Contagion Along the Business Cycle

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Abstract

In this paper, we incorporate a complex network model into a state of the art stochastic general equilibrium framework with an active interbank market. On this market banks exchange funds one another giving rise to a complex network of interbanking relations. With the tools of network analysis it is possible to study how contagion spreads between banks and what is the probability and size of a cascade (a sequence of defaults) generated by a single initial episode. These two variables are a key component to understand systemic risk and to assess the stability of the banking system. In extreme scenarios, the system may experience a phase transition when the consequences of one single initial shock affect the entire population. We show that the size and probability of a cascade evolve along the business cycle and how they respond to exogenous shocks. Financial shocks have a larger impact on contagion probability than real shocks that, however, are long lasting. Additionally we find that monetary policy faces a trade off between financial stability and macroeconomic stabilization. In particular, responding to the contagion probability can reduce risk on financial markets but at the cost of higher volatility of real variables. Government spending shocks, on the contrary, have smaller effects on the volatility of real variables and on cascade probabilities. Finally we analyze a set of contagion-preventing policy specifications.

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1 Introduction

Modern financial markets are populated by agents connected by a deep web of mutual claims and obligations. This web creates an environment that amplifies the response of the financial system to shocks and makes difficult to predict the consequences of even a single (seemingly) isolated event. Crisis that initially involve only few institutions can easily spread to the whole system, generating multiplicative effects (see Figure 1.1). At the same time, however, it is difficult to capture this complexity using a standard representative agent macroeconomic model. The objective of this paper is to extend the reach of the existing literature to include the modelization of financial contagion (in the network literature these default sequences are referred to as cascades) into a state of the art DSGE model with financial frictions (in line with Gertler & Karadi (2011)).

![ Defaults and conditions of banking](image)

**Figure 1.1:** Defaults and conditions of banking

*Notes:* Conditions of banking and default rates in the U.S. source Nationa Bureau of Economic Analysis. During financial crisis, the default probability rises, in some cases even with declining rates. This leads to a reduction of final credit.

The contribution of this paper is two fold. On one hand, it provides a rigorous way to incorporate financial contagion and cascade mechanisms into a standard macro framework. This result fills a gap in the existing literature where contagion and diffusion phenomenons
have been mainly studied in partial equilibrium models.\textsuperscript{1} On the other hand, we highlight the connections between aggregate shocks and financial stability. As it will be shown in the following, the tools of network analysis allow to assess the probability and size of global cascades in the financial system (i.e. sequences of defaults that follow a single initial episode). Simulating the model we can analyze how they evolve along the business cycle and how they respond to exogenous shocks. Our results show that financial shocks impact on the contagion probability more than real shock that, however, are long lasting. Additionally, we highlight a trade off between macroeconomic stabilization and financial stability. Central banks, are not able to achieving financial stability with monetary policy only. If they respond to financial risk -i.e. the probability of contagion in this model- gains in financial stability are connected to higher volatility of real variables. Simulations show that a moderate response to financial risk can, however, reduce volatility after real shocks with small costs in terms of financial stability (the probability and size of contagion). Fiscal authorities, instead, are able to boost output with limited costs in term of risk. The paper will proceed as follows: section 2 presents the model, section 3 describe the structure of the (complex) interbank market, section 4 simulate the model under different shocks and policy specifications, in section 5 we present our conclusions. In the Appendix A and the Online Appendix B we show proofs, additional graphs and complete tables.

\subsection{1.1 Related literature}

The pioneering contribution on the topic of financial contagion is the seminal paper by Allen & Gale (2000). They show how the spread of contagion is crucially dependent on the degree of interconnectedness between agents. In their approach, however, only the direct counterparty of a bank suffers from its default. Additionally, if the banking network is complete,\textsuperscript{2} each institution divides evenly interbank credit between all available partners. In this case, if a

\textsuperscript{1}See for example Allen & Gale (2000), Gai & Kapadia (2010), Battiston, Bersini, Caldarelli, Pirotte & Roukny (2013) and Acemoglu et al. (2015).

\textsuperscript{2}A network is called complete if each agent is connected to every other agent.
default takes place each bank is hit by a negligible (in the limit) shock.

More recent results, such as Acemoglu et al. (2015), have highlighted how the structure of the underlying financial network is not neutral to the size (and consequences) of contagion. On complete networks, each institution divides evenly interbank credit between all available partners. In this case, if a default takes place, each bank is hit by a small shock and there are no additional consequences. On the contrary, when networks are incomplete, with banks being exposed only to a limited number of counterparties, the system is fragile and a single episode can generate a cascade (a series of defaults) that may affect the entire graph. In this case the shock transmitted to each first neighbor is not negligible. If at least one of these succumbs, the contagion is transmitted to the second neighbors bringing them into the “front line of contagion” (Gai & Kapadia (2010)).

To model this scenario it is necessary to rely on the techniques developed by the literature on complex networks (i.e. Strogatz (2001), Newman et al. (2001) and Newman et al. (2001)) and to apply them to a model with an active financial system. However, complex social networks exhibit often highly chaotic properties and fractal structures. This makes difficult to integrate them into a standard rational expectations general equilibrium setting, where variables are simulated around a stable neighborhood of the steady state. This is probably the reason why, despite several empirical papers explored the topology of relations on the interbank market, few attempts were made to integrate these processes in to a standard DSGE model. To do that, we have extended the reach of the existing literature making tractable the chaotic part of the model.

Finally, there is a wide literature on DSGE models with a banking sector in terms of assumptions governing the behaviour of banks. In this paper we follow the setting proposed

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3 As it is in the case of a complete network, where each neighbor of the defaulting bank is exposed by \( \frac{1}{N} \) of the defaulting bank’s assets.

4 See Mandelbrot (2004).

by Gertler & Kiyotaki (2010) and Gertler & Karadi (2011). More specifically, we assume that banks are run directly by a member of a household. This allows to set up an simple agency problems between each bank and its creditors, giving rise endogenously to a credit spread. In other terms this means that households own (indirectly) bank’s capital and, therefore, an agency problem arises.\footnote{This is achieved by assuming that households own (indirectly) bank’s capital and, therefore, can divert part of it. Notice that households do not own deposits on the bank of which indirectly own the capital.}

1.2 What is a cascade?

Before moving on to the description of the model, it is worth to provide an intuition of the process this paper tries to analyze and the methodology used. Network tools are relatively new in the context of macroeconomic DSGE models; this paragraph provides a simple intuition on how these type of model work and what this methodology can capitures.

The tools of network analysis were originally developed in Physics and Engineering to trace how an impulse (i.e. energy on a grid, heat in fluids or gases, viruses between computes) spreads on a complex system of which the connection structure is known or can be (reasonably well) approximated by a function. On financial markets the same tools can be applied to study how contagion spreads between banks.\footnote{Recall Strogatz & Watts (1998), Strogatz (2001), Vega-Redondo (2007).} In this context banks are nodes and the connections between nodes are constituted by debt contracts.

Once defined such network, we can compute the probability that the default of a single institution leads to a sequence of defaults and the number of banks involved in that case (contagion).\footnote{The tools applied here are very similar to those used by engineers to analyze power grids or integrated circuits. Stanley (1971), Barabasi & Albert (1999), Strogatz (2001), Callaway et al. (2000), Newman et al. (2001) provide intuitions on how those methods can be applied on social networks.} These measures can easily be seen as proxies for financial stability as they capture the probability and size of financial contagion. it is worth noting, however, that this type of models capture the consequences of an exogenous default on the financial system. Banks do not systematically (i.e. in the steady state) default in these models. On the
contrary, the initial default is treated as an exogenous shock. If one bank defaults it spreads losses to its counterparties (its neighbors on the graph) that may or may not default as well starting a contagion process. Using the terminology of network analysis the contagion process is called a cascade (i.e. how the impulse given to a single node spreads through the system) and its expected consequences can be traced on the system analyzing the structure of the network.

The size and probability of a default sequence is connected not only to the magnitude of the initial seed\(^9\) (the bank that originally defaulted) but also, and probably more importantly, to the relative position of the initial seed in the network.\(^{10}\) Isolated banks or banks with “resilient” neighbors do not spread contagion. On the contrary, the initial seed grows if and only if it is surrounded by fragile counterparties. Consider the simple network of Figure 1.2.

![Figure 1.2: A simple network model.](image)

**Notes:** The red triangle is the initial seed (i.e. the institution starting the contagion), that moves exogenously from state 0 (no default) to state 1 (default). Circles are vulnerable nodes (i.e. nodes that move to state 1 if hit by a shock) and squares are resilient nodes (i.e. nodes that never move to state 1). Green nodes are in state 0 while blue nodes are in state 1. The cascade is composed by the initial seed and all blue nodes.

The initial seed is connected to only one node. However, this node is vulnerable and is very well connected in the network, therefore the initial seed can grow and infect all blue

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9So to how “large” is the institution that initially defaults.

10The relative position of a node in the network is given by the position and connectivity of the node and the number, position and connectivity of its neighbors.
nodes in the figure. The key component, in this simple example, to explain the diffusion process is not the size of the initial seed, neither its connectivity, but the connectivity of its first neighbor. Notice also that there are vulnerable nodes that are not part of the cascade, as they are connected only to resilient nodes that protect them from contagion. This very simple example is useful to point out four characteristics of network theory that this paper will capture: i) it is important to consider the position of each node in the network to assess its relevance; ii) diffusion processes are affected by the connectivity of any \( k^{th} \) order neighbor of the initial seed; iii) a cascade can grow through those neighbors even if it starts from a rather peripheral node; iv) resilient nodes are important to prevent contagion. In this paper we build on Newman et al. (2001) and Watts (2002) to create a statistical model for the financial network. In this way we can model the network of interbanking relations that arises from the individual decisions of each single institution, taking into account the characteristics i-iv of diffusion processes. ¹¹ Analyzing the characteristics of the web of connections between financial agents we can build an aggregate measure for the system’s stability and the likelihood of contagion.

This modeling takes a step forward from the existing literature as takes explicitly into account that the likelihood of a bank being part of a cascade is affected by bank’s specific characteristics (i.e. its number of links or the quality of its capital), its relative position in the network (i.e. having resilient nodes between its first and second neighbors) and aggregate shocks or macroeconomic dynamics (i.e. demand for loans, TFP, preferences). These dynamics will be addressed more formally in sections 3.1 and 3.2 that are devoted to the application of network theory to a DSGE model with financial frictions.

¹¹Notice that each bank does not know its exact position on the network. Each bank, in fact, has information only on its direct neighbors but knows nothing beyond them.
2 The Model

The general structure of the model follows Gertler & Karadi (2011). The economy is populated by households, firms, the financial sector, a government and a central bank.

Households own firms and banks, supply labor, consume, pay lump-sum taxes and save. Household members can be of two types: workers -i.e. agents who supply undifferentiated labor to firms\(^{12}\)- and bankers, i.e. agents who own and manage banks. Workers earn wages while bankers earn dividends that are transferred to the household only when they exit. Within each household there is full consumption insurance and at any moment in time the fraction \(1 - f\) of workers (bankers) is constant. At the end of each period, with an exogenous probability \(\theta^B\) a banker stays banker for the next period and with the complementary probability ”exits” and becomes a worker \(((1 - \theta^B) f\) is the fraction of household members who switch from banker to worker). At the moment of exit the banker transfers dividends to her household. \((1 - \theta^B) f\) is the fraction of household members who switch from banker to worker. As the composition of each household is constant, this is also the fraction of workers who become bankers. Each household holds deposits at banks but not in the same institution it controls.\(^{13}\)

The private sector of the economy is composed by wholesale producers, retailers and capital producers.

Wholesale producers operate in perfect competition and produce an homogeneous undifferentiated intermediate good which is sold to retailers. Retailers differentiate intermediate goods (with negligible costs) and sell them in consumption bundles to households and capital producers. Therefore, retailers can exploit some degree of market power on final markets as in Dixit & Stiglitz (1977). Following the well-known Calvo formalism,\(^{14}\) we assume that only a fraction of retailers is able to reset (optimally) prices in each period. Wholesale producers

\(^{12}\)More precisely, workers supply labor only to wholesale producers.

\(^{13}\)See Gertler & Karadi (2011). This assumption is needed to set up the agency problem.

\(^{14}\)See Calvo (1983).
combine labor and capital to produce intermediate goods. Capital producers use a fraction of final goods and undepreciated capital as inputs to produce new capital which is sold to wholesale firms. In order to acquire capital wholesale producers demand bank loans. In this setting wholesale producers are in the exactly same position as "entrepreneurs" in Bernanke et al. (1999). Notice, however, that as wholesale producers do not accumulate capital there is no agency problems between banks and firms.  

