Is Equity Home Bias Socially Optimal?

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Abstract

I study the efficiency and aggregate effects of equity home bias using a general equilibrium model with nominal rigidities. Assuming a planner who faces the same frictions than the agents, I find that the source of home bias is key for analyzing the wedge between equilibrium and socially optimal levels of home bias. Surprisingly, when home bias is due to hedging effects, stock positions tend to be approximately efficient despite the aggregate demand externalities induced by the rigidities. On the other hand, home stock positions are excessive when home bias is due to financial frictions or biased expectations. Finally, informational signals concerning home stocks tend to imply positive though smaller benefits from increasing home bias. The gap between equilibrium and socially optimal levels of home bias can be numerically large.

Introduction

Should governments promote the holdings of foreign equity? This paper argues that the answer depends crucially on the cause of home bias.

Home bias and the lack of international risk sharing are one of the key puzzles in international finance and macroeconomics. In seminal work, French and Poterba (1991) find that 94% of US equity wealth is invested in domestic stocks. They argue that the lost diversification benefits result in substantial welfare costs for the investors. While international diversification

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has improved in the last decades, investors still hold portfolios that are heavily tilted towards domestic equity (Coeurdacier and Rey, 2013).

At the same time a large literature has attempted to rationalize the bias in equity portfolios. The leading explanations for the puzzle can be divided into three rough categories. First, home stocks can offer a good hedge to shocks to relative prices (e.g. Cooper and Kaplanis (1994)) or labor income (e.g. Heathcote and Perri (2013)). Second, home bias can be due to financial frictions such as trading costs (e.g. Lewis (1999)). Third home bias can be explained by informational differences (e.g. Brennan and Cao (1997)).

While this literature can potentially explain the bias in equity portfolios, it cannot answer if the bias is good or bad as such. By considering the aggregate effects of equity home bias, this paper attempts to fill that gap.

More specifically, I analyze the wedge between equilibrium and efficient stock positions using a standard macroeconomic model with nominal rigidities. The model with a tradable and non-tradable sector can be seen as a two-country version of Farhi and Werning (2014) adapted to incomplete markets with trading in equity claims. This economy is based on Obstfeld and Rogoff (1995) and Obstfeld and Rogoff (2000a). Moreover, the policy implications of a similar model have been recently considered for example by Kehoe and Pastorino (2016).

The nominal rigidities result in an aggregate demand externality, which implies positive public benefits from macroeconomic stabilization. On the other hand, the agents do not internalize these effects and may engage in an inefficient amount of risk sharing to smooth business cycle fluctuations. This generally also implies a wedge between equilibrium and efficient stock holdings.

As a technical difference to papers such as Farhi and Werning (2014), I use approximation techniques similar to those applied by Devereux and Sutherland (2010) and Coeurdacier and Gourinchas (2013). Such methods can be used to derive closed form solutions for the stock positions as well to obtain corresponding near efficient results. It turns out that, quite generally, the planner solution approximately coincides with an equilibrium with more risk averse agents. Here the positive effects of macroeconomic stabilization can be seen as an increase in the efficient level of risk aversion.

The key contribution of the paper is the finding that the different strands of explanations offered for home bias bear different implications for the efficiency of equilibrium holdings. First, when equity home bias is due to

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1See also Devereux and Sutherland (2011), Tille and van Wincoop (2010), Rabitsch et al. (2015), Coeurdacier and Rey (2013) and Judd et al. (2001)
hedging redistributive shocks, the equilibrium approximately coincides with the constrained efficient solution. Such an explanation for equity home bias has been posited for example by Coeurdacier and Gourinchas (2013) and Heathcote and Perri (2013). This near efficiency result is surprising because of the model externalities and the fact that the public value of risk sharing is always greater than the private value. On the other hand, a financial friction modelled as a simple holding cost on foreign equity tends to result in excessive equilibrium home bias.

Third, I show that overestimation of the return on home stocks results in inefficiencies that are typically similar to those implied by a holding cost based explanation. Fourth, I find that informational signals can imply excessive equilibrium home bias when they result in relative optimism about home stocks. On the other hand, when equity home bias is due to the variance reduction channel of informational signals, increasing equity home bias can increase welfare. Still I find that this channel tends to result in smaller differences between equilibrium and efficient levels of home bias. Finally, a price hedging channel may also imply positive benefits from increasing equity home bias, though such an explanation for equity home bias faces empirical difficulties (Coeurdacier and Gourinchas, 2013).

The structure of this paper is the following. First chapter 1 lays down a simple symmetric two-country model with nominal rigidities. Chapter 2 compares the efficiency implications of two explanations for home bias: hedging redistributive shocks and financial frictions. Chapter 3 considers additional explanations for home bias such as informational signals. Chapter 4 generalizes the analysis. Finally, chapter 5 studies the efficiency of equity home bias using simple numerical examples.

Related Literature

This paper lies at the intersection of two literatures. The first literature consists of papers on equity home bias. The second literature has studied risk sharing in models with externalities.

An important strand of the home bias literature has attempted to explain the bias through investors’ hedging considerations. Cooper and Kaplanis (1994) note that equity home bias can be due to a correlation between home stock returns and domestic inflation. Cole and Obstfeld (1991) detail how exchange rate adjustments can reduce the need to hedge risks through financial markets. Obstfeld and Rogoff (2000b) find that trade costs in goods markets can help create exchange rate dynamics that favor home equity. Van Wincoop and Warnock (2010) argue that empirically the correlation
between prices and equity returns is too small to justify the bias in equity portfolios.

In theory home bias can result from non-tradable income risk. Baxter and Jermann (1997) find that non-tradable income risk should make the home bias puzzle worse because of a positive correlation between stock returns and returns to human capital. Heathcote and Perri (2013) argue that the analysis of Baxter and Jermann (1997) is based on strong assumptions. Their model is able to microfound redistributive shocks that create a negative correlation between returns on labor and capital. Coeurdacier and Gourinchas (2013) emphasize taking optimal bond positions into account when analyzing equity positions. They find that conditional on bond returns the correlation between returns on human capital and equity is negative and argue this can explain the home bias in stock portfolios.

A second strand of literature has emphasized market segmentation and various frictions when explaining equity home bias. Transaction costs, different tax treatment on home and foreign equity and policy induced restrictions on foreign investment can create home bias in equity portfolios and impede international risk sharing (see e.g. Lewis (1996) and Lewis (1999)). While few papers have estimated the exact extent of such costs, the consensus in the literature appears to be that the direct costs of foreign equity investment are low. On the other hand, if home and foreign equity are close substitutes even small costs can create large amounts of home bias. (Coeurdacier and Rey (2013))

A related part of the literature has considered how informational frictions affect portfolio choice. Information costs or natural informational advantages over foreign investors might explain the home bias puzzle. Kang and Stulz (1997) study foreign investors’ holdings of Japanese stocks and find some evidence of an informational disadvantage over Japanese investors. Brennan and Cao (1997) build a model in which home and foreign investors receive differential signals over home and foreign stocks. Van Nieuwerburgh and Veldkamp (2009) add endogenous learning into a model of asymmetric information. Because investors might choose to learn more about stocks they initially know the best, learning can amplify informational advantages.

Finally, some papers have put forth behavioral explanations for equity home bias. Perhaps the best known is the one given by the seminal paper of French and Poterba (1991). They argue that home bias results from investors overestimating the returns of the home market portfolio. Some evidence for such overestimation is provided by Shiller et al. (1991). It will turn out that for the purposes of this paper, a bias that increases the expected return of home stocks has similar effects than a cost that lowers the return of foreign
To my best knowledge, my paper is the first to study whether equity home bias is efficient from a social viewpoint. Here the analysis comes closer to papers which have studied risk sharing and borrowing in models with externalities. My model, which features a two-country setting with nominal rigidities, builds on Farhi and Werning (2014), who study optimal fiscal transfers with two market structures: complete markets and trading in a non-contingent bond; their analysis further builds on Obstfeld and Rogoff (1995). Farhi and Werning (2012) use a similar setting with nominal rigidities to characterize optimal capital controls. Schmitt-Grohe and Uribe (2013) also study capital controls in a model in which a downward rigid wage tends to create excess unemployment.

In a recent report the IMF identifies increasing market-based risk sharing as one of the key challenges for the Eurozone (IMF (2009)). Martinez and Philippon (2014) model a currency union and study how the economy responds to shocks under different risk sharing setups such as a debt or equity union. In contrast to the complete markets approach of Farhi and Werning (2014), this paper shares the spirit of Martinez and Philippon (2014) in allowing for a more realistic risk sharing arrangement. However rather than taking the agents’ portfolios as given as in Martinez and Philippon (2014), I explicitly include portfolio choice into the agents’ problem.

Finally, many papers have analysed the efficiency of equilibrium in models with pecuniary externalities rather than nominal rigidities. In Costinot and Werning (2014) and Brunnermeier and Sannikov (2014) inefficiencies arise due to incomplete markets. In Caballero and Krishnamurthy (2001) and Bianchi (2011) inefficiencies emerge from the interaction of credit constraints and prices. In many cases such models feature positive public benefits from macroeconomic stabilization similar to the New Keynesian models with nominal rigidities.

1 A Simple Model with Nominal Rigidities

I consider a simple two period two country economy with nominal rigidities that builds on Obstfeld and Rogoff (1995), Obstfeld and Rogoff (2000a), Gali and Monacelli (2005) and Farhi and Werning (2014). However, instead of complete markets I assume incomplete markets with trading in equity. For clarity I will first impose restrictive assumptions such as symmetric countries that will be relaxed later.

Assume there are two symmetric countries: home (H) and foreign (F).
Each country is populated by a unit measure of identical households. Assume a mixed endowment-production economy in which the tradable good is given by a random endowment, but the non-traded good is produced in each country using labor as the sole input. It is instructive to think of the non-tradable good as a domestic service sector and the tradable good as industrial production. The appendix describes a dynamic extension of the model in which the tradable good is produced using labor and capital inputs.

Furthermore assume there are two stocks, one for each country. The endowment is distributed as dividends to stockholder and as labor income to residents. The home agents can trade the home stock without further costs. However they receive only a fraction $e^{-f}$ of the returns of the foreign stock. Later I analyze both the case of no frictions ($f = 0$) and a case with frictions ($f > 0$). Moreover, in section 3 we introduce additional frictions such as informational signals.

In the following I will explicitly state the households’, firms’ and the planner’s problem as well as the following equilibrium conditions.

### 1.1 Households

The households make stock trading decisions at $t = 0$ and consumption choice and labor supply decisions at $t = 1$. The household preferences are given by

$$
\mathbb{E}[U(c_{T,i}, c_{NT,i}, N_i)], \quad i = H, F
$$

(1)

where $c_{T,i}$ is tradables consumption, $c_{NT,i}$ is non-tradables consumption and $N_i$ is labor supply. The preferences are separable in consumption and labor

$$
U(c_{T}, c_{NT}, N) = g(c_{T}, c_{NT}) - h(N),
$$

(2)

Here $g$ is a twice differentiable homothetic function such that $g_1 > 0$, $g_1,1 < 0$, $g_2 > 0$ and $g_{2,2} < 0$ and $h$ is a twice differentiable, strictly increasing and convex function, $h' > 0$, $h'' > 0$. Specifically we later consider CRRA preferences over a CES aggregator. Then $g(c_{NT}, c_{T}) = g(C) = \frac{1}{1-\gamma}C^{1-\gamma}$.

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2 This type of simple stock market friction has been considered for example by Lewis (1999). The friction is similar to the iceberg cost model used in the trade literature (Krugman, 1991). Here part of the tradable good is effectively lost due to trade costs. Assuming that part of the cost is rebated back to agents would affect the results quantitatively but not qualitatively.

3 $g(C) = \log(C)$, when $\gamma = 1$. 

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where

\[ C = \begin{cases} \frac{\phi-1}{a} c^{-\phi}_T + \frac{1}{a} (1 - a)^{\frac{\phi-1}{\phi}} c^{-\phi}_{NT}, & \text{if } \phi \neq 1 \\ c^{a} c^{1-a}_T, & \text{if } \phi = 1, \end{cases} \]  

\tag{3}

where \( 0 < a < 1 \). Later we also use the disutility of labor function \( h(N) = \frac{1}{1+\sigma} N^{1+\sigma} \). Without much loss of generality assume each country initially holds the full endowment of domestic stocks. Then the budget constraint at \( t = 0 \) becomes:

\[ S_{ii} p_{S,i} + S_{ij} p_{S,j} = p_{S,i}, \quad i = H, F, j = -i \]  

\tag{4}

where \( S_{ii} \) is country \( i \)'s holdings of country \( i \)'s equity and \( p_{S,i} \) is country \( i \)'s equity price. Here we normalize the supply of each stock to one. The time \( t = 1 \) budget constraint is given by:

\[ p_{NT,i} c_{NT,i} + p_{T,i} c_{T,i} = p_T d_i S_{ii} + p_T d_j e^{-f} S_{ij} + p_T l_i + W_i N_i + \Pi_i + T_i, \]

\[ i = H, F, j = -i \]

Here \( p_T \) and \( p_{NT,i} \) are the price of the tradable and non-tradable good respectively. Moreover, \( W_i \) is the wage from the non-tradable sector. Furthermore \( \Pi_i \) represents profits from the non-tradable sector. Moreover, \( T_i \) represents government tax used to subsidise labor use.\(^4\)

Note that country \( i \)'s total endowment \( c_i \) of the traded good is distributed as labor income and dividends: \( c_i = l_i + d_i \). The assumption that equity represents claims to only tradables endowment but not to non-tradables profits is not required for any of the results but is necessary for obtaining closed form solutions for the stock positions.

