4 Effects of changes in the domestic interest rate:

Under unlimited liability as it is suggested by (5) a higher domestic risk-free interest rate directly affect the marginal cost of capital by increasing it and clearly the results is a higher lower level of capital.

Under limited liability the assessment is more complex. First, by observing (13) as in the unlimited liability case there exist this direct effect on the marginal cost, which is due to the presence of the domestic risk-free interest rate in the denominator inside the parenthesis. The novelty under limited liability is the effects of changes of \( R_B^0 \) on \( \theta_0 \) which is the source of the discrepancy between capital levels under limites and unlimited liability. The result is that a higher \( R_B^0 \) produces a lower \( \theta_0 \) and then lower inefficiency. Next, I do not pretend to provide give an analytical explication since it is no possible to do this, but I provide some intuition and let the rest for the quantitative results after calibration.

Since \( Z^* = \bar{R}_D^0 D_0 / K_0^\alpha \) for a given capital a higher \( R_B^0 \) increases \( \bar{R}_D^0 \) which increases \( Z^* \) and the default probability of banks and hence it increases \( \theta_0 \) increasing the excessive bank risk-taking. The intuition for this is that for a given level of credit if the cost of funding increases the bank will default more times.

However, capital will change as well and it will depend on how \( R_B^0 - \theta_0 \) changes. And this change of capital will affect \( \theta_0 \). Numerical results show that capital will be lower and \( \theta_0 \) is going to be lower, since the lower capital mainly reduce the probability of default by reducing \( Z^* \). The result is that after an increase of domestic interest rate the excessive

\[\text{---}^8\text{The proof for } WL > 0 \text{ is in Appendix D.}\]
bank risk-taking is lower because of the direct effect of a higher $R^B$ on the marginal cost of capital.

This is in line with the literature that suggest that an expansionary monetary policy increases the bank risk-taking in an excessive way, see Jiménez et al. 2014. However, it will be interesting to study the effects of change of funding cost when controlling by the size of loans. In this case as I will show later, a foreign monetary policy expansion reduces the excessive risk-taking of domestic banks.

Next I move to the open economy version where the goal is to isolate the effects of lower cost of funding on the probability of default controlling by the effects on the marginal cost of capital. In other words, I aim to distinguish the direct effect of changes of the interest rates on the cost of capital funding and the effects on the probability of default of banks.

5 Open Economy

Now financial intermediaries can fund their loans with domestic and foreign deposits, $D^F_0$. The balance of the bank becomes,

$$K_0 = D_0 + D^F_0 + N_0,$$

(16)

So now, capital or loans can be funded by domestic or foreign deposits and by the exogenous initial bank equity. I assume that foreign investors have an exogenous opportunity cost of $R^F_0$. In particular, I am assuming

$$R^F_0 < R^B_0.$$

$R^F_0$ can be interpreted as the gross return of safe foreign government bonds. Recall that for simplicity I am assuming that households cannot borrow directly from foreign investors and banks can perfectly identify if the investor is domestic or foreign.

I further assume in the spirit of Kiyotaki and Moore (1997) the following collateral constraint for foreign deposits,

$$D^F_0 \leq \phi,$$

(17)

where $\phi > 0$ is a parameter. This borrowing constraint attempts to highlight two mean features for this small open economy: First, this represents a simple version of the typical borrowing constraint that is imposed in the open economy literature rather than derived from an optimal contract, e.g. Kiyotaki and Moore (1997), Bianchi (2011) and Mendoza (2010), and as in this literature it tries to capture some financial frictions that might exists between the domestic and foreign economy. As it is suggested for this literature,
this type of constraint could result from a more hazard problem. In contrast to most of this literature, as the collateral value does not depend on any endogenous variable that it is determined in equilibrium, it will not generate any inefficient allocation.\textsuperscript{9} For the purpose of this paper this is convenient since I am interested in capturing another type of externality. Since I assume the constraint binds it will create an endogenous premium over the foreign interest rate.

Second, an issue that it is less discussed in these open economy models I believe because they do not model the financial intermediaries system is that I am implicitly assuming that there are not any moral hazard problem or agency problem between the financial intermediaries and domestic investors. And this is something that I want to capture by this borrowing constraint as well. In general, I am assuming that the agency problem (if any) between financial intermediaries and domestic investors is less severe compared to the agency problem between the financial intermediaries and foreign investors. Another way to interpret this is to say that loans funded by foreign debt requires relatively more collateral or that the collateral for foreign debt is relatively less available, Caballero and Krishnamurthyc (2001). It follows the spirit of Gertler et al. (2012) where the moral hazard problem is not symmetric with respect to deposits and outside equity.\textsuperscript{10}

In this context of the moral hazard problem developed at GK (2015) I am implicitly assuming that in the margin it is more difficult to divert assets funded by foreign deposits than by domestic deposits. The explanation is that foreign depositors have less ability to persuade banks to not divert assets. This could be because foreigners have less information (compared to the domestic depositors) about the asset value or it is more expensive for foreign investors to monitor the domestic bank. Empirical literature that studies the ”home bias” puzzle might support this assumption since they argue that the preference for home equity can be explained by information asymmetries between the domestic and foreign investors. For instance, Coval and Moskowitz (2012) states that investors may have easier access to information about the companies located near them. Local investors can talk to employees, managers, and suppliers of the firm all of which may provide them with an information advantage. Choe et al. (2001) finds evidence that domestic individual investors have a short-lived private information advantage.

I also assume that bank cannot be a net lender to domestic depositors,

\[ 0 \leq D_0. \]

It says that bank cannot lend to domestic depositors. This could be because it is difficult to monitor domestic depositors directly and they prefer to invest on firms or projects

\textsuperscript{9}In other words, this simple collateral constraint abstracts from any pecuniary externality that might be created due to the constraint itself. That is something that has been extensively studied in the literature.

\textsuperscript{10}Akinci and Queralto (2014) makes a similar assumption as well.
which financial information is more public. The previous condition ensures that for high values of $\phi$, the bank cannot exhaust all their foreign debt capacity and hence it cannot make profits by borrowing from abroad and lending to domestic depositors.

For illustrative purposes I start assuming

$$\phi = +\infty.$$ 

in order to start assuming that the foreign collateral constraint is not binding. In general, if $\phi_u^{U,LL} < \phi$, the foreign collateral constraint does not bind under unlimited liability and if $\phi_u^{L,LL} < \phi$ the foreign collateral constraint is binding under unlimited and limited liability.\(^{11}\)

As in the closed economy version for didactic purposes first I model the economy with unlimited liability and then the economy with limited liability in order to measure and understand the inefficiency that the limited liability might create in this open economy.

### 5.1 Banks with Unlimited Liability

Since $R^B > R^F$ and domestic deposits cannot be negative, i.e. the domestic agents cannot borrow from banks,

$$D_0 = 0,$$

and then capital is funded only by the initial equity and foreign funds and banks will borrow from abroad as much as they want. Hence, the final equity of the bank is going to be,

$$N_1 = Z_1 K_0^\alpha - \bar{R}_0^F D_0^F,$$

(18)

where $\bar{R}_0^D$ and $\bar{R}_0^F$ are the promised domestic and foreign interest rates respectively at $t = 0$ which is determined in equilibrium and it is taken as given by the individual bank. As in the closed economy framework, the bank never defaults. The expected present value of the future terminal dividend is,

$$V_0 = E_0\{\beta (Z_1 K_0^\alpha - \bar{R}_0^F D_0^F)\}.$$ 

(19)

\(^{11}\)where

$$\bar{\phi}_u^{U,LL} = k_u^{U,LL} - N_0, \quad k_u^{U,LL} = \left(\frac{\alpha \bar{Z}}{R_0^F}\right)^{\frac{1}{1-\alpha}},$$

$$\bar{\phi}_u^{L,LL} = k_u^{L,LL} - N_0, \quad k_u^{L,LL} = \left(\frac{\alpha \bar{Z}}{R_0^F - \theta_u}\right)^{\frac{1}{1-\alpha}},$$

$$\theta_u = \int_0^{Z_u^*} (x_{1,u}^F R_{0,u}^F - \alpha Z_1^u (k_u^{L,LL})^{\alpha-1}) dF > 0, \quad Z_u^* (k_u^{L,LL})^\alpha = R_{0,u}^F (k_u^{L,LL} - N_0),$$

$$R_0^F = (1 - F(Z_u)) R_{0,u}^F + \int_0^{Z_u^*} x_{1,u}^F R_{0,u}^F dF, \quad x_{1,u}^F R_{0,u}^F = \frac{Z_1^u (k_u^{L,LL})^\alpha}{k_u^{L,LL} - N_0}. $$
The representative bank aims to maximize (19) subject to the borrowing constraint (17) and (16) taken as given the interest rate $\bar{R}_F$. Recall that ex-ante I am assuming that $\phi = +\infty$ hence the foreign collateral constraint is not binding and hence it is not taking into account in the maximization problem of the bank. The first order condition for $D^F_0$ yields,

$$\beta(\alpha Z K^\alpha_0 - \bar{R}^F_0) = 0. \quad (20)$$

Since banks have unlimited liability, bank’s deposits are safe assets. Arbitrage in each market assures that the interest rates of the domestic and foreign deposits generate the same return that the corresponding risk-free bond for each market,

$$\bar{R}^F_0 = R^F_0. \quad (21)$$

In equilibrium the capital level is obtained from (20) and (21),

$$K_0 = \left(\frac{\bar{Z} \alpha}{\bar{R}^F_0}\right)^{\frac{1}{1-\alpha}}. \quad (22)$$

Then,

$$D^F_0 = \left(\frac{\bar{Z} \alpha}{\bar{R}^F_0}\right)^{\frac{1}{1-\alpha}} - N_0. \quad (23)$$

In an open economy with unlimited liability and with a non-binding foreign collateral constraint, the risk-free foreign interest rate has a direct effect on the marginal cost of capital as it can be seen by (20) and (22), since in equilibrium the marginal deposit is the foreign one and not the domestic one.

