Abstract

We present a Downsian model of political competition in which parties have a taste for ambiguous strategies. We show that office-seeking incentives discard those equilibria with ambiguous strategies even when ambiguity is attractive for the parties. Equilibrium fails to exist when ambiguity is highly rewarding and, in every other case, parties converge to the median voter position. Our result pinpoints to the uncertainty about voters’ preferences and to voters’ risk-acceptance attitudes as the two key factors that induce ambiguous strategies.

Keywords: Downsian competition; Strategic ambiguity; Lottery stands; Risk-aversion.

JEL classification numbers: D72.
1 Introduction

Ambiguity as a political party strategy was first mentioned by Downs (1957), assuring that in a two-party competition, "it increases the number of voters to whom a party may appeal." In the same vein, some authors refer to "catch-all parties" (Kirchheimer, 1966; Williams, 2009) as those that try to appeal to a broad group of voters by being vague about the preferences of the party leadership.

One of the advantages of ambiguous policy platforms, as opposed to a single policy, is that it can facilitate intra-party politics. That is, political parties may find it easier to agree on a wider set of close policies instead of reaching an agreement on a concrete policy platform. Moreover, a party can address a greater fraction of voters when proposing an ambiguous platform.

We propose a theoretical model in which political parties are office-seeking and have a taste for ambiguity within the framework of the Downsian model of political competition (Downs, 1957).\textsuperscript{1} We solve the proposed model using the concept of Nash equilibrium in weakly undominated strategies and analyze the extent to which ambiguous strategies are sustained in equilibrium.

Political parties in our model simultaneously propose ambiguous policy platforms in the form of lottery stands which assign a probability distribution over certain set of policies (see also Zeckhauser, 1969; Fishburn, 1972; Page, 1976; McKelvey, 1980). We account for discrete and continuous lottery stands, where the former are probability distributions over a discrete set of policies, and the latter are continuous probability distributions over an interval of the policy space. In contrast to Berger et al. (2000), parties propose not only a mean policy but also the variance of its policy platform.

Our main result is that office-seeking incentives discard the existence of equilibria with ambiguous strategies. We find that when office-seeking incentives are high in comparison to parties’ incentives for ambiguity, there is a unique equilibrium in which parties converge to the ideal policy of the median voter. That is, political parties sacrifice ambiguity when competing for votes. When office-seeking incentives are low in comparison to parties’ incentives for ambiguity, there is no equilibrium. The intuition for this last result is that, on the one hand, when parties propose ambiguous platforms with an equal expected value, the less ambiguous proposal gets more votes.

\textsuperscript{1}According to Downs (1957), if two office-seeking parties compete in the one-dimensional policy space, in the unique equilibrium, they locate at the median voter position.
On the other hand, when parties converge to a tying situation in which both propose the ideal policy of the median voter, there are incentives to deviate to an ambiguous platform. Only when the benefits from holding office are null, we find that in the unique equilibrium, parties propose ambiguous platforms.

Previous literature in strategic ambiguity is also related to our results. In particular, voters with risk acceptance attitudes is a key assumption which induces equilibria with ambiguous platforms (Shepsle, 1972; Aragonès and Postlewaite, 2002). The seminal contribution by Shepsle (1972) studies the case in which an incumbent that proposes a single policy competes against a challenger that proposes a lottery stand. He finds that when a majority of voters is risk-acceptant, the challenger chooses a lottery stand when the incumbent locates at the median. Aragonès and Postlewaite (2002) show equilibrium strategies in which parties are ambiguous. Their results rest on two assumptions: intensity of voters preferences over candidates, and some restrictions over the set of lottery stands. They show that when a Condorcet winner is not the first choice for a majority, it can be defeated by an ambiguous strategy. As shown by Shepsle (1972), such result is not possible when voters' preferences are risk-averse.²

There are other related contributions (Alesina and Cukierman, 1990; Aragonès and Neeman, 2000), in which equilibria with ambiguous platforms are deduced when combining two assumptions, parties’ uncertainty about the median voter and parties’ preferences for ambiguity. Alesina and Cukierman (1990) introduce a dynamic electoral model characterized by policy-motivated parties that are uncertain about the location of the median voter and by voters which are not fully informed about the preferences of the incumbent. Politicians, in this case, face a trade-off between the policies that maximize their choices of re-election and the party ideology. Aragonès and Neeman (2000) propose a two-stage political competition model. In the first stage, candidates choose ideologies and in the second, they choose their levels of ambiguity. As in our model, candidates benefit from winning the elections and from proposing ambiguous platforms. In contrast to our model, candidates are uncertain about the median voter position. When office-seeking incentives are sufficiently high, candidates choose unambiguous policies and locate at the median. When candidates assign a greater weight to ambiguity,

²There is recent empirical evidence that tries to estimate voters’ risk attitudes (see Tomz and Van Houweling (2009), Berinsky and Lewis (2007) and Morgenstern and Zeckmeuster (2011), among others).
they choose different ideologies and equal levels of ambiguity.

Our model neither accounts for parties’ uncertainty about voters nor for voters’ risk-acceptance attitudes, but we account for parties with a taste for ambiguous platforms. In this way, we predict the extent to which the combination of parties’ taste for ambiguity and office-holding incentives induce parties to propose ambiguous policy strategies. While most of the contributions to the literature consider continuous lotteries (see, e.g., Aragonès and Neeman, 2000; Berger et al. 2000; Laslier, 2006), we additionally allow for discrete lotteries. As far as we know, our proposal is the first which accounts for these two types of ambiguous strategies — discrete and continuous lottery stands —. Regarding voters’ attitude towards risk, voters in our model are indifferent or strictly prefer the certain expected value of a lottery, to the lottery itself, that is, we account for both types, risk-neutral and risk-averse voters.

The rest of the paper is organized as follows. In the next section we describe the model. Section 3 analyzes voters’ preferences over lottery stands. Section 4 describes equilibrium strategies. Section 5 concludes. We relegate all the proofs to the Appendix.

2 Model

A general election is going to be held in which, by majority voting, voters will elect one out of two political parties, Party A and Party B. There is a continuum of voters. Each voter $i$ has an ideal policy $x_i \in [0, 1]$. The ideal policies of the voters are distributed over the interval $[0, 1]$ according to a continuous and strictly increasing distribution function, which is common-knowledge. We denote the ideal policy of the median voter by $x_M \in [0, 1]$.

The platform of each political party can be a single policy $x \in [0, 1]$ or a lottery stand, which consists of a probability distribution over some policies in $[0, 1]$. For the sake of simplicity, we assume that lottery stands are uniform distributions.

Lottery stands are interpreted by voters as ambiguous policy proposals and every voter equally interprets the probability with which each party will implement each of its proposals once in office. Parties can propose two types of lottery stands: continuous and discrete.

Continuous lottery stands are continuous uniform distributions over subintervals of the policy space. Continuous lottery stands are characterized by
two parameters \((x, \varepsilon)\), where \(x \in [0, 1]\) is the mean of the lottery interval and \(\varepsilon\) is the level of ambiguity that defines the interval \([x - \varepsilon, x + \varepsilon]\) of possible implemented policy. The maximum level of ambiguity is denoted by \(\varepsilon^{\text{max}} = \frac{1}{2}\), which corresponds to a lottery stand centered in the midpoint of the unit interval \(x = \frac{1}{2}\) and such that its extreme policies coincide with the upper and lower bound of the unit interval. Therefore, the level of ambiguity that a party \(j \in \{A, B\}\) can propose is contained in the interval \(\varepsilon_j \in [0, \frac{1}{2}]\).