The intratemporal condition each period is

\[ \frac{U_{NT,i}}{U_{T,i}} = \frac{p_{NT,i}}{p_T} \equiv p_i \quad i = H, F, \]  

\tag{5}

Because \( g \) is homothetic, there is a function \( \alpha(p) \) s.t. \( c_{NT} = \alpha(p) c_T \). Specifically in the CES case we have \( \alpha(p) = \frac{a}{1-a} p^{-\phi} \). The labor choice FOC is

\(^4\)Given this form for the friction, the budget constraint assumes positive stock positions. We mainly consider regions where this is true for both home and foreign stockholdings though the results could be extended to cases where the positions can be negative.
\[
\frac{U_{N,i}}{U_{NT,i}} = \frac{W_i}{p_{NT,i}}, \quad i = H, F \tag{6}
\]

The relative Euler equation, written in terms of the traded good is

\[
\mathbb{E} \left[ \frac{R_{ii} - R_{ij}}{p_T} \right] = 0, \quad i = H, F \tag{7}
\]

Here

\[
R_{ii} = \frac{d_i}{p_{S,i}}, \quad i = H, F \tag{8}
\]

\[
R_{ij} = \frac{d_j}{p_{S,j}} e^{-f}, \quad i = H, F, j = -i \tag{9}
\]

### 1.2 Non-Tradables Producers

The non-tradable good is produced by a competitive firm, which combines a continuum of varieties \( j \in [0, 1] \) using a CES technology. The non-tradables production is given by

\[
Y_{NT,i} = \left( \int_0^1 Y_{NT,i,j}^{1-\frac{1}{\epsilon}} dj \right)^{1 \frac{1}{1-\epsilon}} \tag{10}
\]

where \( \epsilon > 1 \) is the elasticity of substitution. Each variety \( j \) is produced by a monopolistic entrepreneur using the technology \( Y_{NT,i} = AN_i \), where assume \( A \) is a constant, but allow for stochastic productivity shocks later. \(^5\) The demand for variety \( j \) is given by \( c_{NT,i} \left( \frac{p_{NT,i,j}}{p_{NT,i}} \right)^{-\epsilon} \), where \( p_{NT,i} = \left( \int_0^1 p_{NT,i,j}^{1-\epsilon} \right)^{\frac{1}{1-\epsilon}} \) is the price of the tradable good. As in Obstfeld and Rogoff (1995), we assume the price of each variety is set one period in advance. The problem of each entrepreneur is

\[
\max_{p_{NT,i,j}} \mathbb{E} \left[ \Lambda_i \left( \frac{p_{NT,i} - W_i(1 + \tau_{L,i})}{p_{NT,i}} \right) c_{NT,i} \left( \frac{p_{NT,i,j}}{p_{NT,i}} \right)^{-\epsilon} \right] \tag{11}
\]

where \( \Lambda_i \) is a stochastic discount factor. Given that here we assume that the non-tradables producers are fully owned by domestic households, we have \( \Lambda_i = \frac{U_{T,i}}{p_{T,i}} \).

\(^5\)In the appendix, I prove most results with a more general production function \( \chi(N) \), \( \chi'(\cdot) > 0 \) and \( \chi''(\cdot) \leq 0 \)
In equilibrium all entrepreneurs in the same country set the same price. The government can affect firm price setting through tax $\tau_i^L$ set at period $t = 0$. Without uncertainty, $\tau_{L,i} = -\frac{1}{\epsilon}$, which unsets the monopolistic mark-up. This is financed through a lump sum tax from households. The tax guarantees the existence of an equilibrium in which the monetary authority obtains zero labor wedges at any symmetric point. Overall, the exact price setting condition is not important for our results.

1.3 Monetary Policy

For simplicity we assume a central bank can freely alter the price of the tradable good. The key limitation is that due to fixed exchange rates this price must be the same in every country, making it difficult to deal with asymmetric shocks. The monetary policy problem is given by

$$
\max_{\rho_t(s)} \lambda_H V_H + \lambda_F V_F
$$

where $V_H$ and $V_F$ are the value functions of the agents in the home and foreign country in the following competitive equilibrium in which the agents take the chosen tradables prices as given.

1.4 Equilibrium

In this section we define the competitive equilibrium. The state variables are given by $(d_{H}, d_{F}, l_{H}, l_{F})$, a state is denoted by $s$. Denote the state space by $\Omega$. The market clearing conditions are given by

**Goods Markets**

$$
c_{T,H}(s) + c_{T,F}(s) + S_{HF} d_{F,i}(s)(1-e^{-f}) + S_{FH} d_{H,i}(s)(1-e^{-f}) = e_{H}(s) + e_{F}(s), \forall s \in \Omega
$$

$$
c_{NT,i}(s) = AN_i(s) \quad i = H, F, \forall s \in \Omega
$$

**Labor Market**

$$
N_i(s) = N_{i}^{demand}(s), \forall s \in \Omega
$$
Stock Market

\[ S_{ii} + S_{ij} = 1, \quad i = H, F, \quad j = -i, \quad (16) \]

A competitive equilibrium is goods prices \( (p_T(s), p_{NT,H}, p_{NT,F}) \), stock prices \( (p_{S,H}, p_{S,F}) \), consumption decisions \( (c_{T,H}(s), c_{T,F}(s)) \), \( (c_{NT,H}(s), c_{NT,F}(s)) \), stock positions \( (S_{HH}, S_{HF}) \), \( (S_{FH}, S_{FF}) \), labor supply decisions \( (N_H(s), N_F(s)) \) and government policies \( (\tau_{L,H}, \tau_{L,F}) \) and \( (T_H(s), T_F(s)) \) such that

- Given prices, the consumption decisions, stock positions and labor supply decisions solve each households problem characterized by intratemporal conditions and Euler equations.
- Given prices and allocation, the non-tradables prices solve each firm’s problem.
- Given non-tradables prices and allocation, the tradables prices solve the monetary authority’s problem.
- The government budget constraints are satisfied, \( T_i = \tau_{L,i} W_i N_i \), for \( i = H, F \).
- All markets clear.

Because there are four types of shocks and two assets, the market is generally incomplete. However, if some shocks, such as domestic labor and dividend income, were simply linear combinations of each other, the market could be effectively complete. The following assumption rules this case out. Let \( \Sigma = Cov(d_H, d_F, l_H, l_F) \)

**Assumption 1.** \( \Sigma \) is full rank.

The assumption of incomplete markets is relaxed later.

### 1.5 Planner Problem

Generally, the planner problem can be written

\[
\max_{S, p_{NT}, p_T(s), p_S} \lambda_H V_H + \lambda_F V_F \quad (17)
\]

Here, \( V_H = \mathbb{E}[V_H(S, p(s), s)] \) and \( V_F = \mathbb{E}[V_F(S, p(s), s)] \) are the value functions in the following competitive equilibrium, where the agents take the chosen quantities as given. Here we assume the planner can alter also the
non-tradables and stock prices. This simplifies computation as the stock positions can be chosen separately from other quantities. However, we can later show that the planner solution can be implemented with a simple tax on home stock returns.

For clarity let us write the planner problem more explicitly, assuming equity represents a claim only to the tradables endowment. To first simplify the budget constraints, note that

\[ T_i + \Pi_i = p_{NT,i}AN_i - W_iN_i(1 + \tau_{L,i}) + W_iN_i\tau_{L,i} \]

and \( c_{NT,i} = AN_i \). Plugging in and cancelling prices the budget constraint becomes

\[ c_{T,i} = l_i + S_{i,i}d_i + S_{i,j}d_je^{-f}, \quad i = H,F, j = -i \] (18)

Now we can write the planner problem as

\[
\max_{S, p_{NT}, p_T(s), p_S} \sum_{i=H,F} \lambda_i \mathbb{E}[U(c_{i,T}, c_{i,NT}, N_i)] \text{ s.t.}
\]

Equity resource constraint : \( S_{ii} + S_{ij} = 1 \), \( i = H, j = -i \)

Consumption choice FOC : \( c_{NT,i}(s) = \alpha(p(s)c_{T,i}(s), \forall s \)

Budget constraint : \( c_{T,i}(s) = l_i(s) + S_{i,i}d_i(s) + S_{i,j}d_j(s)e^{-f}, \quad i = H,F, j = -i, \forall s \)

Non-tradables resource constraint : \( c_{NT,i}(s) = AN_i(s) \quad i = H,F, \forall s \)

Moreover, the corresponding equilibrium wage is given by the labor supply FOC and the stock prices by the Euler equations. Note that through the budget constraints, the planner faces the stock market friction \( f \). In later sections we introduce further frictions such as informational differences. These modify the planner problem in a straightforward way.

Plugging in the intratemporal FOCs and labor market clearing condition we can write the value function as (I drop time and country subscripts for simplicity),

\[ V(S, p) = U(\alpha(p)c_T(S), c_T(S), \frac{1}{A}\alpha(p)c_T(S)) \] (19)

Where \( c_T(S) \) is given by the budget constraint. As emphasized by Farhi and Werning (2014), the difference between the utility functions of the
planner and agent is that the planner internalizes the multiplier effects of changing consumption. Expressing the value function in terms of consumption of tradable goods rather than stock position, the marginal utility of consumption in each country is

\[ \hat{V}_C(c_T, p) = \alpha(p) U_{NT} + U_T + U_N \frac{\alpha(p)}{A} = U_T(1 + \alpha(p)p) + U_N \frac{\alpha(p)}{A} \quad (20) \]

This can further be written.

\[ \hat{V}_C(c_T, p) = U_T(1 + p\alpha(p)\tau) \quad (21) \]

where

\[ \tau = 1 + \frac{1}{A \frac{U_N}{U_{NT}}} \quad (22) \]

is the labor wedge. Here the planner values extra consumption more than the agent when the labor wedge is positive and less than the agent when it is negative. In the flexible price equilibrium with competitive firms, the labor wedge is always zero. To see this note that by the labor supply FOC: \( \frac{U_N}{U_{NT}} = -\frac{W}{p_{NT}} \). Furthermore, due to zero profits \( A\hat{p}_{NT} = W \). Then the planner’s and agent’s marginal utility coincide. As can be seen from the analysis of the next section, this also implies that the stock positions are efficient. This allocation can be implemented also with monopolistically competitive firms using labor subsidy that offsets the price markup.

### 1.6 Fixed vs. Flexible Exchange Rate

At this point it is instructive to consider the role of the fixed exchange rate. If the exchange rate between the two countries were flexible, the countries could have separate tradables prices. This would in theory enable replicating the flexible price equilibrium exactly. To see this result let \((\hat{p}_{NT,H}(s), \hat{p}_{NT,F}(s), \hat{p}_T(s))\), be the prices in a flexible price equilibrium with zero labor wedge in state \( s \). This is equivalent to an equilibrium with the same allocation but in which home agents face prices \((1, \frac{\hat{p}_{NT,H}(s)}{\hat{p}_{NT,F}(s)})\) and foreign agents prices \((1, \frac{\hat{p}_{NT,F}(s)}{\hat{p}_{NT,H}(s)})\). But these prices are viable if the exchange rate is given by \( \frac{\hat{p}_{NT,F}(s)}{\hat{p}_{NT,H}(s)} \). Because of zero labor wedges the stock positions are also efficient.
Despite such a theoretical result, the empirical literature has found non-zero and cyclical labor wedges also in countries with flexible exchange rates such as the US. Hence we might expect the results of this paper to be relevant also under flexible exchange rates, though the modeling of the frictions causing such behavior is beyond the scope of this paper.

2 Hedging Redistributive Shocks vs Costs

We can now consider the normative implications of the different explanations offered for equity home bias. This section solves the above model and compares the efficiency implications of a holding cost and hedging based explanations. A key result of the analysis is that despite the externality, the question of efficiency is not trivial. Specifically, when equity home bias is due to hedging redistributive shocks, the equilibrium is (approximately) constrained efficient despite the externality. Such an explanation for home bias has been posited in different forms for example by Coeurdacier and Gourinchas (2013) and Heathcote and Perri (2013). The later sections consider additional explanations offered for equity home bias and generalize some of the analysis.

To obtain tractable expressions for the stock positions we follow an approach similar to Devereux and Sutherland (2010). Building on perturbation methods (Judd et al., 2001) they show how a second order approximation of the Euler equation can be combined with a first order solution of other model equations to obtain expressions for zero order stock positions. Here we can combine the second order approximation of the Euler equation with a log-linearization of the budget constraint. Denote log-deviations by tildes, relative values (Home - Foreign) by hats and approximation points (mean values) by bars. We obtain the following proposition

Proposition 1. i) In the base model, the equilibrium (zero order) stock positions $S^{eq}$ are given by

$$S_{HH}^{eq} = S_{FF}^{eq} = S^{eq} = \frac{1}{2} - \frac{1 - \delta}{\delta} \tilde{\beta}_{t,d} + \frac{\tilde{f}}{\gamma \delta \text{Var}(\Delta \tilde{d})}$$

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6See e.g. Karabarbounis (2014). The author defines the labor wedge as MPN over MRS rather than MRS over MPN.

