Markets, Banks and Shadow Banks

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Abstract

We analyze the effect of bank capital regulation on the structure and risk of the financial system. Banks intermediate between entrepreneurs and investors, and can monitor entrepreneurs’ projects. Monitoring is not observed by investors, so there is a moral hazard problem. Banks choose whether to be subject to the regulation, in which case a supervisor certifies their capital, or not be subject to it, in which case they have to resort to more expensive private certification. Market finance, regulated banks, and shadow banks can coexist in equilibrium. Under both flat and risk-based capital requirements, safer entrepreneurs borrow from the market and riskier entrepreneurs borrow from intermediaries. The difference is that flat (risk-based) requirements are especially costly for relatively safe (risky) entrepreneurs which may be better off borrowing from shadow banks. We compare these regulations in terms of welfare, and characterize the optimal requirements taking into account the existence of both market and shadow bank finance.

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“While higher capital and liquidity requirements on banks will no doubt help to
insulate banks from the consequences of large shocks, the danger is that they will
also drive a larger share of intermediation into the shadow banking realm.”

S. Hanson, A. Kashyap, and J. Stein (2011)

1 Introduction

The aftermath of the recent financial crisis resulted in a widespread adoption of tougher
banking regulation, exemplified by the 2010 agreement of the Basel Committee on Banking
Supervision, known as Basel III. However, a concern has emerged about the possibility that
the effects of new regulations may be hindered by a shift of intermediation away from the
regulated banks and into the shadow banking system.

This paper proposes an analytical framework to understand the effects of different types of
bank capital requirements on the structure and risk of the financial system. In particular, we
address issues such as (i) what is the difference between regulated banks and shadow banks,
and how do they differ from direct market finance, (ii) how does bank capital regulation
affects funding through these channels, and (iii) how does the existence of shadow banks
affect the effectiveness of capital regulation.

The framework builds on the model of Martinez-Miera and Repullo (2017), where a set of
heterogeneous entrepreneurs borrow from competitive banks to fund their risky investment
projects, and banks can reduce the probability of default by monitoring/screening these
projects at a cost. Unlike in our previous setup where banks only raised (uninsured) debt
finance from a set of risk-neutral investors, here we introduce the possibility of raising costly
equity finance and analyze the effect of different forms of capital requirements. We consider
three possible modes of funding entrepreneurs’ projects: they may obtain funding directly
from the market, or through intermediaries that can be either regulated banks or shadow
banks. Market finance differs from intermediated finance in that entrepreneurs are not
monitored. Both regulated banks and shadow banks monitor their borrowers, but only the
former comply with the regulation.
A key friction in our model is that monitoring is not observed by investors, so there is a moral hazard problem in intermediated finance. In this setup, (inside) equity capital provides “skin in the game” and hence serves as a commitment device for monitoring. For this reason, banks may be willing to use (more expensive) equity finance in order to ameliorate the moral hazard problem and reduce the cost of debt.\(^1\)

However, for this channel to operate, the capital structure should be observable to outside investors. Given the incentives of banks to save on costly equity, we assume that capital has to be certified by an external (private or public) agent. Public certification is done by a bank supervisor that verifies whether banks that choose to be regulated comply with the regulation. The capital of banks that choose not to be regulated is not certified by the supervisor, so they will have to resort to private certification which we assume to be more expensive. Thus, (cheaper) public certification is tied to complying with a regulation that might be very tough, at least for banks financing certain types of projects. For this reason, (shadow) banks might prefer not to comply with the regulation and resort to private certification. In this manner, the emergence of a shadow banking system is linked to a trade-off between the costs and benefits of public certification.

We consider two different types of regulation, namely risk-insensitive (or flat) and risk-sensitive capital requirements. The former broadly correspond to the 1988 Accord of the Basel Committee (Basel I),\(^2\) while the latter correspond to the 2004 (Basel II) and 2010 (Basel III) Accords.\(^3\) We follow the Basel II and III approach of using a value-at-risk criterion to determine the risk-sensitive requirements. We highlight the different impact that these regulations have on the equilibrium market structure, with especial emphasis on the extent to which they will shift some types of lending from regulated banks into the shadow banking system or direct market finance.

Specifically, under both regulations, safer entrepreneurs will borrow from the market and riskier entrepreneurs will borrow from intermediaries. The difference between them is that

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\(^1\)Note that if capital were not more expensive than debt, banks would be 100% equity financed.

\(^2\)This also corresponds to the leverage ratio proposed by Admati and Hellwig (2013).

\(^3\)See, for example, Gordy, Heitfield and Wu (2015).
flat requirements are especially costly for relatively safe entrepreneurs, that may be better off borrowing from shadow banks, while value-at-risk based requirements are especially costly for risky entrepreneurs, that may be better off borrowing from shadow banks. With flat requirements the equilibrium market structure is such that regulated banks always fund the riskiest projects, while if shadow banks operate they fund projects that are safer than those of the regulated banks. With value-at-risk based capital requirements the equilibrium market structure is such that regulated banks always fund the intermediate risk projects, while if shadow banks operate they fund the riskiest projects.

The results illustrate how the existence of shadow banks affect the effectiveness of the different types of regulation. Tightening flat (value-at-risk based) capital requirements increases the monitoring incentives of banks for which the regulation is binding at the cost of driving some safer (riskier) entrepreneurs to the shadow banking system where they will have lower monitoring and higher default risk.

We then analyze how the structure and risk of the financial system change with key parameters of the model, namely the expected return required by investors (the risk-free rate) and the cost of bank capital. We find that for both types of capital requirements a higher risk-free rate and/or a lower cost of capital reduce the range of entrepreneurs financed by markets and shadow banks, and expand the range of entrepreneurs financed by regulated banks. According to these results, the shadow banking system will thrive when the risk-free rate is low and the cost of bank capital is high. Notice that the latter situation may obtain in an environment of low rates if bank capital is relatively scarce due to the expansion of banks’ balance sheets.

We next compare these regulations in terms of welfare, for a specific parameterization of the model. We compute the optimal flat and value-at-risk based requirements, as well as the optimal unconstrained capital requirements. We show how the optimal capital regulation should be adjusted in response to changes in the risk-free rate and the cost of capital.

