Bank resolution and the mutualization of the public backstop in a banking union

Anatoli Segura                Sergio Vicente
Banca d’Italia                Universidad Carlos III de Madrid
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Abstract

We develop a two-country model in which domestic bank failures may lead to sovereign crises. The resolution of a failing bank involves either the bail-out of its creditors or their bail-in, which can generate local contagion costs that are private information of domestic authorities. The possibility that the bail-out of a bank leads to a sovereign crisis gives risk-sharing motives to mutualize public backstops. Yet, mutualization distorts bank resolution decisions due to the informational asymmetry on bail-in costs. We study the aggregate welfare maximizing bank resolution and public backstop mutualization decisions in this framework. We show that under the optimal banking union the probability of bailing out a failing bank can be either larger or lower than in the first-best. We also find that the probability of sovereign crises may increase in a banking union relative to that in autarky. We extend the model to analyze the interaction between direct and indirect forms of foreign support, the effect of country asymmetry, and the need of stability enhancing fiscal rules.

JEL Classification: G01, G20, G28
Keywords: banking union, bail-in, bail-out, public backstop, mechanism design

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1 Introduction

At the peak of the euro area sovereign debt crisis in June 2012, area members agreed on the creation of a banking union to “break the vicious circle between banks and sovereigns” (Area (2012)). Two of the three pillars of the banking union project were indeed specifically designed to accomplish this objective: the enhancement and centralization of resolution powers (Pillar II of the banking union), and the establishment of a common fiscal backstop to the banking sector during systemic crises (Pillar III).¹ Since then a number of steps have been taken towards their completion.

By the end of 2015, euro area members had transposed into national law the Bank Recovery and Resolution Directive (BRRD, 2014), which provides a common framework for the resolution of failing banks and confers authorities more powers to impose losses on private stakeholders (the so called bail-in tools). In addition, since January 2016 resolution actions are a prerogative of a newly created supranational authority, the Single Resolution Board (SRB). Although this authority has the ultimate say in bank resolution, it operates in strict cooperation with domestic resolution authorities which, at least for a prolonged transitional period, may have better access to information or better knowledge of the domestic legal system. These progresses on Pillar II of the banking union are expected to weaken the banks-sovereign nexus by reducing the likelihood that banks’ problems lead to publicly funded government bail-outs.

Public support is nevertheless allowed when authorities consider private burden sharing would endanger financial stability.² In those cases, member countries have agreed as part of Pillar III of the union that a fraction of the required public funds could be mutualized if the country in which the failing bank is located is experiencing important financial distress.³ Yet, in its recent assessment

¹Pillar I of the banking union instead consisted in the centralization of the supervision of large banks. The role of central supervisor has been assigned to the ECB, which performs it since November 2014
²Even in this case the BRRD imposes strong conditions before public support could be granted. In particular, a minimum private burden-sharing of 8% of total liabilities and a contribution of the privately funded Single Resolution Fund of 5% of total liabilities is necessary before any public aid.
³The mutualization of the public backstop is to be provided by the European Stability Mechanism (ESM), a new
of the progress on the euro area banking union, the IMF has warned that the current progress on
Pillar III falls short of providing sufficient risk-sharing and is likely not to be enough to break the
banks-sovereign nexus (IMF, 2016). 4

How does the presence of better informed domestic authorities affect the capability of a banking
union to sever the banks-sovereign nexus and the frequency of bail-outs and of sovereign crisis?
This paper develops a stylized framework to address these issues. To the best of our knowledge the
paper constitutes the first study of the interaction between Pillars II and III of the euro area banking
union. We study the optimal design of a banking union between countries that can experience simulta-
naneous bank and sovereign crises in an environment characterized by informational asymmetries
between domestic and central resolution authorities regarding the need to provide public support
to institutions in distress. We show how the informational friction affects the decision to bail-in
or bail-out the creditors of a failing bank and the contribution of union members to the funding
of the public backstop. Finally, our analysis highlight the importance of providing the SRB with
the necessary resources to minimize any information or knowledge gap with domestic resolution
authorities as a prerequisite for further progress on the mutualization of the fiscal backstop.

We consider a two-date, two-country economy with one bank and a resolution authority in each
country. At the initial date each bank issues debt to local investors to fund a project. If the project
fails at the final date the bank is not able to repay its debt and it has to be resolved in one out of
two ways: a bail-out or a bail-in. When the bank is bailed-out the resolution authority raises funds
to repay the bank’s creditors. Obtaining these funds is socially costly, as it creates distortions in the
economy. We consider a setting in which a country may be fragile at the final date, so that raising
funds to bail-out the bank may lead to a sovereign crisis, a situation in which the cost of public

4 IMF (2016) also identifies the creation of a common deposit insurance fund as an essential element to complete
the original objective of the banking union, which is currently missing.
funds is particularly high. When the bank is bailed-in, its creditors do not obtain any support from the public and suffer losses. We assume that the bail-in of the bank creditors generates some random domestic contagion costs whose value is realized right before the bank resolution and better appraised by the home authority of the failing bank.

In autarky, that is, when countries resolve banks on their own, the home authority optimally decides to bail-in the creditors of a failing bank when the associated contagion costs are lower than the domestic costs of obtaining the funding for a bail-out. A fragile country may sometimes find it optimal to resolve a troubled bank through a bail-in even when that decision leads to substantial contagion costs to avoid the even worse outcome of a sovereign crisis triggered by a bail-out. In those cases, the resolution authority is constrained to choose between the “lesser of two evils,” and the recourse to the other country’s funds could reduce the overall resolution costs. This gives a rationale for the creation of a banking union in which stable countries contribute to the funding of bank bail-outs in fragile countries. Crucially, optimal resolution policies must take into account that the mutualization of the public backstop gives the home authority incentives to overstate the need of a bail-out.

Formally, we solve a mechanism-design problem in which a central resolution authority (the principal) that aims at maximizing aggregate welfare sets the resolution and public backstop mutualization decisions to be taken for each possible bail-in contagion cost reported by the home authority (the agent). We show that resolution mechanisms can be implemented in a simple manner: the central authority fixes each country’s contribution to the funding of bail-outs and delegates the resolution decision to the home authority that is better informed. The optimal home contribution to bail-outs trades off two considerations. On the one hand, high local contributions curb the home authority’s incentives to overstate the magnitude of bail-in contagion costs so as to convince the central authority of the convenience of a bail-out partially paid by the other country. On the other hand, low home contributions reduce the probability that the fragile country experiences a
sovereign crisis and reduce the overall social cost of bail-outs whenever they are conducted.

The analysis of the optimal resolution policy in a banking union yields three main results. First, although the presence of asymmetric information limits the amount of risk-sharing across sovereigns, it does not completely eliminate it. As a result, the social costs associated with resolutions, which is a natural measure of the strength of the banks-sovereign nexus in our model, get reduced relative to the case in which countries act in isolation.

Second, the foreign contribution to the public backstop reduces the cost of bail-outs for the home country, which increases their probability relative to autarky. Interestingly, such probability may be larger in the presence of asymmetric information than in the first-best even though the informational friction reduces the level of risk-sharing. The reason is that the optimal home country’s contribution to the funding of bail-outs balances the need to curb the local authorities preference for bail-outs and the objective to reduce the social cost of bail-outs when they are conducted. For some distributions of the bail-in contagion costs the latter effect becomes relatively more important, which results in a small optimal home contribution to the public backstop that leads to more bail-outs than in the first-best. We show in fact that this is the case when the mass of the bail-in contagion costs distribution is concentrated at its tails.

Finally, overall welfare maximization requires that the expected social costs associated with the resolution of failing banks in fragile countries be minimized. In the absence of informational frictions, this goal is achieved by demanding a maximal contribution from the stable country to the funding of bail-outs, which in turn brings the probability of sovereign crises down to zero. Hence, there is a perfect alignment between reducing expected resolution costs and minimizing the occurrence of sovereign crises. This is not any more the case when there is asymmetric information because of the double role played by the home contribution to the public backstop. In fact, we provide conditions on the bail-in cost distribution such that the optimal resolution policy in a banking union increases the probability of sovereign crises relative to autarky. Informational frictions thus
induce a wedge between weakening the strength of the banks-sovereign nexus and reducing the probability of sovereign crises.

In the baseline model we have considered risk-sharing schemes in which foreign financial aid consists in disbursements to the debtholders of the fragile country’s failing bank. This can be interpreted as the use of the direct recapitalization instrument of the European Stability Mechanism. In a first extension of the model we also allow the foreign country to grant a loan to the fragile sovereign that is used by the latter to contribute to the funding of the bail-out. Extend the model in this section to allow for indirect recapitalization mechanisms in which the foreign country grants a loan to the fragile sovereign that is used by the latter to contribute to the funding of its bank bail-out. The bridge loan is repaid in a future date in which the sovereign has regained stability. This mechanism resembles the indirect recapitalization instrument of the ESM. The main intuition driving the results of this extension is that indirect recapitalization leads to a double raising of public funds, by the lender to grant the loan and by the borrower to repay it, that also doubles social costs. As a result, we find that the optimal resolution scheme makes always use of some positive amount of home contribution and direct recapitalization. This result suggests that the direct recapitalization instrument of the ESM should not be considered a last resource option.

The mutualization of the public backstop for bank resolution leads to cross-country transfers of funds. When, as in our baseline model, countries are initially identical, these transfers offset each other from an ex-ante perspective and the countries find it optimal at the initial date to create a banking union with ex-post optimal resolution policies. Yet, countries integrating a banking union may not be identical. For instance, in the euro area there is substantial heterogeneity across countries both on the fundamentals of their public finances and on their domestic banking sectors, as some of them still carry unresolved legacy problems from the past crisis. In an extension of the model we show that second-best bank resolution policies are not implementable when countries are sufficiently asymmetric ex-ante, since the country with stronger sovereign or banking sector
would end up making net transfers to weaker countries that overcome the gains from more efficient bank resolution decisions. We also show that the limits on risk-sharing imposed by ensuring the participation of the stronger country can be partially mitigated by setting bank resolution policies that depend on the ex-ante characteristics of the countries. For example, the optimal contribution from a stable foreign country to the bail-out of a failing bank in a fragile country is smaller for the country with stronger ex-ante fundamentals.

In a final extension we study how the presence of a mutualized public backstop affects the countries’ incentives to take costly actions that improve the state of their public finances and reduce their probability of being fragile in the future. We find that when these decisions are taken in a decentralized manner there is a free-riding fiscal problem that reduces the welfare gains from the creation of a banking union. Similarly as with the creation of common economic or monetary areas, this coordination problem could be fixed by establishing rules on the state of public finances that the countries would be required to satisfy to participate in a banking union. Finally, we show that these rules are stricter in a banking union subject to informational frictions.

1.1 Related Literature

This paper contributes to an emerging literature on the banking union in the euro area. Colliard (2014), Carletti, Dell’Ariccia, and Marquez (2016), Calzolari, Colliard, and Lóránt (2016) address this issue and show that national authorities may not acquire or provide all the relevant information necessary for an appropriate bank resolution. A common thread among these papers is that centralizing banks supervision entails a trade-off between a less biased but also less informed supranational supervisor. In an early contribution, Holthausen and Rønde (2004) do also assume that there is a misalignment between national and supranational authorities that distorts information transmission. Our paper is, to the best of our knowledge, the first one to analyze how the mutualization of a public backstop affects national authorities’ incentives to reveal all their relevant information to the
central resolution authority, and its implications for risk-sharing among members of the banking union.

A closely related set of papers has dealt with the regulation and supervision of multinational banks. Calzolari and Lóránt (2011) study the incentives of regulators to monitor multinational banks taking into account the incentives faced by banks to expand abroad and their decision to do it through branches or subsidiaries. Górnicka and Zoican (2016) argue that cross-country spillovers make bailouts more attractive to supranational authorities than to national agencies. In addition, Faia and Weder (2016) and Bolton and Oehmke (2015) assess the convenience of establishing “Single Point of Entry” or “Multiple Points of Entry” bank resolution architectures. Our analysis complements these studies by exploring the supranational aspects of bank resolution where the benefits to form a banking union do not only arise from cross-border spillovers, but also through the provision of a common fiscal backstop to the banking sector in countries that may potentially be in financial distress upon bailing-out a troubled bank. In this sense, our paper is the first one to study banking unions as insurance providers.