7In the context of this simple model, the expression for the budget constraint is exact, ignoring the fee.
\[ \text{and } S_{HF}^\text{eq} = S_{FH}^\text{eq} = 1 - S_{i}^\text{eq}, \text{ where } \delta \text{ is the mean dividend share of endowment, } \beta_{l,d} = \frac{\text{cov}(\tilde{d}, \tilde{l})}{\text{var}(\tilde{d})} \text{ and } \tilde{f} \text{ is a second order approximation of the fee}^{8}. \text{ The (zero order) stock positions } S_{\text{plan}}^\text{eq} \text{ solving the planner problem are given by } \]

\[ S_{HH}^\text{plan} = S_{FF}^\text{plan} = S_{\text{plan}} = \frac{1}{2} - \frac{1}{2} \frac{1-\delta}{\delta} \beta_{l,d} + \frac{\tilde{f}}{\psi \delta \text{Var}(\Delta \tilde{d})} \]  \hspace{1cm} (24)

\[ \text{and } S_{HF}^\text{plan} = S_{FH}^\text{plan} = 1 - S_{\text{plan}}. \]

ii) \[ \psi = \gamma + \frac{\alpha(p)\bar{p}}{1 + \alpha(p)\bar{p} \tau(\bar{p}, \bar{c})} \times \frac{\partial \tau(p,c)}{\partial c}|_{(c,p)=(\bar{c},\bar{p})}, \]  \hspace{1cm} (25)

where \( \bar{c} \) is mean consumption and \( \bar{p} \) is mean relative price. \( \psi > \gamma \), the planner solution is equivalent to an equilibrium with more risk averse agents.

Proof: See appendix.

From the expression for \( \psi \) given in proposition 1 ii), we can see that the planner’s higher risk aversion is a consequence of procyclical labor wedges. The gap between the planner’s and agent’s risk aversion \( \psi - \gamma \) is increasing in \( \frac{\partial \tau(p,c)}{\partial c}|_{(c,p)=(\bar{c},\bar{p})} \), the sensitivity of the labor wedge to consumption and hence economic conditions. Empirical evidence of procyclical labor wedges is given by Karabarbounis (2014). The following lemma helps to understand the solution.

**Lemma 1.** i) The equilibrium (zero order) stock positions solve the mean-variance problem

\[ \max_{S} V^{eq}(S), \text{ where } V^{eq}(S) = -S_{HF} \delta \tilde{f} - S_{FH} \delta \tilde{f} - \frac{1}{2} \gamma \text{Var}(\tilde{c}_{H}) - \frac{1}{2} \gamma \text{Var}(\tilde{c}_{F}) \]  \hspace{1cm} (26)

s.t.

\[ S_{ii} + S_{ij} = 1, i = H, j = -i \]  \hspace{1cm} (27)

\[ \tilde{c}_{H} = (1-\delta)\tilde{l}_{H} + S_{HH}\delta \tilde{d}_{H} + S_{HF}\delta \tilde{d}_{F} \]  \hspace{1cm} (28)

\[ \text{Alternatively assume that the cost is a 2nd order term as in Tille and van Wincoop (2010).} \]
\[ \tilde{c}_F = (1 - \delta)\tilde{l}_F + S_{FH}\delta \tilde{d}_H + S_{FF}\delta \tilde{d}_F \]  

(29)

ii) The planner solution can be similarly represented as

\[
\max_S V_{plan}(S), \text{ where } V_{plan}(S) = -S_{HF}\hat{f}\delta - S_{FH}\hat{f}\delta - \frac{1}{2}\psi Var(\hat{c}_H) - \frac{1}{2}\psi Var(\hat{c}_F) + K
\]

(30)

and the constraints are as in i). We normalize \(K\) so that \(V_{plan}(S_{eq}) = V_{eq}(S_{eq})\)

Proof: See appendix.

It is easy to verify that the maximization problems result in the correct prices. However, it is important to note that the value function corresponding to a 2nd order approximation of the Euler equation is not generally quadratic. Still, assuming a symmetric solution \((S, 1 - S, 1 - S, S)\) and fixed prices the equilibrium and efficient stock positions maximize a 2nd order approximation of the corresponding value functions as explained in the appendix \(^9\).

Here the difference between the equilibrium and efficient stock position, or alternatively the excess home bias, is given by

\[
S_{eq} - S_{plan} = \frac{\hat{f}}{\gamma\delta Var(\Delta d)} - \frac{\hat{f}}{\psi\delta Var(\Delta d)} \geq 0
\]

(31)

To further illustrate the differences between the choices of the agents and the planner, we can express the corresponding value functions as

\[
V_{plan}(S) = -S_{HF}\hat{f}\delta - S_{FH}\hat{f}\delta - \frac{1}{2}\gamma(Var(\hat{c}_H) + Var(\hat{c}_F)) - \frac{1}{2}((\psi - \gamma)(Var(\hat{c}_H) + Var(\hat{c}_F)) + K
\]

\[
= V_{eq}(S) - \frac{1}{2}((\psi - \gamma)(Var(\hat{c}_H) + Var(\hat{c}_F)) + K
\]

(32)

\(^9\)Assuming a flexible tradables price modifies the expressions only slightly.

15
At this point it is interesting to contrast the two explanations offered for home bias. A large home stock position $S$ can emerge either from the good hedging properties of home assets or frictions. In the context of this simple model, the first channel corresponds to making the beta term small. The second channel corresponds to making the fee $f$ large. Note that when we have $f = 0$, equilibrium and socially optimal holdings coincide. At this point ($S^{MV}$) the stock positions minimize consumption variance in both countries, given the budget constraints. The planner still effectively values the risk sharing benefits more than the agents. However, because the equilibrium features the maximal amount of risk sharing possible, the planner cannot gain by altering the stock positions. On the other hand, when $f > 0$ a planner will choose a lower position in the home stock.

The equilibrium necessarily features more consumption variance than would be attainable with complete markets. To illustrate this, note that with complete markets we would have $\hat{c}(s) = 0 \forall s \in \Omega$ and hence $\text{Var}(\hat{c}) = 0$. However, at the minimum variance point (MV), which attains the most risk sharing possible given incomplete markets we have

$$\text{Var}(\hat{c}^{MV}) = (1 - \delta)^2 (1 - \rho^2) \text{Var}(\hat{l}),$$

where $\rho = \text{Corr}(\hat{l}, \hat{d})$. Our assumption of a full rank covariance matrix for the state variables rules out $\delta = 1$, $\rho = 1$ or $\text{Var}(\hat{l}) = 0$.

Figures 1 and 2 illustrate the solution further. Here we can see how the planner and equilibrium solutions equate the marginal risk sharing benefit $-\text{riskaversion} \times \frac{\partial \text{Var}(c)}{\partial S}$ with the marginal loss in expected consumption that equals the fee. However due to higher effective risk aversion the planner penalizes consumption variance more than the agent. Figures 3 and 4 plot the value functions corresponding to the equilibrium and planner solutions.
Figure 1: Risk Sharing vs. Fees, $f = 0$

Figure 2: Risk Sharing vs. Fees, $f > 0$
The solution has some properties that might at first appear puzzling. Due to symmetry the positions are effectively characterized by a single excess Euler equation that sets the price of a home-foreign equity swap to zero. Because in equilibrium both assets have the same expected return, this value does not directly affect stock positions though it affects stock prices. Similarly, the solution is independent of whether the tradables price can
move around its mean value or is entirely fixed at this point. Intuitively, the value of the equity swap depends only on asymmetric states of the world in which the stocks have different returns. However, altering the tradables price can only smooth symmetric shocks.

Above we used equal Pareto weights to derive the planner solution. The following proposition generalizes the above results

**Theorem 1.** In the base model (with no informational signals or belief heterogeneity), the equilibrium zero order stock positions are efficient if and only if there are no fees $f = 0$.

*Proof.* We know when $f = 0$, the equilibrium solves the planner problem with equal weights. When $f > 0$ we saw that the equilibrium and planner solutions are different for equal Pareto weights. When $\lambda_H \neq \lambda_F$, the planner solution is not symmetric. □

### 2.1 Decentralizing the Planner Solution

This paper largely omits a discussion of the government policy tools that can be used to implement the efficient stock positions. However, perhaps the most natural tool is a proportional tax on home equity returns. The following proposition formalizes this result

**Proposition 2.** In the base model, the planner solution can be implemented with a simple proportional tax $e^{-\tau_S}$ on home stock returns rebated back to agents lump-sum. The optimal tax rate, based on a 2nd order approximation, is given by $\tau_S = \tilde{f} \left( 1 - \frac{\gamma}{\psi} \right)$.

Proof: see appendix.

### 2.2 Why is the planner more risk averse than the households?

As mentioned, the planner’s higher risk aversion results from an aggregate demand externality caused by the nominal rigidities. The effect of the externality varies with business cycle conditions along with labor wedges. For better grasp of this mechanism assume for simplicity that all prices are fixed. Then we can write both the marginal utility from the perspective of the household and the country solely as a function of tradables consumption $U'(c)$ and $V'(c)$. Specifically, then we can write:
\[ V'(c) = U'(c)\Theta(c) \] (34)

where \( \Theta(c) \) describes the effect of the aggregate demand externality. As in Farhi and Werning (2014), we have \( \Theta'(c) < 0 \). When consumption is low, increasing it would result in improvements. However with high enough consumption values, the country can benefit from reducing consumption. More generally, the fact that \( \Theta'(c) < 0 \) implies social benefits to macroeconomic stabilization. We can illustrate this by calculating the risk aversion coefficient of the country. First take the second derivative of the above expression:

\[ V''(c) = U''(c)\Theta(c) + U'(c)\Theta'(c) \] (35)

From here we can solve the effective risk aversion coefficient of a country

\[ -\frac{V''(c)}{V'(c)} = -\frac{U''(c)}{U'(c)} - \frac{\Theta'(c)}{\Theta(c)} \] (36)

In theory we can have \( \Theta(c) < 0 \). However, this does not happen at the symmetric approximation point. Then because we always have \( \Theta'(c) < 0 \), the planner is effectively more risk averse

\[ -\frac{V''(c)}{V'(c)} > -\frac{U''(c)}{U'(c)}. \]

### 2.3 A Comparison to Farhi and Werning (2014)

Finally, we have a comparison to Farhi and Werning (2014). In their model the equilibrium with complete markets is constrained inefficient unless labor wedges are always zero. Assuming no financial frictions, the simple model considered in this section can be seen as an incomplete markets adaptation of their model in the special case of two symmetric countries and no productivity shocks in the non-tradable sector.

Here the assumption of incomplete markets is crucial. If the effective number of shocks equaled the number of assets, the equilibrium would attain the complete markets allocation. Due to symmetry, perfect risk sharing would imply that the tradables price needed to attain a zero labor wedge in the home country \( p_{T,H}(s) \) and the foreign country \( p_{T,F}(s) \) would be the same: \( p_{T,H}(s) = p_{T,F}(s) \) \( \forall s \in \Omega \). Then the equilibrium would actually attain the first best allocation and the planner solution would coincide with the equilibrium solution in a more trivial way. However, above we derive a near efficiency result according to which the equilibrium can approximately coincide with the planner solution despite market incompleteness, which

\[ ^{10} \text{At this point } \Theta(c) = 1 \]
gives rise to externalities in risk sharing. Note that due to imperfect risk sharing the solution is still only second best.

In the above symmetric model without financial frictions, market incompleteness is the only source of asymmetries leading to possible imperfections in the market solution. Later we allow for ex ante asymmetries and stochastic productivity shocks between the two countries that tend to lead to non-zero labor wedges even in the case of complete markets.

3 Additional Explanations

In this section we extend the base model considered in the previous section to include three additional explanations for equity home bias: overestimation of domestic market returns, informational signals concerning home stocks and price hedging.

3.1 Overestimation of Home Return

Originally, French and Poterba (1991) argue that the bias in equity portfolios is due to investors overestimating expected returns on domestic assets. A bias that increases the expected returns on home equity has similar effects than a friction that lowers the expected return. However, the matter is complicated by whether the planner respects the beliefs of agents.

The form of overestimation is not important for the results. To map the discussion to the previous section, assume each home agent overestimates the home return by a fraction \( e^\eta \). First consider a non-paternalistic planner who evaluates welfare using each agent’s subjective beliefs.

**Theorem 2.** Assume the welfare criterion is non-paternalistic. A bias in the estimation of returns is equivalent to a holding cost. All the above results hold under such a welfare criterion. The equilibrium is constrained inefficient.

Proof: See appendix.

We can also consider a paternalistic welfare criterion, so that the planner takes some measure \( \mu \) and uses it to evaluate welfare. However, identifying Pareto inefficient situations does not necessarily require choosing a specific measure for welfare calculations. This is the case for example when using the welfare criterion proposed by Brunnermeier et al. (2014). Under their criterion the equilibrium is inefficient if it is so under any convex combination of agents’ beliefs. The following proposition specifies the welfare properties of equilibrium when the welfare criterion is paternalistic.
Proposition 3. The equilibrium is generally constrained inefficient under any paternalistic belief $\mu$. Let the planner assume belief $\mu$. The efficient stock positions are then given by

\[ S_{\text{plan}}^{HH} = S_{\text{plan}}^{FF} = S_{\text{plan}} = \frac{1}{2} \left( 1 - \frac{1 - \delta}{\delta} \beta_{l,d}^\mu \right) \]

and \[ S_{\text{plan}}^{HF} = S_{\text{plan}}^{FH} = 1 - S_{\text{plan}}. \] Here $\beta_{l,d} = \frac{\text{cov}_\mu(\hat{d}, \hat{l})}{\text{var}_\mu(\hat{d})}$ is calculated under measure $\mu$.

When belief differences concern only the means of returns, this stock position can be implemented with a proportional tax $e^{-\tau}$ on home stock return.

Proof: See appendix.

Note that under such a welfare criterion the equilibrium is inefficient even with no externalities.

3.2 Informational Differences

As noted for example by Brennan and Cao (1997) and Van Nieuwerburgh and Veldkamp (2009), home and foreign investors might receive different signals over home and foreign stocks. This can lead to home bias either through the signals’ effect on the means or variances of returns. In this section we consider the efficiency implications of such signals. It turns out the signals affect the efficiency of stock positions in a way that is somewhat different from other explanations.