In contrast with the closed economy, since the bank is able to fund a their loans with cheaper deposits, $V_0$ is going to be higher for an open economy and welfare is going to be higher as well.\textsuperscript{12} As usual the economy is better off when this is open.

\textbf{5.2 Banking with Limited Liability}

Since $Z_1$ has a log-normal distribution, the limited liability is binding by assumption. The equity is now given by

$$N_1 = \max\{0, Z_1 K^\alpha_0 - \bar{R}^F_0 D^F_0\}.$$

Hence, there is going to be a $Z^*$ such that

$$0 = Z^* K^\alpha_0 - \bar{R}^F_0 D^F_0.$$

\textsuperscript{12}Proof in Appendix I.
In the general equilibrium the probability of default of the bank is given hence by \( p_0 = F(Z^*) \). As in the closed economy I define the foreign recovery ratio, \( x^F_1 \), as the fraction of total claims that it is recovered by the foreign investors. When the bank does not default, \( x^F_1 = 1 \) since depositors do receive the full amount of claims, however, when the bank defaults, depositors only receive a fraction, \( x^F_1 < 1 \), and it satisfies \( 0 = Z_1 K_0^\alpha - x^F_1 \bar{R}^F_0 D^F_0 \).

In general, I can rewrite \( x^F_1 \) as

\[
x^F_1 = \min \left\{ 1, \frac{Z_1 K_0^\alpha}{\bar{R}^F_0 D^F_0} \right\}
\]

Hence, \( x^F_1 \bar{R}^F_0 \) represents the effective gross return of domestic debt. The individual bank seeks to maximize,\(^{13}\)

\[
V_0 = E_0 \{ \beta \max \{ 0, Z_1 K_0^\alpha - \bar{R}^F_0 D^F_0 \} \}.
\]

subject to the non-binding foreign collateral constraint (17) and (16). As in the closed economy, I rewrite (23) as

\[
V_0 = E_0 \{ \beta (Z_1 K_0^\alpha - \bar{R}^F_0 D^F_0) - \int_{Z^*}^{Z} \beta (Z_1 K_0^\alpha - \bar{R}^F_0 D^F_0) dF \}
\]

The first order condition for \( D^F_0 \) yields,

\[
\beta (\alpha \bar{Z} K_0^\alpha - \bar{R}^F_0) - \int_{0}^{Z^*} \beta (\alpha Z_1 K_0^\alpha - \bar{R}^F_0) dF = 0,
\]

As in the closed comparison between the optimality conditions (20) and (24) shows that the bank’s choice of capital is indeed distorted. In the ULL case the first term gives zero. When there is limited liability, this decision is not longer optimal since increasing the domestic deposits has an additional advantages represented by the second term of (24).

Domestic and foreign investors will know from the general market observation that the repayment promised of the banks cannot be taken as granted. Hence, the promised interest rates, \( \bar{R}^F_0 \), has to be enough high to compensate the reduced payment when bank defaults. Since all investors are risk neutral, in the equilibrium the expected repayment of the bank deposits is equal to the opportunity cost of each investor. In equilibrium it must holds that

\[
\bar{R}^F_0 D^F_0 = E_0 \{ x^F_1 \bar{R}^F_0 \} D^F_0,
\]

\(^{13}\)Recall that when the bank faces limited liability it only cares about the state of nature where it does not default.
From equations (24) and (25),

\[ K_0 = \left( \frac{Z \alpha}{R_0^F - \theta_0} \right)^{\frac{1}{1-\alpha}}, \]  

(26)

where \( \theta_0 \) is now

\[ \theta_0 = \int_0^{Z^*} (x_1^F R_0^F - \alpha Z_1 K_0^{\alpha-1}) dF(Z_1) = \int_0^{Z^*} Z_1 dF(Z_1) \left( \frac{(1-\alpha)K_0^\alpha + \alpha K_0^{\alpha-1}N_0}{K_0 - N_0} \right) > 0. \]

Hence, the level of capital is going to be higher when banks face limited liability. In an open economy with limited liability and a non-binding constraint, the foreign interest has direct and also an indirect effect on the marginal cost of capital. The direct effect is due to the presence of \( R_0^F \) in the denominator of (26) since the marginal deposit is foreign one rather than domestic one. Changes of \( R_0^F \) affects indirectly the marginal cost of capital through the effects on the inefficient additional marginal benefit of capital that appears under limited liability, \( \theta_0 \). Regarding, the domestic rate, it only affects the marginal cost of capital, indirectly through the effects on \( \theta_0 \).

As in the closed economy if I assume that an individual bank can affect the interest rates of the deposits, the limited liability has not distortionary effects on the economy.\(^\text{14}\)

### 5.3 Domestic Welfare Losses:

As in the close economy a fair measure of the welfare losses is given by difference of the net present value of future dividends,

\[ WL_0 = V_0^{UL} - V_0^{LL} = \beta p Z h \left[ (K_0^{UL})^\alpha - (K_0^{LL})^\alpha \right] - (K_0^{UL} - K_0^{LL}). \]

It can be rewritten as

\[ WL_0 = \left( \frac{\alpha Z}{R_0^F} \right)^{\frac{1}{1-\alpha}} \beta R_0^F \left\{ \left( \frac{1}{\alpha} - 1 \right) - \left( \frac{1}{\alpha} y^\alpha - y \right) \right\}, \]

where \( y = \left( \frac{R_0^F}{R_0^F - \theta_0} \right)^{\frac{1}{1-\alpha}} \). It is proved in Appendix D that the welfare losses, \( WL_0 \), are positive. Hence, the allocation under limited liability is inefficient and it leads to a lower welfare respect to the the unlimited liability case.

\(^{14}\)The formal proof is in the Appendix B.
5.4 Effects of changes in the foreign interest rate:

When the collateral of foreign debt is very large changes on foreign interest rate affects directly the marginal cost of funding and hence of capital. The results is terms of the effects on capital and on the inefficiency of changes of the foreign risk-free interest rate is similar to the effects in a closed economy of changes in the domestic risk-free interest rate. And the mechanisms of how change of foreign risk-free interest rate and domestic risk-free interest rate affects the inefficiency size, \(\theta\), in an open and closed economy respectively are similar as well.

It means that a higher foreign rate for a given capital the probability of default is higher and \(\theta\) is higher since the bank obligations are higher. However, capital is indeed also affected and it will decrease mainly due to the higher marginal cost of capital produces by a higher \(R^F\). A lower capital pushed the size of the obligations and the bank default probability. This effect will dominate the one created by the higher cost of foreign debt. Finally, \(\theta\) is lower.

6 Binding Foreign Collateral Constraint

In this case I assume that the foreign collateral constraint always binds. More specifically the constraint is tight enough such that banks find optimal at the margin to demand domestic deposits. In other words, the marginal productivity of capital, being capital \(\phi + N_0\), is higher that the cost of domestic deposits. Hence, in equilibrium there is going to be a positive value of domestic deposits, \(D_0\).