At the heart of our paper is the fact that bailing out a bank in distress requires a large amount of public funds that may damage the sovereign public finances if funded by a single country. For instance, Schoenmaker (2016) questions the financial capacity of small and medium-sized countries to provide a fiscal backstop of their own to a failing global bank. This feedback loop between bank resolution and sovereign risk has been documented by several papers. For instance, Acharya, Drechsler, and Schnabl (2014) show that bailouts triggered an increase of sovereign risk in 2007 crisis. Alter and Schüler (2012) provide evidence on the interconnection between banks and sovereign risk. We also contribute to the literature linking fiscal capacity and bank regulation. Stavrakeva (2015) shows that capital requirements should be lower in countries with larger fiscal capacity in a framework in which banks can be bailed out with resources levied from consumers through distortionary taxation. Our paper differs from hers in that our focus is on the optimal ex-post
resolution mechanism instead of on ex-ante regulation.

Several papers have analyzed the sovereign-bank nexus created by banks’ holdings of their sovereign debt. For instance, Popov and Van Horen (2015), Acharya, Eisert, Eufinger, and Hirsch (2016), Altavilla, Pagano, and Simonelli (2016) and De Marco (2016) find that the sovereign debt crises had real effects through credit rationing to borrowing firms. Brunnermeier, Garicano, Lane, Pagano, Reis, Santos, Thesmar, Van Nieuwerburgh, and Vayanos (2016) advocate for ESBies as a way of breaking the transmission of sovereign troubles to banks’ balance sheets. While we abstract from banks’ holdings of sovereign debt in our setup, we show that sovereign distress may damage the banking system through the advent of costly bail-in resolutions.

Our paper is also related to the literature on rules versus discretion on bank resolution. Walther and White (2015) show that clear-cut resolution rules that tie regulators’ hands may improve upon discretionary measures, for these may impose too soft standards so as to avoid investors’ runs. Related to this, there is an emerging literature on the political economy of bank bail-outs. For instance, Liu and Ngo (2014) find that bail-outs has historically been less likely in the US as election approached. In a similar study of German banks, Behn, Haselmann, Kick, and Vig (2015) find that local politicians are less likely to bail out banks as elections approach. These paper sheds light on some of the potential distortions from conferring bank resolution prerogatives to authorities that are too close to banks.

One of the aspects that our study highlights is that heterogeneity across jurisdictions restricts the extent of risk-sharing within banking unions, so that the benefits of banking unions increase with the homogeneity of the constituents. Similarly Dell’Ariccia and Marquez (2006) and Beck, Todorov, and Wagner (2013) find that the benefits of supranational regulation increase with country homogeneity. In the same vein, our paper also relates to an incipient literature that investigates the benefits of countries engaging into ex-ante risk-sharing agreements. As in Tirole (2015), we find that asymmetries across countries’ sovereign risk limits the extent of solidarity across countries.
2 The baseline model

We consider a two-identical-country–i ∈ \{1, 2\}–two-date–t ∈ \{0, 1\}–economy where all agents are risk-neutral and have a zero discount factor.\(^5\) In each country there is a bank, a continuum of atomistic investors and a local bank resolution authority. This authority has resolution powers on the country’s bank and the capability to raise funds from domestic agents at t = 1 so as to implement its initiatives. In addition, the domestic resolution authorities may agree at the initial date to create a banking union to transfer their prerogatives to a central resolution authority. We refer to the bank and local authority in country i as bank \(i\) and authority \(i\), respectively, and we use the letter \(j\) to refer to the other country.

At t = 0, each bank has access to a positive NPV project that requires an outlay of 1 unit. Banks raise these funds from local investors, issuing debt claims with face value \(D_i\). The project’s payoff at t = 1 is \(R > 0\), if successful, and 0 otherwise. The probability that the project succeeds is \(p\).

If a bank’s project fails at t = 1, the bank cannot repay its debt. In this case, the competent resolution authority must resolve the bank. There are two types of intervention: a bail-out \((k = 1)\) and a bail-in \((k = 0)\). A bank bail-out consists of the repayment of the unit of principal investment to the bank’s debtholders.\(^6\) Bail-outs are funded with public resources. Local resolution authorities can only levy resources from their countries. If a banking union is created, the central resolution authority can raise funds from either country. The net social cost of public funds in each country depends on whether the country is stable \((s = S)\) or fragile \((s = F)\). In an stable country this cost is \(\lambda_L > 0\) for certain. In a fragile country the funding of a bail-out may lead to a sovereign crisis,

\(^5\)In Section 5.2 we analyze the case in which countries are not identical at the initial date.

\(^6\)We make this assumption for simplicity, as it renders the resolution decisions and expected social costs of resolution independent of the promised repayment \(D_i\). Alternatively, we could consider that a bail-out consists of the repayment of the debt face value \(D_i\). Such assumption would introduce a dependence of the cost of bail-outs on bail-out expectations, which would complicate the analysis without adding further insight.
in which case the cost of funds increases from $\lambda_L$ to $\lambda_H$, where $\lambda_H > \lambda_L$. More precisely, if a fragile country raises $x$ units of funds their social cost is $\lambda_H$ with probability $\mu x$, and $\lambda_L$ with probability $1 - \mu x$, where $\mu \in (0, 1]$ is a parameter that captures the likelihood that the public funds raised to bail-out the bank may trigger a sovereign debt crisis. The state of each of the countries at $t = 1$ is verifiable. We denote $q^{s_1, s_2}$ the probability that the pair of sovereign states $(s_1, s_2)$ is realized at $t = 1$. We let $q^s_t \equiv q^{s, S} + q^{s, F}$ stand for the probability that country 1 is in state $s \in \{S, F\}$ at $t = 1$, and analogously for country 2. Let us note that our symmetry assumption requires that $q^s_1 = q^s_2$, which is equivalent to $q^{S, F} = q^{F, S}$. Finally, we assume that the state of the sovereigns is not perfectly correlated, that is $q^{S, F} > 0$ $(\Leftrightarrow q^{F, S} > 0)$.

When a troubled bank is bailed-in its debtholders suffer losses. We assume that private loss-absorption may disrupt the rest of the financial sector and create some local deadweight contagion costs $c$. As of $t = 0$, $c$ is a random variable with an associated cumulative distribution function $G$ with support $[0, c_{\text{max}}]$, where $c_{\text{max}} > \lambda_H$, and strictly positive density function $g(c) > 0$ in the whole of its support. The bail-in cost $c$ is realized at $t = 1$, right after the bank’s project fails, but before the resolution decision is taken, and is private information of the home authority. This is the main informational friction of the model. In order to reduce the number of cases to analyze, we make the following assumption:

Assumption 1 $\lambda_L > \int_0^{c_{\text{max}}} cdG(c)$.

The assumption asserts that bailing out a bank is (unconditionally) inefficient with the information available at $t = 0$, even if the net social cost of public funds raised to finance the bail-out is $\lambda_L$. This assumption is consistent with the stated objective of the Bank Recovery and Resolution Directive introduced in the EU that attempts to make bail-ins become the norm and bail-outs the exception in bank resolutions.

Domestic authorities take resolution decisions in order to maximize the aggregate utility of
the agents in their respective jurisdictions. Since the resolution of a bank in a fragile country is hampered by the possibility of a sovereign crisis if public support is granted, the local authorities may find it convenient to provide each other with some insurance by agreeing at the initial date to mutualize (to some extent) the funding of a potential public support to failing banks. More precisely, we assume that at $t = 0$ local authorities have the possibility of creating a banking union with the authority to resolve failing banks at $t = 1$. Such arrangement establishes a resolution mechanism to decide whether to bail-out or to bail-in a failing bank based on the bail-in disruption cost revealed by its local authority, as well as the contributions of each country to fund a bail-out if it is conducted.

The mechanism is contingent on both the state $(s_1, s_2)$ at $t = 1$ and the location of the failing bank. Indeed, since there are no gains from public backstop mutualization when the failing bank is located in a stable country, we can assume without loss of generality that a banking union delegates the resolution of bank failures and its funding to home authorities in these contingencies. Moreover, for simplicity we assume that there are political economy constraints that make it unfeasible that a fragile country raises funds to contribute to the bail-out of the other country’s bank. We therefore assume that the resolution of a failing bank and its funding in the state $(F, F)$ is delegated to the home authority. Once this assumption is made, the value of $q^{F,F}$ is irrelevant for our results. To simplify the expressions and the exposition further we assume that $q^{F,F} = 0$.\footnote{This assumption is just made for simplicity, as it shortens the relevant expressions. Notice that, instead of assuming that $q^{F,F} = 0$, we could have alternatively assumed that $q^{S,S} = 0$. However, in a symmetric setup, assuming that both $q^{S,S} = 0$ and $q^{F,F} = 0$ implies that $q^{S,F} = q^{F,S} = \frac{1}{2}$, which would be too restrictive an assumption.}

Formally, a banking union consists of a pair of resolution policies $\{k(\hat{c}), x(\hat{c})\}$ for a failing bank located in a fragile country when the other one is stable (i.e., for the failure of bank 1 in state $(F, S)$ and for the failure of bank 2 in state $(S, F)$). For each bail-in cost $\hat{c}$ reported by the home country, $k(\hat{c})$ is an indicator function that takes a value of 1 if and only if the resolution decision
is a bail-out. The variable $x(\tilde{c}) \in [0, 1]$ stands for the amount (or share) of funds provided by the home country in case of a bail-out—notice that there are no public funds involved when the bank’s debtholders are bailed in. We restrict ourselves to budget-balanced mechanisms, so that the foreign country contributes an amount $1 - x(\tilde{c})$ to the bail-out. For the knife-edge cases in which the home authority is indifferent between reporting a value of $c$ that leads to a bail-out and another one leading to a bail-in, we adopt the convention that it chooses to report the former.\footnote{Without this convention, there could be other incentive compatible schemes on top of those described in Lemma 1 in which the close interval is open on its left boundary. Nonetheless, these other incentive compatible schemes are ex-ante payoff equivalent to those described in Lemma 1.}

3 Autarky and first-best banking union benchmarks

We first analyze the following two benchmarks. First, we consider the situation in which countries cannot form a banking union, which we refer to as autarky. Second, we consider the case in which the bail-in disruption costs are public information, which leads to the first-best banking union. These two benchmarks allow us to assess the potential of a banking union to reduce the banksovereign nexus, as well as the extent to which the incentives of domestic authorities to misreport their private information limits such reduction. We will also focus on two other variables associated with the resolution process: frequency of bail-outs and probability of sovereign crises.

3.1 Bank resolution in autarky

In this section we assume that each country resolves bank failures without any reliance on funds from the other country. Consider first the situation in which the country is stable. A bail-out amounts to a redistribution of funds within local agents at a (net) social cost of $\lambda_L \cdot 1$. Hence, after observing the bail-in cost $c$, the home authority decides to bail-out the bank if and only if $c \geq \lambda_L$.\footnote{The results obtained in the paper generalize to a situation in which we allow for partial bail-outs, that is, when we allow for $k(c) \in [0, 1]$.}
The expected social cost of a bank resolution in a stable country is thus given by:

$$
\Pi^S = \int_0^{\lambda_L} c dG(c) + (1 - G(\lambda_L)) \lambda_L. 
$$ (1)

If the country is instead fragile, the home authority decides to bail-out the bank if the bail-in cost is higher than the expected cost of one unit of home funds, i.e., if and only if $c \geq \bar{\lambda} \equiv \mu \lambda_H + (1 - \mu) \lambda_L$. The expected social cost $\Pi^F$ of a bank resolution in a fragile country is then given by an expression analogous to that in (1) in which $\lambda_L$ is replaced with $\bar{\lambda}$. The expected cost difference of a bank resolution between the two states is therefore given by:

$$
\Pi^F - \Pi^S = \int_{\lambda_L}^{\bar{\lambda}} (c - \lambda_L) dG(c) + [1 - G(\bar{\lambda})] \mu (\lambda_H - \lambda_L). 
$$ (2)

When the bail-in costs are small ($c < \lambda_L$), banks are bailed-in regardless of the shadow cost of funds. Therefore there is no cost difference between a stable and a fragile country. The first term in expression (2) captures the cost difference between a bail-in and a bail-out financed by a stable country in the range of bail-in costs in which the optimal (autarky) policy is to bail-out the bank if and only if the country is stable (i.e., if $\lambda_L \leq c < \bar{\lambda}$). The last term in expression (2) accounts for the cost difference from bailing out a bank in a fragile versus a stable country in the range in which the optimal (autarky) policy is to bail-out the bank regardless of the state of the country (i.e., if $c \geq \bar{\lambda}$). This difference arises due to the (expected) differential cost of public funds between the two states. Recognizing that $q_i^S + q_i^F = 1$, we can write the expected social cost of an autarkic bank resolution in a country at $t = 0$ as:

$$
\Pi_0^A = (1 - p)\Pi^S + (1 - p)q_i^F (\Pi^F - \Pi^S). 
$$ (3)

The expression above decomposes bank resolution costs in two terms. The first one captures the cost of resolution for a country that is always stable, and constitutes a lower bound on resolution costs. The second one accounts for the additional costs due to the possibility that the country is
fragile when its bank fails, which can be naturally interpreted in the context of this model as the measure of the importance of the banks-sovereign nexus.