Because the specifics of the informational structure of the model are not important for the main arguments, we consider information signals in a particularly tractable setting. Now assume there are $N$ ex ante identical stocks in each country with payoffs $d_H$ and $d_F$. For simplicity assume the dividends’ payoffs are independent. To further simplify algebra we now maintain the assumption that each household in the same country is identical and hence receives the same signal about the home stock; we later consider the effects of investor heterogeneity. In particular, each investor in the home country receives a collection of independent signals $s_H$, $\mathbb{E}[s_H] = 0$, about log dividends such that $s_H = \tilde{d}_H + \epsilon_H$. Similarly each investor in the foreign country receives a collection of independent signals $s_F$ about

\[ s_F \]

This is without loss of generality as we can re-express the asset space using principal components.
foreign log dividends such that \( s_F = \tilde{d}_F + e_F, \mathbb{E}[s_F] = 0 \). We assume the home households do not receive any signals concerning foreign dividends and vice versa. This assumption is innocuous as it is necessary to only assume that home investors receive more accurate signals concerning the home stocks. Similarly we abstract away from learning from prices, as it would not change the results qualitatively.

In equilibrium the home investors hold a portfolio of home and foreign stocks \((S_{HH}, S_{HF})'\). Assuming the stocks are in unit supply, the foreign household portfolios are given by: \( 1_{2N} - (S_{HH}, S_{HF})' \). We can further decompose the investments into a domestic market portfolio that weights the stocks equally and a deviation portfolio resulting from signals: \( S_{HH} = S_{HH} \left( \frac{1}{N}, ..., \frac{1}{N} \right)' + S_{HH} - S_{HH} \left( \frac{1}{N}, ..., \frac{1}{N} \right)' \).

\[ S_{HH} = S_{HH} \left( \frac{1}{N}, ..., \frac{1}{N} \right)' + S_{HH} - S_{HH} \left( \frac{1}{N}, ..., \frac{1}{N} \right)' \tag{38} \]

Here the agent overweights asset \( i \) if and only if \( s_i > 0 \). As mentioned by Van Nieuwerburgh and Veldkamp (2009), the signals do not directly lead to home bias, but rather to agents taking large positions, which here means deviating significantly from an equally weighted portfolio. However, the signals always reduce the variance of the corresponding stocks. To see this, consider a multivariate normal framework. Assume the prior distribution of mean corrected log dividend \( i \) is \( N(0, \text{var}(\tilde{d}_i)) \). Conditional on receiving signal \( s_i \), an application of Bayes law gives a conditional distribution \( \tilde{d}_i | s_i \sim N(\rho s_i, (1 - \rho) \text{var}(\tilde{d}_i)) \), where \( \rho = \frac{\text{var}(\tilde{d})}{\text{var}(s)} < 1 \). From here we can see that independent of signal value, the variance of the dividend is reduced to \( \rho \text{var}(\tilde{d}_i) < \text{var}(\tilde{d}_i) \). This further reduces the variance of the home market portfolio, which tends to result in home bias.

To understand the efficiency implications of informational signals, consider a particular type of deviation. Namely assume the home agents invest \( \epsilon \) more in the foreign market portfolio and \( \epsilon \) less in the home market portfolio. In more primitive terms the agents receive \( \frac{\epsilon}{p_{S,F}^M} \) more units of the foreign market portfolio and \( \frac{\epsilon}{p_{S,H}^M} \) less units of the home market portfolio, where \( p_{S,F}^M \) and \( p_{S,H}^M \) are the equilibrium prices of the two market portfolios. The foreign agents take the opposite positions.

Given a 2nd order approximation, the equilibrium satisfies \(^{12} \)

\(^{12}\)Approximation is still around the deterministic steady-state / mean value. Furthermore, we assume the central bank still treats each country symmetrically so that \( \text{cov}(\bar{r}, \tilde{p}_T) = 0 \).
\( \mathbb{E}_H[\hat{r}] + \frac{1}{2} \mathbb{E}_H[\tilde{r}_F^2 - \tilde{r}_H^2] - \gamma \text{cov}_H(\hat{c}_H, \hat{r}) = 0 \) \hspace{1cm} (39)

Given the beliefs of agents, the country benefits from the transaction if

\( \mathbb{E}_H[\hat{r}] + \frac{1}{2} \mathbb{E}_H[\tilde{r}_F^2 - \tilde{r}_H^2] - \psi \text{cov}_H(\hat{c}_H, \hat{r}) > 0 \) \hspace{1cm} (40)

\( \Leftrightarrow \frac{1}{\psi} \left[ \mathbb{E}_H[\hat{r}] + \frac{1}{2} \mathbb{E}_H[\tilde{r}_F^2 - \tilde{r}_H^2] \right] - \text{cov}_H(\hat{c}_H, \hat{r}) > 0 \) \hspace{1cm} (41)

Plugging in the equilibrium condition gives:

\[
\left[ \frac{1}{\psi} - \frac{1}{\gamma} \right] \left[ \mathbb{E}_H[\hat{r}] + \frac{1}{2} \mathbb{E}_H[\tilde{r}_F^2 - \tilde{r}_H^2] \right] > 0
\]

\( \Leftrightarrow \mathbb{E}_H[\hat{r}_F - \hat{r}_H] + \frac{1}{2} \mathbb{E}_H[\tilde{r}_F^2 - \tilde{r}_H^2] < 0 \) \hspace{1cm} (42)

\( \mathbb{E}_H[\tilde{r}_F^2 - \tilde{r}_H^2] > 0 \) is a Jensen’s correction term that is absent in a continuous time limit of a dynamic model or in a 2nd order approximation in levels. It arises because the signals reduce the variance of the home asset relative to the foreign asset. Ignoring this term, we can see from the condition that the country benefits from home bias reduction when it is expecting the home return to be larger than the foreign return. Intuitively the agents who are less risk averse than the planner overweight the high expected return of the home asset relative to the diversification benefits of the foreign asset. By symmetry, the foreign country benefits from taking the opposite positions when:

\( \mathbb{E}_F[\hat{r}_H - \hat{r}_F] + \frac{1}{2} \mathbb{E}_F[\tilde{r}_H^2 - \tilde{r}_F^2] < 0 \) \hspace{1cm} (44)

Home bias tends to be inefficient when both countries expect their own equity market to perform well relative to the foreign market. In theory the equilibrium can be efficient. More specifically this happens when \( \mathbb{E}_F[\hat{r}_H - \hat{r}_F] = -\mathbb{E}_F[\tilde{r}_H^2 - \tilde{r}_F^2] \) and \( \mathbb{E}_H[\hat{r}_F - \hat{r}_H] = -\mathbb{E}_H[\tilde{r}_F^2 - \tilde{r}_H^2] \). If \( \mathbb{E}_H[\hat{r}_F - \hat{r}_H] = 0 \) and \( \mathbb{E}_F[\hat{r}_H - \hat{r}_F] = 0 \), so that the investors expect the same return for each market, increasing home bias in both countries will actually increase welfare due to the Jensen’s correction effect. However, in general we would expect the
inefficiencies to be smaller when home bias is due to a variance reduction caused by better information concerning home stocks.

Finally briefly consider the effect of investor heterogeneity. For simplicity assume there is only one home asset and one foreign asset. Assume each home investor \( j \in [0, 1] \) obtains an independent signal about home asset \( s_{H,j} = \tilde{d}_H + \epsilon_{H,j} \) from an identical normal distribution and each foreign investor \( k \in [0, 1] \) obtains an independent signal about foreign return \( s_{F,k} = \tilde{d}_F + \epsilon_{H,k} \), again from an identical distribution.

Given a 2nd order approximation, the equilibrium values satisfy

\[
E_{H,j}[\hat{\tilde{r}}] + \frac{1}{2} E_{H,j}[\tilde{r}_F^2 - \tilde{r}_H^2] - \gamma \text{cov}_{H,j}(\tilde{c}_H, \hat{\tilde{r}}) = 0 \quad \forall j \in [0, 1] \tag{45}
\]

\[
E_{F,k}[-\hat{\tilde{r}}] + \frac{1}{2} E_{F,k}[-\tilde{r}_F^2 - \tilde{r}_H^2] - \gamma \text{cov}_{F,k}(\tilde{c}_F, -\hat{\tilde{r}}) = 0 \quad \forall k \in [0, 1] \tag{46}
\]

Moreover as second moments are independent of signal values \( E_{H,j}[\tilde{r}_F^2 - \tilde{r}_H^2] = E_H[\tilde{r}_F^2 - \tilde{r}_H^2] \) and \( E_{F,k}[\tilde{r}_F^2 - \tilde{r}_H^2] = E_F[\tilde{r}_F^2 - \tilde{r}_H^2] \). Now let us consider the same deviation as above. Assume each home agent invest \( \epsilon \) more in the foreign market portfolio and \( \epsilon \) less in the home market portfolio; each foreign agent takes the opposite position. Assume the planner weights each agent in the same country equally. Now the home country benefits from the transaction when:

\[
\int_0^1 \left[ E_{H,j}[\hat{\tilde{r}}] + \frac{1}{2} E_{H,j}[\tilde{r}_F^2 - \tilde{r}_H^2] - \psi \text{cov}_{H,j}(\tilde{c}_H, \hat{\tilde{r}}) \right] dj > 0 \tag{47}
\]

Plugging in the equilibrium values we have

\[
\left[ \frac{1}{\psi} - \frac{1}{\gamma} \right] \int_0^1 \left[ (E_{H,j}[\hat{\tilde{r}}] + \frac{1}{2} E_{H}[\tilde{r}_F^2 - \tilde{r}_H^2]) \right] dj > 0 \tag{48}
\]

\[
\Leftrightarrow \int_0^1 (E_{H,j}[\hat{\tilde{r}}]) dj + \frac{1}{2} E_{H}[\tilde{r}_F^2 - \tilde{r}_H^2]) dj < 0 \tag{49}
\]

Because the signals are mean zero and i.i.d., an application of the law of large numbers gives: \( \int_0^1 (E_{H,j}[\hat{\tilde{r}}]) dj = 0 \). Then the above condition becomes

\[
\frac{1}{2} E_{H}[\tilde{r}_F^2 - \tilde{r}_H^2]) < 0 \tag{50}
\]

\footnote{\( \text{For simplicity perform the approximation for each investor around the same point.} \)}
which is false as the signals always reduce relative variance of the home stock. Then again increasing home bias would increase welfare. Still, because the effect is only through a 2nd order correction term, we would expect relatively small inefficiencies.

We conclude that the efficiency implications of informational signals are more complicated than for example those of financial frictions. Still, we can summarize the above discussion in the following proposition.

**Theorem 3.** Assume home bias is caused by informational signals concerning home stocks. The stock positions are generally constrained inefficient in a 2nd order approximation, though may also be constrained efficient. Home bias is excessive when both countries expect higher returns for the domestic market portfolio than for the foreign market portfolio. Increasing home bias improves welfare when there are no differences in beliefs concerning the expected returns of home and foreign market portfolio.

As mentioned, the above result is largely independent of the specific informational structure of the model. However, I abstracted away from learning choices concerning the signals. Van Nieuwerburgh and Veldkamp (2009) note that learning about information signals can amplify initial informational advantages. Such learning decisions can pose some efficiency questions also in a model with no other externalities as discussed by Kurlat and Veldkamp (2015).

### 3.3 Price Hedging

As noted by Cooper and Kaplanis (1994), home equity might offer a good hedge for changes in domestic prices. This price hedging explanation for equity home bias has been considered for example by Obstfeld and Rogoff (2000b) and Coeurdacier and Gourinchas (2013). Here we address price hedging by introducing multiple tradable goods into the model. Assume there are two input goods, one for each country, used to produce the aggregate tradable good. Let the home stock represent claims on the endowment of the home good and the foreign stock a claim on the endowment of the foreign good. The aggregate tradable good is then given by

\[
C_{T,H} = \left( a_l \psi_l \frac{\phi_l}{\phi_{T-1}} c_{I,H} + (1 - a_l) \frac{\psi_l}{\phi_{T-1}} c_{I,F} \right) \frac{\phi_l}{\phi_{T-1}}
\]

where \(a_l\) measures bias towards the home input good. Because of home bias in the goods market, the price indices in the two countries can be
different. The intratemporal conditions imply that the home price for the aggregate tradable good is given by

$$p_{T,H} = \left( a_I p_{1,H}^{1-\phi_I} + (1-a_I)p_{1,F}^{1-\phi_I} \right)^{1-\phi_I}$$  \hspace{1cm} (52)

where $p_H$ and $p_F$ are the prices of the two input goods. We can assume that the monetary authority controls the prices of the two goods, respecting the market clearing conditions for both goods. \footnote{The relative price of the goods is given by the market clearing conditions, hence the monetary authority determines the overall tradables price level.}

Obstfeld and Rogoff (2000b) argue that trade costs in goods market can explain both the home bias in goods and equities. Such costs do not change the above arguments per se though they affect the tradables price dynamics (and market clearing conditions). With no home bias in preferences the price index becomes:

$$p_{T,H} = \left( p_{1,H}^{1-\phi_I} + (1+t)p_{1,F}^{1-\phi_I} \right)^{1-\phi_I}$$  \hspace{1cm} (53)

where $t$ represents an iceberg cost on the foreign good. Coeurdacier (2009) analyses the conditions under which trade costs generate home bias in equity portfolios.