Discrete lottery stands are discrete uniform distributions over a finite number of policy stands in the interval \([0, 1]\), with the particular simplifying assumption that every two adjacent policies gather the same distance. Thus, discrete lottery stands are characterized by a set of \(m\) policies \(X = \{x_1, x_2, ..., x_m\}\) where each single policy belongs to the policy space \([0, 1]\) and where policies can be ordered as follows: \(x_1 < x_2 < ... < x_m\). The smallest proposal (or the leftist) is \(x_1\) and, in the opposite side, the greatest (or rightist) proposal is \(x_m\). As already mentioned, every two adjacent policies gather the same distance, which implies that \(x_2 - x_1 = x_3 - x_2 = ... = x_m - x_{m-1}\).

Thus, if the number of policies \(m\) is an odd number, the mean policy is one of the policies in \(X\) (for example, if \(m = 3\), then the mean policy is \(x_2\)). However, if the number of policies is even then, the mean policy does not belong to the set \(X\). In order to characterize a discrete lottery stand, we use three parameters \((m, x, \varepsilon)\) where \(m\) is an integer number which indicates the number of policies, policy \(x\) is the mean policy which coincides with one of the policies in the lottery when \(m\) is odd and it does not coincide when \(m\) is even, and finally \(\varepsilon\) is the level of ambiguity which satisfies that \(x - x_1 = x_m - x = \varepsilon\).

We use notation \(L_m(x, \varepsilon)\) to represent a discrete lottery stand with \(m\) proposals centered around policy \(x\) and where \(\varepsilon\) measures the distance between the mean and the extreme policies in the lottery stand. For example, the lottery stand \(L_2(x, \varepsilon)\) consists of two policies \(\{x - \varepsilon, x + \varepsilon\}\), where each of them gathers equal probability. The lottery stand \(L_3(x, \varepsilon)\) consists of three policies \(\{x - \varepsilon, x, x + \varepsilon\}\) where each of them gathers equal probability, and so on and so forth. A particular case is \(m = 1\) where the lottery stand is a degenerate distribution, \(\varepsilon = 0\), and this is denoted by \(L_1(x, 0)\). Note that when a party proposes a single policy, there is no uncertainty regarding the policy that this party will implement in the case of holding office. When \(m \to \infty\), the proposed lottery is continuous. In this case, we use notation \(L_C(x, \varepsilon)\) to

\[\text{Related to this assumption, Aragonès and Xefteris (2014) propose a policy space with equidistant policies.}\]
represent the continuous lottery stand centered in \( x \) and with a level of ambiguity \( \varepsilon \). To simplify notation, when possible, we write \( L_m \) when referring to discrete lottery stands and \( L_C \) when referring to continuous lottery stands.

Political parties do not only care about winning the elections but also about their proposed level of ambiguity \( \varepsilon \). When proposing a lottery stand instead of a single policy, the political party represents a wider range of the policy space and this entails certain benefits in terms of internal stability among its members (Alesina and Cukierman, 1990; Aragonès and Neeman, 2000).\(^4\) Besides, parties derive benefits from holding office when winning the elections.

Let \( \mathbb{L} \) denotes the space containing every lottery stand or degenerate lottery that a party can propose, i.e., in terms of a game form, \( \mathbb{L} \) is the strategy space of Party A and Party B. Both parties simultaneously announce their strategies, after which elections are held and voters cast their ballots. The party holding office is the one achieving a majority of votes and, in the case of a tie, both parties face an equal probability of holding office. Once in office, the party can implement one of the policy proposals included in its announced lottery.

Given two lottery stands, the one proposed by Party A and that of Party B, \((L^A, L^B) \in \mathbb{L}^2\), the preferences of each party \( j \in \{A, B\} \) over these proposals are represented by the following utility function \( v_j \):

\[
v_j(L^A, L^B) = \begin{cases} 
\varepsilon_j + \alpha & \text{if Party } j \text{ wins} \\
\varepsilon_j + \frac{\alpha}{2} & \text{if parties tie} \\
\varepsilon_j & \text{if Party } j \text{ loses}
\end{cases}
\]

(1)

where \( \varepsilon_j \in [0, \frac{1}{2}] \) is the level of ambiguity proposed by Party \( j \) and \( \alpha \geq 0 \) represents the benefits derived from holding office. In the case of winning and when \( \alpha > 0 \), the party derives benefits not only from winning but also from its level of ambiguity. In the case of a tie, both political parties gather an equal probability of holding office and therefore, parties just account for the value \( \frac{\alpha}{2} \) plus their proposed level of ambiguity. In the case of losing the elections, the parties also derive a positive payoff when holding certain level of ambiguity.

The preferences of each voter \( i \) over single policies are represented by the negative absolute distance between the policy proposals and the ideal

\(^4\)Our results are robust to the case of including the number of policies \( m \) as an extra-benefit in the parties’ utility function.
policy of the voter, $x_i$. In this way, the preferences of voters are single-peaked over the policy space and can be represented by the utility function $u_i(x) = -|x - x_i|$. Preferences of voters over discrete lottery stands are measured by the von Neumann-Morgenstern utility representation so that the expected utility is

$$U_i(L_m) = E_{L_m}[u_i(x)] = \frac{1}{m} \sum_{x_n \in X_m} u_i(x_n) = -\frac{1}{m} \sum_{n=1}^{m} |x_n - x_i|$$  (2)

where note that every policy is weighted by voters with an equal probability.

In the limit, when $m \to \infty$, the lottery stand is the uniform distribution over the policies in the interval $[x - \varepsilon, x + \varepsilon]$, characterized by the uniform density function $f(x_n) = \frac{1}{2\varepsilon}$ for every $x_n \in [x - \varepsilon, x + \varepsilon]$. In this case, the von Neumann-Morgenstern utility representation over the continuous lottery yields the following expected utility

$$U_i(L_C) = E_{L_C}[u_i(x)] = \int_{x-\varepsilon}^{x+\varepsilon} u_i(x_n) f(x_n) dx_n = -\int_{x-\varepsilon}^{x+\varepsilon} \frac{|x_n - x_i|}{2\varepsilon} dx_n.$$  (3)

Given the strategy of the parties $(L^A, L^B)$, agent $i$ votes for Party A when $U_i(L^A) > U_i(L^B)$, votes for Party B when $U_i(L^A) < U_i(L^B)$ and abstains from voting when $U_i(L^A) = U_i(L^B)$. Another interpretation of this latter case is that citizens are not motivated to vote when parties’ platforms are not substantially different (Hortala-Vallve and Esteve-Volart, 2011).

When elections are held, the party holding office is the one achieving a majority of votes. In the case of a tie, both parties face an equal probability of winning. When both parties propose the same strategy $L^A = L^B$, we assume that there is a tie.

We say that $(L^A, L^B)$ is an equilibrium when none of the political parties can benefit from unilateral deviations of its strategy and moreover, both strategies, $L^A$ and $L^B$, are weakly undominated.

Formally, the pair of strategies $(L^A, L^B) \in \mathbb{L}^2$ is an equilibrium when i) and ii) are satisfied:
i) $v_A(L^A, L^B) \geq v_A(L'^A, L^B)$ for every $L'^A \in \mathbb{L}$ and $v_B(L^A, L^B) \geq v_B(L^A, L'^B)$ for every $L'^B \in \mathbb{L}$,

ii) $\exists L' \in \mathbb{L}$ such that $v_A(L'^A, L) \geq v_A(L^A, L)$ for every $L \in \mathbb{L}$ with strict
inequality for some $L$, and $\exists L^B \in \mathbb{L}$ such that $v_B(L, L^B) \geq v_B(L^B, L)$ for every $L \in \mathbb{L}$ with strict inequality for some $L$.

Requirement (i) implies that $(L^A, L^B)$ is a Nash Equilibrium, and requirement (ii) implies that neither $L^A$ nor $L^B$ are weakly dominated strategies. That is, the proposed equilibrium concept is Nash equilibrium in weakly undominated strategies.

