Modern depository institutions, runs and liquidity∗

David Rivero Leiva†
Universitat Autonoma de Barcelona

Hugo Rodríguez Mendizábal‡
Instituto de Análisis Económico (CSIC), Barcelona GSE, and MOVE

June 14th, 2017
(Preliminary and incomplete.
Please do not quote without permission)

Abstract

We show in this article that bank runs caused by self-fulfilling panics do not occur in an economy exposed to idiosyncratic liquidity shocks with endogeneous money creation. To do so, we modify the model by Diamond and Dybvig (JPE, 1983) to include inside money. Our depository institutions issue nominal demandable claims when new loans are originated, and manage a demand for reserves to offset liquidity risk. Unlike the previous literature wherein banks ensure the optimal amount of real liquidity, we argue that the uncontingent nature of nominal deposit contracts does not allow banks to provide efficient risk-sharing.

JEL classification: G21, E42
Keywords: Bank runs, inside money, risk-sharing.

∗Rodríguez Mendizábal would like to acknowledge the financial support from the Spanish Ministry of Economy and Competitiveness, through the Severo Ochoa Programme for Centres of Excellence in R&D (SEV-2015-0563) and through grant ECO2016-76734-P (AEI/FEDER, UE) as well as that of the Generalitat de Catalunya through Grant 2014SGR-1446. He is also grateful for financial support from the ADEMU project, “A Dynamic Economic and Monetary Union”, funded by the European Union’s Horizon 2020 Program under grant agreement No 649396. All errors are our responsibility.
†Address: Department of Applied Economics, Universitat Autonoma de Barcelona, 08193 Bellaterra, Barcelona, Spain. E-mail: david.rivero.leiva@gmail.com.
‡Address: Instituto de Análisis Económico. Campus UAB. 08193 Bellaterra Barcelona, Spain. E-mail: hugo.rodriguez@iae.csic.es.
1 Introduction

We study modern runs in a banking system where depository institutions issue nominal demandable claims to facilitate transactions. Failures of financial institutions are a recurrent event in the history of banking. A consistent explanation of the fundamental reasons for banking instability, however, is still an open question. According to recent developments in financial markets, bank runs seem to be provoked by the tightening of constraints on bank creditors rather than triggered exclusively by a classic coordination failure. The collapses of Bear Stearns and Lehman Brothers on 2007 are explained by their heavy reliance on short-term debt, which made both institutions more vulnerable when creditors and other investment banks lost confidence in their ability to redeem short-term loans after an increase in perceived risk (Lucas et al. [14]). The run of the UK bank Northern Rock on 2007 was caused by its high leverage coupled with reliance on institutional investors for short-term funding, and it did not seem to be related to a coordination failure (Shin [18]).

In a modern economy, deposit contracts are repayable in terms of money and withdrawals usually take the form of electronic transfers. Banks create the commitment to implement a payment system by which transactions are cleared. This vision contrasts with traditional banking models based on the seminal work of Diamond and Dybvig [7], which has been widely accepted as the cornerstone to study the roots of bank runs. Diamond and Dybvig [7] showed that, with no aggregate uncertainty, depositors can coordinate and lead to a self-fulfilling equilibrium wherein uninsured depositors rush to withdraw their savings from the banking system. When it happens, banks are unable to honour their repayment obligations. The withdrawal decision of each depositor depends on her preferences and beliefs about the behavior of the rest of the population. The role of banks is the design of contingent debt arrangements by which individuals with different time preferences choose the bundle of deposits designed to them. Most of the posterior literature has focused on the mechanism design approach and the incentives of depositors to coordinate to a panic.1

Assuming that banks use real contracts, traditional banking models share two features. First, the essentiality of banking is the superior liquidity insurance achieved through the intermediation process. Second, the vulnerability of banks to runs is explained by the maturity transformation of long-maturity, illiquid assets into short-maturity, liquid liabilities.

We incorporate endogenous money creation into the standard liquidity problem of Diamond and Dybvig [7], wherein risk-adverse households face uncertainty about their specific consumption needs. The aim is to evaluate the implications of introducing inside money over the withdrawal incentives of depositors and the optimality of the risk-sharing deposit contracts. Several features must be remarked. First, instead of considering the traditional description of banking as financial vehicles that take assets from savers and lend them to ultimate borrowers, we consider that when banks originate a new loan they are creat-

1 See Jacklyn [13], Wallace [22], Green and Lin [12], Peck and Shell [16], and Ennis and Keister [9].
ing inside money and purchasing power rather than intermediating pre-existing money. Second, this intermediation process starts on the asset side of the bank’s balance sheet. The liquidity risk faced by the banking institution is due to the transfer of funds between banks, which is solved by managing a demand for outside money. Third, the maturity mismatch between bank assets and liabilities arises automatically when a loan is originated. The counterpart of the longer maturity associated to the asset the loan creates is the overnight disposable nature of the means of payment of the liability linked with it.

Under this nominal setting, we show that the real value of deposit contracts is contingent on the mass of withdrawals, which adjusts the demand for consumption at each date. This mechanism leads to the uniqueness of equilibrium. That is, in an economy where consumers withdraw money to make transactions, the real value of the deposit contract is contingent on the beliefs of patient households, and the withdrawal game depends on the price of goods at each period. We also provide some insights about the inefficiency of nominal deposit contracts. A distinctive feature of modern monetary financial institutions is the way in which their liabilities are used as means of payment. In a world with no record-keeping, this specialized financial vehicles are essential since they make a commitment to facilitate transactions among agents, clearing payments at the end of the trading day. A key feature of the present work, however, is the lack of commitment of depository institutions to ensure future consumption. With nominal deposit contracts repayable in cash, banks cannot set credible promises of repayment in the present because of their inability to set prices.

The present paper is related to models that have introduced money to explore bank runs in a setting with nominal deposit contracts. In sort, this strand of the literature states that nominal contracts can improve risk-sharing through the commitment’s ability of a third party. In most cases, the literature focuses on the commitment provided by central bank intervention by issuing at money. Banks are able to meet their commitments since the monetary authority, by regulating the price level, ensures the real value of deposits at each date. The adjustments in the price level guarantee that depositors receive the optimal allocation of consumption at each date according with their type.

