Eurace Open: an agent-based multi-country model

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Abstract

In recent years the European Union (EU) has revealed a structural fragility and inadequacy to tackle some of the main challenges ahead. In this paper, we mainly focus on the migration challenges, further integrations of a monetary union, spatial inequality, and sovereign debt crisis. To address the relevant issues, we use a multi-country model that is an extension of the Eurace agent-based model. We design a flexible modeling framework that can host many countries allowing for different levels of integration among them. In this paper, the model hosts four countries. It is designed as a monetary union of two countries with the international labor market, international trade, and international financial market, and two independent countries that are always identical to the union members and serve as control states. We found that when countries are identical it is always beneficial to join the union, however, an excess of workers mobility can weaken the performance of the union and even create persistent inequality between countries. Moreover, the monetary policy of the union central bank can deteriorate if the difference between countries growth and in general it tends to overshoot the inflation target. When the two countries differ in productivity, the union on aggregate performs again better than the sum of the isolated countries counterparts. However, taking into account the welfare in both countries of the union, it is not always convenient to form the union. The performance of the union is strongly affected by the level of labor market frictions and the size of the productivity gap among countries. The real sovereign debt per capita in the union increases with low mobility frictions and high productivity gap. Stronger fiscal integration (fiscal transfers) reduces inequality between countries, allowing the low productivity country and, hence, the total union to sustain even with a higher mobility of workers.

Keywords: Agent-based multi-country model, fiscal policy, integration, migration, monetary policy, monetary union.

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1 Introduction

The global economic and financial worldwide crises, whose genesis is often associated with the collapse of Lehman Brothers (the largest corporate bankruptcy in U.S. history), had a pervasive impact on all the leading economies in the world. However, the place where it might have been more disruptive has been the European Union (EU), which revealed a structural fragility and inadequacy to tackle some of the main challenges ahead. The crisis on the middle east and the high migration flows into Europe is another example that contributed to reveal the weakness and the lack of coordination among countries in the EU. The geographical, cultural, and economic diversity of the countries of the union has not been harmonized in a well balanced and integrated project, and the whole union risked the collapse faced with these shocks and in particular handling the financial, sovereign debt and immigration crisis. The north and the south of Europe, with Greece and Germany as representative countries, started a confrontation that put Europe in danger. Threatening anti-European winds begun to blow, putting at risk the permanence of several countries in the union, finally claiming the first victim: the United Kingdom. A frequent topic during the years of the crisis, which nowadays seems temporarily abandoned, has been the two-speed (or multi-speed) Europe, whose basic ideas is that different parts of the EU should integrate at different levels depending on the political and economic situation in each country. The north vs. south or core vs. periphery are typical examples which have been proposed and discussed. Furthermore, the very weak fiscal integration in the EU does not allow for a common “federal” policy strategy but often, fosters conflicts among its member states, which pursue short term individual goals at the expense of one common goal in the union. Prioritizing the individual goals creates a competitive environment between countries and leads to further confrontations and sub-optimal outcomes. For instance, high unemployment rate in southern “periphery” countries results in a huge waste of human resources and the social welfare loss for the entire union. The European Central Bank (ECB), which is actually the only supranational institution able to take effective decisions, is not able, alone, to conduct economic policy in Europe; both because monetary policy is not enough, if not accompanied by integrated fiscal strategies (more coordinated than the mechanical Maastricht fiscal constraints), and because the ECB safeguards the security of the banking system, which is not the only actor in the economy. In the Spanish case (see for instance Petrović et al., 2016), there has been a clear conflict between the interest of banking system and the interest of the households, and we think that the Bank of Spain (and probably the ECB) did not put enough effort into guaranteeing households’ rights with proper regulations and controls. On the top of this, there is also an intrinsic issue related to a common monetary policy in a union of countries which have several heterogeneities.

Since the EU is continuously facing challenges in the last decade, it is important to investigate under what conditions is it optimal for a country to join a monetary union and what is the optimal design of the union to maximize social welfare. Indeed, several streams of literature tackled this important topic from different perspectives (a complete literature review is provided in the next section), but the peculiarity of our work is to start from a well-established agent-based modeling technique for a single country model and extend it to a multi-country framework. We are not devising a simplified setting, dedicated from the beginning to examine stylized open economies,
but we start from complex single country descriptions, which have an intrinsic value, trying to understand what happens when these countries join in union. Certainly, this approach has both advantages and limitations. On the one hand, it allows for a realistic analysis of the problem, as agent-based models mimic with greater detail and realism the functioning of modern economies. On the other hand, the complexity of the model and its computational nature makes it difficult to trace the causal relationship in the system and it comes at the expense of transparency. One way to overcome this is to take the model to the data.

In this paper we are taking our first steps into this complex field, trying to formulate some basic questions and to give them some appropriate answers. The first research question we tackle is under what conditions it would be convenient for two identical countries to join a union. To answer the question, we design a setup with four identical countries, two isolated and two in a union. We study how mobility of workers can affect the economic performance of the union and of its members. Our most important result is that the economic performance of the union is nearly always better than the performance of the individual countries. However, lack of labor regulation (or frictions) can destabilize the union during crises, creating permanent unbalances which undermine economic activity in the member countries.

The second research question is whether \textit{ex-ante} productivity heterogeneities amongst member states can weaken the performance of the union, or even create unfavorable conditions for joining the union in the first place. This question resembles the Germany-Greece issue, or any other where the productivity gap between countries is high. Further we investigate is it convenient for the high productivity country to form a union? And what about the low productivity one? Is there a threshold in productivity gap after which the union does not work? What about the distributional effects of the union?

These are some of the questions we aim to answer in this work, finding that high productivity gaps, combined with low mobility frictions, can bring about massive migrations that can rise inequality and harm the performance of the union. Finally, we design a stylized fiscal integration mechanism that we call “fiscal pool”, consisting of a centralized deposit account where member countries of the union are obliged to put a part of their budget surplus (if any). On the other hand, countries that need to finance budget deficit can ask (and obtain) money from the fiscal pool. We show that this simple transfer mechanism can improve the conditions of the “poor” country of the union allowing higher government provisions to its citizens and consequently give them greater incentives to stay in the country. The governments also finance their budget deficit raising new public debt, but the austerity policy limits it. If sovereign debt reaches a level that is greater than a policy given threshold, the governments enter austerity regime reducing their public spendings and consequently households’ well-being, since public transfers and benefits are part of households’ disposable income. Therefore, when fiscal transfers are available, countries can use funds from the fiscal pool to finance their public expenditures, instead of raising new debt, and potentially avoid austerity measures. Conversely, without fiscal transfers, union countries can only finance their budget deficit through new public debt. This may lead to austerity measures that would reduce government expenditures and thus households’ income. Finally, lower incomes would increase the likelihood of migration since households are not willing to stay in their countries if they can find better earning opportunities abroad. The outflow of workers also acts as a sort of accelerator mechanism. With a lower amount of production factors (workers)
the economic activity declines, and so tax revenue, which may force the government to raise new public debt and even further decrease public expenditures.

2 Literature review

Agent based models (ABMs) are a tool that facilitates the analysis of economic crises, given that they can easily address non-linearities such as liquidity problems, bankruptcies, domino effects, systemic risk, speculative bubbles, and credit crunches (see e.g. Raberto et al., 2012). For instance, Delli Gatti et al. (2010) propose a network based accelerator where small local shocks can trigger large systemic effects, because of contagion and default cascade mechanisms, showing that not only can a firm’s bankruptcy spread to other firms directly through the market for trade credit, but also indirectly weakening the banking sector (bad debt), thus constraining the supply of bank loans. Riccetti et al. (2013) enriched this framework with the analysis of the leverage cycle, following the recent literature on leverage (Adrian and Shin, 2010; Brunnermeier and Pedersen, 2009; Fostel and Geanakoplos, 2008) that shows the importance of changes in leverage during an economic cycle as amplification mechanism of shocks. Other agent-based models, such as Iori et al. (2006) or Battiston et al. (2012), focus on the interbank market. Those papers allow to understand that in a banking system composed of homogeneous banks, the interbank market unambiguously stabilizes the system, but, with heterogeneous banks, knock-on effects become possible and the interbank market plays an ambiguous role. Indeed the increasing connectivity of the interbank network implies a more severe trade-off between the stabilization effect of risk diversification and the higher systemic risk associated with potential bankruptcy cascades triggered by stronger connectivity.

Financial markets have been the first research domain where agent-based micro-models have provided meaningful results starting from late 90s. While the efficient market hypothesis is unable to catch many interesting stylized facts, agent-based micro-models have provided some insights on volatility clustering and fat tails of return distributions. Following the pioneering Santa Fe Artificial Stock Market, SF-ASM (LeBaron et al., 1999), several artificial stock markets have been developed over the last years. For instance, Lux and Marchesi (2000), Chiarella et al. (2002), Kirman and Teyssiere (2002), LiCalzi et al. (2003), Chiarella et al. (2009) and Tedeschi et al. (2009) are examples of agent-based micro-models able to explain the influence of traders’ behaviors on the persistence of asset price volatility. The models cited above have looked at the effect of coordination of traders’ strategies via market-mediated interactions (for example when agents follow common chartist trading rules). Collective behavior nonetheless could reflect the phenomenon known as herding which occurs when agents take actions on the basis of directly imitating each other. The studies on herding effects have focused on how herding can lead to large price fluctuation and on its role as a source of volatility clustering (Stauffer and Sornette, 1999; LeBaron and Yamamoto, 2008). Differently from the Santa Fe model, the Genoa Artificial Stock Market (GASM) (see for details Raberto and Cincotti, 2005), being populated by nearly zero-intelligence traders with limited financial resources, focuses more on the market structural properties than on the agents’ behavioral features. In particular, the GASM, by successfully reproducing the main financial stylized facts, demonstrates that budget constraints
and market micro-structure properties, such as the clearing mechanism, are at least as important as the traders’ behavioral features in explaining the emergence of the statistical properties of financial time series.

Besides agent-based micro-models that are widely used in studies on financial (stock) markets and specifically traders’ behavior and coordinations, De Grauwe and Gerba (2017) use a behavioral bounded rationality macro-model including financial frictions on the supply side to study how imperfect credit and stock markets affect allocations on the production side of the economy. They found that when behavioral aspects are jointly included with supply-side financial frictions in a standard financial accelerator model, the transmission of shocks is significantly intensified via the stock prices mechanism. A sharp increase in stock prices not only allows firms to increase their credit and capital demand but also can reduce the input costs for firms. Conversely, a sharp drop in asset prices can limit the supply of credit to firms, raise their production costs, and reduce the supply of capital and their production capacity (or productivity) over time.

Among the present agent-based macro-models, see e.g. Popoyan et al. (2017); Gaffeo et al. (2011); Gabbi et al. (2015); Gualdi et al. (2015); Wäckerle et al. (2014); Riccetti et al. (2013), it is worth citing the Keynesian model by (Dosi et al., 2010), which has been recently used to test different fiscal and monetary policy scenarios (Dosi et al., 2013, 2015, 2016). Results show that the introduction of constrained fiscal rules mimicking the Stability and Growth Pact or the Fiscal Compact worsen the performance of the economy as well as public finances, while the best policy mix able to stabilized macroeconomic fundamentals is the combination of an unconstrained fiscal policy with a dual-mandate monetary policy, targeting both inflation and unemployment. Among other agent-based studies on the recent crisis, Assenza et al. (2015) focus on the emergence of a crisis from micro behavioral interactions, while Klimek et al. (2015) focus on crisis resolution mechanisms, finding that there are no economic conditions under which a taxpayer-funded bail-out outperforms the bail-in mechanism with private sector involvement.

Summing up, agent-based models are a promising way to develop an alternative micro-founded theory both for descriptive and normative (policy) purposes1, taking the Lucas critique (see Lucas, 1976) seriously. ABMs have the advantages of flexibility and modularity in model building and dispense with theoretical consistency requirements. The ABM approach avoids the oversimplification of the optimizing agent assumption by relying on a more coherent description of agents’ behavior, which is based on heuristics and behavioral patterns instead of rationality, as pointed out by the increasing literature on behavioral economics (Kahneman and Tversky, 2000; Gigerenzer and Selten, 2002; De Grauwe and Gerba, 2015). ABMs also easily allow for adaptive rules and learning (Simon, 1979; Kirman, 2011).

The Eurace agent-based model that we use in this work provides a rich scenario of interactions between real and financial variables. It is a large-scale agent-based model and simulator representing a fully integrated economy consisting of three economic spheres: the real sphere (consumption goods, investment goods, and labor markets), the financial sphere (credit and financial markets), and the public sector (Government and Central Bank), see Cincotti et al. (2012). Following the standard agent-based approach, Eurace economic agents are character-

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1See for instance Dawid and Fagiolo (2008).
ized by bounded rationality and adaptive behavior as well as pairwise interactions in decentralized markets. The balance-sheet approach and the stock flow consistency checks have been followed as a key modeling paradigm in Eurace. The computational results show the real effects on the artificial economy of the dynamics of monetary aggregates, i.e. endogenous credit money supplied by commercial banks as loans to firms, and fiat money created by the central bank (Cincotti et al., 2010; Raberto et al., 2012). In particular, Eurace shows the emergence of endogenous business cycles which are mainly due to the interplay between real economic activity and its financing through the credit market, thus shedding light on the relation between debt, leverage and main economic indicators (Raberto et al., 2012; Teglio et al., 2012). Moreover, Eurace shows that a quantitative easing monetary policy coupled with a loose fiscal policy generally provides better macroeconomic performance in terms of real variables, despite higher wage and inflation rates. In particular, the government is able to finance budget deficit through the quantitative easing mechanism without increasing tax rates, which retains the economic activity at the higher level, however, at the expense of higher wage and inflation rates (see e.g. Cincotti et al., 2010). The Eurace model has also been employed to test regulatory policies providing time varying capital requirements for banks, based on mechanisms that enforce banks to build up or release capital buffers, according to the Basel III regulatory framework and the countercyclical capital buffer (CCyB) principles. Cincotti et al. (2012) have shown that the dynamic regulation of capital requirements is generally more successful than fixed tight capital requirements in stabilizing the economy and improving the macroeconomic performance. It affects the economic performance in the medium-long run reducing the financial fragility (or systemic risk) in the economic system and potentially prevents triggering chains of firms insolvency bankruptcies.

Due to emergence of new challenges in the European Union such as immigration crisis, further integrations, spatial inequality, sovereign debt crisis, etc., agent-based models have started to incorporate a spatial dimension. A new wave of agent-based models have shifted the focus of the analysis from closed to open economies allowing for an international dimension in the agent-based literature. Recent works explore how policies affect convergence and the economic performance of countries in the union using a multi-country model with heterogeneous economies. On the one hand, Dawid and Neugart (2011) use the Eurace@Unibi model to study the convergence between the two regions with different productivity levels including the physical capital mobility, the labor mobility, and international trade. They found that different labor market policies may impact the convergence between the regions. Using the same model, Dawid et al. (2014) extend the analysis by studying how policies aimed at human capital and adoption of technologies may impact the convergence between regions with different productivity levels. They advocate that the government may use above mention policies to encourage convergence, however, necessarily taking into account the labor market policies. On the other hand, Caiani et al. (2017) use a multi-country model, to study how the contractionary and expansionary fiscal policies can impact the performance of the countries in the union. In particular, they show that the increase in the maximum deficit-to-GDP ratio (expansionary policy) improves the dynamics of GDP, labor productivity, and employment, however, at the cost of a higher level of public debt and inflation.

The model that we present in this work is similar to the model given by Dawid and Neugart (2011) and Dawid et al. (2014). We have expanded the original Eurace model (given by Teglio
et al., 2012; Cincotti et al., 2012; Raberto et al., 2012) to a multi-country setting. Our model can accommodate $N$ heterogeneous economies (countries) allowing for a Monetary Union, international trade, international stock market and labor mobility. Our analysis focuses not only on the setup with countries that exhibit different productivities but also on the setup with identical countries. The main difference with the previous literature is that we do not allow for the technology growth. Thus, in the case when countries have different but constant productivity levels, we study the optimal policies to adopt given the constant productivity gap among countries. This assumption is inspired by the empirical evidence which shows that countries in the union have not converged on average (Sondermann, 2014; Tsionas, 2000). We study how various fiscal and labor market policies may affect the dynamics of each country in the union and on aggregate. We propose redistribution of income in the union via a fiscal pool, showing that the pool is beneficial for the union and in particular for the low productivity country in the presence of large productivity gap. In the analysis, we assess major economic aggregates such as GDP (per capita), consumption (per capita), sovereign debt (per capita) and its structure, government expenditures, unemployment rate, inflation, etc.

The agent-based model and the analysis that we provide in this study focuses on economic and integration issues as well as policy analysis within a setup that is inspired by the European Union. However, the paper contributes not only to the literature on heterogeneous agents but also to the literature on Optimal Currency Areas (OCA), international trade and the economic effects of migration especially in the context of the EU Eastern enlargement. Therefore, although the aim of this paper is the policy analysis in the monetary/political union using the agent-based model, we also show how our work links to the standard literature related to international trade, integration, and migration.

