This is how we do: how social norm and diachronic social identity shape decision making under risk.∗

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Abstract

The current study aims to investigate how the presence of social norms defines beliefs formation on future changes in identity, i.e. diachronic identity, and how those beliefs affect individual decisions under risk. The paper proposes a theoretical model in which individuals have preferences over their own attributes and over specific information structures. The individual preferences are motivated by the presence of social norms. The norms, while establishing the social acceptable attributes of an individual identity, also drive individuals’ preferences to information acquisition or avoidance. The model incorporates social norms as empirical expectations and provides a prior dependent theory that allows for prior-dependent information attitudes. Firstly, the model implies that decisions are mitigated by socially grounded behavioral and cognitive biases and secondly, that it can create an incentive to avoid information, even when the latter is useful, free, and independent of strategic considerations. These biases bring out individual trade-offs between internal consistency, or self-image, and social conformity. The two behavioral motivations are represented through an intra-personal model of choice under risk in which self-deception and memory manipulation mechanisms are used to overcome the individuals’ internal trade-off.

Keywords: Identity; Social and Gender Norms; Decision Making under Risk.

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1 Introduction

The standard assumption in economics is that decision makers (DMs) are endowed with perfect reflection. They are likely to embark on a process of conscious mental revision of their available alternatives, in order to characterize each choice and evaluate their consequences. To sustain the perfect reflection assumption, the theory of decision making postulates that firstly, individual preferences are exogenous and secondly, that information is always (weakly) valuable even when it is a bad news. Yet, the robustness of the assumption of perfect reflection depends on both the complexity of the decision problem and on the individual’s subjective beliefs about the value of information. Complex decision problems are those in which payoff relevant variables are defined endogenously and are entangled as part of the choice. On the same account, standard economics assumes that the value of information is just instrumental to make better decisions. However, a growing theoretical and experimental literature suggests that information has also an affective value and that may directly enter the agent’s utility function, a phenomenon known as "belief-based utility". This can create an incentive to avoid information, even when it is useful, free, and independent of strategic considerations. In such cases, the assumption of perfect reflection becomes difficult to justify. The difficulty stems from the relaxation of one or more conditions of exogeneity, rationality and self-knowledge.

To cast light on the process of relaxation of the assumption of perfect reflection, let us start with a simple scenario and complicate it little by little. In many situations real life is complex since choices bring with themselves aspirations, motivations, emotions, dispositions and beliefs that alter the characteristics of the alternatives as well as the nature of the decision problem. Imagine to choose between investing in education to get a career A or B. In a simple framework, the DM chooses A over B only if the pleasure or the profit received from A is greater than that received from B. It follows that she will choose to invest in A when both the possibilities are available. In this scenario she has a precise definition of the objects of his choice, career A over B. Suppose, now, that availability of the alternatives are uncertain, as the outcomes are, and the preferences over them are endogenous, e.g. explained through mental states (i.e. motivations and beliefs). Specifically, we assume that the individual has incomplete information about the outcomes of the alternatives and decides to undertake a conscious mental revision about which is the best, inferring, also, which one is the most socially acceptable. The object of her choice is therefore changed. It is no longer presented to her eyes on the basis of the intrinsic value of the alternatives but also on the value that society attributes to the alternatives. The nature of the decision problem is now changed since the DM’s structure of motivations is changed because it is informed by the heuristics "the most socially acceptable". The heuristic represents a short cut that DM chooses in order to solve the uncertainty.
The previous example shows how decision problems can easily be made more complex. It suffices to characterize the object of the choice from a multi-dimensional perspective. For the DM, her decision problem is no longer between the outcomes of each career path but between careers path together with their degree of social acceptance. Specifically, the career path finds its value in the social recognition as ideal career for a specific type of DM that belongs to a given community. The "socially acceptable" mental states (i.e. beliefs and motivations) are originated by the presence of norms, which, ultimately define the mental representation that DM owns of herself and her identity. Consequently, DM has to internalize the consequences of both action and motivations. A DM with imperfect reflection will not anticipate that her conformist motivation might play a role, whereas, a perfect reflective DM will know how to handle the endogeneity. Hence, given the objections to the assumption of bounded rationality, it seems reasonable to ask ourselves few more questions. Where does DM’s mental states come from? and what are the consequences for a cognitive theory of social identity, on how the DM forms judgment about identity and individual concepts? Considering again the "heuristics" example. It has two features that are worthy of further analysis: the mental states and the sampling of the information.

According to Elster [28, 29], mental states, as beliefs and motivations, define behavior. Beliefs and motivations help the DM to navigate in life, to justify the undertaken action and to experience emotions. If we want to order the states in a causal relation, motivations are causal determinants of the states and, then, of the belief formation. A complete analysis of the mental states should look at both costs and constraints of the motivated mental states. Costs of mental states are the false or inaccurate beliefs that even though they are pleasant they can induce suboptimal choices or emotional drawback. Constraints, in turn, are the information processing that can be biased and affected by overconfidence, or optimism, or be constrained by priors. While the earlier attempts of bounded rationality theory were based on the costs [1, 49], recent theory has focused on the investigation of the constraints. When motivated mental states enter the framework, the traditional exogeneity assumption no longer holds, nor the other assumptions of accurate knowledge of the preferences and of fixed inter-temporal preferences. The "heuristic" example supports an analysis of both costs and constraints. The choice of career A over B could be explained by a motivated mental state that favors a conformist behavior. This motivated belief drives the DM to strategically sample only the information that is considered important for the conformist extent and to disregard the information that does not match it. That is, the DM organizes the information structure to favor the priors that career A is better.

On the last account, the example can be taken to a higher level of sophistication that specifies a further particular mechanism. Let us suppose that beliefs about the future event

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1 Carrillo and Mariotti [21] consider also desire as mental states.
2 According to Elster there are three constraints: consistency, plausibility and imperfection.
of “being a professional A” will have impact on DM’s current utility through the arousing of positive feelings. This particular way of framing the effect of beliefs about future events over current feelings is called Anticipatory Utility [20] (henceforth AU). In the framework of AU, beliefs enter directly the utility function via emotional predisposition, highlighting the importance of information gathering and avoidance. This result is more interesting in light of Simon’s perspective of the role of information. In fact, in the general case of self-conflicting goals, or intra-personal conflict motives, as in Brocas and Carillo [17] or Bénabou and Tirole [9, 12], an individual may choose to avoid information even when it is freely available. Moreover, even if being informed will improve the DM’s current decision, the information could be completely or partially shared with her future incarnations affecting her future optimal choice, and creating, then, an intra-personal conflict. Ignorance would be a strategical behavior.

This paper is devoted to combine the information ordering, AU and intra-personal conflict models, and to apply them to social and economic problems such as female investment in education or female labor supply decisions. The motivations to model the investment in education by using this type of theoretical framework resides in the willingness of acknowledging a role of causal variables to social norms and social identity. In fact, all of these are important to the extent of either a theoretical analysis and a research policy aim. From the theory perspective, it enlarges the debate about the relations between mental states, information acquisitions and the persistence of social inefficient outcomes. On the policy side, the model aims to inform interventions aimed at easing the perpetuation of empirical expectations that limit the natural empowerment and freedom of the DM.

The rest of the paper is organized as follow. In section 2 a literature review accommodating all the above phenomena is presented. To follow, sections 3 and 4 introduce and solve the model, section 5 provides comments and possible extensions. Finally, section 6 offers the conclusions.

2 Literature Review

The paper contributes to the literature in identity economics. Following Akerlof and Kranton [2, 3], although we slightly move away from their core assumptions in three directions, the paper considers the social categories as variables that enter in the Decision Maker’s utility function.

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3 If career path A indicates choosing law school, the belief about the future that DM holds is "being a lawyer".

4 According to Simon [51], the capacity of the human mind of formulating and solving complex problems is small if compared to with the dimension of the problems humans are called to solve. In his theory, time dimension, knowledge and availability of alternatives are given an important role in the decision making process. A non reflective DM is simply acting upon bounded rationality mechanisms, that means, her decision process is a search process guided by aspiration levels.
On diachronic identity. First of all, our DM cares about her diachronic multidimensional identity. The diachronic identity refers to changes in the content and consequences of the identity over time, as opposed to the synchronic identity which refers to relative salience of different aspects of an individual at one point in time that depends on the presence of relevant cues. Let us consider an example of the synchronic identity in gender studies. The category of being a woman has different sub-categories like being a mother, a professional, a spouse, etc. In a specific point in time, being a mother and a professional can be in clash and one of the sub-categories could be more salient than the other, for that specific point in time the DM finds herself.

Diachronic identity theory [7] mostly focuses on how DM tracks continuity of her identities over time and on how the aim to reach this perceived continuity affects her choices and information set. Hence, the DM is not depicted just as a student, but also as future worker and as parent and she wants that the two dimension of her identity, i.e. her present and her future self, would be persistent across time. For example, in gender issues, the ideal of being a good woman will be valued both when the DM is a student and for the next years, hence her choices need to be consistent to maintain such continuity. As a consequence, the decisions that she takes now will impact not only on the single current dimension, but on her future ones, too. Consequently, the way the people determine the persistence of their own identity may affect how they think about the future, about their aspirations and the choice they make. So, understanding how people make judgments about the continuity of their identity could give us different reasons to comprehend why people do not act as often as they should and might suggest us new way to help people to act in accordance with their long-term interests. The way this mechanism is presented in the model occurs by using a particular functional form, called AU function [20, 41, 42, 43], that takes into account utility over beliefs and outcomes with particular intertemporal implications.

On descriptive norms. The matching mechanism between behavior and ideal identity is retained from Akerlof’s analytical framework. Henceforth, the matching mechanism is not determined by fixed preferences but is the outcome of a deliberative process. In fact the matching mechanism in this paper is not a generic endorsement of the prescriptions imposed by the ideal categories to which individuals decide to belong. Rather, the current work specifies the origin of those prescriptions. The origin lies in the existence of descriptive norms [14]. The descriptive norms regulates the mechanism of interaction between the constraints through the work that empirical expectations produce on the definition of “similar” and “attainable”. Those who belong to her ideal categories. In the model individual draws her aspirations from the lives, achievements, or ideals of those who exist in her cognitive window. Hence, the window is important in educational choice.

5In accordance also with [31, 46, 36, 37].
The definition of norms as empirical expectations helps to understand the problem of social identity and educational investment, for three reasons.

First, the presence of descriptive norms, that defines a given identity as ideal, reduce the incentive to experiment and search for information on the alternatives to the ideal identity. In other words, if the descriptive norm and the social context make specific traits of identity salient, then the preferences of individuals can be shaped by cues leading to a observation of inconsistent choice and behavior\(^6\), even in the absence of any explicit awareness of how such cues trigger and arrange the goal pursued. In this case is the individual will be trapped in self-fulfilling expectations situations.

Second, descriptive norms are helpful in playing the role of heuristics during a decision making process and in doing so, they characterize the decision making under risk as a prior dependent process. Namely, people dealing with investment decision problems, as with any decision under risk, appeal to empirical expectations to handle the imperfect information [33, 15] typical of this kind of problems. When information towards the future outcomes of an action is difficult to gather or imperfect, the norms are the best and less expensive way to extract that information. However, this information can be biased, or not true.

