Search and Monitoring in Credit Cycles*

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February 16, 2017

Preliminary draft, please do not circulate

Abstract

We develop a model where the market for commercial loans is characterized by search frictions, together with a costly-state verification contract à la Bernanke, Gertler and Gilchrist (1999). New entrepreneurs undertake a costly search for a financial intermediary with whom they can contract on a loan. The terms of this contract, including a default threshold, are optimally determined and depend, among other things, on expected future idiosyncratic level and volatility shocks of business activities. An “extensive margin” of financial acceleration emerges from this endogenous credit conditions to new risky firms. We embed this mechanism into a DSGE model that we estimate in order to quantify both the role of search frictions in firm dynamics and the importance of this mechanism along the business cycle.

1 Introduction

A vast modern macroeconomics strand studies the role of financial intermediation in driving and amplifying business cycles. In particular, the seminal paper by Bernanke, Gertler, and

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*The views expressed in these papers are the authors’ and do not represent those of the Bank of Finland or the Eurosystem. We thank Gene Ambrocio, Andrea Caggese, Francesco Furlanetto, Adam Gulan, Nobu Kiyotaki, Antti Ripatti, and Aino Silvo for fruitful comments at this early stage of the paper. Isoré acknowledges the financial support of the OP Pohjola Research Group Foundation.

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Gilchrist (1999), BGG henceforth, has shown that the presence of agency costs in borrowing relationships creates a “financial accelerator” which amplifies the size and persistence of the effects of shocks on macroeconomic aggregates. As adverse shocks hit the economy, more firms default on their loans, such that banks have to pay higher monitoring costs, which in turns increases the risk premium and cause all aggregate quantities, such as credit, investment and output in particular, to drop. This ‘costly state verification’ mechanism amplifies the initial effect of negative shocks and thereby reinforce the severity and length of recessions.

In spite of many extensions, this framework is so far mostly limited to an intensive margin of financial frictions in the sense that it refers to the existing firms in the economy only. Yet, one can naturally think that this mechanism may have an extensive margin dimension as well, in the sense of affecting the creation of new firms in the economy. Indeed, Cooley and Quadrini (2001), Black and Strahan (2002), and Lelarge et al. (2010) have documented that access to external finance is a major difficulty in the process of firm creation. In turn, firm dynamics matter for the business cycle. For instance, Bergin and Corsetti (2008), and Jaimovich and Floetotto (2008) document that variations in the number of active firms in the economy is an important source of macroeconomic fluctuations.

Meanwhile, a newer macroeconomic literature argues that search and matching frictions in the credit markets may also provide a source of financial acceleration. Indeed, this is the main result in the seminal papers by Den Haan et al. (2003) and Wasmer and Weil (2004) who consider that entrepreneurs without proper wealth have to invest time and/or effort in prospecting for commercial loans, such that production is conditional on such a successful search on the credit market. These papers are however very stylized and the mechanism not assessed along the business cycle. Empirically, Dell’ Ariccia and Garibaldi (2005) and Craig and Haubrich (2013) have supported the coexistence of creation and destruction of credit flows of credit at all points in time over the business cycles. This suggests that some firms start while other cease receiving loans at each date, such that the observation of net
credit flows may not be informative enough about the transmission of shocks to the real economy.

In this paper, we build a model where credit markets are simultaneously characterized by a costly-state verification mechanism à la BGG and search and matching frictions between entrepreneurs and banks. Specifically, new entrepreneurs have to search for banks from whom they could obtain a loan in order to start their business. Yet, the terms of this debt contract, as well as those of the existing firms in the economy, are determined by an optimal contracting problem in the presence of agency costs. A costly-state verification mechanism accounts for the presence of future uncertain returns and potential default from the entrepreneur. We further embed this mechanism into a DSGE model close to BGG and Christiano, Motto, and Rostagno (2014). Thereby, we are able to nest their results as a particular case where only the intensive margin of financial accelerator is at play, but introduce an additional extensive margin otherwise. Finally, we run estimations of this model on US data with Bayesian techniques.

Our contribution is both theoretical and empirical. On the one hand, we analyze how the optimal financial contract in the presence of agency costs is affected by the presence of search and matching frictions. In particular, we compare how the terms of this contract, i.e. the amount of the loan, the interest rate on the loan, and the default threshold are modified from a search-frictionless but otherwise similar setup. On the other hand, we assess the empirical relevance of this mechanism by quantifying the extensive and intensive (BGG-like) margins of the aggregate financial accelerator resulting from this framework along the business cycle. In particular, we observe how much does the CSV-search contract affect firm creation and destruction flows, and how much does this affect the macroeconomic aggregates over time. This also allows us to estimate the parameters of the search and matching approach of the credit market, which, to the best of our knowledge has not been done previously.

The intuition of our mechanism relates to the existence of long-term relationships,
stemming from a costly search process, on the credit markets. When entrepreneurs undergo a costly search for a loan, they tend to default less on that loan. Indeed, assuming there is a probability that their long-term financial relationship may be severed in case of default, incentives to default are lowered. In turn, this modifies the effect on the macro variables that generate a financial acceleration phenomenon.

The remainder of this paper is as follows. Section 2 describes the Model, especially the description and equilibrium of the credit market that is later on embedded into an estimated DSGE models. Section 3 describes the steady-state and give more intuition for the mechanisms at play in this economy. Section 4 provides simulations and Bayesian estimations. In particular, we consider most standard New Keynesian as well as search-specific shocks, in particular a separation rate shock, a matching efficiency shock, and a flow search cost shock, to evaluate their relative contribution. Finally, Section 5 concludes.

**Literature review**

Our work builds on and contributes to three strands of the literature.

First, we contribute to the literature on credit market search frictions. Seminal papers on search and matching frictions on the credit market are Wasmer and Weil (2004); Den Haan et al. (2003). These papers highlight the attractive properties of the search and matching framework in terms of amplification and propagation to study the credit market. Following Dell’Ariccia and Garibaldi (2005), the importance of the reallocation of credit between banks and firms has been documented in this literature by Herrera et al. (2011); Craig and Haubrich (2013); Hyun and Minetti (2014). Recently, Petrosky-Nadeau and Wasmer (2013, 2015) have extended this literature to the business cycles implications of credit-market search frictions. In particular, they assess the quantitative importance of these frictions to explain the puzzling dynamics of the labor market. We contribute to this literature by extending the scope of the analysis beyond the labor market. Indeed, we embed the credit market search frictions into an otherwise standard DSGE model with traditional real, nominal, and financial frictions. We can then assess the role of credit-market
search friction for the fluctuations in the labor input, but also in output, investment, consumption, credit, inflation or any other relevant variables generally considered in the literature based on DSGE models. We can also investigate the links between these various frictions. That is: how taken into account credit market search frictions alter the importance of other traditional real, nominal, and financial frictions?  

Second, we contribute to the literature on the financial accelerator based on the Costly-State Verification (CSV) framework of Townsend (1979), as originally developed by BGG. Credit-market search frictions per se give birth to a specific financial accelerator mechanism as demonstrated by Wasmer and Weil (2004). Our contribution to the literature on the financial accelerator is to show how search frictions modify the outcome of the financial contract based on the CSV imperfection. Various types of financial contract have been considered in the credit market search literature based on the Nash solution (e.g. Wasmer and Weil (2004)), on moral hazard issue (e.g. Den Haan et al. (2003)) or on adverse selection problem (e.g. Chamley and Rochon (2011)). However, to our knowledge, we are the first that use exactly the assumptions originally proposed by BGG and repeatedly implemented in macroeconomic models since. Actually, we show that the financial contract described by BGG can be viewed as particular case of our financial contract with search frictions. By doing so, we extend the financial accelerator mechanism in two directions. The first direction is the role of lending relationships. Our financial accelerator takes into account that borrowers and lenders establish lending relationships which are costly to destroy for both agents. Our approach is different from Melina and Villa (2014) who assume habit formation to give birth to a financial accelerator of lending relationship. Growing evidence demonstrate the importance of lending relationship to account for the macroeconomic effect of the financial crisis in the United-Sates, e.g. Chodorow-Reich (2014); Darmouni

\footnote{Previous attempt have been made in the literature to incorporate credit-market search frictions in DSGE models, see the working papers of Nicoletti et al. (2006); Liberati (2014); Reza (2013). Distinctive features of our approach are to rely on the estimation strategy recently applied to DSGE models with financial frictions by CMR and Del Negro et al. (2015) and to keep the core assumptions of the BGG financial accelerator mechanism in the model.}
(2016), and in Europe, e.g. Sette and Gobbi (2015). The second direction is the implications of the financial accelerator firm dynamics. Indeed, credit-market search frictions make endogenous the decision of entry for firms since searching for external finance is a costly activity. Moreover, in the presence of credit-market search frictions, when an entrepreneur makes default, she may lose her financial intermediary and then being excluded from the credit market (we interpret this event as an exit from the production sector) until she successfully matches with a new financial intermediary.