The OCA theory investigates the costs and benefits of being in a common currency area. Some of the most important criteria for this analysis, which is also essential for our work, include labor mobility, wage and price flexibility, the effectiveness of monetary policy, correlation and variation of shocks and synchronization of business cycles. According to this theory, factor mobility (especially labor mobility) is an important adjustment mechanism in the absence of exchange rates and when prices and wages are rigid (Mundell, 1961). Furthermore, De Grauwe (2003) shows the importance of the labor market institutions in the integration process, emphasizing that countries may have different labor market centralization. He uses the theory of Bruno et al. (1985) and explains that in the centralized labor markets with the centralized wage bargaining (labor) unions a supply shock will not produce an excessive increase in nominal wages since the labor unions know that the increase in nominal wages will lead to higher inflation without affecting real wages. Moreover, Calmfors and Driffill (1988) show that a similar result comes from highly decentralized labor markets where wage bargaining is kept at the firm level. For instance, if a supply shock hits the country, the excessive nominal wage increase would have a direct effect on the competitiveness of companies. However, labor markets with an intermediate labor union centralization have a different approach to supply shocks. In this setting, a labor union expects that the increase in wages will have only a small effect on inflation, so it starts the bargaining process. Since all labor unions do the same, they finally end up with higher inflation than expected and consequently with lower real wages. De Grauwe (2003) advocates that different labor market institutions may be costly for the countries in currency unions.
Our model incorporates identical labor market institutions among countries in the union with highly decentralized labor markets\(^2\). However, if firms increase labor demand to augment their production capacity due to a positive demand shock, they will compete for workers in the labor market by increasing nominal wages since 1) they will aim to attract more workers, and 2) the increase in wages for those who hire new workers will not affect the production costs in the same proportion\(^3\). Our results suggest that in the case of asymmetric shocks, labor mobility helps to restore equilibrium in the system, always providing lower unemployment rates in the economies. Nevertheless, new equilibrium may not be efficient, given that the performance of the total union is worsening when the labor force is fully mobile, and the countries in the union are identical. In this case, countries tend to polarize regarding the total population, reaching a new stable path which is characterized by the extremely unequal distribution of households resulting in the suboptimal use of the production factors. On the other hand, the moderate labor mobility is beneficial for the union with heterogeneous countries and mainly for small and less productive economies.

Diverse shocks that may hit the members of common currency areas can be also mitigated through the fiscal integration between countries. Kenen (1969) emphasizes that the fiscal transfers between regions can absorb the impact of asymmetric shocks. We also perform a fiscal policy experiment in the last part of the paper showing that financial transfers in the monetary union mitigates diverse shocks and reduce inequality between the high-tech and low-tech countries. However, while the integrated fiscal policy may have a positive impact on the performance, Corden (1972) argues that countries in the monetary union lose direct control over the monetary policy and exchange rate. This means that if a country undergoes a negative demand shock, it will not be able to efficiently use its monetary policy and exchange rate mechanism to facilitate the adjustment of relative wage and prices. Therefore any adjustment will be conducted through increased unemployment, reduction of nominal wages and prices or fiscal policy measures. Our results show that the union central bank is not able to attain the inflation target. This outcome emerges due to incompatible monetary policy. For instance, when a country in the union undergoes a negative demand shock the labor market adjusts in the short run, while the price adjustment takes a longer time to reach a new stable path. During the price adjustment period, the monetary policy is less effective in reaching the inflation target due to the temporary divergence in prices over countries since the union central bank always considers the average inflation level among countries. Thus, countries suffer from the unsuited monetary policy. Regarding the independence and efficiency of monetary policy in common currency areas, the general theory states that the higher the association of shocks among the countries the lower the costs of losing independent monetary policy are (Alesina et al., 2002).

The international trade literature point out the issue of endogeneity vs. specialization. Frankel and Rose (1997) explain that rising the trade volume among countries in the monetary union can have two side effects. On the one hand, it can cause industrial specialization among countries in the goods in which they have comparative advantage, leading to industry specific shocks and asynchronous business cycles. On the other hand, increased trade may generate a higher cost of losing independent monetary policy are (Alesina et al., 2002).
correlation between countries’ business cycles if intra-industry trade accounts for the most of the trade, resulting in common demand shocks, which is the authors’ standpoint. Frankel (1999) also emphasizes that neither openness nor income correlation is fixed over time and that they can change in response to the endogenous and exogenous factors. The author argues that the income correlation is likely to increase over time once the country is in the monetary union. In our model, we do not allow for the industry specialization among countries since all firms produce homogeneous products, hence the intra-industry trade accounts for all trade. We design a perfect international goods market showing that the trade improves the synchronization of business cycles over time even in the absence of the labor mobility. In particular, we show that the union members exhibit higher synchronization in outputs, unemployment rates, wage, and prices.

The economic implications of labor mobility have been addressed meanwhile by many scholars. Empirical and theoretical studies which are focused on the labor market effects of immigration find that immigrants lower the price of factors that are a perfect substitute, and increase the price of factors if they are complements (Okkerse, 2008). Thus, in general, immigration slightly lowers wages of less-skilled workers and earlier immigrants (see for instance Jaeger, 1995, Sarris and Zografakis, 1999, Orrenius and Zavodny, 2003 and Greenwood et al., 1997). Furthermore, Gross (2002) provides the evidence that in the long run immigrants create more jobs than they occupy, thus unemployment lower permanently. Complementary to this, the probability that immigrants will rise unemployment is zero in the long run, and negligibly low in the short term (see the evidence for the US and Canada Marr and Siklos, 1994, for the EU Simon et al., 1993 and for Australia Gang Tian and Shan, 1999).

In our model workers are always perfect substitute. Moreover, we do not address effects of labor mobility on different labor market segments assuming that firms hire workers regardless to their education level. Thus, given the downward nominal wage rigidity, real wages may decrease in the net receiving country in the short term due to the inflation effect, while in the long-run immigrants create more jobs than they occupy, lowering the unemployment rate and pushing the nominal and real wages up.

Further effects of labor mobility on macroeconomic aggregates were widely examined in the theoretical literature using the computable general equilibrium (CGE) framework, especially in the context of the EU Eastern enlargement. This type of macroeconomic modeling enables the analysis of the interaction between migration, capital movements, and trade which is quite similar to our study. The primary focus of the CGE literature is on the changing skill composition of the workers via migration assuming that labor moves from a sending to a receiving country, while in our model the workforce is meant to fluctuate in both directions. Irrespective of countries differences in productivity and size we may observe net sending or net receiving countries as an emerging result of individual decisions. Overall, the CGE literature finds larger implications of migration on wage and unemployment than those found in the empirical literature. Notably, the negative effects of immigration, in particular for low-skilled workers, are outweighed by positive and strong effect coming from the integration of goods markets (e.g. Baldwin-Edwards, 1997). Therefore, most of the models predict that the EU Eastern enlargement results in higher wages and lower aggregate unemployment in both receiving and sending countries.
countries which is in line with the results that we provide in this paper. Besides, CGE models predict that GDP will increase in the receiving country and the total EU. This effect is even amplified if the trade creation between existing and new member states is taken into account (Boeri and Brücker, 2005). However, the gains in aggregate and per capita income might be reduced due to labor market rigidities. Baas and Brücker (2008) analyses possible diversion effects due to transitional periods in labor mobility. In particular, Germany has kept restrictions on labor mobility with new member states after the EU Eastern enlargement in 2004, while the UK has opened the job market. The authors show that the closure of labor markets in Germany has diminished the GDP effect, while the opening of the UK labor market has resulted in larger GDP.

Nevertheless, our study suggests that a completely open and flexible labor market may worsen the economic performance of a union in aggregate and its members, especially if we consider the setup with two identical countries in the union. The discrepancy between our results and the evidence from the previous literature emerges mainly due to different modeling setups. While the CGE literature imposes global-institutional and asymmetric constraints on the job market, we impose local restrictions on the households’ behavior. Thus we analyze what the impact of labor rigidity/flexibility on the economic performance having symmetric labor market policies among countries is.

In general, international migration leads to the more efficient use of human capital and hence, should increase global output. Many studies advocate that the gains from opening labor market can outperform those from a further liberalization of international goods and capital market (Boeri and Brücker, 2005). However, an important issue for the evaluation of migration impacts is the adjustment of other markets in the economy. Namely, while many studies assume that the capital stocks are fixed over time, some empirical evidence shows that capital-output ratio and hence, the capital intensity of production remains constant in the long-run (Kaldor, 1961). In fact, in our model, both outcomes might endogenously emerge and may have several different implications on the global result. If a net receiving country is not able to maintain the capital-output ratio while a net sending country wastes capital stocks due to the outflow of workers the total union will operate in the suboptimal distribution of employees with the lower aggregate and per capita production. Contrary, if the net receiving country manages to maintain the capital-output ratio, the whole union will operate at the higher output level. In the following sections, we present the model as well as analyze and discuss the results in details.

3 The model

The agent-based multi-country model presented in this paper is given in discrete time $t = \{1, 2, ..., T\}$. The time unit $t$ represents one iteration and it is a proxy for one working day. The model consists of a set $C$ of heterogeneous countries where each country has the government $\{g|g \in G\} : G \leftrightarrow C$, a set $H_g$ of Households $\{h|h \in H_g\}$, a set $F_g$ of consumption goods producers – firms $\{f|f \in F_g\}$, a single capital producer ($k$), a set $B_g$ of commercial banks.

\footnote{In the model one week has five days, one month has twenty days while one year contains twelve months.}
\{b|b \in B_g\}$ where we set for simplicity $\forall g \implies B_g = 1$, the central bank ($cb$), the Union central bank ($ucb$), and the statistical office ($stat$). The agents demand and supply real assets, financial claims and monetary flows interacting on five markets: the consumption goods market, the labor market, the financial market (bonds and shares market), the credit market and the physical capital market. The summary of agents’ interactions is provided in table 1.

### Table 1: Interaction Matrix

<table>
<thead>
<tr>
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<th>F</th>
<th>CP</th>
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<th>Bank</th>
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<td>CP</td>
<td>capital-bill (money)</td>
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<tr>
<td>HH</td>
<td>vacancies, wage, shares, dividends</td>
<td>shares, dividends</td>
<td>shares, dividends</td>
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<tr>
<td>Bank</td>
<td>interests and principals, deposits</td>
<td>deposits</td>
<td></td>
<td>loans</td>
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<tr>
<td>CB</td>
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<td>interests and principals, deposits</td>
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<td>GOV</td>
<td>tax</td>
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<td>tax</td>
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<tr>
<td>STAT</td>
<td>Fs’ data</td>
<td>CPs’ data</td>
<td>HHs’ data</td>
<td>Banks’ data</td>
<td>CBs’ data</td>
<td>GOV’s data</td>
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Table 1. Agents in rows demand real assets (denoted in capital letters); financial assets and their related monetary commitments (denoted in bold and italics respectively); and money and data flows (given in small caps). Agents in columns supply corresponding real assets, financial assets and their related monetary flows as well as money and data flows.

The model is designed as a credit economy where firms and the financial system are the engine of the model dynamics. Firms produce durable, homogeneous consumption goods employing physical capital and labor. The purchase of physical capital is investment and it is determined by the expected demand of consumption goods and the expected profitability of the project (Net Present Value investment function). Physical capital is infinitely supplied, and its productivity is constant over time implying that the model has constant productivity growth at zero. Therefore determining country-specific productivity levels as an initial condition provides constant productivity gap among countries in the model. This assumption is the opposite of the one used in studies on convergence where different regions in the union are assumed to converge in productivity over time, and where policies may foster the convergence (Dawid et al., 2014). Therefore, here we examine what policies should be adopted in the Union to sustain having the everlasting productivity gap.
Firms participate in all markets. They buy physical capital from the capital goods producer, hire workers participating in the labor market, demand new bank loans on the credit market, issue new shares on the financial market and finally sell their products on the consumption goods market.

Firms require labor to fulfill their planned productions. They post vacancies on the labor market and set the wage offers to hire new workers. The model assumes a downward nominal wage rigidity which means that employers are unwilling to reduce salaries in nominal terms, while in real terms wages can decrease; on the other hand, employers increase the wage offers depending on demand and supply on the labor market. For instance, an excess in labor supply will force firms to raise wages to attract new workers. This will, in turn, have a positive feedback on the unit production costs and consequently drive higher prices. Companies finance their investments through internal funds, bank loans, and new equity shares following the pecking order theory (Myers and Majluf, 1984). However, they can be rationed on the credit and capital markets. On the one hand, the credit allocation rule in the model is built upon Basel II and Basel III accords and in particular the rationing on the credit market is the result of firm financial fragility and bank capital requirements. On the other hand, issuing new equity on the financial market is uncertain and depends on households’ expected profitability of investing in new shares.

Firms produce homogeneous consumption goods, set firm-specific prices based on the unit production costs and deliver the products to all malls within the union.

Households supply labor to firms and the government and receive monthly wage if they are employed or receive unemployment benefit otherwise. Unemployment benefit is lower than wage, thus households always prefer to work than to be unemployed. They also receive other incomes such as government transfers, bond coupon payments as well as the dividend payments. The government provides monthly lump-sum transfer to all residents and pays monthly bond coupons to the bond holders, while firms, the commercial bank, and the physical capital producer pay dividends to their stakeholders. All households are endowed with an equal amount of the government bonds and stocks at time $t = 0$. However, they can either trade with the government bonds and the firm and bank stocks on the secondary market or buy new bonds and firm stocks through an initial public offering on the primary market. The commercial bank and the physical capital producer do not issue new equity shares, and the households do not trade with the stocks of the physical capital producer on the secondary market, implying that all households always hold an equal and constant number of capital producer’s stocks. Consequently, since the capital producer does not employ any inputs while providing equipments, all its earnings after paying tax is redistributed back to the households in equal shares. Households’ disposable income can be consumed, invested on the financial market or retained on their payment account without earning any interests on the balance.

The capital producer supplies infinitely physical capital to all firms in the economy. It creates new physical capital without employing inputs in the production process. Thus, having no production costs its gross profit is equal to its revenue which is given as: $\sum_f K_{f,t} \cdot P_{Kt}$, where $K_{f,t}$ is the amount of physical capital provided to firm $f$, and $P_{Kt}$ is the capital price given as:

$$P_{Kt+1} = \theta \cdot P_{Kt} \cdot \frac{\bar{\pi}_t}{12} + (1 - \theta) \cdot P_{Kt} \cdot \frac{\pi_t}{12}, \quad (1)$$
where $\theta \in [0, 1]$ is the parameter that accounts for the trust in the central bank announcements, $\pi_t$ is the yearly inflation target announced by the central bank at time $t$, and $\pi_t$ is yearly inflation at time $t$. The capital producer pays the capital tax and redistribute its net profit to the shareholders in terms of dividend payments.

The artificial economy is further composed of one commercial bank which represents the banking sector, the government, the central bank and the Union central bank. The bank issues new loans to firms following the endogenous money creation principle and charge the risk-based interest rate above the policy rate. Profit is then either redistributed back to households in terms of dividend payments or accumulated to recapitalize the banking sector if the value at risk (risk-weighted assets portfolio) is over the policy threshold. The monetary policy is conducted by the central bank or the Union central bank, in the case of the currency union. Both institutions are modeled as the lender of last resort, and their main tasks are to set the bank capital requirement, to establish the interest rate using the Taylor rule (Taylor, 1993a) and to maintain the payment account of the government and commercial banks. The Government creates country-specific fiscal policies, hire public workers and redistribute income. It collects the V.A.T and capital tax, sells bonds on the financial market and finally pays the unemployment benefits, public wages and lump-sum transfers. The transfers are provided to all households that reside in the economy and they represent a proxy for public goods.

Each agent has its balance sheet where all physical and monetary stocks are recorded and updated at the moment of their transaction. Agents’ balance sheets ensure that the model is stock and flow consistent at any point in time. The detailed overview of agents’ balance sheets is given in the table 2 in the appendix.

3.1 From a single to multi-country setting

Before we describe economic agents and their behaviors, it is worth mentioning several general features of our multi-country model. As we mentioned before, we designed a flexible multi-country agent-based model following the Eurace framework. The model can host $N$ heterogeneous countries that can run independently, as closed economies, or in Unions. The countries may differ in many characteristics. For instance, quantitatively, they may have a different number of households, a different number of firms or banks. Whereas, qualitatively, countries can have a different household population structure in terms of their education levels or their consumption preferences and constraints; banks may have different credit granting mechanisms; firms may differ in their innovation strategies, physical capital producers can provide machines with different productivity levels and at the same time determine a productivity level of a country, and finally, each country can have unique or common fiscal and monetary policies. These are some of the features that may be used to capture the heterogeneity among countries. However, note that any behavioral role or interaction of the agents that we will describe in the next section can be a country specific.

The Unions can integrate countries at different levels: 1) international goods markets, 2) international labor markets, 3) international financial markets, where stock and bonds can be traded by all the citizens of the Union, 4) a common monetary policy (monetary Union). We can simulate
any combination of independent countries and Unions whereas the Unions can be characterized by any number of the four cited properties. We present the example of one possible configuration of the model in figure 34 in the appendix. However, in this study we use the setup with four countries. The model in this paper is designed as a monetary union of two countries with international labor market, international trade, and international financial market; and another two independent countries that are always identical to the union members and serve as control states.

In the following section we describe in details agents’ behaviors and interactions.

### 3.2 Firms

Firms employ physical capital $K$ and labor force $n$ to produce homogeneous durable consumption goods. We denote by $q_{f,t}$ real monthly output generated by firm $f$ at time $t$. The consumption goods are freely traded among countries in the union such that firms deliver their products to all malls within the union, while households shop only in the country of residence. Each country $g$ has one representative mall; therefore, $g$ indicates a country, the government, a mall or a country specific market.

The production is modeled using the standard Cobb-Douglas production function with a positive firm-factor productivity (FFP) $\alpha$ and the constant returns to scale given as:

$$q_{f,t} = \alpha f_t \cdot (n_{f,t})^\alpha \cdot (K_{f,t})^\beta.$$  \hspace{1cm} (2)

Firms solve the production planning problem with stochastic demand and stock-out costs. Their production plans depend on the expected demand and the level of inventories inherited from the past. We denote the total production plan of firm $f$ at time $t$ as $\tilde{q}_{f,t}$, the stock of inventories of firm $f$ at mall $g$ at time $t$ as $I_{f,g,t}$, and the expected demand of firm $f$ on the market $g$ at time $t$ as $E_t[q_{f,g}]$. Firms form beliefs about future demand based on past sales and after taking into account the current stocks they calculate market specific planned production such as: $\tilde{q}_{f,g,t} = E_t[q_{f,g}] - I_{f,g,t}$. Since the production is centralized in the country of origin, firms calculate the total planned production summing up all positive market specific planned outputs: $\tilde{q}_{f,t} = \sum_g \tilde{q}_{f,g,t}$. However, the final output is still uncertain and depends on firms’ production capacity and available funds. If companies are not able to produce the planned quantity with the current capacity, they will seek for additional workers and investments in new physical capital. On the one hand, the number of new employees will be determined on the labor market, and it will depend on the aggregate demand and supply for labor, which means that firms not only bargain with workers but also compete with other companies on the market. On the other hand, firms’ investment decision is built upon the Net Present Value (NPV) calculation which states that firms will choose the amount of investment that maximizes the present value of all future returns. Nevertheless, the selected investment will be realized if firms have sufficiently available funds. In the case when they are financially constrained, i.e. they do not have sufficient internal resources, and yet they are rationed either on the credit or financial markets, investments will be adjusted accordingly.

