Monetary Policy Rule, Exchange Rate Regime, and Fiscal Policy Cyclicality in a Developing Oil Economy

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Abstract

According to Frankel and Catao (2011), a commodity exporting developing economy is advised to target the product price rather than consumer price, as the former monetary policy is automatically countercyclical against the volatile terms of trade shock. This paper constructs a dynamic stochastic general equilibrium model of joint monetary and fiscal policies for a developing oil economy, to find an appropriate monetary rule combined with a pro/counter/acyclical fiscal stance based on a loss measure. The foreign exchange interventions distinguish between a managed and flexible exchange rate regime, while fiscal policy cyclicality depends on the oil output response of public consumption and public investment. The study reveals that the best policy combination is a countercyclical fiscal stance and product price monetary targeting under a managed exchange rate regime to stabilize the domestic price inflation, aggregate output, and real exchange rate in a small open economy. However, if a flexible exchange rate regime is institutionally chosen, then the consumer price targeting should be adopted, since it effectively stabilizes the domestic price inflation and aggregate output.

Keywords: oil economy, monetary policy, fiscal policy, exchange rate, oil price shock, interventions, SWF

JEL Classification: E31, E52, E62, E63, F31, F41, H54, H63, Q33, Q38

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1 Introduction

Most macroeconomic DSGE models are constructed for the developed world, incorporating its advanced market structure and relevant policy environment. Emerging market economies have their own unique features, which can modify the existing core frameworks in several respects. First, public investment should be considered separately from public consumption as a growth inducing instrument of fiscal policy (Berg, Portillo, Yang & Zanna, 2013), since it is usually associated with infrastructure and human capital, which developing countries often lack (Rioja, 2003; Sab & Smith, 2002). Second, monetary policy is typically a hybrid of inflation targeting and a managed exchange rate regime; thus, interest rate and foreign exchange interventions represent the two separate instruments of monetary policy (Ghosh, Ostry & Chamon, 2016). Third, in an underdeveloped domestic financial market, the investments of firms are often financed by foreign funds, so that physical capital and foreign debt can be linked through a collateral constraint (Faia & Iliopulos, 2011 linked durables goods with the foreign debt). Fourth, households are heterogeneous in their income and access to a financial market; a certain portion of the population may be liquidity constrained with only wages, without savings (Mankiw, 2000; Gali, Lopez-Salido & Valles, 2007). These four structural specifics are incorporated in the model of Algozhina (2012) calibrated for Hungary as the first emerging market economy to be severely hit by the global financial crisis of 2008.

This paper extends that model for a subset of emerging open economies which export oil. The oil-exporting developing economies obviously differ from other emerging countries and need to be examined through their own DSGE framework. The particular features of an oil economy are as follows: The oil and non-oil production sectors should be specified separately. The economy is exposed to a volatile exogenous world oil price shock. SWF is established collecting the oil tax revenues, saving them abroad, and partly transferring to the government budget\(^1\). Finally, motivated by Frankel and Catao (2011), monetary policy can follow product price targeting (PPT) as an alternative to CPI; thus, these two anchors need to be compared in a general equilibrium framework jointly with fiscal policy based on some welfare measure, to determine which one is preferred.

Frankel and Catao (2011) argue that commodity exporting economies are better off targeting the output price index, which includes export commodities and excludes import products; such monetary policy is automatically countercyclical against the volatile terms of trade shock. The argument is that if the world oil price increases and there is PPT, then monetary policy tightens by raising its interest rate, thus causing exchange rate appreciation, which is the objective of offsetting the initial positive terms of trade shock. Conversely, an adverse terms of trade shock, such as a fall in oil price, can be mitigated

\(^1\)The mechanism of SWF accumulation differs across countries, but since the model is calibrated for Kazakhstan, its experience is specifically captured.
by the exchange rate depreciation under PPT. The CPI inflation targeting, in contrast, does not respond to export prices, but to import prices. If there is an adverse terms of trade shock, such as an increase of import prices, CPI targeting brings exchange rate appreciation, further exacerbating the initial negative shock for producers of tradable goods, who use imports as their intermediate inputs. "Bottom line: a Product Price Targeter would appreciate in response to an increase in world prices of its commodity exports, not in response to an increase in world prices of its imports. CPI targeting gets this backwards." (Frankel & Catao, 2011, p. 4).

The aim of this paper is, therefore, to construct a DSGE model for a developing oil economy capturing its structural specifics, as defined above, to examine the CPI/PPT monetary policy rule under a flexible/managed exchange rate regime combined with a pro/counter/acyclical fiscal policy. In order to assess whether an anchor of price stability should be the CPI or PPT, a welfare measure is adopted. According to De Paoli (2009), the welfare for a small open economy is represented as a loss function of three variables: variations in domestic price inflation, aggregate output, and real exchange rate. Based on this loss measure, inflation and output responses of the Taylor rule are optimized for managed and flexible exchange rate regimes, distinguished by the presence of foreign exchange interventions.

Fiscal policy cyclicality is associated with the oil output response of public spending. This is because the business cycle of an oil producing economy tends to correlate more with its oil sector's output rather than aggregate output; thus, commodity boom/bust is the cycle, to which fiscal policy responds. Since this model focuses on the oil price shock affecting the real oil output, the latter needs to be directly included in fiscal rules to ensure that fiscal policy transmits the shock into the economy. Acyclical fiscal policy assumes the zero oil output response of public spending and is taken as a benchmark to calculate loss in deviation from it; thus, the pro/countercyclical fiscal stance corresponds to the positive/negative oil output response, respectively. The impulse-response functions to a fall in world oil price shock, also referred to as adverse terms of trade shock, are analyzed to understand its transmission mechanism.

The calibration is based on Kazakhstan as a small open, oil exporting economy hit by the global financial crisis of 2008 due to high private sector's foreign debt. Since 2006, the IMF has included Kazakhstan in its "fuel exporters" group analyzed in the World Economic Outlook. In 2000, Kazakhstan established its SWF managed by the National Bank on behalf of the Ministry of Finance. Oil tax revenues directly accumulate the SWF which is invested abroad, but regularly, there are ad hoc transfers from SWF to the government budget. Monetary policy is independently conducted by the National Bank.

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2The classification is made on the evidence that over five years the average share of fuel exports in total exports exceeds 40 percent.
pursuing a primary goal of price stability and intervening in the foreign exchange market to avoid speculative attacks.

In section 2.2, the model is outlined with its two types of households, standard optimizers and rule-of-thumb households, non-oil firms acting in a monopolistically competitive market, capital-intensive oil producer, two monetary policy rules for two instruments, and respective fiscal policy rules. Section 2.3 describes the model calibration. Section 2.4 examines the main results and sensitivity tests followed by the conclusion.

2 Model

The model has several frictions: an incomplete asset market, investment adjustment costs, collateral constraint, and the Calvo price setting. The underlying assumption is that the rest of the world is a saver, while the domestic economy is a borrower; thus, the foreign discount factor is higher than the domestic discount factor, as the domestic households might be relatively impatient compared to the rest of the world. This implies that the interest rate of an emerging economy is always higher than the foreign interest rate, which is consistent with the evidence (Reinhart & Reinhart, 2008). Therefore, imperfect capital mobility is assumed, since the positive difference between the domestic and foreign interest rates exists and foreign borrowings are restricted. According to the impossible trinity, in turn, an independent monetary policy in terms of inflation targeting and a managed exchange rate regime are feasible under imperfect capital mobility.

The model uniquely specifies the oil production as a capital-intensive sector with only capital input, for simplicity, which is accumulated by FDI that responds to the world oil price. The oil sector is owned by the government and foreigners who pay royalty taxes that accumulate the SWF. There are also transfers from SWF to the government budget. These specifics related to oil sector capture the country case of Kazakhstan, contributing thereby to a limited literature on DSGE models for commodity-exporting, emerging market economies. The following subsections describe in detail the model structure.

2.1 Households

The economy is populated by a continuum of households on the interval [0,1], where the fraction $\mu$ is rule-of-thumb households. They do not have access to financial markets and consume all of their disposable income each period. The other $(1 - \mu)$ fraction of households are forward-looking households who hold government bonds, borrow from abroad, invest in non-oil physical capital, rent the capital to non-oil firms, and receive profits from those monopolistic non-oil firms and transfers from the central bank. The labor market is competitive, wage is the same across all households, and both types
of households work the same number of hours. The superscript \( S \) indicates a variable associated with savers (forward-looking households), while \( N \) is for non-savers (rule-of-thumb households).

The forward-looking household maximizes its utility (Schmitt-Grohe & Uribe, 2003):

\[
E_0 \sum_{t=0}^{\infty} \beta^t \left( C_t^S - \phi^{-1} N_t^{S} \right)^{1-\sigma} \frac{1}{1-\sigma}, \quad \phi > 1, \ \sigma > 1 \tag{1}
\]

subject to the following budget constraint:

\[
C_t^S + I_t + b_t + R_{t-1}^* \frac{RER_t}{RER_{t-1}} \frac{b_t}{\pi_t} + T_t^S = W_t N_t + R_t^{kn} K_{t-1}^{no} + R_{t-1} \frac{b_{t-1}}{\pi_t} + b_t^* + \Pi_t + CB_t, \tag{2}
\]

where \( b_t = \frac{b_t}{\pi_t} \) is the real purchase of government bonds, \( RER_t \) is a CPI-based real exchange rate (the price of a foreign goods basket in terms of the domestic goods basket), \( b_t^* = RER_t \frac{b_t}{\pi_t} \) is the real foreign borrowings expressed in domestic goods (all foreign variables are denoted by an asterisk), \( R_t \) and \( R_{t-1} \) are the nominal gross domestic and foreign interest rates respectively, \( T_t^S \) is the real lump-sum taxes, \( W_t \) is a real wage, \( R_t^{kn} \) is the real rental cost of non-oil physical capital, \( \pi_t = \frac{P_t}{P_{t-1}} \) is inflation, \( \Pi_t \) is the real profits of monopolistic non-oil firms\(^3\), and \( CB_t \) is the central bank’s transfers in a form of real foreign exchange reserves (see equation 27).