Third, and finally, we then contribute to the business cycle literature on firm dynamics. The seminal papers of Jaimovich and Floetotto (2008); Bilbiie et al. (2012) have launched an important literature on the role of firm dynamics in the business cycle. DSGE models have been developed and estimated to assess the origins of the fluctuations of the number of firms in the economy and the role of these fluctuations in the aggregate fluctuations. The contributions of Bergin and Corsetti (2008); Lewis (2009); Lewis and Poilly (2012a); Lewis and Stevens (2015); Lewis and Winkler (2016) illustrate this approach. Clementi and Palazzo (2016) supplement this literature by considering explicitly the demography of firms. A distinctive feature of our approach is to based firm dynamics on search frictions and not on monopolistic competition as generally considered in the literature. We do that because financial frictions are generally introduced in the competitive entrepreneurial sector in the literature and our objective is to link financial frictions and firm dynamics. By doing so, we follow Cooley and Quadrini (2001) who originally emphasized the importance of financial frictions for firm dynamics - without considering search frictions as in the business cycle model of Poutineau and Vermandel (2015). The link between the process of firm creation and credit-market search frictions has been investigated by Becsi et al. (2013) but at the steady-state without studying the business cycle implications as we do. One of the interest of search frictions it to provide a rationale for the congestion externality, which has been shown crucial in the business cycle literature on firm dynamics by Lewis (2009); Bergin and Lin (2012); Bergin et al. (2016). The congestion externality in this literature assumes
that the entry costs depends on the number of active firm. In the search model, the congestion externality in directly associated with the matching function and impacted by the agents’ decision: the matching costs depends on the number of searching firms. Finally, the endogenous processes of entry/exit of firms allow us to decompose the fluctuations in the credit into the intensive margin (by active firms or entrepreneurs) and the extensive margin (the number of active firms or entrepreneurs).

2 Model

The general environment is a dynamic stochastic general equilibrium model with financial frictions à la BGG and CMR, in which we incorporate credit search frictions. In 2.1, we describe the general structure of the model with the main relationships between agents. In 2.2 and 2.3, we develop the specific problems of entrepreneurs and financial intermediaries, respectively. Those agents play a role in the credit market of this economy, which is the center of our focus here, by the coexistence of search frictions and a costly-state verification contract. In 2.4, we analyze the equilibrium optimal contract and endogenous firm entry conditions. In 2.5 and 2.6, we describe the main aggregate conditions of this economy, relegating the problems of agents not impacted by search frictions to the Appendix.

2.1 Structure of the Model

Our decentralized economy is populated by infinitely-lived households and entrepreneurs. Households own firms which operate both in a competitive final good sector and in a monopolistic intermediate good sector. Households are identical and composed of a unitary continuum of differentiated workers whose labor force is, together with accumulated capital, provided to the intermediary good technology. They also own financial intermediaries, which transform bonds into loans (to entrepreneurs), in a competitive market. Therefore, households’ resources consist of labor earnings, revenues of accumulated capital, profits
from intermediate goods, bond yields, lump-sum transfers including transfers net of taxes from the government, and time-varying transfers from and to the entrepreneurial activities described further below. Purchases include consumption goods, short-term and long-term bonds, investment goods, and existing capital in the economy. It maximizes the expected value of the discounted utility of its members, derived from leisure and consumption, with habit formation.

Entrepreneurs engage into a risky business of transforming ‘raw capital’ accumulated by the households into ‘effective capital’ for further use by the production sector. They invest in capital purchases their own wealth as well as a loan contracted with financial intermediaries. The return on capital is risky, both at the idiosyncratic and aggregate levels, which can lead to situations where entrepreneurs have to default on their loan, in which case there exists a costly-state verification mechanism, described further below. In addition to these features, our credit market is characterized by search frictions, which also naturally affect the optimal loan contract. Free entry allows for an endogenous mass of active entrepreneurs in the economy, and ensures zero profit in the long run although individual entrepreneurs may make short-term profits from good realizations of the i.i.d shock.\(^2\)

Finally, a public sector sets the nominal interest rate according to a Taylor-type rule, raises taxes, issue bonds, and consume some final good. Public policy is effective through the assumption of stickiness \textit{à la} Calvo on both wages and prices of the intermediary good producers.\(^3\)

\(^2\)Individual entrepreneurs’ wealth accumulation is however not made explicit, here as in CMR. This can be justified in several ways: (i) individual entrepreneurs belong to an entity such as a parent company in charge of the wealth accumulation, in which case they are \textit{homogeneous}, or (ii) individual entrepreneurs make temporary savings but only use expected future wealth as a collateral for current borrowing, in which case they are \textit{representative} despite the presence of (ex-post) stochastic returns. In either case, only aggregate entrepreneurial wealth matters.

\(^3\)When not re-optimized, with exogenous probability, prices and wages are indexed on the target inflation rate. The final good is a Dixit-Stiglitz aggregator of differentiated intermediate goods produced from effective capital and labor.
2.2 Entrepreneurs

The total population of entrepreneurs is constant and normalized to unity. Yet, a (time-varying) fraction \((1 - \gamma_t)\) of them dies and is born again in every period. During their lifetime, entrepreneurs evolve across three distinct states, respectively ‘passive’, searching for a loan (‘unmatched’), and producing (‘matched’). A new-born entrepreneur is always ‘passive’, and then chooses to transit across the other states as follows:

- He/she can decide to seek a loan from a financial intermediary. In this case, he/she is labeled ‘unmatched’. There is a mass \(u_t^e\) of unmatched entrepreneurs as of time \(t\).
- If the search is successful, the entrepreneur becomes ‘matched’. Then, he/she can borrow from the financial intermediary by setting up a one-period debt contract in each period as long as this ‘financial relationship’ (i.e the match) continues. There is a mass \(m_t^e\) of matched entrepreneurs as of time \(t\).
- Conditional on survival, the matched entrepreneur stays in this state as long as he/she repays the loan. Among defaulting entrepreneurs, a fraction of them separate from the financier, in which case they become ‘unmatched’ again.\(^4\)

Figure 1 summarizes the lives of an unmatched and a matched entrepreneurs, given stochastic transitions across the two states.

2.2.1 Search for a loan

A constant returns-to-scale technology gives the periodic flow of new financial relationships as

\[
credit\ flow_t = z_t^c \left( u_t^e \right)^{\alpha^c} \left( u_t^m \right)^{1-\alpha^c}
\]

\(^4\)For the sake of simplicity, the separation rate is kept exogenous so far.
where \( u_{fi}^t \) denotes the number of financial intermediaries searching for an entrepreneur on the credit market, and \( z^c_t \) measures the efficiency of the matching process, evolving as

\[
\ln z^c_t = (1 - \rho^c) \ln z^c + \rho^c \ln z^c_{t-1} + \varepsilon^c_{t}, \tag{2}
\]

where \( \varepsilon^c_{t} \) is i.i.d \( \text{N} \sim (0, \sigma^c) \) and can be interpreted as an exogenous disturbance of the firm creation process. The periodic probability to match with a financial intermediary is

\[
p^t = \frac{\text{credit flow}_t}{u^i_t} = z^c_t \left( \frac{u^e_t}{u^i_t} \right)^{\alpha^c - 1} \left( \frac{u^i_t}{u^f_i} \right)^{1 - \alpha^c} = z^c_t \left( \frac{u^e_t}{u^f_i} \right)^{\alpha^c - 1} = z^c_t \theta^c_1^{\alpha^c - 1} \tag{3}
\]

where

\[
\theta_t \equiv \frac{u^e_t}{u^f_i} \tag{4}
\]

is the credit market tightness.

Let us assume that an unmatched entrepreneur’s decision to search during period \( t \) is
taken at the end of date $t - 1$, with full information on period $t$ search cost, $D_t^S$, matching probability, $p_t^θ$, and survival rate, $γ_t$, all taken as given.\(^5\) Thus, the value at $t - 1$ of searching at time $t$ is

$$
\mathcal{E}^u_{t-1} = -D_t^S + γ_t β^e \left[ p_t^θ E_m^t + (1 - p_t^θ) \mathcal{E}^u_t \right]
$$

(5)

where $β^e$ denotes entrepreneurs’ discount factor, and where $E_m^t$ is the expected present-value of being matched at the end of period $t$, which depends both on the realization of aggregate shocks during period $t$ and the draw of idiosyncratic productivity at the end of period $t$.\(^6\)

2.2.2 Production

Entrepreneurs who are matched in period $t$ decide at the end of period $t$ over their individual capital holding for the next period, denoted $K_{t+1}$ to be purchased at the market price $Q_{K,t}$, taken as given, from both personal wealth $N_{t+1}$ and a one-period debt amount $B_{t+1}$ contracted optimally with the financial intermediary at the end of time $t$. Thus, the capital purchase constraint satisfies

$$
Q_{K,t} K_{t+1} = N_{t+1} + B_{t+1}
$$

(6)

As in BGG-CMR, entrepreneurs may have different levels of wealth (and thus of capital purchases), yet wealth accumulation is not explicit and both types of funding are always required. Hence, entrepreneurs can never become so rich that they would not need intermediation. Similarly, our newly matched entrepreneurs supposedly hold a minimal amount of

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\(^5\) As individual entrepreneurs take as given the current level of financial tightness and matching probability, the standard congestion externality effect applies.