### 3 Voting over lottery stands

In this section, we evaluate the preferences of voters over different lottery stands. As an example, consider the comparison between two lotteries: $L_2(x, \varepsilon)$ and $L_3(x, \varepsilon)$. These lotteries are equal in terms of the level of ambiguity $\varepsilon$ and the mean policy $x$, and they only differ in the number of policies, $m = 2$ against $m = 3$. Whereas in the lottery $L_2(x, \varepsilon)$ the set of policies is $X_2 = \{x - \varepsilon, x + \varepsilon\}$, in the lottery $L_3(x, \varepsilon)$ the set of policies is $X_3 = \{x - \varepsilon, x, x + \varepsilon\}$. Evaluating the preferences of voters with ideal policy $x_i \in [0, x - \varepsilon]$ over $L_2(x, \varepsilon)$,

$$U_i(L_2) = -\frac{1}{2} (|x - \varepsilon - x_i| + |x + \varepsilon - x_i|) = -|x - x_i|.$$  

The peak of the voters with ideal policy in the interval $[0, x - \varepsilon]$ is out of the range of the lottery $L_2$ (or coincides with the lottery’s lower bound), then both policies $\{x - \varepsilon, x + \varepsilon\}$ are in the decreasing side of the single-peaked shape of the preferences of voters. Therefore, each of these voters is risk-neutral with respect to this lottery stand and $U_i(L_2) = u_i(x)$, i.e., the utility of the lottery is equal to the lottery’s expected value.

A similar reasoning follows for every voter with ideal policy in the interval $[x + \varepsilon, 1]$ and for whom $U_i(L_2) = u_i(x)$.

For every voter $i$ with ideal policy in the open interval $(x - \varepsilon, x + \varepsilon)$, we have that

$$U_i(L_2) = -\frac{1}{2} (|x - \varepsilon - x_i| + |x + \varepsilon - x_i|) = -\varepsilon. \quad (4)$$

Regarding $L_3(x, \varepsilon)$, for every voter $i$ such that $x_i \in [0, x - \varepsilon]$ or $x_i \in [x + \varepsilon, 1]$, we can also apply risk neutrality by which $U_i(L_3) = u_i(x) = -|x - x_i|$. And for every other voter $i$ such that $x_i \in (x - \varepsilon, x + \varepsilon)$, we have that

$$U_i(L_3) = -\frac{1}{3} (|x - \varepsilon - x_i| + |x - x_i| + |x + \varepsilon - x_i|) = -\varepsilon - \frac{2\varepsilon}{3} - \frac{|x - x_i|}{3}. \quad (5)$$
As a result, all the voters with ideal policy in the intervals \([0, x - \varepsilon]\) and \([x + \varepsilon, 1]\) derive an equal utility from the two lottery stands given that \(U_i(L_2) = U_i(L_3) = -|x - x_i|\) and they are indifferent between the two lotteries. For the remaining voters, we compare Expressions (4) and (5). Since \(|x - x_i| < \varepsilon\), for every voter with ideal policy \(x_i \in (x - \varepsilon, x + \varepsilon)\), we deduce that \(U_i(L_2) < U_i(L_3)\).

Thus, we have showed that \(L_3(x, \varepsilon)\) weakly dominates \(L_2(x, \varepsilon)\): both strategies propose an equal level of ambiguity, but \(L_3(x, \varepsilon)\) provides greater utility to a fraction of voters which translates into additional situations in which a party can win when proposing \(L_3(x, \varepsilon)\) instead of \(L_2(x, \varepsilon)\).\(^5\)

Given a lottery stand \(L \in \mathbb{L}\), we refer to the outsider voters as those whose ideal policies are out of the bounds of the lottery, that is, those for whom \(x_i \in [0, x - \varepsilon]\) and \(x_i \in [x + \varepsilon, 1]\). We refer to insider voters as those whose ideal policies are inside the bounds of the lottery, that is, \(x_i \in (x - \varepsilon, x + \varepsilon)\).

In general, given a lottery \(L_m \in \mathbb{L}\), its corresponding set of policies \(X_m\) can be calculated as a sequence of policies defined by

\[
X_m = \left\{ x - \frac{m - 1 - 2j}{m - 1} \varepsilon \right\}_{j=0}^{m-1}.
\]

From Expression (6), we can describe the set of policies for every value \(m \geq 2\), for example \(m = 4\) yields \(X_4 = \{ x - \varepsilon, x - \frac{\varepsilon}{3}, x + \frac{\varepsilon}{3}, x + \varepsilon \}\).

The utility of an outsider voter over a lottery stand is equal to the utility of the lottery’s expected value, that is \(U_i(L) = -|x - x_i|\) for all \(L \in \mathbb{L}\). The utility of an insider voter depends on the number of policies \(m\). According to Expressions (2) and (6), we deduce that the expected utility of an insider voter over every discrete lottery stand is measured by

\[
U_i(L_m) = -\frac{1}{m} \sum_{j=0}^{m-1} \left| x - \frac{m - 1 - 2j}{m - 1} \varepsilon - x_i \right|.
\]

When the lottery stand is a continuous lottery \(L_C\) over the interval \([x - \varepsilon, x + \varepsilon]\), the corresponding density function is \(f(x) = \frac{1}{2\varepsilon}\). Expression

\(^{5}\)The fact that the lottery \(L_3(x, \varepsilon)\) defeats the lottery \(L_2(x, \varepsilon)\) does not contradict the result by Zeckhauser (1969). This author shows that there is always a lottery stand with two policies that can defeat a lottery stand with three policies. The range of the lotteries in the comparison proposed by Zeckhauser, however, is different.
(3) evaluates the expected utility of an insider voter over $L_C$. Decomposing the integral and substituting the absolute values we deduce that

$$U_i(L_C) = -\frac{1}{2\varepsilon} \left[ \int_{x-x}^{x_i} (x_i - x_n) \, dx_n + \int_{x_i}^{x+\varepsilon} (x_n - x_i) \, dx_n \right]$$

where note that $x_i - x_n \geq 0$ for every $x_n \in [x - \varepsilon, x_i]$ and $x_n - x_i \geq 0$ for every $x_n \in [x_i, x + \varepsilon]$. Solving for the integrals and simplifying we derive the following expression$^6$

$$U_i(L_C) = \frac{(x - x_i)^2}{2\varepsilon} - \frac{\varepsilon}{2}$$

Now, we compare the expected utility over the continuous lottery $L_C(x, \varepsilon)$ with the expected utility over the discrete lottery $L_m(x, \varepsilon)$ where $m > 1$. Both lotteries gather an equal level of ambiguity and an equal mean policy position $x$. These two lotteries are equivalent for the outsider voters given that they provide an equal expected value $x$. For the insider voters, however, these two lotteries are not equivalent. Given Expression (7) and Expression (9), the differential utility for every insider voter $i$ over these two lotteries is defined by

$$F_i = U_i(L_C) - U_i(L_m)$$

If $L_m = L_2(x, \varepsilon)$, we have that $U_i(L_2) = -\varepsilon$ and comparing this utility level with that of Expression (9), we deduce that condition $-\frac{(x-x_i)^2}{2\varepsilon} - \frac{\varepsilon}{2} > -\varepsilon$ implies that $F_i > 0$. Simplifying yields, $x - x_i < \varepsilon$, and given that the insider voters are those for whom $x_i \in (x - \varepsilon, x + \varepsilon)$, this condition holds.

We have then shown that, for every voter, when comparing the lotteries $L_C(x, \varepsilon)$ and $L_2(x, \varepsilon)$, $U_i(L_C) \geq U_i(L_2)$ and where this inequality is strict for the insider voters. In addition, the lotteries $L_C(x, \varepsilon)$ and $L_2(x, \varepsilon)$ gather an equal level of ambiguity and therefore, according to the utility function of the parties in Expression (1), strategy $L_2(x, \varepsilon)$ is weakly dominated by $L_C(x, \varepsilon)$ (i.e., whatever the strategy of the opponent political party, the strategy $L_C(x, \varepsilon)$ always provides equal or higher payoffs for the political party than the strategy $L_2(x, \varepsilon)$). We deduce that there cannot be an equilibrium in which one or both parties propose the strategy $L_2(x, \varepsilon)$.