Now, we proceed to the computation of the two remaining objects of interest. On the one hand, the probability at \( t = 0 \) that a troubled bank be bailed out at \( t = 1 \) is given by:

\[
\Pr[Bail-out]^A = (1 - p) \left[ q_i^S \left[ 1 - G \left( \lambda_L \right) \right] + q_i^F \left[ 1 - G \left( \bar{\lambda} \right) \right] \right].
\]  (4)

In addition, the probability at \( t = 0 \) that the country experiences a sovereign crisis at \( t = 1 \) is given by:

\[
\Pr[Sov Crisis]^A = (1 - p) q_i^F \mu \left[ 1 - G \left( \bar{\lambda} \right) \right].
\]  (5)

To conclude our analysis of the autarky case, taking into account that bank’s debt is priced competitively the expected utility of the agents in either country is given by:  

\[
U_0^A = pR - 1 - \frac{\Pi_0^A}{\text{Bank Res Cost}},
\]  (6)

which accounts for the project NPV minus the expected social costs of a bank resolution.

### 3.2 First-best banking union

We now turn to the analysis of the first-best banking union, which corresponds to the resolution decision of an aggregate welfare maximizing central resolution authority with recourse to the two countries’ funds and full information about bail-in costs. As argued above, we can restrict our attention to the case in which bank \( i \) fails at \( t = 1 \) when country \( i \) is fragile and country \( j \) is stable. The expected cost of public funds is lower in country \( j \). Since the central authority can raise funds

\[10\] The equilibrium promised repayment \( D^A \) required by the competitive debtholders anticipates the probability of a bail-out, and satisfies

\[ pD^A + (1 - p) \Pr[Bail-out]^A \cdot 1 = 1. \]

The particular value of such promised repayment does not have welfare implications and we henceforth omit it from our computations.
from either country, whenever a bail-out is conducted its funding will be provided by country $j$. The central authority will hence bail-out the bank if and only if $c \geq \lambda_L$. Hence, the resolution decision coincides with that taken by a stable country when dealing with the failure of its bank in autarky. The aggregate social costs of a bank resolution in either country at $t = 0$ under the first-best resolution policy are therefore given by:

$$\Pi_0^{FB} = (1 - p)\Pi^S.$$  

(7)

This expression shows that the first-best banking union eliminates the banks-sovereign nexus.

In the first-best case, the probability at $t = 0$ that a troubled bank be bailed out at $t = 1$ is given by:

$$\Pr[Bail - out]^{FB} = (1 - p) [1 - G (\lambda_L)],$$

while the probability at $t = 0$ that the country experiences a sovereign crisis at $t = 1$ is $\Pr[Sov Crisis]^{FB} = 0$. Hence, we have that:

$$\Pr[Bail - out]^{FB} > \Pr[Bail - out]^A,$$

and that

$$\Pr[Sov Crisis]^{FB} < \Pr[Sov Crisis]^A.$$

Risk-sharing across countries under perfect information thus increases the frequency of bail-outs and reduces the occurrence of sovereign crises.

After a little bit of algebra, we can write the expected utility of country $i$ at $t = 0$ as:

$$U_{0,i}^{FB} = pR - 1 - \underbrace{\Pi_0^{FB}}_{NPV} - \underbrace{(1 - p)}_{Bank Res Cost} - \underbrace{(1 - p) \left(q^E_j - q^E_i\right)}_{Subsidy} \underbrace{(1 + \lambda_L) (1 - G (\lambda_L))}. $$

(9)

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11 Notice that the fact that the first-best completely eliminates sovereign crises and the banks-sovereign nexus relies on the assumption that $q^{E,F} = 0$. If, alternatively, we had assumed that $q^{E,F} > 0$, then we would not have that those variables are not reduced down to zero but they would still be strictly reduced relative to autarky. Apart from this difference, all our results would remain valid with $q^{E,F} > 0$. 

16
Expression (9) is analogous to expression (6), but there are two differences. On the one hand, the bank resolution costs are strictly smaller, since the central authority can recourse to a stable country to rescue a troubled bank in a fragile country. On the other hand, there is an additional term that corresponds to the social costs of the expected subsidy from one country to the other. This term plays an important role in the case of asymmetric countries, as the larger the expected fragility of a country, the larger the expected subsidy that would receive from the neighboring country. Nonetheless, in the baseline model, where we have that $q_j^F = q_i^F$, the last term is zero. Hence, expression (9) can be written as (note that we have dropped supindex $i$):

$$U_{0}^{FB} = pR - 1 - \frac{\Pi_0^{FB}}{NPV}.$$

Comparing expressions (6) and (10), we have that:

$$U_{0}^{FB} - U_{0}^{A} = \Pi_0^{A} - \Pi_0^{FB} = (1 - p)q_i^F (\Pi^F - \Pi^S) > 0,$$

showing that a first-best banking union improves upon an autarky situation by eliminating the banks-sovereign nexus. Since the banking union increases the expected utility in the two countries, it is feasible at the initial date.

### 4 Equilibrium analysis

We now proceed with the analysis of the baseline model, in which home authorities have private information on the bail-in costs when their bank fails. Consider again the situation in which country $i$ is fragile and country $j$ is stable at $t = 1$. When a bank fails in country $i$, the central authority must decide whether to bail-out the bank and, if so, each country’s funding contribution to the repayment of the bank’s debtholders. Let us briefly describe the problem faced by the central authority. A least cost recourse to public funds prescribes that if the bank is bailed out the required funding should come entirely from country $j$ (cost of bail-outs effect). However, whether a bail-out
is preferable to a bail-in depends on the bail-in contagion cost that is privately observed by local authority \(i\). If bail-outs are entirely funded from country \(j\), authority \(i\) will overstate the bail-in costs to convince the central authority of the need of a bail-out that results in a transfer from country \(j\) (overstatement effect). The optimal resolution scheme designed by the central authority attempts to balance the two effects.

Formally, the central authority designs a resolution mechanism \(\{k(\hat{c}), x(\hat{c})\}\) such that for each \(\hat{c}\) reported by the local authority in country \(i\), \(k(\hat{c})\) takes the value 1 if and only if there is a bail-out and, if there is a bail-out, \(x(\hat{c}) \in [0,1]\) stands for the funds that country \(i\) must contribute with. By the virtue of the revelation principle, we can restrict to truth-telling mechanisms. We first analyze the restrictions that incentive compatibility imposes on the set of resolution schemes. Suppose that the true cost is \(c\). The expected utility of agents in country \(i\) from reporting \(\hat{c}\) is given by:

\[
U(\hat{c}, c) = \underbrace{k(\hat{c})}_{\text{Debtholders' payoff}} - \underbrace{(1 - k(\hat{c}))}_{\text{Bail-in Cost}} \underbrace{c - (1 + \lambda_L + (\lambda_H - \lambda_L)\mu x(\hat{c})) \cdot x(\hat{c})}_{\text{Cost Public Funds}}. \tag{11}
\]

The first term corresponds to the transfer that the bank’s debtholders receive through a bail-out. The second term stands for the bail-in costs. The last term includes the expected gross social cost of the funds country \(i\) has to provide for the resolution of its bank, which takes into account that in a fragile country raising public funds may lead to a sovereign crisis. Incentive compatibility of the resolution mechanism can then be written as:

\[
U(c, c) \geq U(\hat{c}, c) \text{ for all } c \text{ and } \hat{c} \in [0, c_{\text{max}}]. \tag{IC}
\]

The next lemma characterizes incentive compatible resolution mechanisms, some of which cannot be optimal, as we argue below.

**Lemma 1 (Incentive compatible resolution mechanisms)** A resolution mechanism for a failing bank in a fragile country is incentive compatible if and only if it belongs to one of the three following classes:
1. *Unconditional bail-in* \( k(c) = 0 \) for all \( c \in [0, c_{\text{max}}] \).

2. *Unconditional bail-out* There exists \( \bar{x} \in [0, 1] \) such that \( k(c) = 1 \) and \( x(c) = \bar{x} \) for all \( c \in [0, c_{\text{max}}] \).

3. *Threshold* There exists \( x \in (\underline{x}, 1] \) and \( \bar{c}(x) \in (0, c_{\text{max}}) \) such that \( k(c) = 1 \) if and only if \( c \geq \bar{c}(x) \), in which case \( x(c) = x \). Moreover, we have that

\[
\bar{c}(x) = (1 + \lambda_L + (\lambda_H - \lambda_L)\mu x) x - 1,
\]

(12)

where \( x \in (0, 1) \) is uniquely determined by the condition \( \bar{c}(\underline{x}) = 0 \).

The intuition for Lemma 1 is as follows. First, unconditional mechanisms, such as those described in items 1 and 2, disregard any information provided by the home country and lead to a predetermined resolution decision. As such, they are trivially incentive compatible. The threshold mechanism described on item 3 accounts for the mechanisms that make use of the revealed bail-in cost to determine the resolution policy, which can be characterized by the required home contribution \( x \in (\underline{x}, 1] \) to bail-outs. The bank is bailed-out if and only if the reported bail-in cost is above a certain threshold \( \bar{c}(x) \), which is increasing in the bail-out contribution \( x \) required to the home country. The bail-out threshold \( \bar{c}(x) \) is determined by an indifference condition between reporting values that lead to a bail-out and a bail-in from the perspective of the home country. Incentive compatibility is achieved because as the observed bail-in cost increases and such a resolution outcome becomes more disruptive, the home authority is more willing to make the required contribution \( x \) to the funding of bail-outs. Let us notice that, since a threshold mechanism only makes use of a binary partition of the private information of the home authority (i.e., whether \( c \) lies below or above \( \bar{c}(x) \)), the resolution mechanism can be implemented in a simple manner: the central authority fixes the home country contribution \( x \) to bail-outs and delegates the resolution decision to the home
Some of the incentive compatible mechanisms characterized in the previous lemma are not optimal. On the one hand, Assumption 1 implies that a resolution scheme that always leads to bailing out a failing bank is dominated by an unconditional bail-in resolution policy. On the other hand, since $c_{\text{max}} > \lambda_H$, an unconditional bail-in policy is dominated by a threshold mechanism in which bail-outs are entirely locally funded (i.e., such that $x = 1$). Such mechanism in fact induces the autarky resolution outcome.

We can hence focus on the determination of the optimal threshold mechanism or, equivalently, of the optimal home contribution to bail-outs. The expected aggregate social resolution cost associated with a mechanism with home contribution $x$ to bail-outs is given by:

$$\Pi(x) = \int_0^{\eta(x)} \left[ cdG(c) + \left(\lambda_L + (\lambda_H - \lambda_L) \mu x^2\right) \left[1 - G(\tau(x))\right]\right].$$

(13)

The first term captures the expected social cost induced by bailing-in the bank’s debtholders. The second term corresponds to the expected cost of funding a bail-out. Let us highlight that the latter is increasing in the contribution required to the home country due to its fragility. We also note that, by construction, we have that $\Pi(1) = \Pi^F$.

The central authority’s problem is simply given by:

$$\min_{x \in [\underline{x}, 1]} \Pi(x).$$

(14)

The following proposition establishes that the banking union induces some—but not full—risk-sharing in the case a bail-out is conducted.

**Proposition 1 (Optimal resolution mechanism)** The contribution $x^*$ provided by a fragile country to its bank’s bail-out in an optimal resolution scheme is smaller than in autarky but larger than in the first-best, that is, $1 = x^{\text{Aut}} > x^* > x^{FB} = 0$.

\footnote{It can be shown that the optimal incentive compatible mechanism would be of the type in Item 3 of Lemma 1 even if we allowed for partial bail-outs.}
Recall from the previous section that the home contribution of a fragile country to a bail-out in autarky is $x^{Aut} = 1$, while the first-best contribution is given by $x^{FB} = 0$. Proposition 1 states that asymmetric information reduces the amount of risk-sharing across countries in a banking union relative to first-best (i.e., $x^* > 0$) but does not completely eliminate it (i.e., $x^* < 1$). The first inequality has been proved in the text just before the proposition. To gain intuition about the second one, consider the marginal effect over the expected aggregate social cost of resolution $\Pi(\cdot)$ of an increase on the fragile home country’s contribution to a domestic bail-out $x$, which (after some algebra) is given by:

$$\Pi'(x) = 2(\lambda_H - \lambda_L) \mu x \left[ 1 - G(\tau(x)) \right] - (1 + \lambda_L) (1 - x) \bar{c}'(x) g(\tau(x)).$$

The first term accounts for the extensive margin effect and reflects the fact that a higher contribution by the home country increases the expected overall cost of bail-outs. This cost effect is positive and pushes down the optimal home contribution to bail-outs. The second term in turn accounts for the intensive margin effect associated with a reduction on the home authority incentives to overstate the need of a bail-out when its contribution to bail-outs increases. This overstatement effect is negative and pushes up the optimal home contribution to bail-outs.