The financial sector of the economy is composed by a large number of banks. Each bank operates in perfect competition, is operatively divided into a wholesale branch and a retail branch and is run by a banker. The banker’s objective is to maximize the value of the bank’s net worth at the end of each period. We depart here from the standard Gertler & Kiyotaki (2010) framework, following, between others, Dib (2010), Gerali et al. (2010), Gambacorta & Signoretti (2013) in this assumption. This assumptions allows to build a simple model of interbanking relations maintaining the financial frictions set up.

The retail branch provides final loans to firms, using interbank credit and deposits. The wholesale branch manages the bank’s liquidity, issuing and acquiring interbank loans. Banks, therefore, interact on a complex interbank market exchanging funds with each other. The simplest reason to justify the existance of an interbank market consists in assuming that banks are hit at random by a liquidity shock. Each institution experiencing a shortage of liquidity can borrow from a bank with excess of liquidity. In this case, however, each bank can be either a borrower or a lender in each period. For example, in Allen & Gale (2000) or Gertler & Kiyotaki (2010), the banking system is composed by a set of lending banks and a set of borrowing banks (with no intersection). Despite being reasonable and straightforward, this approach fails to capture a key element of real banking networks: banks, in fact, borrow from and lend to other banks simultaneously. Only considering this characteristic it is possible build a true model for the interbank network and analyze how contagion spreads on it. An additional characteristic of the interbank market is that banks tend to borrow from and lend

\[^{15}\text{In other terms, banks acquire a state contingent claim on firm’s profits.}\]
\[^{16}\text{This is equivalent to maximizing the transfer to the household in case of exit.}\]
to only a limited number of counterparties. This constrain may arise, for instance, because there are costs connected to exploring the network. Banks need to invest resources to build successful partnership, therefore each institution can monitor (and do business with) only a limited number of "partners".

Following Acemoglu et al. (2015) and Gai & Kapadia (2010), we allow banks to trade only with a limited number (randomly assigned) of counterparties in each period, this allows to describe the interbank market as a complex network. This constraint may rise from liquidity mismatches (i.e. bank a may have funds to borrow but not when bank b needs them) or asymmetric costs of monitoring. On this network the default of a single bank can reverberate to the entire banking system in a "cascade". Our model has two features: i) characterizes the interbank networks following empirical results, ii) the probability and magnitude of such an event are endogenous.

In this setting the default of an institution may not affect only its direct counterparties. If at least one neighbor of the defaulting bank default as well, contagion may be transmitted, starting a cascade. We will characterize this transmission process later in the paper. At last, we assume that each bank has a specific resistance to exogenous shocks in times of distress, this specific component can be seen as a bank specific discount factor to fire sale assets.

Finally, the government acquires final goods financing its expenses with lump-sum taxes. The central bank sets the policy rate following a Taylor-type rule.

2.1 Households

There is a continuum of identical households, indexed by $j \in [0, 1]$. Households consume, save, take portfolio decisions and supply labor to firms.

The preferences of a representative household are described by:

$$U(C_t, l_t) = \sum_{t=0}^{\infty} E_t \beta^t \left[ e_t^c \ln (C_t - \gamma C_{t-1}) - \frac{\chi}{1 + \varphi} l_t^{1+\varphi} \right]$$

(1)
where $C_t$ denotes consumption at time $t$. There is habit formation in consumption with \( \gamma \in [0, 1] \) the habit parameter. \( l_t \) denotes labor supply in terms of hours worked with \( \chi \) the weight of disutility of labor in the period utility function and \( \varphi \) the inverse of the Frish labor supply elasticity;\(^{17} \) \( \epsilon_i^c \) is an I.I.D. preference shocks that follows an AR(1) process whose steady state value is 1.

The intra-period budget constraint is:

$$C_t + D_{t+1} \leq W_t l_t + R_t D_t + \Pi_t \tag{2}$$

Sources of funds are: the wage bill \((W_t l_t)\), with \( W_t \) the real wage per hour worked; returns on risk free deposits \((R_t D_t)\) with \( D_t \) deposits acquired in period t-1 and \( R_t \) the (real) risk free return on deposits between t-1 and t; \( \Pi_t \) collects profits (from firms and banks) net of the tax paid.\(^{18} \) Uses of funds are consumption \((C_t)\) and savings as deposits.

Each household maximizes equation (1) with respect to consumption, labor supply and deposits subject to the sequence of constraints of equation (2) and non negativity constraints. First order conditions are:

$$\frac{\partial L}{\partial C_t} = \frac{\epsilon_i^c}{C_t - \gamma C_{t-1}} - \beta \gamma E_t \left[ \frac{\epsilon_{t+1}^c}{C_{t+1} - \gamma C_t} \right] - \lambda_t^C = 0 \tag{3}$$

$$\frac{\partial L}{\partial l_t} = -\chi l_t^g + \lambda_t^C W_t = 0 \tag{4}$$

$$\frac{\partial L}{\partial D_t} = E_t (\lambda_{t+1}^C R_{t+1}) - \lambda_t^C \beta^{-1} = 0 \tag{5}$$

where \( \{\lambda_t^C\}_{t=0}^{\infty} \) is the sequence of Lagrangian multipliers associated to the optimization problem. \( \lambda_t^C \) defines a stochastic discount factor as: \( E_t \left( \lambda_{t+1}^C \right) = \beta^t \frac{E_t(\lambda_{t+1}^C)}{\lambda_t^C} \).

Equation (3) describes the optimality condition for consumption. Notice that in the limit case of \( \gamma = 0 \) it boils down to a standard optimality condition. Equation (4) describes the

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\(^{17}\)See Frisch (1959).

\(^{18}\)Notice that profits are collected from exiting banks and retailers.
supply of labor; households take \( W_t \) as given and choose the amount of labor to supply as a function of the marginal utility of consumption \( (\lambda^C) \). Habit formation modifies the standard decision problem as labor supply depends on lagged and expected future consumption. Equation (5) is the classic Euler condition that defines the steady state (risk free) deposit rate as \( \frac{1}{\beta} \).

### 2.2 Wholesale Producers

There is a continuum of identical and perfectly competitive wholesale firms owned by households. Wholesale producers acquire capital from capital producers and labor from households to produce undifferentiated intermediate goods. To finance the acquisition of capital, wholesale firms borrow from the financial sector issuing state contingent securities at the price \( Q_t \).

Defining \( Z_t \) the return on capital, each unit of credit from the banking sector is equivalent to a state contingent claim on future returns \( (Z_{t+1}, (1-\delta) Z_{t+2}, (1-\delta)^2 Z_{t+3} \ldots) \), with \( \delta \) the depreciation rate of capital.

Wholesale producers operate on a perfectly competitive market setting the price \( (P^W_t) \) equal to the marginal cost.

Following Merton (1973), firms produce using a Cobb-Douglas function:

\[
Y_t = A_t (U_t \xi_t K_t)^{\alpha} I_t^{1-\alpha} \tag{6}
\]

with \( \alpha \in (0, 1) \), \( Y_t \) the output of each firm, \( K_t \) the level of capital, \( U_t \) the utilization rate of capital, \( \xi_t \) a shock on the quality of capital. Notice that capital utilized in production is \( \xi_t K_t \).\(^{20}\) \( A_t \) is total factor productivity that follows an AR(1) process whose steady state value

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\(^{19}\)We follow here Gertler & Gilchrist (2007) and Gertler & Kiyotaki (2015).

\(^{20}\)Following Merton (1973) this shock allows the introduction of an exogenous source of variation in the value of capital.
Wholesale firms choose the level of inputs in order to maximize profits. First order conditions are:

\[
\frac{\partial L}{\partial U_t} = \lambda_t^f \alpha \frac{Y_t}{U_t} = \Delta' (U_t) \xi_t K_t = Z_t
\]

(7)

\[
\frac{\partial L}{\partial l_t} = \lambda_t^f (1 - \alpha) \frac{Y_t}{l_t} = W_t
\]

(8)

with \(Z_t\) the marginal product of capital and \(\{\lambda_t^f\}_{t=0}^{\infty}\) the sequence of Lagrangian multipliers associated to the problem.\(^{21}\)

As firms re-sell undepreciated capital at the end of each period, the real return \(R^K_t\) on each unit of capital is the sum of marginal product of capital \((Z_t)\) and capital gain or losses \(((1 - \delta)Q_t - \Delta(U_t))\) divided by the cost of capital \((Q_{t-1})\):\(^{22}\)

\[
R^K_t = \frac{Z_t + (1 - \delta)Q_t - \Delta(U_t)}{Q_{t-1}} \xi_t
\]

(9)

Wholesale firms purchase capital issuing state contingent securities \(S_t\) that equal the number of units of capital. The total demand for loans \((B_t)\) is:

\[
Q_t B_t = Q_t K_{t+1} = Q_t S_t
\]

(10)

From perfect competition, the interest rate on loans equals returns on capital \(R^K_t\).\(^{23}\)

2.3 Retailers

Retailers acquire undifferentiated wholesale goods (at the price \(P^W_t\)), bundle them together to produce differentiated final goods that are sold on the final market at the price \(P_t\).

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\(^{21}\)As this sector of the economy operates in perfect competition \(\lambda_t^f = P^W_t\).

\(^{22}\)Notice that returns are net of non-utilized capital \(\Delta(U_t)\).

\(^{23}\)Notice that in this model bank loans are closer to shares than to standard debt contracts. This approach, however, has the merit of solving the agency problem between banks and wholesale producers.
Each retailer (indexed by $j$), faces a downward sloping demand function in a Dixit-Stiglitz setting\textsuperscript{24} for the variety of good sold, with the CES aggregator for output $Y_t = \left[ \int_0^1 Y_{j,t}^{\frac{1}{\epsilon - 1}} \, dj \right]^{\frac{1}{\epsilon - 1}}$.

$\epsilon > 1$ is the elasticity of substitution between different varieties of final goods with the aggregate price level: $P_t = \left[ \int_0^1 P_{j,t}^{1-\epsilon} \, dj \right]^{\frac{1}{\epsilon - 1}}$.

Under these assumptions, the demand function faced by a $j$–th retailer is:

$$Y_{j,t} = \left( \frac{P_{j,t}}{P_t} \right)^{-\epsilon} Y_t$$

(11)

Following the Calvo formalism, in each period retailers are able to reset prices with probability $(1 - \theta_R)$. If a retailer receives the signal, she will choose optimally the new price $P_{j,t} = P^*_t$ in order to maximize future expected profits. Otherwise she will index prices to lagged inflation. More formally retailers solve:

$$\max \sum_{i=0}^{\infty} E_t \left\{ \theta_R^t \Lambda^C_{t,t+i} \left[ \frac{P^*_t}{P_{t+i}} \Pi_{k=1}^i (1 + \pi_{t+k-1})^\theta - P^W_{t+i} \right] Y_{j,t+i} \right\}$$

with $\pi_t$ the inflation rate, subject to the demand function, equation (11).

The first order condition is:

$$\sum_{i=0}^{\infty} E_{t-1} \left\{ \theta_R^t \Lambda^C_{t,t+i} Y_{j,t+i} \left[ \frac{P^*_t}{P_{t+i}} \Pi_{k=1}^i (1 + \pi_{t+k-1})^\theta - \frac{\epsilon}{\epsilon - 1} P^W_{t+i} \right] \right\} = 0$$

(12)

with the aggregate price level given by:

$$P_t = \left[ \theta_R \left( \pi_{t-1}^\theta P_{t-1} \right)^{1-\epsilon} + (1 - \theta_R) P^*_t \right]^{\frac{1}{\epsilon - \epsilon}}$$

(13)

since all retailers are equal, they will choose the same price, so we can drop the cross-sectional index and call $P^*_t = P^*_j$.