$^5 \alpha + \beta = 1.$
The number of workers needed to achieve the production plan is derived from the equation
(2) and it is given as:

$\bar{n}_{f,t} = \left( \frac{\tilde{q}_{f,t}}{\mathcal{A}_{f,t} \cdot (K_{f,t})^{\beta}} \right)^{\frac{1}{\alpha}},$  \hspace{1cm} (3)

where $\mathcal{A}_{f,t}$ is the FFP calculated as:

$\mathcal{A}_{f,t} = \min[\bar{s}_{f,t}, \gamma_{f,t}],$  \hspace{1cm} (4)

where $\gamma_{f,t}$ is the productivity of firms’ physical capital, while $\bar{s}_{f,t}$ denotes the average specific skill of workers in the firm $f$. Note that in the absence of technology growth the productivity of firms’ physical capital becomes country specific such as $\gamma_{f,t} = \gamma_g$, which means that all companies in the country use physical capital with equal productivity, where $\gamma_g$ represents the technology frontier of a country $g$. The final number of vacancies that the firm $f$ will post is given as a difference between the number of employees needed to fulfill the production plan and the current number of workers ($\bar{n} - n$).

### 3.2.1 Investment

Firms’ investment decision is in the first place determined by the amount of physical capital $\tilde{K}_{f,t}$ required to achieve the production plan for the given number of workers $n_{f,t}$, and it is derived from the equation (2):

$\tilde{K}_{f,t} = \left( \frac{\tilde{q}_{f,t}}{A_{f,t} \cdot (n_{f,t})^{\alpha}} \right)^{\frac{1}{\beta}}.$  \hspace{1cm} (5)

The difference between the desired amount of physical capital and the current capital stock $\Delta\tilde{K}_{f,t} = \tilde{K}_{f,t} - K_{f,t}$ is the maximum amount that firms are willing to invest. Therefore, using the investment upper limit, firms determine the grid of visible investment values $i^f$ such that $0 \leq i^f \leq \Delta\tilde{K}$ and maximize their NPV function with respect to $i^f$.

The NPV function takes into account the discounted values of the future cash flows generated by the augmented production capacity, and the present cost of investments ($p_{K,t,g} \cdot i^f_{f,t}$), where $p_{K,t,g}$ is the actual capital price at time $t$ in the country $g$. In particular, the NPV function is given as:

$NPV(i^f_{f,t}) = (1 - \tau_{ct,t}^{f,g}) \sum_{m=1}^{N} \bar{E}_t[P_{f,m}] \cdot \Delta q_{f,t,m} - p_{K,t,g} \cdot i^f_{f,t}$,  \hspace{1cm} (6)

where the cash flows are discounted over the future $N$ months; $r_{f,t}$ is the yearly loan interest rate for firm $f$ at time $t$; $\tau_{ct,t}^{f,g}$ is the corporate tax rate at time $t$ in the country $g$; $\bar{E}_t[P_{f,m}]$ is the weighted average expected price level for firm $f$ at time $t$ in the future month $m$; and $\Delta q_{f,t,m}$ is the additional

---

6Revenues corrected by the amount of tax.

7$N$ is the number of months for which the new capital investment will depreciates to the 1% of its value.
amount of production in the future month \( m \) given by the capital investment \( i_{f,t} \) after taking into account the capital depreciation rate \( \xi \), i.e.,

\[
\Delta q_{f,t,m} = A_{f,t} \cdot (n_{f,t})^\alpha \cdot (K_{f,t} \cdot (1 - \xi)^m + i_{f,t})^\beta - A_{f,t} \cdot (n_{f,t})^\alpha \cdot (K_{f,t})^\beta.
\] (7)

Each firm calculates the net present value of the future investment taking into account firm-specific as well as country-specific variables. Therefore, an exporter who is planning to invest in new capital should calculate the weighted average tax rate \( \tau_{vat} \) and the weighted average expected price level \( \bar{E}_t[P] \) across the countries using the country-specific weights \( \lambda_{f,g,t} \in [0,1] \), such that the equality \( \sum_g \lambda_{f,g,t} = 1 \) holds. However, for simplicity we set the weights for the domestic market equal to one and the foreign market equal to zero, since the tax rates are identical in all countries, as well as the Union countries have the common monetary policy and the barriers free international goods market.

The weighted average Value-added tax rate is given as:

\[
\bar{\tau}_{vat} = \sum_g \lambda_{f,g,t} \cdot \tau_{vat}.
\] (8)

where for the domestic weights equal to one we have that \( \forall f \in F_g \implies \bar{\tau}_{vat} = \tau_{vat} \).

The weighted average expected price level is calculated as:

\[
\bar{E}_t[P,f,m] = \sum_g \lambda_{f,g,t} \cdot E_t[P,g,m],
\] (9)

where for the domestic weights equal to one we have that \( \forall f \in F_g \implies \bar{E}_t[P,f,m] = E_t[P,g,m] \),

where \( E_t[P,g,m] \) is the expected price level in the future month \( m \) in the country \( g \) given as:

\[
E_t[P,g,m] = P_{g,t} \cdot \left( 1 + \frac{m \cdot E_t[\pi_{g,t+1}]}{12} \right),
\] (10)

where \( P_{g,t} \) is the price level in the country \( g \) at time \( t \), and \( E_t[\pi_{g,t+1}] \) is the expected inflation for the next month in the country \( g \) at time \( t \) which is calculated as a linear combination between the inflation target announced by the central bank and the current inflation, i.e.:

\[
E_t[\pi_{g,t+1}] = \theta \cdot \pi_{g,t} + (1 - \theta) \cdot \pi_{g,t}
\] (11)

where \( \pi_{g,t} \) is the inflation target announced by the central bank in the country \( g \) at time \( t \), \( \pi_{g,t} \) is the current inflation in the country \( g \) at time \( t \), and \( \theta \in [0,1] \) is the trust in central banks’ announcements.

It is worth noting that there is a sort of asymmetry in the way demand for capital and labor is determined. The asymmetry in input demand has been modeled to capture the different way firms usually manage the two production inputs in reality. In particular, labor can be considered as a flow variable for firms, whereas capital is a stock component of the balance sheet. In our model, we assume that companies fire and hire employees with no cost, while investments are irreversible meaning that firms cannot re-sell the physical capital stock. Consequently, we define
labor demand, i.e. demand for the flexible input, as a function of short-term sales expectations, while physical capital demand, i.e. demand for irreversible investments, is modeled based on the opportunity cost of physical capital and the upper bound determined by expected sales (see equations 3, 5, and 6).

Also, note that investment decision indirectly takes into account labor costs. The channel is established through the price level, and the expected inflation since firms’ specific prices are the function of the unit production costs (mark-up pricing rule explained in the section 3.2.3) which, in turn, depend on the number of employees and wage. Thus, the higher the labor costs are, the higher the price level (inflation) and investment demand are - \textit{ceteris paribus}.

Nevertheless, simultaneous and optimal determination of labor and investment demand, as in a standard general equilibrium setting with the production plan and the cost minimization objective, would not be practicable in an agent-based model. The reason is the uncertainty about the effective level of input acquired following the market interactions with other agents, i.e. the hiring process in the labor market, workers turnover, and the outcome of the financing process in the credit and stock markets. For instance, in the case firms are rationed in the credit and stock markets, they need to revise their plans by scaling down investments and productions, i.e. the number of employees. Therefore, from a firm perspective, with imperfect information and foresight about future market outcomes, it is better to determine the “optimal”, or more correctly said “satisfying” levels of labor and physical capital independently from the uncertain market outcomes (see equations 3, 5, and 6). The possible final result of these two independent decisions is higher production capacity with respect to the one required by short-term sales expectations, a spare capacity that however could be behaviorally justified to face the uncertainty of the labor/credit/stock market outcomes. Finally, the process of substitution between the two production factors, in the spirit of the Cobb-Douglas production function, does not occur in one single step as in a general equilibrium setting but is realized in the disequilibrium simulation process through the continuous unfolding of labor and investment demand decisions and market outcomes.

3.2.2 Financing and bankruptcies

Firms demand liquidity to pay salaries, taxes, debt installments, debt interests, dividends and physical capital bills (investments). They give priority to the internal financing over the external funds following the pecking order theory (Myers and Majluf, 1984). Therefore they demand new bank loans and issue new equity shares only after its internal funds dry up. Nevertheless, the external resources are uncertain implying that firms can be rationed on the credit and financial markets which not only can reduce the planned investments, but also cause financial crisis and bankruptcies. The credit and financial market mechanisms will be described in details in the later sections while for the time being, we focus on the bankruptcy mechanism.

Rationed firms face the financial crisis. We distinguish two types of crisis: 1) when firms’ liquidity is sufficient to cover their financial payments, i.e., taxes, debt installments, and interests, then the payments are executed, and the dividend payments and the production schedule
are rearranged to take into account the liquidity constraints; 2) otherwise, if the firms cannot pay their financial commitments, they go into bankruptcy.

The model incorporates two types of bankruptcies: illiquidity and insolvency. On the one hand, illiquid firms are not able to fully pay their financial commitments; however, they still own positive equity which enables them to survive. They become inactive for at least six months destroying all inventories and firing all workers. In this period they try to raise additional liquidity issuing new equity shares on the financial market. If after six months there is enough liquidity they become active again; otherwise, they still stay inactive until cash reaches the target level. On the other hand, insolvent companies not only have no enough liquidity to pay their financial commitments but also negative equity. Therefore, they are fully liquidated and replaced by new businesses. An insolvent firm fires all workers destroys all inventories and removes all shareholders meaning that all shareholders lose their shares. The new company inherits the physical capital, liquidity and the part of the debt of bankrupted firm. The total debt of the new corporation is calculated as:

\[ D_{new} = A_f \cdot \delta, \]  

where \( A_f \) is the total assets inherited from the old company which sums up the total value of physical capital and payment account; and \( \delta \in [0, 1] \) is an exogenous parameter that determines the amount of debt that will be inherited by the new firm. The rest of debt \( (D - D_{new}) \) is written-off and represents a bank loss and the negative shock to the banking sector. In general, banks and shareholders take the all bankruptcy burden. The new company further determines its target equity such as:

\[ \tilde{E}_{new} = D_{new} \cdot \varepsilon, \]  

where \( \tilde{E}_{new} - (A_f - D_{new}) \) is the amount of new equity shares that should be raised on the financial market, and \( \varepsilon \in [0, 1] \) is an exogenous parameter that determines the target leverage ratio of new companies. The new company stays inactive for at least six months. In this period it tries to reach the target leverage ratio issuing new equity shares on the financial market. If after six months the company has sold enough equity shares it becomes active; otherwise, it still stays inactive until the leverage ratio reaches the target level.

### 3.2.3 Pricing and consumption goods supply

Firms set prices based on the unit production costs and the expected price growth using the mark-up pricing rule. The prices are given as:

\[ p_{f,t+1} = \theta \cdot \mathbb{E}_t[p_{f,t+1}] + (1 - \theta) \cdot (1 + \mu) \cdot (1 + \bar{\tau}_{vat,g,t}) \cdot c_{f,t}, \]  

where \( \theta \in [0, 1] \) is the Central Bank (CB) trust parameter, \( \mu \) is the mark-up rate, \( \bar{\tau}_{vat,g,t} \) is the weighted average V.A.T rate given in the equation (8), implying that the cost of tax is passed on to the consumers, and \( c_{f,t} \) is the unit production cost for firm \( f \) at time \( t \). The firms make a linear combination setting the weights \( \theta \) between the price based on the unit production costs and the expected price calculated as:

\[ \mathbb{E}_t[p_{f,t+1}] = p_{f,t} \cdot \left( \frac{\bar{\tau}_{vat,g,t}}{12} + 1 \right), \]
where \( \pi_{g,t} \) is the inflation target announced by the central bank in the country \( g \) at time \( t \). It is worth mentioning that there is a sort of indirect stickiness of price through wage and CB inflation target, however, since the price is a function of unit costs it can change in both directions depending on the production volume.

Firms update their prices always after the production takes place and then they distribute the product to the malls. However, the produced quantity can deviate from the planed one due to the fluctuation of workers or rationing on the credit and stock markets, in which case the malls will be delivered with a proportional share of the planned deliveries given as:

\[
q^g_{f,t} = \frac{q_{f,t}}{\tilde{q}_{f,t}} \cdot \tilde{q}^g_{f,t},
\]

where \( q_{f,t}^g \) is the delivery of firm \( f \) to the mall \( g \), \( q_{f,t} \) is the total produced quantity of firm \( f \) at time \( t \), \( \tilde{q}_{f,t}^g \) is the planed delivery of firm \( f \) to the mall \( g \) and \( \tilde{q}_{f,t} \) is the total planed delivery of firm \( f \) at time \( t \) which is given as:

\[
\tilde{q}_{f,t} = \sum_{g=1}^{N} \tilde{q}_{f,t}^g,
\]

where \( N \) is the number of countries in which the firm \( f \) is going to deliver its products. All products are delivered to the malls without additional cost. In general, we assume frictionless consumption goods market, i.e. there are neither transaction costs nor trade barriers.

Firms record all transactions on their balance sheets ensuring the stock and flow consistency of the model. At the end of the month, they also calculate income statements and pay dividends if the net profit is positive. The following subsection presents the behaviors and interactions of households.

### 3.3 Households

Households are simultaneously taking the roles of workers, consumers, and financial market traders. Each household is endowed with a general skill level which represents the general education degree that is exogenously given and it is constant. We denote the education degree with \( \text{edu} \) such that \( \text{edu} \in [1, 5] \), where 1 stands for the lowest while 5 reflects the highest education level. Households are also endowed with the specific skill level denoted as \( s \), which is the function of their general education degree and employer’s productivity given as:

\[
s_{h,t+1} = \begin{cases} 
    s_{h,t} + (\gamma_{f,t} - s_{h,t}) \cdot \zeta(\text{edu}_h) & \text{if } \gamma_{f,t} > s_{h,t} \\
    s_{h,t} & \text{otherwise}
\end{cases},
\]

where \( s_{h,t} \) is current household’s specific skill level, \( \gamma_{f,t} \) is the productivity of employer \( f \), and \( \zeta(\text{edu}_h) \) is the adjustment function which is increasing in the education level \( \text{edu} \). Therefore, working in a more productive company (or a country) households update their specific skills, while in the case of unemployment or employment in a low productive company (or a country) the households never lose their specific skills, thus there is no forgetting effect.
3.3.1 Labor market

Each household can offer one unit of labor per month. The supply of labor is inelastic, given that wage offer \( w \) is equal or higher than households’ reservation wage \( \bar{w} \). The reservation wage is set to the latest received salary payment once households lose their jobs. However, it decreases during the unemployment period by the constant daily rate \( \delta w \) up to the level of the unemployment benefit assumed that the unemployed households are looking for new jobs every day.

On the international labor market firms and the government post vacancies that are available to all households in the union. Each company determines wage offers for each education degree such that it is an increasing function of \( edu \), while the government defines an identical public wage for all general skill levels. The market is modeled with frictions. First of all, households do not have perfect information about job offers on the market since they observe only a subset of all posted vacancies. Furthermore, we assume mobility frictions which are the results of households’ mobility aversion and other barriers to the labor movement such as geographical distance, migration costs, housing shortages, etc. The mobility friction is captured by monetary costs that households add to their reservation wage to compensate the unwillingness of moving and other mobility barriers. Therefore, the reservation wage is always adjusted for the mobility friction rate \( \rho \) in the case of a job offer from abroad. The job acceptance rule on the labor market is given as:

\[
\begin{align*}
\text{if homecountry} & : w_{f,edu,t} \geq \bar{w}_{h,edu,t} \\
\text{if abroad,} & : w_{f,edu,t} \geq \bar{w}_{h,edu,t} \cdot (1 + \rho)
\end{align*}
\]

where all households reside in their home countries at time \( t = 0 \). Given the role, we assume that households strictly prefer to live in their home countries since they always account for the mobility cost if job offers are from abroad. For instance, if a household from the country of origin \( A \) moves to the state \( B \) consequently becomes the resident of the country \( B \). However, if she loses her job, she still stays in the country \( B \) receiving the unemployment benefit and the transfer payment from the government of the country \( B \). On the job market, she always takes into account the mobility costs for the job offers from all countries but the country of origin \( A \).

3.3.2 Income and payments

Total household’s monthly gross income \( y_h \) consists of salaries or unemployment benefits, dividend payments, bond coupon payments and government transfers given as:

\[
y_h = \begin{cases} 
    w_h + y_{GT,g} + \sum_g n_{B_h}^g \cdot c_{B}^g + \sum_e n_{h}^e \cdot d^e & \text{if employed} \\
    u_h + y_{GT,g} + \sum_g n_{B_h}^g \cdot c_{B}^g + \sum_e n_{h}^e \cdot d^e & \text{if unemployed}
\end{cases}
\]

where \( w_h \) is monthly salary; \( u_h \) is the monthly unemployment benefit payment; \( y_{GT,g} \) is the government lump-sum transfer payment; \( n_{B_h}^g \) is the number of bonds from country \( g \) that is owned by household \( h \) implying that households can buy bonds from different countries on the international financial market; \( c_{B}^g \) is the bond coupon of bond issued by country \( g \) hence, \( \sum_g n_{B_h}^g \cdot c_{B}^g \) denotes the sum of all bond coupon revenues by household \( h \); \( e \) stands for the equity share issuer that can be a firm, a commercial bank or the physical capital producer; \( n_{h}^e \) is the
number of equities held by the household \( h \), and \( d^e \) is the dividend payment per equity paid by the equity issuer, thus \( \sum_e n^e_h \cdot d^e \) is the sum of all dividend revenues by the household \( h \).