Allen and Gale [1] introduce fiat money in a model of banking and show that variations in the price level allow nominal debt to become effectively state contingent so that risk-sharing is improved. In a similar paper, Allen et al. [2] also focus in the role of at money to improve risk-sharing in the economy. They state that the combination of nominal contracts and a central bank policy of accommodating commercial banks demand for money leads to first best efficiency. This result holds when there are aggregate liquidity and asset return shocks and also when there are idiosyncratic liquidity shocks. The closest paper to us is Skeie [19], who states that with nominal deposit contracts payable in money (e.g. currency) bank runs are not explained by liquidity shocks so additional frictions are required to explain a bank turmoil. The no-bank run equilibrium is also explained by price adjustments on the goods’ market. However, he states

---

2 This approach is close to the “new view” of banking mentioned by Tobin [20].
that banks allocate liquidity optimally in the economy, so nominal contracts are a Pareto improvement with respect to real contracts, which are susceptible to multiple equilibria. Diamond and Rajan [8], in contrast, find that nominal contracts cannot prevent bank runs when there are idiosyncratic risk on the bank’s asset side caused by delays in assets returns.

Contrary to this strand of literature, we find that nominal contracts do not provide optimal risk-sharing in an economy exposed to pure liquidity shocks. Previous studies are based on the assumption that the quantity theory of money holds in equilibrium: the price level at each date is proportional to the supply of money extended to the commercial banks by the central bank. Doing so, the monetary authority creates a commitment about the future value of real deposits providing the proper level of contingency to the nominal debt contract designed by the depository institution. In a nominal setting with endogenous inside money, however, we state that banks cannot commit that prices will reflect the optimal allocation of goods to achieve the optimal risk-sharing.

The paper is structured as follows. Section 2 briefly reviews the empirical discussion whether bank runs are driven by self-fulfilling panics or fundamentals. Section 3 describes the real economy. Money is introduced in section 4, where we study the existence of panics in a nominal setup. Finally, section 5 concludes.

2 Causes of bank runs

A crucial debate about financial fragility is whether bank runs are explained by self-fulfilling panics or whether they reflect a deterioration in bank specific variables or fundamentals. In a self-fulfilling panic, depositors, anticipating liquidity shortages, coordinate in a run, causing the bank to actually fail even though the depository institution would otherwise remain solvent. Fundamental-based runs, on the contrary, are caused by a perceived worsening of the balance sheet of the bank.

In this respect, the empirical literature has found a strong link between bank crises and fundamentals. Although bank failures that have occurred in the United States up to the 1930s were initially used as case studies of self-fulfilling runs (Friedman and Schwartz [10]), nowadays it is not clear, however, to what extent illiquidity itself was the cause of those bank closures. For example, Gorton [11] shows how bank crises during the National Banking Era in the US between 1863 and 1914 could be linked to an increased perception of risk on the part of depositors. Another important episode for bank failures is the Great Depression. Calomiris and Mason [5] found that deterioration in bank-specific variables as well as in the economic conditions of the geographical area these banks were operating in, affected the probability of failure during the Great Depression for banks within the Federal Reserve System. These authors also found evidence indicating the importance of fundamental components in the Chicago banking panic of June 1932 (Calomiris and Mason [4]). Another example is the collapse of the Bank of the United States in late 1930. According to Trescott [21], at the time of its failure, the bank was insolvent, not just illiquid.

3
Furthermore, the fragmented banking industry during the 1920s is an important characteristic to support the role of fundamentals. At that time, banks in the US were small and they operated regionally, so the bank failures in 1930 and 1931 might also have been originated by large regional specific shocks (Wicker [23]).

Similar evidence can be found for other countries and time periods. For instance, Demirguc-Kunt and Detragiache [6] estimate the probability of a banking crisis with a multinomial logit model using data for 65 countries over the period 1980-1994. They find that the likelihood of a crisis is affected by aggregate variables like low GDP growth, high real interest rates or high inflation. Similarly, Schumacher [17] tested these two hypotheses of bank runs by studying the bank run that took place in Argentina following the currency run triggered by the Mexican devaluation of December 20, 1994. Her analysis supports the idea that the failures and mergers that were commonly viewed as a consequence of the runs were actually rooted in some pre-existing conditions, such as solvency weakness and poor loan portfolio quality that made them more vulnerable to the currency shock. Finally, Martinez Peria and Schmukler [15] use data from Argentina, Chile and Mexico between the 1980s and 1990s to show that depositors tend to withdraw their funds and require higher interest rates from banks pursuing riskier strategies.

Whether individual or system-wide runs are explained exclusively by a self-fulfilling component is still an open question. One possible problem with the empirical literature on bank crises is that, as Gorton [11] recognizes, the panic hypothesis of bank runs does not produce a set of testable implications while the fundamental approach does. Therefore, it is difficult to empirically confront one hypothesis with the other. One answer to this debate is to theoretically show under which conditions banks are fragile and susceptible to self-fulfilling runs. The demonstration that self-fulfilling panic events are inconsistent with a nominal economy producing inside money is our contribution to this literature.

3 The real setup

3.1 The environment

The model is basically the one described in Diamond and Dybvig but introducing labor in the production technology. As it will be clear below, this modification does not alter the production possibility frontier in any respect.

The economy is characterized by a circle with measure 1. On each point of the circle there is a continuum of identical risk averse households with measure 1. These households are composed of a worker and an entrepreneur. There are three dates indexed by $t = 0, 1, 2$. At $t = 0$, each worker is endowed with a unit of time, whereas entrepreneurs have access to a risk-free, constant return to scale productive technology. A unit of labor employed in this technology at $t = 0$ produces $r > 1$ units of the good at $t = 2$. If a fraction $x$ of the production is interrupted at $t = 1$ it will produce a scrap value equal to $x$. The remaining
fraction \( 1 - x \) left until maturity of the production process will yield \( r(1 - x) \) at \( t = 2 \). In addition to the productive technology, households have also access to storage without any cost. Obviously, nobody will store anything from \( t = 0 \) to \( t = 1 \) because at \( t = 0 \) storing is dominated by the production technology. However, it may be the case that storage could be used between \( t = 1 \) and \( t = 2 \).