\textsuperscript{24}Dixit & Stiglitz (1977).
2.4 Capital Good Producers

Capital good producers operate in perfect competition and combine undepreciated capital \((1 - \delta)K_{t-1}\) with “investment goods” to produce new capital that is sold to wholesale firms at the price \(Q_t\). Letting \(I_t\) be the net capital created, the objective function -i.e. expected lifetime profits- of capital producers is:

\[
E_t \sum_{t=0}^{\infty} \Lambda^C_{t,t+1} \left\{ Q_t I_t \left[ 1 - \frac{\chi t}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right) \right] - \left[ (Q_t - Q_t) (1 - \delta) K_{t-1} \right] \right\}
\]

(14)

with \(Q_t\) the price of undepreciated capital at the end of each period \(^{25}\) and \(\frac{\chi}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right)^2\) physical adjustments costs in the investment production process.\(^{26}\)

The first order condition is:

\[
\frac{\partial L}{\partial I_t} = Q_t \left[ 1 - \frac{\chi t}{2} (x_t - 1)^2 + \chi t (x_t - 1) x_t \right] + E_t \left[ \Lambda^C_t \chi t (x_{t+1} - 1) x_{t+1} \frac{Q_{t+1}}{Q_t} \right] - 1 = 0
\]

(15)

where \(x = \frac{I_t}{I_{t-1}}\). Equation (15) is the well known Tobin’s q equation,\(^{27}\) that links the price of capital to its marginal cost. The law of motion of capital is:

\[
E_t (K_{t+1}) = (1 - \delta) K_t + \left[ 1 - \frac{\chi t}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right)^2 \right] I_t
\]

(16)

Since the banking system finances (undepreciated) capital and investments, the total amount of credit equals capital: \(B_t = E_t(K_{t+1})\).

\(^{25}\)Notice that this is assumed to be equal to \(Q_t\). A similar choice has been made by Bernanke et al. (1999), Angeloni & Faia (2009), Christiano et al. (2010) and Gerali et al. (2010).

\(^{26}\)Physical adjustment costs are a key component to understand fluctuations in investments; in this framework they depend on the parameter \(\chi_t > 0\). See for example Bernanke et al. (1999), Angeloni & Faia (2009) and Friedman & Woodford (2011).

\(^{27}\)See Tobin (1969). Notice that \(Q_t = 1\) in the steady state.
2.5 Banks

The financial sector is populated by a large number of identical banks that combine net worth, interbank loans and deposits to provide final loans to firms and interbank credit. As discussed before, each bank is managed by a banker, with the fraction of bankers in each household constant and equal to $f$. At the end of each period a banker can exit and become a worker with probability $1 - \theta_B$.\(^{28}\) If that happens, the banker transfers all remaining capital to his household as dividends that are pulled together with salaries to finance consumption and savings. At the same time, a fraction $\theta_B f$ of workers become bankers, receiving an initial lump sum transfer from households.

The objective of each banker is to maximize the transfer she makes to her household upon exit. Following Gertler & Karadi (2011), we assume that there are credit frictions in the economy due to an agency problem. Bankers can divert a fraction $\Theta$ of funds intermediated on the retail market to the household.\(^{29}\) In that case, the bank defaults and households receive the amount of funds diverted. Knowing that, the principals of the banker (i.e. depositors and other banks) limit the amount of funds lent to each bank. The credit friction, therefore, generates a constraint on the total amount of funds available to the banking system.

Each bank operates at the same time on the interbank market (through its wholesale branch) and on the loan market (through its retail branch). Markets are perfectly competitive except for the agency problem just discussed. Each bank starts the period with an endowment of capital, that has the role of net worth; the balance sheet of a representative bank can be described as follows:

\(^{28}\)Therefore in each period a fraction $(1 - \theta_B)f$ of household members becomes bankers.

\(^{29}\)We assume here that banks can monitor more efficiently other banks. Therefore, bankers can diverts only fund intermediated on the retail market. This assumption is similar to Gertler & Kiyotaki (2010) and Gertler & Kiyotaki (2015).
the wholesale branch uses net worth to extend loans on the interbank market and acquire interbank credit. The retail branch uses interbank funds and deposits to extend loans to firm.

We depart here from Gertler & Karadi (2011) assuming that banks interact on a complex network of credit relations. To model this network we extend the framework proposed in Acemoglu et al. (2015) (that is based on the results of the "coconut" model by Diamond (1982)). Banks use net worth to supply funds on the interbank, while, at the same time, they borrow funds from other banks. Financial intermediaries, in other terms, cannot invest in their own projects. This assumption captures, in a simple way, the fact that investment opportunities (demand for loans by firms) and the liquidity needed to finance them may not arise simultaneously. As a consequence, banks need to borrow from other intermediaries that have liquidity at the same time.\textsuperscript{30} As stated previously, banks can trade only with a limited (random) number of counterparties in each period on the interbank market. These assumptions capture some key characteristics of the banking activity (see Acemoglu et al. (2015)) that explain why banks trade on the interbank market but approach only a limited number of counterparties.

As a consequence, banks are interconnected through a credit network generated by their mutual claims and obligations. If a bank is shocked, that reverberates to its counterparties. We will (extensively) come back to this point in the following section.

Notice, for now, that interbank loans show up on both sides of bank’s balance sheet.\textsuperscript{31}

\begin{table}[h]
\centering
\begin{tabular}{|c|c|}
\hline
Assets & Liabilities \\
\hline
Loans to Firms ($B$) & Net Worth ($N$) \\
Interbank Loans ($Int$) & Deposits ($D$) \\
& Interbank Loans ($Int$) \\
\hline
\end{tabular}
\end{table}

\textsuperscript{30}The idea is that there are liquidity mismatches in the banking activity. Therefore banks need to borrow from other financial institutions. For a more detailed discussion, and alternative ways to model the same behavior, see Acemoglu et al. (2015).

\textsuperscript{31}Notice that each bank divides evenly its desired amount of interbank loans between all counterparties.
End of period profits for a representative bank are defined as the sum of profits for each branch:\footnote{32}

$$\Pi^B_t = R^K_{t+1}B_t + R^I_{t+1}Int^+_t - R^I_{t+1}Int^-_t - D_tR^D_{t+1}$$

(17)

with $R^K$ the interest rate on loans extended to firms, $R^I$ the interbank interest rate and $R^D$ the interest rate on deposits.

Hence the law of motion of banks’ net worth becomes:

$$N_{t+1} = (R^K_{t+1} - R^D_{t+1})B_t + R^D_{t+1}N_t$$

(18)

2.5.1 The wholesale branch

The wholesale branch is in charge of managing the liquidity of the bank. It acquires interbank loans for the retail branch and invests the group’s liquidity on the interbank market, with the opportunity cost of interbank loans that equals the riskless rate (i.e. $R^D$). Each wholesale branch chooses the best level of interbank loans supplied ($Int^+_t$) and demanded ($Int^-_t$) maximizing:

$$\Pi^{WB}_t = \sum_{i=0}^{\infty} (1 - \theta^B)(\theta^B)^i \Lambda_{t,t+1+i}^C \left[ Int^+_t R^I_{t+1} - N_t R^D_{t+1} - Int^-_t R^I_{t+1} \right]$$

(19)

subject to the resource constraint $Int^+_t \leq N_t$ and the demand of funds of the retail branch $Int^+_t = B_t - D_t$. $Int^+_t R^I_{t+1}$ are the gains from interbank lending, $N_t R^D_{t+1}$ is the shadow cost of bank capital and $Int^-_t R^I_{t+1}$ the cost of borrowing on the interbank market. It is straight with which it can trade. Banks differ only in their position on the credit network and in their haircut rate in case of default. As the expected default probability is zero, all banks are ex-ante equal (so each of them has the same “risk”). Therefore it is optimal for the banker to allocate credit evenly among all available partners.\footnote{32}

Given the assumptions on bankers’ behavior, bankers optimally accumulate profits (if they don’t exit) in each period. See Bernanke et al. (1999) and Gertler & Kiyotaki (2010) for a detailed discussion.
forward to derive first order conditions:\(^{33}\)

\[ R^I_t = R^D_t \]  

\[ Int^+_t = Int^-_t = N_t \]  

These conditions translate the interbank market structure of Acemoglu et al. (2015) in a dynamic general equilibrium setting. As long as 20 and 21 hold (i.e. the interbank rate covers at least the shadow cost of holding capital), banks supply all available liquidity on the interbank market.

2.5.2 The retail branch

The retail branch combines interbank credit and deposit to supply final loans to firms. Each intermediary operates on the market if the expected discounted sum of profits is positive\(^ {34}\) in each period, therefore only if:

\[ E_t (V_t) = \sum_{i=0}^{\infty} (1 - \theta^B)^i \Lambda^C_{t,i+1+i} \Pi^{RB}_{t+1+i} \geq 0 \]  

\(^{33}\)Notice that this condition is equivalent to what derived in Gerali et al. (2010) in the absence of frictions.
\(^{34}\)It is straightforward to define profits as returns on lending \((R^F_t B_t)\) minus the of financing cost \((R^D_t D_t)\).

As assumed by Gertler & Karadi (2011) and Gertler & Kiyotaki (2015), at the beginning of the period the banker can also decide to divert a fraction \(\Theta\) of the bank’s assets to households. In this case the bank defaults. The banker decides not to divert assets only if the follow incentive compatibility constraint (ICC) holds:

\[ E (V_t) \geq \Theta B_t \]  

In other words the expected discounted sum of profits at the end of the period \((V_t)\) must
be larger that the amount of divertable funds (\(\Theta B_t\)) for the banker to "comply". Bearing in mind the budget constraint of the retail branch,\(^{35}\) it is possible to write equation (23) as:

\[
k_t^1 Int_t^- + k_t^2 B_t \geq \Theta B_t
\]  

(24)

using a recursive form for \(E_t(V_t)\) with:

\[
k_1^t = E_t \left[ (1 - \theta^B) + \Lambda_{t,t+1}^C \theta^B \frac{Int_{t+1}^-}{Int_t} k_{t+1}^1 \right]
\]

(25)

\[
k_2^t = E_t \left[ (1 - \theta^B) \Lambda_{t,t+1}^C \theta^B (R_{t+1}^K - R_{t+1}^D) + \Lambda_{t,t+1}^C \theta^B \frac{B_{t+1}}{B_t} k_{t+1}^2 \right]
\]

(26)

Denoting \(\phi_t^L = \frac{k_t^2}{\theta - \theta^B}\) the ratio between deposits and interbank funds, we characterize the retail loan market in the spirit of Gertler & Karadi (2011).\(^{36}\) The constraint given by equation (23) is locally biding around the steady state and the spread \((R^K - R^D)\) is determined by the degree of financial frictions.\(^{37}\)

Defining net worth (for surviving banks) at the end of the period as the sum of undepreciated net worth and profits from banking activity, the law of motion of bank's capital is:

\[
E_t \left( N_{t+1}^{BO} \right) = \left( R_{t+1}^K - R_{t+1}^D \right) B_t + R_{t+1}^D N_t
\]

(27)

Finally, in each period only a fraction \(\theta^B\) of banks survives. An equivalent number banks enter the market receiving an initial endowment of capital from households.\(^{38}\)

Therefore, the total net worth of the banking sector, at the beginning of each period, is the sum of the net worth of surviving and new banks:

---

\(^{35}\)That is \(Int_t^- + D_t = B_t\).

\(^{36}\)Notice, again, that here we have operatively divided each bank in a retail and a wholesale branch. Therefore, at retail level, banks use interbank credit and deposits as inputs.

\(^{37}\)Gertler & Karadi (2011) provides a proof for this statement.

\(^{38}\)The endowment is assumed to be equal to a fraction \(\frac{\omega}{1-\theta^B}\) of the equity of the exiting banks; see Gertler & Karadi (2011).
\[ N_{t+1} = \theta B N_{t+1}^{BO} + \frac{\omega}{1 - \theta B} N_{t+1}^{BO} + e_{t+1}^B \]  

(28)

\( e_{t+1}^B \) is a shock on the bank’s capital that follows an AR(1) process and has steady state value of 0.

### 2.6 Equilibrium

Equilibrium relations complete the model. The well known supply-demand identity must hold on the good market:

\[ Y_t = C_t + I_t + G_t + \Phi_t \]  

(29)

where \( \Phi_t \) gathers additional costs for the economy (adjustment costs in capital goods production and frictions) and \( G_t \) is public expenditure.

The central bank follows a (linearized) Taylor-type rule:\(^{39}\)

\[ R^n_t = (1 - \rho) \left[ \overline{R^n} + \psi_\pi \pi_t + \psi_y (Y_t - Y^{ss}) \right] + \rho R^n_{t-1} + e^R_t \]  

(30)

where \( R^n_t \) is the net nominal interest rate, \( \overline{R^n} \) its steady state value and \( e^R_t \) is a monetary policy shock that follows an AR(1) process with steady state 0. The connection between nominal and real interest rates is provided by the following Fisher relation:\(^{40}\)

\[ 1 + R^n_t = E_t \left( R^D_{t+1} \pi_{t+1} \right) \]  

(31)

\(^{39}\)Taylor & Woodford (1999).