The international financial market allows households to buy and sell the shares of all enterprises and the government bonds of all states within the union no matter in which country they reside or from where they are coming. However, the government \( g \) only pays unemployment benefits and transfers to its residents regardless their nationality. For example, if a foreigner loses her job in the foreign country, she will still stay there and receive unemployment benefit and government transfers. In general, households migrate only in the case of accepting a new job in another country. The transfer payments and unemployment benefits are endogenously determined and depend on the economic conditions in the country. While the transfers are calculated as the 50% of average wage in the economy, unemployment benefits are given as the maximum of the mean salary and last household’s wage, increasing the income for those whose salary was below the average in the country.

Households pay income tax and capital tax on their gross earnings. The total tax payment \( m^\tau_h \) of the households \( h \) is given as:

\[
m^\tau_h = \tau^w \cdot w_h + \tau^c \cdot \sum_e n^e_h \cdot d^e ,
\]

where \( \tau^w \) is the income tax rate and \( \tau^c \) is the capital tax rate. All taxes are paid to the government of the country of residence no matter if households are domestic or foreigners. As a result, households’ monthly net income \( y_{NET}^h \) is given as:

\[
y_{NET}^h = y^h - m^\tau_h .
\]

Each household makes the decision how to allocate its disposable income. It can either consume, invest in the financial market or retain the part of the revenue on the payment account without earning any interests. In the following subsection, we present households’ consumption and investment-saving decisions.

### 3.3.3 Consumption choice

Consumption-saving decision is modeled according to the theory of Buffer-stock saving behavior (Carroll, 2001; Deaton, 1992), which states that households consumption depends on a precautionary saving motive determined by the target level of wealth to income ratio \( \bar{\omega} \). For example, consider a household \( h \) receiving a gross money wage \( w_h \) and having the disposable income \( y_{NET}^h \). Consider also the financial wealth \( W_h \) of the household \( h \) which includes its asset portfolio valued at the most recent market prices, as well as liquidity on its payment account \( M_h \). Following the buffer stock theory of consumption, if \( M_h > 0 \) then the household sets the consumption budget \( \bar{\beta}^h \) in the following month as:

\[
\bar{\beta}^h = y_{NET}^h + \phi^c \cdot (W_h - \bar{\omega} \cdot y_{NET}^h) ,
\]  

where \( \phi^c \) is the capital stock saving motive and \( \bar{\omega} \) is the target level of wealth to income ratio. The target level of wealth to income ratio \( \bar{\omega} \) is constant and set to the ratio of the initialized wealth to the wage reservation value. \(^8\)

---

\(^8\)The wealth to income ratio is given as \( \bar{\omega}_h = \frac{W_h}{y^h_{NET}} \), where \( W_h \) is the total wealth of household \( h \), and \( y^h_{NET} \) is the quarterly average net income of the household \( h \). The target level of wealth to income ratio \( \bar{\omega} \) is constant and set to the ratio of the initialized wealth to the wage reservation value.
where $y_{NET,h}$ is the quarterly average net income, $\phi^c$ is the constant Carrol consumption parameter which sets the speed of the budget adjustment, and $\omega$ is the target wealth to income ratio. However, the planed consumption budget has a floor limit to ensure the minimum monthly consumption of households. Moreover, if households are not liquid, i.e. $M_h < 0$, then they also sets the consumption budget to its minimum level such as:

$$B_h = 0.5 \cdot y_{NET,h}.$$  \hspace{1cm} (24)

The rationale of the rule is as follows: if the present wealth to income ratio is higher than the target one, i.e., $W_h/y_{NET,h} > \omega$, then a households $h$ spends more on its consumption in the following months in order to decrease its wealth and return to the target wealth to income ratio. Therefore households tend to smooth their consumption to the level that is defined by the target wealth to income ratio.

Households visit their local mall in the country of residence once per week. They define their weekly consumption budgets dividing the budget $B_h$ on four equal parts. The purchasing decision mechanism assumes the bounded rational behavior of households, and it is designed as follows: households randomly select a basket of products and attach purchasing probabilities to each visible choice. Since all the products are homogeneous, the purchasing probabilities depend solely on prices such that: the probability of choosing a product is higher if the price is lower. Households then rank the basket of goods according to the purchasing probabilities and aim to spend the entire weekly budget. They buy as much as possible of the first two listed products rolling over the remaining budget to the following weeks if any. The remaining consumption budget of the last purchasing week is retained in the payment account and used for the consumption-saving in the next month. The following subsection presents households’ investment decision on the financial market.

### 3.3.4 Investment-saving decision

A part of the disposable income that households do not consume, they either invest in the financial market or retain on their payment accounts without earning any interests. Households invest on the financial market buying government bonds and tradable shares issued either by firms or banks. Note that households also hold the shares of physical capital producers; however, they are not tradable and the capital producers do not issue new equity stocks. The households portfolio allocation is modeled according to the preference structure based on a key prospect theory insight, i.e., the myopic loss aversion, which depends on the limited foresight capabilities characterizing humans when forming beliefs about financial returns. Benartzi and Thaler (1995) showed that loss aversion combined with mental accounting, i.e., frequent evaluation of portfolio, can explain the equity premium puzzle. That combination has been dubbed myopic loss aversion. In practice, each household forms beliefs about future asset returns considering its forward and backward horizons. The main idea is that households are able to foresee the trend of assets only for a short interval, no matter if they plan to hold their assets for a longer period. We distinguish the three stylized behavior in the beliefs formation: random, chartist and fundamental. Therefore, expected return $E_t[R_{t+1}^e]$ for each stock issued by the $e$-th enterprise, are given
by a linear combination of the three terms: expected random return $E_t[R_{e_{t+1}}]$, expected average past return - chartist return $E_t[R_{c_{t+1}}]$, and expected fundamental return $E_t[R_{f_{t+1}}]$. Using the expected returns households determine their utilities for investing in each stock employing a risk averse utility function. The utilities are then normalized together with the risk-free bank’s interest rate and mapped into assets weights using a linear transformation. Once the assets weights are available, the households can build their desired portfolio and make buying or selling orders. All the orders are submitted to the clearing house which collects them together determining the clearing price for each asset. For the details about the financial market mechanism see Benartzi and Thaler (1995) and Raberto et al. (2008).

The number of daily participants on the financial market is uncertain. While the governments and firms attend the market every time they need to raise new liquidity, the number of households is determined by the daily participation rate as well as households’ wealth. Every day a randomly chosen subset of household is assigned to be active as traders on the financial market. Their assignment is based on a daily participation rate that is set to the 10% of the total population. However, only those that have enough cash finally take part in the trading implying that households do not (fire) sell their assets in the case of limited liquidity.

3.4 The banking sector

The commercial bank offers basic checking accounts to depositors paying no interests on the balance. The depositors of the bank in the country $g$ are firms, the physical capital producer, and households that reside in the state $g$. If a household $h$ moves to another country she closes her current payment account and transfers the money to the newly open bank account in another union country without any cross-border barrier. The bank supplies the medium term loans to the private sector (firms) and charges the risk-based interest rate. It demands the liquidity from the central bank and pays the referent interest rate $r$ on the central bank debt (see the graphical presentation of banks’ balance sheet in the table 2 in the appendix for details). The bank redistributes profits to its shareholders in terms of the dividend payments following the monetary policy regulation which will be explained in this section below. In the following, we present the credit allocation role and monetary policy regulations used in the banking sector.

The bank provides new loans to firms if the risk-reward profiles of the loans are acceptable. The reward is the interest paid by the borrowers while the risk is defined as the likelihood of borrowers’ default. The bank uses the following likelihood function $P(\cdot)$ to access the probability of default:

$$P(LR_{f,t}) = a_b \cdot (LR_{f,t})^{k_0},$$

(25)

where $LR_{f,t} = \frac{D_f}{A_f}$ is the leverage ratio of firm $f$ at time $t$ calculated as the current debt to assets ratio. Borrower’s probability of default is computed along the lines of the Moody’s KMV model (Saunders and Allen, 2010), where the rationale is that the likelihood of the default increases in the debt to total assets ratio. The parameters $a_b$ and $k_0$ are set to 2.5 and 3 respectively, thus the particular cubic function is an approximation of the Basel II internal ratings approach as it is shown in Yeh et al. (2009).
Given the loan request \( \ell_{f,t} \) by a firm \( f \) at time \( t \), the bank calculates the probability \( P(\cdot) \) that the firm will default taking into account the augmented amount of debt and assets i.e., the new leverage level such as:

\[
LR_{f,t}^{\text{new}} = \frac{D_{f,t} + \tilde{\ell}_{f,t}}{A_{f,t} + \tilde{\ell}_{f,t}},
\]

hence, using the equation (25) the bank calculates the probability of the default \( P(LR_{f,t}^{\text{new}}) \) for the given new loan amount \( \tilde{\ell}_{f,t} \). The bank is a price setter on the credit market. It sets the risk-based interest for each requested loan taking into account the credit worthiness of firms such as:

\[
r_{b,t}^f = r_g + P(LR_{f,t}^{\text{new}}) \cdot \sigma_b,
\]

where \( r_g \) is the base interest rate set by the Central Bank in the country \( g \) at time \( t \), and \( P(LR_{f,t}^{\text{new}}) \cdot \sigma_b \) is the risk spread determined by firm’s credit risk \( P(LR_{f,t}^{\text{new}}) \) at time \( t \) and the parameter \( \sigma_b \in [0, 1] \) which sets the spread sensitivity to the credit worthiness of the firms.

The bank is allowed to lend money only if it fulfills the regulatory capital requirement. The regulation is inspired by Basel II accords and states that the bank must hold a minimum percentage as \( \psi \in [0, 100] \) and we call it the capital requirement. The risk-weighted assets portfolio (total value at risk) is calculated as a sum of all loans weighted by the probability of firms’ default given as:

\[
\bar{A}_{b,t} = \sum_f P(LR_{f,t})_{b,t} \cdot L_{f,t}^b,
\]

where \( L_{f,t}^b \) is the total loan that is given to the firm \( f \) by the bank \( b \). Nevertheless, in the case of the one representative commercial bank, the total value at risk represents the systemic risk in the economy. Therefore, taking into account the capital requirement regulation the credit allocation rule is given as:

\[
\ell_{b,t}^f = \begin{cases} 
\tilde{\ell}_{f,t} & \text{if } \psi \cdot E_{b,t} + \bar{A}_{b,t} \geq \bar{A}_{b,t} + P(LR_{f,t}^{\text{new}}) \cdot \tilde{\ell}_{f,t}, \\
\frac{\psi \cdot E_{b,t} - \bar{A}_{b,t}}{P(LR_{f,t}^{\text{new}})} & \text{if } \bar{A}_{b,t} + P(LR_{f,t}^{\text{new}}) \cdot \tilde{\ell}_{f,t} > \psi \cdot E_{b,t} > \bar{A}_{b,t}, \\
0 & \text{if } \bar{A}_{b,t} \geq \psi \cdot E_{b,t}.
\end{cases}
\]

In particular, the equations (29) states that the bank \( b \) at time \( t \) can grant the requested amount of loan \( \tilde{\ell}_{f,t} \) only if it does not push \( \bar{A}_{b,t} \) above the Basel II threshold; otherwise it either provides the reduced amount of credits or does not lend at all.

Grantsing the new loan \( \ell_{b,t}^f \) to the firm \( f \) at time \( t \) inflates the banking system since it generates new bank assets and new deposits at the same time. Hence this process is called endogenous money creation.

The bank further collects the interests on loans, pays the capital tax to the government which is the only cost for the bank and pays dividends out of its net profit. The dividend payments are determined following the capital accumulation rule. The rule enables the self-recapitalization of the banking system and it is the only recapitalization mechanism since the bank does not have access to external funds, i.e., neither issues new equity shares nor corporate bonds. Note that
additional central bank debt increases liquidity but not the capital base (equity) - see the table 2 in the appendix. The bank determines the total dividend payments as:

\[ d_{b,t} = \eta(\psi, E_{b,t}, A_{b,t}) \cdot \text{Net Profit} \]  

(30)

where \( \eta(\psi, E_{b,t}, A_{b,t}) \) is the dividend rate given in the form of a step function of capital requirement, net worth and the value at risk on the interval \([0,1]\). For instance, the bank sets the dividend rate \( \eta(\cdot) = 1 \) if \( \frac{\psi \cdot E_{b,t}}{A_{b,t}} > 2 \), which means that if the capital buffer \( \psi \cdot E_{b,t} \) is more than twice as the total value at risk \( A_{b,t} \), the bank redistributes all its net profit to the shareholders without accumulating capital. The dividend rate decreases and consequently the capital accumulation increases when the capital buffer to total value at risk ratio becomes smaller.

3.5 The union central bank and the central banks

Mimicking the Eurosystem, the central bank offers the commercial banks and the government two standing facilities: 1) Marginal lending facility to obtain overnight liquidity from the central bank paying the facility rate; and 2) Deposit facility to make overnight deposits with the central bank without earning any interest on the balance. The central bank provides unconditional infinite standing facilities and sets the marginal lending facility rate for all borrowings as a part of its monetary policy. The rate is denoted as \( r_g \), and at the same time represents the referent interest rate in the country \( g \).

The central bank sets the monetary policy quarterly defining the referent interest rate \( r_g \) and the capital requirement \( \psi_g \), explained in the previous section. However, in the case of the monetary union, the monetary policy is set by the Union Central Bank (UCB) such that \( r_u \) and \( \psi_u \) apply to all countries in the union.

The UCB sets the interest rate \( r_u \) following the Taylor rule (Taylor, 1993b):

\[ r_u = NFR + \bar{\pi} + t \cdot (\bar{\pi} - \tilde{\pi}) + \kappa \cdot (\bar{\upsilon} - \tilde{\upsilon}), \]  

(31)

where \( NFR \) is a Natural Fund Rate, \( \bar{\pi} \) and \( \bar{\upsilon} \) are the average inflation and unemployment rates among all countries in the union respectively; \( \tilde{\pi} \) is the inflation target announced by the UCB or the central banks; \( \tilde{\upsilon} \) is the natural unemployment rate; \( t \) and \( \kappa \) are positive parameters which reveal the trade-off between stabilizing inflation and stabilizing the welfare-relevant output gap. In the current study we set \( \kappa = 0 \) assuming the "divine coincidence", i.e., that stabilizing inflation is equivalent to stabilizing the welfare-relevant output gap (see Blanchard and Gali (2005) for details). Note that the central bank in the country out of the union sets the country-specific reference interest rate \( r_g \) following the same rule presented in the equation (31), however using \( \bar{\pi} = \pi_g \) as well as \( \bar{\upsilon} = \upsilon_g \). The capital requirement is set as a constant for simplicity as: \( \psi_u = \psi_g = \psi \).

3.6 The government

The government hires workers and pays public wages \( W \), creates fiscal policy and redistributes income. Public employees do nothing and receive a monthly salary that is yearly aligned to the
average wage in the economy. However, the public wage may increase on a monthly basis depending on the current demand and supply on the labor market. For example, if the government requires public workers when there is no available labor force on the market it will increase the public wage offer to enhance its competitiveness. Nevertheless, public salary can also be reduced due to government’s austerity policy as well as due to the yearly adjustment to the average wage level in the state. Each country has a number of public employees \( n_G \) which is a fraction \( \nu \) of the total number of households \( N \). This fraction \( \nu \) is set to 0.2 and it is constant. Therefore, if the total population of the country does not change, also the number of public employees does not change \( (n_G = \nu N) \). The government also pays unemployment benefits \( UB \) and provides lump-sum transfer \( GT \), to all residents. Hence the public expenditures are given as:

\[
G_g = W_g + UB_g + GT_g.
\]  

(32)

Public transfers \((GT)\) are an amount of money proportional to the total labor income that the government redistributes to the households, and they are intended to proxy the welfare system of western economies. In the model, the government does not buy any goods directly from firms but always acts on aggregate demand with the intermediation of households, i.e., controlling the monetary transfers to households, which is, in turn, reflected on their demand for goods. This modeling assumption is not only common in the agent-based related literature but also can be supported by empirical evidence. Regarding theoretical studies, Dosi et al. (2015, 2013, 2010) assume that the government pays a subsidy to unemployed workers, that is a fraction of the current market wage. Further, the model developed by the *Bielefeld* group is very similar to ours concerning public spending (see Dawid and Neugart, 2011; Dawid et al., 2014). While the recent work of the *Ancona* group assumes that: Government public expenditure takes the form of a lump-sum transfer which is equally distributed among households, thus providing additional purchasing power (see Caiani et al., 2016). Concerning empirical evidence, the government expenses in the US\(^9\) show that in the last years the spending on human resources has been around 73%, that added to the national defense gives around a 90% of the total public spending. Summing the net interest payments, they go up to 95%. Therefore, although we do not claim that the direct consumption of the government is not worth to be considered, for the time being, we simplify the government mechanism assuming the expenditures given in equation 32.

The government plans a yearly budget, taking into account expected income and expenditures. It generates revenue collecting taxes and raising new debt, i.e., issuing new perpetual bonds \( GBD \) and taking new loans \( GL \). The tax income is strictly preferred to the debt since it is for free. Nevertheless, issuing new bonds is strictly preferred to raising new loans since bond coupons are with fixed interest rate while loans account for the variable reference interest rate and it is ambiguous. Therefore, if the planned budget balance is positive, the government first repays its loans, and the rest of the money keeps on its payment account without earning any interests on the balance. Otherwise, the government targets a number of bonds that will be issued to cover the planned deficit and to convert its debt loan into bonds. Bonds, with a fixed bond coupon rate, are sold in the financial market at a preferential price which is lower than the current bond price. If the government does not manage to sell bonds on the market, it asks new loans which are

---

provided as the marginal lending facility by the central bank. Total sovereign debt is calculated as $SD = GBD + GL$, where $GBD$ is the total issued bonds evaluated at face value and $GL$ is the total lending facility with the central bank. The government also has financial expenditures which are presented as bond coupon payments $BCP$ and total interest payments on loans $LIP$. Therefore the total government spending is given as:

$$TGE_g = G_g + BCP_g + LIP_g.$$  \(33\)

The government uses the level of public expenditures $G$ to create its fiscal policy, while the tax rates are assumed to be fixed and equal in all countries. Fiscal policy is a function of sovereign debt to GDP ratio. Thus, depending on the level of the ratio the government creates either expansionary or contractionary fiscal policy. The policy rule states that if sovereign debt reaches its critical level, that is higher than yearly GDP, the government is compelled to apply austerity measures reducing the public expenditures $G$ by the annual rate of 5%, which is seen as a contractionary fiscal policy. If the total sovereign debt is in between the interval $60\% - 100\%$ of yearly GDP, the public expenditures stay unchanged. Finally, if the total sovereign debt is less than the $60\%$ of yearly GDP the government applies expansionary fiscal policy increasing the public expenditures by the annual rate of 5%.