Finally, we show that the qualitative results of our model remain unchanged when the advantage of regulated banks relative to shadow banks comes from the existence of under-priced deposit insurance instead of a lower cost of capital certification.
Literature review  TBC

Structure of the paper  Section 2 presents the model of bank lending under moral hazard in which banks are not regulated and have to pay a cost to certify their capital. Section 3 introduces bank capital regulation and supervision, analyzes the effects of flat and value-at-risk based minimum capital requirements on the structure and risk of the financial system, and presents comparative static results with respect to funding costs. Section 4 compares these regulations in terms of welfare, and characterizes optimal capital requirements. Section 5 considers the model with deposit insurance for the regulated banks. Section 6 presents our concluding remarks.

2 The Model

Consider an economy with two dates \((t = 0, 1)\), a large set of penniless entrepreneurs with observable types \(p \in [0, \overline{p}]\), where \(\overline{p} \leq 1\), and a large set of investors characterized by an infinitely elastic supply of funds at an expected return equal to \(R_0\). Entrepreneurs have investment projects that require external finance. Such finance may come directly from investors (market finance) or may be intermediated by banks (bank finance).

Intermediated finance differs from direct market finance in two respects. First, banks monitor their borrowers, whereas markets do not. Second, banks raise funds from investors, in the form of uninsured deposits, and also from (inside) shareholders. We assume that bank capital is costly. Specifically, bank shareholders require an expected return equal to \(R_0 + \delta\), where \(\delta > 0\).

Each entrepreneur of type \(p\) has a project that requires a unit investment at \(t = 0\) and yields a stochastic return \(\tilde{A}_p\) at \(t = 1\) given by

\[
\tilde{A}_p = \begin{cases} 
A(x_p), & \text{with probability } 1 - p + m_p, \\
0, & \text{with probability } p - m_p.
\end{cases}
\]  \hspace{1cm} (1)

\(^4\text{Holmström and Tirole (1997) and Diamond and Rajan (2000) provide agency-based explanations specifically related to banks’ monitoring role.}\)
where the success return $A(x_p)$ is a decreasing function of the aggregate investment $x_p$ of entrepreneurs of type $p$, and $m_p \in [0, p]$ is the monitoring intensity of the entrepreneur’s lender. When $m_p = 0$ we have direct market finance, and when $m_p > 0$ we have bank finance. Thus, the safest type $p = 0$ will always be funded by the market. Monitoring is not observed by the investors, so there is a moral hazard problem.

Monitoring increases the probability of success of entrepreneurs’ projects but entails a cost $c(m_p)$. The monitoring cost function $c(m_p)$ satisfies $c(0) = c'(0) = 0$, and $c'(m_p) > 0$, $c''(m_p) > 0$, and $c'''(m_p) \geq 0$ for $m_p > 0$. A special case that satisfies these assumptions and will be used for our numerical results is the quadratic function

$$c(m_p) = \frac{\gamma}{2} (m_p)^2, \quad (2)$$

where $\gamma > 0$.

To simplify the presentation we assume that (i) for each type $p$ there is a single bank that specializes in funding entrepreneurs of this type, and (ii) the returns of the projects of entrepreneurs of each type $p$ are perfectly correlated.$^5$

Under these conditions, the assumption $A'(x_p) < 0$ may be rationalized by introducing a representative consumer with a utility function over the continuum of goods produced by entrepreneurs of types $p \in [0, \bar{p}]$. Specifically, suppose that one unit of investment produces (if successful) one unit of output, and consider the utility function

$$U(q, x) = q + \frac{\sigma}{\sigma - 1} \int_0^\bar{p} (x_p)^{\frac{\sigma - 1}{\sigma}} dp, \quad (3)$$

where $q$ is the consumption of a composite good, $x = \{x_p\}_{p \in [0, \bar{p}]}$, and $\sigma > 1$. The budget constraint of the representative consumer is

$$q + \int_0^\bar{p} A_p x_p dp = I, \quad (4)$$

where $A_p$ is the unit price of the good produced by entrepreneurs of type $p$, and $I$ is her (exogenous) income. Maximizing (3) subject to (4) gives a first-order condition that implies

$$A_p = A(x_p) = (x_p)^{-1/\sigma}. \quad (5)$$

$^5$We could have one single bank or multiple banks lending to each type of entrepreneur. The key simplifying assumption is that no bank lends to more than one type.
Thus, the higher the investment $x_p$ of entrepreneurs of type $p$ the lower the equilibrium price $A_p$, if the investment is successful. If it is not, output will be zero and the representative consumer will not consume this good.\footnote{An alternative rationalization may be derived from the demand of a set of final good producers that use entrepreneurs’ output as an intermediate input; see Martinez-Miera and Repullo (2017).}

We assume free entry of entrepreneurs in the loan market. Hence, if the lowest loan rate for entrepreneurs of type $p$ offered by either markets or banks is $R_p$, then a measure $x_p$ of these entrepreneurs will enter until $A(x_p) = R_p$. Thus, entrepreneurs will only be able to borrow at a rate that leaves them no surplus.

Since investors are characterized by an infinitely elastic supply of funds at an expected return equal to $R_0$, the equilibrium loan rate $R^*_p$ for entrepreneurs of type $p$ under direct market finance will be the rate that satisfies the participation constraint

$$(1 - p)R^*_p = R_0. \tag{6}$$

Computing the equilibrium loan rate under bank finance is more complicated because one has to derive banks’ decision on capital and monitoring. To do this, we assume that the loan market is contestable. Thus, although there is a single bank that lends to each type, the incumbent could be undercut by another bank (or by the market) if it were profitable to do so.

Despite the assumption that bank capital is more expensive than debt, banks may be willing to use equity finance in order to ameliorate the moral hazard problem and reduce the cost of debt. But this requires that banks’ capital structure be observable to outside investors. Given the incentives of banks to save on costly equity, we assume that capital has to be certified by an external agent at a cost $\eta$ per unit of capital.