Third, another consequences of the presence of the norms on the information process is whether individuals treat information differently when it comes from a social context. That is to say, individuals might suffer from confirmation\(^7\) bias or conservative bias\(^8\) or persuasion bias.\(^9\) Namely they are constrained in their ability to process information making some cues more accessible than others in affecting their preferences. For example, let us consider again the phenomenon of “acting white”. Such phenomenon will allow the subject of a stereotype to notice stereotype-consistent information more likely than others. She would probably construct a sequence of information that gives validity to her beliefs, while discarding or forgetting those signals that do not. Accordingly, the observed choice may be consistent with several combinations of expectations and preferences. These combinations, once again, is the result of the social cognitive process\(^10\) to which everyone is subjected to.

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\(^6\)Many students have documented that when a social identity is made salient, choice and performance in a domain that is stereotyped is altered towards the direction predicted by the stereotyped, e.g. the “Acting White” phenomenon.

\(^7\)According to Evans [30] confirmation bias occurs because decision makers seek and interpret information in ways that support their original beliefs. Coherently, [38] address the presence of the biases in situations in which the salience of identity traits is socially imposed.

\(^8\)As Edwards[26] acknowledges conservatism leads individuals to update their beliefs according to the Bayes’s rule but in a smaller magnitude. [55, 54] proposes an empirical application of the bias to the educational choice.

\(^9\)According to Demarzo[24] persuasion bias can be viewed as a simple, bounded rational heuristic according to which individuals update, as Bayesian, except that they do not accurately account for which information they receive is new and which is repetition. This notion correspond with social psychology theories of political opinion formation.

\(^10\)A social cognitive process [23] is a process (a sequence of operations, such as reasoning, decision making, etc) upon social mental representation (e.g. social beliefs and goals).
On cognitive dissonance. On the latter account, the appeal to the use of descriptive norms as heuristic is motivated by the attempt to resolve what [32] calls cognitive dissonance. Dissonance arises when individuals are confronted with information that is inconsistent with their already existing beliefs, ideas or values. It, hence, produces the “unpleasant experience of being caught between two contradictory forces” (Iyengar [39], pag.97): information conflicting with beliefs. The resolution process of the conflict occurs in self-deceiving way[44]. Self-deception is the temptation of people in believing something that they want to be true. It is to say that individuals have false beliefs not because of lack of information. Rather, the self-deceiver is at some level aware of information that would lead her to form the correct belief, but some motivational bias leads her to hold the opposite. In this model empirical expectations and the desire to respect them represent the motivational bias.

Traditionally, self-deception has been modeled on interpersonal deception, yet the current paper uses an intra-personal approach. However, the traditional model of self-deception raises two paradoxes. One concerns the self-deceiver’s state of mind the so called ?static? paradox. How can a person simultaneously hold contradictory beliefs? The other one concerns the process or dynamics of self-deception the so-called dynamic or strategic paradox. How can a person intend to deceive herself without rendering her intentions ineffective[44]?

The answer to these questions and the strategies to solve the dissonance depends on the type of dissonance that the DM faces. According to [52] the dissonance is called nomothetic when it is generated by the presence of cultural and social customs, whereas it is called idiographic when it is related to individual-specific cognitions. The distinction matters when it comes to identifying the strategies that can be followed to minimize the discomfort caused by the dissonance. In principle, the choice is between three options: avoiding the dissonance by not performing the action leading to it; reducing the dissonance by changing the environment, i.e. by removing (some of) the circumstances that induce dissonance; solving the dissonance by changing one? s own opinions about the circumstances that cause it, i.e. by changing one’s subjective evaluation of these circumstances (i.e. the sour grape evaluation).

When the DM faces the idiographic type of dissonance, the latter two strategies are implemented. Several strategies have been largely found for sustaining a particular self-assessment beliefs: e.g selective recall of information regarding the fact that it possesses attribute that could be correlated with their achievements and personality self-report [50]; dismissing the importance of skills not possesses and emphasizing the value of traits one does. In the economic literature the approach to cognitive dissonance ([1], [48], [49]) studies the change of the DM’s beliefs ex-post to the actions taken. The DM tries to make a coherent mirroring between the result of the action (or the recollection of the result) and the initial beliefs.

In the case of nomothetic dissonance, that is the one discussed in this paper, the best strategy is the first. Because of the strategic paradox, the modification of one’s beliefs is not
useful in solving the cognitive dissonance; instead, it is necessary to intervene either on the
information set - through misconception, refutation or rejection - or through seeking supports
from the “relevant-others” who share the same beliefs[22]. Then, the DM will strategically
select the information received in order to reflect the initial beliefs. In the literature this self-
deception actions are modeled in three different ways. The first is to adjust the Bayesian
model of belief formation. The second is to solve an intra-personal signalling model, in
which the DM is represented as a series of temporal incarnations which manipulate the
beliefs from one period of time to another ([9, 11, 12], [41, 42] and [25]). The third approach
uses a self-signalling model adding psychological structure ([16], [34], [18]). The current work
implements the third approach enriched with a monotone information structure.

The formal structure of the model is therefore different from Akerlof and Kranton’ s. It
presents a dual-self model[34] within a monotonic decision problem [4]. The main structure
posits the decision problem as a game between a short- and a long-run self. The two selves
may incur in time inconsistent decisions. Because of that, they might want to manipulate
the informative structure they posses.

The work is closely related to and borrow from Kószegi[40] and Bénabou and Tirole[9, 12].
As in the model, in both Kószegi ignorance is costly in terms of inefficient actions and the
DM is informative averse. Bénabou and Tirole face the two topics into different papers. In
Bénabou and Tirole[9], the imperfect recall is the tool through which the DM can keep a
high self-esteem of herself, however the scenario with a second stage of action strategy is
not considered and the cases of analysis consider only the situation in which a low signal
is subjected to a recall strategy. While in Bénabou and Tirole[12] they show that a DM
with anticipatory utility function and with preferences for the future outcome can be lead
to make inefficient decisions. The current paper merges the three works obtaining the result
that ignorance is costly in terms of inefficient actions whenever the DM is information-
averse. However, the inefficiencies can be offset by the gains in the DM’s self-image (or self-
esteeem). The aversion for the information is analyzed in both the cases of receiving a good
or bad signals, because both matter for the consequence of inefficient actions and self-esteem
maintenance. The imperfect recall strategy introduces a thicker strategic input to the second
result of Kószegi[40] in which he introduces the possibility of partial information acquisition.

3 The Model

Consider a decision problem in which an intra-personal game is played by a sequence of
temporal incarnations of the DM: a myopic short-run self and a long-run self. Both the
incarnations have preferences over their identity \( \theta \in \Theta \), where \( \Theta \) is a non-empty, compact
subset of \( \mathbb{R} \). The identity is validated according to the group’s expectations in terms of
membership to one of the two social groups in which the society is divided in this current
model. Each social group has its own distribution of type \( H \) or \( L \). The DM’s utility is a function of economic outcomes and anticipatory emotions towards the outcomes, either positive or negative, which depend on the rational beliefs formed about the exact same outcome. The total utility is built over three periods of time according to which at the beginning of period \( t \) DM is endowed with an initial endowment, or amount of human capital, \( S_t \). At this stage the DM receives a signal, \( \sigma_t \in \Sigma = \{H; L\} \), informative about her identity traits, \( \theta \). Denote the probability of observing a \( H \) signal by \( p \in (0; 1) \). The signal is informative in the sense that it can either broaden or restrict the set of available decisions to the DM to fulfill her future plan if it does not clash with the expectations of the group. The evaluation of available opportunities is grounded on the existence of two kinds of expectations that support the presence of social norms associated to the social group to which DM belongs or aspire to take part of.

**Definition 1.** Empirical expectations state what the DM “expects the other in her position will do” in a particular situation; Normative expectations that define what “they think other people believe ought to be done” (Bicchieri[14]).

The presence of the two kinds of expectation transform the DM’s decision in a monotone decision problems, whereby the posterior beliefs induced by the signal can be ordered so that specific actions are chosen in response to their corresponding signal realizations sustained by the norms. This is because

**Definition 2.** A social norms is a code of conduct shared by the society and enforced through internal sanctions, like guilt, shame, loss of self-esteem.

The presence of strong norms dictating a certain type of behavior is a possible source of cognitive dissonance for the individual whenever the realization of the signal does not accord with that norm. Accordingly, the DM, who has internalized the norm, needs to solve the dissonance caused by the signal. Since, in our case, it is a nomothetic dissonance the best option is, normally, to avoid the source of the dissonance that is, to choose the interpretation and recall strategy of the signal, \( \hat{\sigma} \), with probability \( \lambda \in [0, 1] \). At \( t + 1 \) DM updates her beliefs about her identity, \( \theta^* \), according to the Bayes rule and chooses to invest, \( e \in \{1, 0\} \), and experiences emotional utility, \( \hat{\theta}(e) \), from the outcome of the action. In period \( t + 1 \) DM faces a choice regarding the investment that individually deemed socially worthy. The action is the main variable that allows the DM to pursue her plan over the future identity. The plan will have as return, both the material outcome and the self-esteem outcome, the latter comes from the DM’s perception of reaching out her plan. The plan can go toward two directions: maintaining the status quo of her neighborhood, with probability \( 1 - x_i \) or change the status and undertaking a new path with probability \( x_i \). The individual has the possibility, in theory, to choose to not replicate the identity trait of her reference group and
to go for her own identity according to her aspiration level and autonomy. The outcome of the action has physical effect only on the last period, \( t + 2 \), and it is the final amount of human capital gained after the investment choice, \( \theta_i S_{t+2} \). The timing is pictured in figure 1, in 6.

The DM’s incarnations are distributed across the three period in the way that the long-run acts at \( t \) and the short-run at time \( t + 1 \). An important assumption to the model is that it is a non-cooperative game between the long-run self and the short-run incarnation, namely individual cannot precommit to her future decisions. Instead, the long-self chooses a self-control action causing current cost that influences the utility function of the myopic self, producing externalities on her future welfare. The magnitude of externality depends on the individual’s identity trait whose value is subject to imperfect information.

**Time \( t \)** The expected value of the identity conditional on each possible realization of such signals is given by:

\[
\theta_t = \begin{cases} 
\theta_H & \text{with probability } p, \\
\theta_L & \text{with probability } (1 - p) 
\end{cases}
\]  

with \( \theta_H > \theta_L \) and \( \bar{\theta} \equiv p\theta_H + (1 - p)\theta_L \).

**Assumption 1.** The DM has a prior distribution \( f(\theta) \) on \( \Theta \) and she observes a signal \( \sigma \) with likelihoods \( l(\sigma|\theta) \). If \( l(\sigma|\theta) \) satisfies the MLRP, then the posterior belief \( f(\theta|\sigma) \) follows the MLRP. Hence the high signal first-order stochastically dominates the low signal:

\[
f(\theta_t|\sigma_t = H) \leq f(\theta_t|\sigma_t = L) \text{ for all } \theta \in \Theta
\]  

Thus, the stochastic order of the informational structure tells us that for any values of the identity parameter (\( \theta_t \)), higher values of the signal make the higher parameter value relatively more likely. In the absence of empirical expectations and social norms, better information allows for a more accurate match between beliefs and actions. If beliefs are formed through the group’s expectations, then, their match with the actions could be driven by a further specification of the signal structure: there are going to be some signal realizations more preferred than others according to the membership group frequency. For example, a group with a majority of \( L \) type, namely with a biased prior that shows that \( L \) types are more frequent in that group, could tend to welcome more other \( L \) types rather than other, and in terms of the stochastic orderings, the information structure considered more informative will be the one that on average will produce with higher probability a \( L \) signal than any other informative structure. Then, in case in which the signal realization does not respect the ordering, the DM decides how to recall the signal.