\(^6\) The expectation operator is omitted here as both aggregate innovations and individual draws are i.i.d., such that the decision to search at the end of period $t - 1$ is unaffected. The next period Bellman equation for matched entrepreneurs in period $t$ account for those and the law of iterated expectation applies at $t - 1$. 

11
wealth. The level of wealth, borrowing, and capital purchase are \( N \)-type specific, but the leverage resulting from the optimal financial contract will be identical for all entrepreneurs, in BGG-CMR’s spirit.

After capital is purchased, an idiosyncratic productivity shock, \( \omega \), converts capital, \( K_{t+1} \), into efficiency units, \( \omega K_{t+1} \). This random component \( \omega \) follows a cumulative distribution function \( F_t(\omega) \equiv F(\omega, \sigma_{\omega,t}) \) with a unit-mean and a standard deviation of \( \log \omega \) equal to \( \sigma_t \). The standard deviation \( \sigma_t \) is itself the result of an exogenous stochastic process, named “risk shock” as

\[
\log \sigma_t = (1 - \rho_\sigma) \log \sigma_\omega + \rho_\sigma \log \sigma_{\omega,t-1} + \varepsilon_{\omega,t}
\]

where \( \sigma_{\omega,t} \) is determined when the capital purchase is made.

In early period \( t + 1 \), knowing their shock realization, (matched) entrepreneurs rent \( u_{t+1} \omega K_{t+1} \) as capital services to intermediate firms, at the real rate \( r_{t+1}^k \) taken as given. The rate of capital utilization, \( u_{t+1} \), is chosen optimally by entrepreneurs, according to the aggregate conditions, and thus similar across all \( N \)-types. It is equal to 1 in steady-state. The utilization per unit of capital generates a cost \( a(u_{t+1}) \) for the productive entrepreneur, which is increasing and convex. Later in period \( t + 1 \), the entrepreneur sells the non-depreciated capital, \( (1 - \delta) \omega K_{t+1} \) to the households, at the market price \( Q_{K,t+1} \). The average/aggregate return per unit of capital invested in period \( t \) at date \( t + 1 \) is denoted \( R_{t+1}^k \) and \( \omega R_{t+1}^k \) at the individual entrepreneur’s level.

The debt contract between a matched entrepreneur and the financial intermediary is based on the costly-state verification framework. The contract is stated in the end of period \( t \), before the realization of the idiosyncratic shock, and is settled in the end of

\footnote{One can imagine that newly matched entrepreneurs are given a fixed initial amount of wealth to invest in the project, either by households or a parent company. Alternatively, one can interpret \( N_{t+1} \) as the valuation at date \( t \) of firms’ asset at date \( t + 1 \), which only depends on ex post returns and is thus similar for all entrepreneurs, regardless of the duration of their financial relationship, since individual uncertainty is i.i.d.}
period $t + 1$. For every state, defined by the realization of $\omega$, with the associated $R^k_{t+1}$, a matched entrepreneur has to either (i) pay a state-contingent gross interest rate $Z_{t+1}$ on the loan, $B_{t+1}$, or (ii) default, in which case the bank seizes all entrepreneur’s revenue net of a fraction $\mu$ spent on monitoring costs. This determines a threshold value $\overline{\omega}$ such that a productive entrepreneur pays back the loan if $\omega > \overline{\omega}_{t+1}$, where

$$R^k_{t+1}\overline{\omega}_{t+1}Q_{K,t}K_{t+1} = B_{t+1}Z_{t+1}$$

(8)

and default otherwise ($\omega \leq \overline{\omega}_{t+1}$). Note that the optimal default threshold will be the same for all entrepreneurs, and not $N$-type specific, since productivity draws are i.i.d across time and across entrepreneurs, as in BGG-CMR.\(^8\)

A fraction $s_{t+1}$ of credit relationships among the defaulting entrepreneurs are severed. We assume that the separation rate is exogenous and evolves as

$$\ln s_t^c = (1 - \rho_{s^c}) \ln s^c + \rho_{s^c} \ln s_{t-1}^c + \varepsilon_{s^c,t}$$

(9)

where $\varepsilon_{s^c,t}$ is an i.i.d $N \sim (0, \sigma_{s^c})$ separation shock. This simplifying assumption has the advantage to express that defaults resulting from low productivity draws do not necessarily lead to separation.\(^9\) Moreover, whenever $s^c = 0$, our model is able to nest CMR without search friction as a particular case. Finally, note that this separation of defaulting entrepreneurs, together with the death of $(1 - \gamma_t)m_t$ matched entrepreneurs, determines firm destruction in our model.

\(^8\)Depending on the realization of the idiosyncratic shock, ex post income flows and next period status (matched or unmatched) will vary. Were the default threshold $\overline{\omega}$ fixed, we would have an occasionally binding constraint à la Aiyagari (1994) and need to keep track of the income distribution of entrepreneurs in equilibrium. Instead, $\overline{\omega}$ is endogenous and, in the absence of binding constraint, identically chosen by all (representative) entrepreneurs.

\(^9\)This probability of separation could be made endogenous as an extension of the model.
2.2.3 Long-term value of the credit relationship

At the end of period \( t \), a \( N \)-type matched entrepreneur’s value reads as

\[
E^m_t = E_t \left\{ \int_{\omega_{t+1}}^{\infty} R^k_{t+1} \omega_t \left( N_{t+1} + B_{t+1} \right) - B_{t+1} Z_{t+1} + (1 - \gamma_{t+1}) P_{t+1} C^e_{t+1} \right\} + \beta E_t \left\{ \gamma_{t+1} \left[ \int_{\omega_{t+1}}^{\infty} E^m_{t+1} dF_t + s^e_{t+1} \int_0^{\sigma_{t+1}} \sigma dF_t + (1 - s^e_{t+1}) \int_0^{\sigma_{t+1}} E^m_{t+1} dF_t \right] \right\} 
\]

(10)

where the first term shows the nominal payoff of the entrepreneur net of loan reimbursement conditional on a productivity draw higher than the (endogenous) default threshold. If, on the contrary, the draw is below the threshold, then the payoff for the entrepreneur is zero since the financier seizes whatever has been produced. The second term \( C^e_{t+1} \) is the nominal consumption level of the matched entrepreneur if he/she dies in that period, with probability \( (1 - \gamma_{t+1}) \), whose level will be determined later on. This assumption is similar to CMR where all productive entrepreneurs who die consume. Here, it gives entrepreneurs a rationale to undertake costly search activities. Finally, the last three terms in the expression above represent the continuation values if the entrepreneur survives in the next period, with probability \( \gamma_{t+1} \). The continuation value states that the entrepreneur will remain matched in the next period if his/her current productivity draw is higher than the threshold or the default does not lead to a separation, or will become unmatched again otherwise.

After simplification, we can rewrite it as

\[
E^m_t = E_t \left\{ \left[ 1 - \Gamma_t (\omega_{t+1}) \right] R^k_{t+1} \left( N_{t+1} + B_{t+1} \right) + (1 - \gamma_{t+1}) P_{t+1} C^e_{t+1} \right\} + \beta E_t \left\{ \gamma_{t+1} \left[ E^m_{t+1} - F_t (\omega_{t+1}, \sigma_t) s^e_{t+1} \left( E^m_{t+1} - E^u_{t+1} \right) \right] \right\} 
\]

(11)

where, following BGG-CMR’s notation, \( \Gamma_t (\omega_{t+1}) \) stands for the share of entrepreneurial earnings that goes to the financial intermediary and satisfies

\[
\Gamma_t (\omega_{t+1}) = \left[ 1 - F_t (\omega_{t+1}) \right] \omega_{t+1} + G_t (\omega_{t+1}) \quad \text{with} \quad G_t (\omega_{t+1}) = \int_0^{\omega_{t+1}} \omega dF_t (\omega) 
\]

(12)
or, net of monitoring cost, $\Gamma_t (\omega_{t+1}) - \mu G_t (\omega_{t+1}) = [1 - F_t (\omega_{t+1})] \omega_{t+1} + (1 - \mu) G_t (\omega_{t+1})$.