The law of motion for non-oil capital is specified according to Berg et al. (2013), incorporating the investment adjustment costs:

\[
K_t^{no} = (1 - \delta) K_{t-1}^{no} + \left[ 1 - \frac{\kappa}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right)^2 \right] I_t, \quad \text{where } \kappa > 0 \tag{3}
\]

The collateral constraint relates gross foreign liabilities to a future value of capital (durable goods in Faia & Iliopulos, 2011) and always binds, assuming that foreign debt is permanently high in this economy\(^4\):

\[
R_t^* b_t^* = E_t \{ \Omega \frac{Q_t^{1+1} \pi_t^{1+1}}{RER_t^{1+1}} K_t^{no} \}, \tag{4}
\]

where \( Q_t \) is a shadow value of non-oil capital (Tobin’s Q) and \( \Omega \) is an upper bound of leverage ratio.

The problem of the saver is, therefore, to maximize the utility (1) with respect to consumption \( C_t^S \), investment \( I_t \), capital \( K_t^{no} \), government bonds holdings \( b_t \), foreign bor-

\(^3\) \( \Pi_t = Y_t^{no}(p_{n,t} - MC_t) \), where \( Y_t^{no} \) is non-oil output, \( p_{n,t} \) is the relative domestic price of non-oil goods to composite consumption, and \( MC_t \) is the marginal costs of non-oil firms to composite consumption.

\(^4\) Occasionally binding collateral constraint is ruled out because it requires global solution methods, which may be infeasible to apply in this complex model.
rowings \( b_t \), and hours worked \( N_t \) subject to the budget constraint (2), capital accumulation equation (3), and collateral constraint (4). The first-order conditions of this problem are in Appendix 2.6.3.

The rule-of-thumb household has the same preferences as the saver. It chooses only consumption and labor and its budget constraint is simply this:

$$C^N_t + T^N_t = W_t N_t$$  \( (5) \)

Each \( i \in \{ S, N \} \) type of household has the composite CES consumption preferences over domestic and foreign goods with \( \eta > 0 \) as an elasticity of substitution between goods:

$$C_t(i) = \left[ \gamma^\frac{\eta}{\eta+1} C_{H,t}^{\eta+1}(i) + (1 - \gamma)^\frac{\eta}{\eta+1} C_{F,t}^{\eta+1}(i) \right]^\frac{\eta}{\eta-1},$$

where \( \gamma \) is a home-bias parameter, while \( 1 - \gamma \) is a degree of openness. The standard consumption expenditures minimization by a household delivers the following CPI index:

$$P_t^{1-\eta} = \gamma P_{h,t}^{1-\eta} + (1 - \gamma) P_{f,t}^{1-\eta} \quad \text{or} \quad 1 = \gamma p_{h,t}^{1-\eta} + (1 - \gamma) RER_t^{1-\eta},$$  \( (6) \)

where \( p_{h,t} \) is a relative price of domestic goods to composite consumption and \( RER_t \) is also a relative price of foreign goods to composite consumption.

The aggregate consumption in turn is \( C_t = \mu C^N_t + (1 - \mu) C^S_t \). Similar to private consumption, investment is the CES basket with the same home-bias parameter \( \gamma \) and CPI for simplicity.

### 2.2 Firms

Following the basic New Keynesian framework, there are monopolistically competitive non-oil firms producing differentiated intermediate goods, and a perfectly competitive non-oil firm producing a final domestic good. The final domestic non-oil producer has a constant returns technology:

$$Y_t = \left( \int_0^1 X_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}},$$

where \( X_t(j) \) is the input amount of intermediate good \( j \) and \( \varepsilon > 1 \) is the elasticity of substitution between differentiated intermediate goods. It maximizes profit taking as given the domestic final good’s price \( P_t^b \) and intermediate goods’ prices \( P_t^b(j) \) such that
the optimal demand allocation is as follows:

\[ X_t(j) = \left( \frac{P_h(j)}{P_l} \right)^{-\varepsilon} Y_t^{no} \] (7)

Each intermediate goods non-oil firm has an identical Cobb-Douglass production function, which includes the non-oil private capital, labor, and public capital:

\[ Y_t^{no}(j) = u^{no} K^{no}_{t-1}(j)^\alpha N_t(j)^{1-\alpha} K^{\psi}_{G,t}, \] (8)

where the level of technology \( u^{no} \) is just constant (unity) and the usage of public capital is common to all firms.

Intermediate goods producers solve their problem in two stages. First, cost minimization subject to the production function (8) provides the following marginal costs common to all non-oil firms, taking the wage and rental cost of capital, denominated in domestic non-oil goods, as given:

\[ mc_t = \frac{w_t^{1-\alpha} (r_t^{kno})^\alpha}{u^{no} K^{\psi}_{G,t-1}(1-\alpha)^{1-\alpha} \alpha} \] (9)

Second, intermediate non-oil producers choose the price \( P_t^{hop} \) to maximize their discounted real profits:

\[ \sum_{m=0}^{\infty} \theta^m E_t \left\{ D_{t,t+m} Y_{t+m}^{no}(j) \left( \frac{P_t^{hop}}{P_t^{l+m}} - mc_{t+m} \right) \right\}, \] (10)

where \( D_{t,t+m} = \beta^m E_t \left( \frac{U_{c_{t+m}^{\psi}}}{C_t^{\psi}} \right) \) is a stochastic discount factor coming from the forward-looking household’s problem, subject to the demand constraint according to (7):

\[ Y_{t+m}^{no}(j) = \left( \frac{P_t^{hop}}{P_t^{l+m}} \right)^{-\varepsilon} Y_{t+m}^{no} \]

A fraction \((1 - \theta)\) of non-oil firms adjusts their prices each period, while the respective fraction \( \theta \) keeps their prices unchanged; thus, \( \theta \) is an index of price stickiness according to Calvo (1983). The domestic price index evolves as follows:

\[ (P_t^h)^{1-\varepsilon} = \theta(P_{t-1}^h)^{1-\varepsilon} + (1 - \theta)(P_t^{hop})^{1-\varepsilon} \]

The first-order condition of this price setting decision (10) is below:

\[ \sum_{m=0}^{\infty} \theta^m E_t \left\{ D_{t,t+m} Y_{t+m}^{no}(j) \left( \frac{P_t^{hop}}{P_t^{l+m}} - \frac{\varepsilon}{\varepsilon - 1} mc_{t+m} \right) \right\} = 0, \] (11)
where $\varepsilon_{t-1}$ is a frictionless price markup.

The production function of an oil firm has only capital input, assuming that oil production is a capital-intensive sector, and to avoid any complications originating from the possible labor mobility between two sectors:

$$Y^o_t = (K^o_{t-1})^{\alpha_o}$$  \hspace{1cm} (12)

The oil capital is accumulated by FDI which responds to the world oil price:

$$K^o_t = (1 - \delta)K^o_{t-1} + FDI^*_t$$  \hspace{1cm} (13)

$$\tilde{FDI}^*_t = \rho_{FDI} \tilde{FDI}^*_{t-1} + (1 - \rho_{FDI}) \tilde{P}^{os}_t$$  \hspace{1cm} (14)

Hats, hereafter, denote the deviation of variables from their steady state.

The world oil price follows the AR(1) process and has an exogenous shock referred to as the terms of trade shock:

$$\tilde{P}^{os}_t = \rho_o \tilde{P}^{os}_{t-1} + \varepsilon^o_t$$  \hspace{1cm} (15)

The oil firm receives its profits $\Pi^o_t$ net of royalties levied on production quantity at a rate $\tau^o$:

$$\Pi^o_t = (1 - \tau^o)P^{os}_t Y^o_t$$  \hspace{1cm} (16)
where \( T_t = (1 - \mu)T^S_t + \mu T^N_t \) and \( p_t^g \) is a relative price of government purchases to composite consumption with its own home-bias parameter \( \gamma_2 \).

\[
p_t^g = \left[ \gamma_2 p_{h,t}^{1-\eta} + (1 - \gamma_2) RER_t^{1-\eta} \right]^{1-\eta}
\]

Public investment is productive so that the law of motion for public capital is given by:

\[
K^G_t = (1 - \delta^g)K^G_{t-1} + G^I_t
\]

Oil tax revenues, denominated in foreign goods, consist of royalties and government share of the oil sector’s profits

\[
T_t^{os} = \tau^o P_t^o Y_t^o + t^{\text{div}} \Pi_t^{os}
\]

which go directly to the SWF, accumulated according to the equation below.

\[
SWF_t^* = \rho_{swf} \frac{R_t-1}{\pi_t} SWF_{t-1}^* + T_t^{os},
\]

where \( \rho_{swf} \) is a persistence in the SWF process after its interest is accrued, while \( (1 - \rho_{swf}) \) fraction of interest income transfers to the government budget.

Two fiscal instruments, public investment and public consumption, have the following rules, with their oil output response (\( \vartheta^G_I \) and \( \vartheta^G_C \)) associated with fiscal cyclicity. The real oil output is affected by the world oil price shock through FDI, accumulating oil capital; therefore, it is directly included to fiscal rules, so that fiscal policy transmits the shock into the economy, which has the cycle of oil sector’s boom/bust. The SWF transfers, in contrast, are a stock variable representing rather an annuity value which is affected by the real exchange rate.

\[
\tilde{G}^I_t = \rho_{GI} \tilde{G}^I_{t-1} + (1 - \rho_{GI})[\vartheta_{GI} Y_t^o - \gamma_{GI} \hat{b}_{t-1} + \gamma_{TR}^{GI} \tilde{T}_t]
\]

\[
\tilde{G}^C_t = \rho_{GC} \tilde{G}^C_{t-1} + (1 - \rho_{GC})[\vartheta_{GC} Y_t^o - \gamma_{GC} \hat{b}_{t-1} + \gamma_{TR}^{GC} \tilde{T}_t]
\]

This specification of referring pro/counter/acyclical fiscal policy to positive/negative/zero values for \( \vartheta^G_I \) and \( \vartheta^G_C \), respectively, is consistent with a notion of cyclically adjusted or structural fiscal balances, according to which a cyclical component, related to automatic stabilizers, should be removed mostly from taxes and public transfers, while public spending on wages, goods, and services is usually independent of the business cycle, thus not requiring any adjustment (Bornhorst, Dobrescu, Fedelino, Gottschalk & Nakata, 2011).