Formally, this is true under the unlimited and limited liability cases If \(\phi \leq \phi^U_{d LL}\), and this is true for the limited liability if \(\phi \leq \phi^L_{d u}\) 15

The above condition ensures that in equilibrium \(D^F_0 = \phi\). 16 Hence, balance of the

\[
\phi^U_{d LL} = \left( \frac{Z_0}{R_0^B} \right)^{\frac{1}{1-\alpha}} - N_0, \quad k^U_{d LL} = \left( \frac{Z_0}{R_0^B} \right)^{\frac{1}{1-\alpha}} \frac{Z_0}{R_0^B - \theta}, \quad \phi^L_{d L} = \frac{Z_0}{R_0^B - \phi}, \quad k^L_{d L} = \frac{Z_0}{R_0^B - \phi}, \quad k^L_{d u} = \frac{Z_0}{R_0^B - \phi} - N_0
\]

\[
\theta_d = \int_0^{Z_d^2} (x_1^{D,R_{d0,d}} - \alpha Z_1(k^L_{d0,d})^{\alpha-1}) \, dF, \quad Z^2_1(k^L_{d0,d})^{\alpha} = R_{d0,d}(k^L_{d0,d} - N_0) - (R_{d0,d} - R^F_{d0,d})\phi
\]

\[
x_1^{D,R_{d0,d}} = \frac{Z_1(k^L_{d0,d})^{\alpha}}{k^L_{d0,d} - \phi - N_0 + \phi R^F_{d0,d}}, \quad x_1^{F,R_{d0,d}} = \frac{Z_1(k^L_{d0,d})^{\alpha}}{R^F_{d0,d}(k^L_{d0,d} - \phi - N_0) + \phi}
\]

\[
R^B_0 = (1 - F(Z_1^*))R^D_{d0,d} + \int_0^{Z_1^*} x_1^{D,R_{d0,d}} dF(Z_1), \quad R^F_0 = (1 - F(Z_1^*))R^D_{d0,d} + \int_0^{Z_1^*} x_1^{F,R_{d0,d}} dF(Z_1)
\]

15where

16For completeness: Under unlimited liability if \(\phi^U_{d LL} \leq \phi \leq \phi^U_{d u}\), \(K_0 = \phi + N_0\) and \(D^F_0 = \phi\) and
bank is given by,

\[ K_0 = D_0 + D_0^F + N_0. \]

6.1 Under Unlimited Liability

The final equity under unlimited liability is

\[ N_1 = Z_1 K_0^\alpha - \bar{R}_0^D D_0 - \bar{R}_0^F D_0^F, \]

As usual the bank seeks to maximize \( V_0 = E_0 \{ \beta N_1 \} \), subject to \( D_0^F \leq \phi \). The focs for \( D_0 \) and \( D_0^F \) are respectively,

\[ \beta (\alpha Z K_0^{\alpha-1} - \bar{R}_0^D) = 0, \quad \beta (\alpha Z K_0^{\alpha-1} - \bar{R}_0^F) - \lambda = 0. \]

\( \lambda \) is the LM associated with the foreign collateral constraint. Since both debts are riskless, in equilibrium \( \bar{R}_0^D = R_0^B \) and \( \bar{R}_0^F = R_0^F \). The level of capital is

\[ K_{0,\text{ULL}} = \left( \frac{Z_1}{R_0^B} \right)^{\frac{1}{1-\alpha}}. \]

In addition, by construction now the foreign collateral is binding and hence \( D_0^F = \phi \) and then the domestic deposits are

\[ D_{0,\text{ULL}} = \left( \frac{Z_1}{R_0^B} \right)^{\frac{1}{1-\alpha}} - \phi - N_0. \]

In this case of an open economy with limited liability and with a binding collateral constraint only the domestic interest rate affects the marginal cost of capital since the marginal deposit is domestic one.

6.2 Under Limited Liability

The final equity becomes

\[ N_1 = \max \left\{ 0, Z_1 K_0^\alpha - \bar{R}_0^D D_0 - \bar{R}_0^F D_0^F \right\}, \]

There is a \( Z^* \) such that

\[ 0 = Z^* K_0^\alpha - \bar{R}_0^D D_0 - \bar{R}_0^F D_0^F. \]

Hence, if \( Z_1 < Z^* \) the bank defaults, otherwise, the bank does not default. Hence, \( F(Z^*) \) is going to be the default probability of the representative bank. And the domestic and

\[ D_0 = 0. \] And under limited liability if \( \phi_{d,\text{LL}} \leq \phi \leq \phi_{a,\text{LL}} \), \( K_0 = \phi + N_0 \) and \( D_0^F = \phi \) and \( D_0 = 0 \).
foreign endogenous recovery ratio when the bank defaults has to satisfy

\[ 0 = Z_1 K_0^\alpha - x_1^D \tilde{R}_0^D D_0 - x_1^F \tilde{R}_0^F D_0^F, \]

In order to find a solution, I have to assume something about the structure of the recovery ratios: (i) I assume that what they recovery is in proportion with the final value of the debt, and (ii) the seniority of domestic and foreign debt. In particular, I assume that both debts have the same seniority, so the recover ratio are the same, \( x_1^D = x_1^F \). In general,

\[ x_1^D = x_1^F = \min \left\{ 1, \frac{Z_1 K_0^\alpha}{\tilde{R}_0^D D_0 + \tilde{R}_0^F D_0^F} \right\}, \]

Hence, \( x_1^D \tilde{R}_0^D \) and \( x_1^F \tilde{R}_0^F \) represent the effective gross return of domestic and foreign debts respectively. The NPV of future positive dividends, \( N_1 \), is

\[ V_0 = E_0 \{ \beta (\max \{ 0, Z_1 K_0^\alpha - \tilde{R}_0^D D_0 - \tilde{R}_0^F D_0^F \}) \} \]

It can be rewritten as

\[ V_0 = \beta \int_{Z^*}^{+\infty} (Z_1 K_0^\alpha - \tilde{R}_0^D D_0 - \tilde{R}_0^F D_0^F) dF(Z_1), \quad (27) \]

The bank seeks to maximize (27) subject to \( D_0^F \leq \phi \) where

\[ Z^* K_0^\alpha = \tilde{R}_0^D D_0 + \tilde{R}_0^F D_0^F, \]

\[ K_0 = D_0 + D_0^F + N_0. \]

The first order condition for \( D_0 \) is

\[ \beta \int_{Z^*}^{+\infty} \left( Z_1 \alpha K_0^{\alpha-1} - \tilde{R}_0^D \right) dF(Z_1) - \beta (Z^* K_0^\alpha - \tilde{R}_0^D D_0 - \tilde{R}_0^F D_0^F) f(Z^*) \frac{\partial Z^*}{\partial D_0} = 0, \]

Since \( Z^* K_0^\alpha - \tilde{R}_0^D D_0 - \tilde{R}_0^F D_0^F = 0 \),

\[ \beta \int_{Z^*}^{+\infty} \left( \alpha Z_1 K_0^{\alpha-1} - \tilde{R}_0^D \right) dF(Z_1) = 0. \quad (28) \]

Similarly, the first order condition for \( D_0^F \) is,

\[ \beta \int_{Z^*}^{+\infty} \left( \alpha Z_1 K_0^{\alpha-1} - \tilde{R}_0^F \right) dF(Z_1) - \lambda = 0, \quad (29) \]

where \( \lambda \) is the LM associated with the binding foreign borrowing constraint. In equilibrium \( \tilde{R}_0^D \) and \( \tilde{R}_0^F \) has to be high enough to compensate the reduced payment when the bank defaults. In equilibrium the expected payment for domestic and foreign investors
need to be equal to their corresponding opportunity costs, i.e.

\[ R_0^B D_0 = E_0\{x_1^D \tilde{R}_0^D\} D_0, \]

\[ R_0^F D_0^F = E_0\{x_1^F \tilde{R}_0^F\} D_0^F. \]

I rewrite (29) as

\[ \beta(\tilde{Z}\alpha K_0^{\alpha - 1} - \tilde{R}_0^D) + \beta \int_{0}^{Z^*} (\tilde{R}_0^D - Z_1 \alpha K_0^{\alpha - 1}) dF(Z_1) = 0, \]

And using the equilibrium condition of the domestic interest rate,

\[ \beta(\tilde{Z}\alpha K_0^{\alpha - 1} - R_0^B) + \beta \int_{0}^{Z^*} (x_1 \tilde{R}_0^D - Z_1 \alpha K_0^{\alpha - 1}) dF(Z_1) = 0, \]

Hence,

\[ K_0^{LL} = \left( \frac{\alpha \tilde{Z}}{R_0^B - \theta_0} \right)^{\frac{1}{\alpha - 1}}, \]

where

\[ \theta \equiv \int_{0}^{Z^*} (x_1 \tilde{R}_0^D - Z_1 \alpha (K_0)^{\alpha - 1}) dF(Z_1) \]

As before, \( \theta_0 \) represents the inefficient additional marginal benefit of increasing one capital under LL. Since \( \theta_0 > 0 \) (Proof in Appendix H), the LL yields to an inefficiently high level of capital, and level of domestic debt, i.e.

\[ K_0^{LL} > K_0^{ULL}, \quad D_0^{LL} > D_0^{ULL}, \quad D_0^{F,LL} = D_0^{F,ULL} = \phi. \]

It is important to notice that the foreign interest rate has not a direct effect on the marginal cost of capital as in the case of a non-binding foreign collateral constraint since the marginal debt is domestic one rather than foreign one. In this case foreign rate affects capital only through the effects on the inefficient additional marginal benefits, \( \theta_0 \). This economic structure will help me to consider the effects of the interest rates when there is not a direct effect on cost of credit and when the indirect effect is just because of the presence of the inefficiency.

Recall that individual bank has not commitment and hence it cannot manipulate the interest rates in the market, this means that the individual bank does not internalizes the positive effects of a higher capital on the domestic and foreign interest rates in equilibrium, \( \tilde{R}_0^D \) and \( \tilde{R}_0^F \). This results in an inefficiently higher capital and bank risk-taking. It is proved in the Appendix C that if the bank internalizes the effects of their decision on the interest rate the solution under LL is the same that the ULL, so there is not inefficiency.
6.3 Domestic Welfare Losses

In expectation the domestic and foreign investors always receive their corresponding risk-free opportunity costs. However, I claim that banks are going to exhibit a lower NPV of future dividends in equilibrium since the limited liability and the fact that banks cannot internalize the effects of their decisions on the interest distort the efficient allocation.