At period \( t = 0 \), households face uncertainty in the form of the timing of consumption. With probability \( \lambda \in [0, 1] \) the household becomes of type 1 and likes to consume only at \( t = 1 \) (impatient households) whereas with probability \( 1 - \lambda \) the household becomes of type 2 and would prefer to wait until \( t = 2 \) to consume (patient households). Notice that, because of the Law of Large Numbers, \( \lambda \) is also the fraction of households consuming at \( t = 1 \), while \( 1 - \lambda \) is the fraction of households consuming at \( t = 2 \), both in each location and also along the circle. Finally, suppose the household has a period utility function \( u(c) \) with

\[
u'(c) > 0, \quad u''(c) < 0, \quad \lim_{c \to 0} u'(c) = \infty, \quad \text{and} \quad \lim_{c \to \infty} u'(c) = 0.
\]

Assume further that the coefficient of relative risk aversion satisfies

\[
-\frac{cu'(c)}{u''(c)} > 1 \tag{1}
\]
everywhere.

### 3.2 The efficient allocation

Let \( c_{1}^{h} \) be the consumption of a household of type \( h \in \{1, 2\} \) at period \( t = 1, 2 \). A household of type \( h \in \{1, 2\} \) faces the problem of choosing the fraction \( \chi_{h} \in [0, 1] \) of the productive technology to liquidate at \( t = 1 \) in response to the realized type they become, either patient (\( h = 1 \)) or impatient (\( h = 2 \)) together with the sequence of consumptions \( \chi_{h}^{c} \).

If types were publicly observable, it is easy to see that a planner who treats each household identically would choose \( c_{1}^{1*} = c_{2}^{1*} = 0 \), while determining \( c_{1}^{1} \), and \( c_{2}^{2} \), together with the aggregate fraction of the productive technology to be liquidated at \( t = 1 \), \( 0 \leq x \leq 1 \), to maximize

\[
\lambda u(c_{1}^{1}) + (1 - \lambda)u(c_{2}^{2}) \tag{2}
\]
subject to the feasibility constraints

\[
\lambda c_{1}^{1} \leq x, \tag{3}
\]
and

\[
(1 - \lambda)c_{2}^{2} \leq r(1 - x). \tag{4}
\]
Substituting (3) and (4) in (2), the first order condition of this problem is

\[
u'(c_{1}^{1*}) = ru'(c_{2}^{2*})
\]
which denotes the efficient risk-sharing condition, as in Diamond and Dybvig [7]. Because \( r > 1 \) and because of the assumed degree of risk aversion in (1), it turns out that \( 1 < c_{1}^{1*} < c_{2}^{2*} < r \), which provides some insurance to households.
3.3 Non-contingent labor and goods markets

Assume now types are privately observable only and decentralize this economy by introducing labor and asset markets. In the labor market entrepreneurs hire workers at $t = 0$ and put them to work in the productive technology. These workers are all paid at $t = 1$ the competitive real rate $w$. Additionally, at $t = 1$ a goods market opens in which $t = 1$ consumption goods exchange for $t = 2$ consumption goods at the price $q$. In this case, both, patient and impatient households face the budget constraint

$$c_1^h + qc_2^h \leq w + \left[ x^h + qr(1 - x^h) - w \right].$$

The left hand side is the present value of consumption while the right hand side is the total revenue of the household in period $t = 1$ coming either from wage payment, $w$, or from profits by the entrepreneur $[x^h + qr(1 - x^h) - w]$. For the goods market to clear it must be the case that

$$q = \frac{1}{r}$$

which implies $c_1^e = 1$ and $c_2^e = r$ together with $c_1^2e = c_2^1e = 0$ where the $e$ superscript denotes the market equilibrium allocation. Furthermore, with perfect competition in the goods and labor markets, the zero profits condition will imply that entrepreneurs determine the fraction of the production technology to be liquidated, $x^b$, so as to fulfill

$$x^h + q(1 - x^h)r = w$$

so that $w^e = 1$.

Because there are $\lambda$ households consuming at $t = 1$, market clearing in the $t = 1$ goods market means

$$\lambda c_1^r = \lambda x^1 + (1 - \lambda) x^2$$

which implies aggregate liquidation at $t = 1$ equals $\lambda x^1 + (1 - \lambda) x^2 = \lambda$. If we compare this allocation with that of the planner we observe markets do not provide enough insurance.

3.4 A contingent time bank

A possibility to decentralize the first best allocation is to set a contingent time bank. Imagine a time depository institution appears in which workers can deposit their time at $t = 0$. Those workers are put to work by the bank in the productive technology. The bank then promises the households to give $c_1$ to anyone withdrawing deposits at $t = 1$, or $c_2$ if the withdrawal is done at $t = 2$. Notice the bank does not know types and the return on deposits can only depend on the timing of withdrawals. To find the equilibrium allocation, the bank
then solves for the choice of the aggregate liquidation rate, \( x \), to maximize the expected utility of households

\[
\lambda u(c_1) + (1 - \lambda)u(c_2)
\]

subject to the budget constraints faced by the bank

\[
\lambda c_1 \leq x, \quad (5)
\]

\[
(1 - \lambda)c_2 \leq (1 - x)r, \quad (6)
\]

Obviously this time bank is able to attain the same solution as the planner as in the bank problem in Diamond and Dybvig [7].

This time deposit contract is efficient because it determines implicit contingent wages to be paid to households at \( t = 1 \) depending on their realized types. Looking at (5) and (6), for one unit of labor, impatient households obtain an equivalent wage of

\[
w_1 = \frac{x}{\lambda} = c_1
\]

while for patient households the \( t = 1 \) equivalent wage is

\[
w_2 = \frac{1 - x}{1 - \lambda} = \frac{c_2}{r}.
\]

Because of the assumed degree of risk aversion (1), it must be the case that

\[
w_1 = \frac{x}{\lambda} > w = 1 > \frac{1 - x}{1 - \lambda} = w_2,
\]

where \( w \) is the non contingent labor market wage of the previous section.