Accordingly and following Bénabou and Tirole [9, 11] and Rabin [49], I admit that the
individual can decide how to record the signal at time $t$; whether to interpret it realistically $\hat{\sigma} = \sigma$ or to rationalize it $\hat{\sigma} \neq \sigma$ with $\hat{\sigma} \in \hat{\Omega} = \{H, L, \emptyset\}$. Rationalization means that the long-run self at time $t$ may either communicate the signal truthfully to short-run self at stage $t + 1$, or to suppress it. The DM’s decision at time $t$ is to make a recall strategy that is the probability, $\lambda \in [0, 1]$, of transmitting to self-$t + 1$ truthfully the signal, $i \in \{H, L\}$:

$$\lambda = Pr[\hat{\sigma} = i|\sigma = i]$$

The DM will observe the signal if the gain of making an informed decision will exceed the cost of memory manipulation. $\lambda$ will be chosen according to how much reliable or informative the signal is, namely, if it is pivotal in the decision.

The recall strategy has its own justification based on DM’s preferences over $\theta$. If the signal confirms her prior on her initial identity trait ($\theta$), she will not discard the original signal ($\lambda = 1$) and then she will make the investment choice correlated with her characteristics. The incentive is also justified by the fact that the DM will do another evaluation that will trigger her choice. She may envisage what are the consequences of sending specific signals, whether it comes from a recollection or an action, to her circle.

**Examples:** A girl, in a social circle of non-educated women, that decides to continue her education, will signal a message diverging from the circle identity; the heir of a wealthy faculty that decides not to pursue the family profession may find some hurdle in gaining the acceptance of her peers; a woman facing a tradeoff between work and family, when all the people around her seems to have chosen one of the two sphere, will hinder her choice on labor supply.

Then in light of this mechanism we can interpret the probability $\lambda \in [0, 1]$ as expression of the DM’s autonomy in its agentic feature.\(^{11}\)

Hence, at time $t$ the long-run chooses the strategy to recall the signal. Endogenizing the imperfect recall allows the DM to reduce the externalities from one self to another. Following Festinger [32], reducing dissonance is not costless, whether it is implemented via attitude change or via memory manipulation. It requires re-valuing past behavior or information and breaking cognitive habits. Because DM has preferences over her identity, and she may want to solve the cognitive dissonance, she has incentive to manipulate her recollection by exerting an effort. It seems necessary, here, to stress the fact that recall strategies in awareness-management context does not lead the individual to fool herself. Yet, she is aware that what she may have forgotten are not random events but rather the result of an incentivized inference. The memory effort is associated to a cost\(^{12}\) function, $M(\lambda)$, that will impose further restriction to the choice and the agency of the individual\(^{13}\).

\(^{11}\)Following Bavetta and Guala [8], Bandura [5, 6].

\(^{12}\)The range of costs considered here goes from time, to real resources.

\(^{13}\)Following Bénabou and Tirole [10].
Assumption 2. The cost is strictly increasing, convex, twice-continuously differentiable, and if $\lambda = 0$, then $M(0) = 0$ and $M'(0) > 0$.

**Time $t+1$** The short-run self updates her beliefs and chooses the action, $e$, given the signal recalled. The DM is rational and then before updating she evaluate the precision of the signal received by her previous incarnation. She takes into account the possibility that the long-run self at time $t$ may have manipulated the true signal. Accordingly, she needs to estimates the probability that the signal is accurate. If $\lambda$ expresses the probability that Self-$t$ has truly transmitted the signal, $\lambda^*$ represents Self-$t+1$’s belief concerning the Self-$t$’s communication strategy. Yet, the accuracy of the signal is given by:

$$r^*(\lambda) \equiv Pr[\sigma = H|\hat{\sigma} = \emptyset; \lambda] = \frac{p(1 - \lambda_H)}{p(1 - \lambda_H) + (1 - p)(1 - \lambda_L)} \in [p, 1]$$

implying that if Self-$t+1$ receives $\hat{\sigma} = \emptyset$, by using Bayes’ rule, she will assign the probability $(1 - p)(1 - \lambda_L)$ that $\sigma = L$ was observed and the probability $p(1 - \lambda_H)$ that $\sigma = H$ occurred. The incentive constraints for Self-$t+1$ do not change at this stage. However, the degree of self-assessment needs to take into account the accuracy of the recall. Hence the DM’s revised beliefs are:

$$\theta_i^* = \theta(r^*) \equiv r^* \theta_H + (1 - r^*) \theta_L$$

After the updating, the DM will choose the investment actions taking into account both the future physical outcome and the anticipatory emotion of the outcome realized that depends on the action taken.

Individual will implement the action strategy: $e(\theta_i) : \Theta \rightarrow A$, where $A$ is a non-empty, compact and convex set in $\mathbb{R}$. The investment in education will impact the DM’s human capital. Let us assume that each individual at time (t) is endowed with an initial level of human capital, $S_t > 0$. $S_t > 0$ is the initial endowment and is interpreted in such way to permit complementarities between the family’s human capital and the quality of community the family lives in. In this way, according to the initial level we give a representation of initial stratification, that may determine the future one. In this scenario population is divided in two groups according to the income and social status: H-types belongs to the portion of population characterized by high income, good social connections and communities; while L-types represent the complement portion of the population. The human capital accumulates according to the following equation:

$$S_{t+i} = S_{t+i-1} + \epsilon_{t+i-1} r$$
The educational investment will determine the future affiliation of the DM. The sense of affiliation is the motivation to the action. The DM’s probability to invest is of the form

\[ x_i = Pr(e = 1|\theta_i) \]

it is the conditional probability law of the action \( e \) for each possible value of \( \theta \).

Since the decision of investment takes into account the persistence of identity, we need to express how the future identity enters the DM’s decision problem. The anticipatory emotion at time \( t \) of the future identity is derived rationally by calculating the posterior identity the DM gains by the means of the investment choice.

The posterior identity evaluated to the investment decision is

\[ \hat{\theta} = \hat{\theta}(e) \in [\theta_H; \theta_L] \]

where

\[ \hat{\theta} = \begin{cases} \theta(0) & \text{then } \hat{q}\theta_H + (1 - \hat{q})\theta_L \\ \theta(1) & \text{then } \hat{p}\theta_H + (1 - \hat{p})\theta_L \end{cases} \]

(5)

After the action the DM experiences a shift in beliefs. By assuming that she is sophisticated, in foreseeing this shift, by applying the Bayesian calculation.

\[ \hat{p} = Pr(\theta_H|e = 1) = \frac{Pr(e = 1|\theta_H^*)Pr(\theta_H^*)}{Pr(e = 1)} \]

(6)

and

\[ \hat{q} = Pr(\theta_H|e = 0) = \frac{Pr(e = 0|\theta_H^*)Pr(\theta_H^*)}{Pr(e = 1)} \]

(7)

Where the prior beliefs at time \( t + 1 \) is represented by \( Pr(\theta_H^*) = r^* \), the probability of being \( H \) type given the recall. Moreover, recalling the probability of investing given the type, \( x_i = Pr(e = 1|\theta_i) \) then \( Pr(e = 1, 0) \) is defined as

\[ Pr(e = 1) = Pr(e = 1|\theta_H^*)Pr(\theta_H^*) + Pr(e = 1|\theta_L^*)Pr(\theta_L^*) \]

and

\[ Pr(e = 0) = Pr(e = 0|\theta_H^*)Pr(\theta_H^*) + Pr(e = 0|\theta_L^*)Pr(\theta_L^*) \]

Then the posterior probabilities in the two scenarios will be:

\[ \hat{p} = \frac{r^*x_H}{r^*x_H + (1 - r^*)x_L} \]

(8)

and

\[ \hat{q} = \frac{r^*(1 - x_H)}{r^*(1 - x_H) + (1 - r^*)(1 - x_L)} \]

(9)
Table 1, in 6, shows the probabilities of the two signals and the posteriors beliefs of the model.

Preferences The model extends for three periods, from $t$ to $t+2$. On the last period the DM will receive the outcomes of the action taken at time $t+1$. Consistently with Carrillo et al. [21] the instantaneous utility is represented by the cost function related to the action that will produce externalities in the future stages. Hence, the instantaneous utilities at time $t$ and $t+1$ are represented by the current effort the DM has to exercise to perform, respectively, the recall strategy, $M(\lambda)$ first, and the action after, $\Psi(e)$. The DM has perfect foresight, namely she is sophisticated in the sense that she applies Bayesian operators to foresee her dynamic inconsistency.

For what concern the cost function at time $t+1$, it is different whether DM decides to invest in the old path, $k(e)$, or in a new path, $c(e)$. In the first case the DM faces a continuation cost of pursuing the old track, the one her peers and family expect from her, while, in the second case the DM faces the cost of deviation from the marked path. The cost functions of each investment, $c(e)$ and $k(e)$, are assumed to be smooth, increasing, convex and known at time 0. The cost $c(e)$ is uniformly distributed over the intervals $[C_L; C_H]$ with:

$$
c(e) = eC_i = \begin{cases} 
0 & \text{if } e = 0 \\
1 & \text{if } e = 1 
\end{cases}
$$

Similarly the cost $k(e)$ is uniformly distributed over the intervals $[K_L; K_H]$ with:

$$
k(e) = eK_i = \begin{cases} 
0 & \text{if } e = 1 \\
1 & \text{if } e = 0 
\end{cases}
$$

I assume that investing in the old path is always efficient if its cost is sufficiently low for both type of DM. While investing in the new path supports both the participation constraint and the sorting condition, considering also that $C_H < C_L$. A compact expression of the two cost functions, considering the expectation of the investment can be defined as:

$$
\Psi(e) = \begin{cases} 
c(e) & \text{if } x_i \\
k(e) & \text{if } (1-x_i)
\end{cases}
$$

In period $t+2$ the DM will gain a final payoff

$$
U_{t+2}^i = \theta_iS_{t+2}
$$

The utility at time $t + 1$ is represented by the function $u(\theta, \hat{\theta}, S_t) : A \times \Theta \rightarrow \mathbb{R}$ and its
expression is the following:

$$U_{t+1}^i(\theta, \hat{\theta}, \lambda, S_t) = -\Psi(e) + V(\theta, \hat{\theta}, S_t|\sigma)$$

$$= -\Psi(e) + sE_{t+1}^i[U_{t+2}^i] + \delta E_{t+1}^i[U_{t+2}^i]$$

(13)

Then the total utility at time $t$ is:

$$U_t^i(\theta, \hat{\theta}, \lambda, S_t) = -M(\lambda) + \delta E_t^i[U_{t+1}^i] + \delta^2 E_t^i[U_{t+2}^i]$$

(14)

The anticipation at time $t+1$ of the utility at time $t+2$ is expressed by, $V(\theta, \hat{\theta}, S_t|\sigma)$, the value function in which both the future outcome of the investment, $\delta \theta$, and the aspiration level, $\hat{\theta} = \theta(e)$ (i.e. the DM’s expected identity given the investment decision), are represented by

$$V(\theta, \hat{\theta}, S_t|\sigma) \equiv (\alpha\delta \hat{\theta} + \delta \theta) S_{t+1} = (s\hat{\theta} + \delta \theta)S_{t+1}$$

Where $(\delta \theta)S_{t+1}$ is represented by the schooling return times the identity trait. The trait identity is represented in such a way that take into account the anticipatory utility at time $t+1$, $s\hat{\theta}$ and the discounted utility from identity at time $t+2$, $(\delta \theta)S_{t+1}$, where $s$ is the anticipatory factor and $\delta$ is the time discount factor.