Hence, I claim that the domestic welfare losses ($WL$) created by the inefficient allocation are realized in the banking system. As before the $WL$ are defined as $WL = V^{ULL}_{0} - V^{LL}_{0}$. They can be rewritten as (Proof in Appendix F)

$$WL = \beta \bar{Z} \left[ (K^{ULL}_{0})^{\alpha} - (K^{LL}_{0})^{\alpha} \right] - (K^{ULL}_{0} - K^{LL}_{0}).$$

And finally using the equilibrium conditions,

$$WL_{0} = \left( \frac{\alpha \bar{Z}}{R_{F}^{0}} \right)^{\frac{1}{1-\alpha}} \left\{ \left( \frac{1}{\alpha} - 1 \right) - \left( \frac{1}{\alpha} y^{\alpha} - y \right) \right\},$$

where $y = \left( \frac{R_{F}^{B}}{R_{F}^{0} - \theta_{0}} \right)^{\frac{1}{\alpha-1}}$. Appendix F shows that $WL > 0$, hence the allocation under limited liability is inefficient.

6.4 Choosing Parameters Values

I calibrate the open economy version with limited liability which is the one that I am interested in and in the situation of a binding collateral constraint. $R_{F}^{F}$ is calibrated following the suggestion of Kydland and Prescott (1982) and Prescott (1986) for the real interest rate in the US. $\beta$ is set to obtain $R_{F}^{F} - R_{B}^{B} = 0.03$ where the gap is the observed in the data for small open economies. The result is $\beta = 0.93$ is a relatively standard value in the literature.

<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter</th>
<th>Value</th>
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<tbody>
<tr>
<td>Discount parameter of domestic HHs</td>
<td>$\beta$</td>
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<tr>
<td>Capital’s shares in output</td>
<td>$\alpha$</td>
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<tr>
<td>Risk-free foreign interest rate</td>
<td>$R_{F}^{F}$</td>
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<td>Mean of log $Z_{1}$</td>
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<td>Std. Dev. of log $Z_{1}$</td>
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<td>Foreign borrowing collateral</td>
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<td>Initial level of bank equity</td>
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Table 2: Main variables values

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<thead>
<tr>
<th>Description</th>
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<tr>
<td>Credit to GDP ratio $K_0/(\bar{Z}K^\alpha)$</td>
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<td>Bank leverage $K_0/N_0$</td>
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<td>Foreign debt participation $D_0^F/(D_0 + D^F)$</td>
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<td>Probability of default of banks $p_0$</td>
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<tr>
<td>Inefficient additional marginal benefits $\theta_0$</td>
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</tbody>
</table>

The other five parameters, $\{\alpha, \phi, N_0, \mu_z, \sigma_z\}$, are set to make the following variables of the model under LL consistent with average data for small open economies, 2000 - 2013: the leverage ratio ($K_0/N_0$), the credit-to-gdp ratio ($K_0/(\bar{Z}K^\alpha)$), the foreign debt participation ratio ($D_0^F/(D_0 + D^F)$) and the bank default probability ($p_0$), see Table 3.

In particular, $\alpha$ is mainly set to match a credit to gdp ratio of 0.30. It results in $\alpha = 0.32$, very standard in the literature. $N_0$ is essentially calibrated to obtain a bank leverage equals to 9.2. $\phi$ is set to obtain foreign debt participation of 30%. $\mu_z$ and $\sigma_z$ are mainly set to make $p_0 = 0.034$ as in Collard 2016. This small value is consistent with the probability of a banking crisis built from the database of Leaven-Valencia (2013).

Table 3: Average ratios for country annual data, 2000-2013, where average $CC \leq 0.60$

<table>
<thead>
<tr>
<th>Group</th>
<th>HI</th>
<th>UMI</th>
<th>LMI</th>
<th>LI</th>
<th>LAC</th>
<th>MENA</th>
<th>SSA</th>
<th>ECA</th>
<th>EAP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Leverage</td>
<td>15.6</td>
<td>9.6</td>
<td>11.1</td>
<td>8.7</td>
<td>10.3</td>
<td>11.3</td>
<td>10.9</td>
<td>15.5</td>
<td>14.8</td>
</tr>
<tr>
<td>BankCred-GDP (%)</td>
<td>104</td>
<td>36.4</td>
<td>23.9</td>
<td>11.7</td>
<td>33.4</td>
<td>53.0</td>
<td>28.9</td>
<td>102</td>
<td>109</td>
</tr>
<tr>
<td>ForDebt Part (%)</td>
<td>46.1</td>
<td>32.7</td>
<td>51.7</td>
<td>17.7</td>
<td>42.1</td>
<td>64.3</td>
<td>17.7</td>
<td>38.7</td>
<td>60.8</td>
</tr>
<tr>
<td>Prob. BC (%) *</td>
<td>2.2</td>
<td>1.8</td>
<td>1.9</td>
<td>2.4</td>
<td>2.7</td>
<td>1.9</td>
<td>2.2</td>
<td>2.5</td>
<td>-</td>
</tr>
<tr>
<td>Prob. Being BC (%) *</td>
<td>8.5</td>
<td>5.5</td>
<td>4.9</td>
<td>7.8</td>
<td>8.2</td>
<td>5.0</td>
<td>6.3</td>
<td>8.8</td>
<td>-</td>
</tr>
</tbody>
</table>

7 Numerical Results

The figure 1 shows the results for main variables of an open economy under unlimited and limited liability. For illustrative purposes I allow \( \phi \) to take values from 0 to 0.35 and keep the other parameters as in the calibration in order to observe the solutions of the model when the foreign constraint binds (low values of \( \phi \)) and when the foreign constraint does not bind (high values of \( \phi \)).

For the calibration described in the previous section for the economy with limited liability and binding collateral constraint, by construction it presents a small probability of default, \( p_0 = 3.4\% \), which makes the inefficient additional marginal benefits of capital small, \( \theta = 2.2\% \). Then, the inefficient level of capital under LL is going to be only 3\% higher than the efficient one under ULL, and it results in very low domestic welfare losses: bank welfare under LL is just 0.014\% lower than the one under ULL.

In general, in both cases the Non-binding and binding constraint, the capital under limited liability is inefficiently higher. When the foreign collateral constraint binds, in both the ULL and LL cases the capital level is lower than when it does not bind, i.e. \( k_{dULL} < k_u^{ULL} \) and \( k_{dLL} < k_u^{LL} \). This is to the feature of emerging economies of \( R_0^B > R_0^F \).

A higher capital level when the foreign constraint does not bind leads to a higher bank obligations and higher probability of default, \( p_0 \). Hence, the size of the inefficiencies, \( \theta \), is higher and the welfare losses, \( WL \), are higher when the constraint does not bind as it is showed in figure 1.
Next, I show how the main variables and the distortions change upon changes of key parameters as the foreign collateral, $\phi$, the risk-free domestic interest rate, $R^B_0$, and the foreign interest rate, $R^F_0$.

Change of $\phi$: When the foreign constraint does not bind, for both the ULL and the LL cases, changes of $\phi$ are not going to affect in any way the equilibrium (a.1-5) and hence it does not affect the level of bank risk-taking and the welfare losses (a.6).

When the foreign constraint binds, under ULL changes of $\phi$ does not affect capital (a.1), but there is a perfect substitution of domestic debt (a.2) for foreign debt (a.3). For instance, If the foreign collateral $\phi$ increases, $D^F_0$ increases and $D_0$ decreases in the same magnitude. Under LL, changes of $\phi$ affects capital, a higher $\phi$ reduces the size of the obligations for a given capital by substituting expensive domestic debt (a.2) by cheap foreign debt (a.3). It results in a lower default probability, $p_0$, (a.4) and then in a lower inefficient additional marginal benefits, $\theta$, (a.5) and in a lower excessive capital and hence lower domestic welfare losses, $WL$, (a.6). Hence, when the constraint binds under LL a higher $\phi$ indirectly increases the marginal cost of capital and reduces the excessive bank risk-taking.\footnote{The lower capital reduces even further the inefficient additional marginal benefits, $\theta$.}

Change of $R^F_0$: When the foreign constraint does not bind, under ULL changes of $R^F_0$ affects directly the cost of capital since the marginal debt in equilibrium is foreign debt.
For example, a higher $R^F_0$ increases the marginal cost of capital and hence capital and foreign debt decrease (b.1)-(b.3). Under LL, a higher $R^F_0$ affects directly and indirectly the marginal cost of capital. The indirect effect occurs due to the presence of the inefficient additional marginal benefits of capital, $\theta$: A higher $R^F_0$ reduces indirectly the marginal cost of capital by pushing it down by increasing the size of the obligations, the probability of default and then the inefficient additional marginal benefits of capital, $\theta$. It affects directly the marginal cost of capital and push the capital level up. The direct effect dominates, and hence the marginal cost of capital increases and capital decreases (b.1), in equilibrium the probability of default decreases (a.4) but $\theta$ slightly increases (b.5) and domestic welfare losses decreases (b.6).