In equilibrium we have

$$-\gamma \text{cov}(\hat{c}_H, \hat{d}) - \gamma_p \text{cov}(\hat{p}_{T,H}, \hat{d}) = 0$$  \hspace{1cm} (54)

where (dropping country subscripts for simplicity)

$$\gamma_p = -p_T \frac{\partial U_T/p_T}{\partial p_T} \bigg|_{(\hat{c}, \hat{p})} > 0$$  \hspace{1cm} (55)

Using the budget constraint, we can solve the following expression for the home stock position:

$$S = \frac{1}{2} - \frac{1}{2} \beta_{l,d} \frac{1-\delta}{\delta} + \frac{1}{2} \left( 1 - \frac{\gamma_p}{\gamma} \right) \frac{1}{\delta} \beta_{p,d},$$  \hspace{1cm} (56)

where

$$\beta_{p,d} = \frac{\text{cov}(\hat{p}_T, \hat{d})}{\text{var}(\hat{d})}.$$  \hspace{1cm} (57)
The stock position can be increasing or decreasing in $\beta_{p,d}$ depending on
the parameter values. This is because total tradables consumption expendi-
ture can be either decreasing or increasing in tradables price. Given typical
parameter values we have $\gamma_p < \gamma$ so that the coefficient on $\beta_{p,d}$ is positive
15. Then home equity provides a good hedge, when $\beta_{p,d} > 0$. Note however
that here the betas generally depend on the stock positions so that the above
formula is not expressed entirely in terms of structural parameters.

The home country benefits from reducing equity home bias when

$$- \psi \text{cov}(\tilde{c}, \hat{d}) - \psi_p \text{cov}(\hat{p}_{T,H}, \hat{d}) > 0$$

(58)

where

$$\psi_p = -p_T \frac{\partial V_T / p_T}{V_T / p_T}$$

(59)

Plugging in the equilibrium condition we can write this condition as

$$\left[ \frac{\gamma_p}{\gamma} - \frac{\psi_p}{\psi} \right] \text{cov}(\hat{p}_{T,H}, \hat{d}) > 0.$$  

(60)

Now the country benefits from reducing equity home bias when $\text{cov}(\hat{p}_{T,H}, \hat{d}) > 0$ and

$$\frac{\gamma_p}{\gamma} > \frac{\psi_p}{\psi}$$

(61)

This condition seems to be true for standard parameter values. In the
symmetric equilibrium $\text{cov}(\hat{p}_{T,H}, \hat{d}) > 0$ implies $\text{cov}(\hat{p}_{T,F}, -\hat{d}) > 0$. Then
the foreign country also benefits from taking the opposite positions. This
correlation pattern can be created by some parameter values as in Coeurdacier (2009). However, to explain home bias by price hedging effects we
need $\text{cov}(\hat{p}_{T,H}, \hat{d}) < 0$ so that home equity provides a good hedge for do-
mestic prices. Then increasing home bias would actually increase welfare.
However, empirically equity returns are largely uncorrelated with price
changes (Coeurdacier and Gourinchas (2013)). Hence we would expect
$\text{cov}(\hat{p}_{T,H}, \hat{d}) \approx 0$, so that the price channel is largely irrelevant for the effi-
ciency of stock positions.

15Given the calibration below we have $\gamma_p \approx 1.6$ and $\gamma = 3$
4 Generalizing the Results

This section wraps up the results of the previous section and considers some extensions of the simple symmetric two period model. Table 1 summarizes the results from the simple base model considered in the previous sections as well as extends the results to the flexible price case. As seen in the previous section, flexible prices (and wages) imply zero labor wedges in each state. It is straightforward to verify that then the equilibrium and planner solutions coincide. However, due to incomplete markets the equilibrium does not reach first best. As seen from the second row of table 1, we conclude that with flexible prices the equilibrium attains 2nd best irrespective of the explanation for equity home bias.

As illustrated by table 1, the nominal rigidities case has more interesting implications for the efficiency of stock positions. First, we derived a near efficiency result according to which the sticky price model approximately attains the constrained efficient solution absent any stock market frictions or informational signals. Second, we found that a holding cost of foreign equity implies excessive equilibrium home bias. Similarly, overestimation of home stock returns results in excessive home bias both under a paternalistic and non-paternalistic welfare criterion. Finally, we found that given informational signals, the sticky price model may or may not yield the constrained efficient solution. Increasing equity home bias improves welfare when households in both countries expect the same return for the home and foreign market portfolio. However, home bias can also be excessive when households in each country expect a higher return for the corresponding domestic market portfolio.

Table 1: Summary of Results From the Base Model

<table>
<thead>
<tr>
<th>Model</th>
<th>Explanation for Equity Home Bias</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Hedging</strong></td>
<td><strong>Costs / Biased Beliefs</strong></td>
</tr>
<tr>
<td>Sticky Prices</td>
<td>≈ 2nd best</td>
</tr>
<tr>
<td>Flex. Prices</td>
<td>EHB Excessive</td>
</tr>
<tr>
<td></td>
<td>2nd best</td>
</tr>
<tr>
<td></td>
<td>EHB usually too low</td>
</tr>
</tbody>
</table>

---

16 Equilibrium with biased beliefs is 2nd best with a non-paternalistic welfare criterion but not with a paternalistic one
4.1 Steady-State of Dynamic Model with Symmetric Countries

First note that the above analysis is based on what is essentially a static model. However, we can easily generalize almost all of the analysis to concern the steady-state of a corresponding dynamic model with symmetric countries.

In theory the planner problem generalizes to the dynamic case. However, here we must allow the planner to alter the dynamic sequence of prices. Because characterizing these policies is complicated, we here choose a simplified approach. Namely, we assume that instead of two countries, there are two groups of countries, a home group and a foreign group. Each group consists of a continuum of ex ante and ex post identical countries. Now each country can take the sequence of stock prices as given. For a more formal characterization of equilibrium in the case of two groups of countries, see the companion paper Sihvonen (2016).

For simplicity still assume no productivity shocks in the non-tradable sector. The exact notion of steady-state is not important for our results that hold at any symmetric point. However, note that portfolio choice is indeterminate at the deterministic steady-state often considered in macroeconomics. Coeurdacier et al. (2011) define a steady state in which the agents anticipate the effect of future shocks. The efficiency result also hold for the zero-order variables as defined by Devereux and Sutherland (2011) or Tille and van Wincoop (2010). The proposition below generalizes the analysis of the previous section.

Proposition 4. Consider a steady-state of a dynamic model with two groups of countries, home and foreign. Assume there are no productivity shocks in the non-tradable sector. In such a steady-state theorems 1-3 and propositions 2 and 3 hold from the perspective of each small country.

Proof: see appendix.

4.2 Generalized Discussion

This section generalizes the above discussion of the relevant mechanisms. Now let the countries be of different sizes $\omega > 0$ and $1 - \omega$. Also allow for stochastic productivity shocks in the non-tradables sector. Without much loss in generality still consider the two period model. Also allow the statistical properties of $(d_H, l_H, A_H)$ and $(d_F, l_F, A_F)$ to be different. Similarly we allow for differences in the preference parameters between home and foreign countries $(\gamma_H, \sigma_H, \phi_H, a_H)$ and $(\gamma_F, \sigma_F, \phi_F, a_F)$. The equity claims can also
partly represent non-tradables profits. Now the market clearing conditions for the two input goods are:

$$\omega c_{H,T,j} + (1 - \omega)c_{F,T,j} + \omega S_{HF}d_{F,j}(1 - e^{-f}) + (1 - \omega)S_{FH}d_{H,j}(1 - e^{-f}) = e_{H,j} + e_{F,j}, j = 1, 2$$  (62)

Similarly, the market clearing conditions for stock market become

$$\omega S_{ii,t} + (1 - \omega)S_{ij,t} = 1, i = H, F, j = -i$$  (63)

Other conditions such as household and firm problems remain the same. Finally, we may allow for trading in other instruments such as bonds. Because of asymmetries and stochastic productivity shocks, labor wedges might generally be non-zero even in the case of complete markets as in Farhi and Werning (2014). Hence we may generally relax the assumption that the effective number of shocks is greater than the number of assets.

A 2nd order approximation of the relative home Euler equation yields

$$\mathbb{E}_H[\hat{r}] + \frac{1}{2} \mathbb{E}_H[\hat{r}^2_F - \hat{r}^2_H] - \gamma_H \text{cov}_H(\hat{c}_H, \hat{r}) - \gamma_{p,H} \text{cov}_H(\hat{p}_T, \hat{r}) = 0.$$  (64)

Here $\mathbb{E}_H[\hat{r}]$ can depend on fees and informational signals. Similarly the above variances and covariances can depend on such signals. The country benefits from home bias reduction when

$$\mathbb{E}_H[\hat{r}] + \frac{1}{2} \mathbb{E}_H[\hat{r}^2_F - \hat{r}^2_H] - \psi_H \text{cov}_H(\hat{c}_H, \hat{r}) - \psi_{p,H} \text{cov}_H(\hat{p}_T, \hat{r}) + \psi_{A,H} \text{cov}_H(\hat{A}_H, \hat{r}) > 0$$  (65)

where (leaving out country subscripts for simplicity)

$$\psi_A = A \frac{\partial U_T(1 + a(p)\tau_T)/p_T}{\partial A} > 0$$  (66)

Plugging in the equilibrium condition, we can write this as:

17In primitive terms, the deviation is such that each country obtains $\frac{1}{p_{SF}}$ units more of the foreign stock and $\frac{1}{p_{SII}}$ units less of the home stock.
We have already discussed most of the mechanisms above. However, productivity shocks in the non-tradables sector create a new channel that can lead to inefficiencies in the stock positions. When $\text{cov}_H(\tilde{A}_H, \hat{r}) < 0$ so that foreign equity is a better hedge for domestic productivity shocks, increasing home equity holdings can result in improvements due to the productivity shock channel. Effectively, the productivity shocks work in the opposite direction than endowment shocks. When productivity is high, consumption tends to be inefficiently low. On the other hand consumption can be increased in such states by increasing investment in assets that pay well in high productivity states.

The foreign country benefits from taking the opposite positions when

$$
\left[ \frac{1}{\psi_H} - \frac{1}{\gamma_H} \right] \left[ \mathbb{E}_H[\hat{r}] + \frac{1}{2} \mathbb{E}_H[\tilde{r}_F^2 - \bar{r}_F^2] \right] + \left( \frac{\psi_{p,H}}{\psi_H} - \frac{\gamma_{p,H}}{\gamma_H} \right) \text{cov}_H(\hat{p}_{T,H}, \hat{r})
$$

$$< 0 + \psi_{A,H} \text{cov}_H(\tilde{A}_H, \hat{r}) > 0 \quad (67)$$

From this we can see that when both $\mathbb{E}_H[\hat{r}]$ and $\mathbb{E}_F[\tilde{r}]$ are large relative to the other terms, both countries can benefit from home bias reduction. As explained above in greater detail, differences that tend to lead to home bias in both countries can be due to either fees on foreign equity, overestimation of home equity returns or informational signals. The consumption hedging channel does not lead to similar inefficiencies in the stock positions. At typical parameter values, the price hedging channel can imply benefits to increasing home bias. However, the correlation between equity returns and price changes is close to zero (Coeurdacier and Gourinchas, 2013). Hence this channel is unlikely to play a major role for the efficiency of stock positions. Finally, informational signals tend to reduce home market portfolio variance relative to the variance of the foreign market portfolio. This increases the variance correction term $\frac{1}{2} \mathbb{E}_F[\tilde{r}_H^2 - \bar{r}_F^2]$ and tends to result in public benefits from increasing equity home bias. However, we would generally expect this second order effect to result in relatively smaller inefficiencies.
5 Numerical Analysis

The focus of this paper is not quantitative. However, to further illustrate the efficiency implications of the different explanations offered for equity home bias, I now study equilibrium and socially optimal portfolios numerically. For simplicity consider the symmetric model and assume no productivity shocks in the non-tradables sector.

Table 2 shows the baseline values for structural parameters. Note that for simplicity we still impose perfect price rigidity in the non-tradables sector.

Table 2: Structural Parameter Values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \gamma )</td>
<td>3</td>
<td>Mendoza (1991)</td>
</tr>
<tr>
<td>( \phi )</td>
<td>0.7</td>
<td>Gali and Monacelli (2005)</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>( \delta )</td>
<td>0.2</td>
<td>Coeurdacier and Gourinchas (2013)</td>
</tr>
<tr>
<td>( a )</td>
<td>0.5</td>
<td>Stockman and Tesar (1995)</td>
</tr>
</tbody>
</table>

5.1 Hedging Explanations

We first consider the hedging channels discussed in chapters 3 and 4. Ignoring price hedging effects, the home stock position is given by

\[
S = \frac{1}{2} - \frac{1}{2} \frac{1 - \delta}{\delta} \beta_{l,d}
\]  

As seen before this is also the efficient stock position. Stock positions are tilted towards home assets when the correlation between stock returns and labor income is negative: \( \beta_{l,d} < 0 \). This is a structural parameter in our two period model with a tradable endowment. However, the appendix offers two ways to microfound a negative correlation coefficient in a model in which the tradable endowment is produced using labor and capital inputs. Here to replicate a home stock position of 0.8, we need to set \( \beta_{l,d} = -0.15 \).

The question of the right beta parameter is complicated by measurement problems. In a dynamic model \( \beta_{l,d} \) is replaced \( \beta_{h,r} \), which measures the dependency between returns to human capital wealth and stock returns. On the other hand, innovations in human capital wealth are not directly observable. The analysis of Lustig and Nieuwerburgh (2008) suggests a
negative value for $\beta_{h,r}$. However, Coeurdacier and Gourinchas (2013) find support for a positive coefficient.