The overstatement effect has implications on the aggregate cost of resolution to the extent that there is a wedge between the aggregate costs of a bail-out (which is given by $(\lambda_H - \lambda_L) \mu x^2 + \lambda_L$) and the part of this cost that is internalized by the home authority (which amounts to $\tau(x)$). In fact, the factor $(1 + \lambda_L) (1 - x)$ corresponds to the difference between the two. We have that the wedge is zero when $x = 1$, as the home country fully internalizes the cost of bail-outs in this case. As a result, we have that $\Pi'(1) > 0$, which proves that $x^* < 1$. Hence, the optimal resolution mechanism in a banking union provides some—albeit not full—risk-sharing despite the informational problems.

Finally, a derivation similar to that conducted at the end of Section 3.2, shows that the expected
utility at \( t = 0 \) of agents in either country under optimal resolution policies is given by:

\[
U_{0}^{BU} = pR - 1 - \underbrace{(1 - p)\Pi^{S}}_{\text{Minimum Res Cost}} - \underbrace{(1 - p)q_{i}^{F}(\Pi(x^{*}) - \Pi^{S})}_{\text{BU Banks-sovereign Nexus}}.
\]  

(16)

where \( x^{*} \) is the contribution of fragile home countries to bail-outs. Expression (16) decomposes the expected utility in country \( i \) as the NPV of its bank’s project from which the expected aggregate social cost of resolution is substracted. The latter in turn can be split into the resolution costs in a country that were always stable and those stemming from the potential fragility of the country and the resulting banks-sovereign nexus.

Comparing expressions (6) and (16), it follows that:

\[
U_{0}^{BU} - U_{0}^{Aut} = (1 - p)q_{i}^{F}(\Pi^{F} - \Pi(x^{*})) > 0,
\]

(17)

the last inequality following from \( \Pi^{F} = \Pi(1) > \Pi(x^{*}) \). Expression (17) shows that the utility of agents in either country increases due to the reduction in the aggregate social costs of the resolution of its bank when the country is fragile, or, in other words, due to the reduction in the strength of the banks-sovereign nexus. It also shows that the \( t = 1 \) optimal resolution policies are welfare maximizing for both countries from an ex-ante perspective and are thus feasible and optimal at the initial date.\(^{13}\)

In the next two subsections we analyze how two important variables associated with the resolution process in a banking union, namely the probability of bank bail-outs and of sovereign crises, compare to those in the autarky and the first-best benchmarks.

4.1 The frequency of bail-outs in a banking union

Let \( x^{*} \) be the home contribution of fragile countries to bail-outs in an optimal banking union. The probability as of \( t = 0 \) of a bail-out in either country is given by:

\[
\Pr[Bail - out]^{BU}(x^{*}) = (1 - p) \left[ q_{i}^{S}[1 - G(\lambda_{L})] + q_{i}^{F}[1 - G(\overline{\epsilon}(x^{*}))] \right].
\]

\(^{13}\)The extension in Section 5.2 shows that this is not necessarily true when countries are asymmetric.
Comparing this expression with equations (4) and (8) for the autarky and first-best cases, respectively, we observe that the frequency of bail-outs in the banking union relative to these benchmarks depends on how the bail-out threshold \( \overline{c}(x^*) \) compares to \( \overline{\lambda} \) and \( \lambda_L \), respectively.

From Proposition 1 we know that \( x^* < 1 \), which implies that \( \overline{c}(x^*) < \overline{c}(1) = \overline{\lambda} \), so that the risk-sharing provided by the banking union increases the frequency of bail-outs relative to autarky. The proposition also states that \( x^* > 0 \), which means that risk-sharing in the banking union with informational frictions is lower than in the first-best. However, this does not necessarily imply that the frequency of bail-outs is lower than in the first-best. In fact, we can prove that the opposite happens under some conditions. The following proposition establishes the comparison between the likelihood of bail-outs in an optimal banking union and that in autarky and first-best benchmarks:

**Proposition 2 (Likelihood of bail-outs)**

1. Bail-outs are more frequent in an optimal banking union with informational frictions than in autarky.

2. Let \( \kappa \equiv \frac{2(\lambda_H - \lambda_L)\mu x_L[1 - G(\overline{x})]}{[2(\lambda_H - \lambda_L)\mu x_L + 1 + \lambda L](1 + \lambda_L)} \) and \( x_L \in (0, 1) \) be uniquely defined by \( \overline{c}(x_L) = \lambda_L \). If \( (1 - x)g(\overline{c}(x)) < \kappa \) for all \( x \geq x_L \), then bail-outs in an optimal banking union with informational frictions are more frequent than in the first-best banking union.

The proposition 1 illustrates the non-trivial interplay between risk-sharing and the probability of a bail-out under asymmetric information. While the informational friction reduces the risk-sharing capability of a banking union with respect to the first-best, it may do so in a way that increases the frequency of bail-outs. In fact, the proposition provides sufficient conditions on the distribution of the bail-in contagion cost that ensure that this is the case.

We have argued above why bail-outs are more likely in a banking union than in autarky. The intuition for the second result is as follows. The optimal home contribution to bail-outs (and, consequently, the induced bail-out threshold) is shaped by the cost and overstatement effects. Consider the case in which the probability density function is sufficiently small in the threshold interval
\( \bar{c}(x) \in [\lambda_L, \bar{\lambda}] \) associated with home contributions belonging to the interval \( x \in [x_L, 1] \). In this case, the relative importance of the overstatement effect, which operates on the intensive margin, is uniformly small. As a result, the cost effect dominates on the whole of the interval \( x \in [x_L, 1] \). Consequently, an optimal resolution scheme must have \( x^* < x_L \) or, equivalently, \( \bar{c}(x^*) < \lambda_L \). The sufficient condition in the proposition sets an upper bound on the probability density function of the bail-in cost distribution in the mid-interval \( [\lambda_L, \bar{\lambda}] \), leaving complete freedom on the distribution on the tails of the support. The condition is satisfied when bail-in costs are likely to be either very mild (smooth bail-in) or very high (systemic contagion), but unlikely to create intermediate levels of disruption. Moreover, notice that Assumption 1 places no restriction on the distribution shape, but simply on its expectation. Hence, we may well have low-expectation large-tailed distributions for which bail-ins are unconditionally efficient but such that bail-outs are more likely in a banking union than in the first-best.

### 4.2 Probability of sovereign crises in a banking union

We have seen in Section 3.2 that the first-best banking union eliminates sovereign crises. This is because bank bail-outs in a fragile country are entirely financed by the stable foreign country. In the presence of asymmetric information, the need to require home contributions to the funding of bail-outs leads to a positive probability of sovereign crises. In fact, under an optimal resolution scheme with home contribution \( x^* \) the probability of a sovereign crisis in either country is given by:

\[
Pr[Sov Crisis]^{BU} = (1 - p)q_i F \mu x^* [1 - G(\bar{c}(x^*))].
\]  

(18)

A natural question that arises is whether sovereign crises could be more likely within a banking union than in autarky. The following expression, which follows immediately from comparing equations (5) and (18) establishes the grounds for comparison:

\[
Pr[Sov Crisis]^{BU} > Pr[Sov Crisis]^A \iff x^* [1 - G(\bar{c}(x^*))] > [1 - G(\bar{\lambda})].
\]  

(19)
There are two forces at work when comparing the probability of a sovereign crisis in a banking union and in autarky. On the one hand, conditional on there being a bail-out, the probability of a sovereign crisis in the banking union is reduced relative to autarky due to the financial aid from a stable country. This is reflected in expression (19) in the fact that $x^* < 1$. However, bail-outs are more likely in a banking union, as stated in Proposition 2, which is reflected in the fact that $\bar{c}(x^*) < \bar{\lambda}$. While the overall effect is ambiguous, the following proposition states sufficient conditions for the probability of sovereign crises to be larger in an optimal banking union than in autarky. For the sake of completeness, the proposition also states the comparison between the probability of sovereign crises in an optimal banking union and in the first-best.

Proposition 3 (Probability of sovereign crises) 1. Suppose there exists $x' \in (x, 1)$ such that (i) the pdf of the bail-in cost satisfies $(1 - x)g(\bar{c}(x)) < \kappa(x')$ for all $x \geq x'$ with $\kappa(x') \equiv \frac{2(\lambda_H - \lambda_L)ux'}{(2\lambda_H - \lambda_L)ux' + 1 + \lambda_L(1 + \lambda_L)}$; and (ii) $P_r[\bar{c}(x') \leq c \leq \bar{\lambda}] > \frac{1 - 2}{x} P_r[c \geq \bar{\lambda}]$. Then, the probability of sovereign crises induced by any optimal resolution mechanism is larger than in autarky.

2. The probability of sovereign crises induced by any optimal resolution mechanism is larger than in the first-best.

The proposition provides sufficient conditions for the probability of sovereign crises to augment when countries create a banking union. Let us highlight that the objective of the banking union is not to minimize the probability of sovereign crises (which, in the model, are always triggered by the public support to a failing bank) but to minimize the expected social costs associated to bank resolutions. While in the absence of informational friction the two objectives coincide, this is not the case when there are information asymmetries. The reason is that the central authority has to

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14 It is also possible to provide sufficient conditions on the bail-in cost distribution that ensure that the probability of sovereign crises induced by any optimal resolution scheme is lower than in autarky.
require some home contributions to the funding of bail-outs even if this could lead to a sovereign crisis so as to reduce the incentives of the home country to overstate bail-in costs.

4.3 Numerical illustration of the results

In this section, we illustrate numerically the results in Lemma 1 and Propositions 1, 2 and 3. We fix the values for the net social costs of public funds in normal times and in a sovereign crisis to \( \lambda_L = 0.2 \) and \( \lambda_H = 0.9 \), respectively, and the fragility parameter to \( \mu = 1 \). We let the distribution of bail-in contagion costs follow a Beta distribution with support \([0,1]\). The Beta distribution is characterized by its (shape) parameters \( \alpha, \beta > 0 \). Its expected value is given by \( E[c|\alpha, \beta] = \frac{1}{1+\beta/\alpha} \).

Throughout this section we fix the ratio \( \beta/\alpha = 17/13 \), so that \( E[c|\alpha, \beta] = 0.15 \), and vary \( \alpha \) so as to analyze the effect of the distribution shape on the optimal policies. This parameterization ensures that Assumption 1 is met.

By defining \( z \equiv 1/\alpha \), it follows that \( \beta = z \left( \frac{1}{0.15} - 1 \right) \), so that we can write the associated probability density function of \( c \) as follows:

\[
g_z(c) = \frac{1}{B(1/z, z \left( \frac{1}{0.15} - 1 \right))} c^{\frac{1}{15}-1} (1 - c)^{z \left( \frac{1}{0.15} - 1 \right)} (-1),
\]

where \( B(1/z, z \left( \frac{1}{0.15} - 1 \right)) \) is the Beta function.
Figure 1: Probability density function of the bail-in cost for expected value $\frac{1}{1+\beta/\alpha} = 0.15$ and different values of $z \equiv 1/\alpha$. The solid blue line ($z=0.05$) represents cases in which there is little asymmetric information between central and local resolution authorities. The dashed red line ($z=10$) represents cases in which the information asymmetry between central and local resolution authorities is large.

Figure 1 depicts the probability density functions for $z = 0.05$ (solid blue line) and $z = 10$ (dash-dot red line). Small values of $z$ are associated with a probability distribution whose mass is mostly concentrated around the distribution mean. In these cases there is little asymmetric information between central and local resolution authorities. For large values of $z$, the density is U-shaped, concentrating most of its mass at the lower and upper ends of the support. In these cases the degree of asymmetric information between authorities is highest. Finally, note that distributions with small values of $z$ satisfy the sufficient conditions in Proposition 3 while distributions with large
values of $z$ satisfy the sufficient conditions in Proposition 2.

![Graphs of various economic indicators](image)

**Figure 2:** Home contribution to bailouts, bail-out threshold, probability of sovereign crisis and expected aggregate costs of resolution as a function of $z = 1/\alpha$. The dashed red line corresponds to autarky values. The dotted blue line represents first-best values. The solid blue line stands for banking union values.