**Assumption 3.** The value function is a continuous and supermodular: it is strictly increasing and continuous in the aspiration level $\hat{\theta}$, and continuous for every $\theta \in \mathbb{R}$ but it also has increasing differences for all pairs of aspiration level and education choice ($V(\cdot)$ satisfies the Mignron-Shannon’s single-crossing condition). Besides, the value function satisfies the following assumption: $V(\hat{\theta}) > 0$, if $r > 0$, $V_{\theta S_{t+1}} > 0$ and $V_{\theta \hat{\theta}} = 0$.

The requirements enlighten three aspects: the first is a condition of good expected behavior with respect to the decision of investment; the second requirement points out the beneficial effect of the investment on the utility. Finally, the third requirement shows that the action has informational content feeding conformist behavior to a norm. On this account a clarification is due. According to Caplin and Leahy [20] anticipatory emotions affect the behavior when the total utility function is strictly concave. In this way the DM is averse to variation to future anticipatory emotion. However, according to Bernheim and Thomadsen [13], the assumption of non-linearity can be relaxed in case of imperfect recall.

One important property of this set up is that, as [20] pointed out, anticipatory emotions can affect behavior when $U_t$ is non-linear, determining the DM’s preference for information. Whenever the utility function is concave the DM is information averse to variation in future anticipatory emotion, she suffers of anxiety. Consequently, the individual prefers to take an ignorant action rather than receiving information prior to make the decision unless this allows her to select an action that sufficiently improves her outcome. This is true when
the information is perfectly recalled. However, the non-linearity assumption can be relaxed, as [13] showed, when memory of the signal is imperfect and the anticipatory emotions still matter, because the imperfect recall breaks down the law of iterated expectations.

Moreover, considering the fact that aspiration level and education choice are complement, and that $\theta(\cdot)$ is a function of the unknown identity, $\theta$ and of the signal and its recall, if the value function respects specific conditions, according to [4] we can investigate the comparative statics of the educational investment with respect to the original signal

Assumption 4. If $V(\cdot)$ is single-crossing in $e$ and $\theta$ holds then $V(\cdot, \theta(1)) - V(\cdot, \theta(0))$ is non-decreasing in $\theta$, if $\theta(1) > \theta(0)$ and if $\frac{\partial^2 V(\cdot)}{\partial e \partial \theta_i} > 0$. And since by the Assumption 1 we know that $f(\theta_i, \sigma_i)$ satisfies the monotone likelihood ratio property, then the expected value function is single crossing in $e$ and $\sigma$ and, therefore, the optimal choice, $e^*(\cdot)$, is weakly increasing in $\sigma$

Figure 2, in 6, represents the decision tree of the entire model.
4 The DM’s problem

The DM’s problem is solved by backward induction.

Choice of $e$. By the end of $t + 1$ the DM updates her beliefs and chooses the action given the recall of the signal. She does it by maximizing anticipatory utility giving the beliefs set she owns at that time. The optimal action is

$$e^* = \arg\max_{e \in \{0, 1\}} E\left[U_{t+1}(\theta, \hat{\theta}, \lambda, S_{t+1})\right] = E_x \left\{ E_{r^*}(\lambda) \left[U_{t+1}(\theta, \hat{\theta}, \lambda, S_{t+1})\right] - \Psi(e) \right\}$$  \hspace{1cm} (15)

Choice of $\lambda$. At the end of period $t$ the DM chooses how to recall the signal received at the beginning of the stage. It occurs whenever the benefit of memory manipulation both in terms of self-esteem and efficient future actions overcome the cost of the manipulation.

$$\lambda^* \in \arg\max_{\lambda} \left\{ \lambda E_{r^*}(\lambda) \left[U(\theta, \theta(\hat{\sigma}), \lambda, S_t)|\hat{\sigma} = \sigma\right] + (1 - \lambda) E_{r^*}(\lambda) \left[U(\theta, \theta(\hat{\sigma}), \lambda, S_t)|\hat{\sigma} = \emptyset\right] - M(\lambda) \right\}$$ \hspace{1cm} (16)

Then, the decision made by the agent with endogenous imperfect recall is modeled as the PBE of the multi-self game. It is constituted of a strategy profile, that comprises an interpretation strategy ($\hat{\sigma}^*$) and an action strategy ($e^*$), and of a posterior belief.

Definition 3. A PBE of the game is the set of strategy profile ($\hat{\sigma}^*, e^*$), where $\hat{\sigma}^*$ is given by the pair $(\lambda^*, r^*) \in [0, 1] \times [p, 1]$ and $e^* \in \{0, 1\}$, and the posterior beliefs $F(\theta|\hat{\sigma})$ such that

1. $\lambda^* \in \arg\max_{\lambda} \left\{ \lambda E_{r^*}(\lambda) \left[U(\theta, \theta(\hat{\sigma}), \lambda, S_t)|\hat{\sigma} = \sigma\right] + (1 - \lambda) E_{r^*}(\lambda) \left[U(\theta, \theta(\hat{\sigma}), \lambda, S_t)|\hat{\sigma} = \emptyset\right] - M(\lambda) \right\}$
2. $e^* \in \arg\max_{e} E_x \left\{ E_{r^*}(\lambda) \left[U(\theta, \theta(e), \lambda, S_t)|\hat{\sigma}\right]\right\}$
3. $r^*$ is obtained by the Bayes’ rule if $Pr(\hat{\sigma}|\lambda^*) > 0, \forall \hat{\sigma} \in \{L, H, \emptyset\}$.

4.1 Self-$t+1$ updating and action strategy.

Given $\lambda$, at $t + 1$ individual will choose the action that maximizes her future plan\footnote{$e$ has to be considered as $\hat{\theta}$.}. For each individual the Incentive criterion to invest at time $t + 1$ in the new path, $e = 1$, requires that the total anticipatory utility from investing are larger than the cost of investing:

$$IC(1) = V(\theta, \theta(1), S_{t+2}) - C_i \geq 0$$ \hspace{1cm} (17)

On the other hand, the incentive criterion to invest at time $t + 1$ in the old path, that is equivalent to not invest in the new course of events, $e = 0$, requires that the total anticipatory utility from not investing are larger than investing after the cost of investing:

$$IC(0) = V(\theta, \theta(0), S_{t+1}) - K_i \geq 0$$ \hspace{1cm} (18)
Investing in the status quo is always efficient if the benefit received by remaining in it is greater than the cost of remaining it.

Since we can say that investing in old path is equivalent to not invest in the new one, we can evaluate the two incentive constraints relatively to the cost function related to the new path \( c(e) \). Hence the two incentive criteria become:

\[
IC_i(1) = V(\theta, \theta(1), S_{t+2}) - V(\theta, \theta(0), S_{t+1}) - C_i \geq 0
\]

\[
= \delta \theta^* r + s[\theta(1)(S_{t+1} + r) - \theta(0)S_{t+1}] \geq C_i
\] (19)

Whenever the gain of investment, measured in terms of utility differentials, is greater than the cost of facing the investment, then the new path is the strategy to be played. While, the criterion to invest at time \( t + 1 \) in the old path, \( e = 0 \):

\[
IC_i(0) = V(\theta, \theta(1), S_{t+2}) - V(\theta, \theta(0), S_{t+1}) - C_i \leq 0
\]

\[
= \delta \theta^* r + s[\theta(1)(S_{t+1} + r) - \theta(0)S_{t+1}] \leq C_i
\] (20)

That is to say, whenever the utility differentials is not so large for DM to confront the cost of investing in a new path, then it is more efficient to remain in her status quo.

Given the parameters, the individual will maximizes her total plan under the set of strategies and beliefs express above. If at any stage of the game the strategies are sequentially rational, namely optimal given the beliefs, and the beliefs system is consistent given the equilibrium strategies, and considering that \( \Theta \) and \( A \) and \( U(\cdot) \) respect the requirements for the existence of a fixed point according to Kakutani’s theorem, then

**Definition 4.** A PBE of the game exists and it is the set of action strategies \((e^*)\) and posterior beliefs \((\hat{p}^*)\) such that

1. \( e^* = \arg\max_{e \in \{0,1\}} E_x \left[ U(\theta, \hat{\theta}, \lambda, S_{t+1}) \right] = E_x \left\{ E_\lambda \left[ U(\theta, \hat{\theta}, \lambda, S_{t+1}) \right] - \Psi(e) \right\} \)

2. \( r^* \) is obtained by the Bayes’ rule if \( Pr(\hat{\sigma} | \lambda^*) > 0, \forall \hat{\sigma} \in \{L, H, \emptyset\} \) and if the beliefs \( b_i = (x_i, \hat{p}) \geq 0 \).

The first condition states that short-run self at time \( t+1 \) chooses the action that maximize her utility given the beliefs she holds about what would be her future identity and in the light of the information available to her at that time. She might consider the fact that she might have forgotten about previous status. At this point the imperfect recall is not strategic. Hence the short-run self does not have to take in consideration the possibility that her previous self might have suppressed the signal. The second is the consistency assumption requiring that the strategy of the short-run self satisfies the Bayes’ rule given the information available at that time.
Because of the Assumption 3 and Assumption 4, \( x_i^* \) is weakly increasing in \( \theta \), and then respectively in \( p \) imposing that a higher type cannot choose a lower investment choice.

\[
x_L(1 - x_H) = 0
\]  

(21)

Hence, whenever \( x_L > 0 \) then \( x_H = 1 \).

Moreover, monotone comparative statics imply that \( r^* \) is a cutpoint decision rule; there is a parameter value \( r^* \), and respectively \( p \) such that the DM chooses the separating equilibria for some value moving from \( r^* \), and, instead, she can have multiple equilibria at the cutpoint.

Hence,

**Proposition 1.** There exists a unique monotonic undominated equilibrium, characterized by thresholds \( \tilde{p} \) and \( \overline{p} \) with \( 0 < \tilde{p} \leq \overline{p} \leq 1 \) and investment probabilities \( x_H(p) \) and \( x_L(p) \) such that:

\[
x_H(p) = \begin{cases} 
0 & \text{if } p > \overline{p} \\
1 & \text{if } p < \overline{p}
\end{cases}
\]

(22)

\[
x_L(p) = \begin{cases} 
0 & \text{if } p \in [\tilde{p}; 1] \\
1 & \text{if } p \in [\tilde{p}; \overline{p}] \\
\text{nondecreasing} & \text{if } p \in [0; \tilde{p}]
\end{cases}
\]

(23)

**Proof.** See Appendix 6

The equilibrium is evaluated for \( 0 < \tilde{p} < 1 \), for decreasing values of \( C_i \) and of \( K_i \). Moreover the results above show that the probability of receiving a specific signal have a monotonic, hum-shaped, effect on the overall probability investment whose trend will increase in \( p \) on the interval \([0; \tilde{p})\)\(^{15}\), then it will equals to 1 on \([\tilde{p}; \overline{p})\), and after that it will falls to 0.

4.1.1 Characterization of Equilibria.

Let us discuss the incentive criterion 19 and 20 in order to characterize the equilibria.

- Pooling toward null investment.