As in Diamond and Dybvig [7], the promise to pay \((c_1, c_2)\) supports a suboptimal equilibria in which all patient households run the bank. In such scenario, the bank is required to liquidate its long term investment to service deposit withdrawals at \( t = 1 \). The liquidation value of the bank at \( t = 1 \) is \( 1 \). Because \( c_1 > 1 \), all the bank’s assets will be used up at date 1 in the attempt to meet the demands of the early withdrawers. Anyone who waits until the last period will get nothing. Thus, it will be optimal for a late consumer to withdraw at date 1 and store the proceeds until \( t = 2 \).

4 The nominal economy

4.1 The environment

In this section we introduce nominal deposit contracts into the economy. To do so, we incorporate the idea that banks create endogenous money in the form of deposits when providing loans. We then explore the extent to which these nominal contracts achieve optimal risk-sharing and study whether self-fulfilling panics do occur.
Production technologies and preferences remain equal as described in Section 2. That is, entrepreneurs use labor hired at \( t = 0 \) in the productive technologies whose proceeds will be collected at either \( t = 1 \) or \( t = 2 \), respectively. To introduce a role for banks, assume entrepreneurs cannot hire the workers living with them in the same household. This means entrepreneurs need to hire workers in a competitive labor market. Furthermore, when workers are hired, at \( t = 0 \), these entrepreneurs lack the credibility to convince workers they will get paid in the future, when production takes place either at \( t = 1 \) or \( t = 2 \).

The function of banks under this setting is to intermediate between households in this payment process as follows. Assume each location is served only by one bank. At \( t = 0 \), entrepreneurs borrow inside money, \( \mathcal{W} \), from the bank located in the same location they live in. This loan produces a double entry in the bank’s balance sheet. On the asset side, the bank annotates the right associated with the loan taken by the entrepreneur. On the other hand, means of payments are created, and the liability side reflects the right of the entrepreneur to dispose of those funds to make payments. In particular, the loan will be used by entrepreneurs to pay workers. The interest rate of these loans is \( i \) to be paid at the end of period \( t = 2 \). Notice the introduction of banks solves the commitment problem of the entrepreneurs as wages are paid in advance. Furthermore, it also solves any commitment problem on the part of workers as the receipt from these transfers is proof of the wage payments and, therefore, can be used by entrepreneurs to claim the workers’ labor services. Additionally, deposits are homogeneous units of account that can be used by depositors to buy goods from entrepreneurs at either \( t = 1 \) or \( t = 2 \). When a depositor (worker) pays for consumption goods, he transfers these deposits to an entrepreneur. A nominal price for consumption goods will be formed as deposits are exchanged for goods. Entrepreneurs then use these revenues from selling the goods they produce to pay back the loan they asked for at period 0. That is the reason these deposits are accepted back by entrepreneurs in exchange for consumption goods. Thus, with the introduction of depository institutions, loan and deposit creation by banks are used to bridge the intertemporal gap between the wage and goods payments in this economy.

The balance sheet of the bank at the time loans are granted on period 0 is:

| Balance sheet of bank when loans are made at period \( t = 0 \) |
|-----------------|-----------------|
| **Assets** | **Liabilities** |
| Loans (entrep.) | \( \mathcal{W} \) |
| Deposits (entrep.) | \( \mathcal{W} \) |

Still at time \( t = 0 \), the loan is used by entrepreneurs to pay workers in advance for their labor services so that the nominal wage bill is \( \mathcal{W} \). Thus, after the wage payments are done, banks just rename deposits who are now owned by workers, that is,

| Balance sheet of bank when wages are paid at period \( t = 0 \) |
|-----------------|-----------------|
| **Assets** | **Liabilities** |
| Loans (entrep.) | \( \mathcal{W} \) |
| Deposits (workers) | \( \mathcal{W} \) |
At the beginning of the interim period, \( t = 1 \), depositors (workers) receive the remuneration from deposits at the interest rate \( i^1 \). Then, the liquidity shock realizes, so that a fraction \( \lambda \) of the households decide to consume on that period. This means they will transfer part of their deposits to entrepreneurs in exchange for goods produced at \( t = 1 \). At the beginning of period \( t = 2 \), interest \( i^2 \) are paid on deposits carried over from period \( t = 1 \). Then, the remaining fraction \( 1 - \lambda \) of households pay for consumption too. Finally, entrepreneurs collect revenues from selling the goods which are then used by the household to pay back the loan they asked for at the beginning of period \( t = 0 \).

This setup includes several features worth mentioning. First, the intermediation role played by banks starts when a borrower asks for a loan. The loan is produced because the borrower lacks the means of payment to make a purchase. Then, once these means of payment, in the form of deposits, are created, they circulate in the economy as broad money as long as the loan does not mature. Notice this sequence of events is opposite to the one in Diamond and Dybvig [7] where the intermediation process starts when a saver deposit assets in the bank to be loaned out to a borrower.

Second, this intermediation service implies two obligations to depository institutions. From a liability side perspective, the production of deposits means the bank has to service the payment orders of depositors. Obviously, if the owner of the deposits make payments to other clients of the same bank, this service obligation is very easy to fulfill. The bank just renames the owner of the deposits. However, if the destination of the payment is a client of another bank, then the transfer of deposits must be met by a transfer of liquid assets. This will also happen if the depositor wants to convert the deposit into outside money (cash). Unlike Diamond and Dybvig [7], these liquid assets are not deposits that are left idle or invested in an inferior short term technology (remember the intermediation process starts on the asset side of the bank’s balance sheet). These liquid assets are borrowed from the central bank either in the form of cash in banks' vaults or in the form of reserves (current accounts in the central bank). Thus, the liquidity risk banks face, and therefore, the need to borrow outside money from the central bank, is not so much related to depositors disposing of their deposits as of the transfer of funds between banks and into outside money. For example, in the model here, under the assumption that all payments are distributed evenly across all banks, depositors will be disposing of their deposits but there would not be any need for banks to hold liquid assets as all net flows between them will be zero.3

From an asset side perspective, the other obligation has to do with the possibility that the value of the loans falls below that of the liabilities. This may well happen if a fraction of loans are not repaid in full. Notice the value of deposits, and the corresponding value of the obligations they generate, should

---

3In practice, central banks support a payment system through which transactions can be easily settled. According to the BIS [3], the pivotal role of central bank money in payment systems “reflects the layered architecture of financial systems, whereby private individuals and non-financial businesses hold (part of) their liquidity in banks, and banks in turn hold (part of) their liquidity in the central bank”.