Coeurdacier and Gourinchas (2013) posit that home bias can be explained by jointly considering optimal portfolio formation with stocks and bonds. To illustrate the effect of bonds on equity portfolio choice, we now introduce domestic and foreign bonds into the model. For simplicity assume trading bonds entails no cost. We can always project the relative equity returns onto relative bond returns $\hat{r}_b$ so that

$$\hat{d} = \beta \hat{r}_b + \hat{d}_o$$

(70)

where $\hat{d}_o$ is orthogonal to $\hat{r}_b$. The Euler equations characterizing optimal equity and bond portfolio choice are approximately given by

$$\text{cov}(\hat{c}, \hat{d}) = 0$$

(71)

$$\text{cov}(\hat{c}, \hat{r}_b) = 0$$

(72)

Plugging the decomposition into the first Euler equation gives

$$\beta \text{cov}(\hat{c}, \hat{r}_b) + \text{cov}(\hat{c}, \hat{d}_o) = 0$$

(73)

Using the second Euler equation, we get

$$\text{cov}(\hat{c}, \hat{d}_o) = 0$$

(74)

From this we can solve

$$S = \frac{1}{2} - \frac{1}{2} \cdot \frac{\delta}{\delta - \beta_{l,d,o}}$$

(75)

where $\beta_{l,d,o}$ measures the part of the dependency between returns to human capital and equity returns that is orthogonal to bond returns. Coeurdacier and Gourinchas (2013) argue that $\beta_{l,d,o} < 0$ (but $\beta_{l,d} > 0$), which tends to create home bias in equity. Here the Euler equations characterizing the equity and bond choices for the planner are the same as above. Hence the equilibrium remains constrained efficient.

Finally, briefly consider the efficiency implications of the price hedging channel. As seen before, with multiple tradable goods the home stock position becomes

$^{18}$More specifically they find that $\beta_{h,r,o} < 0$ but $\beta_{h,r} > 0$
\[ S = \frac{1}{2} - \frac{1}{2} \beta_{l,d} \frac{1 - \delta}{\delta} + \frac{1}{2} \left( 1 - \frac{\gamma_p}{\gamma} \right) \frac{1}{\delta} \beta_{p,d}, \tag{76} \]

As I noted earlier, we typically have \( \gamma > \gamma_p \). Therefore \( \beta_{p,d} > 0 \) results in equity home bias. Moreover, we saw that with typical parameter values, this tends to create public benefits to increasing equity home bias from equilibrium levels. However, empirically \( \beta_{p,d} \approx 0 \) and \( \beta_{p,d,o} \approx 0 \) (Coeurdacier and Gourinchas, 2013). This implies that both the equilibrium and efficient stock positions are given by

\[ S \approx \frac{1}{2} - \frac{1}{2} \frac{1 - \delta}{\delta} \beta_{l,d} \tag{77} \]

where \( \beta_{p,d} \) is replaced by \( \beta_{p,d,o} \) when allowing for bond trading. Hence we would not expect the price hedging channel to be important for the efficiency of stock positions.

### 5.2 Costs and Overestimation of Home Return

As seen before, in the case of a holding cost the equilibrium and efficient home stock positions are given by:

\[ S^{eq} = \frac{1}{2} - \frac{1}{2} \frac{1 - \delta}{\delta} \beta_{l,d} + \frac{\check{f}}{\gamma \delta \text{Var}(\Delta \hat{d})} \tag{78} \]

\[ S^{plan} = \frac{1}{2} - \frac{1}{2} \frac{1 - \delta}{\delta} \beta_{l,d} + \frac{\check{f}}{\psi \delta \text{Var}(\Delta \hat{d})} \tag{79} \]

Here the equilibrium necessarily features excessive home bias because \( \psi > \gamma \). To see how the friction numerically affects the wedge between equilibrium and efficient stock positions, let us perform a simple calibration exercise.

First, assume \( \beta_{l,d} = 0 \). Moreover, let \( \text{Var}(\hat{d}) = 0.06 \), which corresponds to assuming that each dividend has variance 0.04 (roughly annual stock market variance for US) and a correlation coefficient of 0.5. Now in order to attain a home stock position of 0.8, we need \( \check{f} = 0.0108 \). This means roughly that costs reduce the foreign stock return by roughly 1% relative to home stock return. While the direct costs of foreign equity investment are unlikely to be so large, this cost can partly represent indirect information acquisition costs. Moreover, as seen in the previous section, \( \check{f} \) can also represent overestimation of home return.
Our assumptions for structural parameters imply $\psi \approx 7.9$. This results in an efficient home stock position of $S_{plan} \approx 0.61$, a near 20 percentage point difference to the equilibrium level of 0.8. However, it is important to note that we assumed that the non-tradables price is entirely fixed. When the price can partially respond to shocks, the planner’s risk aversion is plausibly lower, implying a smaller gap between equilibrium and efficient levels of home bias.

### 5.3 Informational Signals

Finally, let us consider the efficiency implications of informational signals numerically. Assume a unit mass $[0,1]$ of investors in each country. The investors in the same country are ex ante identical but receive different signals and hence choose different portfolios. Let there be two stocks: a home stock and a foreign stock. Each home investor $j$ receives a signal about the home dividend $s_{H,j} = \tilde{d}_H + \epsilon_{H,j}$. The signals are independent draws from the same underlying zero mean normal distribution. Similarly each foreign investor receives an independent signal concerning the foreign dividend $s_{F,k} = \tilde{d}_F + \epsilon_{F,k}$ from the same underlying normal distribution.

Given a 2nd order approximation, the equilibrium values satisfy:

$$\mathbb{E}_{H,j}[\hat{r}] + \frac{1}{2} \mathbb{E}_{H}[\hat{r}_F^2 - \hat{r}_{H}^2] - \gamma \text{cov}_{H,j}(\tilde{c}_H, \hat{r}) = 0$$  \hspace{1cm} (80)

as well as

$$\int_{0}^{1} \left[ \mathbb{E}_{H,j}[^{d}][\hat{r}] + \frac{1}{2} \mathbb{E}_{H}[\hat{r}_F^2 - \hat{r}_{H}^2] - \gamma \text{cov}_{H,j}(\tilde{c}_H, \hat{r}) \right] dj = 0$$  \hspace{1cm} (81)

For simplicity assume the domestic and home variables are uncorrelated with each other. Then we can write

$$\text{cov}_{H,j}(\tilde{c}_H, \hat{r}) = \text{cov}_{H,j}((1 - \delta)I_H + S_{HH}\delta d_H + S_{HF}\delta d_F, \hat{r})$$

$$= (1 - \delta)\text{cov}_{H,j}(I_H, \hat{r}) + S_{HH,j}\delta \text{cov}_{H,j}(d_H, \hat{r}) + S_{HF,j}\delta \text{cov}_{H,j}(d_F, \hat{r})$$

$$= -(1 - \delta)\text{cov}_{H,j}(I_H, \hat{d}_H) - S_{HH,j}\delta \text{var}_{H,j}(\hat{d}_H) + S_{HF,j}\delta \text{var}_{H,j}(\hat{d}_F)$$  \hspace{1cm} (82)

To focus purely on the informational signal channel, assume that dividends and labor income are uncorrelated. Now we have:
The above follows from the fact that the posterior dividend variance is constant independent of the signal value. Now applying the integral we can write:

\[
\int_0^1 \left[ \mathbb{E}_{H,j}[^{\hat{r}}] + \frac{1}{2} \mathbb{E}_H[^{r_F^2 - r_H^2}] - \gamma \text{cov}_{H,j}(\hat{c}_H, \hat{r}) \right] dj = \int_0^1 \left[ \mathbb{E}_{H,j}[^{\hat{r}}] \right] dj + \frac{1}{2} \mathbb{E}_H[^{r_F^2 - r_H^2}] \\
- \gamma \left[ - \int_0^1 S_{HH,j} dj \times \delta \text{var}_{H}(\tilde{d}_H) + \int_0^1 S_{HF,j} dj \times \delta \text{var}_{H}(\tilde{d}_F) \right] (84)
\]

Define the average home market portfolio by \( S = \int_0^1 S_{HH,j} dj \). By symmetry and the law of large numbers \( \int_0^1 S_{HF,j} dj = 1 - S \). Similarly \( \int_0^1 \mathbb{E}_{H,j}[\hat{r}] dj = 0 \). Moreover, note that \( \mathbb{E}_H[^{r_F^2 - r_H^2}] = \text{var}_H(\tilde{d}_F) - \text{var}_H(\tilde{d}_H) \). Applying these results, the equilibrium condition becomes:

\[
\frac{1}{2} [\text{var}_H(\tilde{d}_F) - \text{var}_H(\tilde{d}_H)] + \gamma S \times \delta \text{var}_H(\tilde{d}_H) - \gamma (1 - S) \delta \text{var}_H(\tilde{d}_H) = 0 (85)
\]

From this we can solve:

\[
S_{eq} = \frac{1}{2} \frac{\text{var}_H(\tilde{d}_F) - \text{var}_H(\tilde{d}_H)}{\delta \gamma [\text{var}_H(\tilde{d}_H) + \text{var}_H(\tilde{d}_F)]} + \frac{\text{var}_H(\tilde{d}_F)}{\text{var}_H(\tilde{d}_H) + \text{var}_H(\tilde{d}_F)} (86)
\]

Without any informational signals, the home and foreign dividend variances are equal. Then the above expression gives \( S_{eq} = \frac{1}{2} \), i.e. there is no home bias. Note that the first term tends to tilt the stock position towards foreign stocks and the second term towards home stocks. When \( \delta \) is small so that consumption is largely driven by labor income rather than stock returns, a variance reduction tends to result in foreign stock bias. Following Van Nieuwerburgh and Veldkamp (2009), I ignore labor income effects in the calibration and consider the case \( \delta \to 1 \).

I maintain the prior dividend variance at \( \text{var}(\tilde{d}_H) = \text{var}(\tilde{d}_F) = 0.04 \). Because the home investors receive no signals concerning foreign stocks \( \text{var}_H(\tilde{d}_F) = \text{var}(\tilde{d}_F) = 0.04 \). Now in order to achieve a home stock position
of 0.8, we need \( \text{var}_H(\tilde{d}_H) = 0.002 \). That is, the signals need to reduce the home market variance to close to zero, which seems implausible. However, to attain a home stock position of 0.7, we need \( \text{var}_H(\tilde{d}_H) = 0.01 \) and to reach a position of 0.6, \( \text{var}_H(\tilde{d}_H) \approx 0.021 \). These numbers, which imply that the signals reduce variance by roughly 75% and 50% respectively might be more plausible, especially if we allow the investors to learn about the payoffs as in Van Nieuwerburgh and Veldkamp (2009).

To study the efficiency implications of informational signals, note that we can derive an equivalent expression for the average efficient home stock position

\[
S_{\text{plan}} = \frac{1}{2} \delta \psi \left[ \frac{\text{var}_H(\tilde{d}_H) - \text{var}_H(\tilde{d}_F)}{\text{var}_H(\tilde{d}_H) + \text{var}_H(\tilde{d}_F)} \right] + \frac{\text{var}_H(\tilde{d}_F)}{\text{var}_H(\tilde{d}_H) + \text{var}_H(\tilde{d}_F)}
\]

(87)

Plugging in the variance of 0.01 yielding a home stock position of 0.7 and using \( \psi \approx 7.9 \), we get \( S_{\text{plan}} \approx 0.76 \), that is roughly 6 percentage points above the equilibrium level. This is consistent with the theoretical discussion, where I noted that the variance reduction channel of informational signals tends to result in some public benefits from increasing equity home bias. However, the wedge between equilibrium and efficient positions appears smaller when compared to the wedge implied by holding cost or overestimation of home return. Moreover, the effect is somewhat specific to the approximation method. On the other hand, we effectively abstracted away from the signals affecting the expected returns of the home and foreign market portfolio. Plausibly the signals sometimes create expected return differences between home and foreign assets, which might increase the wedges between equilibrium and efficient stock positions.

6 Conclusions

I study whether policies to reduce the home bias in equity portfolios can increase welfare. More specifically, I analyze the efficiency of stock positions in a standard macroeconomic model with nominal rigidities and fixed exchange rates. The model can generate home bias in equity positions through multiple channels that correspond to the different explanations proposed in

\[19\] Here we restrict the planner’s choice set to only include the particular type of deviation where each investor reduces home stock position by the same amount. This is attainable using a simple tax on home equity returns.
the literature. I find that the different strands of explanations bear different implications for the efficiency of equity home bias.

First, when equity home bias is due to hedging redistributive shocks, the equilibrium stock positions tend to coincide with efficient stock holdings. On the other hand, a holding cost on foreign equity or overestimation of the home market return implies excessive equilibrium home bias. Finally, the efficiency implications of informational signals are more complicated, though they typically imply some benefits from further increasing equity home bias.

This paper remains agnostic about the true cause of home bias. At the same time, it suggests that optimal government policy, such as the taxation of foreign capital gains, depends on the cause of home bias. This stresses the importance of disentangling the relevance of the different proposed channels empirically. Moreover, I hope that new theoretical research on the causes of equity home bias would also address the relevant consequences on the efficiency of home bias.

The paper has implications for designing international risk sharing arrangements. Athanasoulis and Shiller (2001) call for the creation of new financial products to facilitate international risk sharing. However in the case of frictions, gains can also be made from optimizing the positions in currently available instruments, especially equity. While establishing new insurance markets might entail costs or legal difficulties, it seems that the positions in available instruments could be adjusted with relatively simple tools such as taxes on capital gains.

An interesting extension would be to add limited stock market participation to the framework considered in this paper. It seems that generally the planner would want to both increase participation in risk sharing as well as alter the positions chosen by the agents.