\(^{40}\)See Fisher (1896).
3 The Interbank Market

In this model banks interact on a complex interbank market and generate a complex network of connections. The objective of this paper is to model that network and study how contagion spreads through it.

As pointed out in many empirical and theoretical works, connections between financial institutions affects significantly the resilience of the system to small and large shocks. We assume that banks are connected only to a limited number of counterparties in each period. The type and characteristics of the distribution that generates the number of partners with which each bank can trade in each period is set to match the results of empirical studies on the interbank network. Given the probability that a bank is connected to $k$ other counterparties, the volume of trade on the interbank market and the value of banks’ assets, it is possible to generate a statistical model that describes the main features of the network and study how contagion spreads between nodes (banks).

3.1 A General Model of Transmission

The properties of transmission on networks have been studied since the beginning of the 2000s in physics and engineering. In this section we present a general model for diffusion on random graphs. In the next paragraph we will incorporate it into the DSGE model.

Consider a random graph composed by $V$ nodes (in our context banks) and $d$ edges (credit linkages). The probability of a node having $k$ edges is defined as $p_k$. In the context of a banking network $k$ is the number of counterparties with which each bank trades. $p_k$, therefore,

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42We make use of the methods proposed by Strogatz & Watts (1998), Callaway et al. (2000), Strogatz (2001), Newman et al. (2001) and Watts (2002). This model can be applied to any random graph independently on the underlying distributions.
is also the share of banks that have exactly $k$ partners.  

Each node can move from state 0 (inactive) to state 1 (active) if at least a fraction $\phi$ of its $k$ neighbors are in state 1.  

$\phi$ is a parameter specific to each node and is drawn from a distribution $f(\phi)$ that is normalized such that $\int_0^1 f(\phi) d\phi = 1$. In this context a cascade is defined as the number of nodes that move from state 0 to state 1 as a consequence of a random node (called the initial seed) moving exogenously from state 0 to 1. In the case of the interbank network, banks in state 1 are defaulted.

The aim of this model is to identify: i) the probability that a global cascade is triggered by a single node, where a global cascade is defined as a cascade that occupies a finite fraction of the total number of agents; ii) the size of the cascade if it takes place. In a banking network that would be: i) the probability of contagion in case of default of a single bank, ii) the number of banks affected.

From the properties of random graphs follows that in large random graphs with small initial seeds the local neighboring of an initial seed does not contain cycles, therefore no vertex neighbor to an initial seed will be adjacent to more than one seed. This property is important to pin down the shock diffusion. Another important property of large random graphs is that they can be regarded as pure branching structures, therefore each subcluster below the transition size may be regarded as independent.

Under these conditions, an initial seed can diffuse through the network if and only if at least one of its initial neighbors has a threshold $\phi \leq \frac{1}{k}$ or $k \leq \bar{k} = \frac{1}{\phi}$; a vertex that meets this condition is called vulnerable as it might be part of the cascade. The existence of a

\[\text{For the law of large numbers.}\]

\[\text{This is equivalent to say that a node moves to state 1 if receives enough shocks ($\phi$) from its neighbors.}\]

\[\text{Notice that this assumption can be relaxed. Each node can be assumed to have the same value of $\phi$. In this case the model is simple, as each node would have the same resistance.}\]

\[\text{The interested reader may refer to Newman et al. (2001), Watts (2002) or Vega-Redondo (2007).}\]

\[\text{See Watts (2002) and Gross & Yellen (2003), this is asymptotically true for random graphs as the number of vertices grows. See Appendix A.2 for an intuition.}\]

\[\text{A cluster is said to be at the transition size if a shock starting there can percolated to the entire network. See Newman et al. (2001) and Watts (2002) for proofs.}\]
cascade depends not only on the magnitude of the initial shock but also, and maybe more importantly, on the position of the initial seed(s). In fact, a seed that is isolated does not threaten the whole system even if large in magnitude. On the contrary, a small seed located in a very “central” position may expand through the entire graph.

Define the probability of a vertex with \( k \) neighbors to be vulnerable as \( \rho_k = P[\phi \leq \frac{1}{k}] \).\(^{49}\) Therefore, it is easy to define the moment generating function of the vulnerable vertex degree as:\(^{50}\)

\[
G_0(x) = \sum_{k=0}^{\infty} \rho_k p_k x^k 
\]

(32)

from this moment generating function we can derive the moments of the vulnerable cluster (i.e. the group of vertices -banks- that are affected by the initial seed), the vulnerable fraction of the population and the average degree of vulnerable vertices.

The vulnerable fraction of the population is simply given by \( P_v = G_0(1) \). Notice that a vertex is vulnerable depending at the same time on its own value of \( \phi \) and on the number and vulnerability of its neighbors. The combination of the two defines weather a vertex (bank) in the model is part of the vulnerable cluster. The average degree of vulnerable vertices (i.e. how many connection vulnerable vertices have on average) is defined as \( z_v = G'_0(1) \). An additional index to compute is the degree distribution of a vulnerable randomly chosen vertex \( a \) that is a neighbor of the initial (active) vertex \( b \). The larger is the degree of \( a \) the more likely it can spread contagion from \( b \). The probability of choosing \( a \) is clearly proportional to \( kp_k \) and the corresponding, well-behaving, moment generating function is:

\[
G_1(x) = \frac{\sum_{k=0}^{\infty} \rho_k k p_k x^{k-1}}{\sum_{k=0}^{\infty} kp_k} 
\]

(33)

the numerator is the first derivative of equation (32) while the denominator is simply the average degree of the whole network (\( z \)). Therefore, it is easy to transform the previous

\(^{49}\)Notice that for \( k = 0 \rho_k = 1 \). Additionally \( \rho_k \) constant is just a special case of this model.

\(^{50}\)The interested reader may refer to Newman et al. (2001), Strogatz (2001), Watts (2002) and Vega-Redondo (2007) for an extensive theory.

24
equation into:

$$G_1(x) = \frac{G'_0(x)}{z}$$

(34)

Finally, define $q_n$ as the probability that a randomly chosen vertex belongs to a vulnerable cluster of size $n$, and $r_n$ as the corresponding probability for a neighbor of the initial vertex.

The corresponding moment generating functions ($H_0(x), H_1(x)$) are:

$$H_0(x) = \sum_{n=0}^{\infty} q_n x^n \text{ and } H_1(x) = \sum_{n=0}^{\infty} r_n x^n$$

(35)

Self-consistency conditions\(^{51}\) call for:

$$H_1(x) = [1 - G_1(1)] + xG_1[H_1(x)]$$

(36)

allowing to derive $H_0(x)$ as:

$$H_0(x) = [1 - G_0(1)] + xG_0[H_1(x)]$$

(37)

$[1 - G_0(1)]$ is the probability that a chosen vertex is not vulnerable, $xG_0[H_1(x)]$ takes into account the size of the vulnerable cluster. From the definition of $H_0(x)$ it is possible to compute all moments of the vulnerable cluster, in particular its average size $\|n\| = H'_0(1)$.

Exploiting equations (36) and (37):

$$\|n\| = G_0(1) + \left[\frac{G'_0(1)^2}{z - G''_0(1)}\right] = P_v + \frac{z_v^2}{(z - G''_0(1))}$$

(38)

\(^{51}\)See Newman et al. (2001) and Strogatz (2001). Self-consistency allows to approximate the distribution of a random vector $X$ by a random vector $Y$ whose structure is less complex without significant loss of information. In particular we can construct a self-consistent approximation of $X$ dividing $X$ into subsamples and defining $Y$ as a random variable with values the means of each subset. Notice that a (random) search process on a graph can be regarded as a pure martingale process. See Tarpey & Flury (1996) for more details.
it is clear that the previous equation diverges ($\|n\| \to \infty$) when:

$$G''_0 (1) = \sum_k k(k-1) \rho_k p_k = z \quad (39)$$

See Appendix A for an extensive proof. $\|n\|$, the average vulnerable cluster size, defines the average number of nodes that participate in the cascade if it takes place. $P_v$ defines the vulnerable fraction of the population and, on large networks, it approximates the probability that a single node is vulnerable. Being vulnerable means that a node may trigger a cascade if it is hit by an exogenous shock. In this model if a node (bank) is eliminated at random, it starts a cascade (default sequence) with probability $P_v$ that has, on average, size $\|n\|$. Notice that these measures depend on the way in which nodes are connected (described by $p_k$) and on the resilience of each single node ($\phi$).\(^{52}\)

Equation (38) is generally known as the cascade condition. On top of defining the average cluster size, it states that whenever $G''_0 (1) < z$ all vulnerable clusters in the network are small; in other words the initial shock is unable to percolate through the whole system. On the contrary, when the condition expressed by equation (39) is verified a percolating vulnerable cluster arises; through it an initial seed may spread to the whole system.\(^{53}\) Equation (39) is also known as phase transition and as $k(k-1)$ is monotonically increasing in $k$, while $\rho_k$ is monotonically decreasing in it, has two solutions or none. In the case of two solutions we should have, according to Watts (2002), an interval in $z$ in which cascades can occur. Paragraph 3.3 provides an overlook of the properties of the network under analysis. In Appendix A.5 the interested reader may find a longer discussion.

\(^{52}\)In the following $p_k$ is chosen to match the empirical literature on interbank networks.

\(^{53}\)On this topic see Stanley (1971), Bollobas (1985) and Stauffer & Ahrony (1991).
3.2 Application to the Model

It is straightforward to identify nodes as banks operating on the interbank market, the initial seed as a defaulting bank and the parameter $\phi$ as the maximum number of defaults that a bank can absorb from its counterparties without defaulting itself. In this case a "global cascade" is nothing else than contagion on the interbank market and its probability and size a good measure for systemic risk.\(^{54}\) In this context equation (32) is the probability that the default of a randomly chosen bank triggers a sequence of episodes. The lower is $P_v$ the less likely a cascade occurs, the more stable is the system. At the same time $\|n\|$, the cascade size, defines the magnitude of the default sequence and, so, the severity of the episode.

This framework has two advantages: at first it integrates, in a rigorous way, a mechanism to describe the systemic behavior of the interbank network into a standard DSGE models. Until now DSGE models were not able to incorporate this kind of mechanisms whose importance, however, has became clear in the last decade. With this method, additionally, we take explicitly into consideration that the position of a bank on the network (i.e. the number and characteristics of all its $k^{th}$ order neighbors) defines the probability that it participate in a cascade. On the other hand, we are able to identify a true measure of systemic risk, for which it is possible to compute the corresponding probability introducing a new and important policy tool in a state of the art model. We can trace how systemic risk moves through the business cycle and assess how it evolves after shocks.

Define $p_k$ as the distribution from which is drawn the number of counterparties with which each bank trades at any time $t$.\(^{55}\)

Given a distribution for $p_k$ it is possible to evaluate equations (32) to (39) that describe the characteristic of the interbank network. To keep things simple one could follow the so call small-world model by Strogatz & Watts (1998) based on Erdos & Renyi (1959) and Erdos &

\(^{54}\)Cascade probability and size define, in terms of macroprudential policy, the probability of contagion spreading from one defaulting institution to the system and the expected number of defaulting banks involved.

\(^{55}\)That number is assumed to be I.I.D between time and banks.
However, results of this approach appear not entirely adequate to describe real world networks, most of which exhibit power law properties. In particular, there are sound and coherent evidences that the structure of the interbank market network is described by a power law distribution; between them: Boss et al. (2004a), Boss et al. (2004b), Caldarelli et al. (2006), Soramaki (2007), Allen & Babus (2007), Alentorn et al. (2007), Clauset et al. (2009), Cohen-Cole et al. (2011), Co-Pierre (2013) and Gabrieli & Co-Pierre (2014). In this paper we follow those evidences to characterize $p_k$.\textsuperscript{57}

Notice also that the domain of a power law is bounded in the interval$[1, \infty]$.\textsuperscript{58} Additionally this particular function describes some of the key characteristics of the interbank market (as well as of other real world networks): i) there are signs of preferential attachment,\textsuperscript{59} ii) the tails of the distribution are not irrelevant.\textsuperscript{60}

Define $p_k$ as:\textsuperscript{61}

$$p_k = Lk^{-\gamma} \quad (40)$$

\textsuperscript{56}See Barabasi & Albert (1999).

\textsuperscript{57}For a detailed description of the properties of random graphs generated by this distribution see Newman et al. (2001), Aiello et al. (2001), Strogatz (2001), ben Avraham et al. (2002) and Vega-Redondo (2007).

\textsuperscript{58}This fits perfectly with the assumptions of the model. In fact, in our model banks need at least one counterparty to operate. If $k$ was 0, than a bank would not participate on the market and its supply of loans would be equal to deposits.