Since fiscal debt clears the government budget constraint, the lump-sum taxes need a separate equation, which includes fiscal debt, public spending, and SWF transfers specific
\[
T_t = \varphi \hat{y}_{t-1} + \varphi_f \hat{G}_t + \varphi_c \hat{G}_t^C - \varphi_{TR} \widehat{TR}_t
\]

2.4 Monetary policy

As domestic interest rate is not equal to foreign interest rate, an independent monetary policy is feasible under imperfect capital mobility. The nominal interest rate responds to its lagged value, CPI inflation, and aggregate output according to the CPI targeting Taylor rule below:

\[
R_t = \rho R_{t-1} + (1 - \rho) \left[ \phi_\pi \pi_t + \phi_y \hat{Y}_t \right],
\]

where \( \rho \) is an interest rate smoothing parameter, \( \phi_\pi \) and \( \phi_y \) are the inflation and output responses, respectively.

The PPT Taylor rule, in contrast, uses the product price inflation, which is a weighted average of oil price inflation in real terms \( \pi_t^o = \Delta \widehat{P}_t^o + \Delta \widehat{RER}_t \) and domestic price inflation \( \pi_t^h = \pi_t - \frac{1 + \gamma}{\gamma} \Delta \widehat{RER}_t \), according to Appendix 2.6.5, with weights corresponding to the GDP share of the oil \( s_o \) and non-oil \( (1 - s_o) \) sectors, respectively.

\[
\widehat{R}_t = \rho \widehat{R}_{t-1} + (1 - \rho) \left[ \phi_\pi \left( s_o \pi_t^o + (1 - s_o) \pi_t^h \right) + \phi_y \hat{Y}_t \right]
\]

After rearranging, the PPT rule boils down to:

\[
\widehat{R}_t = \rho \widehat{R}_{t-1} + (1 - \rho) \left[ \phi_\pi \left( s_o \Delta \widehat{P}_t^o + (1 - s_o) \pi_t + \frac{s_o - 1 + \gamma}{\gamma} \Delta \widehat{RER}_t \right) + \phi_y \hat{Y}_t \right]
\]

Every period, the central bank receives interest on its foreign exchange reserves and invests into a new stock of reserves. This flow of revenues is transferred to forward-looking households, savers:

\[
CB_t = R_{t-1}^* \frac{RER_t}{\pi_t^*} \frac{f_{x_r}^*}{\pi_t^*} - f_{x_r}^*.
\]

where \( f_{x_r}^* = RER_t \frac{FXR_t^*}{\pi_t^*} \) is the real foreign exchange reserves expressed in composite consumption goods.

A managed exchange rate regime is associated with the foreign exchange interventions as an additional monetary policy instrument. They represent the purchases/selling of foreign currency by a central bank, and accumulate the foreign exchange reserves according to their rule (Benes et al., 2015), responding to the exchange rate and its rate of depreciation\(^6\).

\[
\widehat{f_{x_r}} = \rho_{f_{x_r}} \widehat{f_{x_r}}_{t-1} + (1 - \rho_{f_{x_r}})(\alpha_1 \widehat{RER}_t + \alpha_2 \Delta \widehat{RER}_t), \quad \alpha_1 < 0, \ \alpha_2 < 0
\]

\(^6\)The higher \( \widehat{RER}_t \), the more the real exchange rate depreciates.
This rule shows that the more the exchange rate depreciates/appreciates, the more the foreign exchange reserves fall/accumulate, implying the selling/purchases of foreign currency by a central bank, respectively.

A flexible exchange rate regime is associated with the zero values for $\alpha_1$ and $\alpha_2$ parameters in the foreign exchange interventions rule.

2.5 Market clearing conditions

For simplicity, an elasticity of substitution between domestic and foreign goods is assumed to approach one ($\eta \to 1$); thus, the domestic non-oil goods market clearing condition is as follows:

$$p_t^h Y_{t}^{no} = \gamma [C_t + (1 - \mu) I_t] + \gamma_2 p_t^g (G_t^C + G_t^I) \quad (29)$$

The real GDP on its supply and demand sides is:

$$Y_t = p_t^h Y_{t}^{no} + Y_t^o P_t^{ex} RER_t = C_t + (1 - \mu) I_t + p_t^g (G_t^C + G_t^I) + NX_t \quad (30)$$

The labor and capital markets clear according to their conditions:

$$N_t = \int_0^1 N_t(j) dj, \quad (1 - \mu) K_t^{no} = \int_0^1 K_t^{no}(j) dj$$

The balance of payments equates its current account with the financial account, combining the budget constraints of government and households-savers. The current account includes net exports, interest income of SWF assets (as those assets are saved abroad) minus the foreign share of the oil sector’s profits, while the financial account represents the interest payments on foreign debt, a new foreign borrowing of households, foreign exchange reserves transferred to households, and FDI.

$$NX_t + \frac{R_t^{\pi}}{\pi_t^{*}} (1 - \rho_{swf}) SWF_{t-1} RER_t - (1 - \iota^{div}) RER_t \Pi_t^{o*} =$$

$$= (1 - \mu) \left( R_{t-1}^{RER} \frac{b_{t-1}}{\pi_t^{*}} - b_t^{*} \right) + f^{x_t^{*}} - R_{t-1}^{RER} \frac{f^{x_t^{*}}}{\pi_t^{*}} - RER_t FDI_t$$

2.6 The rest of the world

The rest of the world is a large economy governed by three exogenous equations for its output, interest rate, and inflation, respectively:

$$\hat{Y}_t^{*} = \rho_{Y} \hat{Y}_{t-1}^{*} + \epsilon_t^{Y*} \quad (31)$$

$$\hat{R}_t^{*} = \phi_{\pi}^{*} \pi_t^{*} + \phi_{y}^{*} \hat{Y}_t^{*} \quad (32)$$
\[ \pi_t^* = \beta^* E_t \pi_{t+1}^* + \lambda^* \left( \sigma + \frac{\phi^* + \alpha^*}{1 - \alpha^*} \right) \hat{Y}_t^* \]  

(33)

The equilibrium of this model consists of households’ and firms’ optimality conditions (41, 42, 43, 44, 49, and 58), capital accumulation equations (45, 46, and 47), SWF accumulation (48), outputs (50, 51, and 52), the government budget constraint (53), fiscal policy (22, 23, and 24), monetary policy (25 or 26 and 28), the balance of payments (57), FDI process (14), market clearing conditions (55 and 56), price equations (15, 40, and 54), and the rest of the world (31, 32, and 33).

3 Calibration

All parameters can be divided into three sets: standard values borrowed from other studies because of the non-availability of relevant data, estimates from time-series regressions according to the model’s equations, and calibrated parameters based on a steady state of the model. The list of parameters is provided in Appendix 2.6.1, excluding the GDP ratios and parameters for the rest of the world which are described in this section.

The first set includes the depreciation rates for private and public capital \( \delta = 0.025, \delta^g = 0.02 \) (Traum & Yang, 2015), the elasticity of substitution between differentiated intermediate goods \( \varepsilon = 9 \) (Gali, 2015), price stickiness \( \theta = 0.9 \) (Jakab & Vilagi, 2008), the inverse of intertemporal elasticity of substitution for consumption \( \sigma = 2 \) (Schmitt-Grohe & Uribe, 2003), investment adjustment costs parameter \( \kappa = 20 \) (Berg et al., 2013), and the fiscal debt response of lump-sum taxes \( \varphi_b = 0.4 \) (Algozhina, 2012). The foreign parameters are set to their standard values: the elasticity of wages with respect to hours worked \( \phi^* = 1.45 \) (Schmitt-Grohe & Uribe, 2003), discount factor \( \beta^* = 0.99 \), inflation and output responses in the Taylor rule \( \phi_p^* = 1.5, \phi_y^* = 0.125 \) (Gali, 2015), price stickiness \( \theta^* = 0.75 \) (Gali, Lopez-Salido & Valles, 2007), output elasticity to capital \( \alpha^* = 0.32 \), and output persistence \( \rho_{Y^*} = 0.8 \).

The second set consists of significant OLS estimates according to the model’s equations based on quarterly Kazakh data in real terms, described in Appendix 2.6.2. Since the data are trending and integrated of first order, a new method proposed by Hamilton (2017) has been used to extract a cyclical component: 8-quarters future or current value is regressed on a constant and current or 8-lags value. The residuals from this two-year projection would represent cyclical factors and describe a true-data generating process, as opposed to a cyclical component produced by the Hodrick-Prescott filter. These forecast errors from a univariate time series in log can be treated as a data-consistent analogous object to the stationary variables in the model, expressed in deviation from their steady state and matched with trending observed data. A cyclical component of each time series, retrieved
in this way, is then regressed on a cyclical component of other variables according to the model’s equations.

The estimates of the public consumption rule (23) are as follows, with t-statistics in parentheses, suggesting the only significant autoregressive coefficient $\rho_{GC} = 0.44$, which is also set for the persistence in public investment $p_{GI} = 0.44$.

$$\bar{G}^C_t = 0.04 + 0.44\bar{G}^C_{t-1} + 0.14\bar{Y}^o_t - 0.02\bar{Y}^o_{t-1} - 1.3 \cdot 10^{-6} T\bar{R}^o_t$$  
R-sq. 0.37, Adj. R-sq. 0.29

(2.1) (2.5) (1.58) (-0.37) (-0.82) DW 1.6, N 37 obs.

According to the lump-sum taxes equation (24), the regression of cyclical components of non-oil fiscal revenues on public consumption, lagged public debt, SWF transfers to the government budget\(^7\), and public investment, which is proxied by the data of fiscal capital expenditures, produces the significant response to public consumption $\varphi_C = 0.95$.

$$\bar{T}_t = -0.03 + 0.02\bar{h}_{t-1} + 0.08\bar{G}^I_t + 0.95\bar{G}^C_t - 6.1 \cdot 10^{-7} T\bar{R}^o_t$$  
R-sq. 0.34, Adj. R-sq. 0.26

(-0.89) (0.22) (0.68) (3.9) (-0.24) DW 1.15, N 37 obs.

The interest rate smoothing $\rho$, according to the empirical CPI Taylor rule, appears to be 0.88.

$$\bar{R}_t = -0.04 + 0.88\bar{R}_{t-1} + 0.04\pi_t - 1.2\bar{Y}^o_t$$  
R-sq. 0.87, Adj. R-sq. 0.86

(-0.37) (16.6) (0.6) (-0.94) DW 1.6, N 49 obs.