Note the last term in equation (11) with a minus sign, which makes it apparent that there is a loss of surplus from severing a financial relationship. Were $s_t = 0$ (no separation), this term would vanish and the expected profits boil down to BGG-CMR.

### 2.3 Financial Intermediaries

Financial intermediaries’ population is constant over time. As of time $t$, a mass $u_{ti}$ of financiers are “searching” for an entrepreneur, yet at no cost, while others, $m_{ti}$ are engaged in a financial relationship. Each financier thus evolves between two states: unmatched and matched.\(^{10}\) By definition, matched financiers are as numerous as matched entrepreneurs, i.e. $m_{ti} = m_{te} \equiv m_t$.

The value of being unmatched for a financial intermediary in the end of period $t-1$ is

$$F_t^{u} = \beta_{t,t+1}^{u} \left[ \theta_t p_t^{\theta} F_t^{m} + \left( 1 - \theta_t p_t^{\theta} \right) F_t^{u} \right] = \beta_{t,t+1}^{u} \left[ \theta_t p_t^{\theta} (F_t^{m} - F_t^{u}) + F_t^{u} \right]$$

where $\beta_{t,t+1}^{u}$ is the stochastic discount factor from households owning the intermediaries\(^{11}\), $\theta_t p_t^{\theta}$ is financiers’ probability to match with an entrepreneur, and $F_t^{m}$ is financiers’ value of being matched as of time $t$.

When matched, a financier lends $B_{t+1}$ to the entrepreneur by borrowing the same amount from the households at the risk-free rate $R_t$. Even though the financial market is perfectly competitive and intermediaries makes zero profit, an interest rate spread still holds for the monitoring costs associated with borrowers’ risky activity. The financier’s payoff from the lending activity is either $Z_{t+1} B_{t+1}$ if the entrepreneur draws $\omega > \omega_{t+1}$ and does not default, or $(1 - \mu) R_k Q_{K,t} \omega_{K,t+1}$ if the entrepreneur draws $\omega \leq \omega_{t+1}$ and does not default.

\(^{10}\) Unlike entrepreneurs, financial intermediaries are never “passive”, because search is assumed to be costless for them. Endogenous entry of financial intermediaries would be an interesting extension of this setup.

\(^{11}\) Households’ problem is similar to CMR and thus relegated to Appendix. In particular, the expression for the stochastic discount factor is given by equation (A.20).
defaults. If there is no separation with the entrepreneur, and if the entrepreneur survives in \( t+1 \), the lending relationship continues and a new loan contract is initiated at that time. Otherwise, the financial intermediary gets the value of being unmatched (13). Hence, using (12), the value of being matched to a \( N \)-type entrepreneur is

\[
F_m^t = E_t \left\{ \frac{R_k k t+1 [\Gamma_t (\omega_{t+1}) - \mu G_t (\omega_{t+1})]}{R_t B_{t+1}} + \beta^*_{t,t+1} \left[ \gamma_{t+1} F_m^{t+1} + (1 - \gamma_{t+1}) F_t^{t+1} + \gamma_{t+1} F_t (\omega_{t+1}) s_{t+1}^e (F_m^{t+1} - F_t^{t+1}) \right] \right\} 
\]

(14)

### 2.4 Equilibrium conditions

#### 2.4.1 The free entry condition for entrepreneurs

By free entry, \( \mathcal{E}_t^u = 0, \forall t \), in equilibrium. Hence, equation (5) becomes

\[
\frac{D_S}{p_t^\beta} = \gamma_t \beta^e \mathcal{E}_t^m
\]

(15)

and equation (11) becomes

\[
\mathcal{E}_t^m = E_t \left\{ \left[ 1 - \Gamma_t (\omega_{t+1}) \right] R_k^{k t+1} (N_{t+1} + B_{t+1}) + (1 - \gamma_{t+1}) P_{t+1} C_{t+1}^e + \left[ 1 - F_t (\omega_{t+1}) s_{t+1}^e \right] \frac{D_S^{t+1}}{p_t^{\beta_{t+1}}} \right\}
\]

(16)

such that, combining both, the equilibrium condition for a \( N \)-type entrepreneur is

\[
\frac{D_S}{p_t^\beta} = \gamma_t \beta^e E_t \left\{ \left[ 1 - \Gamma_t (\omega_{t+1}) \right] R_k^{k t+1} (N_{t+1} + B_{t+1}) + (1 - \gamma_{t+1}) P_{t+1} C_{t+1}^e + \left[ 1 - F_t (\omega_{t+1}) s_{t+1}^e \right] \frac{D_S^{t+1}}{p_t^{\beta_{t+1}}} \right\}
\]

(17)

where the expected costs of successful search (LHS) equate the expected gains from search (RHS).
2.4.2 The optimal financial contract

The optimal contract is characterized by the threshold value of the idiosyncratic shock, $\omega_{t+1}$, the gross interest rate on loan, $Z_{t+1}$, and the level of debt, $B_{t+1}$, that maximize the expected present-value of a (matched) entrepreneur (16) subject to the financial intermediary’s participation constraint. In presence of search frictions, this constraint is restated as

$$F^m_t - F^n_t \geq 0$$

which binds in equilibrium, i.e $F^m_t = F^n_t = 0$, such that by (14), this implies

$$R_t B_{t+1} = R^k_t Q_{K,t} K_{t+1} [\Gamma_t (\omega_{t+1}) - \mu G_t (\omega_{t+1})]$$

where the left-hand side is known at the end of period $t$. Note that, in spite of the presence of search frictions, this constraint for the participation of financiers is exactly similar as in BGG, expressing that banks’ cost of borrowing on the LHS must be equal in equilibrium to their expected share of the output of the entrepreneur net of monitoring costs. Also note that the funds received by financial intermediaries in each period $(t+1)$ state of nature are assumed to be no less than the funds paid to households in that state of nature, such that (18) holds in each realized $(t+1)$ state of nature and not only in expectation as of time $t$. This assumption holds in CMR with free entry condition of financial intermediaries on the market for household deposits. Here, it still hold because search frictions are featured on the credit market and not the market for household deposits, where the $m$ matched financial intermediaries are in perfect competition.
Hence, the $N$-type entrepreneur’s maximization problem is

$$\max_{B_{t+1}, \omega_{t+1}} E_t \left\{ \left[ 1 - \Gamma_t (\omega_{t+1}) \right] R^k_{t+1} (N_{t+1} + B_{t+1}) + (1 - \gamma_{t+1}) P_{t+1} C^e_{t+1} + [1 - F_t (\omega_{t+1}) s^e_{t+1}] \frac{D^S_{t+1}}{P_{t+1}} \right. \\
+ \left. \lambda^c_{t+1} \left[ -R_t B_{t+1} + R^k_{t+1} (B_{t+1} + N_{t+1}) [\Gamma_t (\omega_{t+1}) - \mu G_t (\omega_{t+1})] \right] \right\}$$

(19)

where $\lambda^c_{t+1}$ denotes the Lagrange multiplier associated with the participation constraint (which depends on the $t+1$ state of nature), and where (6) has been used to introduce the loan variable in the objective function. The first-order condition with respect to $B_{t+1}$ is

$$E_t \left\{ \left[ 1 - \Gamma_t (\omega_{t+1}) \right] R^k_{t+1} - \lambda^c_{t+1} \left[ 1 - \frac{R^k_{t+1}}{R_t} [\Gamma_t (\omega_{t+1}) - \mu G_t (\omega_{t+1})] \right] \right\} = 0$$

(20)

The first order condition with respect to $\omega_{t+1}$ is

$$E_t \left\{ R^k_{t+1} Q_{K,t} K_{t+1} \left[ \lambda^c_{t+1} \left[ \Gamma^c_t (\omega_{t+1}) - \mu G^c_t (\omega_{t+1}) \right] - \Gamma^c_t (\omega_{t+1}) \right] - F^c_t (\omega_{t+1}) s^c_{t+1} E_t \frac{D^S_{t+1}}{P^c_{t+1}} \right\} = 0$$

(21)

with $\Gamma^c_t \equiv \frac{\partial \Gamma(\omega_{t+1}, \sigma_{\omega, t})}{\partial \omega} |_{\omega = \omega_{t+1}}$, $G^c_t \equiv \frac{\partial G(\omega_{t+1}, \sigma_{\omega, t})}{\partial \omega} |_{\omega = \omega_{t+1}}$, and $F^c_t \equiv \frac{\partial F(\omega_{t+1}, \sigma_{\omega, t})}{\partial \omega} |_{\omega = \omega_{t+1}}$.