$^6$Expression (8) yields $-\frac{1}{2\varepsilon} \left[ ((x;x_n - \frac{x^2}{2})^{x_i}_{x-x} + (\frac{x^2}{2} - x_i x_n)^{x+\varepsilon}_{x_i} \right] = -\frac{1}{2\varepsilon} [x_i^2 - 2x_i x + x^2 + \varepsilon^2] = -\frac{(x-x_i)^2}{2\varepsilon} - \frac{\varepsilon}{2}$.
The following proposition analyzes the extent to which every discrete lottery stand \( L_m(x, \varepsilon) \) is weakly dominated by the continuous lottery stand \( L_C(x, \varepsilon) \).

**Proposition 1** According to the preferences of the political parties, every discrete lottery \( L_m(x, \varepsilon) \) with \( m > 1 \) is weakly dominated by the continuous lottery \( L_C(x, \varepsilon) \) with equal mean policy \( x \) and equal level of ambiguity \( \varepsilon \).

In the proof we show that for every insider voter, the utility derived from the continuous lottery is greater than the utility derived from every other lottery stand with the same mean policy and with the same level of ambiguity. Besides, given that outsider voters are indifferent between these two lottery stands, each political party derives a greater utility when proposing the continuous lottery than when proposing every other discrete lottery with equal mean policy and equal level of ambiguity.

For a given value \( \varepsilon > 0 \) and a fixed policy \( x \), Figure 1 illustrates the value \( F_i \) in Expression (10) as a function of \( x_i \), that is, it indicates the differential utility between the continuous lottery \( L_C \) and the discrete lottery \( L_m \) when \( m = 2 \) up to \( m = 10 \). We show how the greater \( m \), the smaller the differential utility and, in all the cases, \( F_i > 0 \) for all \( x_i \in (x - \varepsilon, x + \varepsilon) \).

![Figure 1: Values of the differential utility when \( m = 2 \) up to \( m = 10 \).](image)
Finally, we argue why the lottery $L_C(x, \varepsilon)$ and the single policy $L_1(x, 0)$ cannot be compared in terms of domination. Note that in those situations in which both proposals, $L_C(x, \varepsilon)$ and $L_1(x, 0)$, display the same electoral result, the party strictly prefers the continuous lottery over the single policy since it has a positive level of ambiguity $\varepsilon > 0$. However, in some situations $L_1(x, 0)$ can guarantee the electoral victory but $L_C(x, \varepsilon)$ cannot. This is the case, for instance, when one party proposes $L_1(x, 0)$ and the other $L_C(x, \varepsilon)$. Then, for those agents with ideal policy close to $x$, they strictly prefer $L_1(x, 0)$ over $L_C(x, \varepsilon)$. If a majority of voters has their ideal policies around $x$, this shows that the single policy can guarantee the victory over the continuous lottery. Consequently, in this situation, the party strictly prefers $L_1(x, 0)$ over $L_C(x, \varepsilon)$ if the benefits from holding office are sufficiently high. Thus, the non-weakly dominated set of strategies in our setting contains continuous lotteries and single policies.\(^7\)

4 Equilibrium analysis

In this section we analyze equilibrium existence. We first discard non-equilibrium strategies and we second analyze lottery stands that either differ in the proposed mean policies or in the proposed level of ambiguity.

Our first result shows that there cannot be an equilibrium in which one party wins and the other loses the elections.

**Lemma 1** If an equilibrium exists then, there is a tie.

Lemma 1 shows that every pair of strategies in which one party wins and the other loses cannot be sustained as a Nash equilibrium. The proof distinguishes between two cases: when $\alpha > 1$ (the benefits from holding office are more than two times the maximum benefits derived from ambiguity $\varepsilon^{\text{max}} = \frac{1}{2}$), and when $\alpha \leq 1$. When $\alpha > 1$, the losing party improves inducing a tie by selecting the same strategy as its opponent. When $\alpha \leq 1$, there can be situations in which the losing party has no incentives to induce a tie, but in those cases, the winning party improves by proposing a more ambiguous lottery stand. In particular, in the hypothetical case of an equilibrium in which a party wins, the losing party proposes the maximum level of ambiguity $\varepsilon^{\text{max}}$. We then show that the winning party always has a profitable deviation.

\(^7\) Thus, the non-weakly dominated set of strategies in our setting is equivalent to the set of strategies proposed by Aragonès and Neeman (2000).
in which it increases its proposed level of ambiguity (up to some level below $\varepsilon^{\text{max}}$). Interestingly, this deviation guarantees the electoral victory due to the fact that insider voters, when comparing lotteries with equal mean policies, strictly prefer the less ambiguous one.

From Lemma 1 we deduce that the search for equilibrium strategies can be restricted to tying situations. The following lemma discards certain lottery stands as equilibrium strategies in a tying situation. In particular, we show that in the case of an equilibrium in which parties tie, parties cannot hold different levels of ambiguity.

**Lemma 2** *If there is an equilibrium in which parties tie then, parties propose an equal level of ambiguity, $\varepsilon_A = \varepsilon_B$.\'*

Thus, in a tying situation, parties’ taste for ambiguity makes them propose equal $\varepsilon$.

Some additional strategies can also be discarded as equilibrium strategies when the proposed mean policies do not satisfy certain conditions.

**Lemma 3** *If there is an equilibrium in which parties tie then, parties’ mean policies are either equal $x_A = x_B$, or equidistant to the ideal policy of the median voter, $|x_A - x_M| = |x_B - x_M|$.\'*

The fact that in a tying equilibrium both parties propose an equal level of ambiguity implies that parties mean policies have to be either symmetric around the median or equal. The proof shows that in any other case, there is a party which achieves a majority of votes, in contradiction with a tying equilibrium. When parties propose mean policies that are symmetric around the median voter, all the voters with ideal policy to one of the sides of the median (say right) vote for one of the parties, the voters with ideal policy to the other side of the median voter (left) vote for the other party and the median voter is indifferent between the two proposals. When both parties propose the same mean policy and given that their proposed levels of ambiguity have to be equal, we also obtain a tying situation.

According to the results obtained so far, we next show that we can discard all but one of the parties’ strategies in the case of a tying situation. The strategy that is not discarded is the degenerate lottery in which the party proposes the ideal policy of the median voter.

**Lemma 4** *If there is an equilibrium in which parties tie then, parties propose the unambiguous ideal policy of the median voter, $x_M$.\'*
In the proof we show that equilibrium strategies in which parties propose equal mean policies and certain positive level of ambiguity $\varepsilon > 0$ can be discarded. In fact, when parties propose equal mean policies, one of the parties has incentives to deviate by proposing less ambiguity. We show that the lottery with smaller level of ambiguity $\varepsilon$ is more preferred for those insider voters (of one or the other lottery), whereas outsider voters (of both lotteries) remain indifferent between them. This is very intuitive given that once a voter is within the bounds of a lottery stand, the smaller the level of ambiguity, the higher the probability assigned to policy proposals which are closer to the voter’s ideal policy. From the outsiders’ viewpoint, the two lottery stands yield an equal expected value and these voters are immune to different levels of ambiguity. Therefore, this result reveals that voters dislike ambiguity and that the smaller the range over which the lottery stand is distributed, the greater the electoral support. In every other situation in which the parties propose different mean policies we find that, independent of the level of ambiguity $\varepsilon$, a profitable deviation always exists in which a party moves closer to the median voter.

In our last result, we show that equilibrium existence crucially depends on the magnitude of the office-holding benefits. We distinguish three scenarios: one in which the benefits from holding office are above or equal to one $\alpha \geq 1$, another in which $0 < \alpha < 1$, and the one in which $\alpha = 0$ (i.e., where there are no office-holding benefits).