The foreign exchange interventions rule (28) results in the following, suggesting the foreign exchange reserves are as follows, respectively:

<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>$f_{fxr}^* = 0.008 + 0.77f_{fxr}^*<em>{t-1} - 0.33\bar{R}ER</em>{t} + 0.29 \bar{R}ER_{t}$</td>
<td>(0.45) (8.78) (-3.2) (0.65)</td>
<td>R-sq. 0.76, Adj. R-sq. 0.75</td>
<td>DW 2.1, N 61 obs.</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

An empirical counterpart of the world oil price equation (15) gives the persistence in the oil price process $\rho_{oy} = 0.85$, while the standard deviation of residuals is 0.2.

$$\bar{P}^{os}_{t} = 0.003 + 0.85\bar{P}^{os}_{t-1} + \epsilon^o_t$$  
R-sq. 0.72, Adj. R-sq. 0.717

(0.1) (12.76) (s.d. 0.2) DW 1.3, N 65 obs.

The FDI equation (14) produces the significant FDI persistence $\rho_{FDI} = 0.3$.

$$\bar{FDI}^*_t = 0.007 + 0.3\bar{FDI}^*_{t-1} + 0.01\bar{P}^{os}_{t}$$  
R-sq. 0.09, Adj. R-sq. 0.03

(0.07) (1.7) (0.04) DW 1.92, N 33 obs.

The third set includes the parameters calibrated to a steady state of the model which corresponds to data averages\(^8\). The GDP ratios of private consumption, public consumption, net exports, FDI, foreign debt, fiscal debt, oil output, public investment, and foreign exchange reserves are as follows, respectively: $c_y = 0.61$, $g^C_y = 0.08$, $nx_y = 0.07$.

---

\(^7\) The SWF transfers to the government budget are zeros in 2006-Q3 and 2006-Q4. Thus, it is not possible to take a log of them, and therefore the coefficient in front of this variable is very low.

\(^8\) The steady state is natural and inefficient in Appendix 2.6.4, since it is at flexible prices and with monopolistic competition.
\( f d i_y = 0.09, b_y^* = 2.17, b_y = 0.5, s_o = 0.52, g'_y = 0.07, \) and \( f x r_y = 0.52. \) The degree of openness is calculated as a ratio of imports to GDP, \( 1 - \gamma = 0.32; \) thus, the home-bias parameter in private consumption and investment \( \gamma \) is equal to 0.68, while it is assumed to be higher for public spending \( \gamma_2 = 0.9, \) as its large share may go to the wages of public servants. The domestic discount factor is around 0.978 because the average T-bill rate is used as a proxy for the policy interest rate, 2.3 percent per quarter\(^9\). The upper bound of leverage ratio \( \Omega \) appears to be 0.54. The elasticity of output with respect to private capital \( \alpha \) is equal to 0.3 as a share of capital income to GDP, while with respect to public capital it is \( \psi = 0.16 \) suggested by a steady state wage equation in Appendix 2.6.4. Using data on wages, the elasticity of wages with respect to hours worked \( \phi \) is 1.45 according to the labor supply condition (39), in which hours are obtained from the non-oil production function (8). The royalties rate levied on oil production quantity \( \tau_o = 0.27 \) is calculated as the SWF inflows share in oil output. The dividend share of oil profits that the government receives \( d^{\text{div}} \) is set to 0.05, while the elasticity of oil output with respect to oil capital \( \alpha^o \) is technically feasible at 0.7. The persistence in SWF process \( \rho_{swf} \) is equal to 0.747 to match the GDP ratio of SWF assets \( swf_y = 0.65. \)

There are three types of fiscal policy: procyclical, countercyclical, and acyclical. The acyclical fiscal policy is a benchmark to calculate welfare loss in deviation from it. It is associated with the zero oil output response of public consumption and public investment in their rules \( (\vartheta_{GC} = 0 \) and \( \vartheta_{GI} = 0). \) The procyclical fiscal policy corresponds to the positive oil output response of public spending \( (\vartheta_{GC} = 0.4 \) and \( \vartheta_{GI} = 0.4), \) while the countercyclical fiscal policy is simulated at their negative values \( (\vartheta_{GC} = -0.4 \) and \( \vartheta_{GI} = -0.4). \) Those are the two parameters which differ across fiscal cyclicality, while the rest hold the same. The fiscal debt responses of public consumption \( \gamma_{GC} = 0.3 \) and public investment \( \gamma_{GI} = 0.3 \) are assumed to be equal. The response of public consumption to SWF transfers \( \gamma_{TR}^{GC} \) is set to 0.2, fixing it slightly lower than \( \gamma_{GC} = 0.3, \) whereas public investment response to the SWF transfers \( \gamma_{TR}^{GI} \) is 0.1. The parameters of the lump-sum taxes equation (24) are as follows: public investment response \( \varphi_I = 0.2 \) and SWF transfers response \( \varphi_{TR} = -0.3. \) The latter is calculated according to taxes at a steady state under acyclical fiscal policy:

\[
\varphi_{TR} = \frac{\varphi_b \ln b + \varphi_I \ln G_I + \varphi_C \ln G_C - \ln T}{\ln T R}
\]

\(^9\)The domestic interest rate matters for the government bonds in this model, as investments are financed by foreign funds rather than the domestic financial market.
4 Results

This section describes results in the following order. The welfare measure derived by De Paoli (2009) is explained, based on which a grid search of Taylor rule parameters is made. The model has been simulated with two shocks: the world oil price and foreign output shocks with their standard deviations of 0.2 for both. Given these optimal monetary policy parameters, the impulse-response functions to a negative world oil price shock, interpreted as the worsening of a terms of trade shock, are analyzed to understand its transmission mechanism. The welfare loss components are examined across fiscal policy cyclicality, exchange rate regimes, and monetary policy’s price anchors at calibrated parameters of the foreign exchange interventions rule. Furthermore, these parameters of exchange rate policy are varied at the optimal Taylor rule to find their loss-minimizing values\textsuperscript{10}. Finally, weights for loss components are assumed to be not equal, following the parametrization of De Paoli (2009), to compare with the baseline results.

In a small open economy with monopolistic competition and nominal rigidities, De Paoli (2009) has used a linear-quadratic approach to derive welfare as a second-order approximation of households’ utility. The linear terms in this objective function have been eliminated by a second-order approximation of her model’s equilibrium conditions in order to take into account the effect of second moments on the mean of the variables. As a result, the objective loss function becomes a purely quadratic expression of domestic price inflation, output gap, and real exchange rate.

In this model, such a loss measure is adopted as a sum of variances in domestic price inflation $\pi^h_t$, aggregate output $\hat{Y}_t$, and real exchange rate $\hat{RER}_t$. This loss function is minimized to find the Taylor rule: inflation $\phi_\pi$ and output responses $\phi_y$ across pro/counter/acyclical fiscal stance. Monetary policy can be hybrid, combining a managed exchange rate regime with a CPI/PPT anchor, or pure inflation targeting associated with the CPI/PPT under a flexible exchange rate regime. The foreign exchange interventions rule is set to its calibrated parameters ($\alpha_1 = -1.4$ and $\alpha_2 = -0.1$) for a managed exchange rate regime, while those parameters are zeros under a flexible exchange rate regime.

\textsuperscript{10}The existing literature has not yet explored an optimal exchange rate policy combined with monetary policy in a DSGE framework. Meanwhile, Ilzetzki, Reinhart, and Rogoff (2017) find that less flexible exchange rate arrangements account for around 80 percent of all countries out of 194 over 1946-2016, or about one-half of world GDP.
Table 1. Optimal monetary policy responses

<table>
<thead>
<tr>
<th></th>
<th>Procyclical fiscal policy</th>
<th>Countercyclical fiscal policy</th>
<th>Acyclical fiscal policy</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Managed exchange rate</td>
<td>Flexible exchange rate</td>
<td></td>
</tr>
<tr>
<td></td>
<td>CPI targeting</td>
<td>PPT</td>
<td>CPI targeting</td>
</tr>
<tr>
<td>( \phi_\pi )</td>
<td>11</td>
<td>14</td>
<td>13</td>
</tr>
<tr>
<td>( \phi_y )</td>
<td>2.35</td>
<td>1.1</td>
<td>2.85</td>
</tr>
</tbody>
</table>

Table 1 shows the results of grid search made in a range 0-20 with a step of 1 for inflation response \( \phi_\pi \) and 0.1-5.1 with a step of 0.25 for output response \( \phi_y \). The reason for high monetary policy parameters compared to their standard values, commonly found in the literature, is in the collateral constraint of this model. The Lagrange multiplier to the collateral constraint appears in the first-order conditions of a forward-looking household’s problem in Appendix 2.6.3, the log-linearization of which results in two equations containing this shadow value of relaxing the borrowing constraint (43 and 49 in Appendix 2.6.6). One of this equations is essentially the UIP condition, which determines the real exchange rate that affects inflation, due to its complete pass-through effect, and aggregate output via oil sector or net exports. As a result, the model produces an additional volatility, which tries to be stabilized by a more aggressive monetary policy than it would have been in a standard framework without the collateral constraint\(^{11}\).

According to Table 1, the PPT inflation response is higher than the CPI targeting because the PPT Taylor rule includes oil price inflation, which needs to be properly stabilized in the presence of oil price shock. A flexible exchange rate regime, across CPI/PPT, has higher inflation responses than the price anchors produce under a managed exchange rate. This is because the former exchange rate regime has only one monetary

\(^{11}\)Faia and Iliopulos (2011), using the same collateral constraint for their durable goods, also found that consumption, output, and inflation become volatile in the open economy setting.
policy instrument, the interest rate, which should still respond to a change in the real exchange rate as the terms of trade, due to consumption of foreign goods. A managed exchange rate regime, in contrast, has the foreign exchange interventions in addition to the interest rate that stabilize the exchange rate; therefore, the inflation response of interest rate does not need to be high.

As for optimal output response, Table 1 suggests that it is lower under a PPT rule because oil output can be stabilized by the PPT inflation response, which controls for oil price inflation. Thus, there is no need for a strong reaction to aggregate output, since its volatile oil output is taken care by the inflation response already. This, in contrast, does not hold under a CPI rule. A flexible exchange rate regime, meanwhile, causes the output response to be higher than a managed exchange rate does at respective CPI/PPT anchors. This is because the absence of foreign exchange interventions in the former case, which would otherwise better stabilize the exchange rate and thereby aggregate output, limits the ability of monetary policy to deal with output volatility, leaving this exclusively to the interest rate, which therefore has to respond strongly to output.