When the constraint binds, under ULL changes of $R^F_0$ do not affect capital, domestic and foreign debt (b.1-3) since it does not alter the marginal cost of capital either directly or indirectly. It only increases domestic welfare. Under LL, changes of $R^F_0$ only affect indirectly the marginal cost of capital through $\theta_0$ since the marginal debt is domestic one. A higher $R^F_0$ increases the bank obligations and increases the default probability, $p_0$, (b.4) and hence it increases the size of the distortions, $\theta$, (b.5) and increases the welfare losses, $WL$, (b.6). Finally, $R^F_0$ inefficiently increase the level of capital and hence inefficiently increase the bank risk-taking.

Hence, under this realistic calibration a higher $R^F_0$ increases always the excessive bank risk-taking and the excessive level of capital and this positive effect is higher when the foreign collateral constraint binds, but only decreases the level of capital and welfare losses when the constraint does not bind.

**Change of $R^B_0$:** When the foreign constraint does not bind, under both the ULL and the LL cases changes of $R^B_0$ does not affect the equilibrium of any variable (c.1-4), and hence it does not affect the excessive bank risk-taking (c.5). However, it affects the domestic welfare losses (c.6). For instance, a higher $R^B_0$ or a lower domestic discount factor value less the higher production un the LL case and hence there is higher welfare losses.

When the constraint binds, under ULL changes of $R^B_0$ affects directly the marginal cost of capital since the marginal debt in equilibrium is domestic one. Clearly a higher $R^B_0$ increases increases the marginal cost and reduces the level of capital (c.1). Since foreign debt is fixed (c.3), domestic debt decreases (c.2). Under LL, a higher $R^B_0$ has a direct and indirect effect on the marginal cost of capital. The direct effect pushes up the marginal cost of capital and pushes down the capital level as in the ULL cases. The indirect effect occurs through $\theta$, in particular a higher $R^B_0$ increases the size of bank obligations and then the probability of default is higher and inefficient additional marginal benefits are higher, $\theta$. Hence, it indirect effect pushes down the marginal cost of capital. The direct effect dominates, and capital finally decreases (c.1), this lower level of capital reduces the
bank obligations and the default probability, $p_0$, (c.4) and hence it reduces the size of the inefficiencies, $\theta$, (c.5) and reduces the welfare losses (c.6).\footnote{This lower $\theta$ due to a lower capital, increase by more the marginal cost of capital and reduce the level of capital even further.} Hence, a higher $R_0^B$ reduces the bank risk-taking both its efficient and its inefficient components.

Finally, a higher $R_0^B$ decreases the excessive risk taking when the constraint binds and does not affect this when the constraint does not bind, (c.5).

Figure 2: Elasticities of main variables
To sum up under this realistic calibration if the foreign constraint does not binds banks fund all its credit activity with equity and foreign funds since $R_F^0 < R_B^0$. Hence, changes of $R_F^0$ affects the marginal cost of capital directly and indirectly. It affect directly since the marginal deposits is foreign one. And it will affect indirectly through the inefficient additional marginal benefits due to the limited liability. The direct effect dominates and hence after a lower foreign interest rate, capital increases. In a second round effect the higher capital increases the default probability, but the inefficient additional marginal benefits, $\theta_0$, does not increases since the effective foreign interest rate of debt is going to decrease enough to make $\theta_0$ lower.

However, when the constraint binds, banks fund their credit activities with foreign and domestic funds. Hence, in equilibrium changes of $R_F^0$ does not affect directly the marginal cost of credit since the marginal deposit is domestic one. But, it affect the marginal cost indirectly by affecting the inefficient additional marginal benefit. Hence, a lower $R_F^0$ reduces the probability of default, the marginal costs the are not internalized due to LL are lower reducing the inefficient excessive credit and the excessive bank risk-taking in the economy.

Finally, when the foreign collateral constraint is binding and the economy is more open, the probability of defaults become smaller and the excessive bank risk-taking becomes smaller since the inefficient additional marginal benefit, $\theta_0$. When the foreign constraint does not bind, $\theta_0$ has not effect on the excessive bank risk taking or in any
8 Implementation of the Efficient Allocation

In this section I aim to restore the socially efficient allocation by introducing some macro-prudential policies, first, in an closed economy and, second, in an open economy which is the main interest of this paper. This is to observe the difference between the optimal macro-prudential policy for a closed and open economy.

8.1 Closed Economy:

In a closed economy a simple capital requirement constraint (CRC) restores the efficient allocation. Let say that the individual bank is going to face the following CRC,

\[ \kappa K_0 \leq N_0, \]  \hspace{1cm} (30)

where \( \kappa \in (0, 1) \) is the capital requirement ratio (CRR). By (30) a fixed fraction, \( \kappa \), of total credit must be lower than the exogenous initial equity. If the CRC binds, the capital is immediately determined by \( \kappa \) and \( N_0 \). Clearly, the optimal CRR is \( \kappa^* = N_0/K_0^{ULL} \), where the superscript \( \text{ULL} \) refers to the solution of the unlimited liability case. Since the level of capital (in the limited liability case) is going to be inefficiently higher, the optimal CRC is binding by construction. Hence, this optimal CRR leads to the efficient allocation.

8.2 Open Economy:

In an open economy, the optimal capital requirement leads to the efficient level of capital and to the efficient liability composition. As in the closed economy I impose the CRC, equation (30), for each bank. However, recall that in the open economy, \( K_0 = D_0 + D_0^F + N_0 \).

When the foreign constraint is binding, in particular if \( \phi < \phi^{LL}_d \), the optimal capital requirement ratio is

\[ \kappa = \frac{N_0}{k_d^{ULL}} = N_0 \left( \frac{\bar{Z} \alpha}{R_0^B} \right)^{1/\alpha}. \]

Since the level of capital is inefficiently high under LL, the CRC clearly binds. In addition, since the borrowing constraint for foreign debt is binding, i.e. \( D_0^F = \phi \), the optimal CRC implies a reduction of the domestic deb participation and hence this policy restores the efficient level of capital and liability composition simultaneously.
When the foreign constraint is not binding, in particular if $\bar{\phi}^{ULL} < \phi$, the optimal capital requirement ratio is

$$\kappa = \frac{N_0}{k_u^{ULL}} = N_0 \left( \frac{\bar{Z}_\alpha}{R_0^F} \right)^{1-\alpha}.$$  

Again this policy restore the efficient level of capital and the liability composition simultaneously.

In general, note that $\kappa$ is weakly decreasing on $\phi$ and is weakly increasing on $R_0^F$. The optimal capital requirements depends on the economic conditions. Lower capital requirements will be asked in economies with good economic conditions as higher $\bar{Z}$. Also economies with higher access to foreign resources that allow them to borrow from abroad as much as they want, present a lower capital requirement ratio.

For completeness if $\bar{\phi}^{LL}_d \leq \phi \leq \bar{\phi}^{ULL}_u$, the allocations for domestic and foreign debts is the same under both the limited and Unlimited liability. Hence, there is not need of policy intervention.

9 Discussion: Some interesting issues

Unanticipated Ex-post Capital Injection: In this simple model bank’s owner (domestic households) has not incentives to inject capital at $t = 1$ to avoid bank defaulting. Even the government has not incentives to provide capital injection at $t = 1$ since it will not improve domestic welfare since what determines the level of domestic welfare are the decisions taken at $t = 0$. Hence, in this two period model the inefficiencies created by the limited liability and the fact that the individual bank does not internalize the effects of their decisions on the interest rates cannot be eliminated by unanticipated ex-post policies.

When the ex-post capital injection is anticipated the inefficiency increases. The capital which is excessively high under limited liability becomes even more higher when the agents anticipate the ex-post capital injection. This is because in equilibrium interest rates in the market are the same than the risk-free one, and do not increases since depositors know ex-ante that if they bank default they will be always fully paid. This is in line with the argument that some ex-post policies as capital injection can motivate banks to take more excessive risk and hence reduce welfare.

The exogenous initial equity ($N_0$): In this model it is possible to notice that a higher initial equity, $N_0$, decreases the inefficiencies since the probability of default becomes smaller and then $\theta$ becomes smaller. Hence, at some point the higher the $N_0$
makes yield that bank never default and hence the unlimited liability is not longer binding and the welfare loses will drop to zero.

Regarding the effects of capital requirements, in this simple binding capital requirements can be only satisfied by reducing the level of loans. However, in an infinity period bank equity might become endogenous. Hence, to satisfy the capital requirement constraint the individual bank might reduce the level of capital or to ex-ante ensure a higher equity.