**Proposition 2** When $\alpha \geq 1$, there is a unique equilibrium $(L^A, L^B)$ in which the two parties propose, with no ambiguity, the ideal policy of the median voter, i.e., $L^A = L^B = L_1(x_M, 0)$. When $0 < \alpha < 1$, there is no equilibrium. When $\alpha = 0$, there is a unique equilibrium $(L^A, L^B)$ in which the two parties propose the maximal level of ambiguity, i.e., $L^A = L^B = L_0\left(\frac{1}{2}, \varepsilon^{\text{max}}\right)$.

When office-holding incentives are sufficiently rewarding (in comparison to ambiguity), there is a unique equilibrium prediction in which parties converge to the ideal policy of the median voter. Surprisingly, once the benefits from holding office are not high enough (in comparison to the extra-benefits derived from ambiguity), an equilibrium fails to exist. When office-holding benefits are not present, the only prediction consists of both parties proposing the maximal level of ambiguity.

From a theoretical perspective, the proposed payoff function is discontinuous. For example, when parties’ strategies are the same and equal to
\( L_C(\frac{1}{2}, \varepsilon^{\text{max}}) \), one of the parties strictly improves announcing \( L_C(\frac{1}{2}, \varepsilon) \) where \( \varepsilon \) is slightly smaller than \( \varepsilon^{\text{max}} \), which induces a winning situation. In fact, the discontinuity of the payoff function implies that the standard result by Glicksberg (1952) on existence of mixed-strategy equilibria does not apply to our model. However, according to Dasgupta and Maskin (1986), we cannot discard existence of other type of equilibrium in mixed-strategy.\(^8\) An equilibrium in mixed strategies in which parties do not propose continuous or degenerate lottery stands would imply mixing over different levels of ambiguity or/and mixing over non-connected intervals of the policy space. In both cases, interpretation of such strategies are difficult and this would only apply to a scenario in which \( 0 < \alpha < 1 \) which means that office-holding benefits are less than two times the benefits derived from being ambiguous.

5 Concluding remarks

Our main result states that office-seeking incentives mitigate parties’ taste for ambiguity or, in other words, when office-seeking incentives are present, ambiguous policy platforms cannot be sustained as (Nash) equilibrium strategies. When the intensity of parties’ preferences over ambiguity is low (with respect to winning the elections), parties converge to the ideal policy of the median voter. However, an equilibrium fails to exist when the intensity of preferences over ambiguity is high. Convergence to the median voter is not any more an equilibrium prediction in this last case since parties have incentives to deviate to an ambiguous platform even when this implies losing the elections.

Our proposal accounts for two types of ambiguous strategies — discrete and continuous lottery stands —. Continuous lotteries, we show, weakly dominate discrete lotteries. Among the continuous lottery stands, when comparing lotteries with equal mean policy, we find that the less ambiguous lottery attracts more votes. Intuitively, once the preferred policy of a voter is within the bounds of a lottery stand, the smaller the level of ambiguity, the higher the probability assigned to policy proposals which are not far from the voter’s ideal policy. This result derives from the single-peaked shape of preferences, by which those voters whose ideal policies are within the bounds of a lottery stand behave as risk-averse voters.

\(^8\)Dasgupta and Maskin (1986) maintain that mixed-strategy equilibria can exist when the set of discontinuities is of (Lebesgue) measure cero.
According to our results and the existing literature, we conclude that there are two key features that induce ambiguous strategies — voters with risk-acceptance attitudes and parties with uncertainty about voters’ preferences. In particular, in a setting with uncertainty about voters’ preferences and a taste for ambiguity, parties may sacrifice some probability of winning in exchange for the extra-benefits derived from proposing an ambiguous platform. When there is no uncertainty about voters’ preferences, we show, both parties perfectly predict the election result and this eliminates those profitable trade-offs between ambiguity and votes.

Our theoretical prediction, by which uncertainty about voters’ preferences generates ambiguous platforms whereas ambiguous strategies vanish when there is no uncertainty, is open to empirical scrutiny. We propose a two-round election as a suitable scenario for this purpose. In this setting, the first election round can be interpreted as one in which parties are more uncertain about voters’ preferences, and the second election round as one with less uncertainty. The variance and location of parties’ policy proposals, as perceived by voters in election surveys, reveal information about parties’ strategies. This a natural experiment which, we believe, can provide additional insights into the electoral use of strategic ambiguity.

There are several directions in which our proposed theoretical model can be extended. Among others, we can analyze strategic ambiguity in a setting with more political parties and different electoral rules, from majority rule to proportional representation. The analysis of these scenarios is left for further research.

Acknowledgements: We are grateful to Kenneth Shepsle, David Austen-Smith and participants at the ICOPEAI 2014, Arne-Ryde Symposium 2015 and Summer School in "Interdisciplinary Analysis of Voting rules" (Caen), 2014 for their useful comments. We acknowledge financial support from Junta de Andalucía (SEJ-5980) and the Spanish Ministry of Science and Technology (grant ECO2014-53767-P).
Appendix

This Appendix presents the proofs of Propositions 1 and 2 and Lemmas 1 to 4.

Proof of Proposition 1. For every insider voter, the utility over $L_m(x, \varepsilon)$ is measured by (7). We can decompose and substitute the absolute values of Expression (7) so that

$$U_i(L_m) = \frac{1}{m} \left[ \sum_{j=0}^{j^*} \left(-\gamma_i + \frac{(m-1-2j)\varepsilon}{m-1} \right) + \sum_{j+1}^{m-1} \left(\gamma_i - \frac{(m-1-2j)\varepsilon}{m-1} \right) \right]$$

where $\gamma_i = x - x_i$ and $j^* = k - 1$ with $k$ being the number of policy proposals $x_n \in X_m$ such that $x_n < x_i$. The above expression is equivalent to

$$U_i(L_m) = \frac{1}{m} \left[ -(j^* + 1)\gamma_i + (j^* + 1)\varepsilon - \frac{2\varepsilon}{m-1} \sum_{j=0}^{j^*} j + (m-1-j^*)\gamma_i \right.$$

$$\left. -(m-1-j^*)\varepsilon + \frac{2\varepsilon}{m-1} \sum_{j+1}^{m-1} j \right].$$

Substituting that $\sum_{j=0}^{j^*} j = \frac{j^*(j^*+1)}{2}$, $\sum_{j+1}^{m-1} j = \frac{(m-1)n}{2} - \frac{j^*(j^*+1)}{2}$ and simplifying yields

$$U_i(L_m) = \frac{1}{m} \left[ (m-2-2j^*)\gamma_i + \frac{2\varepsilon (j^* + 1)(m-1-j^*)}{m-1} \right].$$

Substituting the above expression and Expression (9) into (10) we deduce

$$F_i = -\frac{\gamma_i^2}{2\varepsilon} - \frac{\varepsilon}{2} + \frac{(m-2-2j^*)\gamma_i}{m} + \frac{2\varepsilon (j^* + 1)(m-1-j^*)}{m(m-1)}.$$