The bank lending to entrepreneurs of type $p$ will raise $1 - k_p$ funds per unit of loans from investors at a rate $B_p$ (the rest will be funded with capital), set a loan rate $R_p$, and choose a monitoring intensity $m_p \in [0, p]$. By contestability and free entry of entrepreneurs in the loan market, the equilibrium loan rate $R^*_p$ for entrepreneurs of type $p$ will be the lowest feasible rate.
Formally, an equilibrium for entrepreneurs of type $p$ under bank finance is an array $(k_p^*, B_p^*, R_p^*, m_p^*)$ that minimizes the loan rate $R_p$ subject to the bank’s incentive compatibility constraint

$$m_p^* = \arg \max_{m_p} \left[ (1 - p + m_p)[R_p^* - (1 - k_p^*)B_p^*] - c(m_p) \right],$$  

(7)

the shareholders’ participation constraint

$$(1 - p + m_p^*)[R_p^* - (1 - k_p^*)B_p^*] - c(m_p^*) - \eta k_p^* \geq (R_0 + \delta)k_p^*,$$  

(8)

and the investors’ participation constraint

$$(1 - p + m_p^*)B_p^* \geq R_0.$$

(9)

The incentive compatibility constraint (7) characterizes the bank’s choice of monitoring $m_p^*$ given that the bank gets $R_p^*$ and pays $(1 - k_p^*)B_p^*$ with probability $1 - p + m_p^*$ (and with probability $p - m_p^*$ gets zero, by limited liability).\(^7\) The participation constraints (8) and (9) ensure that the shareholders and the investors get the required expected return on their investments.

To ensure that market and bank finance coexist in equilibrium, we assume that parameter values are such that

$$\frac{R_0(R_0 + \delta)}{\delta c''(0)} < 1.$$  

(10)

The following result characterizes the range of entrepreneurs’ types that borrow from the market and from banks, as well as the banks’ equilibrium capital and monitoring decisions and their borrowing and lending rates.

**Proposition 1** There exists a marginal type $\hat{p} \in (0, 1)$ given by

$$\hat{p} = 1 - \sqrt[\delta + \eta]{\frac{R_0(R_0 + \delta + \eta)}{(\delta + \eta)c''(0)}}$$  

(11)

such that entrepreneurs of types $p \leq \hat{p}$ will borrow from the market and entrepreneurs of types $p > \hat{p}$ will borrow from banks.

\(^7\)Note that the assumption that project returns are perfectly correlated implies that the bank’s return per unit of loans is identical to the individual project return, which is given by (1).
The sketch of the proof is as follows. Consider a type \( p \) for which the equilibrium monitoring intensity \( m_p^* \) satisfies \( 0 < m_p^* < p \). Then the bank’s incentive compatibility constraint (7) reduces to the first-order condition

\[
R_p^* - (1 - k_p^*)B_p^* = c'(m_p^*).
\] (12)

From here it can be shown (see the formal proof in the Appendix) that both the shareholders’ participation constraint (8) and the investors’ participation constraint (9) will be binding. Substituting out \( B_p^* \) and \( R_p^* \) from these three equations gives

\[
k_p^* = \frac{(1 - p + m_p^*)c'(m_p^*) - c(m_p^*)}{R_0 + \delta + \eta}.
\] (13)

By the properties of the monitoring cost function \( c(m_p) \) this equation implies that \( k_p^* > 0 \) if and only if \( m_p^* > 0 \). In other words, monitoring banks will always want to have a positive amount of capital.

Next, solving for \( B_p^* \) in the investors’ participation constraint (9), substituting it into the first-order condition (12), and rearranging gives

\[
R_p^* = \frac{(1 - k_p^*)R_0}{1 - p + m_p^*} + c'(m_p^*).
\] (14)

The equilibrium loan rate \( R_p^* \) is found by minimizing (14) with respect to \( m_p \) and \( k_p \) subject to (13). Finally, we show that for entrepreneurs of types \( p \leq \hat{p} \), the loan rate (14) is minimized by setting \( m_p^* = k_p^* = 0 \), so they will borrow from the market, and for entrepreneurs of types \( p > \hat{p} \), the loan rate (14) is minimized by setting \( m_p^* > 0 \) and \( k_p^* > 0 \), so they will borrow from banks.

In what follows we introduce two possible institutions that may certify banks’ capital. One is a private auditor that charges a rate \( \eta_1 \) per unit of capital. The other is a public auditor that charges a rate \( \eta_0 \) per unit of capital. The presence of a public auditor may be justified by introducing bank capital requirements and associating the public auditor to a bank supervisor that verifies whether the bank complies with the regulation. We will assume that private certification is costlier than public certification, \( \eta_1 > \eta_0 \). This may be
rationalized by assuming that supervisors have lower agency problems than private auditors or have better access to relevant bank information.

But if private auditors are more expensive than the public auditor, why would banks want to resort to them? The answer is that using the public auditor is tied to complying with a regulation that might be very tough, at least for banks financing certain types of entrepreneurs. These (shadow) banks might then prefer not to comply with the regulation and resort to private auditors. In this manner, the possible emergence of a shadow banking system is linked to a trade-off between the costs and benefits of public certification.

Bank capital requirements will be introduced in the next section. Here we present, for future reference, the comparative static properties of the model with respect to the cost of certification.

**Proposition 2** An increase in the certification cost \( \eta \) expands the range \([0, \hat{p}]\) of market finance, and for types \( p > \hat{p} \) reduces banks’ equilibrium capital and monitoring and increases their probability of failure.

Figure 1 illustrates this result for the quadratic monitoring cost function (2) and two values of the certification cost, \( \eta_0 \) and \( \eta_1 \), corresponding respectively to a public and a private auditor. To simplify the presentation, in what follows we will normalize to zero the cost of the public auditor (\( \eta_0 = 0 \)), and drop the subindex for the cost of the private auditor (\( \eta_1 = \eta \)). Panel A shows that an increase in the certification cost shifts to the right from \( \hat{p}_0 \) to \( \hat{p}_1 \) the marginal type that is indifferent between market and bank finance. As capital becomes more expensive, due to the higher certification costs, banks reduce their capital per unit of loans. Panel B shows the effect on the probability of failure \( p - m_p \). Under market finance, \( m_p = 0 \), so the probability of failure coincides with the 45° line. The reduction in the level of capital under high certification costs worsens the banks’ moral hazard problem and leads to an increase in their probability of failure.