Governments of countries belonging to a union face additional decisions related to the migration flows which can raise or decrease the population of the country. It is known that governments can show rigidities when they have to downsize the public sector, and we take into account this specific factor in the model. We introduce a parameter $\chi$ which accounts for the rigidities of the government when adapting the number of public employees ($n_G$) to the total population $N$. Therefore, a country could face an increase or a decrease in its population and a gap between the target level of public employees and its current level ($\Delta n_G = n_G - \nu N$). If $\Delta n_G$ is positive it means that there is an excess of public employees and the government has to fire them. The quantity that is fired monthly by the government is $\chi \cdot \Delta n_G$. So, if $\chi = 0$ the government is completely rigid and never fires public employees, while if $\chi = 1$ the government immediately fires all the public employees in excess. The values in between account for different scales of rigidity in the public sector. It is worth noting that, after a sufficient period of time (or asymptotically), if $\chi \neq 1$, the level of public employees should converge to the target one. On the other hand we do not suppose any exogenous rigidity when $\Delta n_G$ is negative. In this case the government tries to hire the needed workers in the labor market; nevertheless, rigidities could be endogenous because the government could be rationed.

### 3.7 The fiscal pool

In the case of countries belonging to a union, we design a fiscal instrument that we call “fiscal pool” with the aim to create a simple integrated fiscal system within the union. The fiscal pool mechanism works as follows. Every country that runs a budget surplus in any given year, has to deposit a fraction of this surplus to a “fiscal pool” account by the union central bank (UCB). The UCB has been chosen...
just for technical reasons, because it is the only super-national agent in the system. This choice
has no particular political meaning, in the sense that we do not suggest that the UCB should
keep the fiscal pool money, but it could also be another ad-hoc institution. When a country of
the union needs to finance its budget deficit, it can ask money to the fiscal pool. In this way, the
country does not have to issue new bond, raising debt. The money taken from the fiscal pool
should not be repaid, but every country has the obligation to contribute to the fiscal pool when
running public surplus. If the total amount of “fiscal pool” money requested is higher than the
available one, each country will be rationed according to its size and the money requested.

In the following sections we introduce the computational experiments as well as analyze and
discuss the results in details.

4 Results

In this section, we present the results of computational experiments, which are designed to
study the macroeconomic implication of forming a union of countries. We aim at understanding
when it is convenient for two countries to join a union where consumption goods can be sold in
an international goods market, workers can be hired in an international labor market, and stocks
and bonds can be traded in an international financial market. To reach this goal, we design a set
of experiments with different scenarios that will be explained in the next section.

4.1 Computational experiments (simulations)

The computational experiments presented in this work are always designed with four coun-
tries. Two of them belong to a monetary union, and two of them are completely isolated. The
general methodology of our work is to compare the performance and/or characteristics of the
countries in the union with the performance and/or characteristics of the countries out of the
union. This procedure, which uses to some extent the isolated countries as control groups, al-
 lows us to understand which are the particular advantages or drawbacks of joining a union. We
present three distinct computational experiments.

In the first one (section 4.3), the four countries are all identical at the beginning, being the only
difference the fact to belong to a union or not. Therefore, we have two countries that belong to
a union and two countries that are isolated. The union is a monetary union, where the central
bank sets a common monetary policy, and it is characterized by a frictionless international goods
market, i.e., households can buy products from both countries without any additional cost, and
they are indifferent. The union shares a common financial market where households can buy
assets from each country. We study different scenarios concerning households’ mobility, letting
the mobility frictions (represented by the parameter $\rho$ of equation 19) vary between a minimum
of 0 and a maximum of 1.8 with a 0.2 step. When $\rho = 0$ households are completely indifferent
between working at home or abroad, while when $\rho = 1.8$ they have a strong preference for
working at home. In the middle, we have intermediate cases. We also differentiate two cases,
concerning the rigidity of the public sector. In the first case, we suppose a total rigidity of the
public sector, which never fires or hires new public employees, thus keeping the number of public workers constant over time. In the second case, we suppose a low rigidity of the public sector which adapts the number of public employees to the current population. This two modes are represented by the parameter $\chi$ introduced in section 3.6, which assumes the value of $\chi = 0$ and $\chi = 0.5$, respectively. Thus, in this first experiment, we study 20 ($10 \times 2$) scenarios.

The second experiment (section 4.4) is similar to the first one, but we introduce a technology difference between the two countries of the union. Therefore, the configuration of this second experiment considers two countries in the union, which differ only in the level of technology $\gamma_c$ (see equation 4), and two additional countries which are identical to the two countries in the union, but isolated. This technological gap between the two countries remains constant over all the studied scenarios. In the introduction of the paper, we motivated the rationale for this choice. In the country with higher technology, $\gamma_H$ is fixed for every scenario to a value of 3, while in the country with lower technology, $\gamma_L$ increases over the different scenarios, from a value of $\gamma_L = 1.5$ to a value of $\gamma_L = 2.9$ with a 0.2 step. Thus, we consider eight levels of the technology gap between the high-tech country and the low-tech country of the union. Concerning the mobility frictions, we study a range of $\rho$ from 0 to 1, with 0.1 step. Thus, in this second experiment, we study 88 ($11 \times 8$) scenarios.

The third experiment (section 4.5) introduces a fiscal policy set-up (called “fiscal pool”) where we try to address the inequality problems emerged in the second experiment (see section 4.4) by creating a common pool of liquidity that comes from the surplus of each country of the union and can be redistributed in the case a country has to finance its budget deficit. The mechanism has been introduced and explained in details in the section 3.7. We, therefore, compare the performance of the countries in the union with or without the activation of the “fiscal pool” device.

The general setup of the model, concerning the number and types of agents and regions, includes four regions where each region hosts 750 households, 16 firms, a single physical capital producer, one commercial bank, the central bank and the government. For each scenario presented above, we ran 30 independent Monte Carlo seeds, where each seed consists of 24,000 iterations which in our model stands for 1,200 months or 100 years. In the following, we present the statistical measures that we use and provide the results.

### 4.2 Statistical measures

To analyze the outcome of the computational experiments, we make use of several statistical measures, which allow us to explore the differences between the union and the separate countries, and also within the countries belonging to the same union. The simulations have been run for a time span $T$. We present here a list of the main measures that will be used to show and comment results of the next section.

**The average over seeds**
\[ Y_t = \frac{1}{|\mathcal{S}|} \sum_{s \in \mathcal{S}} Y^s_t \]  

where \( Y \) can be any economic indicator generated by the simulations, and \( \mathcal{S} \) is the set of seeds used in the computational experiment. Note that \( |\mathcal{S}| \) indicates the cardinality of the set, which here coincides with the number of elements, i.e., months.

The mean of the sum

\[
\begin{align*}
\Sigma_{12}(Y) &= \frac{1}{|\mathcal{T}|} \sum_{t \in \mathcal{T}} (Y_{1,t}^s + Y_{2,t}^s) \\
\Delta \Sigma_{12,34}(Y) &= \Sigma_{12}(Y) - \Sigma_{34}(Y)
\end{align*}
\]  

Given one random seed \( s \), the statistical measure \( \Sigma_{12}(Y) \) represents the average value in the time subset \( \mathcal{T} \subset \mathcal{T} \) of the sum of the observed economic indicator \( Y \) in the two considered countries (1 and 2). We often use this measure to compare the value of an economic indicator in the union with the value of the same indicator in the isolated countries, therefore computing the difference \( \Delta \Sigma_{12,34}(Y) \). In general countries, 1 and 2 are in the union, while 3 and 4 are the isolated ones. If \( Y \) is real GDP, then \( \Sigma_{12}(Y) \) will be the average real GDP level in the union (if countries 1 and 2 belong to the union), and \( \Delta \Sigma_{12,34}(Y) \) will be the difference between the average real GDP level of the union and the average of the sum of real GDP of the isolated countries.

For the sake of clarity, sometimes we use the relative difference instead of the difference, defined as \( [\Sigma_{12}(Y) - \Sigma_{34}(Y)] / \Sigma_{34}(Y) \). In this way, the gap between the union and the isolated countries is in relative terms (given in percentage) and can be grasped at first sight.

The mean of the difference

\[
\begin{align*}
\Psi_{12}(Y) &= \frac{1}{|\mathcal{T}|} \sum_{t \in \mathcal{T}} (Y_{1,t}^s - Y_{2,t}^s) \\
\Delta \Psi_{12,34}(Y) &= \Psi_{12}(Y) - \Psi_{34}(Y)
\end{align*}
\]  

Given one random seed \( s \), the statistical measure \( \Psi_{12}(Y) \) represents the average value in the time subset \( \mathcal{T} \subset \mathcal{T} \) of the difference of the observed economic indicator \( Y \) between the two considered countries (1 and 2). If the setting is entirely symmetric, i.e., if all the countries are identical at the beginning, we expect that the mean value of this indicator across seeds should be around zero.

Polarization, or local divergence

\[
\begin{align*}
\Upsilon_{12}(Y) &= \frac{1}{|\mathcal{T}|} \sum_{t \in \mathcal{T}} |Y_{1,t}^s - Y_{2,t}^s| \\
\Delta \Upsilon_{12,34}(Y) &= \Upsilon_{12}(Y) - \Upsilon_{34}(Y)
\end{align*}
\]  

Given one random seed \( s \), the statistical measure \( \Upsilon_{12}(Y) \) captures the divergence of two countries 1 and 2 concerning the observed indicator \( Y \). It is computed as the average value in the time subset \( \mathcal{T} \) of the absolute difference of the observed economic indicator \( Y \) between the two
considered countries (1 and 2). When the local divergence \( \Upsilon_{12}(Y) \) is high, it means that the values of \( Y \) in the two countries are different and that the countries tend to “diverge”. In general, we consider the difference \( \Delta \Upsilon_{12,34}(Y) \) to compare the local divergence of countries in the union with a reference divergence, which is usually the one observed in the case of isolated countries, e.g., 3 and 4.

We designed our simulations to study two different regimes, the first one is a capital accumulation regime, while the second is a close-to-steady-state capital per capita regime (of course, being an endogenous business cycles model, we can have fluctuations, and even crises, also in the close-to-steady-state regime). In this way, we can analyze two important conditions of the economy, and we can try to disentangle the effects of the proposed scenarios on the two economic regimes. As we will show in this section, the first phase of capital accumulation is generally characterized by far from the equilibrium and more fragile economies, where endogenous shocks can violently propagate across the markets. On the other hand, the second phase is more stable but always characterized by business cycles that can become turbulent in some cases. Figures 1a and 1b show the average time series of capital per capita and real GDP for a representative scenario (government flexibility \( \chi = 0.5 \) and mobility friction \( \rho = 1.4 \)). These figures reveal the two regimes of the economies.

![Figure 1: Average time series; the statistical measure \( \Upsilon_t \) given in equation 34. A scenario with government flexibility \( \chi = 0.5 \) and mobility friction \( \rho = 1.4 \). Panel 1a presents the real capital stock per capita; panel 1b depicts real GDP. Time series on both panels exhibit two regimes. The regime in the period of first 400 months is characterized by the capital accumulation and huge fluctuation in real GDP, while in the period after 400 months the economy enters a stable path.](image)

4.3 Identical countries

**Result 1.1** *The union clearly outperforms the isolated/independent countries.*

In general, the economic performance of the two countries in the union is better than the performance of two separate countries. Figure 2 presents a direct comparison between the performance of the union and the aggregate performance of the isolated countries, for each considered scenario. In particular, it plots the statistical measure \( \Delta \Sigma_{12,34}(Y) \) described in equation 35, representing the difference of economic indicators in the union and the two countries out of the union.
Real GDP, real GDP per capita, real investments, real consumption, capital stock, the unemployment rate, and the number of active firms are presented on figure 2. The overall picture shows that the economic activity is significantly higher in the case of the union (around 10%-20% higher), and that, therefore, belonging to a union constitutes a considerable value-added for two identical countries. This outcome is explained by the presence of international goods...
Figure 3: Average time series; the statistical measure $Y_t$ given in equation 34. A scenario with government flexibility $\chi = 0.5$ and mobility friction $\rho = 0.8$. C1 and C2 are countries in the union, whereas C3 and C4 are independent countries.

and labor markets in the union. In particular, the international goods market allows for a better allocation of products across the two countries, where a potential decrease in the local demand is compensated by the demand of the foreign country, leading to a more stable path. To understand this mechanism, we show in figure 3 some exemplary times series of one sample scenario, i.e., with government flexibility $\chi = 0.5$ and an intermediate mobility friction value ($\rho = 0.8$), aggregated for all the 30 random seeds. The balance of payment, in figure 3a, shows the continuous activity of import and export between the two countries of the union, which eases the sales of firms’ products. A similar dynamics characterizes the labor market, where countries in the union count on a larger basin of workers, hiring foreigners if needed, as shown in figure 3b, where foreigners are defined as citizens of country A in the union, who emigrate to country B in the union. This flexibility leads to a lower, and more stable, unemployment rate, as shown in the boxplots of figure 2f and in the aggregated time series of figure 3c. To better characterize the situation, we remind here that the model exhibits endogenous business cycles, as explained in Raberto et al. (2012); Cincotti et al. (2010), which can show phase differences between countries. Therefore, the union configuration allows for a more efficient economic adjustment in the cases of excess demand or supply in both the labor and goods market, finally outperforming the configuration with isolated countries.

Figure 2g shows that in the union there is, on average, a higher number of active companies with respect to the isolated countries (or a lower number of defaulted firms). This result applies to every scenario and indicates a better stability of the economy of the union, due to the higher
efficiency of the goods and labor markets.

As a final remark, we mention that the performance of the union depends on the considered scenario, as figure 2a, 2b, 2c, 2d, 2e clearly show. In particular, it seems that the advantage of the union, with respect to the isolated countries, becomes weaker when mobility frictions $\rho$ are very low. This point will be addressed with more attention in Result 2.I.

**Corollary 1.2.I** Prices and money supply, both in nominal and real terms, are higher in the union.

Prices, wages, and inflation tend to be higher in the case of countries in the union. Figure 4a shows that the average nominal wage is around 50% higher in the union, while inflation (see figure 4g) is also larger by approximately a percentage point. Real wage, in figure 4b, is also higher in the union, by a 15% on average. Money supply (approximated by total deposits, as there is no cash in the model), is higher in the union, both in nominal and real terms, as shown in figures 4c and 4d.

These observed outcomes are strictly related with the one discussed in result 1.I. The higher economic activity raises labor demand in the countries of the union, while lower unemployment rate forces firms to increase nominal wages. Prices, which are set by a mark-up rule, as described in equation 14, also increase, but less than wages because the other costs for firms do not rise proportionally. From figure 2g, showing a lower bankruptcy rate in the union, we can argue that the financial statement of firms in the union is more balanced and their unit financing costs are lower. In this respect, it is very interesting to point out that, although the total bank credit is higher in nominal terms in the union (see figure 4e), in real terms it is lower (figure 4f). This probably means that firms in the union are more able to finance their production plan with internal resources (we remind here that they follow the pecking order theory) and do not need as much credit as firms in the isolated countries. This, of course, implies also lower interest payment costs, and therefore, lower prices with respect to the paid wage.

Finally, the stronger economic activity of the union generates both more endogenous money and more wealth, which is reflected in the level of real deposits in figure 4d.

**Corollary 1.3.I** The monetary policy in the union tends to overshoot inflation target.

Figure 5 presents the divergence in the price level between the two countries of the union compared with the divergence for isolated countries. It is computed using the statistical measure described in equation 37. The figure shows that on average the price coordination is much better in the union for every considered scenario, which is the consequence of the market coordination in the union. However, although the coordination helps the union central bank while setting a common interest rate for the two member countries, the asynchrony in the price dynamics of the two countries of the union is still present, and it does not allow the union central bank to reach the target inflation of 2%. Note that the monetary policy follows an intermediate line, as described in equation 31. Inflation tends to be higher than the target, because the interest rate is set considering an average inflation level, and thus it is less effective in reducing prices in the country with higher inflation. Figure 6 shows that the average gap between inflation and target inflation in the two union countries tends to be positive for most scenarios, whereas fig-
Figure 4: The differences between the union and the countries out of the union; the statistical measure $\Delta \Sigma_{12,34}(Y)$ given in equation 35. Panels a-f show relative differences, while panels g and h show differences. C1&C2 denotes countries in the union, while C3&C4 denotes closed economies. X-axis indicates scenarios with respect to government flexibility $\chi$. Colors indicate scenarios with respect to mobility friction $\rho$. The solid lines in the bars represent median while dots denote mean.

Figure 4h shows that the interest rate is on average higher in the union with respect to the isolated countries.
Figure 5: The graph shows differences in the local divergence of the price levels of countries in the union C1&C2 and the local divergence of the price levels of countries out of the union C3&C4. A statistical measure $\Delta \gamma_{1234}(\bar{Y})$ given in equation 37. X-axis indicates scenarios with respect to the values of government flexibility $\chi$. Colors indicate scenarios with different values of mobility frictions $\rho$ which are given in the legend. The solid lines in the bars represent median while dots denote mean.

Figure 6: Inflation deviation from the target. Left panel shows the inflation deviation from the target of the country in the union C1, while right panel presents the inflation deviation from the target of the country in the union C2. X-axis indicates scenarios with respect to the values of government flexibility $\chi$. Colors indicate scenarios with different values of mobility frictions $\rho$ which are given in the legend. The solid lines in the bars represent median while dots denote mean.