In these two equations, the variables specific to the $N$-type entrepreneur are the next period level of capital $K_{t+1}$, the default threshold $\omega_{t+1}$, and the Lagrange multiplier $\lambda^c_{t+1}$. Using the entrepreneur’s equilibrium condition (??) to eliminate the level of capital, we are left with $\omega_{t+1}$ and $\lambda^c_{t+1}$, which are thus pinned down by (20) and (21), in equilibrium. This implies, as in BGG-CMR, that the optimal default threshold is chosen the same by all entrepreneurs in equilibrium, regardless of their level of net worth. Further, the level of capital $K_{t+1}$ and the level of borrowing $B_{t+1}$ are $N$-type specific, but entrepreneurs’ leverage is not since, by (6) and (18), we have

$$L_t \equiv \frac{Q_{K,t} K_{t+1}}{N_{t+1}} = \frac{1}{1 - \frac{R^k_{t+1}}{R_t} [\Gamma_t (\omega_{t+1}) - \mu G_t (\omega_{t+1})]}$$

(22)
which holds the same for all entrepreneurs. The third term of the optimal contract is the loan interest rate, \( Z_{t+1} \), as determined by (8). Note that (18) and (20) are identical to the BGG-CMR models, while the last term in (21) accounts for the effect of search frictions on the optimal contract. However, the BGG-CMR case is still nested as a particular case of our model, should the search cost \( D^S \) or the separation rate \( s^c \) be nil. Search frictions make the value of existing lending relationships positive by the existence of a positive probability \( F_t(ω_{t+1}) s^c_{t+1} \) that the entrepreneur must pay search costs again to find a new financial intermediary.

2.5 Aggregate dynamics of net worth

Aggregate variables are affected by the mass of entrepreneurs across different states (passive, searching, and producing). In particular, market-clearing for the physical capital requires \( \bar{K}_t = m_t K_t \) where \( \bar{K}_t \) is the aggregate capital supply from households, \( m_t \) the number of matched entrepreneurs where each is demanding \( K_t \) units of capital at time \( t \). Similarly for bonds, \( \bar{B}_t = m_t B_t \) and for the aggregation of net worth, \( \bar{N}_t = m_t N_t \).

This implies that the aggregate net worth evolves according to

\[
\bar{N}_{t+1} = \frac{m_{t+1}}{m_t} \left\{ \gamma_t [1 - \Gamma_{t-1} (ω_t)] R^k_t Q_{t+1} \bar{K}_t + W^e_t \right\}
\]

(23)

where the first term in the curly brackets is the production revenues from matched entrepreneurs at the end of period \( t - 1 \), who survive with probability \( \gamma_t \), and the second term \( W^e_t = m_t W^e_t \) is the aggregate transfer from households to matched entrepreneurs at the end of period \( t \).

This curly bracketed part is similar to CMR, except that the aggregate capital stock \( \bar{K}_t \) and transfer \( W^e_t \), is proportional to the amount of matched entrepreneur at time \( t \). The additional term \( \frac{m_{t+1}}{m_t} \) makes it clear that the accumulation of aggregate net worth also depends on the growth of matched entrepreneurs in this economy.

\[\text{12} \text{This assumption is similar to BGG-CMR and reflects the idea that a minimum level of wealth is necessary for the feasibility of external finance with asymmetric information.}\]
The aggregate leverage is thus identical to the individual leverage since

\[ \tilde{L}_t \equiv \frac{Q_{K,t} \tilde{K}_{t+1}}{N_{t+1}} = \frac{Q_{K,t} \tilde{K}_{t+1} m_{t+1}}{N_{t+1} m_{t+1}} = \frac{Q_{K,t} \tilde{K}_{t+1}}{N_{t+1}} = L_t \] (24)

The aggregate resource constraint of this economy, expressed in real terms, is given by

\[ Y_t = C_t + G_t + \frac{I_t}{\bar{Y}_t \mu_Y} + a(u_t) \gamma^{-t} K_t + \bar{D}_t^M + \bar{C}_t^e \] (25)

where the first four terms on the right-hand side stand for households’ consumption, public consumption, households’ investment in raw capital, and capital utilization costs, which are determined as in CMR and thus relegated to Appendix here. The fifth term stands for the monitoring costs, now proportional to the mass of matched entrepreneurs as

\[ \bar{D}_t^M = \mu G(\omega_t)(1 + R_k^t) \frac{Q_{t-1} \bar{K}_t}{P_t} \] (26)

and finally the last term is the aggregate non-survival payoff, in real terms, as

\[ \bar{C}_t^e = \frac{1 - \gamma_t}{\gamma_t} \Theta \frac{\bar{N}_{t+1} m_{t+1}}{P_t} - \bar{W}_t^e \] (27)

Indeed, the aggregate level of entrepreneurial assets, hold by matched entrepreneurs, in nominal terms, at the end of period \( t \), is \( [1 - \Gamma_{t-1} (\omega_t)] R_k^t Q_{K,t-1} \tilde{K}_t \). By (23), this is equal to \( (\bar{N}_{t+1} m_{t+1} / P_t) - \bar{W}_t^e / \gamma_t \). A fraction \( (1 - \gamma_t) / \gamma_t \) of it is hold by those who die, and a fraction \( \Theta \) itself consumed. Equivalently, at the individual (matched entrepreneur) level, \( C_t^e = \overline{C}_t^e / m_t \) is thus the non-survival payoff entering the Bellman equations, from (10) onward.13 15

13The households, monopolistic producers, and public authority are kept unchanged from CMR here.
14CMR’s main text page 39 (and footnote 17) does not have this dying entrepreneur consumption term in the resource constraint but it is indeed present in CMR’s Appendix equation (B23) (while (B13) is the model without entrepreneurs). If entrepreneurs were households’ members, it could alternatively be assumed that the non-survival consumption \( \bar{C}_t^e \) is part of household’s consumption \( C_t \).
15Note that the credit search costs are assumed non-pecuniary and thus do not appear in (25).
2.6 Aggregate dynamics of firms

The number of productive entrepreneurs evolves as

\[ m_{t+1} = \gamma_t \left[ \int_{\omega_t}^{\infty} m_t dF_{t-1} (\omega_t) + \int_0^{\omega_t} (1 - s_t^c) m_t dF_{t-1} (\omega_t) + \text{creditflow}_t \right] \]  

(28)

where the total number of matched entrepreneurs at the beginning of period \( t+1 \), denoted \( m_{t+1} \), is equal to the number of matched entrepreneurs that did not separate in period \( t \) plus the number of new matches. Old and newly matched entrepreneurs face the risk of dying with probability \( (1 - \gamma_t) \). Using (3), this can be rewritten as

\[ m_{t+1} = \gamma_t \left( [1 - F_{t-1} (\omega_t) s_t^c] m_t + z_t^c (u_t^f)^{\alpha^e} \left( u_t^f \right)^{1-\alpha^e} \right) \]  

(29)

The net creation of firms is measured as

\[ net_t = m_{t+1} - m_t \]  

(30)

and can be expressed as the difference between gross firm creation and destruction flows, i.e

\[ net_t = \gamma_t (\text{creditflow})_t - [(1 - \gamma_t) + \gamma_t F_{t-1} (\omega_t) s_t^c] m_t \]  

(31)

where firm creation as of time \( t \) is the flow of newly matched firms during period \( t \) which survive in fraction \( t \), and where firm destruction as of time \( t \) is the sum of dying firms, with a probability \( (1 - \gamma_t) \), and defaulting firms that separate with the financial intermediary with probability \( F_{t-1} (\omega_t) s_t^c \).

3 Steady-state Analysis

This section aims at providing more intuition about the role of search for the credit market outcomes in our economy. First, we examine how the presence of search affects the optimal
loan contract derivation, as compared to the search-frictionless BGG case. Second, we relate our findings with the traditional search literature by observing the determinants of the aggregate mass of unmatched entrepreneurs in the steady-state of our economy.

To simplify the analysis, we take prices, especially the interest rate, $R$, and the return on capital, $R^k$, as given here, which is indeed the case when deriving the optimum contract from the viewpoint of individually maximizing agents in Section 2.\textsuperscript{16}

### 3.1 Financial contract optimality conditions

At steady-state, the first-order condition with respect to the amount of debt reads as

$$\left(1 - \Gamma (\omega)\right) \frac{R^k}{R} = \lambda^c \left[1 - \frac{R^k}{R} \left(\Gamma (\omega) - \mu G (\omega)\right)\right]$$

The left-hand side of this expression is the entrepreneur’s marginal gain of holding debt. Additional capital purchases increase profits, with a share $(1 - \Gamma (\omega))$ left for the entrepreneur. The right-hand side is the entrepreneur’s marginal cost of holding debt, which is equal to the inverse of leverage, $L \equiv \frac{QK}{N} = \frac{B + N}{N} = \left[1 - \frac{R^k}{R} \left(\Gamma (\omega) - \mu G (\omega)\right)\right]^{-1}$ from (22), times the shadow value of wealth, $\lambda^c$.