\textsuperscript{59}In the literature on random graphs preferential attachment is a characteristic of networks where agents with more links are more likely to form new ones. This characteristic is embodied by power law distributions while it is missing in networks generated by normal distributions. In the terms of the interbank market, it means that larger banks tend to have more connections that small banks. This is consistent with empirical evidences on the the banking literature and the literature on interbank networks (see Caldarelli et al. (2006) and Boss et al. (2004a)).

\textsuperscript{60}This last property is particularly important. Graphs characterized by power law distributions have typically a significant number of nodes located in the tails of the degree distribution contrary to graphs described by normal distributions. In our context this means that there is a significant number of banks populating the upper tail of the distribution -i.e. having many links therefore being more connected- that are systemically important institutions. Therefore, a simple average number of connection is not informative and, as we have done before, it is necessary to rely on additional statistics to describe the network structure. The interested reader can, again, rely on Newman et al. (2001).

\textsuperscript{61}Notice that in this case the power-law does not exhibit an exponential cutoff. A possible extension of the model would be to incorporate it.
where $L$ is a standardization parameter. The parameter $\tilde{\gamma} > 2$ defines the degree of "concentration" in the system; as $\tilde{\gamma}$ grows, the probability associated to large $k$ decreases, therefore there are less "well-connected" banks. Substituting (40) into a standard moment generating function (i.e $G_0(x) = \sum_k p_k x^k$):

$$G_0(x) = \frac{Li_{\tilde{\gamma}}(x)}{\zeta(\tilde{\gamma})}$$

Despite being coherent with empirical evidences, this class of functions presents a main drawback: they exhibit fractal and chaotic properties. This is the main reason why it is so difficult to incorporate them into a standard DSGE framework. Fractals have not always closed form solutions and are heavily affected by chaotic behavior. In this case, $Li_{\tilde{\gamma}}(x)$, the $\tilde{\gamma}^{th}$ order polylogarithm of $x$, has a chaotic behavior. The polylogarithm, additionally, has closed-form solution only for some specific choice of parameters; in most of the cases it can be computed, at best, iteratively.

In order to incorporate the cascade mechanism into this model it is necessary to derive a more tractable solution.

Recall the definition of a definite integral as the limit of a Riemann sum and that the number of agents in our model is very large. In this case, as shown by Vega-Redondo (2007), it is possible to approximate the discrete distribution to its continuous counterpart. Given an interval $[a, b]$:

$$\int_a^b f(x) \, dx = \lim_{n \to \infty} \frac{b - a}{n} \sum_{i=1}^n f(x_i)$$

with $x_i = a + i \frac{b - a}{n}$.

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62 Defined $L = \zeta(\tilde{\gamma})$ in equation (32) with $\zeta$ the Reinmann $\zeta$-function.

63 See Newman et al. (2001), ben Avraham et al. (2002), Watts (2002) and Vega-Redondo (2007). From the previous chapter it is clear that the graph is not defined for $\tilde{\gamma} >= 2$

64 See Falconer (2003) and Mandelbrot (2004).

65 See Krantz (2005).
The generating function of the entire graph is \( \sum_k f(k) \). Applying this definition it is straightforward to show that \( a = 0 \) and \( b = n \).\(^{66}\)

\( n \), the upper bound of the interval, is the maximum number of loans (edges) that a bank may have. In this context can be thought as if each lends one unit of capital to a different counterparty until it runs out of funds. Therefore the upper bound of the interval is simply the maximum amount of fund lent, equal to \( \text{Int}_t^+ \). As it is easy to see, the amount of loanable funds evolves with the business cycle (increases in booms and decreases in recessions). As a consequence, all network variables depend on the state of the economy affecting the stability and riskiness of the system.

Rewriting equation (32) we have that:\(^{67}\)

\[
G_0(1) = \int_1^{\text{Int}_t} \rho_k Lk^{-\tilde{\gamma}} x^k dk
\]

(43)

with \( \rho_k = P[\phi \leq \frac{1}{k}] \) and \( \phi = f(V_t^e) \). To close the model it is necessary to evaluate \( \phi \). It can be proved\(^{68}\) that \( \phi = \frac{V_t}{\text{Int}_t} \) with \( V_t \) the maximum amount of funds that a bank can lose without defaulting. The definition of \( V_t \) is straightforward from the budget of a single bank:

\[
V_t = - \frac{R^K_t B_t - R^D_t D_t - R^I_t \text{Int}_t}{R^I_t}
\]

(44)

Assume now that banks face an haircut on the value of their assets when they are forced to sell them during crisis. This haircut is specific to each bank reflecting bank-specific conditions.\(^{69}\) This assumption is in line with empirical results and introduce heterogeneity during

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\(^{66}\)Consider the moment generating function \( G_0(x) = \sum_{k=0}^{\infty} \rho_k p_k x^k = \sum_{i=1}^{n} f(x_i) \). Additionally \( x_i = k \), as \( k \) is the argument of the MGF \( G_0 \). This implies that \( \frac{k-a}{n} = 1 \) and \( a + k \frac{b-a}{n} = k \). This approximation holds when \( n \) tends to a large number. A similar procedure is used by Newman et al. (2001) and Vega-Redondo (2007).

\(^{67}\)Notice that the power law distribution is defined over the interval \([1, \infty]\).

\(^{68}\)See Appendix A.

\(^{69}\)It takes into account the specific managerial ability of some banks or the specificity of the assets in which they have invested.
crisis between banks.\textsuperscript{70} Relaxing this assumption simplifies the model without changing the derivation.\textsuperscript{71}

We assume that $V_t$ is affected by a bank specific random shock $\eta$ that follows a uniform distribution over $[0, 1]$ and is I.I.D. through banks and time. $\eta$ measures the haircut on assets that a specific banks faces if it has to sell them during a cascade.\textsuperscript{72} Each bank is able to use only:

$$V_t^e = \eta_t V_t$$ \hfill (45)$$

However, the limit case of $\eta_t = 1$ for each $t$ and bank is simply a special case of the model under consideration, with a much simpler network structure.\textsuperscript{73}

Using equation (44) it is possible to derive the probability of having a specific level of $V_t^e$. It is easy to define $P[\phi \leq \frac{1}{k}]$ as:

$$P[\phi \leq \frac{1}{k}] = P\left[\frac{V_t^e = \eta_t V_t}{I_{nt_t}} \leq \frac{1}{k}\right] = P\left[\eta_t \leq \frac{I_{nt_t}}{kV_t}\right]$$ \hfill (46)$$

Therefore, equation (43) becomes:

$$G_0(1) = \int_{1}^{I_{nt_t}} \Omega_t L k^{-\tilde{\gamma}} x^k dk$$ \hfill (47)$$

with $\Omega_t \equiv P[\eta_t \leq \frac{I_{nt_t}}{kV_t}]$. The stability of the financial system is affected not only by the total amount of credit exchanged by banks but also by a bank-specific component. Notice that each bank differs in terms of resilience to external shocks not only by its realized value of $\eta$ but also by its number of connections $k$; both these characteristics are important to determine if it belongs or not to a vulnerable cluster. From equation (47) it is possible to derive all

\textsuperscript{70}See for example Kumhof & Jakab (Bank of England working papers), Angeloni & Faia (2009), Altunbas et al. (2015).

\textsuperscript{71}In that case, in fact, $\phi$ will be constant across all banks and $P_v$ would depend only on the connection of each bank.

\textsuperscript{72}See Christopher (1991), Coval & Stafford (2005), Brunnermeier (2009) and Duarte & Eisenbach (2013).

\textsuperscript{73}$\rho_k$ would be always 0 or 1 depending on $k$. 

31
relevant informations about the network structure of the interbank market, following the procedure proposed in the previous paragraph:

$$z_t = \int_{1}^{\text{Int}_t} L k^{-\tilde{\gamma}} x^k dk$$  \hfill (48)$$

$$G'_{0,t}(x) = \int_{1}^{\text{Int}_t} \Omega_t L k^{-\tilde{\gamma}} k x^{k-1} dk$$  \hfill (49)$$

$$G''_{0,t}(x) = \int_{1}^{\text{Int}_t} \Omega_t L k^{-\tilde{\gamma}} (k-1) x^{k-2} dk$$  \hfill (50)$$

These equations, evaluated at each time $t$, describe the aggregate characteristics of the network that arises from the lending and borrowing contracts between banks. Notice that we are able to identify both the probability and size of a default sequence. These variables are the result of individual decisions of a single bank and of the collective actions of the banking system.\(^{74}\) They can be used to describe in the aggregate the expected probability and size of systemic events. These events are triggered by the exogenous default of one institution; in that scenario the relevant question for policy makers is whether that default would lead to others and how many they might be.

### 3.3 Probabilities, Cascades and the Banking Network

Equation (32) describes the probability that a defaulting bank triggers a cascade. In other words, it gives the probability that a bank $a$ is connected to at least one fragile neighbor $b$; if $a$ defaults a cascade is triggered (i.e. at least another bank -b- defaults). Generally, the more a network is sparse (the more $\tilde{\gamma}$ is large), the more likely a randomly chosen bank starts

\(^{74}\)Notice that each banker cannot computed her own exposure to risk. In fact, to compute her own exposure to risk a banker, a banker should know her own set of neighbors on the interbank market, their characteristics and the set of connections (and characteristics) of any $k^{th}$ order neighbor.
a cascade if it defaults.\textsuperscript{75} Determining the likelihood of contagion is not sufficient; policymakers should also focus on expected number of institutions involved in it. This variable is captured by equation (38). It is the result of a complex convolution of the deep parameters of the network structure; depends, quite intuitively, positively on the probability of being part of a vulnerable cluster ($P_v$) and on the average degree of vulnerable vertices.\textsuperscript{76} On the contrary $\| n \|$ depends negatively on the whole network average connectivity $z$ and positively on $G'_0(x)$. Their combination leads to the so called “phase transition equation” (39).\textsuperscript{77} This equation defines the conditions by which the entire system is affected by the initial seed. Similarly to previous contributions on financial networks, we find that the condition is very rarely met, but the effects of a phase transition are potentially devastating.\textsuperscript{78}

A key question for policymakers is how it is possible to reduce the probability and size of contagion, minimizing the likelihood of a phase transition. An extensive literature on diffusion on social networks shows that the only effective way to prevent diffusion of an epidemic is to immunize the population.\textsuperscript{79} In the context of the interbank market immunization is carried on through subsidies to banks’ capital. As availability of funds and public support for these measures are limited, policymakers should identify which bank is more efficient to target with capital subsidies. Simulations show that it is generally better (for reasonable values of the immunized share of banks) to intervene on the higher part of the distribution, targeting the more connected banks. Preventing the default of a very central node, in fact, stops the cascade before it reaches more nodes than targeting many (peripheral) small agents.\textsuperscript{80} Results are reported in Figures 3.1 and A.II; Appendix A.5 provides a more detailed discussion.

\textsuperscript{75}Notice that larger is $\tilde{\gamma}$ the lower is the number of highly connected agents. The resulting network, therefore, is more sparse in the sense that there are relatively more agents less connected between them. Notice, also, that $\tilde{\gamma}$ must fall in the interval $[2, 5]$, see Appendix A.5.

\textsuperscript{76}The more a vulnerable vertex is connected, in fact, the easier it can be "infected" and spread contagion to other vertices.

\textsuperscript{77}See Appendix A.5 for more details.

\textsuperscript{78}See for example Acemoglu et al. (2015). Appendix A.5 provides a more extensive analysis.


\textsuperscript{80}Notice also that well connected nodes are more likely to be part of the cascade rather than peripheral nodes.
Figure 3.1: Probability of a cascade, lower versus top.

Notes: the graph shows the difference in the share of banks that participate in a cascade (that equals the probability of a cascade) between the case in which the same share of banks is subsidies in the bottom or in the top of the distribution. A positive value means that $P_v$ is larger in the case of subsidies to the least connected institutions.

4 Simulation Exercises

The model is calibrated following the standard DSGE literature, see in particular Gertler & Karadi (2011), Friedman & Woodford (2011). The discount parameter $\beta$ is set to 0.99, $\gamma$ and $\varphi$ to 0.815 and 0.276 respectively. The weight of labor in the utility function ($\chi$) is 3.409.

The share of capital in the production function, $\alpha$, is calibrated to 0.33. $\delta$ and $\epsilon$, the depreciation rate of capital and the elasticity of demand, take the values of 0.025 and 4.167. The Calvo parameter is set to 0.779 and $\chi_I$ to 12.\textsuperscript{81} The key network parameter ($\tilde{\gamma}$) is initialized to 2.5 following the evidences of a robust empirical literature on social networks.\textsuperscript{82}

\textsuperscript{81}Following in this case Dib (2010).