9
not be affected by that event. This obligation is met by a new category of liabilities, capital, which should absorb fluctuations in the value of assets. In the model here, because loans are to be repaid, there are no solvency issues and capital is not needed.

The third point to stress is the maturity mismatch between bank assets and liabilities. In the model, loans take two periods to mature while deposits are available to depositors anytime. Notice this maturity mismatch is an inevitable consequence of loan provision. Of course, the bank can manage its balance sheet to reduce or eliminate that maturity mismatch. However, because loans are provided essentially to produce the means of payments a borrower lacks and, therefore, to be disposable immediately, automatically the asset the loan creates will have a longer maturity than the liability associated with it.

Finally, the loan the bank holds provides a right to a future flow of funds to the bank. This flow of funds originates from the production and selling activities of the borrowers (the entrepreneurs in the model). An important point to make is that forcing the bank to liquidate that asset to respond to a deposit outflow, does not necessarily imply the production activity financed with that loan has to be liquidated too. The bank cannot recall the loan or force the borrower to pay earlier than what is established in the loan contract. What the bank can do is to try to sell the asset in the market, possibly at a discount. This means the bank is only obtaining a fraction of the present discounted value of cash flows produced by the loan. But selling the loan this way, per se, has nothing to do with the ability of the borrower to pay back the loan. Thus, liquidation of bank assets is just a redistribution of future flows between market participants and does not need to imply a real cost for society as a whole.

4.2 Only inside money

In the previous section, we have described what the balance sheet of banks would look like at the end of time $t = 0$. In this and the next section, we will evaluate the consequences for this balance sheet and final net worth of the bank derived from the flow of funds between banks at $t = 1$ and $t = 2$. In this sense, we have previously argued that there are two possible uses of deposits. One appears when the transfers of deposits happen between clients of the same bank. The second occurs when there is a net flow of funds between depository institutions. This net flow of funds must be met with outside money in the form of either reserves or cash previously obtained from a central bank loan. In this section, we consider the case in which deposits cannot be converted into cash. This means that withdrawals take the form of electronic transfers among depository institutions. Because these transfers are equal among banks, the net flow of funds is zero and there is no need for outside money. The next section will cover the case where net flows between banks are not zero.

We focus on a symmetric equilibrium in which individuals of the same type follow the same strategy. Without cash convertibility, impatient agents do not have incentives to withdraw. However, patient households may have reasons to rush and use their deposits to buy consumption goods at $t = 1$. We first solve
for the no bank run equilibrium and then wonder whether an equilibrium with a bank run exists.

4.2.1 No bank run equilibrium

Let $P_1$ be the nominal price level of the consumption good at $t = 1, 2$. Then, any of the impatient households, those of type $h = 1$, faces the following budget constraint at $t = 1$

$$P_1 c^1_t + S^1 \leq (1 + i_1^t)W + P_1 x^1,$$

while at $t = 2$ it must be the case that

$$(1 + i^t)W \leq P_2(1 - x^1)r + (1 + i_2^t)S^1.$$

At the beginning of $t = 1$, the household starts with the return on deposits carried over from period $t = 0$, $(1 + i_1^t)W$. During the period $t = 1$, the entrepreneur sells, at the current price $P_1$, the production from the liquidation of the productive technology, $P_1 x^1$, and the household buys consumption goods, $P_1 c^1_t$. Thus, $S^1$ is their nominal savings accumulated at $t = 1$. At the beginning of period $t = 2$, the household starts with the return on those deposits, $(1 + i_2^t)S^1$. These resources together with the revenue from selling the fraction of the production technology that remains in place, $P_2(1 - x^1)r$, should be enough to pay back the loan, $(1 + i^t)W$. Notice here we impose that, because the household is impatient, it will consume all goods purchased at $t = 1$ and will not be optimal for the household to buy any consumption goods at $t = 2$ ($c^2_1 = 0$). These budget constraints collapse to

$$c^1_t \leq x^1 + (1 - x^1)r - (1 + i_1^t)W - 1 + i^t P_1 (W/P_1).$$

On the other hand, any of the patient households, those of type $h = 2$, faces the budget constraint at $t = 1$

$$P_1 c^2_t + S^2 \leq (1 + i_1^t)W + P_1 x^2,$$

while at $t = 2$ the budget constraint is

$$P_2 c^2_t + (1 + i^t)W \leq P_2(1 - x^2)r + P_2 c^2_t + (1 + i_2^t)S^2,$$

where $c^2_2$ is consumption done at $t = 2$ and $S^2$ is the nominal savings of patient households. Notice, we include the possibility of patient households buying consumption goods at $t = 1$, $c^2_1 > 0$, and storing them to be consumed at $t = 2$. Similarly, these constraints collapse to

$$c^2_1 \leq \left[1 - (1 + i_1^t) \left(\frac{P_1}{P_2}\right)^2 \right] c^2_1 + (1 + i_2^t) \left(\frac{P_1}{P_2}\right) x^2 + (1 - x^2)r$$

$$+(1 + i_1^t)(1 + i_2^t) \left(\frac{W}{P_2}\right) - (1 + i^t) \left(\frac{W}{P_2}\right).$$
In this problem it is important to notice that the decision to liquidate the productive technology, that is, the choice of $x^h$, $h = \{1, 2\}$, is taken by the households, not by the bank, as in Diamond and Dybvig [7]. At this point, the decision to run the bank has to do with the possibility of using deposits at $t = 1$ to buy goods and carry them to $t = 2$ instead of holding the deposits themselves. Of course, the change in demand for goods, and possibly prices, may affect the liquidation choice in general equilibrium. But this is different than assuming that the bank run automatically forces productive investment to be liquidated as done in Diamond and Dybvig [7].