**Corollary 1.4.1** The real sovereign debt of countries in the union is lower.

Figure 7a shows that the real sovereign debt of countries in the union is on average around 20% lower with respect to countries out of the union. We remind here that the expenditures of the government are public wages, public transfers (which are per capita transfers, equal to a fraction of the average wage), unemployment benefits and bonds’ interest payment. We show all the components of the government expenditure (see figures 7b, 7c, 7d and 7e), to outline that the higher prosperity of the union countries allow the governments to transfer more resources to the citizens (or to offer more services), but at the same time, the lower unemployment in the union allows the government to reduce the expenses in unemployment benefits and to finally accumulate less real debt, and bond’s interest payment.

Until this point we compared the countries belonging to the union with the isolated ones, getting a clear indication of the better performance of the union’s countries. However, our results also showed some differences among the proposed scenarios, which we didn’t take into account so far. In the following, we analyze these differences.

**Lemma 2.1.1** As the two countries of the union are identical at the beginning; we do not observe any systematic difference between them.
Before considering the effect of the proposed scenarios, we need to state that, on average, there are no differences between the two countries in the union. This, of course, is a trivial result but it is also an important validation check. It is trivial because the two countries are identical, and therefore there is no reason why one of them should systematically perform better (or different) than the other; on average, their behavior should be statistically identical. It is an important check because it allows us to interpret and explain the following result Result 2.1. To show that there is no systematic difference between the behavior of the two union countries, we use the statistical measure of equation 36, computing the mean of the differences between the two countries. Using GDP, in figure 8a, and the total number of households, in figure 8b, as examples, we observe that for every scenario there is no significant difference between the two countries of the union. We only observe a variable dispersion around the median.
Figure 8: The mean of differences of economic indicators between two countries: the statistical measure $\Psi_{12}^s(Y)$ given in equation 36. The left panel of the figure 8a shows the mean of difference of real GDP between countries C1 and C2 in the union, while the right panel of the same figure indicates the mean of difference of real GDP between isolated countries C3 and C4. The left panel of the figure 8b shows the mean of difference of the number of households between countries C1 and C2 in the union, while the right panel of the same figure presents the mean of difference of the number of households between isolated countries C3 and C4. X-axis indicates scenarios with respect to the values of government flexibility $\chi$. Colors indicate scenarios with different values of mobility frictions $\rho$ which are given in the legend. The solid lines in the bars represent median while dots denote mean.

We would like to point out here that Lemma 2.1.1 just states that, on average, we don’t observe any difference between countries, but still we can have cases where one country performs better than the other. For instance, if for a particular random seed, country A of the union performs better than country B, and for another random seed country B performs better than country A, we will only observe a higher dispersion in the box-plot, but no difference in the mean/median. Moreover, if country A performs better for half of the simulation and country B performs better for the other half, on average we will observe the same performance, and the
box-plots would not reveal this difference at all. However, as shown for example by Raberto et al. (2012) and Teglio et al. (2012), the Eurace model exhibits endogenous business cycles which could affect the economic dynamics of the union’s countries differently. In other terms, we could think of endogenous idiosyncratic shocks that affect each country, and that could potentially produce different effects, depending on the scenario. For instance, a particular scenario could bring about a situation where one country becomes systematically bigger than the other for some simulation runs or for specific time intervals. Of course, being country A and country B identical at the beginning, there is no reason to observe a general dominance of country A with respect to country B (or the other way around), but we could observe that one of the two countries (no matter which) becomes bigger than the other. To measure this divergence between countries we use the statistical measure described in equation 37, which assesses if the two countries become polarized, concerning the observed economic indicator $Y$.

**Result 2.1** When mobility frictions are low, the polarization between the two countries of the union increases, and the economic performance of the union is negatively affected

From figure 9 we observe that the number of foreign workers in the union, defined as citizens of country A of the union who emigrate to country B of the union (or vice-versa), is much higher when the mobility frictions are low. Moreover, the divergence in the number of households between the two countries of the union is much higher, as figure 10a clearly shows. It is worth noting again that figure 10a plots the statistical measure described in equation 37, which emphasizes the local dominance of one country with respect to the other one, and which is very different from the average difference (equation 36) represented in figure 8b. Actually, the average difference is always around zero, given the perfect symmetry of the experiment, while the local dominance depends strongly on the scenario.

![Figure 9: Number of foreigners. The differences between the union and the countries out of the union; the statistical measure $\Delta\Sigma_{12,34}(Y)$ given in equation 35. C1&C2 denotes countries in the union, while C3&C4 denotes closed economies. Note that the sum of foreigners in countries C3&C4 is always zero since there is no labor flow between the isolated countries. X-axis indicates scenarios with respect to government flexibility $\chi$. Colors indicate scenarios with respect to mobility friction $\rho$. The solid lines in the bars represent median while dots denote mean.](image)

In the case of low mobility frictions, the wider gap in population between the two union countries affects all the aggregate economic indicators like real GDP (figure 10b), real consumption (figure 10c), or real banks’ deposits (figure 10d), which show the dominance of one country with respect to the other one. At the same time, when the mobility frictions are very low, we
observe a worse performance of the whole union, as we can notice from the lower GDP and consumption in figures 2a, 2d. Summarizing, we observe that for low mobility frictions the two countries of the union diverge and the economic performance of the union declines.

We can explain these results by considering that the endogenous idiosyncratic shocks affecting the economies, generate more or less immigration depending on the level of mobility frictions $\rho$. When frictions are low, workers tend to abandon their home country more easily, and they have little incentive to go back, as equation 19 shows. In particular, as the simulation starts from a far from equilibrium amount of capital per capita, at the beginning the economies are more unstable and subject to stronger shocks and oscillations (as already shown in figures 1a and 1b). Therefore, it can happen that an economy incurs into a big economic recession before the other one, and many of the households emigrate to the other country. Later, when the economies
get closer to a medium run equilibrium, business cycles are in general not so strong to cause large workers flows, and the country that received more workers tends to remain larger. On the other hand, when mobility frictions are higher, workers prefer to go back to their countries of origin if the real wage difference is not too high, and we have a reverting mechanism that tends to equilibrate the number of workers in the two countries. This mechanism can be observed in figure 11. The figure 11a shows that when mobility frictions are higher, the initial flow of migrants tends to go back to the home country. While the figure 11b shows that when there are no frictions, the initial migration of workers is more consistent, and the final number of foreign workers stabilizes at a higher value. This difference in the number of households drives therefore similar gaps in all the economic indicators that depend on workers.

Figure 11: The number of foreigners. Average time series; the statistical measure \( Y_t \) given in equation 34. The left panel depicts the scenario with government flexibility \( \chi = 0 \) and mobility friction \( \rho = 0.4 \), while the right panel shows the scenario with government flexibility \( \chi = 0 \) and mobility friction \( \rho = 0 \).

The weaker performance of the union in the case of low mobility frictions mainly depends on the inefficient use of the capital stock between the two countries. As the model does not allow for capital stock displacement, the smaller country does not fully use its available capital stock, while the larger country has an excess of households concerning the available capital stock. The Union, in this case, reaches a sort of sub-optimal equilibrium where there is one small country producing with an excess of capital and one big country producing with an excess of workers. Figures 2e and 10 confirm our statement. The interesting point about this situation is that both countries seem to reach a similar economic performance level, which consolidates the sub-optimal equilibrium. In fact, the real wage, the GDP per capita, and the unemployment rate do not show any polarization when the mobility frictions are low (see figures 10e, 10f and 2f). This means that the two countries of the union (both inefficient in exploiting production factors) reach a very similar economic state. In particular, the undercapitalized big country is not able to accumulate more capital because it does not face enough demand, which is satisfied by both countries. The small country, which has, of course, a smaller but stable fraction of the market, undergoes a capital depreciation process, and finally in the union the amount of capital for low mobility friction is quite lower, even lower than in the isolated countries, as figure 2e clearly shows. This finally determines the lower performance of the Union. It is also worth noting that, when the government is flexible, there is more immigration because countries do not keep a constant number of public workers and therefore the fired public em-
employees are eligible to leave the country too, potentially increasing the migration flows. Figure 10a shows a larger gap in population when the government is more flexible.

4.4 Different countries

In this section we extend the previous analysis to a setting where the two countries of the union have a different level of technology, implying a different productivity. The country with the higher level of productivity (henceforth high-tech country) has a fixed technology level $\gamma_H$ (see equation 4 and the explanation below), which remains constant over all the studied scenarios. The country with lower productivity has the fixed technology level, but it increases over the different scenarios, from a value of $\gamma_L = 1.5$ to a value of $\gamma_L = 2.9$. Thus, we consider eight levels of productivity gap between the high-tech country and the low-tech country of the union. In the following, we will call this difference both as a productivity gap and as a technology gap, without distinction.

Figure 12 shows the real GDP in the four considered countries. At the top of the figure, there are countries in the union, while at the bottom there are countries out of the union. On the left side of the figure, there are high-tech countries, while on the right side there are low-tech countries. Therefore, for instance, the high-tech country of the union is the sub-figure at the top left corner. We will present in this section several figures sharing the same format of figure 12, but related to other economic indicators. For this reason, it is worth spending a few more lines to comment its general characteristics.

Every one of the four sub-figures shows eighty-eight box-plots, corresponding to the eight values of productivity $\gamma_L$, represented in the $x$ axis, and the eleven values of mobility frictions $\rho$, represented by a color code. The box-plots display the variation across the different seeds $s \in \mathcal{S}$ of the average value of the considered economic indicator (real GDP in this specific case) over a time span $\mathcal{T} \subset \mathcal{T}$. In the case of figure 12, and in general, unless otherwise stated, $\mathcal{T}$ is the second half of the simulation, i.e., the last 50 years.

We can observe very different patterns, by comparing the countries which are in and out of the union. Let us examine them briefly. In the high-tech country out of the union, we do not observe any significant variation for all the eighty-eight box-plots. This is due to the fact that mobility does not affect countries out of the union, and that the technological level is constant for the high-tech country. In the low-tech country out of the union, we naturally do not observe any significant variation related to the mobility frictions, as there is no migration, but we see jumps in the average level of production, generated by the different productivity scenario of the low-tech country. Finally, in the countries of the union, we can observe patterns that depend both on the mobility frictions and on the productivity gap.

Together with figure 12 we introduce also figure 13a, presenting the mean of the real GDP sums $\Sigma_{12}(Y)$, according to equation 35. The left side of 13a shows the total real GDP in the union, whereas the right side shows the same indicator for the countries out of the union. It is worth noting that, given the population in the union constant (and equal to the sum of the
households of the isolated countries), the real GDP per capita indicator is given by the real GDP divided by the total number of households. Therefore, figure 13a could also represent the GDP per capita pattern over the different scenarios.

**Result 3.D** The economic performance of the union is always better than the one of the isolated/independent countries.

This result is very similar to **Result 3.I**, and it can be proven by a simple visual inspection of figure 13a. If we consider aggregated economic indicators as the real GDP (or consumption, or investments, or any other one), the performance of the union is always better. The explanation of this result recalls the one used for **Result 3.I**, and it is based on the presence of a common goods market which allows companies to easily sell their products, thus increasing GDP. The common labor market can also help, but its role is more complex, and we need to introduce several distinctions about the labor mobility issue later. In the case of countries with different technologies, an additional reason for higher overall production in the union is that, whenever a household migrates from the low-tech country to the high-tech country, its productivity raises, along with the overall production in the union. And **Result 4.D** shows exactly that net migration is always toward the high-tech country.

**Result 4.D** Net migration always flows toward the high-tech country, which becomes more populated than the low-tech country.

Figure 14 proves the result.

**Corollary 4.1.D** The high-tech country is performing better than the low-tech one, especially when the productivity gap is large and when mobility frictions are small.
Figure 13: The average real GDP level in the union and isolated countries; the statistical measure $\Sigma_{i=1}^{n}(Y)$ given in equation 35. The left panels show the average of the sum of real GDPs of the countries in the union C1 and C2; the right panels show the average of the sum of real GDPs of the isolated countries C3 and C4. On the figure 13a, x-axis indicates scenarios with respect to the productivity level of the low-tech country $\gamma_L$. Colors indicate scenarios with different values of mobility frictions $\rho$ which are given in the legend. On the figure 13b, x-axis indicates scenarios with respect to the values of mobility frictions $\rho$. Colors indicate scenarios with different productivity levels of the low-tech country $\gamma_L$ which are given in the legend. The solid lines in the bars represent median while dots denote mean.

Figure 15 presents the difference between the real GDP of the high and low tech countries in the union (along with the same plot about the isolated countries, on the right), according to equation 36. It shows that the real GDP in the high tech country is almost always greater than the one in the low-tech country, but the difference decreases both with a smaller productivity gap and with larger mobility frictions. The reasons for the higher production in the high-tech country can be resumed by two factors: more productivity and more population, as stated in result 4.D.
Figure 14: The number of households by countries. X-axis indicates scenarios with respect to the productivity level of the low-tech country $\gamma_l$. Colors indicate scenarios with different values of mobility frictions $\rho$ which are given in the legend. The solid lines in the bars represent median while dots denote mean.

Figure 15: The mean of differences of real GDP between two countries; the statistical measure $\Psi_{12}(Y)$ given in equation 36. The left panel shows the mean of difference of real GDP between countries $C1$ and $C2$ in the union, while the right panel indicates the mean of difference of real GDP between isolated countries $C3$ and $C4$. X-axis indicates scenarios with respect to the different productivity levels of the low-tech country $\gamma_l$. Colors indicate scenarios with different values of mobility frictions $\rho$ which are given in the legend. The solid lines in the bars represent median while dots denote mean.

**Result 5.D** When the technology gap is large, workers mobility improves the overall economic performance of the union. When the technology gap is small, workers mobility worsen it.

Figure 13b and 13a contain the same information, presenting the variation of the real GDP in the union with respect to mobility frictions and productivity gap with two different graphical solutions. A visual inspection of the figures clearly demonstrates the result. When the technol-
ogy gap is shallow, the scenario tends to collapse to the two identical countries case, already presented in the previous section. Therefore, the explanation for the second part of result 5.D, i.e., when the technology gap is small, workers mobility worsen the performance of the union, is analogous to the one of result 2.I. We remind it shortly here, by remarking that excessive workers movement creates a polarized situation where the country with the excess of workers is not able to accumulate more capital because it does not attract sufficient demand to justify new investments. The two countries are in a sort of inefficient but stable equilibrium and the union’s performance is harmed.

However, in the first part of the result, i.e., when the technology gap is large, whether workers mobility improves the economic performance of the union depends on the following argument: if the productivity gap is sufficiently large, the massive inflow of workers in the more productive high-tech country allows for a higher production of goods in the union, completely offsetting any other effect, as the bad use of capital which is the argument of the case of a small gap.

Until this point we analyzed the differences between the union and the two isolated countries, considering the characteristics of the union as a whole. However, now we would like to extend the analysis and to access the performance of each country. So, we study the differences between similar countries in or out of the union, i.e., the differences between the two high-tech countries (one in the union and the other out), or low-tech countries. This approach will allow us to study under which conditions it is convenient for a high-tech (low-tech) country to join the union. In this phase of the analysis, we will use mainly the statistical measure of eq. 36, considering the difference between the high-tech (low-tech) country in the union and the high-tech (low-tech) out of the union. So, concerning equation 36, country 1 is the country in the union while country 2 is the one out of the union. In particular, we want to investigate how joining the union affects the per capita economic indicators, and therefore the welfare of a country; real wage and GDP per capital are good indicators for our scope.

In order to gain some insight into the effects of joining the union for both the high-tech and low tech countries, we draw special attention to figures 16a, 16b, 17a and 17b, representing respectively, the differences (as in eq. 36) in real GDP, real GDP per capita, real wage, and unemployment rate. These figures allow us to make some remarks.

**Result 6.D** Joining the union decreases the unemployment rate of both the high and low tech countries, in almost every scenario. Two main reasons can explain this result. The first one, and more important, is that the mobility of workers allows for a more efficient allocation of the labor force. We see in figure 17b that, especially in the most critical scenarios, i.e., when the productivity gap is large, and the GDP per capita of the low-tech country is small (see figure 16b), joining the union leads to a lower unemployment rate in both countries. In the low-tech country, in particular, we see how joining the union decreases the level of unemployment when the productivity level is very low. This is of course due to the possibility to emigrate and find a job in a stronger economy. The second reason is that the international goods market, implying a better allocation of products among the union’s countries, generally enhance the economic activity, as stated in Result 3.D, therefore creating more employment.
Figure 16: The mean of differences of economic indicators between two countries; the statistical measure $\Psi_{12}^s(Y)$ given in equation 36. The left panel of the figure 16a shows the mean of difference of real GDP between high-tech countries, C1 in the union and C3 out of the union, while the right panel of the same figure indicates the mean of difference of real GDP between low-tech countries, C2 in the union and C4 out of the union. The left panel of the figure 16b shows the mean of difference of real GDP per capita between high-tech countries, C1 in the union and C3 out of the union, while the right panel of the same figure indicates the mean of difference of real GDP per capita between low-tech countries, C2 in the union and C4 out of the union. X-axis indicates scenarios with respect to the productivity level of the low-tech country $\gamma$. Colors indicate scenarios with different values of mobility frictions $\rho$ which are given in the legend. The solid lines in the bars represent median while dots denote mean.

Result 7.D A lack of mobility frictions severely damages the welfare conditions of the low-tech country in the union, especially in the case of large productivity gap. Joining the union is not convenient for the low-tech country in this case. In general, the welfare of the low-tech country is negatively affected by an excessive mobility. The low-tech country is not able to compete in terms of real wage and a strong migration toward the high tech country occurs. Workers have no incentive to go back to their countries of origin and the population gap becomes persistent,
(a) Real wage.

(b) Unemployment rate.