7 Appendix A: Proofs

7.1 Proof of Proposition 1

i) Because the solution is symmetric \((S, 1 - S, S, 1 - S)\), the stock position \(S\) can be solved similarly to Devereux and Sutherland (2011) and Coeurdacier and Gourinchas (2013). The approximation consists of a 2nd order log-linearization of the Euler equation and a first order log-linearization
of the budget constraint (BC)\(^\text{20}\). Denote log deviations by tildes, relative (Home - Foreign) log values by hats and approximation points by bars. Approximating the fee around 0, the BC becomes

\[
\tilde{c}_H = (1 - \delta) \hat{l}_H + S \delta \bar{d}_H + (1 - S) \delta (\bar{d}_F - \tilde{f})
\]

(88)

where \(\delta\) is the mean dividend share of income. Subtracting the corresponding BC for the foreign country we get the relative BC:

\[
\hat{c} = (1 - \delta) \hat{l} + (2S - 1) \delta \hat{d}
\]

(89)

Where \(\hat{c}\) is relative log-consumption, \(\hat{d}\) is relative log-dividend, \(\hat{l}\) is relative log-labor income, \(\delta\) is mean dividend share of endowment and \(\tilde{f}\) is a second order approximation of the fee. Deduct the Euler equations for home and foreign stock. This gives

\[
\mathbb{E}_0 \left[ U_T (R_{HH} - R_{HF}) \frac{1}{p_T} \right] = 0
\]

(90)

where \(R_{HF} = e^{-\tilde{f}} R_{FF}\). Then consider a second order approximation around the mean values\(^\text{21}\). Again approximate the cost around 0. After deducting the conditions for home and foreign investor, we get

\[
\Leftrightarrow \text{Cov}(-\gamma(\bar{p}, \bar{c}) \Delta \tilde{c}_T, \hat{d}) + \text{Cov}(-\gamma_s(\bar{p}, \bar{c}) \bar{p}_T, \hat{d}) = -2(f - 0.5 f^2) = -2\tilde{f}
\]

(91)

where

\[
\gamma(\bar{p}, \bar{c}) = -\frac{\partial U_T (\bar{a}_T, \bar{c}_T, N)}{\partial c_T} \bigg|_{\bar{c}, \bar{N}} \hat{c} = -\frac{\alpha g_{21}(\bar{a}_T, \bar{c}_T) + g_{22}(\bar{a}_T, \bar{c}_T)}{g(\bar{a}_T, \bar{c}_T)} \hat{c}
\]

(92)

Note that at any symmetric solution \(\text{Cov}(\bar{p}_T, \hat{d}) = 0\). Hence we obtain

\[
\text{Cov}(-\gamma(\bar{p}, \bar{c}) \Delta \tilde{c}_T, \hat{d}) = -2(f - 0.5 f^2) = -2\tilde{f}
\]

(93)

From here we can solve the stock position using the relative BC

\[
S^c_T = \frac{1}{2} - \frac{1 - \delta}{2} \beta_{l,d} + \frac{\tilde{f}}{\gamma(\bar{p}, \bar{c}) \delta \text{Var}(\Delta \hat{d})}
\]

(94)

\(^{20}\)A first order approximation of the BC is sufficient for solving the stock position up to 2nd order accuracy because we are approximating a product in the Euler equation.

\(^{21}\)We can also approximate around “zero order variables”. The result is otherwise the same except that the fee is also multiplied by the difference between zero order and mean values.
For generality assume a production function \( \chi(N), \chi'( ) > 0, \chi''( ) \leq 0. \) Then the marginal utility in the country becomes

\[
\dot{V}_c(c_T, p) = U_T(1 + \alpha(p)p) + U_N \frac{\partial\chi^{-1}}{\partial c_T} \alpha(p)
\]

(95)

The planner’s FOC w.r.t. \( S_{HH} \) gives

\[
\mathbb{E}_0 \left[ \frac{\partial V_H(S, p)}{\partial S_{HH}} - \frac{\partial V_F(S, p)}{\partial S_{FH}} \right] = 0
\]

(96)

\[
\Leftrightarrow \mathbb{E}_0 \left[ \frac{\partial \dot{V}_H(c_{HH}, p)}{\partial c_{HH}} d_{HH} - \frac{\partial \dot{V}_F(c_{FF}, p)}{\partial c_{FF}} d_{HH} e^{-f} \right] = 0
\]

(97)

Now a second order approximation gives

\[
\text{Cov}(-\psi(\bar{p}, \bar{c}) \Delta \hat{c}_T, \hat{d}) = -\tilde{f}
\]

(98)

where

\[
\psi(\bar{p}, \bar{c}) = \bar{c}_T(1 + \bar{\alpha} \bar{p}) \times
\]

\[
(\bar{\alpha} g_{2,1}(\bar{\alpha} \bar{c}_T, \bar{c}_T) + g_{2,2}(\bar{\alpha} \bar{c}_T, \bar{c}_T)) - \frac{h''(\chi^{-1}(\bar{\alpha} \bar{c}_T)) \left( \frac{2\chi^{-1}(\bar{\alpha} \bar{c}_T)}{\alpha} \right)^2 + h'(\chi^{-1}(\bar{\alpha} \bar{c}_T)) \frac{\partial^2 \chi^{-1}}{\partial c_T^2} \bar{\alpha}^2}{1 + \bar{\alpha} \bar{p}}
\]

(99)

Now the stock position can solved similarly to above.

\[
\psi(\bar{p}, \bar{c}) = \bar{c}_T \times
\]

\[
(\bar{\alpha} g_{2,1}(\bar{\alpha} \bar{c}_T, \bar{c}_T) + g_{2,2}(\bar{\alpha} \bar{c}_T, \bar{c}_T)) - \frac{h''(\chi^{-1}(\bar{\alpha} \bar{c}_T)) \left( \frac{2\chi^{-1}(\bar{\alpha} \bar{c}_T)}{\alpha} \right)^2 + h'(\chi^{-1}(\bar{\alpha} \bar{c}_T)) \frac{\partial^2 \chi^{-1}}{\partial c_T^2} \bar{\alpha}^2}{1 + \bar{\alpha} \bar{p}}
\]

\[
g_{2}(\bar{\alpha} \bar{c}_T, \bar{c}_T) - \frac{h'(\chi^{-1}(\bar{\alpha} \bar{c}_T)) \frac{\partial \chi^{-1}}{\partial c_T} \bar{\alpha}}{1 + \bar{\alpha} \bar{p}}
\]

(100)

ii) The representation for \( \psi \) in the case of a linear production function follows directly from the representation \( U_T(1 + \alpha p \tau) \). To see the second part notice:
where we used $h'( ) > 0$ and $h''( ) > 0$, $\frac{\partial x^{-1}}{\partial c} > 0$ and $\frac{\partial^2 x^{-1}}{\partial c^2} \geq 0$ and (the denominator is positive).

Assuming CRRA preferences over a CES aggregator, we have $U_T = \left( \frac{p_T}{c_T} \right)^{1-\gamma} \frac{1}{a} \frac{d}{d} c_T^{-\gamma}$. $\gamma(\bar{p},\bar{c}) = \gamma$. Then the planner’s risk aversion is globally higher. Assuming a linear production function we have

$$
\psi(\bar{p},\bar{c}) = \gamma + \left( \frac{\bar{p}}{\bar{c}} \right)^{\gamma-1} a^{-\frac{1}{\gamma}} h''(\bar{\alpha} g_2(\bar{\alpha} g_2,\bar{c})) \bar{c}^{1+\gamma}(1 + \bar{\alpha} \bar{p})^{-1}
$$

At any symmetric point, the planner can achieve a zero labor wedge so that the planner’s marginal utility is $U_T$ for each country. At this point we have

$$
\psi = \gamma(1 + \bar{\alpha} \bar{p}) \bar{c}_T + (1 + \bar{\alpha} \bar{p}) \bar{c}_T g_2(\bar{\alpha} \bar{c}_T,\bar{c}_T) \left[ h''(\bar{\alpha} \bar{c}) \bar{c}_T \right] \left[ \bar{\alpha} \bar{c}_T \right] (102)
$$

This expression is used when deriving most of the numerical results.

### 7.2 Proof of Lemma 1

It is easy to verify that the proposed maximization problems result in the correct stock positions. Here we verify the proposed value functions more generally assuming a symmetric solution. More specifically we assume stock positions are of the form $(S,1-S,1-S,S)$ and compute utilities for different values of $S$. Moreover, for simplicity we assume that all prices are fixed, incorporating them modifies the expressions slightly. First consider the value function of a home agent who does not internalize how his choices affect labor demand. We can write his utility in terms of tradable consumption so that

$$
V_{H}^{c_{T}} = \mathbb{E} \left[ U(c_{T,H}(s),\alpha(p(s))c_{T,H}(s)) \right]
$$

We can approximate this around mean tradables consumption. Using a 2nd order approximation, we obtain

$$
V_{H} = \mathbb{E} \left[ \tilde{V} + U_{c}(\bar{c},\alpha(\bar{p})\bar{c})\bar{c}_{H} + \frac{1}{2} U_{cc}(\bar{c},\alpha(\bar{p}))\bar{c}^2 \bar{c}_H^2 \right]
$$
The stock portfolios do not affect $\tilde{V}$, if we approximate the fee around 0 or assume the fee is a 2nd order term (see Tille and van Wincoop (2010)). Now we can write:

$$\tilde{V}_H(S) = \mathbb{E}[\tilde{c}_H \mathcal{U}_c(\tilde{c}, \alpha(\tilde{p}))\tilde{c} + \frac{1}{2} \mathcal{U}_{cc}(\tilde{c}, \alpha(\tilde{p}))\tilde{c}^2 + t.i.p] \quad (105)$$

Note that $\mathcal{U}_c(\tilde{c}, \tilde{c}\tilde{\alpha}) = g_1(\tilde{c}, \tilde{c}\tilde{\alpha}) + g_2(\tilde{c}, \tilde{c}\tilde{\alpha})\tilde{\alpha}$. However, by the intratemporal condition $g_2 = g_1\tilde{p}$. Therefore $\mathcal{U}_c(\tilde{c}, \tilde{c}\tilde{\alpha}) = g_1(\tilde{c}, \tilde{c}\tilde{\alpha})(1 + \tilde{\alpha}\tilde{p})$. Moreover, $\mathcal{U}_{cc}(\tilde{c}, \tilde{c}\tilde{\alpha}) = g_{11}(\tilde{c}, \tilde{c}\tilde{\alpha}) + g_{12}(\tilde{c}, \tilde{c}\tilde{\alpha})\tilde{\alpha} + g_{21}(\tilde{c}, \tilde{c}\tilde{\alpha})\tilde{\alpha}^2$. Derivating the intratemporal condition one more time gives $g_{21} = g_{11}\tilde{p}$. Moreover, $g_{22} = g_{12}\tilde{p}$. Therefore, $\mathcal{U}_{cc}(\tilde{c}, \tilde{c}\tilde{\alpha}) = (g_{11}(\tilde{c}, \tilde{c}\tilde{\alpha}) + g_{12}(\tilde{c}, \tilde{c}\tilde{\alpha})\tilde{\alpha})(1 + \tilde{\alpha}\tilde{p})$. Now by dividing by $\mathcal{U}_c(\tilde{c}, \alpha(\tilde{p}))\tilde{c}$ and redefining the value function we obtain:

$$V_H(S) = \mathbb{E}[\tilde{c}_H] - \frac{1}{2} \gamma \text{Var}(\tilde{c}_H(S)) + t.i.p \quad (106)$$

To obtain the 2nd order term $\text{Var}(\tilde{c}_H(S))$, we use the 1st order approximation of the budget constraint. The expression for the foreign value function is similar. Note that the 2nd order term $\mathbb{E}[\tilde{c}_H + \tilde{c}_F] = -2(1 - S)\tilde{f}\delta$. Now we obtain

$$V_H(S) + V_F(S) = -2(1 - S)\tilde{f}\delta - \frac{1}{2} \gamma \text{Var}(\tilde{c}_H(S)) - \frac{1}{2} \gamma \text{Var}(\tilde{c}_F(S)) + t.i.p \quad (107)$$

Moreover, it is easy to verify that $S^{eq}$, maximizes the sum of $V_H(S) + V_F(S)$ given the budget constraints.

The value functions of the planner can be derived using the same arguments.

7.3 Proof of Proposition 2

Because taxes are distributed back to agents, the equilibrium budget constraints will remain unchanged. Including the taxes, the equilibrium home stock position becomes

$$S^{eq}_{FF} = S^{eq} = \frac{1}{2} - \frac{1}{2} \frac{1 - \delta}{\delta} \beta_{l,d} + \frac{\tilde{f} - \tau_S}{\gamma \delta \text{Var}(\Delta \tilde{d})} \quad (108)$$

Plugging in $\tau_S = \tilde{f}(1 - \frac{\psi}{\bar{q}})$. We get:

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That is the efficient stock positions are attained when the tax rate is as above.

Due to symmetry, the equilibrium non-tradables prices will be the same in both countries. The same is true of the planner problem with equal weights. Because the equilibrium is fully characterized by relative prices and the tradables price can be chosen freely, the common non-tradables price level is irrelevant. The stock prices are not in the problem, so the planner cannot gain by changing them.

7.4 Proof of Theorem 2

It is easy to verify that the equilibrium stock position will take the same form as in proposition 1 with the cost \( \tilde{f} \) replaced by overestimation \( \tilde{\eta} \). The planner evaluates the welfare of each country using the biased probability measures employed by the agents. Hence the stock positions chosen by the planner correspond to those in proposition 1, with \( \tilde{f} \) replaced by overestimation \( \tilde{\eta} \).