\textsuperscript{82}Between them we can remember Bollobas (1985), Wormald (1999), Boss et al. (2004a), Boss et al. (2004b), Caldarelli et al. (2006), Soramaki (2007), Clauset et al. (2009), Bluhm et al. (2013), Gabrieli & Co-Pierre (2014), Oliver et al. (2014).
The monetary policy function parameters ($\rho$, $\psi_\pi$ and $\psi_y$) are calibrated to 0.8, 1.5 and 0.5 respectively. Finally, the share of government consumption on total output is equal to 0.2 in the steady state and the autocorrelation coefficient of each shock is set to 0.5.

4.1 Impulse Response to Standard Shocks

We simulate the effect of a 1% positive exogenous shock on the interest rate (i.e. a contractionary monetary policy surprise), the TFP, consumption preferences, government spending, capital quality and banks’ net worth. On this part, the model resembles standard New Keynesian results for output, consumption and employment.\textsuperscript{83}

Figure 4.1: Output response to TFP and monetary policy shocks.

Output responds positively to a TFP shock and negatively to a monetary policy tightening. TFP shocks die out after 12 periods, while monetary policy shocks have longer lasting effects.

A positive consumer preference shock, Figure B.I, increases output as $\lambda$ rises, following equation (3). However, after an initial positive effect, in order to drive back $\lambda$ to its steady state value, consumers are forced to decrease consumption. Government spending shocks boost significantly output on impact, but die much faster than TFP shocks. Finally, a positive capital quality shock, Figure B.II, increases output for a prolonged period of time. In this

\textsuperscript{83}These results are in line with previous works such as: Angeloni & Faia (2009), Christiano et al. (2010), Gerali et al. (2010), and Gertler & Karadi (2011).
model, in fact, the economy is constrained by the amount of capital available. The more efficiently capital is used, the higher is output and the lower prices leading to an increase of households’ real balance sheets and boosting demand. The combined effects of these variables lead to the significant and persistent increase of output. The process is self reinforcing as growth in output straighten banks’ capital and the credit supply.

A similar reasoning explains the positive impact of a banks’ net worth (positive) shock. The more financial resources are available to banks, the more credit grows and, therefore, the more capital is available for production.\(^{84}\).

A TFP shock increases output and wages leading to an higher level of consumption. The increase dies out slowly over time as the initial impact of the shock is reinforced by financial gains for households as returns on capital and deposits grow. The opposite is true for a tightening monetary policy shocks.

Following the reasoning outlined in the previous case, a positive preference shock leads to an increase in consumption, Figure B.IV. Government spending shocks reduce consumption, as the marginal utility of consumption decreases and households increase the amount of resources invested in deposits. At last, consumption increases (following output) after a positive capital quality shock, Figure B.V in line with what discussed before.

![Figure 4.2: Inflation response to TFP and monetary policy shocks.](image)

As expected, inflation responds negatively to both TFP and monetary policy shocks as a

\(^{84}\)See the Online Appendix B.1.
result of lower production costs generated by the TFP shock and a fall in aggregate demand induced by the increased interest rate. On the contrary, inflation rises following consumption preference and government spending shocks, see Figure B.VI. As capital quality or banks’ net worth shocks, Figure B.VII, increase production, they also lead to an increase in inflation. The impact of the first shock is (again) larger, as the second is dampened by financial frictions.

Total loans increase after a positive TFP and decrease after a contractionary monetary policy shock. This is due to higher (lower) demand and a lower (higher) cost of funding for banks. Interbank loans, Figure 4.3, follow a similar pattern.

![Figure 4.3: Interbank loans response to TFP and monetary policy shocks.](image)

Finally, the central bank conducts an easing policy after a positive TFP shock as the drop in inflation dominates the increase in output. Similarly, after an initial positive monetary policy shock, the central bank is forced to ease as a consequence of the fall of both output and inflation. After positive consumption preference and government spending shocks, Figure

![Figure 4.4: Policy rate response to TFP and monetary policy shocks.](image)
B.XIII, the central bank increases rates to react to the raise of output and inflation. The reaction is similar in the case of capital quality or banks’ net worth shocks, Figure B.XIV.

4.2 Network Impulse Responses

We now leave the safe shores of standard IRFs to enter the uncharted territory of the inter-bank network structure. In this case, there are not standard results in the literature.\footnote{Although there is a wide network literature, grown faster in the last years, on financial markets (such as Gai & Kapadia (2010), Cohen-Cole et al. (2011), Battiston, Delpini, Caldarelli, Gabbi, Pammolli & Riccaboni (2013), Co-Pierre (2013), Bluhm et al. (2013), Gabrieli & Co-Pierre (2014), Capponi & Chen (2015)) few models analyze them in a general equilibrium set up assessing the evolution of banking relations over the business cycle.}

The main question to be answered is how more or less likely a cascade (contagion) can occur following exogenously driven changes in the macro variables. This class of model tries to identify the relationship between shocks and financial stability, highlighting an additional trade off channel in public policies. In this setting, as has already been pointed out, financial stability is defined as the likelihood of systemic crisis (a crisis that involves 2 or more institutions) after a single default episode and the number of banks involved. This variables show what would happen if a bank exogenously defaults at any point in time. It is worth remembering that in this model defaults \textit{do not occur systematically}, but are defined as exogenous shocks.

The first variable we analyze is the share of banks belonging to a vulnerable cluster, $P_v$. For the law of large numbers this variable approximates the probability of a single bank being vulnerable and able, if it defaults, to trigger a general cascade.

Before moving on it is useful to remember what drives this variable. According to equations (32), (44) and (45), $P_v$ is influenced not only by the size of the interbank market, but also by interest rates and by the propensity to save of consumers. Therefore, the interaction between financial and real markets are complex and not completely predictable a priori.

Financial shocks (monetary policy, capital quality and banks’ net worth) have a stronger
Figure 4.5: Cascade probability $P_v$ response to shocks.

impact on $P_v$ than real shocks (TFP, government spending and consumer preference). There are two reasons for that. On one hand $P_v$ depends directly from financial variables and only indirectly from real ones. Their effect is mediated through the frictions of the model and, therefore, dampened. On the other, real shocks do not have a direct impact on $\Omega_t$ that affects the share of vulnerable banks. Recall, in fact, that the probability of contagion depends both on the number of connections ($k$) and on the resilience of each institution ($\phi$). The probability of being able to start a global cascade increases after a TFP or government spending shocks. Their effect is not extremely large in magnitude but is very persistent in time, especially for the case of TFP shocks. The more the economy is active (with pressure on demand or an increase in productivity) the larger financial markets are, with banks supplying more loans and having higher profits. This increases the amount of funds banks are willing to trade one another and, therefore, the overall connectivity of the system. As a result, the probability of being part of a vulnerable cluster increases. On the contrary, a consumer preference shock decreases $P_v$, as reduces the amount of final and interbank loans. Banks become less connected ($z$ decreases) and the number of banks being part of a vulnerable cluster is reduced.

A contractionary monetary policy surprise drives down significantly the probability of contagion -i.e. being part of a vulnerable cluster. As the policy rate increases, $\Omega$ increases, leading to, ceteris paribus, an increase in $P_v$. This effect, however, does not overcome the

\[86\text{See Figures B.XVII and B.XVIII in Online Appendix B.1.}\]
impact of the output drop caused by a contractionary policy. As output falls banks finance less loans to firms and, consequently, demand less interbank loans. This is reflected on the interbank market, where banks are less connected; as financial institutions share less links, the cascade probability declines.

Capital quality and banks’ net worth shocks, on the contrary, have a significant and positive impact on $P_v$. The reasoning is similar to the previous case. Banks finance more loans to firms and, therefore, demand more funds on the interbank market. At the same time, they increase the volume of assets held; the combination of the two increases interbank lending, connections on the interbank market, and the likelihood that a crisis grows. This last result is quite significant as highlights, from a theoretical point of view, the connections between an asset price boom and an increase of systemic fragility.

From the analysis of the response of $P_v$ to shocks we can conclude that: i) the probability of being able to trigger a cascade -i.e. contagion- is affected not only by financial shocks but also by real ones, ii) real shock impact less on the cascade probability, as they influence only indirectly network variables, iii) real shocks, even if small in the initial magnitude tend to last longer, iv) policy makers face a trade off when they are forced to directly stimulate the economy as an output increase comes at the price of a more risky financial system, v) there is not a “divine coincidence” in this case, as an expansionary monetary policy has a cost in terms of financial stability, vi) government spending appears to enjoy the benefits of an increase in output with a very limited deterioration of financial stability.

It is not sufficient, however, to consider only the probability of being able to trigger a cascade; economists and policy makers should also be concerned by its size. This variable is captured by $\| n \|$ that is a complex convolution of the variables describing the network nested. It grows, almost intuitively, in $P_v$ and in the probability of having a vulnerable neighbor and decreases in the total size of the interbank market and in $L''_0(x)$. In the case of large financial markets, so when $z$ grows, banks are more distant and, therefore, clusters are more likely to be isolated.
The cascade size follows a path similar to the probability of contagion, only larger in magnitude. However, the dynamics involved are complex and highly non linear. An intuition to explain them is that $\|n\|$ depends positively on $P_v$, $G'_0(x)$ and $G''_0(x)$; these network measures are affected in the same way by shocks as the moments of $G(x)$ are increasing in the same set of variables. The only variable that can push on the opposite direction is $z$ as the larger and less interconnected is the network the less likely a seed is able to grow. However, looking back at equation (38), its impact is mitigated by $G''_0(x)$. Finally, it is worth to notice that this model does not generates a phase transition. It is intuitively sound as this is a general equilibrium model that is simulated around the steady state. In the steady state there are not phase transitions and, therefore, we can not observe them in the IRFs. Relying on the conclusions drawn in Appendix A and Appendix A.5 it is possible to infer how these dynamics make us closer to the transition threshold or not.

To conclude, IRFs of the cascade size reinforce the conclusions drawn before. Real shocks have an effect on the stability of financial markets and policy makers face an additional trade off implementing policies. Additionally, asset price booms have the undesirable effect of leading to a more unstable financial system, ceteris paribus.
4.3 Leaning Against the Cascade

The previous paragraph shows how a tightening policy shock leads to a more stable financial system with lower $P_v$ and $\| n \|$. In this paragraph we show what are the costs in terms of output and inflation for a more stable financial system if central banks try to intervene. Recall that each bank is not able to infer its own position in the network, because it would need informations on all its $j^{th}$ order neighbors. Regulators, on the contrary, have access to the aggregate data on the financial sector, monitor the stream of transactions on the interbank market and, therefore, can easily compute the probability of cascades at each point in time.

A straightforward approach for the central bank would be to use the interest rate to affect the portfolio choices of banks. In practice, central banks can modify their interest rate rule to react to changes in the cascade probability ($P_v$). In other terms, central banks might be worried that low interest rates generate instability on financial market. This model provides a simple measure for risk that can be added used to augment a standard Taylor rule.

More formally, we can modify the interest rate rule to:

$$R^n_t = (1 - \rho) \left[ \bar{R}^n_t + \psi_\pi \pi_t + \psi_y (Y_t - Y^{ss}) + k_{pv} (P_{v,t} - P_v^{ss}) \right] + \rho R^n_{t-1} + \epsilon_t R$$

(51)

where $P_v^{ss}$ is the steady state for the contagion probability. In this case the central bank reacts tightens ($pv > 0$) when the risk of contagion between banks increases. As the interest rate rises, the volume of trade on the interbank market should decline, leading to a safer system.

We simulate the model under different parametrization for $k_{pv}$. The main question to answer is weather any of these policies is able to reduce risk on the interbank market and what are (if any) the costs associated to them. We report the changes in variances and the absolute change of output and inflation with respect to the baseline. As preferences of

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87 Notice that $P_v$ and $\| n \|$ commove.

88 See Caruana (2016).
regulators and central bankers can vary significantly between output or inflation, we take an agnostic approach and compare the two. 89

It is well known that ad hoc simple rules are highly dependent on the shock under consideration. Therefore, we decide to present each shock separately.

Results, for a subset of parameters, are reported in table 4.1.90

Table 4.1: Variance of output ($Y$), inflation ($\pi$) and cumulative change in the contagion probability ($P_v$) relative to the baseline ($k_pv = 0$), using the monetary policy rule (equation (51)).