[FIGURE 1]

Summing up, we have presented a model in which a heterogeneous set of entrepreneurs
seek funding from either banks or the market. The difference between bank and market finance is that banks monitor their borrowers, which leads to a reduction in their probability of failure. Bank monitoring is subject to a moral hazard problem that can be ameliorated by equity capital. However, capital is costlier than deposits, and also requires paying a certification cost. We have shown that safer entrepreneurs borrow from the market while riskier entrepreneurs borrow from banks, and that banks will always want to fund part of their lending with capital. We concluded analyzing the effect of certification costs on the equilibrium of the model, and in particular the effect of having a public auditor with lower certification costs.

3 Bank Capital Regulation

This section introduces a bank regulator that sets minimum capital requirements and a bank supervisor that verifies whether banks that choose to be regulated comply with the regulation, in which case their capital is certified at a cost that is normalized to zero. Banks that choose not to be subject to the regulation will be called shadow banks. Since their capital is not certified by the supervisor, they will have to resort to a more expensive private certification.

Two types of minimum capital requirements, flat and risk-based, will be analyzed. A flat requirement does not vary with the bank’s risk, whereas a risk-based requirement is increasing in the bank’s risk. The risk-insensitive regulation broadly corresponds to the 1988 Basel Capital Accord, known as Basel I, while the risk-sensitive regulation corresponds to the 2004 Revised Capital Framework, known as Basel II. In particular, we follow the Basel II approach of using a value-at-risk criterion to determine the requirements for the different types of banks. We highlight the different impact that these regulations have on the equilibrium market structure of our model, with especial reference to the extent to which they will shift some types of lending into the shadow banking system.

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8See Basel Committee on Banking Supervision (2015).
3.1 Flat capital requirements

Suppose that regulated banks are required to fund at least a proportion $k$ of their lending with capital, independently of their type $p$. In this case, we show that when the requirement $k$ is low, safer entrepreneurs borrow from the market while riskier entrepreneurs borrow from regulated banks. However, when the requirement $k$ raises above a threshold the equilibrium of the model changes, with safer entrepreneurs borrowing from the market, lower risk entrepreneurs borrowing from shadow banks, and higher risk entrepreneurs borrowing from regulated banks.

To characterize the equilibrium under flat capital requirements, consider a bank lending to entrepreneurs of type $p \geq \hat{p}_0$ whose capital is certified at zero cost by the supervisor.\(^9\) Clearly, if the bank would like to have more capital than the minimum required by regulation, that is if $k_p \geq \bar{k}$, the capital requirement would not have any effect on the equilibrium for entrepreneurs of type $p$. However, if the bank would like to have less capital than $\bar{k}$, one can show that if $k_p$ is very close to zero, then complying with the regulation has high costs so these entrepreneurs will shift to market finance. On the other hand if $k_p$ is very close to $\bar{k}$, then complying with the regulation has low costs so these entrepreneurs will be funded by regulated banks. What happens when $k_p$ is between zero and $\bar{k}$ depends on the level of the capital requirement. When $\bar{k}$ is low there is a marginal type $p_m$ that switches from market to regulated bank finance. When $\bar{k}$ is high shadow banks can profitably enter the market, and there is a marginal type $p_m$ that switches from market to shadow bank finance and a marginal type $p_s > p_m$ that switches from shadow to regulated bank finance. Thus, we can state the following result.

**Proposition 3** If the minimum capital requirement $\bar{k}$ is below a threshold $\hat{k}$, there is a marginal type $p_m > \hat{p}_0$ such that entrepreneurs of types $p \leq p_m$ will borrow from the market and entrepreneurs of types $p > p_m$ will borrow from regulated banks. If the minimum requirement $\bar{k}$ is above the threshold $\hat{k}$, there are two marginal types, $p_m$ and $p_s > p_m$, such that entrepreneurs of types $p \leq p_m$ will borrow from the market, entrepreneurs of types $p_m < p \leq p_s$...

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\(^9\)Entrepreneurs of types $p \leq \hat{p}_0$ are always funded by the market.
will borrow from shadow banks, and entrepreneurs of types \( p > p_s \) will borrow from regulated banks.

Figure 2 illustrates the result for the case of a low minimum capital requirement \( (k \leq \hat{k}) \). Panel A shows equilibrium bank capital. Two regions may be distinguished. To the left of the marginal type \( p_m \) entrepreneurs borrow from the market. To the right of the marginal type \( p_m \) entrepreneurs borrow from regulated banks, with the safer ones borrowing from banks with capital \( k \) and the riskier ones borrowing from banks with \( k_p > k \). Panel B shows the corresponding probabilities of failure \( p - m_p \), which jump down at \( p_s \) because of the effect of the binding capital requirements.

[FIGURE 2]

Figure 3 illustrates the result for the case of a high minimum capital requirement \( (K > \hat{k}) \). Panel A shows equilibrium bank capital. Three regions may be distinguished. To the left of the marginal type \( p_m \) entrepreneurs borrow from the market, between \( p_m \) and \( p_s \) entrepreneurs borrow from shadow banks, and to the right of the marginal type \( p_s \) entrepreneurs borrow from regulated banks.\(^{10}\) Panel B shows the corresponding equilibrium probabilities of failure \( p - m_p \), which bend down at \( p_m \) where shadow banks start to operate, and jump down at \( p_s \) because of the effect of the binding capital requirements.

[FIGURE 3]

Thus, although tightening flat capital requirements reduces the probability of failure of relatively safe banks in the regulated banking system, this comes at the cost of pushing some entrepreneurs toward alternative sources of funding (market finance or shadow banks), which reduces monitoring and increases their probability of failure.

It should be noted that with flat capital requirements the equilibrium market structure of the financial sector is such that regulated banks always fund the riskiest projects, while if

\(^{10}\) Notice that \( p_m \) coincides with the marginal type \( \hat{p}_1 \) that is indifferent between market and shadow bank finance.
shadow banks operate they fund projects that are ex-ante safer than those of the regulated banks (although not necessarily ex-post, given their different monitoring incentives).

### 3.2 Value-at-risk based capital requirements

The risk-sensitive minimum capital requirements of Basel II are based on the criterion that capital should cover the losses of a sufficiently diversified loan portfolio with a given confidence level $1 - \alpha$ (say, 99.9%). To translate this criterion to our model setup, in which loan defaults are perfectly correlated, we will define a capital requirement $\bar{K}_p$ such that the probability of default $p - m_p^*$ of the loans to entrepreneurs of type $p$ is equal to $\alpha$.