Different seniority assumption: In an open economy with limited liability and a binding foreign collateral constraint it is possible to modify item (ii) of the recovery structure assumptions and to assume that domestic and foreign debts have different seniorities. For instance, if the domestic debt is paid first, then

$$x^D_1 = \min \left\{ 1, \frac{Z_1 K_0^\alpha}{R_0^D D_0} \right\}, \quad x^F_1 = \min \left\{ 1, \max \left\{ 0, \frac{Z_1 K_0^\alpha - \tilde{R}_0^D D_0}{R_0^F D_0^F} \right\} \right\},$$

and if the foreign debt is paid first, then

$$x^F_1 = \min \left\{ 1, \frac{Z_1 K_0^\alpha}{R_0^F D_0^F} \right\}, \quad x^D_1 = \min \left\{ 1, \max \left\{ 0, \frac{Z_1 K_0^\alpha - \tilde{R}_0^F D_0^F}{R_0^D D_0} \right\} \right\},$$

Note that these always satisfy $0 = Z_1 K_0^\alpha - \tilde{R}_0^D D_0 - \tilde{R}_0^F D_0^F$ when the bank defaults. Regarding the results of the model, note that under unlimited liability capital is independent of the seniority assumption.

In terms of the inefficiency, the higher the seniority of foreign debt, the lower the size of the inefficiency, $\theta$, and hence the lower the capital and the welfare losses, and vice-verse, Proof in Appendix G. The explanation is the following: When foreign debt has higher seniority, there are less resources available to pay the domestic debtors, hence for a given capital level the effective return of domestic debt when the bank defaults, $x^D_1 \tilde{R}_0^D$, become smaller and then $\theta$ becomes smaller. Hence, the level of capital becomes lower or closer to the unlimited liability case.

Regarding item (i) of the recovery structure assumptions It is possible to assume that in case of default the available income of banks can be distributed among depositors in proportion of the value of the debt at time $t=0$ and not in proportion to the face value of the debt as it is done in the model. This will not affect the key results of the model.

10 Conclusions

I develop a two-period partially open economy model with domestic and foreign investors who have different opportunity costs, the foreign one has a lower opportunity cost, and cannot lend to each other or invest in risky assets directly, they only can intermediate
resources through the financial intermediaries.

The novelty in this model is to assess the externality that appears in an open economy when banks have limited liability and private agents do not internalize the effects of their decisions on the interest rates. These two features makes that individual bank underestimate the marginal cost of funding which leads to an inefficiently higher supply of loans. In addition, under a realistic calibration a lower foreign interest rate always reduce the excessive bank risk-taking, the reduction is more significant when the constraint binds, and the excessive level of capital, but only when the constraint does not bind it increases the level of capital. And a higher access to foreign funds always reduces the excessive bank risk-taking and the excessive level of capital.

Finally, in this two period model with equity exogenous a macro-prudential policy oriented to reduce the level of risky investment automatically restore the efficient liability composition.

References


A Banks manipulate the interest rates: Closed Economy

Recall the objective function for the bank facing limited liability is
\[ V_0 = E_0\{\beta(max\{0, Z_1K_0^\alpha - \bar{R}_0^D D_0\})\}. \]
which can be rewritten as
\[ V_0 = \int_{Z^*}^{\infty} \beta(Z_1K_0^\alpha - \bar{R}_0^D D_0)dF(Z_1) \]
Recall that in equilibrium the domestic gross interest rate satisfies \( R_0^B = E_0\{x_1^D\bar{R}_0^D\} \), that can be rewritten as
\[ R_0^B = \int_{0}^{Z^*} \frac{Z_1K_0^\alpha}{R_0^D D_0} \bar{R}_0^D dF(Z_1) + \int_{Z^*}^{\infty} \bar{R}_0^D dF(Z_1) \]
Hence, if the bank can manipulate the interest rates, it internalizes de effects of their decisions on the interest rate, it consider the above condition for the domestic interest rate. Hence, considering the above condition, the expected present value of future dividends can be rewritten as
\[ V_0 = \int_{Z^*}^{\infty} \beta Z_1K_0^\alpha dF(Z_1) + \beta \left( \int_{0}^{Z^*} \frac{Z_1K_0^\alpha}{R_0^D D_0} \bar{R}_0^D dF(Z_1) - R_0^B \right) D_0. \]
Then, it results
\[ V_0 = E_0\{\beta(Z_1K_0^\alpha - R_0^B D_0)\}. \]
which is the same objective function of the bank when there is unlimited liability, equation (2), when considering the fact that the domestic debt interest rate is risk-free, \( \bar{R}_0^D = R_0^B \). Hence, the optimality condition is going lead to the efficient domestic debt allocation.

B Banks manipulate the interest rates: Open Economy & NonBinding Constraint

Recall the objective function for the bank facing limited liability is
\[ V_0 = E_0\{\beta(max\{0, Z_1K_0^\alpha - \bar{R}_0^F D_0\})\}. \]
which can be rewritten as
\[ V_0 = \int_{Z^*}^{+\infty} \beta (Z_1 K_0^\alpha - \bar{R}_0^D D_0) dF(Z_1) \]

Recall that in equilibrium the foreign gross interest rate satisfies \( R_0^F = E_0 \{ x_1^F \} \bar{R}_0^F \), that can be rewritten as
\[ R_0^F = \int_0^{Z^*} \frac{Z_1 K_0^\alpha}{\bar{R}_0^F D_0^F} \bar{R}_0^F dF(Z_1) + \int_{Z^*}^{+\infty} \bar{R}_0^F dF(Z_1). \]

Hence, if the bank can manipulate the interest rates, it internalizes the effects of their decisions on the interest rate, it consider the above condition for the foreign interest rate. Hence, considering the above condition, the expected present value of future dividends can be rewritten as
\[ V_0 = \int_{Z^*}^{+\infty} \beta Z_1 K_0^\alpha dF(Z_1) + \beta \left( \int_0^{Z^*} \frac{Z_1 K_0^\alpha}{\bar{R}_0^D D_0^D} \bar{R}_0^D dF(Z_1) - R_0^B \right) D_0^F. \]

Then, it results
\[ V_0 = E_0 \{ \beta (Z_1 K_0^\alpha - R_0^F D_0^F) \}. \]

which is the same objective function of the bank when there is unlimited liability, equation (19), when considering the fact that the foreign debt interest rate is risk-free, \( \bar{R}_0^F = R_0^F \). Hence, the optimality condition is going lead to the efficient domestic debt allocation.

C Banks manipulate the interest rates: Open Economy & Binding Constraint

Recall the objective function for the bank facing limited liability is
\[ V_0 = E_0 \{ \beta (\max \{ 0, Z_1 K_0^\alpha - \bar{R}_0^D D_0 - \bar{R}_0^F D_0^F \} ) \}. \]

which can be rewritten as
\[ V_0 = \int_{Z^*}^{+\infty} \beta (Z_1 K_0^\alpha - \bar{R}_0^D D_0 - \bar{R}_0^F D_0^F) dF(Z_1) \]

Recall that in equilibrium the domestic and foreign gross interest rates satisfy \( R_0^B = E_0 \{ x_1^D \} \bar{R}_0^D \) and \( R_0^F = E_0 \{ x_1^F \} \bar{R}_0^F \) respectively which can be rewritten as
\[ R_0^B = \int_0^{Z^*} \frac{Z_1 K_0^\alpha}{\bar{R}_0^D D_0 + \bar{R}_0^F D_0^F} \bar{R}_0^D dF(Z_1) + \int_{Z^*}^{+\infty} \bar{R}_0^D dF(Z_1). \]
\[ R_0^F = \int_0^{Z^*} \frac{Z_1K_0^K}{R_0^D + R_0^F} R_0^F dF(Z_1) + \int_{Z^*}^{+\infty} R_0^F dF(Z_1). \]

Hence, if the bank can manipulate the interest rates, it internalizes the effects of their decisions on the interest rate, it consider the above condition for the domestic interest rate. Hence, considering the above condition, the expected present value of future dividends can be rewritten as

\[ V_0 = \int_{Z^*}^{+\infty} \beta Z_1K_0^K dF(Z_1) + \beta \left( \int_0^{Z^*} x_1^D R_0^D dF(Z_1) - R_0^D \right) D_0 + \beta \left( \int_0^{Z^*} x_1^F R_0^F dF(Z_1) - R_0^F \right) D_0^F. \]

Then, it results

\[ V_0 = E_0 \{ \beta (Z_1K_0^K - R_0^B D_0 - R_0^F D_0^F) \}. \]

which is the same objective function of the bank when there is unlimited liability, equation (23), when considering the fact that the domestic and foreign debt interest rates are risk-free, \( \bar{R}_0^D = R_0^B \) and \( \bar{R}_0^F = R_0^F \). Hence, the optimality condition is going to lead to the efficient domestic debt allocation.