<table>
<thead>
<tr>
<th>$k_pv$</th>
<th>Int. Rate</th>
<th>TFP</th>
<th>Inflation</th>
<th>Gov. Spending</th>
<th>Preference</th>
<th>Cap. Quality</th>
<th>Banks’ Net Worth</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>2.34</td>
<td>-0.07</td>
<td>0.15</td>
<td>0.00</td>
<td>0.04</td>
<td>-1.33</td>
<td>-0.11</td>
</tr>
<tr>
<td>0.10</td>
<td>2.91</td>
<td>-0.08</td>
<td>0.18</td>
<td>-0.01</td>
<td>0.04</td>
<td>-1.68</td>
<td>-0.20</td>
</tr>
<tr>
<td>0.15</td>
<td>3.18</td>
<td>-0.08</td>
<td>0.20</td>
<td>-0.01</td>
<td>0.04</td>
<td>-1.87</td>
<td>-0.29</td>
</tr>
<tr>
<td>0.20</td>
<td>3.34</td>
<td>-0.08</td>
<td>0.21</td>
<td>-0.01</td>
<td>0.04</td>
<td>-2.02</td>
<td>-0.41</td>
</tr>
<tr>
<td>0.25</td>
<td>3.45</td>
<td>-0.07</td>
<td>0.22</td>
<td>-0.01</td>
<td>0.03</td>
<td>-2.17</td>
<td>-0.55</td>
</tr>
</tbody>
</table>

Ratio between the variances of $Y$ and $\pi$ under a specific rule and the baseline

<table>
<thead>
<tr>
<th>$Y$</th>
<th>$\pi$</th>
<th>$Y$</th>
<th>$\pi$</th>
<th>$Y$</th>
<th>$\pi$</th>
<th>$Y$</th>
<th>$\pi$</th>
<th>$Y$</th>
<th>$\pi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>0.11</td>
<td>0.06</td>
<td>0.61</td>
<td>1.08</td>
<td>0.11</td>
<td>0.06</td>
<td>1.02</td>
<td>0.74</td>
<td>1.47</td>
</tr>
<tr>
<td>0.10</td>
<td>0.05</td>
<td>0.18</td>
<td>0.64</td>
<td>1.02</td>
<td>0.05</td>
<td>0.18</td>
<td>1.04</td>
<td>0.91</td>
<td>1.80</td>
</tr>
<tr>
<td>0.15</td>
<td>0.04</td>
<td>0.36</td>
<td>0.78</td>
<td>1.01</td>
<td>0.04</td>
<td>0.36</td>
<td>1.07</td>
<td>1.67</td>
<td>2.18</td>
</tr>
<tr>
<td>0.20</td>
<td>0.04</td>
<td>0.60</td>
<td>1.04</td>
<td>1.18</td>
<td>0.04</td>
<td>0.60</td>
<td>1.12</td>
<td>3.49</td>
<td>2.69</td>
</tr>
<tr>
<td>0.25</td>
<td>0.06</td>
<td>1.00</td>
<td>1.51</td>
<td>1.76</td>
<td>0.06</td>
<td>1.00</td>
<td>1.18</td>
<td>7.61</td>
<td>3.49</td>
</tr>
</tbody>
</table>

Monetary policy alone is able to reduce the risk of contagion only for 4 shocks (TFP, gov. spending, capital quality and banks’ net worth) but at the cost of a significant increase in the volatility of inflation and, in most of the cases, also of output. Notice that in the case of government spending and preferences the gain (loss) in terms of financial stability is negligible, but costs for real economy are sizable. It is, therefore, not sufficient as macroprudential tool. Monetary policy alone, therefore, seems not entirely adequate as a macroprudential tool. The cost for a safer financial system are generally large in terms of volatility and absolute losses. However, these simulations suggest that a weak response to the probability of financial contagion can reduce volatility for real shocks. At the contrary, after financial shocks the central bank should not tighten more.

89 Results are extensively reported in Online Appendix B.
90 Extensive results are reported in Online Appendix B.2.
It is possible, however, to draw at least three conclusions from this experiment: i) there is a trade-off between financial stability and economic stability when the central bank uses the policy rate to achieve a macroprudential objective, ii) the nature of the shock is crucial to achieve the desired objective, iii) the optimal strength of the response is different depending on the shock, therefore central bankers should be cautious exploiting this channel, iv) a mild reaction to financial variables can improve economic stability after real shocks, v) monetary policy alone seems not able to fully achieve a macroprudential objective.

As central banks face a complex and highly non-linear system, it is necessary to know precisely the nature of the shock that has hit the economy before implementing any policy action. An intervention, following what proposed in Appendix A, targeting banks in distress (or potentially in distress) seems more adequate in this context.

5 Conclusion

This paper delivers three main results. At first, it provides a rigorous way to incorporate a network model into a standard general equilibrium framework. This procedure can be adapted to any model and translated to other sectors of the economy where diffusion processes and cascades dynamics matter (i.e. technological diffusion, information diffusion...). Our procedure can provide the backbone necessary for this type of analysis.

As we have pointed out through the model, the characteristics of the starting node are not sufficient to understand the entire process. Its position in the network and the position of its first and second neighbors matter much more. On the interbank market there is a trade-off between the overall connectivity of the network and the stability of the system. In addition, for reasonable values of the parameters, we have seen how the system can get dangerously close to a phase transition. This model analyzes the reasons behind these phenomena and pinpoints their causes; the main result is that policy makers should try to target, in case of
need, the most connected institutions.91

With this model it is possible to analyze how contagion (in terms of likelihood and severity) evolves along the business cycle, integrating a complex network structure into a DSGE framework. The main objective is to identify how the network structure evolves along the business cycle. Simulations shows how systemic risk is influenced by financial and real shocks (with the latters having a smaller impact) and, in particular, we identify a “divine coincidence” for government spending shocks, that boost output at the price of negligible changes in the stability of the system. Central banks, on the contrary, face a trade-off when implementing monetary policy. Easing policies have a positive effect on output but only at the price of a greater instability on financial markets. This is due to the effect on banks’ balance sheets of easing surprises. Additionally we show that monetary policy alone is not able to achieve financial stability without costs in terms of growth. However a moderate response to financial variable can improve stability after real shocks. Finally, as the effects of a stability-targeting rule are highly shock-specific, it appears to be difficult to implement them in practice. This model, therefore, suggests that a good understanding of the shocks driving the observed fluctuations in the economic cycle is needed to effectively improve financial stability through the interest rate.

The overall result is that the network structure is deeply affected by other economic variables. In this complex framework there is the possibility that a marginal episode leads to consequence of systemic relevance. Policy makers, on one hand, can work on new financial regulations to affect the incentives bankers face. This would translate into a change of the parameters governing the net work (i.e. a change of $\tilde{\gamma}$ or reducing the vulnerability of each bank trough $V$). Even small improvements in that direction would have sizeable consequences.

Future extensions of this work may be the integration into an open economy model and the estimation of the model itself. In particular this last exercise could lead to the identification of the key network parameter ($\tilde{\gamma}$) without the need to access to restricted data.

91See Appendix A.5.
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Gambacorta, L. & Signoretti, F. M. (2013), Should monetary policy lean against the wind? an analysis based on a dsge model with banking, Temi di Discussione 921, Bank of Italy.


Appendix A  Network Construction

Appendix A.1  Construction of $H_0(x)$ and $H_1(x)$

To define the average cluster size we follow the procedure developed by Baumann & Stiller (2005). Assume that:

- the network contains no cycles, that is asymptotically true for large networks\(^{92}\)
- for any edge between two nodes, i.e. $a$ and $b$, the degree of $b$ does not depend on $a$’s neighbors or on the degree of $a$

Consider a general moment generating function $D_0(x)$ and $k$, the degree distribution, as a random variable. The sum of $m$ realization of a random variable $X$ is nothing else than:

\[ S = X_1 + X_2 + \ldots + X_m \]

in terms of a generating function, the generating function of $S$ is:

\[ D_S(x) = D_{X_1}(x) D_{X_2}(x) \ldots D_{X_m}(x) \]

summing over the degree distribution of $m$ randomly chosen variables (nodes in case of a graph), we have:

\[ [D_0(x)]^m = \left[ \sum_k p_k x^k \right]^m = \left[ \sum_k p_k x^k \right] \left[ \sum_j p_j x^j \right] = \sum_{jk} p_j p_k x^{j+k} \tag{52} \]

the coefficient of the power of $x^n$ is the sum of the products $p_j p_k$ such that $j + k = n$ and gives the probability that the sum of the degrees of the two vertices is $n$.\(^{93}\) Therefore, to

\(^{92}\)See Newman et al. (2001).

\(^{93}\)Intuitively, the sum over $p_j p_k$ gives the probability that any combination of two random extractions
compute the sum of the distribution of $n$ randomly chosen vertices, extracted by a generating function $S$ it is necessary to compute $[D_0(x)]^m$. Define the average degree of second neighbors of a vertex $\lambda$ as the sum over the neighbors of the $n$ first neighbors of $\lambda$, so as $[D_0(x)]^m$. Using equation (52), we can write $D_1(x)$, the average degree of the first neighbors of $\lambda$, as:

$$D_1(x) = \sum_k kp_k x^{k-1} \left[ \sum_j j p_j \right] = D_0 \left( D_1(x) \right)$$ (53)

Consider now the probability distribution $H_1(x)$ of the cluster size of a neighbor of a randomly chosen initial vertex $\lambda$. This is the probability that a cluster attach to one neighbor of a vertex $a$ has exactly size $s$. If $q_k$ is the probability of one of this neighbor to have $k$ edges in addition to the one connected to the starting node, $H_1(x)$ must satisfy:

$$H_1(x) = x q_0 + x q_1 H_1(x) + x q_2 [H_1(x)]^2 + \ldots$$

in other words, the distribution of the cluster size attached to a neighbor of an initial vertex is given by the average number of connection of the neighbor times the cluster connected to each of them. Using again equation (52) and realizing that $q_k$ is just the coefficient on $x^k$ of $D_1(x)$, we can write this distribution as:

$$H_1(x) = x D_1 \left( H_1(x) \right)$$ (54)

Define now $H_0(x)$ as the size of the whole component, so the size of the component constituted by $\lambda$, its first neighbors and all the components (described by $H_1(x)$) attached to them. It is clear that this component is nothing else than the summation of the components attached to all neighbors of $\lambda$. Therefore, using the previous reasoning, it is:

$$H_0(x) = \sum_k H_1^k$$

gives exactly $n$. 

52
Consider now the diffusion model. $H_0(x)$ represents the average vulnerable cluster size attached to an randomly chosen vulnerable vertex $\lambda$ and $H_1(x)$ the average cluster size for a neighbor of $\lambda$. In this case, however, $H_1(x)$ must satisfy:

$$H_1(x) = x \{ P_0^1 q_0 + [1 - P_0^0] \} + x \{ P_1^1 q_1 H_1(x) + [1 - P_1^1] \} + x \{ P_2^2 q_2 [H_1(x)]^2 + [1 - P_2^0] \} + \ldots$$  \hspace{1cm} (56)

with $P_k^v$ the probability of a vertex with $k$ edges being vulnerable. The previous equation states that the vulnerable cluster attached to a first neighbor of $\lambda$ is proportional to the probability of this neighbor to be vulnerable given its degree distribution ($P_k^v q_k$). In case the neighbor is not vulnerable, than the vulnerable cluster attached to it is just $\lambda$. Using the properties seen so far, it is easy to write equation (56) as:

$$H_1(x) = x G_1 (H_1(x)) + \sum_k [1 - P_k^v]$$  \hspace{1cm} (57)

$P_k^v$ is equation (32) and therefore $\sum_k [1 - P_k^v]$ is the probability of a vertex to be non vulnerable. In this way we have recovered equation (36). Using the same argument of equation (55) we can get to equation (37).

**Appendix A.2 Large networks contain no cycles**

We follow here Gross & Yellen (2003). Define $x$ the number of cycles in a random graph $V$ with $n$ vertices and $p$ the edge probability. Given $k$ vertices in the cycle, there are $\binom{n}{k} \frac{(k-1)!}{2}$ cycles of length $k$. The expected number of cycles is:
\[ E(x) = \sum_{k=3}^{n} \binom{n}{k} \frac{(k-1)!}{2} p^k \leq \sum_{k=3}^{n} (np)^3 = (np)^3 \frac{1 - (np)^{n-2}}{1 - np} \] (58)

as \( \lim_{n \to \infty} np = 0 \), a \( p \) is asymptotically less than \( 1/n \) thus the graph has no cycles.