Figures 2.a and 2.b represent the home contribution to a bail-out and the bail-out thresholds, respectively, for different distributions parameterized by $z$. In autarky, bail-outs are always fully-funded by the home country and the bail-out threshold is given by the expected cost of funds $\bar{X}$. The first-best bail-out resolution prescribes that the stable country always funds the bail-out in its entirety, so that the threshold is determined by the cost of funds $\lambda_L$ at the stable country. In the optimal banking union with informational frictions the home contribution to bail-outs and the resulting bail-out thresholds depend on the distribution of bail-in costs. When $z$ is small (mass
hump-shaped around mean) and, by assumption, below the cost of funds of the stable country, the central authority has a strong a priori that it is not convenient to conduct bail-outs. Consequently, the optimal resolution mechanism requires a high home contribution to convince the central authority that bail-in costs are large and the public support warranted. This results in a high bail-out threshold. When \( z \) is large (mass U-shaped with high concentration in the tails), the central authority only needs to acquire information about which of the tails the bail-in cost lies on. To get convinced of the need of a bail-out the central authority only has to discard the possibility that bail-in costs are in the left tail, and hence a lower home contribution to bail-outs is required. This results in a lower bail-out threshold that when \( z \) is sufficiently large that can be even below that in the first-best banking union.

Figure 2.c highlights the results stated in Proposition 3. Sovereign crises are more likely in autarky than in the first-best resolution. Also bank resolutions within a banking union may lead to a higher occurrence of sovereign crises than in autarky when \( z \) is not too large. Finally, Figure 2.d illustrates two facts concerning the welfare effects of the banking union. First, the banking union does always reduce aggregate resolution costs relative to autarky, although not as much as in the first-best union. Second, the gains from a banking union are larger the higher \( z \), that is, the more concentrated the probability mass in the tails. Recall from above that the main inefficiency associated with the banking union arises from the need to impose large contributions from the fragile country to fund bail-outs, so as to curb down the overstatement effect. However, home contributions get reduced as \( z \) increases, which reduces aggregate resolution costs.

5 Extensions

In this section we address several extensions of the model. In Section 5.1 we analyze the optimal resolution mechanism when foreign support to the funding of bail-outs can be a mix of transfers to the failing bank’s debtholders (direct recapitalization, as in the baseline model), and one period
loans to the fragile sovereign (indirect recapitalization). Section 5.2 allows for country asymmetry at the initial date and shows that a banking union with ex-post optimal resolution policies may not be feasible. In such circumstances, the analysis shows that there are benefits from designing resolution policies that depend on the initial characteristics of the countries. Finally, Section 5.3 endogenizes the probability that sovereigns become fragile and finds that due to a fiscal free-riding problem stemming from the risk-sharing provided by a banking union imposing fiscal discipline on the sovereigns is aggregate welfare enhancing. Moreover, asymmetric information renders the fiscal rule tighter than in the first-best.

5.1 Direct versus indirect recapitalization

In the baseline model we have considered financial aid from the foreign country in the form of a transfer to the debtholders of the failing bank. Such external support resembles the Direct recapitalization instrument (DRI) established by the ESM in 2014 and that has not yet been used. It differs from the indirect recapitalization used for example in the banking crisis in Spain in 2012 by which the ESM can grant a loan to the fragile sovereign, which is then channeled to the troubled institutions. The new DRI is nevertheless conceived to be used only “if indirect recapitalization is not possible”. In this section we extend the model to allow for both types of foreign financial aid and analyze the trade-offs involved in either one. Our most important result is that some amount of direct recapitalization is always part of the optimal resolution of failing banks in fragile countries.

We consider another date $t = 2$ in which sovereigns can also raise funds. For simplicity we assume that sovereigns are stable at that date with certainty, so that the social cost of obtaining funds for them is $L$.\footnote{The assumption that with some positive probability the sovereign remains fragile at date 2 would only strengthen our results.} We allow for an additional possibility to fund the bail-out of a bank (indirect recapitalization): the foreign country makes a one-period loan of $x_2$ units of funds to
the home country. A resolution mechanism is a tuple \( \{ k(\hat{c}), x_1(\hat{c}), x_2(\hat{c}) \} \) such that, for each \( \hat{c} \) reported by local authority 1, \( k(\hat{c}) \) takes the value 1 if and only if there is a bail-out. In the case of a bail-out, \( x_1(\hat{c}) \) stands for the funds that the home country must contribute to, \( x_2(\hat{c}) \) corresponds to the indirect recapitalization provided by the foreign country through a one period loan to the home country. The remaining amount \( 1 - x_1(\hat{c}) - x_2(\hat{c}) \) accounts for the direct recapitalization of the foreign country to the failing bank.

An analogous to Lemma 1 can be obtained. In particular, threshold mechanisms are described by pairs \((x_1, x_2)\) of home contribution at \( t = 1 \) and indirect recapitalization (to be repaid by the home country at \( t = 2 \)) such that that \( k(c) = 1 \) if and only if \( c \geq \bar{c}(x_1, x_2) \), where:

\[
\bar{c}(x_1, x_2) = ((1 + \lambda_L + (\lambda_H - \lambda_L)\mu x_1) x_1 + (1 + \lambda_L)x_2 - 1, 0)^+. \tag{20}
\]

Relative to the analogous expression in (12), there is an additional term \((1 + \lambda_L)x_2\) that captures the fact that the indirect foreign contribution \( x_2 \) must be repaid by the home country at \( t = 2 \).

The expected aggregate social resolution costs associated with a threshold mechanism with home contributions \((x_1, x_2)\) to bail-outs is given by:

\[
\Pi(x_1, x_2) = \int_0^{\bar{c}(x_1, x_2)} cdG(c) + [(\lambda_H - \lambda_L)\mu x_1^2 + \lambda_Lx_2 + \lambda_L] [1 - G(\bar{c}(x_1, x_2))]. \tag{21}
\]

Note that, relative to the analogous expression in (13), there is an additional term \( \lambda_Lx_2 \) that captures the fact that indirect recapitalization involves double raising of public funds. First, at \( t = 1 \), funds must be raised by the foreign (stable) country to repay the bank’s debtholders. Second, at \( t = 2 \), the home country must repay the loan provided by the foreign country. The following proposition establishes the conditions for the optimal resolution to involve a mix of both instruments.

**Proposition 4 (Optimal resolution mechanism with direct and indirect recapitalization)**

Let \((x_1^*, x_2^*)\) be optimal home contributions to a bail-out at \( t = 1 \) and \( t = 2 \), respectively. Then, we have that \( x_1^* > 0 \) and \( 1 - x_1^* - x_2^* > 0 \). Moreover if \( \frac{2c_{\max} - \lambda_L}{1 + c_{\max}} \mu > (1 + \lambda_L)\lambda_L \) then there exists \( \lambda_h^* \in (\lambda_L, c_{\max}) \) such that, if \( \lambda_H > \lambda_h^* \), then \( x_2^* > 0 \).
The proposition states that optimal resolution mechanisms always exhibit some direct foreign recapitalization, i.e. $1 - x_1^* - x_2^* > 0$. The rationale for this result is the same as in the baseline model: the overstatement incentives of the home authority vanish as it faces the repayment of its bank debtholders, that is, as $x_1 + x_2$ approaches one. As a result, the cost effect dominates and there are aggregate welfare gains from a reduction on the overall repayment from the home authority, that is, $1 - x_1^* + x_2^* > 0$. Moreover, optimal resolution mechanisms always require a positive home contribution at $t = 1$, that is, $x_1^* > 0$. The reason is that indirect recapitalizations have a fixed marginal aggregate social cost $2\lambda_L$, while the marginal social cost of each unit of home contribution when such contribution is small is close to $\lambda_L$. Finally, if $\lambda_H$ is sufficiently large, then optimal resolution mechanisms exhibit a positive amount of foreign indirect recapitalization. The reason is that in such a case exclusively relying on home contributions at $t = 1$ to provide the home authority incentives not to overstate the need of a bail-out would be excessively costly. The main takeaway from this analysis is that the DRI should not be regarded as a last resort mechanism, but as an integral part of the optimal resolution scheme.

5.2 A banking union with asymmetric countries

So far we have assumed that countries are symmetric. This need not necessarily be the case. For instance, the Euro area is formed by a group of quite heterogeneous countries. While all countries may potentially benefit from risk-sharing, countries with more solid public finances or safer banking systems may end up funding an excessive amount of bail-outs in weaker neighboring countries. Hence, the efficiency gains in the resolution process associated with the creation of a banking union may not suffice for certain countries. In this section we analyze how the optimal banking union changes when the two countries differ at the initial date. Countries may be heterogenous in the probability of being fragile at the final date or in the likelihood that their bank fails. Since both cases are qualitatively identical, we focus only on the first dimension of heterogeneity. To fix our
ideas, we assume that $q^F_1 < q^F_2$ and we refer to countries 1 and 2 as (ex-ante) strong and weak, respectively.\textsuperscript{16}

In our analysis of the symmetric case carried out in Section 4, we have shown that in the optimal banking union resolution policies are ex-post (second-best) efficient. We first inquire whether such arrangement is still implementable at the initial date. The only difference with respect to the baseline situation is that when countries are asymmetric, the expected cross-country transfers at $t = 1$ determined by the banking union do not offset from an ex-ante perspective. Indeed, the strong country makes strictly positive net expected transfers to the weak one. We can rewrite expression (16) for the case in which there is country asymmetry as follows:

\begin{equation}
U_{i;\text{BU}}^0 = pR - 1 - (1-p) \left[ \Pi^S + q^F_i (\Pi(x^*) - \Pi^S) \right] \\
- (1-p) \left( q^F_j - q^F_i \right) (1-x^*) (1 + \lambda_L) \left( 1 - G(\bar{v}(x^*)) \right).
\end{equation} (22)

The last term accounts for the net expected cost for country $i$ stemming from the cross-country transfers associated with bail-outs in the banking union and, taking into account that $q^F_2 > q^F_1$, it is negative for country 1.

From the expressions for $U_{0;\text{BU}}^{1,\text{BU}}$ and $U_{0;\text{Aut}}^{1,\text{Aut}}$ in equations (6) and (16), respectively, and using equation (3), we can write the difference between the expected utility for the agents in the strong country under the \textit{ex-post optimal resolution policies} and in autarky as:

\begin{equation}
U_{0;\text{BU}}^{1,\text{BU}} - U_{0;\text{Aut}}^{1,\text{Aut}} = (1-p)q^F_1 \left( \Pi^F - \Pi(x^*) \right) - (1-p) \left( q^F_2 - q^F_1 \right) (1-x^*) (1 + \lambda_L) \left( 1 - G(\bar{v}(x^*)) \right).
\end{equation} (23)

The first term in equation (23) captures the gains achieved by the creation of a banking union, which provides risk-sharing to improve the efficiency of the resolution process. The second term, which is proportional to the difference $q^F_2 - q^F_1$ between the probabilities of being fragile in the weak and the strong country, captures the cost experienced by the strong country resulting from

\textsuperscript{16}Let us remind that under our assumption that $q^{F,F} = 0$ we have that $q^F_1 = q^{F,S}$ and $q^F_2 = q^{S,F}$. 

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its position as a net positive contributor to the funding of bail-outs. If the latter term dominates
the former, a banking union with ex-post optimal resolution policies is not feasible, as the strong
country will prefer to remain in autarky. The next proposition states that a banking union with
optimal ex-post resolution policies is feasible if and only if the asymmetry between the countries
involved is not too large.

**Proposition 5 (Infeasibility of ex-post resolution efficiency with large country asymmetry)**

There exists $a \in (0,1)$ such that a banking union with ex-post optimal resolution policies is not fea-
sible if and only if $q_1^F < aq_2^F$.

When country asymmetry is large, the optimal feasible banking union thus benefits from the
definition of resolution policies that depend on the initial strength of the country that obtains
financial aid. These policies might be ex-post suboptimal, but in their absence a strong country
would not be willing to join the banking union. Taking into account that, for simplicity, we have
ruled out the possibility that fragile countries make transfers abroad, a general banking union can
be described by a tuple of resolution mechanisms in the following four contingencies: i) failure of
bank 1 in state $(F,S)$; ii) failure of bank 2 in state $(S,F)$; iii) failure of bank 1 in state $(S,S)$;
and iv) failure of bank 2 in state $(S,S)$. Using an analogous result to that in Lemma 1 for the
resolution of banks in state $(S,S)$, a banking union is characterized by a tuple of home contributions
$(x_{1,(F,S)}, x_{2,(S,F)}, x_{1,(S,S)}, x_{2,(S,S)})$ to the funding of bail-outs in the set of contingencies i)-iv) just
described.

For a given resolution policy $(x_{1,(F,S)}, x_{2,(S,F)}, x_{1,(S,S)}, x_{2,(S,S)})$ the expected utility of agents in
each of the countries can be derived. Its analytical expression is more complicated than expression
(16), as the cross-country transfers in states $(S,F)$ and $(F,S)$ do not necessarily offset and also
because it includes additional terms capturing the costs of the resolution process for each country.
in state \((S, S)\).\(^{17}\) Using that expression we can prove the following result.