Figure 1. Impulse-response functions to a negative world oil price shock of 1%: countercyclical fiscal policy combined with PPT monetary rule under a managed exchange rate regime

Figures 1 and 2 show the impulse-response functions to a negative world oil price shock produced at the optimized monetary policy parameters according to Table 1. A flexible exchange rate regime, across all cases of policy combination, produces no changes...
in the foreign exchange reserves, otherwise the impulse-response functions stay the same. The figures, in text, display the dynamics of a countercyclical fiscal stance and managed exchange rate regime, while the rest can be found in Appendix 2.6.7. A sudden drop in oil price discourages the oil sector’s FDI and decreases net exports. A fall in FDI implies that the economy is less indebted to the foreign world, whereas an increase of trade deficit needs to be financed externally. These two effects on the balance of payments front lead to a fall in the Lagrange multiplier to collateral constraint, since the shadow value of relaxing the borrowing constraint goes down, as this borrowing limit extends. The interest rate also decreases, because the UIP condition (49 in Appendix 2.6.6) includes the Lagrange multiplier, suggesting that a difference between domestic real interest rate and foreign real interest rate should be equal to a sum of expected change in the real exchange rate and this shadow value of relaxing the borrowing constraint. Therefore, the Lagrange multiplier positively affects the domestic interest rate, and its fall means that the marginal cost of borrowing declines, triggering a substitution effect from consumption, which immediately drops, to investment, as foreign debt and non-oil capital accumulate over time. Low consumption decreases the domestic prices and contributes to a fall in non-oil output, as hours worked are discouraged by low wages that are production costs to be covered by low domestic prices. Since non-oil output and oil price decline, the aggregate output falls as a result of combining two production sectors.

Figure 1, in contrast to Figure 2, shows a larger drop in the interest rate because a decrease in oil price shock affects the oil price inflation, to which the PPT Taylor rule responds in Figure 1. The Lagrange multiplier falls to a larger extent under PPT, influenced by the more decreased interest rate. As the interest rate goes down, inflation has an obvious spike in the second period. These spikes, due to the lower interest rate of PPT, are also observed in consumption, domestic prices, hours worked, and non-oil output as opposed to the dynamics of CPI rule in Figure 2.
Figure 2. Impulse-response functions to a negative world oil price shock of 1%: countercyclical fiscal policy combined with CPI monetary rule under a managed exchange rate regime

Table 2 summarizes the numerical results of loss measure $L$ as a sum of variances in domestic price inflation, aggregate output, and real exchange rate. The results are produced at the optimal Taylor rule parameters and calibrated foreign exchange interventions rule ($\alpha_1 = -1.4$ and $\alpha_2 = -0.1$ in the equation 28). All entries are in percent deviation from a benchmark policy combination: acyclical fiscal stance and a CPI monetary anchor under a flexible exchange rate regime. Positive values mean the percentage increase in loss relative to the benchmark, while negative values indicate lower loss contributed by a respective entry.
Table 2. **Loss components (in %)**

<table>
<thead>
<tr>
<th>Procyclical fiscal policy</th>
<th>Managed exchange rate</th>
<th>Flexible exchange rate</th>
<th>Countercyclical fiscal policy</th>
<th>Managed exchange rate</th>
<th>Flexible exchange rate</th>
<th>Acyclical fiscal policy</th>
<th>Managed exchange rate</th>
<th>Flexible exchange rate</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CPI targeting</td>
<td>PPT</td>
<td>CPI targeting</td>
<td>PPT</td>
<td>CPI targeting</td>
<td>PPT</td>
<td>CPI targeting</td>
<td>PPT</td>
</tr>
<tr>
<td>$L$</td>
<td>-4.56</td>
<td>-5.29</td>
<td>1.55</td>
<td>2.13</td>
<td>-0.66</td>
<td>-0.3</td>
<td>0.00</td>
<td>0.46</td>
</tr>
<tr>
<td>$Var(\pi_t)$</td>
<td>-0.009</td>
<td>0.01</td>
<td>0.01</td>
<td>0.03</td>
<td>0.01</td>
<td>2.05</td>
<td>0.01</td>
<td>0.022</td>
</tr>
<tr>
<td>$Var(\hat{Y}_t)$</td>
<td>-3.67</td>
<td>-2.12</td>
<td>1.21</td>
<td>3.9</td>
<td>-0.5</td>
<td>2.05</td>
<td>0.5</td>
<td>2.18</td>
</tr>
<tr>
<td>$Var(\hat{RER}_t)$</td>
<td>-0.88</td>
<td>-3.18</td>
<td>0.35</td>
<td>-1.78</td>
<td>-0.17</td>
<td>-2.38</td>
<td>0.00</td>
<td>1.74</td>
</tr>
</tbody>
</table>

Values are in the percent deviation of corresponding entry from the benchmark acyclical fiscal policy combined with a CPI targeting monetary rule and flexible exchange rate regime.

Table 2 reports that a countercyclical fiscal stance and PPT monetary rule under a managed exchange rate regime is the best policy combination, delivering lower variances in aggregate output and real exchange rate than the benchmark does. This is because fiscal stimulus dampens a negative terms of trade shock by countercyclically offsetting it and stabilizing the ultimate effects on the economy better than procyclical and acyclical stances. In fact, a procyclical fiscal policy is the worst, since it transmits an external terms of trade shock to the domestic economy, which becomes dependent on volatile foreign shocks rather than staying resilient to them. A managed exchange rate regime is strongly preferred across all fiscal cyclicalities because a stabilization of exchange rate matters for a small open economy with its collateral constraint. Such an economy imports foreign goods and borrows from abroad, so that fluctuations in the exchange rate exacerbate not
only relative price of imports (terms of trade), but also foreign debt. Therefore, foreign exchange interventions, which stabilize the exchange rate, are beneficial and they better smooth the aggregate output volatility at the same time.

The PPT anchor helps to achieve a more stable exchange rate than the CPI target shows in Table 2. This is because the interest rate of PPT responds to a change in the real exchange rate more than the CPI target, due to expressing the oil price inflation in real terms. Yet, the CPI rule provides a better stabilization of aggregate output and domestic price inflation, as the impulse-response functions do not display spikes in consumption, domestic prices, and non-oil output unlike PPT. It is driven by the CPI Taylor rule, which has higher optimal output and inflation responses, since the latter is not adjusted by the GDP share of non-oil output like under PPT. Therefore, if there is a flexible exchange rate regime, the CPI targeting outperforms the PPT.

Overall, Table 2 supports Frankel and Catao (2011) who recommends the PPT monetary anchor for a commodity exporting economy, since the exchange rate depreciates more in response to a fall in the world oil price. Note that the exchange rate depreciation does not occur immediately and is delayed due to sticky prices. However, PPT performs well under a managed exchange rate regime by improving the exchange rate stabilization, at the cost of higher volatilities in aggregate output and domestic price inflation. The latter two variables are better stabilized by the CPI targeting which is preferred under a flexible exchange rate regime.

Table 3. Loss components at optimal $\alpha_1$ and $\alpha_2$ (in %)

<table>
<thead>
<tr>
<th></th>
<th>Procyclical FP</th>
<th>Countercyclical FP</th>
<th>Acyclical FP</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CPI targeting</td>
<td>PPT</td>
<td>CPI targeting</td>
</tr>
<tr>
<td>$Var(\pi^h_t)$</td>
<td>-0.02</td>
<td>-0.03</td>
<td>-0.028</td>
</tr>
<tr>
<td>$Var(\hat{Y}_t)$</td>
<td>-24.27</td>
<td>-25.8</td>
<td>-26.5</td>
</tr>
<tr>
<td>$Var(\hat{RER}_t)$</td>
<td>-0.64</td>
<td>-0.98</td>
<td>-0.21</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>-9</td>
<td>-4</td>
<td>-9</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>-5</td>
<td>-6</td>
<td>-6</td>
</tr>
</tbody>
</table>

Values are in the percent deviation of corresponding entry from the benchmark acyclical fiscal policy combined with a CPI targeting monetary rule and flexible exchange rate regime.

The results of managed exchange rate regime, discussed so far, apply to the calibrated foreign exchange interventions rule ($\alpha_1 = -1.4$ and $\alpha_2 = -0.1$ in the equation 28). However, this rule can be optimized given the monetary policy responses provided by Table 1. The grid search of parameters $\alpha_1$ and $\alpha_2$ is therefore made in a range between
Table 3 summarizes the numerical results of loss components under a managed exchange rate regime at optimal values for $\alpha_1$ and $\alpha_2$. Since a flexible exchange rate regime is associated with zero $\alpha_1$ and $\alpha_2$, its respective columns would be the same as in Table 2, thus they are omitted in this case.

The results show that output stabilization is significantly improved by a more active exchange rate policy, in which interventions strongly respond to the exchange rate now. This is because a stable exchange rate is preferred for this open economy, which borrows from abroad to accumulate physical capital, imports foreign goods for consumption and investment, and exports oil that is dependent on the world oil price. It has been assumed, though, that weights for loss components are equal to one, implying that policymakers have equal stabilization goals over domestic price inflation, aggregate output, and real exchange rate.