## D Domestic Welfare Losses: Closed Economy

Recall that under unlimited liability,

\[ V_0^{ULL} = E_0 \{ \beta (Z_1(K_0^{ULL})^\alpha - R_0^B D_0^{ULL}) \}. \]

and under limited liability,

\[ V_0^{LL} = E_0 \{ \beta (\max(0,Z_1(K_0^{LL})^\alpha - \bar{R}_0^D D_0^{LL})) \}. \]

And recalling that

\[ R_0^B = \int_0^{Z^*} \frac{Z_1K_0^K}{R_0^D} R_0^D dF(Z_1) + \int_{Z^*}^{+\infty} R_0^D dF(Z_1). \]

Then, using the above information I rewrite \( V_0^{LL} \) as

\[ V_0^{LL} = E_0 \{ \beta (Z_1(K_0^{LL})^\alpha - R_0^B D_0^{LL}) \}. \]

Hence,

\[ WL_0 = V_0^{ULL} - V_0^{LL} = \beta \bar{Z} \left[ (K_0^{ULL})^\alpha - (K_0^{LL})^\alpha \right] - (D_0^{ULL} - D_0^{LL}). \]

Then,

\[ WL_0 = V_0^{ULL} - V_0^{LL} = \beta \bar{Z} \left[ (K_0^{ULL})^\alpha - (K_0^{LL})^\alpha \right] - (K_0^{ULL} - K_0^{LL}). \]
Recalling the expressions for $K_{0}^{ULL}$ and $K_{0}^{LL}$, $WL_0$ is rewritten as

$$WL_0 = \beta \tilde{Z} \left[ \left( \frac{\alpha \tilde{Z}}{R_{0}^{B}} \right)^{\frac{1}{1-\alpha}} - \left( \frac{\alpha \tilde{Z}}{R_{0}^{B} - \theta_0} \right)^{\frac{1}{1-\alpha}} \right] - \left[ \left( \frac{\alpha \tilde{Z}}{R_{0}^{B}} \right)^{\frac{1}{1-\alpha}} - \left( \frac{\alpha \tilde{Z}}{R_{0}^{B} - \theta_0} \right)^{\frac{1}{1-\alpha}} \right].$$

Then,

$$WL_0 = \left( \frac{\alpha \tilde{Z}}{R_{0}^{B}} \right)^{\frac{1}{1-\alpha}} \left\{ \beta \tilde{Z} \left[ \frac{R_{0}^{B}}{\alpha \tilde{Z}} - \frac{R_{0}^{B} - \theta_0}{\alpha \tilde{Z}} \left( \frac{R_{0}^{B}}{R_{0}^{B} - \theta_0} \right)^{\frac{1}{1-\alpha}} \right] - \left[ 1 - \left( \frac{R_{0}^{B}}{R_{0}^{B} - \theta_0} \right)^{\frac{1}{1-\alpha}} \right] \right\}. $$

Then,

$$WL_0 = \left( \frac{\alpha \tilde{Z}}{R_{0}^{B}} \right)^{\frac{1}{1-\alpha}} \left\{ \frac{1}{\alpha} - \frac{R_{0}^{B} - \theta_0}{\alpha R_{0}^{B}} \left( \frac{R_{0}^{B}}{R_{0}^{B} - \theta_0} \right)^{\frac{1}{1-\alpha}} - 1 + \left( \frac{R_{0}^{B}}{R_{0}^{B} - \theta_0} \right)^{\frac{1}{1-\alpha}} \right\}. $$

Finally, I rewrite $WL_0$ as

$$WL_0 = \left( \frac{\alpha \tilde{Z}}{R_{0}^{B}} \right)^{\frac{1}{1-\alpha}} \left\{ \left( \frac{1}{\alpha} - 1 \right) - \left( \frac{1}{\alpha} y^{\alpha} - y \right) \right\}. $$

where $y = \left( \frac{R_{0}^{B}}{R_{0}^{B} - \theta_0} \right)^{\frac{1}{1-\alpha}}$. Hence, it is easy to see that the $y$ that maximizes $(\frac{1}{\alpha} y^{\alpha} - y)$ is $y = 1$. In this case, $WL_0 = 0$. However, since $y > 1$, then $(\frac{1}{\alpha} - 1) - (\frac{1}{\alpha} y^{\alpha} - y) > 0$, and then $WL_0 > 0$.

**E Domestic Welfare Losses: Open Economy & Non-Binding Constraint**

Recall that under unlimited liability,

$$V_{0}^{ULL} = E_{0} \{ \beta \left( Z_{1}(K_{0}^{ULL})^{\alpha} - R_{0}^{F} D_{0}^{F,ULL} \right) \}. $$

and under limited liability,

$$V_{0}^{LL} = E_{0} \left\{ \beta \left( \max(0, Z_{1}(K_{0}^{LL})^{\alpha} - R_{0}^{F} D_{0}^{F,LL}) \right) \right\}. $$

And recalling that

$$R_{0}^{F} = \int_{0}^{Z^*} \frac{Z_{1} K_{0}^{\alpha}}{R_{0}^{B} D_{0}^{F}} R_{0}^{F} dF(Z_{1}) + \int_{Z^*}^{+\infty} R_{0}^{F} dF(Z_{1}). $$

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Then, using the above information I rewrite \( V_0^{LL} \) as

\[
V_0^{LL} = E_0 \{ \beta \left( Z_1 (K_0^{LL})^\alpha - R_0^F D_0^{F,LL} \right) \}.
\]

Hence,

\[
WL_0 = V_0^{ULL} - V_0^{LL} = \beta \bar{Z} \left[ (K_0^{ULL})^\alpha - (K_0^{LL})^\alpha \right] - \beta R_0^F (D_0^{F,ULL} - D_0^{F,LL}).
\]

Then,

\[
WL_0 = V_0^{ULL} - V_0^{LL} = \beta \bar{Z} \left[ (K_0^{ULL})^\alpha - (K_0^{LL})^\alpha \right] - \beta R_0^F (K_0^{ULL} - K_0^{LL}).
\]

Recalling the expressions for \( K_0^{ULL} \) and \( K_0^{LL} \), \( WL_0 \) is rewritten as

\[
WL_0 = \beta \bar{Z} \left[ \left( \frac{\alpha \bar{Z}}{R_0^F} \right)^{\frac{1}{\alpha}} - \left( \frac{\alpha \bar{Z}}{R_0^F - \theta_0} \right)^{\frac{1}{\alpha}} \right] - \beta R_0^F \left[ 1 - \left( \frac{R_0^B}{R_0^F - \theta_0} \right)^{\frac{1}{\alpha}} \right].
\]

Then,

\[
WL_0 = \left( \frac{\alpha \bar{Z}}{R_0^F} \right)^{\frac{1}{\alpha}} \left\{ \beta \bar{Z} \left[ \frac{R_0^F}{\alpha \bar{Z}} - \frac{R_0^F}{R_0^F - \theta_0} \left( \frac{R_0^F}{R_0^F - \theta_0} \right)^{\frac{1}{\alpha}} \right] - \beta R_0^F \left[ 1 - \left( \frac{R_0^B}{R_0^F - \theta_0} \right)^{\frac{1}{\alpha}} \right] \right\}.
\]

Finally,

\[
WL_0 = \left( \frac{\alpha \bar{Z}}{R_0^F} \right)^{\frac{1}{\alpha}} \beta R_0^F \left\{ \left( \frac{1}{\alpha} - 1 \right) - \left( \frac{1}{\alpha} y^\alpha - y \right) \right\},
\]

where \( y = \left( \frac{R_0^F}{R_0^F - \theta_0} \right)^{\frac{1}{\alpha}} \). Hence, it is easy to see that the \( y \) that maximizes \( \left( \frac{1}{\alpha} y^\alpha - y \right) \) is \( y = 1 \). In this case, \( WL_0 = 0 \). However, since \( y > 1 \), then \( \left( \frac{1}{\alpha} - 1 \right) - \left( \frac{1}{\alpha} y^\alpha - y \right) > 0 \), and then \( WL_0 > 0 \).

### F Domestic Welfare Losses: Open Economy & Binding Constraint

Recall that under unlimited liability,

\[
V_0^{ULL} = E_0 \{ \beta \left( Z_1 (K_0^{ULL})^\alpha - R_0^B D_0^{ULL} - R_0^F D_0^{F,ULL} \right) \}.
\]
and under limited liability,

\[ V^{\text{LL}}_0 = E_0 \left\{ \beta \left( \max(0, Z_1(K^{\text{LL}}_0)^\alpha - \tilde{R}^D_0 D^{\text{LL}}_0 - \tilde{R}^F_0 D^{F,\text{LL}}_0) \right) \right\}. \]

And recalling that

\[ R^B_0 = \int_0^{Z^*} \frac{Z_1 K^{\alpha}_0}{R^D_0 D_0 + R^F_0 D^F_0} \tilde{R}^D_0 dF(Z_1) + \int_{Z^*}^{+\infty} \tilde{R}^D_0 dF(Z_1). \]

\[ R^F_0 = \int_0^{Z^*} \frac{Z_1 K^{\alpha}_0}{R^D_0 D_0 + R^F_0 D^F_0} \tilde{R}^F_0 dF(Z_1) + \int_{Z^*}^{+\infty} \tilde{R}^F_0 dF(Z_1). \]

Then, using the above information I rewrite \( V^{\text{LL}}_0 \) as

\[ V^{\text{LL}}_0 = E_0 \left\{ \beta \left( Z_1(K^{\text{LL}}_0)^\alpha - R^B_0 D^{\text{LL}}_0 - R^F_0 D^{F,\text{LL}}_0 \right) \right\}. \]