Next, we show that $F_i > 0$ for all $\gamma_i \in (-\varepsilon, \varepsilon)$. When $\gamma_i \to \varepsilon$, the value of $j^* = 0$ so that $F_i(\gamma_i) = \frac{\gamma_i^2}{2\varepsilon} - \frac{\varepsilon}{2} + \frac{(m-2)\gamma_i}{m} + \frac{2\varepsilon}{m}$. This function is continuous in $\gamma_i$ from where

$$\lim_{\gamma_i \to \varepsilon} F_i(\gamma_i) = F_i(\varepsilon) = 0.$$
The value of the parameter $j^*$ changes with $\gamma_i$. Expression (6) defines the values $x_n$ in $X_m$. Then, we can define the values $\gamma_i$ for which $j^*$ changes, as the ones in the sequence \( \{ \frac{m-1-2i}{m-1} \}^{m-1}_{j=0} \). In particular, for every agent $i$ such that $\gamma_i \in \left[ \frac{m-1-2(j^*+1)}{m-1} \varepsilon, \frac{m-1-2j^*}{m-1} \varepsilon \right]$, the value of $j^*$ is the same (e.g., $j^* = 0$ for all $\gamma_i \in \left[ \frac{m-3}{m-1} \varepsilon, \varepsilon \right]$), and $j^* = 1$ for all $\gamma_i \in \left[ \frac{m-5}{m-1} \varepsilon, \frac{m-3}{m-1} \varepsilon \right]$). We show that in each of these intervals, $F'_i$ is strictly increasing when evaluated in its lower bound and strictly decreasing when evaluated in its upper bound. Solving for the derivative evaluated in its lower bound

\[
\frac{\partial F_i(\frac{m-1-2j^*+1}{m-1} \varepsilon)}{\partial \gamma_i} = -\frac{m-1-2(j^*+1)}{m-1} + \frac{m-2-2j^*}{m} = \frac{2(j^*+1-m)}{(m-1)m},
\]

where given that $0 \leq j^* \leq m-2$ then, $j^* + 2 \leq m$, from where $j^* + 1 < m$ and the above derivative is strictly negative. Solving for the derivative evaluated in its upper bound

\[
\frac{\partial F_i(\frac{m-1-2j^*}{m-1} \varepsilon)}{\partial \gamma_i} = -\frac{m-3-2j^*}{m-1} + \frac{m-2-2j^*}{m} = \frac{2+2j^*}{(m-1)m} > 0.
\]

Given that \( \frac{\partial^2 F_i}{\partial \gamma_i^2} = -\frac{1}{\varepsilon} < 0 \), $F_i$ is concave in each of the proposed intervals.

Besides, $\frac{\partial^2 F_i}{\partial j^*} = 0$ implies that in each of the intervals, function $F_i$ is symmetric around its maximum value. Let $\gamma_0^{\max}$ be the agent with maximum $F_i$ in the interval where $j^* = 0$. Starting from the interval where $j^* = 0$ and therefore, $\gamma_i \in \left[ \frac{m-3}{m-1} \varepsilon, \varepsilon \right]$, we first show that the agent in the upper bound, $\gamma_i = \frac{m-3}{m-1} \varepsilon$ is closer to $\gamma_0^{\max}$ than the agent in the lower bound, $\gamma_i = \varepsilon$. This feature, together with the symmetry of $F_i$ and the fact that $F_i(\varepsilon) = 0$ implies that $F_i > 0$ in the interval $\gamma_i \in \left[ \frac{m-3}{m-1} \varepsilon, \varepsilon \right]$. Second, we show that in every subsequent interval, where $j^* > 0$, the agent in the upper bound is not further from $\gamma_{j^*}^{\max}$ than the agent in the lower bound. By continuity of $F_i$ this implies that $F_i > 0$ in the interval $\gamma_i \in (-\varepsilon, \varepsilon)$.

In each interval, defined by a value of $j^*$ where $0 \leq j^* \leq m-2$, the lower and upper bounds are defined by $\gamma_L = \frac{m-1-2(j^*+1)}{m-1} \varepsilon$ and $\gamma_U = \frac{m-1-2j^*}{m-1} \varepsilon$, respectively. We want to show that $\gamma_{j^*}^{\max} - \gamma_L \leq \gamma_U - \gamma_{j^*}^{\max}$ for every $j^*$, which is equivalent to showing that $\gamma_{j^*}^{\max} \leq \frac{2(\gamma_U + \gamma_L)}{2}$. The maximum in each interval is deduced from the first order condition $\frac{\partial F_i}{\partial j^*} = -\frac{n_i}{\varepsilon} + \frac{(m-2-2j^*)}{m} = 0$ which yields $\gamma_{j^*}^{\max} = \frac{\varepsilon m-2-2j^*}{m}$. Calculating the mean value $\frac{\gamma_U + \gamma_L}{2} = \frac{\varepsilon m-2j^* - 2}{(m-1)}$. Substituting in condition $\gamma_{j^*}^{\max} \leq \frac{\gamma_U + \gamma_L}{2}$ yields $(m-1) \leq m$ which always holds. This shows that $F_i > 0$ for every insider voter, which implies that $L_C(x, \varepsilon)$ weakly dominates $L_m(x, \varepsilon)$ for every $m > 1$. \( \blacksquare \)
Proof of Lemma 1. When \( \alpha > 1 \), the minimum payoff that a political party derives from tying \( \frac{\alpha}{2} \) is always greater than the maximum payoff associated to losing \( \varepsilon_{\text{max}} \) and therefore, a party always strictly prefers tying to losing. When \( \alpha \leq 1 \), however, this is not the case. Suppose to the contrary, that there is an equilibrium in which one party loses and the other wins. We distinguish two cases depending on the value of \( \alpha \).

When \( \alpha > 1 \), the losing party can deviate by selecting the same strategy as the winning party, which guarantees a tie. This is a profitable deviation in contradiction with an equilibrium situation.

When \( \alpha \leq 1 \), consider \( \text{wlog} \) that Party A wins and Party B loses. Then, because Party B cannot improve deviating, its unique optimal strategy is \( L^B = L_C(\frac{1}{2}, \varepsilon_{\text{max}}) \) with \( v_B = \varepsilon_{\text{max}} = \frac{1}{2} \) since for every other lottery stand \( x \), such that \( \varepsilon < \varepsilon_{\text{max}} \), \( v_B = \varepsilon < \frac{1}{2} \). Given that Party A is winning, its strategy has to be different from that of Party B. Let \( L^A = L_C(x, \varepsilon) \) with \( \varepsilon \in [0, \varepsilon_{\text{max}}) \), then, Party A’s utility is \( v_A = \alpha + \varepsilon \). We find, however, that \( L^A = L_C(x, \varepsilon) \) with \( \varepsilon \in [0, \varepsilon_{\text{max}}) \) and \( L^B = L_C(\frac{1}{2}, \varepsilon_{\text{max}}) \) is not a Nash equilibrium: if Party A deviates to \( L^A' = L_C\left(\frac{1}{2}, \varepsilon'\right) \) where \( \varepsilon < \varepsilon' < \varepsilon_{\text{max}} \), it still wins (as we show next) and its payoffs increase \( v_A(L^A', L^B) = \alpha + \varepsilon' > v_A(L^A, L^B) = \alpha + \varepsilon \). We show that Party A wins when \( L^A' = L_C\left(\frac{1}{2}, \varepsilon'\right) \) and \( L^B = L_C(\frac{1}{2}, \varepsilon_{\text{max}}) \).

By Expression (9), \( \frac{\partial U_i}{\partial \varepsilon} = \frac{(\frac{1}{2} - x_i)^2}{\varepsilon^2} - 1 \) where the fact that \( (\frac{1}{2} - x_i)^2 < \varepsilon^2 \) implies that \( \frac{\partial U_i}{\partial \varepsilon} < 0 \), that is, insider voters prefer the lottery with smaller \( \varepsilon \) whereas outsider voters remain indifferent. Thus, voters with ideal policy \( x_i \in \left(\frac{1}{2} - \varepsilon', \frac{1}{2} + \varepsilon'\right) \) prefer the lottery with smaller \( \varepsilon \), i.e., \( L^A' = L_C\left(\frac{1}{2}, \varepsilon'\right) \).

Then, for \( \varepsilon' \) sufficiently close to \( \varepsilon_{\text{max}} \); Party A obtains a majority of votes and wins, in contradiction with an equilibrium situation.

Proof of Lemma 2. Suppose that there is an equilibrium in which both parties tie and where \( \varepsilon_A < \varepsilon_B \). Then, \( v_A(L^A, L^B) = \varepsilon_A + \frac{\alpha}{2} < v_B(L^A, L^B) = \varepsilon_B + \frac{\alpha}{2} \). In this case, Party A can benefit by selecting the same strategy as Party B, which also guarantees a tie and besides, this increases its payoffs. This contradicts an equilibrium situation.