By Proposition 1, there is an equilibrium relationship between capital and monitoring given by (13). Solving for $m_p^*$ in the condition $p - m_p^* = \alpha$, and substituting it into (13) then gives the model equivalent of the Basel II formula

$$
\bar{K}_p = \begin{cases} 
0 & \text{if } p \leq \alpha, \\
\frac{(1 - \alpha) c'(p - \alpha) - c(p - \alpha)}{R_0 + \delta} & \text{otherwise}.
\end{cases}
$$

(15)

Notice that for $p > \alpha$ we have

$$
\frac{d\bar{K}_p}{dp} = \frac{(1 - \alpha) c''(p - \alpha) - c'(p - \alpha)}{R_0 + \delta}.
$$

Thus, riskier banks will be required to have more capital if $(1 - \alpha) c''(p - \alpha) - c'(p - \alpha) > 0$. For the quadratic monitoring cost function (2) this condition simplifies to $\gamma (1 - p) > 0$, which always hold.\footnote{The certification cost $\eta$ does not appear in the denominator of $\bar{K}_p$, since we assume that certification by the bank supervisor is costless.}

We next show that when the confidence level $1 - \alpha$ is low, safer entrepreneurs borrow from the market while riskier entrepreneurs borrow from regulated banks. However, when the confidence level $1 - \alpha$ raises above a threshold the equilibrium of the model changes, with safer entrepreneurs borrowing from the market, intermediate risk entrepreneurs borrowing from regulated banks, and higher risk entrepreneurs borrowing from shadow banks. Thus, in contrast with the equilibrium under flat capital requirements, here if shadow banks operate they fund projects that are riskier than those of the regulated banks.
Proposition 4 If the confidence level \(1 - \alpha\) is below a threshold \(1 - \hat{\alpha}\), there is a marginal type \(p_m\) such that entrepreneurs of types \(p \leq p_m\) will borrow from the market and entrepreneurs of types \(p > p_m\) will borrow from regulated banks. If the confidence level \(1 - \alpha\) is above the threshold \(1 - \hat{\alpha}\), there are two marginal types, \(p_m\) and \(p_s > p_m\), such that entrepreneurs of types \(p \leq p_m\) will borrow from the market, entrepreneurs of types \(p_m < p \leq p_s\) will borrow from regulated banks, and entrepreneurs of types \(p > p_s\) will borrow from shadow banks.

Figure 4 illustrates the result for the case of a low confidence level \((\alpha > \hat{\alpha})\). Panel A shows equilibrium bank capital. Two regions may be distinguished. To the left of the marginal type \(p_m\) entrepreneurs borrow from the market. To the right of the marginal type \(p_m\) entrepreneurs borrow from regulated banks, with the safer ones borrowing from banks with capital \(k_p > \overline{k}_p\) and the riskier ones borrowing from banks with capital \(\overline{k}_p\). Panel B shows the corresponding probabilities of failure \(p - m_p\), which become equal to \(\alpha\) for high risk banks.

[FIGURE 4]

Figure 5 illustrates the result for the case of a high confidence level \((\alpha < \hat{\alpha})\). Panel A shows equilibrium bank capital. Three regions may be distinguished. To the left of the marginal type \(p_m\) entrepreneurs borrow from the market, between \(p_m\) and \(p_s\) entrepreneurs borrow from regulated banks, and to the right of the marginal type \(p_s\) entrepreneurs borrow from shadow banks. Panel B shows the corresponding probabilities of failure \(p - m_p\), which jump up at \(p_s\) when lending switches to shadow banks.

[FIGURE 5]

Thus, although tightening value-at-risk based capital requirements reduces the probability of failure of relatively risky banks in the regulated banking system, this comes at the cost of pushing the riskiest entrepreneurs toward the shadow banking system, which reduces monitoring and increases their probability of failure.
It should be noted that with value-at-risk based capital requirements the equilibrium market structure of the financial sector is such that regulated banks always fund the medium risk projects, while if shadow banks operate they always fund the riskiest (ex-ante and ex-post) projects.

### 3.3 Changes in funding costs

This subsection analyzes the effects of changing two key parameters of the model, namely the expected return required by investors $R_0$ and the excess cost of bank capital $\delta$. The results illustrate the implications of the model for the structure and risk of the financial system along the credit cycle.

We first use the result in Proposition 1 to show the effects of these changes on the marginal types $\hat{p}_0$ and $\hat{p}_1$ that are indifferent between market and bank finance for two benchmark values of the certification cost corresponding to the public and the private auditor, namely $\eta_0 = 0$ and $\eta_1 = \eta$. Differentiating (11) it is immediate to show that $\hat{p}_0$ and $\hat{p}_1$ are decreasing in the the expected return required by investors $R_0$, and increasing in the excess cost of bank capital $\delta$.

We then analyze how the equilibrium structure of the financial system varies with different funding costs in the presence of capital regulation. We find that for both types of capital requirements a higher risk-free rate and/or a lower cost of capital reduce the range of entrepreneurs financed by markets and shadow banks, and expand the range of entrepreneurs financed by regulated banks. According to these results, the shadow banking system will thrive when the risk-free rate is low and the cost of bank capital is high. Notice that the latter situation may obtain in an environment of low rates if bank capital is relatively scarce due to the expansion of banks’ balance sheets.

Figure 6 shows how the equilibrium structure of the financial system varies when there is an increase in funding costs under flat capital requirements. An increase in the risk-free rate $R_0$ results in higher capital and lower probabilities of failure for the regulated banks. On the other hand, an increase in the excess cost of capital $\delta$ results in lower capital and differential effects on probabilities of failure: entrepreneurs that move out of the regulated
banking system increase their probability of failure, whereas those that remain with the regulated banks reduce it, due to the higher monitoring incentives associated with higher loan rates.