The first-order condition with respect to the default threshold $\omega$,

$$R^k QK (\omega) \left\{ \lambda^c \left[\Gamma' (\omega) - \mu G' (\omega)\right] - \Gamma' (\omega) \right\} = F' (\omega) s^c \frac{D^S}{p^\theta}$$

holds for each individual (heterogeneous) entrepreneur’s level of capital, and thus requires to use the free-entry condition, simplified with $C^e = 0$ without loss of generality here,

$$\frac{D^S}{p^\theta} = \frac{(1 - \Gamma (\omega)) R^k QK (\omega)}{(\gamma \beta^e)^{-1} - 1 + F (\omega) s^c}$$

\textsuperscript{16}The impulse response functions of Section 4.4 confirm that the full general equilibrium model exhibit dynamic properties consistent with these analytical properties of the partial equilibrium steady-state. The appendix provides detailed calculations.
the solution for the shadow value of wealth, $\lambda^c$, is given by

\[
\lambda^c = \frac{\Gamma' (\omega)}{\Gamma' (\omega) - \mu G' (\omega)} + \frac{(1 - \Gamma (\omega))}{\Gamma' (\omega) - \mu G' (\omega)} \frac{F' (\omega) s^c}{(\gamma \beta^c)^{-1} - 1 + F (\omega) s^c}
\]

(33)

The first term of this expression is identical to BGG. It stands for the fact that an entrepreneur chooses the optimal value of the default threshold $\omega$ depending on its impact on his or her own share of profits. Indeed, an increase of $\omega$ implies a fall in the entrepreneur’s share \((1 - \Gamma (\omega))\) since \(\Gamma' (\omega) > 0\), and an increase in the bank’s share \((\Gamma (\omega) - \mu G (\omega))\), since we restrict the model to the cases where \((\Gamma' (\omega) - \mu G' (\omega))\) is positive as in BGG.

The second term is specific to the search friction and can be interpreted as the expected search cost of increasing the risk of default. An increase of $\omega$ leads to a higher risk of default $F$, the magnitude of this increase being given by $F'$, which leads to a separation of the financial relationship with a probability $s^c$. Upon separation, an entrepreneur loses the value of the financial relationship, which is equivalent to the expected total search costs (the per-period search costs $D^s$ divided by the matching probability $p^\theta$). It shows that the cost of defaulting is higher in an economy with search frictions than without since, in addition to a reduction in the share of returns, measured by $\Gamma'$, the return of past investment in search activities are lost in case of a separation. This last effect increases the shadow value of wealth as compared to the search-frictionless case.

There exists no explicit analytical solution for the optimal $\omega$ here as in BGG. Hence, we follow them in introducing the cutoff function $\rho (\omega)$ solving for

\[
\frac{R^k}{R} = \rho (\omega)
\]

(34)

Hence, combining (32) and (33) to substitute out $\lambda^c$ and dropping out the argument $\omega$, we
get

\[ \rho = \frac{\left( \Gamma' - \mu \Gamma \right) \left[ 1 + \frac{1 - \Gamma}{\Gamma'} \frac{F' s^c}{(\gamma \beta)^{-1} - 1 + F_s c} \right]}{(1 - \Gamma) + \left( \Gamma' - \mu \Gamma \right) (\Gamma - \mu G) \left[ 1 + \frac{1 - \Gamma}{\Gamma'} \frac{F' s^c}{(\gamma \beta)^{-1} - 1 + F_s c} \right]} \]

This function has several appealing properties. First, it depends on a single endogenous variable, the cutoff \( \overline{w} \), in this partial equilibrium analysis. In particular, it does not depend on the credit market tightness \( \theta \). Therefore, it gives an implicit solution for the equilibrium value of \( \overline{w} \) and, therefore also the leverage. Second, it nests BGG as a particular case, whenever \( s^c = 0 \), i.e whenever defaulting does not sever the financial relationship. Third, the impact of search frictions on the equilibrium financial contract are unambiguous.

To make this last point clear, let us compare the solutions with and without separation for defaulting entrepreneurs. Let us assume that a solution exists when there is no separation, \( s^c = 0 \), and denote it \( \overline{w}^*_{s^c=0} \), which satisfies \( \rho(\overline{w}^*_{s^c=0}) = R^k/R \). Let us also assume that a solution exists for a positive separation rate, \( s^c > 0 \), and denote it \( \overline{w}^*_{s^c>0} \), which satisfies \( \rho(\overline{w}^*_{s^c>0}) = R^k/R \). Then \( \overline{w}^*_{s^c>0} < \overline{w}^*_{s^c=0} \) since \( \rho'(\overline{w}^*_{s^c=0}) > 0 \), as in BGG. The economy with separations has a lower equilibrium default cutoff because of the associated probability that the matching costs of the relationship are lost. Implications for the leverage are also straightforward: \( L(\overline{w}^*_{s^c>0}) < L(\overline{w}^*_{s^c=0}) \) since \( L'(\overline{w}^*) > 0 \).

### 3.2 Long-run firm creation flow

We follow here the search and matching literature in order to determine the steady-state value of firm creations as the intersection of two equilibrium conditions. First, (28) reads, in steady-state, as

\[ p^\theta u^c \gamma = m (1 - \gamma + \gamma F(\overline{w}) s^c) \]

(36)
where the left-hand side stands for the firm creation flow in this long-run economy, while the right-hand side stands for the firm destruction flow, stemming from two sources, namely the exogenous death with probability \((1 - \gamma)\) among matched entrepreneurs, \(m\), and the separation with probability \(s^c\) among surviving matched entrepreneurs who default on the loan with probability \(F(\varpi)\). Using the matching function in (1), it can be rewritten as

\[
u_{BC}^{e}(u_{fi}, \varpi) = \left[ \frac{\text{pop}_{fi} - u_{fi} z_{c}(u_{fi})}{(1 - 1 + F(\varpi) s^c)} \left( \frac{1}{\gamma} - 1 - F(\varpi) s^c \right) \right]^{1/\alpha^c} \]

where \(z^c\) is the efficiency parameter of the matching function, \(\alpha^c\) the elasticity of the matching function with respect to searching entrepreneurs. (37) is the counterpart of the so-called ‘Beveridge curve’ for the labor market, i.e a decreasing and convex relationship between the two stocks of searching agents on the market, here \(u^e\) unmatched entrepreneurs and \(u^{fi}\) unmatched banks. A higher default cutoff \(\varpi\) implies a higher firm destruction rate \(F(\varpi)\) and thus a higher firm creation rate by this relationship, everything else equal.

The second equilibrium condition substitutes the definition of the tightness (4) out of the free-entry condition (17), which, taken at steady-state, gives

\[
u_{FE}^{e}(u_{fi}, \varpi) = u_{fi} \left[ \frac{z^c}{\lambda^S (\gamma \beta^e)^{-1}} - 1 + F(\varpi) s^c \right]^{1/(1 - \alpha^e)} \]

\(\lambda^S\) is the slope of this line depends on structural parameters and on the equilibrium cutoff \(\varpi\). The impact of \(\varpi\) is however ambiguous because of three effects. First, an increase in \(\varpi\) implies a reduction in the share of profits for the entrepreneur, \(1 - \Gamma(\varpi)\), which decreases the mass of searching entrepreneurs on the credit market. Second, an increase in \(\varpi\) increases the loan default rate, \(F(\varpi)\), which also diminishes the return of search activity and, then, the mass of searching entrepreneurs. The third effect acts in the opposite

\(^{17}\text{Recall that the flow of new matches, } p^a u^e, \text{ is not equal to the firm creation flow in this economy since only a share } \gamma \text{ of these new matches } (p^a u^e) \text{ survive in each period.}\)
direction, alleviating the participation constraint of banks by an increase in their profit share, \( \Gamma (\bar{z}) - \mu G (\bar{z}) \).

4 Estimation (Preliminary)

The model is estimated through Bayesian procedures surveyed by An and Schorfheide (2007).

4.1 Data

We use quarterly observations on twelve variables covering the period 1988Q1-2016Q1. First, we include eight macroeconomic aggregates, quite standard in Bayesian estimation of DSGE models, namely GDP, consumption, investment, inflation, wage, price of investment, hours worked and the short-term risk-free interest rate. We then have three financial variables similar to CMR, i.e credit, entrepreneurial net worth, and the credit spread. Finally, we add firm entry data from the Bureau of Labor Statistics.\(^{18}\)

Figure 2 depicts the particular time period of the Great Recession and the subsequent weak recovery, with the deviation of our series with respect to their values in 2007Q4. This pattern of firm creation is particularly interesting for our purpose. The fall in firm creation has been severe during the great recession – more than 15% in 2010Q2 – and the recovery has been very sluggish. In 2014Q1, the index of firm creation was still about 10% below the pre-crisis level. According to the last data, firm creation has still not recovered its pre-crisis level.