7.5 Proof of Proposition 3

Assuming symmetric beliefs, it is straightforward to verify that the equilibrium and planner solutions take the following forms

\[
S_{eq,\mu} = \frac{1}{2} - \frac{1}{2} \frac{1 - \delta}{\delta} \beta_{l,d}^{\mu} + \frac{\tilde{f}}{\psi \delta \operatorname{Var}(\Delta d)} = S_{plan}^{\mu}
\]  

(109)

where \( \mu' \) and \( \mu \) are the beliefs assumed by the agents and the planner respectively. From this we can see that generally \( S_{eq,\mu} = S_{plan,\mu} \). Assume that the belief differences concern only mean returns. Then \( \beta_{l,d}^{\mu'} = \beta_{l,d}^{\mu} = \beta_{l,d} \). This implies that home bias is excessive \( S_{eq} > S_{plan} \). Now at an interior solution we can offset the bias by setting a tax \( \tau = \eta \). Then \( S_{eq} = S_{plan} \).
7.6 Proof of Proposition 4

Results 2-6 can all be proven using a deviation argument resulting in two approximate Euler equations that correspond to the equilibrium stock positions and the optimal stock positions from the perspective of a small country.

Here we prove proposition 2 in the case of a dynamic model and two groups of countries. That is we show that with symmetric countries, the frictionless solution is optimal from the perspective of each country. Moreover, we show that with frictions the equilibrium features excessive home bias from the perspective of each country. Up to 2nd order the frictionless equilibrium stock position is still characterized by the Euler equation

\[ \text{Cov}(\psi(\bar{p}, \bar{c})\Delta \tilde{c}_T, \hat{r}) = 0 \quad (112) \]

\[ \iff \text{Cov}(\Delta \tilde{c}_T, \hat{r}) = 0 \quad (113) \]

and the, now dynamic, budget constraint. The optimal solution from the perspective of a country using is characterized by this same condition. The budget constraint remains the same. We conclude that at a symmetric point

\[ S_{\text{sym}}^{\text{country}} = S_{\text{sym}}^{eq} \quad (114) \]

Hence the frictionless equilibrium is optimal from the perspective of a small country. Consider the case of frictions \( f > 0 \). A second order approximation of each Euler equation gives

\[ \iff \text{Cov}(\gamma(\bar{p}, \bar{c})\Delta \tilde{c}_T, \hat{r}) = \tilde{f} \quad (115) \]

, where \( \gamma(\bar{p}, \bar{c}) \) is as in proposition 1. On the other hand the country benefits from reducing the home stock position and increasing the foreign position by a small amount \( \epsilon > 0 \) when

\[ \text{Cov}(\psi(\bar{p}, \bar{c})\Delta \tilde{c}_T, \hat{r}) - \tilde{f} > 0 \quad (116) \]

\[ \iff \text{Cov}(\Delta \tilde{c}_T, \hat{r}) - \frac{\tilde{f}}{\psi(\bar{p}, \bar{c})} > 0, \quad (117) \]

where \( \psi(\bar{p}, \bar{c}) \) is as in proposition 1. But plugging in the equilibrium condition, we get

\[ \tilde{f} \left[ \frac{1}{\gamma(\bar{p}, \bar{c})} - \frac{1}{\psi(\bar{p}, \bar{c})} \right] > 0 \quad (118) \]
But this is true because $\psi(\bar{p}, \bar{c}) > \gamma(\bar{p}, \bar{c})$. □

Extending the proofs of propositions 4, 5 and 6 to a symmetric point of a dynamic symmetric model proceeds using the same arguments as above. □

### 7.7 Expressions for $\gamma_p$, $\psi_p$ and $\psi_A$

With CRRA preferences over a CES aggregator we have

$$U_T = a^\gamma \left( \frac{PT}{P} \right)^{1-\gamma} \phi^{-\gamma} c_T$$  \hspace{1cm} (119)

Where the price index $P$ is given by

$$P = \left( ap_{T}^{1-\phi} + (1-a)p_{NT}^{1-\phi} \right)^{\frac{1}{1-\phi}}$$  \hspace{1cm} (120)

To solve for $\gamma_p$ note

$$\frac{\partial U_T}{\partial p_T} = (1-\gamma\phi) \frac{P}{p_T} U_T \left[ \frac{1}{p} - \frac{p_T}{P^2} \frac{\partial P}{\partial p_T} \right] = (1-\gamma\phi) U_T \left[ \frac{1}{p_T} - \frac{1}{P} \frac{\partial P}{\partial p_T} \right]$$  \hspace{1cm} (121)

Here

$$\frac{\partial P}{\partial p_T} = a \left( \frac{P_T}{P} \right)^{-\phi}$$  \hspace{1cm} (122)

Hence

$$\gamma_p = (\gamma\phi - 1) \left[ 1 - a \left( \frac{P_T}{P} \right)^{1-\phi} \right] + 1$$  \hspace{1cm} (123)

Next consider solving $\psi_p$. First note,

$$\frac{\partial \hat{V}_C}{\partial p_T} = \frac{\partial U_T}{\partial p_T} [1 + p\alpha(p)] + U_T \frac{\partial [\alpha(p)p]}{\partial p_T} + \frac{1}{A} \frac{\partial [U_N \alpha(p)]}{\partial p_T}$$  \hspace{1cm} (124)

We get

$$\frac{\partial \hat{V}_C}{\partial p_T} = (1-\gamma\phi)U_T \left[ 1 + a \left( \frac{P_T}{P} \right)^{1-\phi} \right] \frac{1}{P_T} \left[ 1 + p\alpha(p) \right] + U_T \alpha(p) p (\phi - 1) \frac{1}{P_T} + \frac{1}{A} U_N (1 + \sigma) \alpha(p) \phi \frac{1}{P_T}$$  \hspace{1cm} (125)

$$\hspace{1cm} + \frac{1}{A} U_N (1 + \sigma) \frac{1}{P_T}$$  \hspace{1cm} (126)
At a symmetric point $\hat{V}_C = U_T$. Then we get

$$\psi_p = (\gamma \phi - 1) \left[ 1 + a \left( \frac{P_T}{P} \right)^{1-\phi} \right] \left[ 1 + p \alpha(p) \right] + \alpha(p) \phi(1 - \phi) - \frac{1}{A} \frac{U_N}{U_T} (1 + \sigma) \alpha(p) \phi - 1$$  \hspace{1cm} (127)

Again using the fact that by symmetry $\tau = 1 + \frac{U_N}{pAU_T} = 0$, we have

$$\psi_p = (\gamma \phi - 1) \left[ 1 + a \left( \frac{P_T}{P} \right)^{1-\phi} \right] \left[ 1 + p \alpha(p) \right] + \alpha(p) \phi - 1 - p(1 + \sigma) \alpha(p) \phi$$  \hspace{1cm} (128)

Finally to solve for $\psi_A$, note

$$\frac{\partial \hat{V}_C}{\partial A} = U_{NN} \left( \frac{\alpha(p)c_T}{A} \right) \frac{\alpha(p)^2}{A^3} c_T + U_N \left( \frac{\alpha(p)c_T}{A} \right) \frac{\alpha(p)^2}{A^2} c_T$$  \hspace{1cm} (129)

Again using the fact that at a symmetric point $\tau = 0$, we have

$$\psi_A = - \frac{U_{NN}}{U_T} \left( \frac{\alpha(p)c_T}{A} \right) \frac{\alpha(p)^2}{A^2} c_T + p \left( \frac{\alpha(p)c_T}{A} \right) \frac{\alpha(p)^2}{A^2} c_T$$  \hspace{1cm} (130)

8 Appendix B: Numerical Examples Using a Dynamic Model

Proposition 4 shows that the key qualitative results of this paper extend to a dynamic setting. Moreover, the formulas for equilibrium and socially optimal stock positions take a similar form in a dynamic model as those in a two period model. Here we calculate the effect of the friction using a simple dynamic model. It is seen that the difference between equilibrium and socially optimal stock positions is similar to that in a two period model. For a more general dynamic extension of the two period model of this paper, see the companion paper Sihvonen (2016).

Consider the case of symmetric countries. Assume the relative log-dividends and labor income (tradable goods) follow

$$\hat{d}_{t+1} = \phi d_t \hat{d}_t + \epsilon_{d,t+1}$$  \hspace{1cm} (131)
\[ \hat{l}_{t+1} = \varphi l_t + \varepsilon_{l,t+1}. \quad (132) \]

Where the shocks are zero mean. However, the exact process for these variables is not important for the results. A 2nd order approximation of the Euler equations corresponding to the planner and equilibrium solutions are

\[ 2f - \gamma \text{cov}_t(\hat{c}_{t+1}, \hat{r}_{t+1}) = 0 \quad (133) \]

\[ 2f - \psi \text{cov}_t(\hat{c}_{t+1}, \hat{r}_{t+1}) = 0. \quad (134) \]

To evaluate these expressions, first order solutions for \( \hat{c}_{t+1} \) and \( \hat{r}_{t+1} \) are needed. A first order approximation of the Euler equations gives

\[ \mathbb{E}_t[\hat{c}_{t+1}] = \hat{c}_t. \quad (135) \]

We also have

\[ \mathbb{E}_t[\hat{r}_{t+1}] = 0. \quad (136) \]

Also \( \hat{r}_{t+1} \) is a structural parameter given by

\[ \frac{1}{\beta} \hat{r}_{t+1} = \hat{p}_S^t \hat{r}_{t+1} + \frac{\bar{d}}{\bar{p}_S} \hat{d}_{t+1} - \frac{1}{\beta} \hat{p}_t^S \quad (137) \]

and

\[ \hat{p}_t^S = \frac{1 - \beta}{\beta} \sum_{i=0}^{\infty} \mathbb{E}_{t+1} \left[ \beta^i \hat{d}_{t+1+i} \right]. \quad (138) \]

A first order approximation of the budget constraint yields

\[ \tilde{W}_{t+1} = \frac{1}{\beta} W_t + \tilde{y}_{t+1} - \tilde{c}_{t+1} + \tilde{\alpha}_{HF} \hat{r}_{t+1}, \quad (139) \]

where

\[ \tilde{\alpha}_{HF} = \frac{\bar{d}}{1 - \beta} \frac{S_{HF}^{eq}}{\bar{y}}. \quad (140) \]

We can solve

\[ 0 = \frac{1}{\beta} W_t + \sum_{i=0}^{\infty} \beta^i \mathbb{E}_{t+1} \left[ \tilde{y}_{t+i+1} - \tilde{c}_{t+i+1} \right] + \tilde{\alpha}_{HF} \hat{r}_{t+1}. \quad (141) \]
Deducting home and foreign conditions

\[ 0 = \frac{2}{\beta} \bar{W}_t + \sum_{i=0}^{\infty} \beta^i \mathbb{E}_{t+1} [\hat{y}_{t+i+1} - \hat{c}_{t+i+1}] + 2 \bar{\alpha}_{HF} \hat{r}_{t+1}. \]  \hspace{1cm} (142)

Using the first order condition for consumption

\[ \hat{c}_{t+1} = \frac{1 - \beta}{\beta} \bar{W}_t + (1 - \beta) \sum_{i=0}^{\infty} \beta^i \mathbb{E}_{t+1} [\hat{y}_{t+i+1}] + 2(1 - \beta) \bar{\alpha}_{HF} \hat{r}_{t+1}. \]  \hspace{1cm} (143)

Plugging into the 2nd order approximation of the Euler equation we obtain

\[ \frac{f}{\gamma \text{Var}_t(\hat{r}_{t+1})} - \beta_{y,r} + 2(1 - \beta) \bar{\alpha}_{HF} = 0. \]  \hspace{1cm} (144)

Here

\[ \beta_{y,r} = \frac{\text{Cov}_t((1 - \beta) \sum_{i=0}^{\infty} \beta^i \mathbb{E}_{t+1} [\hat{y}_{t+i+1}], \hat{r}_{t+1})}{\text{Var}_t(\hat{r}_{t+1})} \]  \hspace{1cm} (145)

and \( \text{Var}_t(\hat{r}_{t+1}) \) are structural parameters. We can solve

\[ \frac{f}{\gamma \delta \text{Var}_t(\hat{r}_{t+1})} - \frac{1}{\delta} \beta_{y,r} + 2 S_{HF}^{eq} = 0 \]  \hspace{1cm} (146)

or

\[ S_{eq}^{plan} = 1 - \frac{1}{2 \delta} \beta_{y,r} + \frac{f}{\delta \gamma \text{Var}_t(\hat{r}_{t+1})}. \]  \hspace{1cm} (147)

Similarly we can solve

\[ S_{plan} = 1 - \frac{1}{2 \delta} \beta_{y,r} + \frac{f}{\delta \psi \text{Var}_t(\hat{r}_{t+1})}. \]  \hspace{1cm} (148)

The excess home bias is given by

\[ S_{eq} - S_{plan} = \frac{f}{\delta \gamma \text{Var}_t(\hat{r}_{t+1})} - \frac{f}{\delta \psi \text{Var}_t(\hat{r}_{t+1})}. \]  \hspace{1cm} (149)
Therefore the difference in the effect of the friction is similar to that in a two period model, with the exception that \( \text{Var}_t(\hat{d}_{t+1}) \) is replaced by \( \text{Var}_t(\hat{r}_{t+1}) \). In the numerical examples section we already set \( \text{Var}_t(\hat{d}_{t+1}) \) to correspond to the typical annual return volatility \( (0.2^2) \). Therefore the effect of the friction is similar to that in a two period model.

References