Notice that for an equilibrium to exist at $t = 1$, households have to liquidate part of the productive technology, $x^h > 0$. Otherwise impatient households will not have anything to consume. For that to happen, it must be the case that

$$(1 + \nu_2) \left(\frac{P_1}{P_2}\right) \geq r.$$  

However, because $r > 1$, this necessarily means that the optimal choice for patient households is to set $c_2 = 0$. In other words, deposit holding dominates storage between $t = 1$ and $t = 2$. Then the intertemporal budget constraint for patient households become

$$c_2 \leq (1 + \nu_2) \left(\frac{P_1}{P_2}\right) x^2 + (1 - x^2)r + (1 + \nu_1)(1 + \nu_2) \left(\frac{W}{P_2}\right) - (1 + \nu_1)\frac{W}{P_2}. \quad (10)$$

Thus, the choice of $x^h$, $h = \{1, 2\}$, is such that

$$x^h = \begin{cases} 
1 & \text{if } (1 + \nu_2) \left(\frac{P_1}{P_2}\right) > r \\
\in (0, 1) & \text{if } (1 + \nu_2) \left(\frac{P_1}{P_2}\right) = r. 
\end{cases} \quad (11)$$

In either case, substituting this expression back in (9) we have

$$c_1^1 = 1 + \left(1 + \nu_1^d - \frac{1 + \nu_1}{1 + \nu_2} \right) \left(\frac{W}{P_1}\right) \quad (12)$$

so that

$$S^1 = (1 + \nu_1^d)W + P_1 x^1 - P_1 c_1^1$$

$$= W + P_1 x^1 - P_1 \left(1 + \nu_1^d - \frac{1 + \nu_1}{1 + \nu_2}\right) W$$

$$= \frac{1 + \nu_1^d}{1 + \nu_2} W - (1 - x^1)P_1. \quad (13)$$

On the other hand,

$$S^2 = (1 + \nu_1^d)W + P_1 x^2. \quad (14)$$
To close the model, the net worth of banks at the end of $t = 2$ is

$$NW = (1 + i^d)W - \lambda(1 + i^d_2)S^1 - (1 - \lambda)(1 + i^d_2)S^2.$$  

That is, all households return the loan $W$ at the rate $i^d$ while the bank has to pay a return $i^d_2$ to the savings $S^2$ of the $\lambda$ impatient households as well as the savings $S^2$ of the $1 - \lambda$ patient households. Substituting for $S^1$ and $S^2$ from (13) and (14), respectively, and imposing the zero profit condition implies that

$$\left(1 + i^d_1 - \frac{1 + i^d}{1 + i^d_2}\right)\left(\frac{W}{P_1}\right) = \frac{\lambda(1 - x^1) - (1 - \lambda)x^2}{1 - \lambda}. \quad (15)$$

As mentioned above, there are two possibilities regarding the liquidation choice, expression (11). If

$$(1 + i^d_2)\left(\frac{P_1}{P_2}\right) = r$$

then $x^h \in (0, 1)$ so that from (12) $c^1_1$ becomes

$$c^1_1 = 1 + \frac{\lambda(1 - x^1) - (1 - \lambda)x^2}{1 - \lambda}.$$  

Provided that the supply of goods at $t = 1$ is $\lambda x^1 + (1 - \lambda)x^2$, market clearing for goods

$$\lambda c^1_1 = \lambda x^1 + (1 - \lambda)x^2,$$

implies

$$c^1_1 = 1,$$

so that

$$c^2_2 = r,$$

and

$$\lambda = \lambda x^1 + (1 - \lambda)x^2.$$  

This means

$$(1 + i^d_1)(1 + i^d_2) = 1 + i^d.$$  

On the other hand, it may be the case that

$$(1 + i^d_2)\left(\frac{P_1}{P_2}\right) > r,$$

so that $x^h = 1$ for both, $h = \{1, 2\}$. In such a case, combining expressions (10), (12) and (15) we can see that the consumption of both impatient and patient households is $c^1_1 = c^2_2 = 0$. However, provided that all projects are liquidated, the market clearing condition for goods at $t = 1$ and $t = 2$ is $\lambda c^1_1 + (1 - \lambda)c^2_2 = 1$. So, this cannot be an equilibrium.
Thus, the banking solution implies that deposits between $t = 1$ and $t = 2$ must produce a real return equal to the productive technology

$$(1 + i_2^d) \left( \frac{P_1}{P_2} \right) = r$$

so that the consumption allocation ($c_1^1 = 1$, $c_2^2 = r$) coincides with the suboptimal equilibrium associated with the noncontingent labor and asset market. The reason for this inefficient allocation is the same both in the market as well as in this banking solution. With its nominal deposit contract the bank cannot promise real returns which are contingent on household types.

Finally, given that everyone is following this policy and keeping their deposits in the bank, we have seen that there are no incentives for any patient household to accumulate goods in period $t = 1$ and store them until $t = 2$ since deposits dominate storage in rate of return, $c_1^2 = 0$. Therefore, it does not pay to deviate in this case and the bank equilibrium is stable.

### 4.2.2 Is there a bank run equilibrium?

In the previous section we have found out that it is not in the interest of an individual patient household to dispose of their deposits at $t = 1$ and transfer goods to $t = 2$. The reason is that deposits pay a gross real return of $r > 1$, see (16), while storing goods implies a gross real return of 1. Notice that if it does not pay for an individual household to run, for the very same reason it seems that it will also not pay for all households of a single bank to run. With a run in only one bank, prices will still be the same as there is a continuum of banks and, again, maintaining deposits dominates carrying goods from one period to the next. This logic, however, only applies if all payments at $t = 1$ are done among the clients of the bank that is run. It therefore misses the point that if there is a run in a single bank, funds, most likely, will be transferred to another depository institution. Thus, the bank may be facing liquidity risks associated with the run. For that reason, the analysis of this case is delayed to the next section where we allow for cash convertibility.