Figure 17: The mean of differences of economic indicators between two countries; the statistical measure $\Psi_{12}^{\mu_1}(Y)$ given in equation 36. The left panel of the figure 17a shows the mean of difference of real wage between high-tech countries, C1 in the union and C3 out of the union, while the right panel of the same figure indicates the mean of difference of real wage between low-tech countries, C2 in the union and C4 out of the union. The left panel of the figure 17b shows the mean of difference of the unemployment rate between high-tech countries, C1 in the union and C3 out of the union, while the right panel of the same figure indicates the mean of difference of the unemployment rate between low-tech countries, C2 in the union and C4 out of the union. X-axis indicates scenarios with respect to the productivity level of the low-tech country $\gamma_L$. Colors indicate scenarios with different values of mobility frictions $\rho$ which are given in the legend. The solid lines in the bars represent median while dots denote mean.

as figure 18 shows for a particular scenario (low-tech productivity $\gamma_L = 2.3$, mobility frictions $\rho = 0.2$). Figures 16b and 17a highlight the possible negative effects of joining the union concerning the welfare of the low-tech countries. For any productivity level, $\gamma_L < 2.7$, i.e., any productivity gap $\Delta \gamma > 0.3$, citizens receive lower real wages and the per capita GDP level is lower with respect to the isolated country case.

Figure 19a and 19b show the difference in nominal wage and prices between the high and low
Corollary 7.1.D \textit{In the presence of low mobility frictions, the high-tech country has no clear advantage in joining the union, as the welfare of its citizens does not improve significantly. Therefore, we conclude that the union needs some mobility frictions between the member countries to constitute a value-added. This becomes stronger (weaker) in the case of large (small) technology gap.} To explain our statement, we need to enter into some details related to the GDP per capita concept. Figure 13a presents the variation of real GDP in the union across the different scenarios. However, it also represents the variation of GDP per capita, as the population in the union is constant. As stated in result 3.D, GDP per capita in the union is always higher, and this is because many citizens of the low-tech country emigrate to the high-tech country and find there a higher living standard. However, the average difference in GDP per capita between the high-tech and the low-tech country, shown in figure 20 (indistinguishable by construction from figure 13a for the countries out of the union), tells us that the union is performing poorly for low mobility frictions and that the low-tech country is strongly and negatively affected, as stated in result 7.D. Figure 16b shows, under a different perspective, that the low-tech country should not join the union when the productivity gap is large. In other words, even though the average welfare (measured as GDP per capita) is higher in the union, the citizens remaining in the low-tech country decrease sharply their life standards (again measured as GDP per capita). Therefore we can state:

\textbf{Corollary 7.2.D} \textit{Too low mobility frictions increase the inequality between the high-tech country and the low-tech country, especially when the productivity gap is large.}

\textbf{Result 8.D} \textit{For the low-tech country, it is always convenient to join the union, if there are sufficient mobility frictions. The same can also be stated for the high-tech country.} This result clearly emerges from figures 16b and 17a. The welfare of low-tech country citizens,
measured as real wage and per capita GDP, significantly improves when there are sufficient
mobility frictions and their country belongs to the union. Interestingly, this improvement does
not depend on the productivity gap. So we can state that, irrespectively of the productivity gap,
the low-tech country improves its welfare by joining the union. Given that there are mobility
frictions a similar result holds for the high-tech country, with a distinction: the lower the
productivity gap, the higher the incentive to join the union for the high-tech country. In other
words, the high-tech country seems to have a strong interest in reducing the productivity gap in

Figure 19: The mean of differences of economic indicators between two countries; the statistical measure \( \Psi_{12}^{\text{m}}(Y) \) given in equation 36. The left panel of the figure 19a shows the mean of difference of the wage level between countries C1 and C2 in the union, while the right panel of the same figure indicates the mean of difference of the wage level between countries C3 and C4 out of the union. The left panel of the figure 19b shows the mean of difference of the price level between countries C1 and C2 in the union, while the right panel of the same figure indicates the mean of difference of the price level between countries C3 and C4 out of the union. X-axis indicates scenarios with respect to the productivity level of the low-tech country \( \gamma_L \). Colors indicate scenarios with different values of mobility frictions \( \rho \) which are given in the legend. The solid lines in the bars represent median while dots denote mean.
order to take full advantage of the union. Figure 21, presenting the GDP per capita in the four countries of our experiment, clearly shows that it is not only the low-tech country in the union to obviously benefit from a productivity increase but also the high-tech country.

Figure 20: The average real GDP per capita in the union and isolated countries; the statistical measure $\Sigma_{12}^1(Y)$ given in equation 35. The left panel shows the average of the sum of real GDP per capita of the countries in the union C1 and C2; the right panel shows the average of the sum of real GDP per capita of the isolated countries C3 and C4. X-axis indicates scenarios with respect to the productivity level of the low-tech country $\gamma$. Colors indicate scenarios with different values of mobility frictions $\rho$ which are given in the legend. The solid lines in the bars represent median while dots denote mean.

Figure 21: Real GDP per capita by countries. X-axis indicates scenarios with respect to the productivity level of the low-tech country $\gamma$. Colors indicate scenarios with different values of mobility frictions $\rho$ which are given in the legend. The solid lines in the bars represent median while dots denote mean.
Result 9.D  The real sovereign debt per capita in the union increases with low mobility frictions and high productivity gap.

Corollary 9.1.D  For low mobility frictions the real debt of the union is higher than the real debt of isolated countries

Corollary 9.2.D  The higher the low-tech country productivity, the lower the real sovereign debt in the union. The opposite holds for isolated countries

We comment result 9.D and its corollaries together in this paragraph. Figure 22 compares the debt in the union (as the sum of member countries’ sovereign debt) with the debt in the separate countries, and shows several interesting issues. First, when the productivity gap is large, and the mobility frictions are low, the public debt in the union is much higher than in the isolated countries. The debt of the union becomes lower than the debt of the isolated countries only when the productivity gap and the mobility of workers are reduced. We explain this result by highlighting that when the technological gap is high, and mobility frictions are low, we observe a strong migration from the low-tech to the high-tech country, as stated in result 4.D. The dimension of the public debt of the low-tech country, before the migration, is commensurate with the population of the country pre-migration, which is in turn, proportional to the isolated low-tech country. However, when the migration takes place, the low-tech country is not able to reduce its debt because, despite a sharp decrease in public expenditures, it also faces a sharp decrease in tax income, which is proportional to the GDP, and the public debt does not decrease,
but actually it even increases due to the unsustainable burden of interest on public debt. From figure 23a, presenting the difference in total public spending between countries in or out of the union, we can notice that the low-tech country in the union reduces its public expenditures with respect to the isolated low-tech country drastically. However, it is not able to reduce sovereign debt, which is even higher with respect to the isolated low-tech country, as figure 23b proves. The problem is the per capita interest payment on the outstanding debt, represented in figure 24, which is overwhelming for a country that experienced a significant contraction in population. This component of the government expenditure prevents any reduction of public debt, even though the other expenditure components are reduced.

Therefore, the higher debt of the union with respect to the isolated countries in the case of large productivity gap, as shown in figure 22, can be explained by an increase in the debt of the high-tech country, that is significantly increasing its GDP due to a massive immigration, which is not compensated by a decrease of debt in the low-tech country, due to the reasons that have been just formulated.

Concerning Corollary 9.2.D, when countries are isolated, the total real sovereign debt increases with total GDP, as in figure 22. So, when the low-tech country becomes more productive, raising its real GDP, the total debt also increases. However, in the union, the mechanism is more complex. It is true that, as for the isolated countries, when the productivity of the low-tech country in the union grows, total real GDP of the union grows as well (see again figure 13a). Nonetheless, some other aspects should be kept into account. First, raising the productivity of the low-tech country mitigates the problem of sustainability concerning its public debt (the problem is solved when the gap is low). Second, the virtuous mechanism of markets interaction in the union improves the employment conditions in the private sector (see figure 25), reducing the number of people that receive unemployment benefits, while increasing the number of taxpayers.

We will finally examine qualitatively some average time series representing some specific scenarios, in order to better explain our results. The economic indicator that we present in the following figures is the one of equation 34. In figure 26a and 26b we present consumption and production for one of the eighty eight scenarios where mobility frictions $\rho$ are at median value ($\rho = 0.4$) and the technology gap is high $\Delta \gamma = 1.1$, with the technology of the high-tech country $\gamma_H = 3$ and the technology of the low-tech country $\gamma_L = 1.9$. The figures show how the majority of the goods are produced in the high-tech country, while part of them is consumed also in the low-tech country. The balance of payments in figure 26c reflects this configuration of net exports. We can also notice in figure 26d that the high tech country is able to attract workers from the low tech country and consequently achieve its higher total production as it is reflected in figure 26b. Figure 27a displays the real wage in the four countries. It is interesting to notice that the difference in real wage between the two isolated countries is almost constant and it does not converge at all. However, the presence of common markets allows the real wage of the two countries in the union to be more close and to converge slowly. This convergence of the real wage is an essential element in our analysis because when the real wage converges, the economy reaches a sort of equilibrium in the labor market.

For lower levels of mobility frictions, the real wage is no more able to converge. See for instance figure 27b where the mobility frictions changes from $\rho = 0.4$ to $\rho = 0.2$. The effect is
that the real wage is no more able to converge, and the emigration of workers from the low-tech to the high-tech country is more massive, as depicted in figure 28. Contrary, if the productivity of the low-tech country is increased to $\gamma_L = 2.4$ (decreasing the technology gap to $\Delta \gamma = 0.7$), the real wage converges earlier and in a more stable way, as figure 29a clearly shows. The population of the two countries is also much more balanced, but it persists a biased balance of trade due to the higher productivity of one country that has, therefore, positive net-exports. In
order to see more balanced net exports we have to raise further the productivity of the low-tech country ($\gamma_L = 2.9$), see figure 29b, where the high-tech country is still generally a net exporter, but the situation is much more balanced.
4.5 The fiscal pool

To analyze the effects of the fiscal pool in the union, we need to discuss the two following types of plots briefly.

Figure 30 shows the statistical measure $\Delta T_{12,34}(Y)$, presented in equation 35, where $Y$ is the real GDP. The left part of the figure presents the difference between the sum of real GDP of the countries in the union with a fiscal pool (countries 1 and 2 in $\Sigma_{12}(Y)$) and without the fiscal pool.
(countries 3 and 4 in $\Sigma_{34}(Y)$). The part on the right presents the difference between the sum of real GDP of the isolated countries with the fiscal pool (countries 1 and 2 in $\Sigma_{12}(Y)$) and without the fiscal pool (countries 3 and 4 in $\Sigma_{34}(Y)$). Of course, we should observe no difference for the isolated countries as the mechanism of fiscal pool only concerns the union. In other words, figure 30 compares the real GDP in the union when the fiscal pool mechanism is enabled with the real GDP in the union when the fiscal pool mechanism is not enabled. If we observe a positive (negative) value, it means that the real GDP is higher (lower) when the fiscal pool is activated.

Figure 31 shows the statistical measure $\Delta\Psi_{12,34}(Y)$, presented in equation 36, where $Y$ is the real GDP. The left part of the figure presents the difference between the gap in real GDP between the high-tech country and the low-tech country in the union with fiscal pool (countries 1 and 2 in $\Psi_{12}(Y)$) and the same gap without fiscal pool (countries 3 and 4 in $\Psi_{34}(Y)$). The part on the right presents the same indicator for the isolated countries. Of course, again, we should observe no difference for the isolated countries as the mechanism of fiscal pool only concerns the union. In other words, figure 31 compares the inequality (or gap) in real GDP between the high-tech country and the low-tech country in the case of the active and inactive fiscal pool. If we observe a positive (negative) value, it means that the inequality in real GDP is higher (lower) when the fiscal pool is enabled.
Figure 30: The differences of real GDP between the setups with and without the fiscal pool; the statistical measure $\Delta \Sigma_{12,34} (Y)$ given in equation 35. The left panel shows the difference of the sum of real GDP in countries C1 and C2 in the union with the fiscal pool and the sum of real GDP in countries C1 and C2 in the union without the pool. The right panel shows the difference of the sum of real GDP in isolated countries C3 and C4, with the fiscal pool and the sum of real GDP in isolated countries C3 and C4 without the pool. X-axis indicates scenarios with respect to the productivity level of the low-tech country $\gamma_L$. Colors indicate scenarios with different values of mobility frictions $\rho$ which are given in the legend. The solid lines in the bars represent median while dots denote mean.

**Result 10.** The total, and per capita, level of real GDP in the union is reduced by the activation of the fiscal pool, when mobility frictions are low, especially for a large technology gap. In the other scenarios, the fiscal pool does not affect total production.

Figure 30 proves this result.

**Result 11.** The inequality between real GDP in the high-tech and low-tech countries is reduced by the activation of the fiscal pool, when mobility frictions are low, especially for a large technology gap. In the other scenarios, the fiscal pool does not affect this inequality.

Figure 31 proves this result.

So, from our experiment, it seems that the activation of the fiscal pool gives conflicting results. On the one hand, it reduces the inequality between the high and low tech countries, highlighted in corollary 7.2D, on the other hand, it decreases the total production in the union. However, we need to know if this reduction of inequality is given by an improvement in the GDP of the low-tech country or by a decrease in the GDP of the high-tech country, or both. In order to give an answer to this question, we present figure 32a, based on the same statistical indicator $\Delta \Psi_{12,34} (Y)$.

Figure 32a shows the statistical measure $\Delta \Psi_{12,34} (Y)$, presented in equation 36, where $Y$ is the real GDP. The left part of the figure presents the difference between the gap in real GDP between the high-tech country in the union with fiscal pool and high tech country out of the union. The part on the right presents the difference between the gap in real GDP between the low-tech country in the union with fiscal pool and low-tech country out of the union. In other words, figure 32a compares the real GDP of the high-tech (low-tech) country in the
Figure 31: The difference in setups with and without the fiscal pool of the mean of differences in Real GDP between two countries; the statistical measure $\Psi_{12}(Y)$ given in equation 36. The left panel shows the difference between countries C1 and C2 in the union, while the right panel shows the difference in isolated countries C3 and C4. X-axis indicates scenarios with respect to the productivity level of the low-tech country $\gamma$. Colors indicate scenarios with different values of mobility frictions $\rho$ which are given in the legend. The solid lines in the bars represent median while dots denote mean.

Corollary 11.1. The fiscal pool increases real GDP in the low-tech country, whereas it decreases GDP in the high-tech country, when mobility frictions are low, especially for a large technology gap. In the other scenarios, the fiscal pool does not affect real GDP. Figure 32a proves this result. The main reason for this result is that the fiscal pool, given the mobility frictions, manages to reduce the migration from the low-tech country to the high-tech country, as figure 32b clearly shows.

Corollary 11.2. The fiscal pool increases the welfare in the low-tech country, whereas it slightly decreases it in the high-tech country, when mobility frictions are low, especially for a large technology gap. In the other scenarios, the fiscal pool does not affect welfare. Figures 33a, showing the real wage, 33b, showing per capita consumption, and 32a, showing the per capita GDP, prove this result.

The previous results show that the fiscal pool can reduce the gap between the high and low tech countries in the union. The cost is a lower immigration to the more productive high-tech country, and therefore a loss in the overall production in the union. However, concerning welfare, the loss of real wage in the high-tech country, due to the activation of the fiscal pool, seems less significant than the gain for the low-tech country. So, we could comment that the fiscal pool can be useful if the target is reducing inequalities among the union countries, whereas it is not
number of households between high-tech countries C1 in the union and C3 out of the union, while the right panel shows the difference in real GDP of low-tech countries C2 in the union and C4 out of the union. On the figure 32a the left panel shows the difference in real GDP between high-tech countries C1 in the union and C3 out of the union, while the right panel shows the difference in real GDP of low-tech countries C2 in the union and C4 out of the union. X-axis indicates scenarios with respect to the productivity level of the low-tech country C2. Colors indicate scenarios with different values of mobility frictions $\rho$ which are given in the legend. The solid lines in the bars represent median while dots denote mean.

useful if the goal is raising the total production in the union, for instance in the perspective of competing with other foreign countries or unions.

Figure 32: The difference in setups with and without the fiscal pool of the mean of differences between two countries; the statistical measure $\Psi_{12}(Y)$ given in equation 36. On the figure 32a the left panel shows the difference in real GDP between high-tech countries C1 in the union and C3 out of the union, while the right panel shows the difference in real GDP of low-tech countries C2 in the union and C4 out of the union. On the figure 32b the left panel shows the difference in the number of households between high-tech countries C1 in the union and C3 out of the union, while the right panel shows the difference in the number of households of low-tech countries C2 in the union and C4 out of the union. X-axis indicates scenarios with respect to the productivity level of the low-tech country C2. Colors indicate scenarios with different values of mobility frictions $\rho$ which are given in the legend. The solid lines in the bars represent median while dots denote mean.
Real consumption
Real wage level

(a) Real wage level.

(b) Real consumption per capita.

Figure 33: The difference in setups with and without the fiscal pool of the mean of differences between two countries; the statistical measure $Ψ_{13}(Y)$ given in equation 36. On the figure 33a the left panel shows the difference in the wage level of high-tech countries C1 in the union and C3 out of the union, while the right panel shows the difference in the wage level of low-tech countries C2 in the union and C4 out of the union. On the figure 33b the left panel shows the difference in the real consumption per capita of high-tech countries C1 in the union and C3 out of the union, while the right panel shows the difference in the real consumption per capita of low-tech countries C2 in the union and C4 out of the union. X-axis indicates scenarios with respect to the productivity level of the low-tech country $γ_L$. Colors indicate scenarios with different values of mobility frictions $ρ$ which are given in the legend. The solid lines in the bars represent median while dots denote mean.
5 Conclusions

In this work, we designed an extension of the Eurace agent-based economic model, which allows for the inclusion of multiple countries. This extension has been devised in a very flexible way, in order to embrace a broad spectrum of economic integration among countries. The model can include isolated countries and cluster of countries that have different degrees of integration, sharing the goods market and/or the labor market and/or the financial market. In this way, we created a flexible instrument, which are able to mimic several real-world configurations. We start our analysis in this thesis with a simple configuration two by two, where we have two countries in a fully integrated union and two countries that are isolated control states. Nevertheless, this simple setting allows us to get several interesting insight.

Our work has been inspired by the current economic and political challenges related to the increasing integration of the leading economies and, in particular, by the European Union. Among these challenges, we can remind the economic and financial integration, the inequality issues, immigration and the sovereign debt crises. We used our specific approach to investigate how the economies can better deal with these current problems, trying to examine also strategies for a long-term sustainable path.