**Proposition 6 (Optimal banking union with large country asymmetry)** Suppose that the function \(\Pi(x)\) capturing the aggregate social cost of resolution in states \((S, F)\) and \((F, S)\) is convex and achieves its minimum at the home contribution \(x^*\). Let \(a\) be the constant defined in Proposition 5. Then, if \(q^F_1 < a q^F_2\), in any optimal feasible banking union \((x^*_1,(F,S), x^*_2,(S,F), x^*_1,(S,S), x^*_2,(S,S))\) we have that:

(i) Only the participation constraint of the strong country is binding.

(ii) The bail-out contribution of the strong (weak) country when it is fragile and its bank fails is smaller (larger) that in the ex-post optimal resolution policy, that is, \(x^*_1,(F,S) < x^* < x^*_2,(S,F)\).

(iii) If the weak country is stable at \(t = 1\), then it entirely funds a bail-out of its bank, that is, \(x^*_2,(S,S) = 1\).

As anticipated in the discussion above, whenever optimal ex-post policies are not implementable the banking union must incorporate some suboptimal ex-post policies so as to soften the strong country participation constraint at the initial date. In particular, when the strong country is fragile the financial aid it obtains to support its bank from the from the weak country is larger than ex-post optimal \((x^*_1,(F,S) < x^*)\), and the converse happens when the weak country is fragile \((x^*_2,(S,F) > x^*)\). In addition, if the level of country asymmetry is very important another source of inefficiency comes from the possibility that the weak country provides financial aid to the strong one when both are stable (i.e., \(x^*_1,(S,S) < 1\)), while the converse (i.e., \(x^*_2,(S,S) = 1\)) cannot happen. This extension shows the importance of recognizing the ex-ante differences across countries in order to design an optimal banking union. Resolution schemes that fail to do so might be more efficient ex-post but unfeasible from an ex-ante perspective.

\(^{17}\) The analytical expression can be found in the proof of Proposition 6.
5.3 Fiscal free-riding in a banking union

So far we have assumed that the probability that a country becomes stable or fragile is exogenously given. Countries, however, can affect these chances by adopting a tough or a loose fiscal policy. A tough fiscal policy may reduce the chances that a country becomes fragile at the time of a banking crisis, but it entails some costs. For instance, a country may reduce its public deficit at the expense of foregoing some productive infrastructure investments or social expenditures. In this section we extend the model to endogenize the countries’ fiscal strength. We highlight the need of enforcing stability enhancing rules on the countries’ public finances and show the extent to which the informational frictions induce tougher enforcing rules than in the first-best banking union.

Consider a situation in which countries determine at the initial date their fiscal strength facing the following trade-off: a reduction in the likelihood of being fiscally fragile at \( t = 1 \) when bank resolution may require public support can be achieved with a short-term “austerity” cost. More precisely, we assume that the cost for agents in country \( i \) of the measures associated with ensuring that the sovereign is stable at \( t = 1 \) with probability \( q_i \) is \( \frac{k}{2} q_i^2 \). Finally, we depart from our simplifying assumption that \( q_{F;F}^{F} = 0 \) and instead assume that the stability or fragility of a country is independent of the situation in the other one. This in particular implies that \( q_{F;F}^{F} = (1 - q_1)(1 - q_2) \).

In autarky, country \( i \) chooses its fiscal strength so as to minimize the sum of the costs of resolving banks and those associated to austerity measures, that is:

\[
\min_{q_i \in [0,1]} \ (1 - p) \left[ q_i \Pi^S + (1 - q_i) \Pi^F \right] + \frac{k}{2} q_i^2.
\]

We assume that the cost parameter of fiscal austerity \( k \) satisfies \( k \in \left[ (1 - p) \left( \Pi^F - \Pi^S \right), (1 - p) \Pi^F \right] \), which ensures that the solution for the problem above is interior and given by:

\[
q_{Aut}^i = \frac{(1 - p) \left( \Pi^F - \Pi^S \right)}{k}.
\]  \hspace{1cm} (24)

We now analyze the fiscal strength problem when countries join a banking union with asymmetric information and optimal resolution schemes as described in Section 4. Suppose the optimal
resolution scheme when the home country of the failing bank is fragile is characterized by a home contribution to bail-outs of $x^*$, and, for notational simplicity, let us denote by $T^*$ the associated expected social cost for a foreign (stable) country of the transfers it makes upon failure of a bank in a fragile country, that is:

$$T^* \equiv (1 - G(\bar{c}(x^*))) (1 - x^*) (1 + \lambda_L).$$

A Nash equilibrium of the fiscal game in a banking union consists of a pair $(q^*_1, q^*_2)$ such that each $q^*_i$ minimizes the costs for country $i$ of resolving banks in a banking union and those associated to austerity measures:

$$q^*_i \in \arg\min_{q_i \in [0,1]} \left( (1 - p) \left[ q_i \Pi_S + (1 - q_i) q^*_j (\Pi (x^*) - T^*) + (1 - q_i) (1 - q^*_j) \Pi^F \right] + (1 - p) q_i (1 - q^*_j) T^* + \frac{k}{2} q^2_i \right). \quad (25)$$

The terms in the first line in the minimization problem of country $i$ account for the costs associated with resolving a failure of its bank. In fact the three terms in the brackets account for the resolution costs when the country is stable, when it is fragile and the foreign country is stable and thus contributes to the funding of bail-outs, and when both of them are fragile, respectively. The first term in the second line captures the transfers country $i$ makes when it is stable and the other one is fragile and its bank fails, and the second term accounts for the short-term austerity costs suffered by the country.

From (25) it is easy to derive the unique equilibrium of the fiscal game, which is symmetric and satisfies:

$$q^*_i = q^{\text{Aut}} - \frac{(1 - p) (\Pi^F - \Pi (x^*))}{k + (1 - p) (\Pi^F - \Pi (x^*))} q^{\text{Aut}} - \frac{(1 - p) T^*}{k + (1 - p) (\Pi^F - \Pi (x^*))}. \quad (26)$$

The expression shows that the equilibrium value of the fiscal stand is lower than that in autarky because of two forces. A risk-sharing effect among countries, which diminishes the social benefits of being fiscally stable, as a fragile country can recourse to the stable one to finance a bail-out. The more the banking union reduces aggregate resolution costs (i.e., the higher $\Pi^F - \Pi (x^*)$), the more
risk-sharing pushes down equilibrium fiscal policy. Additionally, a free-riding effect that further reduces the level of the fiscal stand as countries benefit in two ways from being fragile: should their bank fail and the other country be stable, they obtain a transfer in case there is a bail-out, and should the other country be fragile and its bank fail they do not have to contribute to the funding of an eventual bail-out.

In contrast, the aggregate welfare-maximizing fiscal policy choices within the banking union are given by the solution to the problem:

$$\min_{q_1,q_2 \in [0,1]^2} \sum_{i=1,2,i\neq j} (1-p) \left( q_i \Pi^S + (1-q_i) q_j \Pi (x^*) \right) + \left( (1-q_i) (1-q_j) \Pi^F \right) + \frac{k}{2} q_{i}^{2},$$

in which relative to (25) the terms capturing transfers among countries have canceled out. The solution to the problem consists in a common fiscal policy level $\hat{q}_i$ whose expression, after some algebra, is found to be:

$$\hat{q}_i = q^{Aut} - \frac{(1-p) \left( \Pi^F - \Pi (x^*) \right)}{k + (1-p) \left( \Pi^F - \Pi (x^*) \right)} q^{Aut}. $$

Comparing with the expression in (26) we observe that

$$q_i^* < \hat{q}_i < q^{Aut}. $$

In fact, the reduction in the fiscal stand from the autarky case to the socially optimal level in a banking union ($q^{Aut} - \hat{q}_i$) corresponds to what we have previously called the risk-sharing effect. The additional reduction under the equilibrium of the fiscal game ($\hat{q}_i - q_i^*$) stems from the free-riding effect.

Finally, an analogous computation of the equilibrium and socially optimal level of the fiscal policy in a banking union with perfect information can be conducted. In fact, it suffices to replace in all the expressions the term $\Pi^F - \Pi (x^*)$ capturing resolution costs and the term $T^*$ capturing bailout transfers, by the corresponding ones when there is perfect information, which are given by $\Pi^F - \Pi^S$ and $(1 - G (\lambda_L)) (1 + \lambda_L)$, respectively. In particular, the socially optimal fiscal level in a
banking union with perfect information is given by:

$$q^{FB}_i = q^{Aut} - \frac{(1 - p) (\Pi^F - \Pi^S)}{k + (1 - p) (\Pi^F - \Pi^S)} q^{Aut}.$$ 

Since $$\Pi^F - \Pi^S > \Pi^F - \Pi(x^*)$$ we have that:

$$\hat{q}^{FB}_i < \hat{q}_i,$$

so that the presence of the informational friction increases the socially optimal fiscal level. The intuition is that the strength of the risk-sharing effect that reduces the socially optimal fiscal level relative to autarky is weakened in the presence of informational asymmetries. We summarize these findings in the following proposition.

**Proposition 7** Due to a free-riding problem associated with transfers across countries to fund bailouts in a banking union there are benefits from enforcing stability enhancing rules on the countries’ public finances, that is, $$q^*_i < \hat{q}_i.$$ Moreover, the optimal common fiscal rule is tighter due to the informational friction than it would be in its absence, that is, $$\hat{q}^{FB}_i < \hat{q}_i.$$

### 6 Conclusion

In this paper, we have analyzed the optimal mechanism for bank resolutions within banking unions. Our assessment is based on a model in which countries can benefit from a risk-sharing agreement by which they can aid each other in case one of the countries have a fragile fiscal stand at the time of having to resolve a troubled bank. In our model, countries have private information about the disruption costs that a bail-in may generate. We have shown that this informational friction limits the extent of risk-sharing, because countries face an incentive to overstate the bail-in costs so as to obtain funding from the neighboring country to bail-out home troubled banks. In particular, the optimal mechanism entails co-funding of bail-outs by home (fragile) countries and neighboring (stable) countries to incentivize countries to reveal the true extent of bail-in costs.
We have found that risk-sharing makes bail-outs relatively more desirable, so that bail-outs are more likely in a banking union than in its absence. Indeed, bail-outs may be more likely than in the first-best, in which there are no informational frictions limiting the extent of risk-sharing. Moreover, the combination of a higher occurrence of bail-outs within a banking union and the home country contribution to partially fund those may lead to a higher likelihood of sovereign crises, which could be totally eliminated in the absence of informational asymmetries.

We have also addressed a number of extensions of the baseline model. We have considered the possibility of including indirect recapitalization in the form of a loan to the sovereign as part of the optimal mechanism. When this option is considered, the optimal mix of funds continues to include foreign direct recapitalization and a home country contribution, but also some indirect recapitalization to alleviate the present home country contribution when the cost of the home sovereign funds are large. We have also assessed the optimal mechanism when countries are asymmetric, finding that the banking union must incorporate some suboptimal ex-post policies so as to soften the strong country participation constraint at the initial date. We have finally analyzed the impact of the banking union on the incentives of countries to loosen their fiscal policies and shown that there are benefits from enforcing stability enhancing tighter rules on the countries’ public finances than in the absence of informational frictions.

The findings in this paper highlight the benefits of banking unions as risk-sharing instruments, but also stress the importance of co-funding of bail-outs to provide appropriate incentives. Our analysis does also draw attention on the need to establish asymmetric rules for different countries to be willing to participate in banking unions with heterogenous participants.
Proof of Lemma 1  

Necessity  Suppose that \( \{ k(\tilde{c}), x(\tilde{c}) \} \) is an incentive compatible (truth-revealing) mechanism. Let us define the set \( \Gamma^1 = \{ c \in [0, c_{\text{max}}] \mid k(c) = 1 \} \). If \( \Gamma^1 \) is empty then case 1 in the lemma is satisfied. Let us suppose that \( \Gamma^1 \) is non empty. Taking into account that 

\[
(1 + \lambda_L + (\lambda_H - \lambda_L)\mu x) x
\]

is strictly increasing in \( x \), it is immediate to show that incentive compatibility implies that there exists a constant \( \overline{x} \) such that \( x(c) = \overline{x} \) for all \( c \in \Gamma^1 \).