A second possibility has to do with all patient households coordinating into buying goods at $t = 1$ and running all banks. With an economy wide bank run prices will change both at $t = 1$ and $t = 2$. To see whether this is an equilibrium, we need to check the incentives for a patient agent to run the bank, given that all other patient agents are deviating and buying goods at $t = 1$. Because all these payments cancel out between banks, deposits of impatient agents at the end of period $t = 1$, will be, respectively

$$S^{1R} = (1 + i_{1j}^d)W + P_{1R}^R x^{1R} - P_{1R} c_{1R}^1,$$

and

$$S^{2R} = (1 + i_{1j}^d)W + P_{1R}^R x^{2R} - P_{1R} c_{2R}^1,$$

where the superscript $R$ in aggregate prices denotes the solution when the aggregate bank run takes place and the superscript $R$ in the individual choices
denotes the household is also running the bank.\footnote{Notice interest rates on deposits from \( t = 0 \) to \( t = 1 \), \( i_{t1} \), are fixed at \( t = 0 \) and, therefore, do not include the possibility that there is a run at \( t = 1 \). The same thing happens with nominal deposits, \( W \) and the loan rate \( i \).}

In this case, because the total measure of households buying goods at \( t = 1 \) is equal to 1 and the aggregate supply of goods is only \( \lambda x^{1R} + (1 - \lambda)x^{2R} \equiv x^R < 1 \), each agent (independently of being patient or impatient) will buy \( c_{t1}^R = x^R < 1 \) goods with their \( t = 1 \) deposits. Impatient agents consume it, clearly making them worse off with respect to the equilibrium with no run. Patient households store it for consumption at \( t = 2 \). Substituting these purchases in the deposit holdings implies

\[
S^{1R} = (1 + i_{t1})W + P_{1R}x^{1R} - P_{1R}x^R
= (1 + i_{t1})W + P_{1R}(1 - \lambda) (x^{1R} - x^{2R}),
\]

and

\[
S^{2R} = (1 + i_{t1})W + P_{1R}x^{2R} - P_{1R}x^R
= (1 + i_{t1})W - P_{1R} \lambda (x^{1R} - x^{2R}).
\]

As mentioned above, although everyone is running on their bank, because there are no flows outside the banking system, the transfer of deposits just cancel out among depository institutions and households still maintain deposits from \( t = 1 \) until \( t = 2 \).

The net worth of the bank at \( t = 2 \) when there is a run, would be

\[
NW^R = (1 + i')W - \lambda (1 + i_{t2}^{dR})S^{1R} - (1 - \lambda)(1 + i_{t2}^{dR})S^{2R}
= (1 + i')W - \lambda [(1 + i_{t1})W + P_{1R}(1 - \lambda) (x^{1R} - x^{2R})]
- (1 - \lambda)(1 + i_{t2}^{dR}) [(1 + i_{t1})W - P_{1R} \lambda (x^{1R} - x^{2R})]
= (1 + i')W - (1 + i_{t2}^{dR})(1 + i_{t1})W.
\]

Again, the zero profit condition implies

\[
(1 + i_{t1})(1 + i_{t2}^{dR}) = 1 + i',
\]

which means that \( i_{t2}^{dR} = i_{t2}' \) and the bank still is solvent even with the run. Obviously this is because no bank looses funding if deposits are just reshuffled between them.

In this situation, consumption of any patient agent will be

\[
c_{t2}^R = \left[ 1 - (1 + i_{t2}^{dR}) \left( \frac{P_{1R}}{P_{2R}} \right) \right] c_{t2}^R + (1 + i_{t2}') \left( \frac{P_{1R}}{P_{2R}} \right) x^R + (1 - x^R)r
+ (1 + i_{t2}')(1 + i_{t2}^{dR}) \left( \frac{W}{P_{2R}} \right) - (1 + i') W
= \left[ 1 - (1 + i_{t2}^{dR}) \left( \frac{P_{1R}}{P_{2R}} \right) \right] x^R + (1 + i_{t2}^{dR}) \left( \frac{P_{1R}}{P_{2R}} \right) x^R + (1 - x^R)r.
\]
Notice that for patient households to run the bank and spend all their accumulate deposits buying goods so as to make \( c_{1}^{R} > 0 \), it must be the case that

\[
(1 + i_{2}^{R}) \left( \frac{P_{1}^{R}}{P_{2}^{R}} \right) \leq 1.
\]

However, at the same time, for both patient and impatient households to liquidate some of the productive technology (so that \( x^{h^{R}} > 0, h = \{1, 2\} \)) and have goods to be purchased at \( t = 1 \), it still have to be the case that

\[
(1 + i_{2}^{R}) \left( \frac{P_{1}^{R}}{P_{2}^{R}} \right) \geq r > 1.
\]

Thus, without cash convertibility it is impossible to have both a run on banks and liquidation of the productive technology so there is no equilibrium with a bank run in this case.

### 4.3 Cash convertibility

We analyze now the possibility of bank runs with cash convertibility. Two points must be remarked. First, with this option, apart from deposits, patient households could use also either goods or cash to transfer purchasing power from \( t = 1 \) until \( t = 2 \). However, in the event of a run, we have already shown that it is not worth for patient households to buy goods at \( t = 1 \) and store them until \( t = 2 \). The existence of cash does not change that result. Thus, a run is now associated with cash withdrawals at \( t = 1 \). Below we analyze whether this is indeed an equilibrium. Second, allowing for cash convertibility forces banks to demand outside money. Banks gather this liquidity from an open market operation conducted by a central bank at \( t = 1 \) at the interest rate \( i^{e} \). This interest rate is set exogenously by the monetary authority.

#### 4.3.1 No bank run equilibrium

In section 3.2.1 we have shown the stability of the nominal deposit equilibrium as patient households do not have any incentive to buy goods at \( t = 1 \) and store them until \( t = 2 \) given that nobody is running the bank. Here we show whether that is the case when the patient household which is deviating hoards cash instead.

Again, because this is a single household, prices remain at their equilibrium values. Let \( M^{2} \) be the cash accumulated at the end of period \( t = 1 \) by this household. It must be the case that

\[
M^{2} = (1 + i_{1}^{d})W + P_{1}x^{2}.
\]

Then, their budget constraint at \( t = 2 \) must read

\[
P_{2}c_{2}^{2D} + (1 + i^{l})W = P_{2}(1 - x^{2})r + M^{2},
\]

16
where, again, the superscript $D$ means a household deviating, so that

$$c_{2}^{2D} = (1 - x^2)r + \frac{P_1}{P_2} \Delta x^2 - (i^d - \bar{i}_1^d) \frac{W}{P_2}.$$  

Using the equilibrium prices

$$r = \frac{1 + i^d}{P_2/P_1}$$

and

$$(1 + \bar{i}_1^d)(1 + i^d) = 1 + i^d,$$

it turns out that

$$c_{2}^{2D} = (1 - x^2)r + \frac{x^2}{1 + \bar{i}_2^d} - (i^d - \bar{i}_1^d) \frac{W}{P_2} < r = c_2^2.$$  

Thus, still it does not pay to deviate and hoard cash instead of deposits. The reason is that since $i^d \geq 0$, deposits dominate cash as store of value between $t = 1$ and $t = 2$. This means the nominal deposit equilibrium still is stable under cash convertibility.