Proof of Lemma 3. Let \( x_A \) and \( x_B \) be the mean policies of the lottery stands proposed by Party A and Party B, respectively. Suppose to the contrary that there is an equilibrium in which parties tie and where \( x_A \) and \( x_B \) are neither equidistant to \( x_M \) nor equal. We take, \( \text{wlog} \), the case \( x_B < x_A \), where \( x_A \) is closer to \( x_M \) than \( x_B \), i.e. \( (|x_A - x_M| < |x_B - x_M|) \). By Proposition 1, equilibrium strategies if they exist, can only be continuous lotteries.
or single policy stands. Moreover, by Lemma 2, there cannot be a tying situation in which one party proposes a single policy and another a continuous lottery. Therefore, either both parties propose a single policy or both parties propose a continuous lottery with an equal level of ambiguity. In the former case, Party A wins the election in contradiction with a tying situation. Thus, we just account for tying situations in which both parties propose continuous lottery stands with equal $\varepsilon$. Then, the group of voters with ideal policy $x_i \in [x_M, 1]$ fits into one of the three following cases:

- **Case 1.** Agents with $x_i \in [x_M, 1]$ are all outsider voters of the lottery stands of Party A and Party B. This can only occur in a situation where $x_B < x_A < x_M$. For these agents, $U_i(L^A) = - |x_A - x_i|$ and $U_i(L^B) = - |x_B - x_i|$ and they prefer the party which mean policy is closer to their ideal policy. Given that $x_B < x_A < x_M$, all outsider agents prefer $x_A$. Therefore, a majority votes for Party A, in contradiction with a tying situation.

- **Case 2.** Agents with $x_i \in [x_M, 1]$ are of two types: agents with $x_i \in [x_A + \varepsilon, 1]$ are outsider voters of Party A and Party B, and agents with $x_i \in [x_M, x_A + \varepsilon]$ are insider voters of Party A. This can occur either in a situation where $x_B < x_A < x_M$ or $x_B < x_M < x_A$. For every outsider voter, $U_i(L^A) = - |x_A - x_i|$ and $U_i(L^B) = - |x_B - x_i|$ and they all prefer Party A over Party B since their ideal policies are closer to $x_A$ than to $x_B$. Insiders of Party A evaluate $L^A$ according to Expression (9), i.e. $U_i(L^A) = - \frac{(x_A - x_i)^2}{2} - \varepsilon$. By definition, an insider voter satisfies that $|x_A - x_i| < \varepsilon$ and so, $-(x_A - x_i)^2 > -\varepsilon^2$, from where $U_i(L^A) > -\frac{\varepsilon^2}{2} - \varepsilon = -\varepsilon$. For these voters $U_i(L^B) = - |x_B - x_i|$ and, given that they are outsider voters of Party B, then $- |x_B - x_i| < -\varepsilon$. We deduce that $U_i(L^B) < -\varepsilon < U_i(L^A)$, i.e. insider voters of Party A strictly prefer Party A over Party B. Thus, a majority votes for Party A, in contradiction with a tying situation.

- **Case 3.** Agents with $x_i \in [x_M, 1]$ are of three types: agents with $x_i \in [x_A + \varepsilon, 1]$ are outsider voters of Party A and Party B, agents with $x_i \in [x_B + \varepsilon, x_A + \varepsilon]$ are insider voters of Party A and agents with $x_i \in [x_M, x_B + \varepsilon]$ are insider voters of both parties. This can occur either in a situation where $x_B < x_A < x_M$ or $x_B < x_M < x_A$. For every outsider voter, $U_i(L^A) = - |x_A - x_i|$ and $U_i(L^B) = - |x_B - x_i|$ and they all prefer Party A since $x_A$ is closer to their ideal policy than $x_B$. Insider voters of Party A evaluate $L^A$ according to Expression (9), i.e. $U_i(L^A) = - \frac{(x_A - x_i)^2}{2\varepsilon} - \frac{\varepsilon}{2}$, and evaluate Party B by $U_i(L^B) = - |x_B - x_i|$. Since $|x_A - x_i| < \varepsilon$ and $-(x_A - x_i)^2 > -\varepsilon^2$, then $U_i(L^A) > -\frac{\varepsilon^2}{2\varepsilon} - \frac{\varepsilon}{2} = -\varepsilon$. Given that $- |x_B - x_i| < -\varepsilon$ we deduce that
Given that \( \varepsilon \) is equal in \( L^A \) and \( L^B \), these voters prefer the party which mean policy is closer to their ideal policy, which is \( x_A \). Thus, a majority votes for Party A in contradiction with a tying situation. ■

**Proof of Lemma 4.** Consider to the contrary a tying equilibrium in which the strategies of the parties differ from \( L_1(x_M, 0) \). By Proposition 1, Lemma 2 and Lemma 3 the possible equilibrium strategies in a tie situation are restricted to one of the following three cases:

- **Case 1.** Suppose that parties gather equal mean policies in their lotteries and that \( L^A = L^B = L_C(x, \varepsilon) \) where \( \varepsilon > 0 \). Then, parties tie and obtain the payoffs \( \varepsilon + \frac{\alpha}{2} \). Consider that Party A deviates to \( L^A = L_C(x, \varepsilon') \) where \( \varepsilon' < \varepsilon \). According to Expression (9), \( \frac{\partial U_i}{\partial \varepsilon} = \frac{(x_A-x_i)^2}{\varepsilon^2} - 1 \) where the fact that \( (x_A-x_i)^2 < \varepsilon^2 \) implies that \( \frac{\partial U_i}{\partial \varepsilon} < 0 \), that is, insider voters prefer the lottery with smaller \( \varepsilon \), while outsider voters remain indifferent. Thus, for \( \varepsilon' < \varepsilon \) sufficiently close to \( \varepsilon \), Party A can win the elections and obtain a payoff \( \varepsilon' + \alpha \), that is greater than \( \varepsilon + \frac{\alpha}{2} \), in contradiction with an equilibrium situation.

- **Case 2.** Suppose that parties gather equal mean policies in their lotteries and that \( L^A = L^B = L_1(x, 0) \) where \( x \neq x_M \). Then, parties tie and obtain the payoffs \( \frac{\alpha}{2} \). Suppose that Party A deviates to \( L^A = L_1(x', 0) \) where \( |x_M - x'| < |x_M - x| \). Then, Party A can win with this strategy since the median voter now strictly prefers Party A over Party B. This implies that Party A obtains a greater payoff with this deviation, in contradiction with an equilibrium situation.