Both agent will choose the strategy of non-investment in the new path. From the assumption on \( V(\cdot) \) and the incentive criteria specific for each type, the H types are those who might have some strategic interest in not investing in a different path, since for them the decision to invest could be less costly than for the L type. Assume that then it means that

\[
\delta \theta^* + s[\theta_H S_{t+2} - \overline{S}_{t+1}] < C_H
\]

(24)

\(^{15}\)This is called also mixing region and its slope depend on the magnitude of the initial cost L-types have to face.
In this equilibrium, we consider only the incentive constraint of a H-type, since for L-type will be always costly to invest. We obtain the incentive constraint by knowing that if \( p \) is high enough (namely the self-image is behind the threshold \( \bar{p} \)) H-type can afford not to invest. Then \( x_H = x_L = 0 \) and the posterior beliefs are \( \theta(0) = \bar{\theta} \) and \( \theta(1) = \theta_H \) since \( \hat{q} = \frac{1}{2} \) and \( \hat{p} = \varepsilon \in [0; 1] \). Then, type H can afford not to invest if \( p > \bar{p} \), but when the latter relation does not hold, then H will take some action to distinguish herself from L. In this equilibrium H-type will not invest in education for other several motivations. The fact that in this scenario \( p > \bar{p} \) means that the agent is surrounded by people following the old path and it would be too costly for him to take action to prove being the eccentric one. If she will deviation from the norm, this deviation will be self-justified as the malleability of the beliefs will increase. In turn,

- Separating for \( \theta_H \):
  \[
  \delta\theta^* + s[\theta_H S_{t+2} - \theta_L S_{t+1}] > C_H
  \]  
  (25)

- Separating for \( \theta_L \):
  \[
  \delta\theta^* + s[\theta_H S_{t+2} - \theta_L S_{t+1}] < C_L
  \]  
  (26)

In this equilibrium, we consider both the incentive constraints of a H-type and L-type. We get the incentive constraint by knowing that if there is the possibility to separate the types, then \( x_H = 1 \) and \( x_L = 0 \) and the posterior beliefs are \( \theta(0) = \theta_L \) and \( \theta(1) = \theta_H \) since \( \hat{q} = 0 \) and \( \hat{p} = p \). Then \( \theta(1) > \theta(0) \) type H will invest and L will not since \( 0 < p < \bar{p} \) and the \( C_{L0} \) is so high that for L is not worthy to invest. In this scenario the incentive constraints bind for the two types with \( IC_L \leq 0 \) and \( IC_H \geq 0 \).

- Semi-separating (hybrid) by randomizing for \( \theta_L \):
  \[
  \delta\theta^* + s[\theta(1) S_{t+2} - \theta_L S_{t+1}] = C_L
  \]  
  (27)

This scenario occurs when the cost of investment for L-type is small enough to imitate the H-type. However the imitation is always constraint by the prior and the credibility of signalling to be H-type depends on \( p \) and \( x_L \): the lower the former, the more truth the strategy. Then we have \( x_H = 1 \) and \( 0 < x_L < 1 \). Then, the posterior probabilities are \( \hat{p} = \bar{p} \) while \( 0 < \hat{q} < \hat{p}(1) \) with the posterior beliefs represented by \( \theta(0) = \theta_L \) and \( \bar{\theta} < \theta(1) < \theta_H \). Accordingly, type H will not invest and L will only depending on how credible the posterior probabilities are, \( IC_L = 0 \).

- Pooling towards total investment:
  \[
  \delta\theta^* + s[\bar{\theta} S_{t+2} - \theta_L S_{t+1}] > C_L
  \]  
  (28)

In the case of both type investing in further education, the incentive constraint that we need to take in consideration is the one of the L-type, since by definition H-type
will always have incentive to invest. In this scenario we have that the probability to invest is \( x_H = x_L = 1 \) and then, recalling (8), the posterior beliefs are \( \theta(0) = \theta_L \) and \( \theta(1) = \bar{\theta} \) since \( \hat{q} = \varepsilon \in [0; 1] \) and \( \hat{p} = \frac{1}{2} \). Then \( \theta(1) > \theta(0) \) type H will invest and L will only if her \( IC_L \geq 0 \) binds and in that case \( \hat{p} \leq p \leq \bar{p} \). The incentive holds since for cost faced even a small gain from self-image is worth to pursue: as the cost increases so the threshold \( \hat{p} \) does.

### 4.1.2 Comparative Analysis.

The probability to invest \( x \) is determined by both the internal \((p, \lambda, s)\) and external \((S_t, r, C_t)\) constraints. Moreover both individual invest more when \( x_H \) and \( x_L \) (weakly) increase. In all the cases H-type will go either for a separating, a full investment or hybrid equilibrium, while L-type will invest only in hybrid or full situation. Then, to show for what values of the main parameters, the probability to invest increases, we need to study how the incentive constraint varies with respect to a variation of parameters. From

\[
IC_i = \delta \theta^* r + s[\theta(1)(S_{t+1} + cr) - \theta(0)S_t] - C_i
\]

if individuals invest in education, then \( e = 1 \), we have:

\[
\frac{\partial IC_i}{\partial S_{t+1}} = s(\theta(1) - \theta(0)) > 0
\]

\( S_{t+1} \) is a measure of initial affiliation. Its level and quality is higher for the H-type driving him to show more incentive in eliciting identity-affirming investment. The positive impact of the stock on the willingness to invest is consequence of the so called *escalating commitment*, expressed by the requirement on \( V_{\theta S_{t+1}} \): individuals have a higher demand for optimistic beliefs when they have more at stake (in this case when they have higher identity-specific capital \( S_t \) already invested). From here it is worthy to make another consideration.

\[
\frac{\partial^2 IC_i}{\partial S_{t+1} \partial s} = (\theta(1) - \theta(0))
\]

The higher the initial investment, the more positive is the emotion attached to future identity and the higher is the incentive to invest.

\[
\frac{\partial IC_i}{\partial r} = \delta \theta^* + s \theta(1) > 0
\]

An increase in the evaluation of the interest rate means an enlargement of the perception and awareness of the opportunity available. The interest rate of education is an important
information for the decision mechanism and its lack depends on how connected the society is in vertical and horizontal way. Whenever the society is highly stratified this peace of information is lost.

\[
\frac{\partial IC_i}{\partial s} = [\theta(1)S_{t+1} - \theta(0)S_t] > 0
\]

the more pleasant the sentiment related to the future identity conveyed by the investment possibility, the higher the incentive to invest. The positive feelings depends on how much the DM feels her current self connected with her future self, so that she can engage in forward-looking choice.

Finally, the impact of the cost on the probability of investment is inverse related regarding the type: for H-type a decreasing of the initial cost increases \( p \) (it gets closer to 1), decreasing her willingness to invest. For L type a decreasing of cost means a decreasing of \( \tilde{p} \) (it gets closer to 0) reducing the mixing region, then increasing her willingness to invest \((x_L)\).

**Proposition 2.** The probability to invest weakly increases when
- \( S_{t+1} \) and \( r \) increase;
- the lower the investment costs;
- \( \lambda \) decreases (people becomes more malleable/progressive);
- \( s \) increases.

### 4.2 Self-t strategy of imperfect recall and the coping mechanism for solving the cognitive dissonance

Once the signal has been received, how will the DM interpret the new information in order to safeguard her beliefs? Here, the Self-t performas an *ex-post* denial strategy. She has incentives to process the \( H \) and \( L \) signal asymmetrically. In order to do so let us define what are the expected utilities given the signals and the recollections.\(^\text{16}\)

The expected utility given the signal is the following:

\[
U_i(\theta; \theta(\sigma), \sigma_i) = E[U(\theta; \theta(\sigma); S_t; \sigma)|\sigma]
\]

If the signal is recall and interpreted realistically, the expected utilities given either the signal and the recollection are the same, then:

\[
U_i(\theta; \theta(\hat{\sigma}_i), \sigma_i) = E[U(\theta; \theta(\hat{\sigma}_i); S_t; \sigma_i)|\hat{\sigma}]
\]

\(^\text{16}\)I will compress the equation (14) for reasons of clarity in the exposition and to put in evidence the role of the signal and the recollection.
If, instead, the DM decides to rationalize the signal (i.e., if she recollects $\hat{\sigma} = \emptyset$), the expected utilities will be:

$$U_{\emptyset}(\theta; \theta(\emptyset), \sigma_i) = E_r \left[ U(\theta; \theta(\hat{\emptyset}); S_i; \sigma_i)|\hat{\sigma} \right] = r(\lambda^*) \times U_H(\theta(\emptyset)) + [1 - r(\lambda^*)] \times U_L(\theta(\emptyset))^{17} \quad (31)$$

Thus as illustrated in Fig.(1).

Rationalization of the signal represent an incentive for the DM who has preferences over her identity. However, the manipulation of her beliefs is still defined according to Bayes’s rule, then DM makes correct inferences given the recollection strategy though the updating process is relatively slower than it is implied by the Bayes’ rule. This situation is described as conservatism bias, according to which all new information is insufficiently weighted in the updating process.

4.2.1 Memory manipulation

Individuals maximize utility not only through behavior but by adopting a view of reality consistent with their well-being. These views have direct effect on behavior. The adoption passes by the imperfect recall of information and manipulation of the beliefs. The main consequence is that the long-run self will recall the information received in such a way to manipulate the information set$^{18}$ that the short-run self will inherit. Hence after observing a signal $\sigma = \{H; L\}$ the long-run self chooses $\lambda$ to maximize the following function

$$\lambda U_i(\theta(\hat{\sigma}_i), \sigma_i) + (1 - \lambda) U_{\emptyset}(\theta; \theta(\emptyset), \sigma_i) - M(\lambda)$$

By adding equation (31), we get

$$U_i(\theta; \theta(\emptyset), \sigma_i) + \lambda [U_i(\theta; \theta(\hat{\sigma}_i), \sigma_i) - U_i(\theta; \theta(\emptyset), \sigma_i)] + (1 - \lambda) r(\lambda^*) [U_j(\theta; \theta(\emptyset), \sigma_i) - U_i(\theta; \theta(\emptyset), \sigma_i)] - M(\lambda) \quad (32)$$

**Definition 5.** $\Delta DM = U_i(\theta; \theta(\hat{\sigma}_i), \sigma_i) - U_i(\theta; \theta(\emptyset), \sigma_i)$ represents the decision making factor

Given the recall strategy, will the DM take suboptimal decision?

**Definition 6.** $\Delta U = U_j(\theta; \theta(\emptyset), \sigma_i) - U_i(\theta; \theta(\emptyset), \sigma_i)$ represents the self-image differential in utility

Given the recall strategy, how much utility DM gains through a more favorable inference of her identity? Let us consider the two cases.

$^{18}$In Bernheim and Thomadsen [13] the manipulation will result in the break down of the law of iterated expectations.
The expected utility of Self-t conditional on receiving a signal $\sigma = L$ is:

$$U_L(\theta; \theta(\emptyset), L) + \lambda[U_L(\theta; \theta(L), L) - U_L(\theta; \theta(\emptyset), L) + (1 - \lambda)r(\lambda^*)[U_H(\theta; \theta(\emptyset), L) - U_L(\theta; \theta(\emptyset), L)] - M(\lambda)$$

Because of the assumption on revealed preferences $U_L(\theta; \theta(L), L) < U_L(\theta; \theta(\emptyset), L)$, hence the decision making factor is negative for the $L$ type. That is to say forgetting $L$ signal might lead to suboptimal choice of the future action. However the self-esteem will boost since, because of the stochastic ordering of the signal, $U_H(\theta; \theta(\emptyset), L) > U_L(\theta; \theta(\emptyset), L)$.