Let us now prove that if \( c' \geq c \) for some \( c \in \Gamma^1 \) then \( c' \in \Gamma^1 \). In fact, incentive compatibility implies

\[
U(c', c') \geq U(c, c') \iff k(c') - (1 - k(c'))c' - (1 + \lambda_L + (\lambda_H - \lambda_L)\mu x(c')) x(c') \geq
\]

\[
\geq 1 - (1 + \lambda_L + (\lambda_H - \lambda_L)\mu \overline{x}) \overline{x},
\]

\[
U(c, c) \geq U(c', c) \iff 1 - (1 + \lambda_L + (\lambda_H - \lambda_L)\mu \overline{x}) \overline{x} \geq
\]

\[
\geq k(c') - (1 - k(c'))c - (1 + \lambda_L + (\lambda_H - \lambda_L)\mu x(c')) x(c'),
\]

and combining the two inequalities we have

\[
-(1 - k(c'))c' \geq -(1 - k(c'))c,
\]

which can only be satisfied if \( k(c') = 1 \).

Let \( \overline{c} = \inf(\Gamma^1) \). We must have \( \overline{c} \in \Gamma^1 \). In fact, if that is not the case then \( k(\overline{c}) = 0 \). By definition of \( \overline{c} \), for any \( \varepsilon > 0 \) there exists \( c' \in \Gamma^1 \) and such that \( 0 < c' - \overline{c} < \varepsilon \). Incentive compatibility implies:

\[
U(\overline{c}, \overline{c}) \geq U(c', \overline{c}) \iff -\overline{c} \geq 1 - (1 + \lambda_L + (\lambda_H - \lambda_L)\mu \overline{x}) \overline{x}
\]

\[
U(c', c') \geq U(\overline{c}, c') \iff 1 - (1 + \lambda_L + (\lambda_H - \lambda_L)\mu \overline{x}) \overline{x} \geq -c'.
\]

Combining the two inequalities and taking limits \( \varepsilon \to 0 \) we conclude that

\[
-k = 1 - (1 + \lambda_L + (\lambda_H - \lambda_L)\mu \overline{x}) \overline{x}.
\]

But then when the local authority observes \( \overline{c} \) it is indifferent between reporting \( \overline{c} \), which leads to a bail-in, and reporting any \( c \in \Gamma^1 \), which leads to a bail-out. Our convention established at the end of Section 2 then states that the local authority would report \( c \in \Gamma^1 \) and the mechanism would not be truth-revealing. Hence we must have \( \overline{c} \in \Gamma^1 \) and the fact that for all \( c \geq \overline{c} \) we have \( c \in \Gamma^1 \) then implies that \( \Gamma^1 = [\overline{c}, 1] \). If \( \overline{c} = 0 \) then case 2 in the lemma is satisfied. If \( \overline{c} > 0 \) then for any \( c < \overline{c} \) it has to be the case that \( k(c) = 0 \) and incentive compatibility implies

\[
U(\overline{c}, \overline{c}) \geq U(c, \overline{c}) \iff 1 - (1 + \lambda_L + (\lambda_H - \lambda_L)\mu \overline{x}) \overline{x} \geq -\overline{c}
\]

\[
U(c, c) \geq U(\overline{c}, c) \iff -c \geq 1 - (1 + \lambda_L + (\lambda_H - \lambda_L)\mu \overline{x}) \overline{x}.
\]
Taking limits \( c \to \bar{c} \) and combining the two inequalities we have that
\[
1 - (1 + \lambda_L + (\lambda_H - \lambda_L)\mu\bar{x}) \bar{x} = -\bar{c},
\]
and then case 3 in the lemma is satisfied.

**Sufficiency** It is straightforward to check that the mechanisms in the 3 cases in the lemma satisfy incentive compatibility.

**Proof of Proposition 1** The proposition has been proved in the main text.

**Proof of Proposition 2** The first statement in the proposition has been proved in the main text. Let us move to the proof of the second statement.

Using the expression for \( \bar{c}(x) \) in (12) we have for \( x \geq x_L \) that
\[
\bar{c}'(x) = 1 + \lambda_L + 2(\lambda_H - \lambda_L)\mu x,
\]
as (27) and after some algebra we obtain from (15) that
\[
\Pi'(x) = 2(\lambda_H - \lambda_L)\mu x [1 - G(\bar{c}(x)) - (1 + \lambda_L) (1 - x) g(\bar{c}(x))] - (1 + \lambda_L)^2 (1 - x) g(\bar{c}(x)).
\]
Let us define \( \kappa_1 \equiv \frac{1-G(\bar{\lambda})}{1+\lambda_L} \). Suppose that the proposition conditions are satisfied for some \( \kappa \leq \kappa_1 \). From the expression above we have the following inequalities for \( x \geq x_L \)
\[
\Pi'(x) > 2(\lambda_H - \lambda_L)\mu x [1 - G(\bar{c}(x)) - (1 + \lambda_L) \kappa] - (1 + \lambda_L)^2 \kappa
\geq 2(\lambda_H - \lambda_L)\mu x [1 - G(\bar{\lambda}) - (1 + \lambda_L) \kappa] - (1 + \lambda_L)^2 \kappa
\geq 2(\lambda_H - \lambda_L)\mu x [1 - G(\bar{\lambda}) - (1 + \lambda_L) \kappa] - (1 + \lambda_L)^2 \kappa,
\]
where in the last inequality we have used that \( 1 - G(\bar{\lambda}) - (1 + \lambda_L) \kappa \geq 0 \).

Finally, let us define \( \kappa \equiv \frac{2(\lambda_H - \lambda_L)\mu x [1 - G(\bar{\lambda})]}{2(\lambda_H - \lambda_L)\mu x + 1 + \lambda_L (1 + \lambda_L)} \) and suppose that the proposition conditions are satisfied for this \( \kappa \). Since by construction \( \kappa < \kappa_1 \), the inequalities above imply that \( \Pi'(x) > 0 \) for \( x \geq x_L \) and then any optimal mechanism must satisfy \( x^* < x_L \), which by construction means that \( \bar{c}^* \equiv c(x^*) < \lambda_L \).
Proof of Proposition 3  Let $x^*$ be an optimal home contribution to the funding of bail-outs. We must have $x^* > x$. Following the proof of Proposition 1, we have that the first statement in the proposition implies that $x^* < x'$. Using the second statement in the proposition the following sequence of inequalities follow immediately:

$$x^* \Pr[\bar{c}(x^*) \leq c \leq \bar{\lambda}] \geq \Pr[\bar{c}(x^*) \leq c \leq \bar{\lambda}] \geq \Pr[c > \bar{\lambda}] \geq (1 - x') \Pr[c > \bar{\lambda}].$$

Taking into account that the probability of sovereign crises under the resolution scheme with home contribution to bail-outs $x^*$ is $\mu \cdot x^* \Pr[c \geq \bar{c}(x^*)]$, and that in autarky is $\mu \cdot \Pr[c \geq \bar{\lambda}]$, the statement in the proposition is an immediate consequence of the inequalities above. 

Proof of Proposition 4  Let $(x_1^*, x_2^*)$ be some optimal home contributions to a bail-out at $t = 1$ and $t = 2$, respectively. We must have $\bar{c}(x_1^*, x_2^*) > 0$ because otherwise the resolution scheme is dominated by $x_1 = 1, x_2 = 0$. Using the expressions in (20) and (21) we have that if $\bar{c}(x_1, x_2) > 0$ then

$$\frac{\partial \Pi(x_1, x_2)}{\partial x_1} = 2(\lambda_H - \lambda_L) \mu x_1 [1 - G(\bar{c}(x_1, x_2))] - (1 + \lambda_L) (1 - x_1 - x_2) (1 + \lambda_L + 2(\lambda_H - \lambda_L) \mu x_1) g(\bar{c}(x_1, x_2));$$

$$\frac{\partial \Pi(x_1, x_2)}{\partial x_2} = \lambda_L [1 - G(\bar{c}(x_1, x_2))] - (1 + \lambda_L) (1 - x_1 - x_2) (1 + \lambda_L) g(\bar{c}(x_1, x_2)).$$

We prove the different statements in the proposition sequentially.

1) $1 - x_1^* - x_2^* > 0$

Suppose that $1 - x_1^* - x_2^* = 0$. From (28) we have that $\frac{\partial \Pi(x_1, x_2)}{\partial x_2} > 0$ and if $x_2^* > 0$ the pair $(x_1^*, x_2^*)$ cannot be a local minimum of $\Pi(x_1, x_2)$. We must then have $x_2^* = 0$ and since $\bar{c}(x_1^*, x_2^*) > 0$ it has to be the case that $x_1^* > 0$. From (28) we have that $\frac{\partial \Pi(x_1, x_2)}{\partial x_1} > 0$ which since $x_1^* > 0$ implies that the pair $(x_1^*, x_2^*)$ cannot be a local minimum of $\Pi(x_1, x_2)$.

2) $x_1^* > 0$

Suppose that $x_1^* = 0$. Using 1) we have from (28) that $\frac{\partial \Pi(x_1, x_2)}{\partial x_1} < 0$ and the pair $(x_1^*, x_2^*)$ cannot be a local minimum of $\Pi(x_1, x_2)$.

3) If $2 \frac{c_{\text{max}} - \lambda_L}{1 + c_{\text{max}}} \mu > (1 + \lambda_L) \lambda_L$ then there exists $\Delta_H \in (\lambda_L, c_{\text{max}})$ such that if $\Delta_H > \Delta_H$ then $x_2^* > 0$. 

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Suppose the first inequality is satisfied and suppose that \( x_2^* = 0 \). Taking into account \( i) \) and \( ii) \), we must have that

\[
\frac{\partial \Pi(x_1^*, x_2^*)}{\partial x_1} = 0 \quad \text{and} \quad \frac{\partial \Pi(x_1^*, x_2^*)}{\partial x_2} \geq 0.
\]

Using the expressions in (28) we deduce that

\[
\frac{2(\lambda_H - \lambda_L)\mu x_1^*}{1 + \lambda_L + 2(\lambda_H - \lambda_L)\mu x_1^*} < \frac{\lambda_L}{1 + \lambda_L}.
\]  

(29)

Since \( \bar{c}(x_1^*, 0) > 0 \) we must have \( x_1^* > x \) where \( x \) satisfies \( \bar{c}(x, 0) = 0 \). Moreover, we have that \( x \geq \frac{1}{1+\lambda_H} \). Taking into account that the LHS in the inequality (29) is increasing in \( x_1^* \) and using that \( x_1^* > \frac{1}{1+\lambda_H} \) we have that

\[
\frac{2(\lambda_H - \lambda_L)\mu x_1^*}{1 + \lambda_L + 2(\lambda_H - \lambda_L)\mu x_1^*} > \frac{2(\lambda_H - \lambda_L)\mu}{(1 + \lambda_L)(1 + \lambda_H) + 2(\lambda_H - \lambda_L)\mu}.
\]  

(30)

One can check after some algebraic manipulation that

\[
\frac{2c_{\max} - \lambda_L}{1 + c_{\max} < (1 + \lambda_L)\lambda_L} \Leftrightarrow \frac{2(c_{\max} - \lambda_L)\mu}{(1 + \lambda_L)(1 + c_{\max}) + 2(c_{\max} - \lambda_L)\mu} \geq \frac{\lambda_L}{1 + \lambda_L}.
\]

Since the LHS in the inequality above is strictly increasing in \( c_{\max} \), we deduce that there exists \( \Lambda_H \in (\lambda_L, c_{\max}) \) univocally defined by

\[
\frac{2(\Lambda_H - \lambda_L)\mu}{(1 + \lambda_L)(1 + \Lambda_H) + 2(\Lambda_H - \lambda_L)\mu} = \frac{\lambda_L}{1 + \lambda_L}.
\]

Taking into account the definition of \( \Lambda_H \), inequality (30), and that the RHS in (30) is increasing in \( \lambda_H \), we have that if \( \lambda_H > \Lambda_H \) then (29) cannot be satisfied. So our assumption that \( x_2^* = 0 \) leads to a contradiction. \( \blacksquare \)

**Proof of Proposition 5** Let us first consider the case with informational asymmetries. Let \( \Lambda \) denote the finite set \( \arg \min_{x \in [0,1]} \Pi(x) \). Using (17) we have that a banking union with ex-post optimal resolution policies satisfies the participation constraint of country 1 if and only if:

\[
(1 - p)q_1^F (\Pi^F_1 - \Pi(x^*)) \geq (1 - p) (q_2^F - q_1^F) (1 - x^*)(1 + \lambda_L) (1 - G(\bar{c}(x^*))) \quad \text{for some } x^* \in \Lambda
\]
which is equivalent to

\[ q_1^F (\Pi^F - \min \Pi(x)) \geq (q_2^F - q_1^F) (1 + \lambda_L) \min_{x^* \in \Lambda} (1 - x^*) (1 - G(\bar{c}(x^*))) \iff q_1^F \geq a q_2^F, \quad (31) \]

with

\[ a \equiv \frac{(1 + \lambda_L) \min_{x^* \in \Lambda} (1 - x^*) (1 - G(\bar{c}(x^*)))}{\Pi^F - \min_{x \in [2,1]} \Pi(x) + (1 + \lambda_L) \min_{x^* \in \Lambda} (1 - x^*) (1 - G(\bar{c}(x^*)))} \in (0, 1). \]

Finally, from (22) and using that \( q_2^F > q_1^F \) we have that if the participation constraint of country 1 is satisfied then that of country 2 is also satisfied. We conclude that a banking union with ex-post optimal resolution policies satisfies the participation constraint of the two countries if and only if (31) is satisfied.■

**Proof of Proposition 6**  An analogous to Lemma 1 to describe the incentive compatible resolution mechanisms in the state \((S, S)\). In particular, if the required home contribution to bail-outs is \(x\) then the bail-out threshold in the bail-in cost observed by the home authority is:

\[ \bar{c}(x) \equiv \max \{ (1 + \lambda_L)x - 1, 0 \}, \quad (32) \]

and the associated aggregate social costs of resolution are given by:

\[ \tilde{\Pi}(x) = \int_0^{\bar{c}(x)} cdG(c) + \lambda_L (1 - G(\bar{c}(x))) \]. \quad (33) \]

Using (32) we can check that \( \tilde{\Pi}'(x) < 0 \) for all \( x < 1 \).