[FIGURE 6]

Figure 7 shows how the equilibrium structure of the financial system varies when there is an increase in funding costs under value-at-risk based capital requirements. An increase in the risk-free rate $R_0$ results in higher capital and lower probabilities of failure for those banks that keep a capital buffer, but lower capital and no change in the probabilities of failure for those banks for which the regulation is binding.\(^{12}\) On the other hand, an increase in the excess cost of capital $\delta$ results in lower bank capital and increases in the probability of failure for those banks for which the regulation is not binding.\(^{13}\)

[FIGURE 7]

4 Optimal Bank Capital Regulation

This section characterizes optimal capital requirements. We start by determining the optimal flat and value-at-risk based requirements, taking into account that the funding of some entrepreneurs will endogenously take place through markets and/or shadow banks. Then, we characterize the optimal unconstrained capital requirements, showing that they are risk-sensitive, but with a slope smaller than that of the value-at-risk based requirements (except for safer types for which the optimal value-at-risk based capital requirement is zero).

To derive the social welfare function, we first note that by the proof of Proposition 1 the shareholders’ participation constraint (8) and the investors’ participation constraint (9) are satisfied with equality, which means that they exactly receive the opportunity cost of their funds. Moreover, by the assumption of free entry of entrepreneurs, they will only be able to

\(^{12}\)Notice that $\bar{k}_p$ in (15) is decreasing in $R_0$.

\(^{13}\)Notice that when the regulation is binding the probability of failure is, by construction, equal to $\alpha$. 
borrow at a rate that leaves them no surplus. Finally, due to contestability bank profits are equal to zero.

Hence, social welfare reduces to the utility of the representative consumer. Substituting the budget constraint (4) into the utility function (3), and taking into account the fact that the goods of entrepreneurs of type \( p \) are produced with probability \( 1 - p + m_p \), yields

\[
W(x) = I + \frac{\sigma}{\sigma - 1} \int_0^\infty (1 - p + m_p) (x_p)^{\frac{\sigma - 1}{\sigma}} dp - \int_0^\infty (1 - p + m_p) A_p x_p dp,
\]

where \( x = \{x_p\}_{p \in [0,\bar{p}]} \) denotes an investment allocation. Substituting the first-order condition (5) into this expression then gives the social welfare function

\[
W(x) = I + \frac{1}{\sigma - 1} \int_0^\infty (1 - p + m_p) (x_p)^{\frac{\sigma - 1}{\sigma}} dp.
\]  

(16)

Any bank capital regulation is described by a function \( \bar{k}_p \) that gives the minimum capital requirement for loans to entrepreneurs of type \( p \in [0,\bar{p}] \). If \( \bar{k}_p = \bar{k} \) we have flat capital requirements, whereas if \( \bar{k}_p \) is given by (15) we have value-at-risk based capital requirements for a confidence level \( 1 - \alpha \).

By our previous results, a bank capital regulation \( \bar{k}_p \) implies an equilibrium market structure and a corresponding equilibrium loan rate \( R_p \) for each type \( p \) of entrepreneur. Since entrepreneurs of type \( p \) will enter the market until \( R_p = A(x_p) = (x_p)^{-1/\sigma} \), the equilibrium aggregate investment \( x_p \) of entrepreneurs of type \( p \) will be

\[
x_p = (R_p)^{-\sigma}.
\]  

(17)

Hence, we can compute the social welfare \( W(x) \) associated with any bank capital regulation.

### 4.1 Optimal flat and value-at-risk based capital requirements

Optimal flat capital requirements are given by

\[
\bar{k}^* = \arg \max \limits_{\bar{k}} W(\bar{x}),
\]

where \( \bar{x} \) denotes the equilibrium investment allocation corresponding to the flat capital requirement \( \bar{k} \).
The confidence level of the optimal value-at-risk based capital requirements is given by

$$\alpha^* = \arg \max_{\alpha} W(x^{\alpha}),$$

where $x^{\alpha}$ denotes the equilibrium investment allocation corresponding to a value-at-risk based capital requirement with confidence level $1 - \alpha$.

Figure 8 shows the optimal flat and value-at-risk based capital requirements, together with the corresponding probabilities of failure $p - m_p$, for the parameterization used in Section 3. As previously argued, flat capital requirements are such that relatively safe entrepreneurs may borrow from shadow banks, while value-at-risk requirements are such that high risk entrepreneurs may borrow from shadow banks. However, in our parametrization the optimal confidence level $1 - \alpha^*$ is sufficiently low so that no shadow banks operate in equilibrium.

[FIGURE 8]

To compare these two regulations we can compute the social welfare $W_p$ associated with the equilibrium investment $x_p$ of entrepreneurs of type $p$, which is given by

$$W_p = \frac{1}{\sigma - 1} (1 - p + m_p) (x_p)^{\frac{\alpha - 1}{\alpha}}$$

Let $W^f_p$ and $W^v_p$ denote the value of $W_p$ for the optimal flat and value-at-risk based requirements. Figure 9 shows that the difference $W^v_p - W^f_p$ is positive for relatively low and high risk entrepreneurs, and it is negative for intermediate risk entrepreneurs.

[FIGURE 9]

4.2 Optimal capital requirements

Optimal unconstrained capital requirements are defined by

$$k^*_p = \arg \max_{k_p} W(x^*),$$

where $x^*$ denotes the equilibrium investment allocation corresponding to the capital requirement $k_p$. 

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Figure 10 shows the optimal unconstrained capital requirements, together with the corresponding probabilities of failure $p - m_p$, for the parameterization used in Section 3. Such requirements are risk-sensitive, but they are different from the ones that come out from a value-at-risk based regulation. In particular, they leave no room for shadow banks in order to save the higher certification costs, which result in lower welfare. In order to achieve this, the optimal requirements are such that banks' probabilities of failure are increasing in the type $p$. This should be contrasted with value-at-risk based requirements where the probability of failure of the regulated banks is equal (when the requirements are binding) to the given confidence level $1 - \alpha$.

[FIGURE 10]

We next analyze how optimal capital requirements vary with key determinants of banks' funding costs, namely the expected return required by investors $R_0$ and excess cost of bank capital $\delta$. Figure 11 shows the results. An increase in $R_0$ leads to higher optimal capital requirements, an expansion of the range of regulated banks, and consequently a safer financial system. This effect is explained by the fact that an increase in $R_0$ for a given $\delta$ reduces the relative cost of bank capital. Not surprisingly, an increase in the excess cost of capital $\delta$ leads to the opposite effects, reducing optimal capital requirements and the range of regulated banks, and increasing the risk of the financial system.