Upon receiving a signal $\sigma = H$, and after few manipulations, the expected utility will be:

$$U_H(\theta; \theta(\emptyset), H) + (1 - r(\lambda^*))U_L(\theta; \theta(\emptyset), H) + \lambda[U_H(\theta; \theta(H), H) - U_H(\theta; \theta(\emptyset), H)] + (\lambda)r(\lambda^*)[U_H(\theta; \theta(\emptyset), H) - U_L(\theta; \theta(\emptyset), H)] - M(\lambda)$$

In this case forgetting the $H$ signal has negative effect on both decision making and self-esteem. Because of both of the revealed preferences and of the stochastic ordering, receiving $H$ signal should be a situation in which memory manipulation should not be implemented.

**4.2.2 Memory manipulation in hedonic case**

If we consider the decision of recalling the signal as in pure hedonic value\(^{19}\) then the DM will choose to take into account only the self-image differential. The recall strategy will be dependent on it and on the cost function of memory and on the content of the initial signal. In the 6 it is possible to find the full characterization of the equilibria coming from the manipulation when both types of agent will choose the amount of information to recall. Let us consider case by case. When $\sigma_t = L$ Self-t maximizes the following:

$$\lambda U_L(\theta; \theta(\bar{\sigma}_i), \sigma_i) + (1 - \lambda)U_\emptyset - M(\lambda) = U_L + r(\lambda^*)(1 - \lambda)\Delta U - M(\lambda)$$

Thus, Self-t will choose the amount of manipulation, then $\lambda$, so to maximize the previous formula. Then for L-type we may observe that

$$\max_\lambda \lambda U_L + r(\lambda^*)(1 - \lambda)U_H + [1 - r(\lambda^*)](1 - \lambda)U_L - M(\lambda)$$

\(^{19}\)The node in which Self-t is positioned, that is the initial node, is a singleton information set.
then by Kuhn-Tucker Theorem,
\[
\frac{\partial U_L}{\partial \lambda} : -r(\lambda^*)\Delta U - M'(\lambda) = 0 \tag{35}
\]

The recollection strategy would be different for H and L-type. Let’s start by studying the L-type. her inference about the accuracy\(^{20}\) of the signal, with some manipulation is

\[
r(\lambda^*_L) = \frac{p}{p + (1 - p)(1 - \lambda^*_L)}
\]

Whose values depend on whether \(\lambda^* = 0\), then \(r^* = p\), or \(\lambda^* = 1\), then \(r^* = 1\), hence, \(r(\lambda^*_L) \in [p, 1]\). By applying Kuhn-Tucker’s theorem the FOC (33) becomes:

\[
M'(\lambda) = -\left[\frac{p}{p + (1 - p)(1 - \lambda^*_L)}\Delta U\right] \tag{36}
\]

By the implicit function theorem \(\lambda^*_L \in \mathbb{R}\) exists.

**Proposition 3.** The PBEs for belief manipulation are defined by\(^{40}\)

\[
\lambda^*_L = \begin{cases} 
0 & \text{if } \Delta U \leq \frac{M'(0)}{p}, \\
1 & \text{if } \Delta U \geq M'(1) 
\end{cases} \tag{37}
\]

The amount of belief manipulation will be:

- increasing in \(\Delta U\)
- decreasing in \(M(\lambda)\)
- increasing in \(p\)

**Proof.** See Appendix 6

Analogously, the expected utility of Self-t after observing a high signal is:

\[
\lambda U_H(\theta; \theta(\hat{\sigma}_i), \sigma_i) + (1 - \lambda)U_0 - M(\lambda) \\
= \lambda U_H + r(\lambda^*)(1 - \lambda)U_H + (1 - r(\lambda^*))(1 - \lambda)U_L - M(\lambda) \tag{38}
\]

\[
\max_{\lambda} \lambda U_H + r(\lambda^*)(1 - \lambda)U_H + [1 - r(\lambda^*)](1 - \lambda)U_L - M(\lambda) \tag{39}
\]

and

\[
\frac{\partial U_H}{\partial \lambda} : [1 - r(\lambda^*)]\Delta U - M'(\lambda) = 0 \tag{40}
\]

\(^{20}\)The accuracy should be

\[
r(\lambda^*_L) = \frac{p(1 - \lambda^*_H)}{p(1 - \lambda^*_H) + (1 - p)(1 - \lambda^*_L)}
\]
$H$-type’s inference about the accuracy\textsuperscript{21} of the signal, with some manipulation is

$$r(\lambda^*_H) = \frac{p(1 - \lambda^*_H)}{p(1 - \lambda^*_L) + (1 - p)}$$

Whose values depend on whether $\lambda^* = 0$, then $r^* = p$, or $\lambda^* = 1$, then $r^* = 0$, hence, $r(\lambda^*_L) \in [0, p]$. By applying Kuhn-Tucker’s theorem the FOC (38) becomes:

$$\left[1 - \frac{p(1 - \lambda^*_H)}{p(1 - \lambda^*_L) + (1 - p)}\right] \Delta U = M'(\lambda)$$

(41)

By the implicit function theorem $\lambda^*_H \in \mathbb{R}$ exists.

**Proposition 4.** The PBEs for belief manipulation follows

$$\lambda^*_H = \begin{cases} 
0 & \text{if } \Delta U \leq \frac{M'(0)}{(1-p)}, \\
1 & \text{if } \Delta U \geq M'(1).
\end{cases}$$

(42)

The amount of belief manipulation will be:

- increasing in $\Delta U$
- decreasing in $M(\lambda)$
- increasing in $(1 - p)$

In both cases, for any increase of $\lambda$ both sides of (33) and (38) are increasing in the belief strategy, then there may be multiple interior equilibria.

## 5 Discussion

Like in the investment decision, the possibility of different equilibria depends on the probability of investing that, in turn, rely on the decision of how recollecting the signal, $\lambda$, in accordance with the prevailing empirical expectations on the identity types $l(\sigma|\theta)$. Moreover, the incentive to engage in manipulation are higher the higher is the self-image payoff gain, $\Delta U$. The assumption on the manipulation cost function allows us to have equilibria with both manipulation and no manipulation according to the magnitude of $\Delta U$. In a pure strategy with accurate recall the initial division of the population will be reflected in the following periods, then no undermining of the self-esteem in terms of peer recognition.

In a pure strategy with suppression of the signal, the updated beliefs at time $t + 1$ converges to the average level of $\bar{\theta}$ showing that DM has either overconfident or underconfident

\textsuperscript{21}The accuracy should be

$$r(\lambda^*_H) = \frac{p(1 - \lambda^*_H)}{p(1 - \lambda^*_L) + (1 - p)(1 - \lambda^*_L)}$$
beliefs in absolute terms. Moreover, if we consider the initial proportion of H-type and L-type in the population, then the sense of overconfidence or underconfidence will be held in relative terms: e.g. in a population where \( p < \frac{1}{2} = (\bar{p}) \) the median identity will be the one of \( \theta_L \). The norm is to be L-type. Accordingly, if we assume that the entire population is motivated to endorse the socially accepted type, then we may have L-type DM, exercising \( \lambda = 1 \), and H-type choosing \( \lambda = 0 \), reinforcing the identity with trait L. Let’s see how it works. As already stated at the beginning of the model in the population a proportion \( 1 - p \) of DM are identified by \( \theta_L \) having received the signal \( \sigma_t = L \). The proportion of \( \theta_H \) having receives \( \sigma_t = H \) is \( p \). The average identity is given by \( \bar{\theta} = p\theta_H + (1 - p)\theta_L \) that can be written as function of the proportion of H-type: \( \theta(p) \). Now let us assume that the the median identity is given by \( \theta_L \). What would be the distribution of the self-evaluation? Let us assume that \( \sigma_t = L \) is a signal reducing the self-image of DM and that everyone will use the choose the same probability of recall the signal, \( \lambda^* \in (0, 1) \), and accordingly they will impute the reliability of the memory by \( r^* \). Now, the self-assessment considering the accuracy of the recall will be \( \theta(r^*) \) no longer \( \bar{\theta} \). Then, the fraction of agents who will manipulate the recall is represented by \( (1 - p)(1 - \lambda^*) \) and they are those who overestimates their identity by \( \theta(\lambda^*) - \theta_L = r^*(\theta_H - \theta_L) \). To those individuals, we have to add the fraction of people who did receive a good signal, and they are \( p \): the latter and the former will enlarge the proportion of people considering beyond the average, \( (1 - \lambda^*)(1 - p) \). While the fraction of those underestimating their ability by \( \theta_L - \theta(\lambda^*) = (1 - r^*)(\theta_H - \theta_L) \) is represented by the minority \( p \), who will see confirmed their self-assessment of agent behind the average. Moreover, the less the manipulating cost, the higher the number of H-type repressing the good signal. Another perspective from where to look at the problem can be expressed in term of homogeneity of the social context in which the DM lives. Whenever \( p = \frac{1}{2} \) the social system pounds the highest level of entropy and diversity and then the individual should not incur in any dissonance problem; that is to say, in a society in which, when you toss a coin you have equally likely chance to move in one direction or another, then the self esteem problem related to the quest of belonging could be downsized. Whenever, instead, the context is highly homogeneous then the choices of either the action and of the signal rationalization are driven by the sense of belonging to a precise social group.

Yet, the manipulation costs can be very high, to leave individual in self-trap situations caused by their beliefs. Then, as already proved, the initial proportion of identity has an effect in both recalling and action strategy. One question that arises is the following: if every strategy can be brought back primitively to \( p \), would be DM able to identify a signal as informative for her identity and aspirations and then taking a decision accordingly? And, why this question is so important? Because, the initial beliefs work as mental insurance tool affecting the propensity towards a new piece of information. Indeed, the cognitive neighborhood, or the the information about \( p \) through the experience of a “similar” other,
matters. As it matters also what kind of fraction of information on $p$ the DM draws from whom.

5.1 Characterizing beliefs updating heuristics

The work on how the DM’s information processing can be restrained is very extensive. In the studies of belief updating (Tversky and Kahneman [53]; Cameron [19]; Eil and Rao [27]; Mobius et al. [45]), agents are provided with signals about the same quantity over which revision of beliefs are being analyzed. For example, in Eil and Rao [27], Mobius et al. [45], and Grossman and Owens [35] respondents are revising their beliefs about either their own intelligence or beauty, and receiving feedback about the same underlying entity for which beliefs are being reported. In Nguyen [47] and Zafar [55, 54] information on future earnings has been provided to students in other contexts, and it has been shown to have an impact on actual schooling choices. This literature classifies agents’ updating according to the heuristic used, that ranges from Conservative to Representative, which takes distance from a Bayesian updating depending on the weight assigned to the recent information or on the adjustment mechanism.

The Conservative heuristics is the one that better applies to the context of the model. Individuals are subjected to Conservatism bias if they fail to sufficiently adjust their beliefs in light of new information. Or in different words, they update in the right direction but less than a Bayesian updater. On this regard people suffering of conservatism bias are those sensitive to new information, and then they prefer to infer their identity from their recollection rather than from the actual signal. If we consider $\hat{\theta}_\sigma$ as the expected value of $\theta$ given the information structure based on the recalled signal $\hat{\sigma}$, while $\theta_\sigma$ is the expected valued of the identity parameter given the information structure produce by the observed signal $\sigma$, that is the Bayes estimate of $\theta$ given the original signal, then $\hat{\theta}_\sigma$ dominates $\theta_\sigma$ in the sense of second-order stochastic dominance. Let us considering $G(\theta_\sigma)$ as the cumulative distribution of the expected value of $\theta$ conditional on the actual signal $\sigma$.