Table 4. Loss components with different weights at optimal $\alpha_1$ and $\alpha_2$ (in %)

<table>
<thead>
<tr>
<th></th>
<th>Procyclical fiscal policy</th>
<th>Countercyclical fiscal policy</th>
<th>Acyclical fiscal policy</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Managed exchange rate</td>
<td>Flexible exchange rate</td>
<td>Managed exchange rate</td>
</tr>
<tr>
<td></td>
<td>CPI targeting PPT</td>
<td>CPI targeting PPT</td>
<td>CPI targeting PPT</td>
</tr>
<tr>
<td>$L$</td>
<td>-3.27</td>
<td>-4.13</td>
<td>0.84</td>
</tr>
<tr>
<td>$0.54 Var(\pi_t^h)$</td>
<td>-0.03</td>
<td>-0.05</td>
<td>-0.009</td>
</tr>
<tr>
<td>$0.03 Var(\hat{Y}_t)$</td>
<td>-1.84</td>
<td>-1.95</td>
<td>0.09</td>
</tr>
<tr>
<td>$0.86 Var(RER_t)$</td>
<td>-1.4</td>
<td>-2.13</td>
<td>0.76</td>
</tr>
<tr>
<td></td>
<td>-2.5</td>
<td>-5.41</td>
<td>-0.4</td>
</tr>
<tr>
<td>$0.54 Var(\pi_t^h)$</td>
<td>-0.04</td>
<td>-0.02</td>
<td>0.01</td>
</tr>
<tr>
<td>$0.03 Var(\hat{Y}_t)$</td>
<td>-2</td>
<td>-2.13</td>
<td>-0.04</td>
</tr>
<tr>
<td>$0.86 Var(RER_t)$</td>
<td>-0.46</td>
<td>-3.26</td>
<td>-0.37</td>
</tr>
<tr>
<td></td>
<td>-1.56</td>
<td>-6.12</td>
<td>0</td>
</tr>
<tr>
<td>$0.54 Var(\pi_t^h)$</td>
<td>-0.03</td>
<td>-0.04</td>
<td>0</td>
</tr>
<tr>
<td>$0.03 Var(\hat{Y}_t)$</td>
<td>-2.01</td>
<td>-2.03</td>
<td>0</td>
</tr>
<tr>
<td>$0.86 Var(RER_t)$</td>
<td>0.48</td>
<td>-4.05</td>
<td>0</td>
</tr>
</tbody>
</table>

Values are in the percent deviation of corresponding entry from the benchmark acyclical fiscal policy.
combined with a CPI targeting monetary rule and flexible exchange rate regime.

As a sensitivity test, the weights for loss components can differ, following, for example, the parametrization of De Paoli (2009): 0.54, 0.03, and 0.86 for variances in the domestic price inflation, output, and real exchange rate, respectively. These weights are used to calculate entries in Table 4 at the same Taylor rule and foreign exchange interventions rule optimally found earlier. Ideally, the weights should be derived in terms of deep parameters based on a second-order approximation of households’ utility, suggesting future research on finding a model-consistent loss function. At this stage, Table 4 is rather a preliminary exercise to assess properties of the model and ensure that the model has been constructed correctly. An analytical representation of welfare derivation may yet require a substantial simplification of the model, to go forward.

The results indicate that even a flexible exchange rate regime should be combined with PPT, since a weight on the real exchange rate is relatively high now. This supports the previous finding that a PPT monetary rule stabilizes the exchange rate better than a CPI anchor. A further analysis of Table 4 should be avoided though, since the variances are produced by the Taylor rule and foreign exchange interventions rule that are optimized based on the loss function with equal weights for its components.

5 Conclusion

This paper develops a DSGE model for an emerging oil economy to study the loss-minimizing monetary policy jointly with a pro/counter/acyclical fiscal stance. The study reveals that the best policy combination is a countercyclical fiscal stance and managed exchange rate regime with the PPT monetary anchor. This allows the fiscal policy to countercyclically offset a volatile terms of trade shock, to which developing countries are often exposed. It also allows the exchange rate to be managed by the central bank’s interventions, which seem beneficial in providing a stable exchange rate since the economy borrows from abroad, imports foreign goods, and depends on the world oil price. It also suggests the monetary policy to target product price inflation, which includes oil price inflation that is important for the oil sector’s exports and delivers a better stabilization of exchange rate than the CPI anchor. However, if a flexible exchange rate regime is institutionally chosen, then the CPI targeting should be adopted, since it effectively stabilizes the domestic price inflation and aggregate output. In conclusion, the adverse effects of a volatile terms of trade shock can be mitigated by an appropriate fiscal and monetary policy combination to smooth variances in the domestic price inflation, aggregate output, and real exchange rate in a small open economy.
References


## A Appendix: Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta = 0.978$</td>
<td>discount factor</td>
</tr>
<tr>
<td>$\gamma = 0.68$</td>
<td>home-bias in consumption and investment</td>
</tr>
<tr>
<td>$\gamma_2 = 0.9$</td>
<td>home-bias in government purchases</td>
</tr>
<tr>
<td>$\Omega = 0.54$</td>
<td>upper bound of leverage ratio</td>
</tr>
<tr>
<td>$\mu = 0.5$</td>
<td>fraction of rule-of-thumb households</td>
</tr>
<tr>
<td>$\alpha = 0.3$</td>
<td>non-oil output elasticity to private capital</td>
</tr>
<tr>
<td>$\psi = 0.16$</td>
<td>non-oil output elasticity to public capital</td>
</tr>
<tr>
<td>$\alpha^o = 0.7$</td>
<td>oil output elasticity to private capital</td>
</tr>
<tr>
<td>$\phi = 1.45$</td>
<td>wage elasticity to hours worked</td>
</tr>
<tr>
<td>$\sigma = 2$</td>
<td>inverse of intertemporal elasticity of substitution for consumption</td>
</tr>
<tr>
<td>$\delta = 0.025$</td>
<td>depreciation rate of private capital (oil and non-oil)</td>
</tr>
<tr>
<td>$\delta^g = 0.02$</td>
<td>depreciation rate of public capital</td>
</tr>
<tr>
<td>$\theta = 0.9$</td>
<td>index of price stickiness</td>
</tr>
<tr>
<td>$\varepsilon = 9$</td>
<td>elasticity of substitution between differentiated intermediate goods</td>
</tr>
<tr>
<td>$\kappa = 20$</td>
<td>investment adjustment costs parameter</td>
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<tr>
<td>$\phi_x = 14$</td>
<td>inflation response in the Taylor rule</td>
</tr>
<tr>
<td>$\phi_y = 1.35$</td>
<td>output response in the Taylor rule</td>
</tr>
<tr>
<td>$\alpha_1 = -1.4$</td>
<td>exchange rate response in the interventions rule</td>
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<tr>
<td>$\alpha_2 = -0.1$</td>
<td>exchange rate change response in the interventions rule</td>
</tr>
<tr>
<td>$\tau^o = 0.27$</td>
<td>oil royalty rate</td>
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<tr>
<td>$\iota^{div} = 0.05$</td>
<td>dividend share of oil profits accrued to the government</td>
</tr>
<tr>
<td>$\gamma_{GC} = \gamma_{GI} = 0.3$</td>
<td>response of public consumption/investment to fiscal debt</td>
</tr>
<tr>
<td>$\vartheta_{GC} = \vartheta_{GI} = -0.4$</td>
<td>response of public consumption/investment to oil output</td>
</tr>
<tr>
<td>$\gamma_{TR} = 0.2$</td>
<td>response of public consumption to SWF transfers</td>
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<tr>
<td>$\gamma_{GI} = 0.1$</td>
<td>response of public investment to SWF transfers</td>
</tr>
<tr>
<td>$\varphi_b = 0.4$</td>
<td>response of lump-sum taxes to fiscal debt</td>
</tr>
<tr>
<td>$\varphi_{TR} = -0.3$</td>
<td>response of lump-sum taxes to SWF transfers</td>
</tr>
<tr>
<td>$\varphi_C = 0.95$</td>
<td>response of lump-sum taxes to public consumption</td>
</tr>
<tr>
<td>$\varphi_I = 0.2$</td>
<td>response of lump-sum taxes to public investment</td>
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<td>$\rho_{GC} = \rho_{GI} = 0.44$</td>
<td>persistence in public consumption/investment</td>
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<tr>
<td>$\rho_{FDI} = 0.3$</td>
<td>persistence in the FDI process</td>
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<tr>
<td>$\rho_{swf} = 0.747$</td>
<td>persistence in the SWF process</td>
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<tr>
<td>$\rho = 0.88$</td>
<td>interest rate smoothing in the Taylor rule</td>
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<tr>
<td>$\rho_{Int} = 0.77$</td>
<td>persistence in the foreign exchange reserves of a central bank</td>
</tr>
<tr>
<td>$\rho_o = 0.85$</td>
<td>persistence in the world oil price process</td>
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<tr>
<td>$\sigma_q = \sigma_r = 0.2$</td>
<td>standard deviation of the world oil price and foreign output shocks</td>
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## B Data description

Data are outlined here which are used in the OLS regressions, calculated GDP ratios, and other calibrated parameters. Most data are manually retrieved from the non-English websites of respective institutions, indicated in parentheses below. Some data are obtained based on a formal request to those institutions.

Real GDP is the GDP at constant prices of 1994 in mln tenge according to the National

Private consumption is the consumption expenditures of households at constant prices of 1994 in mln tenge according to the National Accounts over 1994Q1-2012Q2 (Agency of Statistics).

Public consumption is public consumption at constant prices of 1994 in mln tenge according to the National Accounts over 1994Q1-2012Q2 (Agency of Statistics).

Fixed capital formation is gross fixed capital formation at constant prices of 1994 in mln tenge according to the National Accounts over 1994Q1-2012Q2 (Agency of Statistics).

Imports are the imports of goods and services at constant prices of 1994 in mln tenge according to the National Accounts over 1994Q1-2012Q2 (Agency of Statistics).

Net exports are the net exports at constant prices of 1994 in mln tenge according to the National Accounts over 1994Q1-2012Q2 (Agency of Statistics).

Real public debt is the CPI-deflated public debt, including debt guaranteed by the state, in mln tenge over 1999Q4-2012Q2 (Ministry of Finance).

Real SWF transfers to the government budget represent the CPI-deflated oil revenues of government budget till 2007 and SWF transfers to the government budget since 2007 in mln tenge over 2001Q2-2012Q4 (Ministry of Finance).

Real fiscal capital expenditures are the CPI-deflated capital expenditures of government budget in mln tenge over 2000Q1-2012Q2 (Ministry of Finance).

Real non-oil fiscal revenues are the CPI-deflated difference between total fiscal revenues and SWF transfers to the government budget in mln tenge over 2000Q1-2012Q2 (Ministry of Finance).

T-bill rate is an effective annual return on medium-term Treasury bills in percent over 1998Q1-2012Q2 (Statistical Bulletin of the National Bank).

Producer price index is a quarterly producer price index over 1994Q1-2012Q2 (Agency of Statistics).

World oil price is a petroleum UK Brent price USD/barrel over 1993Q4-2012Q2 (International Financial Statistics of the IMF).