### 4.3.2 Is there a bank run equilibrium?

We turn now to the case where all households of a single bank are worried about a possible run. The difference with respect to the case in section 3.2.2. is that now all depositors, independently of their type, will convert nominal deposits into cash. Thus, the demands for cash by both impatient, call it $M^1$, and patient, call it $M^2$, households at $t = 1$ are given by

$$P_1c_{1}^{1R} + M^1 = (1 + i^d_1)W + P_1x^{1R}$$

and

$$M^2 = (1 + i^d_1)W + P_1x^{2R},$$

respectively. At $t = 2$, the budget constraint for impatient households is

$$(1 + i^2_1)W = P_2(1 - x^{1R})r + M^1,$$

while the constraint for patient households reads

$$(1 + i^2_1)W + P_2c_{2}^{2R} = P_2(1 - x^{2R})r + M^2,$$

where the superscript $R$, again represents the fact that households are running the bank. Notice since there is a run only on a single bank, prices remain at their equilibrium levels

$$r = \frac{1 + i^d_2}{P_2/P_1}$$

and

$$(1 + \bar{i}_1^d)(1 + i^d_2) = 1 + i^d.$$  

17
Substituting for $M^1$, the consumption level of impatient households is
\[ c^1 = (1 + \bar{i}^1) \frac{W}{P_1} + x^1 - M^1 \frac{W}{P_1} = (\bar{i}^d - \bar{i}) \frac{W}{P_1} + x^1 + P_2 (1 - x^1) r. \]
On the other hand, substituting for $M^2$ implies that consumption for patient households is
\[ c^2 = (1 - x^2) r - (1 + \bar{i}^1) \frac{W}{P_2} + M^2 \frac{W}{P_2} = (\bar{i}^d - \bar{i}) \frac{W}{P_2} + P_1 (1 - x^2) r. \]

The allocation $(c^1, c^2)$ will not be an equilibrium if two conditions hold simultaneously. First, given that everyone is running the bank, $(c^1, c^2)$ will not be an equilibrium if both impatient and patient households have an incentive not to withdraw. Second, given that everyone is running the bank, for $(c^1, c^2)$ not to be an equilibrium we also need not withdrawing to be feasible for a household, that is, the bank to be still solvent. To check the first condition, notice that for the aggregate prices (17) and (18) and using that $\gamma > 0$, it can be shown that
\[ c^1 = 1 + (1 - x^1) \bar{i}^1 - (1 + \bar{i}^1) \frac{W}{P_1} \bar{i}^d < 1 = c^1, \]
and
\[ c^2 = r - \left( 1 - \frac{1}{\bar{i}^2} \right) x^2 r - (\bar{i}^d - \bar{i}) \frac{W}{P_2} < r = c^2. \]
Thus, neither impatient nor patient households have an incentive to join the run. They may still do it, however, if they foresee the bank to be insolvent on period $t = 2$. To check that, the net worth of the bank in the event of a run is,
\[ NW^R = (1 + i^1) W - \lambda (1 + i^o) M^1 - (1 - \lambda) (1 + i^o) M^2 \]
since the bank needs to borrow from the central bank, at the rate $i^o$, $M^1$ units of cash to be delivered to impatient households and $M^2$ units of cash for patient households. Using the expressions for $c^1$ and $c^2$ in the budget constraints, the bank’s net worth is
\[ NW^R = \left[ 1 + \bar{i} - \lambda (1 + i^o)(1 + i^1) - (1 - \lambda) (1 + i^o)(1 + \bar{i}^1) \right] W + P_1 \left[ \lambda (1 - x^1) - (1 - \lambda) x^2 R \right]. \]
Because there are $\lambda$ impatient households and patient households do not demand goods at $t = 1$, for an equilibrium to exist it must be the case that
\[ \lambda = \lambda x^{1R} + (1 - \lambda) x^{2R}. \]
Then, the net worth of the bank will be positive as long as the central bank’s refinancing rate satisfies
\[ 1 + i^o < \frac{1 + \bar{i}^d}{1 + \lambda \bar{i}^2} \equiv 1 + \tilde{r}. \]
Thus, as long as the refinancing rate of the central bank is low enough, i.e. $0 \leq i^o \leq \tilde{r}$, there will be no equilibrium with a bank run.
5 Conclusions

Banking failures are a complex phenomena. The large variety of factors that foster the difficulties of the banking system, and the inherent characteristics of each financial institution make the evaluation of bank runs a hard task. In this sense, a relevant discussion about financial instability is whether runs of banking institutions are driven by self-fulfilling panics that force solvent banks to fail, or whether they reflect a fundamental deterioration of bank specific variables.

In this paper we show that it is hard to consider self-fulfilling panics as the only cause that explain bank runs in a modern economy where depository institutions issue nominal demandable claims to facilitate transactions. Based on the traditional liquidity problem of Diamond and Dybvig [7], our framework incorporates three additional elements into the theoretical literature of bank runs. First, the chain of intermediation starts when borrowers need money to make payments. Once a loan is originated, the bank creates deposits that borrowers use as means of payments that circulate through the economy as broad money. Second, depositors withdraw money making electronic transfers to other banking institutions or when they decide to hoard cash. To offset this liquidity risk, banks manage a demand for reserves from the central bank. Third, the maturity mismatch between banks assets and liabilities is inherent to the creation of new loans, and the liquidation of bank assets implies the redistribution of future flows these loans produce between market participants.

In such nominal setting, the existence of a unique equilibria is fundamented by the uncontestable nature of nominal deposit contracts. In addition, we show that depositors have no incentives to withdraw their funds independently of whether cash convertibility is allowed or not. Although coordination failures might contribute to bank instability, these results do not support the critical role played by the self-fulfilling hypothesis as a significant force of bank runs.

References