**Proposition 5.** Whenever the DM is risk averse and her decision making is justified by the presence of social norms, and whenever she is exposed to two informational structures, one of which it is in conflict with the information released by the norm, she will order the two structure according to the SOSD. Namely, she will choose the information structure that is less variable with respect to the social norm. In our model, $\hat{\theta}_\sigma$ is less variable than $\theta_\sigma$. And the updating process of the information produced by $\sigma$, used by the DM, is conservative, in the sense that it is less than what Bayes’ rule implies.

**Proof.** See Appendix 6
5.2 Welfare analysis of awareness

In order to further assess the analysis the perspective to consider is an *ex ante* one that will try to answer the following question: if the DM could choose whether to engage in a manipulating strategy before receiving the initial signal, what would she do? *Ex ante* it would be optimal to commit not to engage in rationalization strategy because the expected cost related to the strategy would outweighs the expected gain. Moreover the DM will be willing to observe the signal if the expected cost of making an uninformative decision are greater than the premium of engagement, otherwise she would prefer to gain a lower expected payoff but also avoid the self-deception costs. If this does not happen, then the DM will follow one of the four methods\textsuperscript{22} that in psychology are used as resolution of the dissonance: the selective exposure to information, namely the tendency people have to avoid information that would create cognitive dissonance because it is incompatible with their current beliefs.

6 Conclusions

The paper proposes a diachronic approach to the study of the identity. The work identifies empirical expectations and monotone information structure to be the tools through which people form the conception of their identity. The social norms, behind these instruments, are the determinant of the genesis of the identity conception. The model shows that because of the presence of the norms people decide to conform or react against them by manipulating the information structure in a self-serving way. In doing so, it addresses both the demand and supply sides of motivated cognition. The manipulation is justified also to reach intertemporal goals, that is to say, the work shows how people make choices between identity-relevant options and how deliberately shape and manage not only their current identity but also their future one coherently with a specific informational structure.

The findings highlight that the relation between norms and identity is not as simple as commonly used in the literature of bounded rationality and synchronic identity theory. The presence of norms does not only define the action strategies that it is expected to be taken but it also shapes the mental structure that people use to read the reality. The latter findings are important in the lights of the debate about the micro level consequences that social norms, such as gender or group norms, have on macro level inefficiencies as decreasing female labour participation, persistence of gender and social and economic inequalities. Interventions aiming to reduce the material cost of the action (e.g. reducing economic barriers, quota interventions, scholarships etc.) might not be effective as expected if also the individual cognitive costs are not addressed too. However, no internal costs nor distorted belief will be reduced if social norms are not considered and studied extensively.

\textsuperscript{22}Those are Selective Exposure to Information, Minimal Justification, Hypocrisy Induction and Postdecision Dissonance.
Appendix A

Proof. Proposition 1 For Kakutani’s fixed point theory to hold the requirements are that $A$ to be non-empty, compact and convex set and $e^*$ to be a set-valued function on $A$. Since $e^* : \theta \rightarrow A$ is a best-response correspondence on the action set for each possible player, then $e^*$ exists as a fixed point, hence as PBE.

- **Separating.**
  In this scenario $\hat{p} = 1$ and $\hat{q} = 0$. By expressing $x_i$ in function of the priors $p$ and of the recall strategy $r^*$ I have to solve the following system:

\[
\begin{cases}
1 &= \frac{ar^*}{ap+b(1-r^*)} \\
0 &= \frac{(1-a)r^*}{(1-a)r^*+(1-b(1-r^*))}
\end{cases}
\]

where $a = x_H$, $b = x_L$ and $r^* = \lambda^*$.

\[
\begin{cases}
b(1 - r^*) &= 0 \\
(1 - a)r^* &= 0
\end{cases}
\]

\[
\begin{cases}
r^* < 1 \text{ and } b = 0, \\
r^* > 0 \text{ and } a = 1
\end{cases}
\]

From which I obtain the relationship between recall probability and the priors.

\[
\begin{cases}
p < 1 \text{ and } b = 0, \lambda_L = 1 \\
p > 0 \text{ and } a = 1, \lambda_H = 1
\end{cases}
\]

Yet, from the results above I envisage there will be a ”critic” level of $p$ that will make $x_i$ to switch from a decision to another.

- **Pooling:** universal investment.
  In this case $a = b = 1$ and $\hat{p} = \frac{1}{2}$, hence $\hat{\theta}(1) = \bar{\theta}$. I have to discuss the value of $\hat{q}$ because Bayes rule does not work and I need to assign arbitrarily beliefs regarding the off-equilibrium path behavior that supports a pooling equilibrium. Moreover, I need to study, also, for which value of $p$ the universal investment is supported by the two players.

\[
\begin{cases}
\frac{1}{2} &= \frac{ar^*}{ar^*+b(1-r^*)} \\
\hat{q} &= \varepsilon \in [0, 1]
\end{cases}
\]

\[
\begin{cases}
ar^* + a(1 - r^*) &= 2ar^* \\
\hat{q} &= \varepsilon \in [0, 1]
\end{cases}
\]
since \( a = b = 1 \), I can substitute one with the other,

\[
\begin{aligned}
&\begin{cases}
r^* = \frac{1}{2} \quad \text{and } a > 0, \lambda_L = \lambda_H \\
\hat{q} = \varepsilon \quad \in [0, 1]
\end{cases}
\end{aligned}
\]

From where it occurs that whenever \( \lambda_H = \lambda_L \) \( p = \frac{1}{2} = \bar{p} \). Hence, \( x_H = 1 \) will occur only for certain value of the prior, \( 0 \leq p \leq \bar{p} \).

- **Pooling:** null investment.

In this case \( a = b = 0 \) and \( \hat{q} = \frac{1}{2} \), hence \( \hat{\theta}(0) = \bar{\theta} \). I have to discuss the value of \( \hat{p} \) because Bayes rule does not work and I need to assign arbitrarily beliefs regarding the off-equilibrium path behavior that supports a pooling equilibrium. Moreover, I need to study, also, for which value of \( p \) the universal investment is supported by the two players.

\[
\begin{aligned}
&\begin{cases}
\hat{p} = \varepsilon \in [0, 1] \\
\frac{1}{2} = \frac{(1-a)r^*}{(1-a)r^* + (1-b)(1-r^*)}
\end{cases}
\end{aligned}
\]

\[
\begin{aligned}
&\begin{cases}
\hat{p} = \varepsilon \in [0, 1] \\
\hat{q} = \varepsilon \in [0, 1]
\end{cases}
\end{aligned}
\]

\[
\begin{aligned}
&\begin{cases}
r^* = \frac{1}{2} \quad \text{and } a < 1, \lambda_L = \lambda_H \\
\hat{q} = 0
\end{cases}
\end{aligned}
\]

Hence \( p = \frac{1}{2} \) is the critic level for DM \( H \) to switch from the strategy to invest to the one of not investing. She invests anytime \( p \in [0, \frac{1}{2}) \) and she retains her self to invest in the new path whenever \( p \in [\frac{1}{2}, 1] \). The DM, instead, has another threshold to consider when she is to choose the investment strategy. The second threshold is reached when DM decides to randomize between the pure strategies.

- **Randomization:** for L-type.

In randomization case the updating process is imperfect, and it consists of H-type playing the pure strategy of investing \( (a = 1) \) and the L-type applying a mixed strategy between the possible two actions \( (b \in [0; 1]) \). In this case both investing and non-investing strategy are played with positive probability along the equilibrium path. Moreover, since \( \hat{q} = 0 \), then \( \hat{\theta}(0) = \theta_L \) and \( \bar{\theta} \leq \hat{\theta}(1) \leq \theta_H \). Because of consistency requirement with respect to \( \hat{p}, b \) and \( p \) need to support each others. Hence,

\[
\begin{aligned}
&\begin{cases}
\hat{p} = \frac{ar^*}{ar^* + b(1-r^*)} \\
\hat{q} = 0
\end{cases}
\end{aligned}
\]

Since \( a = 1 \), then I will solve only the first equation by \( b \in [0; 1] \). I have already discuss
the case in which \( b \) takes on one of the two extreme, that will conduct either to the separating equilibrium or to the full investment. Let us take a \( b = 0.5 \) then

\[
r^*(1 - \hat{p}) = \frac{1}{2}(1 - r^*)\hat{p}
\]

and

\[
r^* = \frac{\hat{p}}{2 - \hat{p}}
\]

where \( \hat{p} \leq 1 \). Again if \( \hat{p} = 0 \), then \( r^* = 0 \) and the system falls in the full investment equilibrium; if \( \hat{p} = 1 \), \( r^* = 1 \) the system reproduces a separating one; for intermediate values of \( \hat{p} = \frac{1}{2} \) a further threshold for the prior, \( r^* = \frac{1}{3} \), is reached.

\[\Box\]

**Remark 1.** Assumption 2 The condition satisfies two requirements. The first is related to ensure that both manipulating and non-manipulating strategy will occur. The first is the Kuhn-Tacker constraint qualification necessary for the uniqueness of the equilibrium. A further specification of the cost function that may drive the DM towards an equilibrium rather than another would be the follow:

- \( M(\lambda) = \epsilon(1 - \lambda) \) for which \( \lambda = 0 \)
- \( M(\lambda) = \epsilon(\lambda) \) for which \( \lambda = 1 \)

In both case the constrained qualification holds but conceptually the difference between one or another is justified by the emphasis that the social context gives to one of the two strategy.

**Proof.** Proposition 3 The proof for the existence of the PBE proceeds in three steps following the direct approach:

1. For Kakutani’s fixed point theory to hold the requirements are:
   - for the interpretation strategy: \( \Omega \) has to be non-empty, compact and convex set and \( \sigma^* \) to be a set-valued function on \( \Omega \). Since \( \sigma^* : \hat{\Omega} \to L; H \) is a best-response correspondence on the interpretation set for each possible player, then \( \lambda^* \) exists as a fixed point, hence as PBE.
   - for the action strategy: \( A \) has to be non-empty, compact and convex set and \( e^* \) to be a set-valued function on \( A \). Since \( e^* : \hat{\Omega} \to A \) is a best-response correspondence on the action set for each possible player, then \( e^* \) exists as a fixed point, hence as PBE.

2. Kuhn-Tacker’s constraint qualification condition holds since all the cost functions are linear, then automatically satisfied.
3. By the implicit function theorem:

\[ X(r^*; \lambda) = \lambda U_i + (1 - \lambda) U_\emptyset - M(\lambda) = U_L + r(\lambda^*)(1 - \lambda)\Delta U - M(\lambda) \]

Recall that \( X(r^*; \lambda) \) is continuous and differentiable in \( r^* \) and because of the implicit function theorem, then \( \lambda^* \) exists.

Thus, since the right properties for the existence hold, then \( \lambda^* \) is a PBE.