Real foreign exchange reserves are the CPI-deflated net foreign exchange reserves of the National Bank in mln tenge over 1994Q4-2012Q2 (National Bank).

Real exchange rate is a bilateral real exchange rate, tenge per 1 USD, over 1995Q1-2012Q2 (National Bank).

Real FDI is the US CPI-deflated foreign direct investment in mln USD according to the balance of payments statistics over 2002Q1-2012Q2 (National Bank). The US CPI index is retrieved from the International Financial Statistics of the IMF.

Foreign debt is an external debt of banks and other private entities in mln USD according to the balance of payments statistics over 1995Q1-2012Q2 (National Bank).

Wage is an average monthly wage of a hired employee in tenge over 1994Q1-2012Q4 (Agency of Statistics).

SWF inflows represent the oil revenues of government budget till 2007 and inflows into SWF since 2007 in mln tenge over 2001Q2-2012Q4 (Ministry of Finance).

SWF assets are the stock of SWF at the end of period in mln tenge over 2001Q2-2012Q4 (Ministry of Finance).
C First-order conditions

The first-order conditions of the forward-looking household’s problem are listed in this appendix, where $\lambda_t$, $\lambda^k_t$, and $\lambda^c_t$ are the Lagrange multipliers to the budget constraint (2), capital accumulation (3), and collateral constraint (4), respectively. In particular, equations (34), (35), (36), (37), (38), (39) below are the first-order conditions with respect to consumption, investment, non-oil capital, government bonds, foreign debt, and hours worked, respectively.

\[
\frac{1}{C^S_t - \frac{N^o_t}{\sigma}} = \lambda_t 
\]

\[
\frac{1}{Q_t} = 1 - \frac{\kappa}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right)^2 - \kappa \left( \frac{I_t}{I_{t-1}} - 1 \right) \frac{I_t}{I_{t-1}} + \beta \kappa E_t \left\{ \frac{Q_{t+1} \lambda_{t+1}}{Q_t \lambda_t} \left( \frac{I_{t+1}}{I_t} - 1 \right) \left( \frac{I_{t+1}}{I_t} \right)^2 \right\},
\]

where $Q_t = \frac{\lambda^k_t}{\lambda_t}$ is a shadow value of non-oil capital.

\[
Q_t = E_t \left\{ \beta \frac{\lambda_{t+1}}{\lambda_t} \left[ R^e_{t+1} + Q_{t+1} (1 - \delta) \right] + \lambda^c_t \Omega \frac{Q_{t+1} \pi^*_{t+1}}{RER_{t+1}/RER_t} \right\}
\]

\[
\frac{1}{R_t} = \beta E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t \pi_{t+1}} \right\}
\]

\[
\frac{1}{R^*_t} = \beta E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \frac{RER_{t+1}}{RER_t \pi^*_{t+1}} \right\} + \lambda^c_t
\]

\[
W_t = N^o_t - 1
\]

The first-order conditions of a rule-of-thumb household with respect to consumption $C^N_t$ and hours worked $N_t$ are identical to the saver’s solutions above. Thus, a non-saver faces the same labor supply condition (39).

D Steady state

The model’s steady state assumes zero inflation, thus it is at flexible prices. Variables at steady state are denoted by bars and presented in this appendix.

The first-order condition of a forward-looking household with respect to the government bonds (37) gives that $\bar{R} = \frac{1}{\beta}$, while with respect to foreign debt (38) suggests $\bar{R}^* = \beta^* - \beta$ at steady state. Similarly, $\bar{R}^o = \frac{1}{\beta^o}$.

An oil producer equalizes the marginal product of capital to its price:

\[
\alpha^o (1 - \tau^o) (\bar{K}^o)^{\alpha^o - 1} = \bar{R}^o = \frac{1}{\beta} - (1 - \delta),
\]

from which the steady state of oil capital can be found.

\[
\bar{K}^o = \left[ \frac{1/\beta - (1 - \delta)}{\alpha^o (1 - \tau^o)} \right]^{\frac{1}{\alpha^o - 1}}
\]
Since oil capital is known, the oil output, FDI, and SWF are obtained from their respective equations (12), (14), and (16, 20, 21):

\[ Y^o = (K^o)^{\alpha_o}, \quad FDI^* = \delta K^o, \quad SWF = \frac{[\tau^o + \epsilon \text{div}(1 - \tau^o)]Y^o}{1 - \rho_{swf}/\beta^*} \]

The law of one price holds. There is also an assumption of symmetric steady state \( \left( \frac{P_f}{P_h} = 1 \right) \) and a unit elasticity of substitution between domestic and foreign goods \( (\eta = 1) \). Thus, the real exchange rate and relatives prices at steady state are equal to one.

The SWF transfers to the government budget are as follows:

\[ TR = R^* (1 - \rho_{swf}) \cdot SWF \cdot RER \]

The public capital accumulation equation (19) gives public investment at steady state:

\[ G_I = \delta g K_G \]

Fiscal debt is represented in terms of public capital, using the public investment equation (22) and the expression above:

\[ b = \left( \frac{Y^o}{\delta g K_G} \right) \frac{1}{G_I} \]

Public consumption is as follows based on its rule (23), in which fiscal debt can be plugged from the previous equation:

\[ G_C = \frac{Y^o}{b G_I} \frac{1}{G_I} \]

The lump-sum taxes equation (24) suggests taxes at steady state:

\[ T = \frac{b G_I G_C}{TR^{\rho_{TR}}} \]

The government budget constraint (17) can be used to obtain public capital by substituting the taxes, SWF transfers, public consumption, public investment, and fiscal debt with their respective previous expressions:

\[ T + TR = G_C + G_I + (1 - \mu)(R - 1)b \]

The first-order condition with respect to non-oil capital (36) yields the following rental cost of capital:

\[ R^{kno} = \frac{1}{\beta} - (1 - \delta) - \frac{\lambda^\omega}{\beta} \]

The price setting problem of a non-oil firm suggests that the marginal costs (9) equate
with the inverse of price frictionless mark-up \( \frac{\varepsilon}{\varepsilon - 1} \) at steady state; thus, wages are:

\[
\bar{W} = (1 - \alpha) \left[ \frac{K_G^{\psi} \alpha^\alpha (\varepsilon - 1)}{(R^{kao})^\alpha} \right]^{\frac{1}{\alpha - 1}}
\]

The labor supply condition (39) gives \( \bar{N} = \bar{W}^{\frac{1}{\varepsilon - 1}} \).

As aggregate output is a sum of non-oil and oil output \( \bar{Y} = \bar{Y}^{no} + \text{RER} \bar{Y}^o = \bar{N}^{1 - \alpha} K_G^{\psi} K^{n^o} + \text{RER} \bar{Y}^o \), the non-oil capital is obtained in terms of aggregate output:

\[
\bar{K}^{no} = \left( \frac{\bar{Y} - \text{RER} \bar{Y}^o}{\bar{N}^{1 - \alpha} K_G^{\psi}} \right)^{\frac{1}{\alpha}}
\]

The law of motion for capital (3) relates private investment with the non-oil capital:

\[
I = \delta \bar{K}^{no}.
\]

The collateral constraint (4) allows finding the foreign debt:

\[
\bar{b}^* = \frac{\Omega \bar{K}^{no} R^*}{\bar{R}}
\]

The steady state foreign exchange reserves, according to their rule (28), are equal to 1 given that \( \text{RER} = 1 \).

The balance of payments equation provides net exports:

\[
\bar{NX} = (1 - \mu)(\bar{R}^s - 1) \bar{b}^* + (1 - \bar{R}^s) \bar{fxr}^* - \text{RERFDI}^s - \bar{R}^s(1 - \rho_{swf}) \text{SWFRER} + (1 - \varepsilon) \text{RER} (1 - \tau^o) \bar{Y}^o
\]

The taxes of rule-of-thumb households are equal to:

\[
\bar{T}^N = \frac{T - (1 - \mu) T^S}{\mu}
\]

given that \( T_t = \mu T_t^N + (1 - \mu) T_t^S \), while the taxes of savers can be derived from their budget constraint (2), assuming that both types of household have equal consumption at steady state:

\[
\bar{T}^S = \mu \left[ (R^{kao} - \delta) \bar{K}^{no} + \bar{b}(\bar{R} - 1) + \bar{b}^* (1 - \bar{R}^s) + (1 - \frac{\varepsilon}{\varepsilon - 1}) \bar{Y}^{no} + (\bar{R}^s - 1) \bar{fxr}^* \right] + T
\]

The budget constraint of a rule-of-thumb household (5) provides its consumption \( \bar{C}^N = \bar{W} \bar{N} - \bar{T}^N \), which is assumed to be equal to the saver’s consumption, thus to aggregate consumption as well due to the sum of both households’ consumption: \( \bar{C} = \mu \bar{C}^N + (1 - \mu) \bar{C}^S \).

The real GDP condition (30) can be utilized to derive the aggregate output, by plugging into variables expressed in terms of output according to their steady state equations above:

\[
\bar{Y} = \bar{C} + (1 - \mu) \bar{I} + \bar{G}_C + \bar{G}_I + \bar{NX}
\]
\[ \pi_t^h = \beta E_t \pi_{t+1}^h + \lambda \frac{1 - \alpha}{1 - \alpha + \alpha \varepsilon} \hat{MC}_t, \]

where \( \hat{MC}_t \) is the log deviation of the economy’s average real marginal costs from their steady state and \( \lambda = \frac{(1 - \beta \theta)(1 - \theta)}{\theta} \).