4.2 Parameter Values

Table 1 summarizes the parameters of the model that we calibrate and treat as fixed in the estimations. Table 2 shows the prior and posterior of estimated structural parameters and shock processes, for both models with search frictions and without (BGG-like).

\(^{18}\)See Appendix C for further details about the different series.
Some results are specific to the presence of search frictions and particularly worth mentioning here. First, according to our estimation results, the separation rate of matched defaulting entrepreneurs is equal to 3.26%, meaning that a substantial share of entrepreneurs lose their financial relationship after a default. Our estimation results also suggest that the separation rate is subject to substantial persistent shocks. The standard deviation of the shocks to the separation rate is estimated at 0.5871 with an coefficient of auto-correlation estimated at 0.92.

Second, recall that entrepreneurs who separate have to subsequently search for a new financial intermediary. The cost of search for this new relationship affects the optimal financial contract and the financial accelerator mechanism. Hence, some parameters which characterize the credit market equilibrium can differ quite significantly in the presence of search frictions. This is especially the case for monitoring costs, measured by the parameter...
Table 1: Calibrated parameters

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Name</th>
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<th>with search</th>
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<tr>
<td>$\beta$</td>
<td>Discount rate</td>
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<td>$\delta$</td>
<td>Depreciation rate of the economy</td>
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<td>$\alpha$</td>
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<td>1.60</td>
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<td>$\lambda_w$</td>
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<td>1.05</td>
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<tr>
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<td>$\pi_{target}$</td>
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<td>Tax rate on labor income</td>
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<tr>
<td>$u^e$</td>
<td>Share of searching entrepreneurs</td>
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<tr>
<td>$\beta^e$</td>
<td>Discount factor of entrepreneurs</td>
<td>-</td>
<td>0.899</td>
</tr>
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</table>

$\mu$, at the core of the financial accelerator mechanism described by BGG. In our version of the search-frictionless (BGG-like) economy, our estimated $\mu$ is 21.06%, very close to values previously obtained in the literature, in particular in BGG and CMR. However, in the presence of search frictions, the estimated value is clearly smaller, at 5.88%. There is a sort of reallocation between the monitoring costs and the search costs in the steady-state of this economy. Other parameter values are also substantially affected from the presence of search frictions, such as the monetary policy rule or costs associated with physical capital.

4.3 Historical Decomposition

Figure 3 depicts the historical growth rate of real GDP per capita and the contribution of three financial shocks, namely the equity shock, the risk shock, and the separation rate shock, depending whether search frictions are present or not in the model. Figure 4 does the same for the historical growth rate of real credit per capita.

Consistently with numerous narratives of the Great Recession, financial shocks are found to play a key role during this episode. However, whether search frictions are present or not matters. First, separation shocks are found to play a substantial role during the
Great Recession, but also before it. Indeed, during the 2000s decade, separation shocks seem to be at the origin of higher growth rates for both the real GDP and credit per capita.
Figure 5: Historical contribution of financial shocks to firm creation growth rate, year-on-year (1988Q1-2013Q4), with and without search frictions.

If we interpret separation shocks as credit supply shocks, this result is consistent with a less regulated financial sector where an ex ante less rigorous selection of ex post defaulting entrepreneurs takes place. Second, the presence of separation shocks does not reduce the role of other financial shocks. This especially holds for the risk shock which role during the Great Recession is even amplified when search frictions are at play as compared to the BGG-like economy.

As far as firm creation is concerned, Figure 5 reproduces the contribution of the same financial shocks, as well as an additional non-financial shock, namely a shock to the efficiency of the investment sector. This last shocks is found to play a particularly important role in the fall of firm creation during the Great Recession. This seems quite natural since, in the model, firm creation takes place in the entrepreneurial sector where physical capital is the primary input.
4.4 Impulse Response Functions

4.5 Effect of a separation shock

Figures 6 and 7 present the responses to a separation rate shock, $s^c_t$. As for the main macroeconomic variables in Figure 6, the separation shock has the same qualitative effect than the risk shock, i.e an increase in the risk premium and recessionary effects on credit growth, investment, output, labor, and consumption. As for the firm dynamics variables in 7, the same pattern than for the risk shock holds as well, with less entrepreneurs searching and less firm creation on the one hand, and an increase in firm destruction on the other hand, which both contribute to the slower net firm growth.

The shape of the responses of firm creation and destruction fits quite well to the data. Indeed, the destruction tends to spike in the beginning of recessions and return quite rapidly to its steady-state level, whereas the firm creation reaction is more delayed but prolonges the recession. One can see a similar pattern from the data we have collected on births and deaths of firm establishments in the US. This may tell that both risk shocks and separation shocks contribute to this empirical observation, which we have to confirm from the estimation.

5 Conclusion (Preliminary)

This paper builds a general equilibrium model where search and matching frictions in the credit market interact with the costly state-verification loan contract à la BGG-CMR. Search frictions modify the outcome of the financial contract as it increases the borrower’s cost of default by impairing its long-run financial relationship. The decision to search thus creates an endogenous extensive margin of financial acceleration in the model. The model is able to replicate the cyclical behaviour of firm dynamics observed in the data in response to risk shocks, as well as in response to some search-specific shocks as the separation rate of long-term financial relationships.
References


35


Mathematical Appendix

A  General equilibrium

Here, we describe the stationarization procedure and apply it to the full equilibrium set, a part of it being from Section 2 and the other reproduced from the CMR framework.

A.1 Stationarization

We have to account for both the trend of technical progress in the final good sector, \( z^*_t \), and, whenever necessary, the trend of technical progress in the sector of physical capital accumulation, \( \Upsilon^t \). Nominal variables are also divided by the price level, \( P_t \), to be expressed in real terms. Let us denote \( \mu_{z,t} \equiv z^*_t/z^*_t - 1 \) the rate of productivity growth in the final good sector and \( \pi_t \equiv P_t/P_{t-1} \) inflation. Unless otherwise stated, we use lower case letter for denoting detrended real variables, with an upper bar standing for the aggregate level.

First, output, consumption, investment, and public expenditures thus become \( y_t \equiv Y_t/z^*_t \), \( c_t \equiv C_t/z^*_t \), \( i_{z,t} \equiv I_t/(z^*_t \Upsilon^t) \), and \( g_t \equiv G_t/z^*_t \), respectively. The marginal utility of consumption is further denoted \( \lambda_{z,t} \equiv \lambda_t z^*_t P_t \) Aggregate capital, bonds, and entrepreneurial net worth are then \( \bar{k}_{t+1} \equiv \bar{K}_{t+1}/(z^*_t P_t) \), \( \bar{b}_{t+1} \equiv \bar{B}_{t+1}/(z^*_t P_t) \), and \( \bar{n}_{t+1} \equiv \bar{N}_{t+1}/(z^*_t P_t) \), respectively. Moreover, the price of capital is \( q_t \equiv Q_K \Upsilon^t/P_t \), such that stationarized capital purchases are \( q_t \bar{k}_{t+1} \equiv Q_K \bar{K}_{t+1}/(P_t z^*_t) \). Other prices, i.e wages and the rental rate of capital, become \( w_t \equiv W_t/(z^*_t P_t) \) and \( r^k_t \equiv \tilde{r}^k_t \Upsilon^t \). Net transfers from households to matched entrepreneurs are \( \bar{w}_t \equiv \bar{W}_t/(z^*_t P_t) \), the non-survival real consumption payoff is \( \bar{c}^e_t \equiv \bar{C}^e_t/z^*_t \), and real monitoring costs are \( \bar{d}^M_{t+1} \equiv \bar{D}^M_{t+1}/(z^*_t) \). Finally, individual non-pecuniary search costs are stationarized as \( \bar{d}^S_{t+1} \equiv D^S_{t+1}/(P_{t+1} z^*_t) \) to be substituted out in the individual free entry condition.
A.2  Equilibrium equations from the search part of the model (Section 2)

First, we have 6 search-specific variables as \{θ, p, m, u, u^f\} which are pinned down by 6 search-specific equations, stationarized as follows:

- the credit market tightness
  \[
  θ_t = \frac{u^c_t}{u^c_{t-1}},
  \]  
(A.1)

- entrepreneurs’ matching probability
  \[
  p^θ_t = z^c_t (θ_t)^{α^c - 1},
  \]  
(A.2)

- the evolution of the mass of matched entrepreneurs
  \[
  m_{t+1} = γ_t \left\{ [1 - F_t (ω_t) s^c_t] m_t + z^c_t (u^c_t)^{α^c} (u^f_t)^{1-α^c} \right\},
  \]  
(A.3)