[FIGURE 11]

These results provide a rationale for the cyclical adjustment of capital requirements. In particular, when capital is scarce and the cost of capital is high, the requirements should be lowered; see Repullo (2013).

5 Deposit Insurance

This section shows that our main qualitative results remain unchanged when we replace the assumption that private certification of capital (of shadow banks) is costlier than public
certification (of regulated banks) by the assumption that regulated banks (but not shadow banks) are able to raise insured deposits at an underpriced deposit insurance premium that is normalized to zero. In our original setup, shadow banks entered the market when the higher cost of resorting to private certification was compensated by the lower cost of not complying with capital regulation. In this setup, shadow banks enter the market when the lower cost of insured deposits is compensated by the lower cost of not complying with capital regulation.

Clearly, the equilibrium loan rate \( R_p^* \) for entrepreneurs of type \( p \) under direct market finance will be the rate that satisfies the participation constraint (6). Similarly, the equilibrium loan rate \( R_p^* \) for entrepreneurs of type \( p \) under shadow bank finance will be the minimum rate that satisfies the bank’s incentive compatibility constraint (7), the shareholders’ participation constraint (8), and the investors’ participation constraint (9) for a certification cost \( \eta = 0 \). So the only loan rate that needs to be determined is the one corresponding to the regulated banks.

A regulated bank lending to entrepreneurs of type \( p \) will raise \( 1 - k_p \) funds per unit of loans from investors at a the risk-free rate \( R_0 \) (the rest will be funded with capital), set a loan rate \( R_p \), and choose a monitoring intensity \( m_p \in [0, p] \), subject to the capital constraint \( k_p \geq \bar{k}_p \). By contestability and free entry of entrepreneurs in the loan market, the equilibrium loan rate \( R_p^* \) for entrepreneurs of type \( p \) will be the lowest feasible rate. Formally, the problem is very similar to the one in Section 2, except for the fact that there is no certification cost in the shareholders participation constraint (8), due to deposit insurance the investors’ participation constraint (9) is replaced by \( B_p^* = R_0 \), and we now have the capital constraint \( k_p \geq \bar{k}_p \). In the case of a flat capital requirement we would have that \( k_p = \bar{k} \) for all \( p \), while as before in the case of a value-at-risk based requirement \( \bar{k}_p \) would be given by (15).

Figures 12–15 show the equilibrium structure of the financial system under low and high flat and value-at-risk based capital requirements. It should be noted that with full deposit insurance regulated banks have no incentive to have capital buffers, since they would not have any effect on their borrowing costs. Hence regulated banks will always have binding capital requirements. With low flat and value-at-risk based capital requirements (Figures
safe entrepreneurs are funded by the market while intermediate and high risk entrepreneurs are funded by regulated banks. Interestingly, the probability of failure of these intermediate and high risk entrepreneurs need not be lower than the one that would obtain in a laissez-faire economy, since in such an economy banks could choose to have higher capital than the one required by the regulation. Figures 13 and 15 show that, as in the certification model, high flat (value-at-risk based) capital requirements move intermediate (high) risk entrepreneurs to shadow banks, which compensate the higher cost of deposits with lower capital than the one required by the regulation. TBC

[FIGURES 12–15]

6 Concluding Remarks

TBC
7 Figures

**Figure 1. Public and private capital certification**

This figure shows the equilibrium of the model with public and private capital certification. Panel A exhibits equilibrium capital and Panel B equilibrium probabilities of failure. Solid (dashed) lines represent equilibrium values with public (private) certification.
Figure 2. Low flat capital requirements

This figure shows the equilibrium of the model with low flat capital requirements. Panel A exhibits equilibrium capital and Panel B equilibrium probabilities of failure. Solid (dashed) lines represent equilibrium values with low (no) flat capital requirements.
Figure 3. High flat capital requirements

This figure shows the equilibrium of the model with high flat capital regulation. Panel A exhibits equilibrium capital and Panel B equilibrium probabilities of failure. Solid (dashed) lines represent equilibrium values with high (no) flat capital requirements.
This figure shows the equilibrium of the model with low value-at-risk based capital regulation. Panel A exhibits equilibrium capital and Panel B equilibrium probabilities of failure. Solid (dashed) lines represent equilibrium values with low (no) value-at-risk based capital requirements.
Figure 5. High value-at-risk based capital requirements

This figure shows the equilibrium of the model with high value-at-risk based capital regulation. Panel A exhibits equilibrium capital and Panel B equilibrium probabilities of failure. Solid (dashed) lines represent equilibrium values with high (no) value-at-risk based capital requirements.
Figure 6. Changing funding costs with flat capital requirements

This figure shows the effect of changes in funding costs under flat capital requirements. Panel A exhibits equilibrium capital and Panel B equilibrium probabilities of failure. Dotted (dashed) lines represent equilibrium values with a higher excess cost of capital (risk-free rate). Bold lines represent the equilibrium before these changes.
Figure 7. Changing funding costs with value-at-risk based capital requirements

This figure shows the effect of changes in funding costs under value-at-risk based capital requirements. Panel A exhibits equilibrium capital and Panel B equilibrium probabilities of failure. Dotted (dashed) lines represent equilibrium values with a higher excess cost of capital (risk-free rate). Bold lines represent the equilibrium before these changes.
Figure 8. Optimal flat and value-at-risk based capital requirements

This figure shows the equilibrium of the model with optimal flat and value-at-risk based capital requirements. Panel A exhibits equilibrium capital and Panel B equilibrium probabilities of failure. Bold (dashed) lines represent equilibrium values with the optimal value-at-risk based (flat) capital requirements.
Figure 9. Welfare difference between optimal value-at-risk based and flat capital requirements

This figure shows the welfare difference between the optimal value-at-risk based and flat capital requirements corresponding to lending to the different types of entrepreneurs.
This figure shows the equilibrium of the model with optimal unconstrained capital requirements. Panel A exhibits equilibrium capital and Panel B equilibrium probabilities of failure.
Figure 11. Changing funding costs with optimal capital requirements