Hence,

\[ WL_0 = V^{\text{ULL}}_0 - V^{\text{LL}}_0 = \beta \tilde{Z} \left[ (K^{\text{ULL}}_0)^\alpha - (K^{\text{LL}}_0)^\alpha \right] - (D^{\text{ULL}}_0 - D^{\text{LL}}_0). \]

Then,

\[ WL_0 = V^{\text{ULL}}_0 - V^{\text{LL}}_0 = \beta \tilde{Z} \left[ (K^{\text{ULL}}_0)^\alpha - (K^{\text{LL}}_0)^\alpha \right] - (K^{\text{ULL}}_0 - K^{\text{LL}}_0). \]

Recalling the expressions for \( K^{\text{ULL}}_0 \) and \( K^{\text{LL}}_0 \), \( WL_0 \) is rewritten as

\[ WL_0 = \beta \tilde{Z} \left[ \left( \frac{\alpha \tilde{Z}}{R^B_0} \right)^{\frac{\alpha}{\alpha - 1}} - \left( \frac{\alpha \tilde{Z}}{R^B_0 - \theta_0} \right)^{\frac{\alpha}{\alpha - 1}} \right] - \left[ \left( \frac{\alpha \tilde{Z}}{R^B_0} \right)^{\frac{1}{\alpha - 1}} - \left( \frac{\alpha \tilde{Z}}{R^B_0 - \theta_0} \right)^{\frac{1}{\alpha - 1}} \right]. \]

Then,

\[ WL_0 = \left( \frac{\alpha \tilde{Z}}{R^B_0} \right)^{\frac{1}{\alpha - 1}} \left\{ \beta \tilde{Z} \left[ \frac{R^B_0}{\alpha R^B_0 - \theta_0} - \left( \frac{R^B_0}{R^B_0 - \theta_0} \right)^{\frac{1}{\alpha - 1}} \right] \right\} - \left[ 1 - \left( \frac{R^B_0}{R^B_0 - \theta_0} \right)^{\frac{1}{\alpha - 1}} \right]. \]

\[ WL_0 = \left( \frac{\alpha \tilde{Z}}{R^B_0} \right)^{\frac{1}{\alpha - 1}} \left\{ \frac{1}{\alpha} - \frac{R^B_0 - \theta_0}{\alpha R^B_0} \left( \frac{R^B_0}{R^B_0 - \theta_0} \right)^{\frac{1}{\alpha - 1}} \right\} - \left[ 1 - \left( \frac{R^B_0}{R^B_0 - \theta_0} \right)^{\frac{1}{\alpha - 1}} \right]. \]

Finally,

\[ WL_0 = \left( \frac{\alpha \tilde{Z}}{R^B_0} \right)^{\frac{1}{\alpha - 1}} \left\{ \left( \frac{1}{\alpha} - 1 \right) - \left( \frac{1}{\alpha} y^\alpha - y \right) \right\}, \]

where \( y = \left( \frac{R^B_0}{R^B_0 - \theta_0} \right)^{\frac{1}{\alpha - 1}} \). Hence, it is easy to see that the \( y \) that maximizes \( \left( \frac{1}{\alpha} y^\alpha - y \right) \) is \( y = 1 \). In this case, \( WL_0 = 0 \). However, since \( y > 1 \), then \( \left( \frac{1}{\alpha} - 1 \right) - \left( \frac{1}{\alpha} y^\alpha - y \right) > 0 \), and then \( WL_0 > 0 \).
G Different Seniority Assumption:

Recall that \( \theta_0 \) is

\[
\theta_0 = \int_0^{Z^*} \left( x_1^D \tilde{R}_0^D - \alpha Z_1 K_0^{\alpha-1} \right) dF
\]

and if the domestic debt is paid first, then

\[
x_1^D = \min \left\{ 1, \frac{Z_1 K_0^{\alpha}}{R_0^D D_0} \right\}, \quad x_1^F = \min \left\{ 1, \frac{\max\{0, Z_1 K_0^{\alpha} - \tilde{R}_0^D D_0\}}{R_0^F D_0} \right\},
\]

and if the foreign debt is paid first, then

\[
x_1^F = \min \left\{ 1, \frac{Z_1 K_0^{\alpha}}{R_0^F D_0} \right\}, \quad x_1^D = \min \left\{ 1, \frac{\max\{0, Z_1 K_0^{\alpha} - \tilde{R}_0^F D_0\}}{R_0^D D_0} \right\},
\]

Hence, if domestic debt has higher seniority, \( \theta_0 \) is rewritten as

\[
\theta_0 = \int_0^{Z^*} \left( \frac{Z_1 K_0^{\alpha}}{K_0 - \phi - N_0} - \alpha Z_1 K_0^{\alpha-1} \right) dF, \tag{31}
\]

and if domestic debt has higher seniority, \( \theta_0 \) is rewritten as

\[
\theta_0 = \int_0^{Z^*} \left( \frac{Z_1 K_0^{\alpha} - \tilde{R}_0^F D_0}{K_0 - \phi - N_0} - \alpha Z_1 K_0^{\alpha-1} \right) dF. \tag{32}
\]

Assuming that for a given capital, \( Z^* \) is not affected by the different seniority assumption, then clearly a higher foreign debt seniority leads to lower \( \theta_0 \) and then lower excessive capital. The lower excessive capital reduces \( Z^* \) which reduces even by more \( \theta_0 \).

H LL & Binding Foreign Collateral Constraint: Proof

\( \theta_0 > 0 \)

Recall

\[
\theta_0 = \int_0^{Z^*} \left( x_1^D \tilde{R}_0^D - \alpha Z_1 (K_0)^{\alpha-1} \right) dF.
\]

\( \theta \) can be rewritten,

\[
\theta_0 = \int_0^{Z^*} \left( \frac{Z_1 K_0^{\alpha}}{D_0 + \frac{R_0^F}{R_0^D} \phi} - \alpha Z_1 K_0^{\alpha-1} \right) dF.
\]

Recall \( R_0^B = E_0 \{ x_1^D \} R_0^D \) and \( R_0^F = E_0 \{ x_1^F \} R_0^F \) and under the assumption stated about the recovery ratio,

\[
\theta_0 = \int_0^{Z^*} \left( \frac{Z_1 K_0^{\alpha} R_0^B}{R_0^B D_0 + R_0^F \phi} - \alpha Z_1 K_0^{\alpha-1} \right) dF.
\]
Finally, $\theta$ can be rewritten as

$$\theta_0 = K_0^{a-1} \left( \frac{R^B_0 (1 - \alpha) + (R^B_0 - R^F_0) \phi \alpha + \alpha R^B_0 N_0}{R^B_0 D_0 + R^F_0 \phi} \right) \int_0^{Z^*} Z_1 dF.$$  

where clearly $\theta_0 > 0$. It can be also proved analytically when $D_0$ has higher seniority. However, when $D^F_0$ has higher seniority, it is not possible to prove analytically.

I Welfare Gains when opening the economy under ULL

A fair measure of the domestic welfare is the present expected value of the future bank dividends. In the closed economy this is

$$V^{ULL,C}_0 = \beta \left( \tilde{Z} \left( K^{ULL,C}_0 \right)^\alpha - R^B_0 D^{ULL,C}_0 \right) = \beta \left[ \tilde{Z} \left( \frac{\alpha \tilde{Z}}{R^B_0} \right)^\frac{\alpha}{1-\alpha} - \frac{R^B_0}{R^B_0} \left( \frac{\alpha \tilde{Z}}{R^B_0} \right)^\frac{1}{1-\alpha} - N_0 \right]$$

And in the open economy is

$$V^{ULL,O}_0 = \beta \left( \tilde{Z} \left( K^{ULL,O}_0 \right)^\alpha - R^F_0 D^{ULL,O}_0 \right) = \beta \left[ \tilde{Z} \left( \frac{\alpha \tilde{Z}}{R^F_0} \right)^\frac{\alpha}{1-\alpha} - \frac{R^F_0}{R^F_0} \left( \frac{\alpha \tilde{Z}}{R^F_0} \right)^\frac{1}{1-\alpha} - N_0 \right]$$

Then,

$$V^{ULL,O}_0 - V^{ULL,C}_0 = \beta \left[ \tilde{Z} \left( \frac{\alpha \tilde{Z}}{R^F_0} \right)^\frac{\alpha}{1-\alpha} - \frac{R^F_0}{R^F_0} \left( \frac{\alpha \tilde{Z}}{R^F_0} \right)^\frac{1}{1-\alpha} + R^B_0 \left( \frac{\alpha \tilde{Z}}{R^B_0} \right)^\frac{1}{1-\alpha} \right]$$

Then,

$$V^{ULL,O}_0 - V^{ULL,C}_0 = \left( \frac{\alpha \tilde{Z}}{R^F_0} \right)^\frac{1}{1-\alpha} \left[ (R^F_0 \beta)^{\frac{\alpha}{1-\alpha}} - 1 \right] \left[ \frac{1}{\alpha} - 1 \right].$$

Since by assumption $R^F_0 \beta < 1$, $V^{ULL,O}_0 - V^{ULL,C}_0 > 0$.  

Since by assumption $R^F_0 \beta < 1$, $V^{ULL,O}_0 - V^{ULL,C}_0 > 0$.  

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