The existence of the PBE in the proposition follows the previous proposition. However, the uniqueness of the equilibria follows by applying the contradiction approach. For

\[ M'(\lambda_L^*) = r(\lambda_L^*)\Delta U \]

there exists a unique PBE if

\[ \frac{M'(0)}{p} < \Delta U < M'(1) \]

To prove it, let us assume that there exists two interior equilibria:

1. \( \lambda_L = 0 \) and \( \lambda_L \in (0, 1) \) according to which:
   - \( \lambda_L = 0 \implies \Delta U \leq \frac{M'(0)}{p} \)
   - \( \lambda_L \in (0, 1) \implies \Delta U > \left( 1 + \frac{(1-p)(1-\lambda_L)}{p} \right) M'(0) \)

Then

\[ \left( 1 + \frac{(1-p)(1-\lambda_L)}{p} \right) M'(0) < \Delta U \leq \frac{M'(0)}{p} \]

would happen only if

\[ \left( 1 + \frac{(1-p)(1-\lambda_L)}{p} \right) < \frac{1}{p} \]

namely only if

\[ \lambda_L > 0 \]

Hence, the assumption of having two interior equilibria is false.

2. \( \lambda_L = 1 \) and \( \lambda_L < 1 \) then,
   - \( \lambda_L = 1 \implies \Delta U \leq M'(1) \)
   - \( \lambda_L \in (0, 1) \implies \Delta U > \left( 1 + \frac{(1-p)(1-\lambda_L)}{p} \right) M'(1) \)
Yet,
\[
\left(1 + \frac{(1 - p)(1 - \lambda_L)}{p}\right) M'(1) < \Delta U \leq M'(1)
\]
would happen only if
\[
\left(1 + \frac{(1 - p)(1 - \lambda_L)}{p}\right) \leq 1
\]
namely only if
\[
\lambda_L \leq 1
\]
which is a contradiction.

**Proof of Claims 1-3**

By the implicit function theorem:

\[
G(p^*; \lambda^*) \equiv \lambda U_i + (1 - \lambda)U_0 - M(\lambda)
\]

\[
= U_L + r(\lambda^*)(1 - \lambda)\Delta U - M(\lambda)
\]
hitherto \(G(p^*; \lambda^*) = G(\cdot)\) we can calculate

- **Claim 1**

\[
\frac{\partial \lambda}{\partial U} = \frac{\partial G(\cdot)}{\partial U} \quad = -\frac{\partial G(\cdot)}{\partial \lambda} = \frac{r(\cdot)(1 - \lambda^*)}{\partial \lambda} - \left[\Delta U(1 - \lambda^*)\frac{\partial r(\cdot)}{\partial \lambda} - \Delta U r(\cdot) + M'(\lambda)\right] > 0
\]

- **Claim 2**

\[
\frac{\partial \lambda^*(M(\lambda^*))}{\partial M(\lambda)} \equiv -\frac{\partial G(\cdot)}{\partial M(\lambda)} = \frac{\partial G(\cdot)}{\partial \lambda} = -\frac{1}{\partial \lambda} - \left[\Delta U(1 - \lambda^*)\frac{\partial r(\cdot)}{\partial \lambda} - \Delta U r(\cdot) + M'(\lambda)\right] < 0
\]

- **Claim 3**

\[
\frac{\partial \lambda}{\partial p} = \frac{\partial G(\cdot)}{\partial p} \quad = -\frac{\partial G(\cdot)}{\partial \lambda} = \frac{\Delta U \frac{\partial r^*(\cdot)}{\partial p}}{\partial \lambda} - \left[\Delta U(1 - \lambda^*)\frac{\partial r(\cdot)}{\partial \lambda} - \Delta U r(\cdot) + M'(\lambda)\right]
\]
since:
\[
\frac{\partial r^*}{\partial p} = \frac{1 - \lambda^*}{[p + (1-p)(1 - \lambda^*)]^2}
\]

\[
\frac{\partial r^*}{\partial \lambda} = \frac{p(1-p)}{[p + (1-p)(1 - \lambda^*)]^2}
\]

Then,
\[
\frac{\partial \lambda}{\partial p} = \Delta U \frac{1 - \lambda^*}{[p + (1-p)(1 - \lambda^*)]^2} - \left\{ \Delta U(1 - \lambda^*) \frac{p(1-p)}{[p + (1-p)(1 - \lambda^*)]^2} - \Delta U(\cdot) + M'(\lambda) \right\} \geq 0
\]

Now we have to discuss how the values of \(\frac{\lambda(p^*)}{p}\) as \(\lambda^*\) changes. Now, if:
- \(\lambda^* = 0 \Rightarrow \frac{\partial \lambda(p^*)}{\partial p} > 0\) depending on \(p\)
- \(\lambda^* = 1 \Rightarrow \frac{\partial \lambda(p^*)}{\partial p} = 0\)

\(\blacksquare\)

**Proof.** Proposition 4 It holds by following the same steps of the previous one. \(\blacksquare\)

**Proof.** Proposition 5 Because of Assumption 1, we know that the signal structure follows the FOSD. Then \(\hat{\theta}_s\) SOSD \(\theta_s\) if and only if, for any given concave function \(g: \Theta \rightarrow \mathbb{R}\), then:
\[
\int g(\hat{\theta}_s) dF(\hat{\theta}_s) \geq \int g(\theta_s) dG(\theta_s)
\]

(43)

Where
\[
\int g(\hat{\theta}_s) dF(\hat{\theta}_s) = p\lambda_H g(\theta_H) + (1-p)\lambda_L g(\theta_L) + [p(1 - \lambda_H) + (1-p)(1 - \lambda_L)] g(\theta_0)
\]

and
\[
\int g(\theta_s) dG(\theta_s) = pg(\theta_H) + (1-p)g(\theta_L)
\]

Substituting the two equations into the stochastic dominance inequality, and after few simplification, we get:
\[
[p(1 - \lambda_H) + (1-p)(1 - \lambda_L)] g(\theta_0) \geq p(1 - \lambda_H)g(\theta_H) + (1-p)(1 - \lambda_L)f(\theta_L)
\]

substituting the value of \(\theta_0\) and simplifying for \(\alpha\) we have:
\[
g(\alpha \theta_H + (1 - \alpha)\theta_L) \geq \alpha g(\theta_H) + (1 - \alpha)g(\theta_L)
\]
that is true because of the functional form of \( g(\dot{\lambda}) \).

Appendix B

Extended Proofs. In this section I discuss how many PBE exists for manipulation \( \lambda_i^* \in [0; \hat{\lambda}_i] \) with \( 0 \leq \hat{\lambda}_i < 1 \). Remembering that \( r(\lambda_H, \lambda_L) \) is defined in terms of both types’ \( \lambda_i \), and that the cost function is such that \( M'(0) > 0 \), there exist multiple equilibria.

- **Pooling manipulation.**

  In this case \( \lambda_H^* = \lambda_L^* = 0 \) and \( r(0; 0) = p \) iff

  \[
  r(0; 0) \Delta U \leq M_L'(0)
  \]

  and

  \[
  (1 - r(0; 0)) \Delta U \leq M_H'(0)
  \]

  Everytime the two conditions are satisfied there exist a PBE with the maximal manipulation or minimal dissonance. Everytime the DM steps away from them, she will the trade it with a higher dissonance. The conditions fix a lower bound for the existence of a PBE. To have a more compact representation of the boundary, let us define

  \[
  \tilde{\lambda} = \min \left\{ \frac{M_L'(0)}{r(0; 0)} \frac{M_H'(0)}{1 - r(0; 0)} \right\}
  \]

  Then whenever \( \Delta U \leq \tilde{\lambda} \) there exist a PBE with \( \lambda_H^* = \lambda_L^* = 0 \)

  However pooling manipulation can occur at different rate of \( \lambda_i \) every time \( \Delta U \leq \hat{\lambda}_i \), where,

  \[
  \hat{\lambda}_i : max \{ \lambda_L^*; \lambda_H^* \} > 0
  \]

  There are three possible cases of PBE for different \( r(\lambda_H^*; \lambda_L^*) \)

  1. \( \lambda_H^* = \lambda_L^* = \hat{\lambda}_i \) that is when, \( r(\hat{\lambda}_H; \hat{\lambda}_L) \) Let us consider \( \hat{\lambda}_i = 0.5 \). In this case

      \[
      \Delta U = M_L'(0.5) \frac{1}{r(0.5; 0.5)} = M_H'(0.5) \frac{1}{(1 - r(0.5; 0.5))}
      \]

      hence there exists a PBE.

  2. \( r(0; \hat{\lambda}_L) \) In this scenario there exists a PBE such that if \( \hat{\lambda}_L = 0.5 \)

      \[
      M_L'(0.5) \frac{1}{r(0; 0.5)} = \Delta U \leq M_H'(0) \frac{1}{(1 - r(0; 0.5))}
      \]
3. If $\hat{\lambda}_H = 0.5$ then a PBE will exist whenever 

$$M_U'(0.5) \frac{1}{(1 - r(0.5; 0))} = \Delta U \leq M_L'(0) \frac{1}{(r(0.5; 0))}$$

- **Pooling non manipulation.**
  It will occur whenever $\Delta U > \hat{\lambda}$

- **Mixed manipulation**
  In this scenario the DM will mixed between the two pure strategies. It occurs if $\hat{\lambda} \leq \Delta U \leq \hat{\lambda}$. Let us discuss how many of the possible combinations can be considered PBE given the assumption on the cost function $M'(0) > 0$.

  1. Whenever $H$-type manipulates completely and $L$-type recall properly, the incentives of the two type should be $\Delta U > M_H'(1)$ for $(1 - r(1; 0) = 1)$ and consequently $0 = M_L'(0)$ for $(r(1; 0) = 0)$. However, the last incentive does not hold because of the requirement of the cost function. Then this combination can not be considered an equilibrium.

  2. In the opposite case it occurs that $0 \leq M_H'(0)$ for $(1 - r(0; 1) = 0)$ and consequently $\Delta U > M_L'(1)$ for $(r(0; 1) = 1)$. The first condition respect the assumption imposed on the cost function, hence there is going to be a PBE for such a combination of strategies.

  3. If $H$-type partially rationalizes the signal while $L$-type chooses to recall the signal then, $(r(0.5; 1) = 1)$ and consequently the incentive conditions will be $0 = M_H'(0.5)$ and $\Delta U > M_L'(1)$. Both conditions support the cost function assumption, thus a PBE will exist.

  4. While, when the types swap their strategy, the opposite occurs with respect to the previous combination. In such a case it results that $0 = M_L'(0.5)$ for $(r(1; 0.5) = 0)$ and $\Delta U > M_H'(1)$ for $(1 - r(1; 0.5) = 1)$. Hence, since none of the two conditions fails to respect the assumption on the cost function, also in this case a PBE will exist.

**Appendix C**

**Fig. and Tab.**

Figure 1: Timing

<table>
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<tr>
<th>$T$</th>
<th>$\sigma$</th>
<th>$\hat{\sigma}$</th>
<th>$\theta^*$</th>
<th>$\epsilon$</th>
<th>$\theta_i S_{t+2}$</th>
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<td>$t+2$</td>
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</tbody>
</table>
Figure 2: DM’s decision Tree

Table 1: Posterior beliefs

|       | prob(σ) | prob(σ = i|σ = i) | prob(σ = H|σ = ∅) | prob(θ_e|e = 1) |
|-------|---------|----------------|----------------|----------------|
| H     | p       | λ              | r^*            | p              |
| L     | 1 - p   | λ              | 1 - r^*        | p              |
References