For the sake of compactness in the expressions to come, we denote \( \bar{t}(x) = (1-x)(1+\lambda_L) (1 - G(\bar{c}(x))) \) and \( \tilde{t}(x) = (1-x)(1+\lambda_L) (1 - G(\bar{c}(x))) \). The two functions are decreasing in \( x \).

For a banking union described by the tuple of home contributions to bail-outs in the different contingencies \( (x_{1,(F,S)}, x_{2,(S,F)}, x_{1,(S,S)}, x_{2,(S,S)}) \), it can be shown that the expected utility as of \( t = 0 \)
of agents in country 1 and 2 is given by:

\[
U_{0}^{1,BU}(x_{1,(F,S)}, x_{2,(S,F)}, x_{1,(S,S)}, x_{2,(S,S)}) = pR - 1 - (1 - p)q_{1}^{F} \Pi_{S} - (1 - p)q_{2}^{F} \Pi(x_{1,(F,S)}) + (1 - p)q_{1}^{F} \tilde{t}(x_{1,(F,S)}) - (1 - p)q_{1}^{S} \Pi(x_{1,(S,S)}) - (1 - p)q_{2}^{S} \tilde{t}(x_{1,(S,S)})
\]

\[
U_{0}^{2,BU}(x_{1,(F,S)}, x_{2,(S,F)}, x_{1,(S,S)}, x_{2,(S,S)}) = pR - 1 - (1 - p)q_{1}^{F} \Pi_{S} - (1 - p)q_{2}^{F} \Pi(x_{2,(S,F)}) + (1 - p)q_{2}^{F} \tilde{t}(x_{2,(S,F)}) - (1 - p)q_{1}^{S} \Pi(x_{2,(S,S)}) - (1 - p)q_{2}^{S} \tilde{t}(x_{2,(S,S)})
\]

The derivation of this expression is similar to that of (??), albeit more involved because: i) the home contributions of fragile countries \(x_{1,(F,S)}\) and \(x_{2,(F,S)}\) may differ; and ii) there can be financial aid in state \((S,S)\).

Adding up (34) and (35), the aggregate welfare in the banking union can be written in the compact and intuitive form:

\[
W_{0}(x_{1,(F,S)}, x_{2,(S,F)}, x_{1,(S,S)}, x_{2,(S,S)}) = 2(pR - 1) - (1 - p)(q_{1}^{F} + q_{2}^{F})\Pi_{S} - (1 - p)q_{1}^{F} \Pi(x_{1,(F,S)}) - (1 - p)q_{2}^{F} \Pi(x_{2,(S,F)}) - (1 - p)q_{1}^{S} \Pi(x_{1,(S,S)}) - (1 - p)q_{2}^{S} \Pi(x_{2,(S,S)})
\]

Let \((x_{1,(F,S)}, x_{2,(S,F)}, x_{1,(S,S)}, x_{2,(S,S)})\) be an optimal feasible banking union. Let us denote \(PC_{i}\) the participation constraint of country \(i\). We structure the proof of the proposition as a sequence of steps.

i) If \(PC_{1} (PC_{2})\) is not binding then \(x_{1,(S,S)}^{*} = 1 (x_{2,(S,S)}^{*} = 1)\).

For \(x_{1,(S,S)} < 1\) we have that

\[
\frac{\partial W_{0}(x_{1,(F,S)}, x_{2,(S,F)}, x_{1,(S,S)}, x_{2,(S,S)})}{\partial x_{1,(S,S)}} > 0 \quad \text{and} \quad \frac{\partial U_{0}^{2,BU}(x_{1,(F,S)}, x_{2,(S,F)}, x_{1,(S,S)}, x_{2,(S,S)})}{\partial x_{1,(S,S)}} > 0,
\]

which implies that if \(PC_{1}\) is not binding then \(x_{1,(S,S)} < 1\) cannot be optimal.

ii) Either \(PC_{1}\) or \(PC_{2}\) is binding.

Suppose none of the participation constraints is binding. Then from (36) we deduce that the resolution policies induced by the banking union \((x_{1,(F,S)}, x_{2,(S,F)}, x_{1,(S,S)}, x_{2,(S,S)})\) have to be optimal at \(t = 1\). But \(q_{1}^{F} < aq_{2}^{F}\), and Proposition 5 states that this cannot be the case.
iii) Both $PC_1$ and $PC_2$ cannot be binding.

From (34) and (35) we have that

\[
\frac{\partial U_{0}^{BU}(1,1,1,1)}{\partial x_{1}(F,S)} = -(1-p)q_{1}^{F}\Pi'(1) - (1-p)q_{1}^{F}(1 + \lambda_{L})(1 - G(\tau(1)))
\]

\[
= -(1-p)(2(\lambda_{L} - \lambda_{L})\mu + 1 + \lambda_{L})(1 - G(\tau(1)))q_{1}^{F},
\]

\[
\frac{\partial U_{0}^{BU}(1,1,1,1)}{\partial x_{2}(S,F)} = (1-p)(1 + \lambda_{L})(1 - G(\tau(1)))q_{2}^{F},
\]

\[
\frac{\partial U_{0}^{BU}(1,1,1,1)}{\partial x_{1}(F,S)} = -(1-p)q_{2}^{F}\Pi'(1) - (1-p)q_{2}^{F}(1 + \lambda_{L})(1 - G(\tau(1)))
\]

\[
= -(1-p)(2(\lambda_{H} - \lambda_{L})\mu + 1 + \lambda_{L})(1 - G(\tau(1)))q_{2}^{F}.
\]

Let us define $x_{1}(F,S)(y) = 1 - y, x_{2}(S,F)(y) = 1 - ky$ with $k \equiv \frac{(2(\lambda_{H} - \lambda_{L})\mu + 1 + \lambda_{L})q_{1}^{F} - \varepsilon}{(1 + \lambda_{L})q_{2}^{F}}$ and $\varepsilon > 0$, and let us consider the functions $\hat{U}_{0}^{1}(y) = U_{0}^{1,BU}(x_{1}(F,S)(y), x_{2}(S,F)(y), 1, 1)$, and $\hat{U}_{0}^{2}(y) = U_{0}^{2,BU}(x_{1}(F,S)(y), x_{2}(S,F)(y), 1, 1)$. Using the expressions in (37) we have that

\[
\frac{d\hat{U}_{0}^{1}(y)}{dy} \bigg|_{y=0} = \frac{\partial U_{0}^{1}(1,1,1,1)}{\partial x_{1}(F,S)} - k\frac{\partial U_{0}^{1}(1,1,1,1)}{\partial x_{1}(F,S)} = \\
= \varepsilon(1-p)(1 + \lambda_{L})(1 - G(\tau(1)))q_{1}^{F} \text{ and}
\]

\[
\frac{d\hat{U}_{0}^{2}(y)}{dy} \bigg|_{y=0} = \frac{\partial U_{0}^{2}(1,1,1,1)}{\partial x_{1}(F,S)} - k\frac{\partial U_{0}^{2}(1,1,1,1)}{\partial x_{1}(F,S)} = \\
= (1-p)(1 - G(\tau(1))) \left[ \frac{(2(\lambda_{H} - \lambda_{L})\mu + 1 + \lambda_{L})^{2} - (1 + \lambda_{L})^{2}}{1 + \lambda_{L}}q_{1}^{F} - \varepsilon(2(\lambda_{H} - \lambda_{L})\mu + 1 + \lambda_{L})q_{2}^{F} \right].
\]

We have that for $\varepsilon$ sufficiently small $\frac{d\hat{U}_{0}^{1}(y)}{dy} \bigg|_{y=0} > 0$ and $\frac{d\hat{U}_{0}^{2}(y)}{dy} \bigg|_{y=0} > 0$. Which means that there exist $x_{1}(F,S), x_{2}(S,F)$ close to 1 such that in a banking union described by the tuple $(x_{1}(F,S), x_{2}(S,F), 1, 1)$ both countries are strictly better off than under the autarky tuple $(1,1,1,1)$. As a result both participation constraints cannot be binding under an optimal feasible banking union.

iv) $PC_2$ is not binding.

Suppose $PC_2$ is binding. Then $PC_1$ is not binding and $i)$ implies that $x_{1}^{*}(S,S) = 1$. Since an increase in $x_{1}(F,S)$ increases $U_{0}^{2,BU}(x_{1}(F,S), x_{2}(S,F), 1, x_{2}(S,F))$ and thus relaxes $PC_2$, from (36) aggregate welfare maximization implies that $\Pi'(x_{1}^{*}(F,S)) > 0$. Since by assumption $\Pi(x)$ is convex
and achieves its minimum at $x^*$; we must have $x_{1,(F,S)}^* > x^*$. Analogously optimality implies that 
$\Pi'(x_{2,(F,S)}^*) < 0$ and thus $x_{2,(F,S)}^* < x^* < x_{1,(F,S)}^*$.

For $x_{1,(S,S)}^* = 1$ and using (34) we can write $PC_1$ as:

$$q_1^F \left[ \Pi^F - \Pi(x_{1,(F,S)}^*) + \tilde{t}(x_{1,(F,S)}^*) \right] \geq q_2^F \tilde{t}(x_{2,(F,S)}^*) + q^{S,S} \tilde{t}(x_{2,(S,S)}^*),$$

and then using that $q_2^F > q_1^F$, $x_{2,(F,S)}^* < x_{1,(F,S)}^*$, that $\Pi(x) - \tilde{t}(x)$ is increasing in $x$, and that $\tilde{t}(x)$ is decreasing in $x$ we have that

$$q_2^F \left( \Pi^F - \Pi(x_{2,(F,S)}^*) + \tilde{t}(x_{2,(F,S)}^*) \right) + q^{S,S} \left( \Pi^S - \tilde{t}(x_{2,(S,S)}^*) + \tilde{t}(x_{2,(S,S)}^*) \right) >$$

$$> q_1^F \left( \Pi^F - \Pi(x_{2,(F,S)}^*) + \tilde{t}(x_{2,(F,S)}^*) \right) > q_1^F \left( \Pi^F - \Pi(x_{1,(F,S)}^*) + \tilde{t}(x_{1,(F,S)}^*) \right) \geq$$

$$\geq q_2^F \tilde{t}(x_{2,(F,S)}^*) + q^{S,S} \tilde{t}(x_{2,(S,S)}^*) > \tilde{t}(x_{1,(F,S)}^*)$$

Comparing the first and last term of this sequence of inequalities and using (35) we deduce that

$$U_0^{2,BU}(x_{1,(F,S)}^*, x_{2,(F,S)}^*, 1, x_{2,(S,S)}^*) > U_0^{2, Aut},$$

which contradicts the fact that $PC_2$ is binding.

$v) PC_1$ is binding.

Immediate consequence of $ii)$ and $iv)$.

$vi) x_{2,(S,S)}^* = 1$.

Immediate consequence of $i)$ and $iv)$.

$vi) x_{1,(F,S)}^* < x^* < x_{2,(S,F)}^*.$

From $v)$ we have that $PC_1$ is binding. Since a decrease in $x_{1,(F,S)}^*$ increases $U_0^{1,BU}(x_{1,(F,S)}^*, x_{2,(F,S)}^*, x_{1,(S,S)}^*)$, and thus relaxes $PC_1$, from (36) aggregate welfare maximization implies that $\Pi'(x_{1,(F,S)}^*) < 0$. Analogously optimality implies that $\Pi'(x_{2,(F,S)}^*) > 0$. The statement then results from the assumption that $\Pi(x)$ is convex and achieves its minimum at $x^*$.

References