Appendix A.3  Computing the average cluster size

Recall equations (36) and (37). Taking the first derivative of \( H_1(x) \) for its argument one gets:

\[ H_1'(x) = G_1(H_1(x)) + xG_1'(H_1(x)) H_1'(x) \]

\[ H_1'(x) = \frac{G_1(H_1(x))}{1 - xG_1'(H_1(x))} \] (59)

with a similar process it is possible to define \( H_0'(x) \) as:

\[ H_0'(x) = G_0(H_1(x)) + xG_0'(H_1(x)) H_1'(x) \]

plugging it into equation (59):

\[ H_0'(x) = G_0(H_1(x)) + xG_0'(H_1(x)) \frac{G_1(H_1(x))}{1 - xG_1'(H_1(x))} \] (60)

recall now equation (33) and derive it for its argument to get \( G_1'(x) \). The previous equation becomes:

\[ H_0'(x) = G_0(H_1(x)) + \frac{x [G_0'(H_1(x))]^2}{z - G_0'(H_1(x))} \]
the average cluster size is given by \( H'_0(x) \mid x = 1 \), therefore for the previous equation:

\[
H'_0(x) = \|n\| = P_v + \frac{z^2_v}{(z - G''_0(1))}
\] (61)

since \( H_1(x) \) is a well behaving moment generating function with \( H_1(1) = 1 \).

### Appendix A.4 Definition of \( \phi \)

Define \( \phi \) as the fraction of neighbors of a bank that can default without leading to the default of the bank itself.

\( i = \frac{\text{Int}_k}{k} \) is the share of funds landed to each counterparty on the interbank market. As banks are all equal, each bank is indifferent between any counterparty. In order to reduce risk, therefore, it is optimal to differentiate.

We can further compute \( F \), the maximum number of clients that may default without threatening the bank itself:

\[
F = \frac{V}{i}
\]

Recall the definition of \( \phi \):

\[
\phi = \frac{F}{k}
\]

use the definition of \( F \) to get:

\[
\phi = \frac{V}{i} \frac{1}{k}
\]

using the definition of \( i \) we get:
Appendix A.5 Network analysis

To analyze how a change in the parameters of the network structure affects the cascade equations, we abstract from the DSGE model (therefore we will discuss the properties of cascade size and probability instead of speaking explicitly of financial contagion) and evaluate the core set of equations (32)-(38) describing the network structure for different values of their parameters: $\tilde{\gamma}$ and the average network degree $z$. With these results we can verify how cascade probabilities and sizes change if the network becomes more or less connected.

Equation (32) describes the probability that a defaulting bank triggers a cascade. Its main parameter is ($\tilde{\gamma}$) that defines how concentrate is the network. Low values of $\tilde{\gamma}$ mean that the share of agents in the network with high degree is relatively larger, and vice versa. For the properties of power law distributions, values of $\tilde{\gamma}$ used in this simulation fall in the interval [2, 5]. However, from the empirical results on real word and banking networks, we know that reasonable values for $\tilde{\gamma}$ lie in the interval (2, 3).

The straightforward interpretation of the simulation is that the more a network is sparse, the more is likely that an agent belongs to a cluster. In sparse networks, in facts, agents are divided between many small cluster. If an agent in the group defaults it is more likely that his default affects at least another member of the cluster. However, Figure A.I should not be misleading. Equation (32) describes the probability that a randomly chosen agent belongs to a vulnerable cluster, however it tells nothing about the cluster size.

With more sparse networks clusters are smaller. Therefore the correct interpretation of Figure A.I is the following: as the network becomes more sparse, there is an higher probability that a randomly chosen vertex belongs to a cluster, however the dimension of the

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95 See for example Boss et al. (2004a), Boss et al. (2004b), Caldarelli et al. (2006) and Soramaki (2007).
cluster decreases with $\tilde{\gamma}$. This means that as the probability of a cascade increases with $\tilde{\gamma}$ the consequences of contagion become are smaller, as nodes are more isolated and an initial seed can infect fewer neighbors. To verify this intuition, we set up a simulation exercise evaluating equation (38) at different values of $\tilde{\gamma}$ and $z$. Before moving on, it is probably worth to examine the cascade condition more in details. It defines the size of contagion (i.e. the number of banks that are part of it) if a chain of defaults start. It is influenced by all network variables: $P_v$, $z_v$ (the average degree of vulnerable vertices) and $z$ (the average degree of the whole network). These variables are combined by equations (37) and (36) to define $\| n \|$. Their effects push in opposite directions as, for example, equation (33) clearly shows how $z_v$ decreases as $\tilde{\gamma}$ increases. The higher is $\tilde{\gamma}$ the fewer neighbors each vulnerable vertex has. On the contrary, $\| n \|$ depends positively on $G'_0(x)$. This term has not a clear economic interpretation, but captures the second moment of the distribution of vulnerable vertices.\footnote{See Newman et al. (2001) and Watts (2002).} Notice that it depends positively on $\tilde{\gamma}$.

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure_A.I}
\caption{Probability of starting a cascade after an initial default, from equation (32).}
\end{figure}
Finally $\| n \|$ depends negatively on the whole network average connectivity $z$. Intuitively, the larger is $z$ the more agents are interconnected; as connectivity rises each agent gets a smaller share (negligible in the limit) of default costs from the initial seed.\footnote{A similar argument is developed in Acemoglu et al. (2015).}

Notice that $z-G'_0(x)$ defines the so called “phase transition equation”.\footnote{Equation (39).} The network literature on this topic is vast and, sometimes, enters the field of chaos and fractal mathematics.\footnote{See Stanley (1971), Stauffer & Ahrony (1991), Callaway et al. (2000), Watts (2002), Falconer (2003), Alentorn et al. (2007), Falconer (2003), Mandelbrot (2004), Krantz (2005), Alentorn et al. (2007).} As can clearly be seen, when $z = G'_0(x)$ equation (39) has a singularity as $\| n \| \to \infty$. If the phase transition condition is met, the initial seed grows up to the point that it covers the entire graph, independently by its initial size. In the terms of the interbank network that means that the default of a single institution leads to the collapse of the entire market structure. The conditions for a phase transition root deep into the network structure, and are not easily pinned down. What is clear, however, is that it occurs only at specific combinations of variables. Additionally, as it is common in chaotic systems, the shift to phase transition is sudden as an apparently small change of the variables may suddenly trigger the phase transition condition. Despite its rarity, therefore, policy makers should be aware of its existence and develop policies that pushes the economy away from its border (\footnote{See Ott (2002).}defined by equation (39)).

Figure A.II shows $\| n \|$ evaluated at different values of $z$ and $\widetilde{\gamma}$. The spikes in the graph are connected to the regions close to a phase transition. As $z - G'_0(x) \to 0$, $\| n \| \to \infty$. The close the values of $z$ and $\widetilde{\gamma}$ are to the actual combination that generates a phase transition, the higher is the share of nodes participating in the cascade and the higher is the spike. In the limit, for that combination of parameters such that $z = G'_0 \| n \|$ is infinite.

Phase transitions are concentrated around low values of $\widetilde{\gamma}$. Figure A.II is also a good example of the consequence of chaotic behavior. With slight changes in the parameters’ values the system moves from almost a zero cascade size to a phase transition, the worst
Notes: simulations for different values of $z$ and $\tilde{\gamma}$ of the size of contagion. The spikes in the graphs are associated to those regions where $z - G'_0 (x) \to 0$. In those areas the system is getting closer to a phase transition (as $\| n \| \to \infty$).

possible outcome. Small changes can modify the state of the system drastically; this is the reason why economic theory should start to analyze more these type of models. Policy makers, in addition, should try to avoid the chaotic area of the system because, as this simulation shows, even a slight change in the true value of the parameters can lead to significant changes.

To build on what we discussed before, the cascade size decreases with $\tilde{\gamma}$ for the intuitive reason that on more sparse networks vulnerable seeds are less connected. The opposite reasoning can explain why it decreases with $z$. As the average connectivity grows, on one hand it is more likely that vulnerable nodes are connected to non vulnerable ones on the other each node receives a smaller share of the shock. These two effects reinforce each other.

A last word of warning on the specific type of network under consideration. As pointed

$^{101}$Notice that this means that the effect of $\tilde{\gamma}$ on $P_v$ prevails on that on $G'_0$. 

Figure A.II: Average cascade size, equation (38).
out before, there are sound and coherent empirical evidences that the interbank network is described by a power law distribution; however the choice of the distribution function is not neutral. Chaotic behavior is deeply routed into the power law function and probably for this reason it fits so well real world networks. A different moment generating function would have led to different results in terms of likelihood and severity of cascades. Therefore these results should not be extended to *any other type* of network, but are typical of the interbank market. The methodology we developed, on the contrary, can be easily used in other contexts (i.e. with different PDFs) to produce a similar analysis.

Consider now a public institution that wants to reduce the likelihood of cascades, their size and the probability of a phase transition. The standard tool to prevent diffusion on social network is immunization. Consider the cascade as an epidemic spreading through the economy, in this case between banks. The objective of public authorities is to ensure that, in case there is a breakthrough, only a minimum share of agents is infected. The rational for this is, obviously, the desire to prevent reductions in the loan supply and a fall in asset prices that vehicle to the real economy the consequences of a financial crisis. To achieve that objective public authorities need to provide fundings to vulnerable banks so that they can refund their debts and prevent the spreading of contagion. This type of actions are typically enforced with bailouts, bail-ins by stock holders or with emergency loans by central banks. There are two types of constraints to their implementation. On one hand, it might be complicate to gather sufficient funds for the program. At the same time, there are political economy issues in their implementation (i.e. political support is generally scarce leading to inefficient solutions).\footnote{See for example Grossman & Woll (2014) and Behn et al. (2015).}

This experiment wants to shed some light on the most efficient way to guarantee a reasonable level of immunization to the system, given a limited amount of funds available.

To be more formal, we assume that the public authority has funds to provide liquidity only to a limited fraction ($\varrho \in (0, 1)$) of the population of banks. In this case, the authority may decide to allocate them randomly (with a first-come first-served type of rule) or by...
targeting specific banks. The trade off is between providing liquidity to the most connected banks, that are fewer but of greater systemic importance, or to the least connected, that are more but with fewer neighbors. In the first case, the the top $\varrho$ percent of nodes get the treatment and in the second the bottom $\varrho$ percent. Evaluating equations (32) to (39), given the share of immunized population, allows to compare the effectiveness of each type of policy, given a specific parameter $\tilde{\gamma}$. Intuitively, the algorithm computes $P_v$ and $\| n \|$ removing the target share of nodes from the vulnerable cluster size. In this way, we reduce the vulnerable share of the population by the targeted group of banks.

Figure 3.1 shows the difference in $P_v$ given by targeting the $\varrho$ lower or top percentage of banks. If the value is positive, targeting larger banks is more convenient and the other way around.

For low shares of immunization and low $\tilde{\gamma}$, the simulation shows that it is generally preferable to target larger banks, however as $\tilde{\gamma}$ grows, it becomes more efficient to target the least connected institutions. This result is quite intuitive considering that for high $\tilde{\gamma}$ the average connectivity decreases. Therefore there are less well-connected banks and the lower $\varrho$ percentage of the population contains, proportionally, a larger share of the links between financial institutions.

In the next exercise we compare the cascade size $\| n \|$ under the two regimes: targeting the lower $\varrho$ percent against top $\varrho$ percent. Figure A.III shows the difference between the two. A positive value means that targeting the top $\varrho$ percentage is more convenient than the bottom $\varrho$ percentage.
Simulations show clearly how it is generally better to intervene on the higher part of the
distribution, with the gain of these policies that decrease as $\bar{\gamma}$ increases. In this context, as it has been largely discussed before, the network becomes less concentrated and, therefore, the two cases become closer. However, given reasonable percentage of the immunized population, the first policy is preferable. 103

\footnotetext{103It is, of course, very unrealistic to believe that public authorities have funds to immunize the top 40% of the banking sector.}
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Figure B.XXVII: Differences in inflation from the baseline ($k_{pv} = 0$) after preference and government spending shocks.
### Table B.I: Cumulative difference in the % change of output from the baseline ($k_{pv} = 0$) ×100

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Table B.II: Cumulative difference in the % change of inflation from the baseline ($k_{pv} = 0$) ×100.
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**Table B.III:** Cumulative difference in the % change of $P_v$ from the baseline ($k_{pw} = 0$)
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Table B.IV: Variance ratio for output ($Y$) and inflation ($\pi$) with respect to the baseline variance ($k_{pv} = 0$), using the monetary policy rule