- **Case 3.** Suppose that parties mean policies are different and equidistant to \( x_M \) so that \( L^A = L_C(x_A, \varepsilon) \) and \( L^B = L_C(x_B, \varepsilon) \) where, wlog \( x_B < x_M < x_A \) with \( x_A - x_M = x_M - x_B \), and where \( \varepsilon \geq 0 \). In this tying situation each of the parties obtains the payoff \( \varepsilon + \frac{\alpha}{2} \). Suppose that Party A deviates to \( L_C(x'_A, \varepsilon) \), where \( x_M < x'_A < x_A \), reducing the distance to the median voter location. Then, Party A can win with this strategy since the median voter now strictly prefers Party A over Party B: if the median voter is an outsider voter, then \( x'_A \) is closer to \( x_M \) than \( x_B \), which implies that Party A is more preferred than Party B; if \( x_M \) is an insider voter of both lottery stands, by Expression (9), \( U_i = -\frac{(x_i - x_M)^2}{2\varepsilon} - \frac{\alpha}{2} \), \( j = A, B \), so that the fact that \( x'_A \) is closer to \( x_M \) than \( x_B \), together implies that Party A is more preferred than Party B for the median voter. Party A wins the elections and obtain the payoff \( \varepsilon + \alpha \),
that is greater than \( \varepsilon + \frac{\alpha}{2} \), in contradiction with an equilibrium situation. ■

**Proof of Proposition 2.** First, we show existence of equilibrium when \( \alpha \geq 1 \).

By Lemma 1, if there exist some equilibrium strategies, these induce a tie. Moreover, by Lemma 4, all the strategies but \( L_1(x_M, 0) \) are discarded as equilibrium strategies in a tying situation. Thus, it remains to show that \( L^A = L^B = L_1(x_M, 0) \) are equilibrium strategies. Suppose to the contrary that \( (L^A, L^B) \) is not an equilibrium and consider, \( \text{wlog} \), that Party A has a profitable deviation. There are up to three possible deviations: i) \( L_1(x'_A, 0) \) where \( x'_A \neq x_M \); ii) \( L_C(x_M, \varepsilon_A) \) where \( \varepsilon_A \in (0, \varepsilon_{\text{max}}] \) and iii) \( L_C(x'_A, \varepsilon_A) \) where \( x'_A \neq x_M \) and \( \varepsilon_A \in (0, \varepsilon_{\text{max}}] \). This is sufficient to analyze these deviations given that, by Proposition 1, a discrete lottery is weakly dominated by a continuous lottery.

i) Suppose that Party A deviates to \( L^A' = L_1(x'_A, 0) \), where, \( \text{wlog} \), \( x'_A < x_M \). Given that Party B’s platform is the median voter location \( x_M \), those voters with ideal policy \( x_i \in [x_M, 1] \) vote for Party B. Thus, Party A loses the election and its utility is \( v_A(L^A', L^B) = 0 \), whereas \( v_A(L^A, L^B) = \frac{\alpha}{2} \geq \frac{1}{2} \), in contradiction with a profitable deviation.

ii) Suppose that Party A deviates to \( L^A' = L_C(x_M, \varepsilon_A) \) where \( \varepsilon_A \in (0, \varepsilon_{\text{max}}] \). Those voters with ideal policy \( x_i \in [0, x_M - \varepsilon_A] \), \( x_i \in [x_M + \varepsilon_A, 1] \) are outsider voters so that \( U_i(L^A') = U_i(L^B) = -|x_M - x_i| \) and they abstain from voting. Insider voters of Party A compare \( U_i(L^B) = -|x_M - x_i| \) with the utility of the lottery stand which, according to Expression (9), is \( U_i(L^A') = -\frac{(x_M-x_i)^2}{2\varepsilon_A} - \frac{\varepsilon_A}{2} \). We want to show that \( U_i(L^A') < U_i(L^B) \) for every insider voter. Take, \( \text{wlog} \), \( x_i \in (x_M - \varepsilon_A, x_M) \) and let \( \beta = |x_M - x_i| \). Then, condition \( U_i(L^A') < U_i(L^B) \) can be rewritten as \( -\frac{\beta^2}{2\varepsilon_A} - \frac{\varepsilon_A}{2} + \beta < 0 \), from where \( -\beta^2 + 2\varepsilon_A\beta - \varepsilon_A^2 < 0 \). We define \( F_1(\beta) = -\beta^2 + 2\varepsilon_A\beta - \varepsilon_A^2 > 0 \) for \( \beta \in (0, \varepsilon_A) \), \( F_i(0) = -\varepsilon_A^2 \) and \( F_i(\varepsilon_A) < 0 \), we deduce that \( F_1(\beta) < 0 \) for every \( \beta \in (0, \varepsilon_A) \). This implies that \( U_i(L^A') < U_i(L^B) \) for every voter such that \( x_i \in (x_M - \varepsilon_A, x_M) \) and, in an equivalent way, for those voters such that \( x_i \in (x_M, x_M + \varepsilon_A) \). Thus, Party A loses the elections with this deviation and its utility is \( v_A(L^A', L^B) \leq \varepsilon_{\text{max}} = \frac{1}{2} \), whereas \( v_A(L^A, L^B) = \frac{\alpha}{2} \geq \frac{1}{2} \), in contradiction with a profitable deviation.

iii) Suppose that Party A deviates to \( L^A' = L_C(x'_A, \varepsilon_A) \) where \( x'_A \neq x_M \) and \( \varepsilon_A \in (0, \varepsilon_{\text{max}}] \). Take, \( \text{wlog} \), \( x'_A > x_M \). Then, those agents with ideal policy \( x_i \in [0, x'_A - \varepsilon_A] \) are outsider voters and for them, \( U_i(L^A') = -|x'_A - x_i| < U_i(L^B) = -|x_M - x_i| \), so they vote for Party B. For those agents such that
$x_i \in [x'_A - \varepsilon_A, x_M]$, their utility over Party A is measured by Expression (9) so that $U_i(L^A) = -\frac{(x'_A - x_i)^2}{2\varepsilon_A} - \frac{\varepsilon_A}{2}$ and $U_i(L^B) = -|x_M - x_i|$. We want to show that $U_i(L^A) < U_i(L^B)$ for every insider voter. In the analysis of deviation (ii), we have shown that $-\frac{(x_M - x_i)^2}{2\varepsilon_A} - \frac{\varepsilon_A}{2} < -|x_M - x_i|$ for every agent with $x_i \in (x_M - \varepsilon_A, x_M]$. Since $(x'_A - x_i) > (x_M - x_i)$ then, $(x'_A - x_i)^2 > (x_M - x_i)^2$, which implies that $-(x'_A - x_i)^2 < -(x_M - x_i)^2$ and $-\frac{(x'_A - x_i)^2}{2\varepsilon_A} - \frac{\varepsilon_A}{2} < -\frac{(x_M - x_i)^2}{2\varepsilon_A} - \frac{\varepsilon_A}{2} < -(x_M - x_i)$. Then, those voters to the left of the median (including the median) prefer Party B over Party A. Thus, Party A loses the elections and its utility is $v_A(L^A, L^B) \leq \varepsilon_{max} = \frac{1}{2}$, whereas $v_A(L^A, L^B) = \frac{\alpha}{2} \geq \frac{1}{2}$, in contradiction with a profitable deviation. We have shown that all the deviations from $L_1(x_M, 0)$ (when the strategy of the opponent is $L_1(x_M, 0)$), are such that the party deviating loses the election. This also proves that $L_1(x_M, 0)$ is not weakly dominated by any other strategy and this completes the proof.

Second, we show non existence of equilibrium when $0 < \alpha < 1$. By Lemmas 1 and 4, this is sufficient to show that $L^A = L^B = L_1(x_M, 0)$ is not an equilibrium in this case. Consider that Party A deviates to $L^A = L_C(\frac{1}{2}, \varepsilon_{max})$. As shown above, in deviations (ii) and (iii), Party A cannot defeat Party B when proposing $L_C(\frac{1}{2}, \varepsilon_{max})$ where either $\frac{1}{2} \leq x_M$ or $\frac{1}{2} > x_M$. However, when $\alpha < 1$, Party A derives more utility when proposing $L^A = L_C(\frac{1}{2}, \varepsilon_{max})$ than when proposing $L^A = L_1(x_M, 0)$: $v_A(L^A, L^B) = \varepsilon_{max} = \frac{1}{2}$ and $v_A(L^A, L^B) = \frac{\alpha}{2} < \frac{1}{2}$. Therefore, $L^A = L^B = L_1(x_M, 0)$ is not a Nash equilibrium and this completes the proof.

Third, we show that $L^A = L^B = L_C(\frac{1}{2}, \varepsilon_{max})$ is an equilibrium when $\alpha = 0$. In this case, parties only derive benefits from being ambiguous and therefore, the strategy $L_C(\frac{1}{2}, \varepsilon_{max})$ is a strictly dominant strategy for each party and the unique Nash equilibrium.
References