The CPI inflation includes the domestic inflation \( \pi_t^h \) and the terms of trade, which can be alternatively represented by the real exchange rate \( RER_t \):

\[ \pi_t = \pi_t^h + \frac{1 - \gamma}{\gamma} \Delta \hat{RER}_t \] (40)

The Phillips curve then is as follows:

\[ \pi_t = \beta E_t \pi_{t+1} + \lambda \frac{1 - \alpha}{1 - \alpha + \alpha \varepsilon} \hat{MC}_t + \frac{1 - \gamma}{\gamma} \Delta \hat{RER}_t - \beta \frac{1 - \gamma}{\gamma} E_t \Delta \hat{RER}_{t+1}, \]

where \( \hat{MC}_t = \hat{W}_t - (\hat{Y}_t^{no} - \hat{N}_t) + \frac{1 - \gamma}{\gamma} \hat{RER}_t \). Wages can be substituted with the log-linearized labor supply condition (39), so that the Phillips curve used in the model is:

\[ \pi_t = \beta E_t \pi_{t+1} + \frac{1 - \gamma}{\gamma} \left( \lambda \frac{1 - \alpha}{1 - \alpha + \alpha \varepsilon} + \beta + 1 \right) \hat{RER}_t - \beta \frac{1 - \gamma}{\gamma} \hat{RER}_{t-1} \]

(41)

\[ -\beta \frac{1 - \gamma}{\gamma} \hat{E}_t \hat{RER}_{t+1} + \lambda \frac{1 - \alpha}{1 - \alpha + \alpha \varepsilon} \left( \phi \hat{N}_t - \hat{Y}_t^{no} \right) \]

\[ \Phi (1 + \beta) \hat{I}_t = \left( \beta (1 - \delta) + \Omega \bar{X} \right) \left[ \kappa (1 + \beta) E_t \hat{I}_{t+1} \right] + \left( 1 - \beta (1 - \delta) - \Omega \bar{X} \right) E_t \left[ \phi \hat{N}_{t+1} - \hat{K}_{t+1}^{no} \right] \]

(42)

where \( \Phi = \left[ \mu \bar{N}^\phi \phi + (1 - \mu) \bar{N}^\phi \right] \bar{C}^{-1} \) and \( \Theta = (\sigma \bar{C})^{-1} (1 - \mu) (\bar{C} - \phi^{-1} \bar{N}^\phi) \).

The combination of the first-order condition with respect to non-oil capital (36) and investment (35) given that \( \hat{R}_t^{kno} = \phi \hat{N}_t - \hat{K}_t^{kno} \) delivers the following:

\[ \kappa (1 + \beta) \hat{I}_t = \left( \beta (1 - \delta) + \Omega \bar{X} \right) \left[ \kappa (1 + \beta) E_t \hat{I}_{t+1} - \kappa \beta E_t \hat{I}_{t+2} - \kappa \hat{I}_t \right] + \kappa \beta E_t \hat{I}_{t+1} \]

(43)

\[ + (1 - \beta (1 - \delta) - \Omega \bar{X}) E_t \left[ \phi \hat{N}_{t+1} - \hat{K}_{t+1}^{no} \right] - (1 - \Omega \bar{X}) \left( \hat{R}_t - E_t \pi_{t+1} \right) \]

\[ + \kappa \hat{I}_{t+1} + \Omega \bar{X} (E_t \pi_{t+1} + \hat{RER}_t - E_t \hat{RER}_{t+1} + \lambda \hat{I}_t) \]
The collateral constraint (4) combined with the first-order condition with respect to investment (35) yields:

\[ b_t^* = E_t \pi_t^{*+1} - \hat{R}_t^* + K_t^{no} + RER_t - E_t RER_{t+1} + \kappa(1 + \beta)E_t I_{t+1} - \kappa \beta E_t I_{t+2} - \kappa \hat{I}_t \] (44)

The law of motion for non-oil capital (3) is as follows:

\[ \hat{K}_t^{no} = (1 - \delta) \hat{K}_{t-1}^{no} + \delta \dot{I}_t \] (45)

Similarly, the public capital accumulation (19) in its log-linearized form is below:

\[ \hat{K}_t^G = (1 - \delta^G) \hat{K}_{t-1}^G + \delta^G \hat{G}_t \] (46)

The oil capital is accumulated by FDI according to its equation (13):

\[ \hat{K}_t^o = (1 - \delta) \hat{K}_{t-1}^o + \delta \hat{FDI}_t \] (47)

The combination of oil tax revenues equation (20), SWF accumulation (21), and the profits of oil producer (16) corresponds to:

\[ \hat{SWF}_t = \rho_{swf} R_t (SWF_{t-1} - \pi_t^*) + \frac{[\tau^o + \ell^{div}(1 - \tau^o)]s_o(Y_t^o + P_t^{po}) + \rho_{swf} \hat{R}_t^{*+1}}{s_w f_y} \] (48)

The log-linearization of the first-order condition of a saver with respect to foreign debt (38) provides the following UIP condition:

\[ \hat{R}_t = E_t \pi_t^{*+1} + \frac{\beta^*}{\beta} \hat{R}_t^* - E_t \pi_{t+1}^* + E_t RER_{t+1} - \hat{R}ER_t + \left( \frac{\beta^*}{\beta} - 1 \right) \hat{\lambda}_t \] (49)

The non-oil and oil production functions (8 and 12) give respectively:

\[ \hat{Y}_t^{no} = \alpha \hat{K}_t^{no} + (1 - \alpha) \hat{N}_t + \psi \hat{K}_G,t \] (50)

\[ \hat{Y}_t^o = \alpha \hat{K}_t^o \] (51)

The aggregate output is as follows:

\[ \hat{Y}_t = (1 - s_o)\hat{Y}_t^{no} + (1 - s_o)\hat{Y}_t^o + s_o(\hat{Y}_t^o + \hat{RER}_t + \hat{P}_t^{po}) \] (52)

The government budget constraint (17) in terms of fiscal debt results in:

\[ \hat{b}_t = \hat{R}(\hat{b}_{t-1} - \pi_t) + \hat{R}_{t-1} + \frac{g_y^I}{(1 - \mu) b_y} \hat{G}_t^I + \frac{g_y^C}{(1 - \mu) b_y} \hat{G}_t^C + \frac{g_y^C + g_y^I}{(1 - \mu) b_y} \hat{P}_t^{po} \]

\[ - \frac{T}{b(1 - \mu)} \hat{\lambda}_t - \frac{(1 - \rho_{swf}) s_w f_y}{b_y(1 - \mu)} \hat{R}_{t-1}^{*+1} - \frac{(1 - \rho_{swf}) RER_s f_y}{b_y(1 - \mu)} (RER_t + \hat{SWF}_{t-1} - \pi_t^*) \] (53)

The log-linearized relative price of government purchases to composite consumption

33
(18), assuming $\eta \to 1$, is this:

$$\hat{p}_t^r = \gamma_2 \hat{p}_t^h + (1 - \gamma_2) RER_t \tag{54}$$

The domestic goods market clearing condition (29) can be rewritten as:

$$\bar{Y}_t^{n} + \hat{p}_t^r = \frac{\gamma c_y}{(1 - s_o)} \hat{C}_t + \frac{(1 - \mu)(1 - i_y)\gamma \hat{I}_t}{(1 - s_o)} + \frac{\gamma_2 g_y^C}{(1 - s_o)} \hat{G}_t^C + \frac{\gamma_2 g_y^l}{(1 - s_o)} \hat{G}_t^l + \frac{\gamma_2 (g_y^C + g_y^l)}{(1 - s_o)} \hat{p}_t^r, \tag{55}$$

where $(1 - i_y) = 1 - c_y - g_y^C - g_y^l - nx_y$.

The real GDP (30) is represented in terms of investment:

$$\hat{I}_t = \frac{1}{(1 - i_y)(1 - \mu)} \left[ \bar{Y}_t - c_y \hat{C}_t - g_y^C \hat{G}_t^C - g_y^l \hat{G}_t^l - (g_y^C + g_y^l) \hat{p}_t^r - nx_y \bar{N}_t \right] \tag{56}$$

The log-linearization of the balance of payments equation results in:

$$\bar{N}_t = \frac{R^\prime b_{y}^* (1 - \mu) \hat{p}_t^r}{n x_y} + \frac{R^\prime (1 - \rho_{swf}) swf_y - (1 - \mu) R^\prime b_{y}^* + R^\prime f x r_y}{n x_y} \pi_t^* + \frac{R^\prime b_{y}^* (1 - \mu)}{n x_y} \hat{p}_t^r - \hat{p}_t^r \rho_{swf} \hat{p}_t^r - \hat{p}_t^r (1 - \mu) R^\prime \hat{p}_t^r - \hat{p}_t^r + \frac{f x r_y}{n x_y} f x r_t^* - \frac{b_{y}^* (1 - \mu) \hat{p}_t^r}{n x_y} - \frac{f d_i y}{n x_y} F D I_t^* \tag{57}$$

The budget constraint of a saver (2) is log-linearized as well, by combining the aggregate relationships for consumption $C_t = \mu C_t^N + (1 - \mu) C_t^S$ and taxes $T_t = \mu T_t^N + (1 - \mu) T_t^S$, and the budget constraint of a rule-of-thumb household (5):

$$\hat{C}_t = \frac{\phi \bar{N}^\phi + (1 - \mu) K^{n o} K^{n o} \phi}{C} - \frac{(1 - \mu)(1 - s_o)(\varepsilon - 1)\phi}{c_y} \hat{N}_t + \frac{(1 - \mu) b_y}{c_y} \hat{R}_t \tag{58}$$

$$+\frac{T}{C} \hat{I}_t + \frac{(1 - \mu) b_y}{c_y} \left[ \frac{b_{y}^* \hat{p}_t^r - (1 - i_y)\hat{I}_t - b_y \hat{b}_t}{c_y} \right] - \frac{(1 - \mu) b_{y}^* R^\prime}{c_y} \hat{b}_t - \frac{(1 - \mu) b_{y}^*}{c_y} \frac{R^\prime f x r_y}{\pi_t^*} \hat{R}_t - \frac{f x r_y f x r_t^*}{c_y} + \frac{f x r_y f x r_t^*}{c_y} f x r_t^* - \frac{b_{y}^* (1 - \mu) \hat{p}_t^r}{n x_y} - \frac{f d_i y}{n x_y} F D I_t^* \tag{58}$$
G Impulse-response functions

Figure 1. Impulse-response functions to a negative world oil price shock of 1%: acyclical fiscal policy combined with PPT monetary rule under a managed exchange rate regime

Figure 2. Impulse-response functions to a negative world oil price shock of 1%: acyclical fiscal policy combined with CPI monetary rule under a managed exchange rate regime
Figure 3. Impulse-response functions to a negative world oil price shock of 1%: procyclical fiscal policy combined with PPT monetary rule under a managed exchange rate regime

Figure 4. Impulse-response functions to a negative world oil price shock of 1%: procyclical fiscal policy combined with CPI monetary rule under a managed exchange rate regime

A flexible exchange rate regime, across all figures of policy combination, produces no changes in the foreign exchange reserves, otherwise the impulse-response functions stay the same.