We performed a first experiment, where all countries are identical, finding that it is always convenient for identical countries to join in a union. Not only the union as a whole outperforms the independent countries, but also each state in the union is better off with respect to the isolated version. However, an excess of workers mobility can weaken the performance of the union and even create persistent inequality between countries. Furthermore, the monetary policy of the union central bank can deteriorate if the difference between countries growth and in general it tends to overshoot the inflation target. This is because, although there is a better price coordination among countries in the union, the business cycles are not completely synchronized and the central bank is less efficient in reducing prices in the country with higher inflation. Finally, we find that the real sovereign debt in the countries of the union is always lower.

When the two countries differ in productivity, the union on aggregate performs again better than the sum of the isolated countries counterparts. However, taking into account the welfare in both countries of the union, it is not always convenient to form the union. The performance of the union is strongly affected by the level of labor market frictions and the size of the productivity gap among countries. Results suggest that, when the technology gap is large, workers mobility improves the overall economic performance of the union; while when the technology gap is small, the labor mobility worsens it. The main idea is that the high technology country tends to attract workers when the mobility is high, and the union benefits by a more productive employment of the labor force. However, a lack of mobility frictions can damage the welfare conditions of the low-tech country, increasing inequality. The real sovereign debt per capita in the union increases with low mobility frictions and high productivity gap.

Concerning the sustainability of the union, especially in the critical case when mobility frictions are low and the technology gap between countries is significant, we found that a stronger fiscal integration reduces inequality between countries, allowing the low-tech country and, hence, the total union to sustain even with a higher mobility of workers. This, however, comes at the cost of harming the overall output of the union, because it limits the emigration from the low-tech
to the high-tech country. In particular, the fiscal pool increases real GDP and welfare in the low-tech country whereas, slightly reduces real GDP and well-being of the high-tech country, mainly when mobility frictions are small, and the productivity gap is big. To sum up, our results suggest that a better fiscal integration would be useful if the policy target in the union is reducing inequalities among the countries, whereas it would not be beneficial if the goal is to maximize the overall production of the union. This raises some interesting political and ethical issues about the kind of development strategy that a union (with particular reference to the European Union) want to pursue. Our study seems to underline that the objective of maximizing the economic growth is not always in harmony with the objective of reducing inequality and preserving the structural and cultural difference that exist among countries. We personally consider that a wise mixture between these two lines should be adopted.
References


De Grauwe, P. and Gerba, E. (2017). Rationalising the irrational: How relevant are beliefs for supply side frictions and business cycle fluctuations?


Appendix

From a single to multi-country setting

In this section we present an example of one possible configuration of the model. Figure 34 shows, for instance, that we can create a global economy that includes one labor union that integrates two heterogeneous countries which differs in terms of the number of households, the number of firms, and the number of banks; one monetary union with labor mobility and the international goods market, which integrates three heterogeneous countries; and finally one isolated country.

![Figure 34: A multi-country setting.](image)

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### Table 2: Balance sheets

<table>
<thead>
<tr>
<th>Agent</th>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Household</td>
<td>Liquidity: $M_h$</td>
<td>Equity: $E_h$</td>
</tr>
<tr>
<td><em>abbrev.: HH</em></td>
<td>Equity Shares: $ES^{f,k}_h$</td>
<td></td>
</tr>
<tr>
<td><em>index: h = 1,....,N_{HH}</em></td>
<td>Gov Bonds: $GB_h$</td>
<td></td>
</tr>
<tr>
<td>Consumption Goods Producers</td>
<td>Liquidity: $M_f$</td>
<td>Debt: $D_f = \sum_b D_{f,b}^b$</td>
</tr>
<tr>
<td><em>abbrev.: F</em></td>
<td>Capital goods: $K_f$</td>
<td>Equity: $E_f$</td>
</tr>
<tr>
<td><em>index: f = 1,....,N_{F}</em></td>
<td>Inventories: $I_f$</td>
<td></td>
</tr>
<tr>
<td>Capital Goods Producers</td>
<td>Liquidity: $M_k$</td>
<td>Equity: $E_k$</td>
</tr>
<tr>
<td><em>abbrev.: KP</em></td>
<td></td>
<td></td>
</tr>
<tr>
<td><em>index: k = 1,....,N_{KP}</em></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bank</td>
<td>Liquidity: $M_b$</td>
<td>Deposits (Liquidity of HH, F and KP): $D_{b,h} = \sum_{h,f,k} M_{h,f,k}^b$</td>
</tr>
<tr>
<td><em>abbrev.: B</em></td>
<td>Loans: $L_b = \sum_f D_{f,b}$</td>
<td>Standing facility with the Central Bank: $D_{c,b}$</td>
</tr>
<tr>
<td><em>index: b = 1,....,N_{B}</em></td>
<td></td>
<td>Equity: $E_b$</td>
</tr>
<tr>
<td>Government</td>
<td>Liquidity: $M_g$</td>
<td>Bond Debt: $BD_{g}$</td>
</tr>
<tr>
<td><em>abbrev.: GOV</em></td>
<td></td>
<td>Loan Debt: $LD_{g}$</td>
</tr>
<tr>
<td><em>index: g</em></td>
<td></td>
<td>Equity: $E_g$</td>
</tr>
<tr>
<td>Central Bank</td>
<td>Liquidity: $M_{cb}$</td>
<td>Outstanding fiat money: $FM_{cb}$</td>
</tr>
<tr>
<td><em>abbrev.: CB</em></td>
<td>Loans to the government and banks: $L_{cb} = LD_{g} + \sum_b D_{h,b}$</td>
<td>Deposits (Liquidity of the GOV and B): $D_{cb} = M_g + \sum_b M_{h,b}$</td>
</tr>
<tr>
<td><em>index: cb</em></td>
<td></td>
<td>Standing facility with the UCB: $D_{c}$</td>
</tr>
<tr>
<td>Union Central Bank</td>
<td>Liquidity: $M_{ucb}$</td>
<td>Equity: $E_{ucb}$</td>
</tr>
<tr>
<td><em>abbrev.: UCB</em></td>
<td>Loans to the Central Banks, $L_{ucb} = \sum_b D_{cb}$</td>
<td></td>
</tr>
<tr>
<td><em>index: ucb</em></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2. The balance sheets of agents populating the Eurace economy. Balance sheet entries in the table are named with capital letters. A subscript character is the index of the agent that the variable refers to, while the superscript refers to other agents’ balance sheets. For instance, $D_f$ refers to the total debt of firm $f$ (liability), and $L_b$ refers to the total loans of bank $b$ (asset), while $D_{f,b}^b (L_{f,b}^b)$ refers to the debt (loan) that firm $f$ (bank $b$) has with bank $b$ (firm $f$). Hence, there is the identity: $D_f = \sum_b D_{f,b}$ and $L_b = \sum_f L_{f,b}$. 

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### Table 3: The description of parameters and symbols - Firms

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Name</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(n_f)</td>
<td>Labor force</td>
<td>The number of workers used in the production process.</td>
</tr>
<tr>
<td>(\tilde{n}_f)</td>
<td>Desired labor force.</td>
<td></td>
</tr>
<tr>
<td>(K_f)</td>
<td>Capital</td>
<td>The capital stock of firm (f).</td>
</tr>
<tr>
<td>(\tilde{K}_f)</td>
<td>Desired Capital</td>
<td>The desired capital stock of firm (f).</td>
</tr>
<tr>
<td>(\gamma_f)</td>
<td>Productivity parameter</td>
<td>The productivity of firm’s physical capital.</td>
</tr>
<tr>
<td>(edu)</td>
<td>General skill level</td>
<td>The education degree of households (workers).</td>
</tr>
<tr>
<td>(A_f)</td>
<td>The productivity factor</td>
<td>The productivity parameter used in the Cobb Douglas production function.</td>
</tr>
<tr>
<td>(q_f)</td>
<td>Consumption good quantity</td>
<td>The amount of homogeneous consumption good that is produced by firm (f).</td>
</tr>
<tr>
<td>(t)</td>
<td>Time</td>
<td>The counter of the basic time unit in the model which is day or iteration.</td>
</tr>
<tr>
<td>(\xi)</td>
<td>Physical capital depreciation rate</td>
<td>Monthly physical capital depreciation rate</td>
</tr>
<tr>
<td>(i_f)</td>
<td>Investment</td>
<td>The amount of money that firms invest to buy new physical capital.</td>
</tr>
<tr>
<td>(i^a_f)</td>
<td>Investment grid</td>
<td>The grid of visible investment values.</td>
</tr>
<tr>
<td>(\mathbb{E}<em>t[q</em>{f,t+1}])</td>
<td>Expected demand</td>
<td>The quantity of goods that the firm (f) expects to be demanded next month.</td>
</tr>
<tr>
<td>(I_f)</td>
<td>Inventories</td>
<td>The quantity of the current inventories of firm (f).</td>
</tr>
<tr>
<td>(\tilde{q}_f)</td>
<td>Planned production quantity</td>
<td>Planned production quantity for the next month.</td>
</tr>
<tr>
<td>(\bar{s}_f)</td>
<td>Average workers’ specific skill</td>
<td></td>
</tr>
<tr>
<td>(r^b_{f,t})</td>
<td>Interest rate</td>
<td>Interest rate associated to the loan of bank (b) taken at time (t).</td>
</tr>
<tr>
<td>(p_K)</td>
<td>Capital price</td>
<td>The price of one unit of physical capital.</td>
</tr>
<tr>
<td>(\tau^c)</td>
<td>Corporate tax</td>
<td>The corporate tax rate.</td>
</tr>
<tr>
<td>(\tau^{vat})</td>
<td>Value-added tax</td>
<td>The Value-added tax rate.</td>
</tr>
<tr>
<td>(\Delta q^{f,m}_{t})</td>
<td>Additional monthly production</td>
<td>The additional amount of monthly production given including the new capital investment (i^a_f).</td>
</tr>
<tr>
<td>(\lambda_{f,g})</td>
<td>Market specific weights</td>
<td></td>
</tr>
<tr>
<td>(\mathbb{E}_t P)</td>
<td>Expected price level</td>
<td></td>
</tr>
<tr>
<td>(P_{g,t})</td>
<td>Price level</td>
<td>The price level in the country (g) at time (t).</td>
</tr>
<tr>
<td>(\mathbb{E}<em>t[\pi</em>{g,t+1}])</td>
<td>Expected inflation</td>
<td></td>
</tr>
<tr>
<td>(\pi_t)</td>
<td>Target inflation</td>
<td>The inflation target announced by the Central Bank at time (t).</td>
</tr>
<tr>
<td>(m)</td>
<td>The future month of discounting the NPV</td>
<td>It shows how much agents believes in the Central Bank’s announcements.</td>
</tr>
<tr>
<td>(\theta)</td>
<td>The trust factor</td>
<td></td>
</tr>
</tbody>
</table>

*Continued on next page*
### Table 3 – Continued from previous page

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Name</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_f$</td>
<td>Total debt</td>
<td>Firms’ balance sheet item.</td>
</tr>
<tr>
<td>$A_f$</td>
<td>Total assets</td>
<td>Firms’ balance sheet item.</td>
</tr>
<tr>
<td>$\delta$</td>
<td></td>
<td>Debt write-off parameter It determines the amount of debt that will be written-off during the bankruptcy procedure.</td>
</tr>
<tr>
<td>$E_f$</td>
<td>Total equity</td>
<td>Firms’ balance sheet item.</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td></td>
<td>Target leverage ratio parameter It determines the leverage ratio level of the new company in the bankruptcy procedure.</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Mark-up parameter</td>
<td>Mark-up rate in the firms’ price setting.</td>
</tr>
<tr>
<td>$c_f$</td>
<td>Firms’ unit production costs</td>
<td></td>
</tr>
</tbody>
</table>

### Table 4: The description of parameters and symbols - Households

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Name</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s_h,\text{edu}$</td>
<td>Specific skill</td>
<td>The specific skill level of households.</td>
</tr>
<tr>
<td>$\zeta(\text{edu})$</td>
<td>Specific skill adjustment parameter</td>
<td>The parameter (function) determines the speed of the improvement of specific skill level. It is an increasing function of the general skill level $\text{edu}$.</td>
</tr>
<tr>
<td>$w_{f,\text{edu}}$</td>
<td>Wage offer</td>
<td></td>
</tr>
<tr>
<td>$\bar{w}_{f,\text{edu}}$</td>
<td>Households’ reservation wage</td>
<td></td>
</tr>
<tr>
<td>$\rho$</td>
<td>Mobility friction rate</td>
<td>The sum of total dividend payments, total bond coupons payments, the unemployment benefit or wage and the government transfer.</td>
</tr>
<tr>
<td>$y_h$</td>
<td>Total household’s gross income</td>
<td></td>
</tr>
<tr>
<td>$w_h$</td>
<td>Wage</td>
<td>Monthly wage payment.</td>
</tr>
<tr>
<td>$u_h$</td>
<td>Unemployment benefit</td>
<td>Monthly payment of the unemployment benefit.</td>
</tr>
<tr>
<td>$y_{\text{GT}}$</td>
<td>Government lump-sum transfer</td>
<td></td>
</tr>
<tr>
<td>$n_{Bg,h}$</td>
<td>The number of bonds</td>
<td>The number of bonds from country g that is owned by household h.</td>
</tr>
<tr>
<td>$c_{Bg}$</td>
<td>Bound coupon payment</td>
<td>The bond coupon payment by country g.</td>
</tr>
<tr>
<td>$e$</td>
<td>An equity share issuer</td>
<td>Can be a firm, a bank or the physical capital producer.</td>
</tr>
<tr>
<td>$n_{e}^h$</td>
<td>The number of equities held by the</td>
<td></td>
</tr>
<tr>
<td>$d_e$</td>
<td>The dividend payment per equity $e$</td>
<td></td>
</tr>
<tr>
<td>$m_T^h$</td>
<td>Total tax payment</td>
<td>The sum of all tax payments.</td>
</tr>
<tr>
<td>$\tau^w$</td>
<td>Wage tax rate</td>
<td>The tax rate on wages.</td>
</tr>
<tr>
<td>$\tau^c$</td>
<td>Capital tax rate</td>
<td>The tax rate on the received dividend payments.</td>
</tr>
<tr>
<td>$y_{\text{NET}}^h$</td>
<td>Total net income</td>
<td>Household’s total net income in the.</td>
</tr>
</tbody>
</table>

Continued on next page
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Name</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y_{NET,h} )</td>
<td>Quarterly average total net income</td>
<td>Household’s average total net income in the last three months.</td>
</tr>
<tr>
<td>( W_h )</td>
<td>Households’ wealth</td>
<td>The sum of asset portfolio valued at the most recent market prices and liquidity on Households’ payment account.</td>
</tr>
<tr>
<td>( \beta )</td>
<td>Households’ consumption budget.</td>
<td></td>
</tr>
<tr>
<td>( R^e )</td>
<td>The total return of assets ( e ).</td>
<td></td>
</tr>
<tr>
<td>( \phi^c )</td>
<td>Carrol consumption parameter</td>
<td>It sets the adjustment of speed and it is constant.</td>
</tr>
<tr>
<td>( \omega_h )</td>
<td>Current wealth to income ratio</td>
<td>It is the ratio of current household’s wealth to average total net income.</td>
</tr>
<tr>
<td>( \bar{\omega} )</td>
<td>Target wealth to income ratio</td>
<td>It is the constant target wealth to income ratio.</td>
</tr>
</tbody>
</table>

**Table 5: The description of parameters and symbols - Commercial Banks**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Name</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma_b )</td>
<td>The spread sensitivity to the credit worthiness of the firms.</td>
<td></td>
</tr>
<tr>
<td>( \tilde{A}_b )</td>
<td>The risk-weighted loan portfolio of banks.</td>
<td>The sum of all loans of bank ( b ) weighted by firms’ probability of default.</td>
</tr>
<tr>
<td>( \psi )</td>
<td>The capital requirement.</td>
<td>The equity capital ( E_b ) must be at least as large as the percentage ( \psi ) of the risk-weighted assets portfolio ( \tilde{A}_b ).</td>
</tr>
<tr>
<td>( \tilde{\ell}_f )</td>
<td>The loan request of firm ( f ).</td>
<td></td>
</tr>
<tr>
<td>( \ell^f_b )</td>
<td>The loan granted by the bank ( b ) to the firm ( f ).</td>
<td></td>
</tr>
<tr>
<td>( \eta(.) )</td>
<td>The dividend rate step function.</td>
<td></td>
</tr>
<tr>
<td>( a_b )</td>
<td>The parameter in the likelihood function.</td>
<td></td>
</tr>
<tr>
<td>( \kappa_b )</td>
<td>The parameter in the likelihood function.</td>
<td></td>
</tr>
</tbody>
</table>

**Table 6: The description of parameters and symbols - The Central Bank**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Name</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \pi )</td>
<td>Inflation</td>
<td>The yearly inflation rate.</td>
</tr>
<tr>
<td>( \hat{\pi} )</td>
<td>Desired (target) inflation</td>
<td>The desired rate of inflation.</td>
</tr>
<tr>
<td>( \nu )</td>
<td>The unemployment rate</td>
<td>Monthly unemployment rate.</td>
</tr>
<tr>
<td>( \delta )</td>
<td>The natural rate of unemployment.</td>
<td></td>
</tr>
</tbody>
</table>
Table 7: The description of parameters and symbols - The Governments

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Name</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G$</td>
<td>Public expenditures.</td>
<td></td>
</tr>
<tr>
<td>$W$</td>
<td>Public wage expenditures.</td>
<td></td>
</tr>
<tr>
<td>$UB$</td>
<td>Unemployment benefit expenditures.</td>
<td></td>
</tr>
<tr>
<td>$GT$</td>
<td>Transfer expenditures</td>
<td></td>
</tr>
<tr>
<td>$\chi$</td>
<td>Government flexibility parameter</td>
<td></td>
</tr>
<tr>
<td>$\nu$</td>
<td>The share of public workers to total</td>
<td>population in the country</td>
</tr>
</tbody>
</table>