This figure shows the effect of changes in funding costs under optimal unconstrained capital requirements. Panel A exhibits equilibrium capital and Panel B equilibrium probabilities of failure. Dotted (dashed) lines represent equilibrium values with a higher excess cost of capital (risk-free rate). Bold lines represent the equilibrium before these changes.
Figure 12. Low flat capital requirements with deposit insurance

This figure shows the equilibrium of the model with low flat capital requirements and deposit insurance. Panel A exhibits equilibrium capital and Panel B equilibrium probabilities of failure. Solid (dashed) lines represent equilibrium values with low (no) flat capital requirements.
This figure shows the equilibrium of the model with high flat capital requirements and deposit insurance. Panel A exhibits equilibrium capital and Panel B equilibrium probabilities of failure. Solid (dashed) lines represent equilibrium values with high (no) flat capital requirements.
Figure 14. Low value-at-risk based capital requirements with deposit insurance

This figure shows the equilibrium of the model with low value-at-risk based capital requirements and deposit insurance. Panel A exhibits equilibrium capital and Panel B equilibrium probabilities of failure. Solid (dashed) lines represent equilibrium values with low (no) value-at-risk based capital requirements.
Figure 15. High value-at-risk based capital requirements with deposit insurance

This figure shows the equilibrium of the model with high value-at-risk based capital requirements and deposit insurance. Panel A exhibits equilibrium capital and Panel B equilibrium probabilities of failure. Solid (dashed) lines represent equilibrium values with high (no) value-at-risk based capital requirements.
Appendix

Proof of Proposition 1 Suppose that the equilibrium monitoring intensity \( m^*_p \) satisfies \( m^*_p \in (0, p) \). Then, by the convexity of the monitoring cost function \( c(m_p) \), the bank’s incentive compatibility constraint (7) reduces to the first-order condition (12).

To show that in this case the investors’ participation constraint (9) is binding, note that if it were not we could slightly reduce the borrowing rate \( B^*_p \) and the loan rate \( R^*_p \) so that (12) would hold for the same \( m^*_p \), in which case the shareholders’ participation constraint (8) would still be satisfied, which contradicts the definition of equilibrium. To show that the shareholders’ participation constraint (8) is also binding, note that if it were not we could slightly increase the bank’s capital \( k^*_p \) and reduce the loan rate \( R^*_p \) so that (12) would hold for the same \( m^*_p \), in which case the investors’ participation constraint (9) would still be satisfied, which contradicts the definition of equilibrium.

Solving for \( B^*_p \) in the investors’ participation constraint (9) (written as an equality), substituting it into the first-order condition (12), and rearranging gives (14). Adding up the two participation constraints (8) and (9) (written as equalities) and rearranging gives

\[
R^*_p = \frac{R_0 + (\delta + \eta)k^*_p + c(m^*_p)}{1 - p + m^*_p}.
\] (18)

Putting together (14) and (18) implies

\[
\frac{(1 - k^*_p)R_0}{1 - p + m^*_p} + c'(m^*_p) = \frac{R_0 + (\delta + \eta)k^*_p + c(m^*_p)}{1 - p + m^*_p}.
\]

This equation shows the combinations of capital \( k^*_p \) and monitoring \( m^*_p \) that satisfy the incentive compatibility constraint (7) and the participation constraints (8) and (9) for the case where \( 0 < m^*_p < p \). Solving for \( k^*_p \) in this equation gives (13). By the properties of the monitoring cost function \( c(m_p) \) we have \( c'(m^*_p) = c(m^*_p) = 0 \) for \( m^*_p = 0 \), and \( m^*_pc'(m^*_p) > c(m^*_p) \) for \( m^*_p > 0 \). This implies that \( m^*_p > 0 \) if and only if \( k^*_p > 0 \).

The equilibrium loan rate \( R^*_p \) is given by

\[
R^*_p = \min_{m_p,k_p} \left[ \frac{(1 - k_p)R_0}{1 - p + m_p} + c'(m_p) \right]
\] (19)
subject to (13). The first-order condition that characterizes the solution to this problem is

$$\frac{dR_p^*}{dm_p^*} = -\frac{(1 - k_p^*)R_0}{(1 - p + m_p^*)^2} + \frac{(\delta + \eta)c''(m_p^*)}{R_0 + \delta + \eta} = 0.$$  \hspace{1cm} (20)

The second-order condition is

$$\frac{d^2R_p^*}{d(m_p^*)^2} = \frac{R_0}{(1 - p + m_p^*)^3} \left[ \frac{(1 - p + m_p^*)^2c''(m_p^*)}{R_0 + \delta + \eta} + 2(1 - k_p^*) \right] + \frac{(\delta + \eta)c'''(m_p^*)}{R_0 + \delta + \eta} > 0,$$

which holds by our assumptions on the monitoring cost function $c(m_p)$.

Notice that the first-order condition (20) implies

$$\left. \frac{dR_p^*}{dm_p^*} \right|_{m_p = k_p = 0} = -\frac{R_0}{(1 - p)^2} + \frac{(\delta + \eta)c'(0)}{R_0 + \delta + \eta} < 0$$

if and only if $p > \hat{p}$, where $\hat{p}$ is defined in (11). From here it follows that riskier entrepreneurs of types $p > \hat{p}$ will borrow from (monitoring) banks, while safer entrepreneurs of types $p \leq \hat{p}$ will borrow from the market. Note that $\hat{p} < 1$, and since $\hat{p}$ is increasing in $\eta$ assumption (10) ensures that $\hat{p} > 0$.

It only remains to show that $m_p^* < p$. Since $m_p^* = 0$ for $p = \hat{p}$, it suffices to show that $dm_p^*/dp < 1$ for $p > \hat{p}$. Differentiating the first-order condition (20) taking into account (13) gives

$$\frac{dm_p^*}{dp} = \frac{2(\delta + \eta)(1 - p + m_p^*)c''(m_p^*) + R_0c'(m_p^*)}{(\delta + \eta) \left[ 2(1 - p + m_p^*)c''(m_p^*) + (1 - p + m_p^*)^2c'''(m_p^*) \right] + R_0(1 - p + m_p^*)c''(m_p^*)},$$

which implies the result given that $c''(m_p^*) > 0$, $c'''(m_p^*) > 0$, and $m_p^*c''(m_p^*) > c'(m_p^*)$ by the properties of the monitoring cost function $c(m_p)$. □
References