- the free entry
  \[
  \frac{d^S_t}{p^θ_t} = γ_t β^θ E_t \left\{ [1 - Γ_t (ω_{t+1})] R_{t+1}^k \frac{c^k_{t+1}}{m_{t+1}} + π_{t+1} μ_{t+1} \left[ \frac{c^y_{t+1}}{m_{t+1}} + [1 - F_t (ω_{t+1}) s^c_{t+1}] \frac{d^S_{t+1}}{p^θ_{t+1}} \right] \right\},
  \]  
(A.4)

- the mass of searching banks
  \[
  u^f_t = \text{pop}^f_t - m_t,
  \]  
(A.5)

- the mass of passive entrepreneurs (neither searching for credit nor producing)
  \[
  \text{passive}_t = 1 - m_t - u^c_t.
  \]  
(A.6)

Then, optimality conditions from the financial contract are stationarized as follows. First, the participation constraint of financial intermediaries is identical to CMR and can be rewritten as

\[
\frac{q_t R_{t+1}}{m_{t+1}} = \left[ 1 - \frac{R_{t+1}}{R_t} [Γ_t (ω_{t+1}) - μG_t (ω_{t+1})] \right]^{-1}
\]  
(A.7)
The first-order condition with respect to the amount of debt is also similar to CMR before the substitution of the constraint multiplier $\lambda_{t+1}^c$, i.e.

$$
E_t \left\{ \left[ 1 - \Gamma_t(\varpi_{t+1}) \right] \frac{R_{t+1}^k}{R_t} - \lambda_{t+1}^c \left[ 1 - \frac{R_{t+1}^k}{R_t} \left[ \Gamma_t(\varpi_{t+1}) - \mu G_t(\varpi_{t+1}) \right] \right] \right\} = 0 \quad (A.8)
$$

However, the first order condition with respect to the default threshold is directly modified from search as

$$
E_t \left\{ R_t^k q_t k_{t+1} \left\{ \lambda_{t+1}^c \left[ \Gamma_t(\varpi_{t+1}) - \mu G_t(\varpi_{t+1}) \right] - \Gamma_t'(\varpi_{t+1}) \right\} \right\} = E_t \left\{ \frac{G_t'(\varpi_{t+1})}{\varpi_{t+1}} s_{t+1}^{c} \frac{d_{t+1}^c}{p_{t+1}^{c}} \right\},
$$

with
$$
\frac{G_t'(\varpi_{t+1})}{\varpi_{t+1}} = F_t'(\varpi_{t+1}) \quad (A.9)
$$

The time-varying number of matched entrepreneurs also implies to modify some aggregate expressions from CMR. In particular, the aggregate resource constraint is

$$
g_{z,t} = g_t + c_t + \frac{i_t}{\mu_{\gamma,t}} + a(u_t) \frac{\mathcal{E}_t}{\mu_{z,t}^*} + d_t^M + c_t^e \quad (A.10)
$$

where the aggregate detrended monitoring cost is

$$
d_t^M = \mu G(\omega_t)(1 + R_t^k q_t - \frac{\mathcal{E}_t}{\pi_t^*}) \quad (A.11)
$$

and the aggregate detrended non-survival payoff is

$$
c_t^e = \Theta \left( 1 - \gamma_t \right) \left( \frac{m_t}{\pi_{t+1}} - \mathcal{W}_t^c \right) \quad (A.12)
$$

Finally, the law of motion for aggregate entrepreneurial net worth is

$$
\pi_{t+1} = \frac{m_{t+1}}{m_t} \left\{ \gamma_t \left[ 1 - \Gamma_{t-1}(\varpi_t) \right] \frac{R_{t+1}^k q_t - \mathcal{E}_t}{\mu_{z,t}^* \pi_t} + \mathcal{W}_t^c \right\}
$$
\[
\frac{m_{t+1}}{m_t} = \frac{\gamma_t}{\mu_{z,t} \pi_t} \left\{ q_{t-1} \bar{e}_t \left[ R_t^k - R_{t-1} - \mu G_{t-1} (\tilde{\omega}_t) R_{t-1}^k \right] + \bar{\pi}_t R_{t-1} \right\} + \bar{w}_t \tag{A.13}
\]

where the last equality uses the participation constraint (A.7).

### A.3 Other equilibrium equations

This part reproduces the equilibrium conditions of CMR which are not modified from the presence of search frictions.\(^{19}\)

#### A.3.1 Households

The representative household maximizes the expected discounted sum of utilities given by

\[
\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \zeta_{c,t} \left\{ \log (C_t - b C_{t-1}) - \psi_L \int_0^1 \frac{h_{1,1}^{1+\sigma_L}}{1 + \sigma_L} \, di \right\}
\]

with \(b\) the degree of habit formation and \(\zeta_{c,t}\) a consumption preference shock, subject to

the law of capital accumulation

\[\bar{K}_{t+1} = (1 - \delta) \bar{K}_t + \left( 1 - S \left( \zeta_{l,t} \frac{I_t}{I_{t-1}} \right) \right) I_t\]

and the budget constraint

\[R_t B_t + (1 - \tau) \int_0^1 W_{i,t} h_{i,t} d_i + Q_{K,t} \bar{K}_{t+1} + \Pi_t = B_{t+1} + (1 + \tau^c) P_t C_t + Q_{K,t} (1 - \delta) \bar{K}_t + \frac{P_t}{\Gamma_t \mu_{T,t}} I_t\]

\(^{19}\)CMR incorporates an additional working capital channel in some expressions but activate it only when financial frictions are mute, as a particular case of their model. Hence, we omit these terms \(\Psi_L\) and \(\Psi_K\) here. Similarly, the tax rates are time-varying in some versions of their model but are treated as parameters in the estimated version and so they are here.
where \( \tau^c \) and \( \tau^l \) are consumption and labor tax rates respectively. After stationarization, the first-order conditions read as

\[
\frac{\mu^*_c \zeta_{c,t}}{\epsilon_t \mu^*_{z,t} - bc_{t-1}} - (1 + \tau^c) \zeta_{c,t} \lambda_{z,t} = b\beta \mathbb{E}_t \left( \frac{\zeta_{c,t+1}}{\epsilon_{t+1} \mu^*_{z,t+1} - bc_t} \right)
\]

(A.14)

for consumption, with \( \lambda_{z,t} \equiv \lambda_t \zeta^*_t \) \( P_t \) where \( \lambda_t \) is the Lagrange multiplier on the constraint;

\[
\zeta_{c,t} \lambda_{z,t} = \beta \mathbb{E}_t \left( \frac{\zeta_{c,t+1} \lambda_{z,t+1}}{\pi_{t+1} \mu^*_{z,t+1}} R_{t+1} \right)
\]

(A.15)

for short-term bonds, with \( R_t \) the nominal interest rate on those bonds; and

\[
\frac{1}{\mu^*_{z,t}} \left[ 1 - S \left( \frac{\zeta_{c,t} \mu^*_{z,t} \bar{Y}_t}{i_{t-1}} \right) - S' \left( \frac{\zeta_{c,t} \mu^*_{z,t} \bar{Y}_t}{i_{t-1}} \right) \right] = \beta \mathbb{E}_t \left[ \frac{\zeta_{c,t+1} \lambda_{z,t+1}}{\zeta_{c,t} \lambda_{z,t}} \frac{q_{t+1}}{\mu^*_{z,t+1} \bar{Y}} S'' \left( \frac{\zeta_{c,t+1} \lambda_{z,t+1} \bar{Y}_{i_{t+1}}}{i_t} \right) \zeta_{c,t} \left( \frac{\mu^*_{z,t+1} \bar{Y}_{i_{t+1}}}{i_t} \right)^2 \right]
\]

(A.16)

for investment, after replacing \( Q_{K,t} \bar{K}_{t+1} - Q_{K,t}(1 - \delta) \bar{K}_t \) by \( \beta \mathbb{E}_t \left[ Q_{K,t} \bar{K}_{t+1} - Q_{K,t}(1 - \delta) \bar{K}_t \right] \) \( I_t \) from the capital accumulation into the budget constraint.

After stationarization, the law of capital accumulation reads as

\[
\bar{K}_{t+1} = (1 - \delta) \frac{1}{\mu^*_{z,t} \bar{Y}} \bar{K}_t + \left[ 1 - S \left( \frac{\zeta_{c,t} \lambda_{z,t}}{i_{t-1}} \right) \right] \bar{Y}_t
\]

(A.17)

where the adjustment cost function is of the form

\[
S \left( x_t \right) = e^{\sqrt{\left(S''/2\right)(x_t - \bar{x})}} + e^{-\sqrt{\left(S''/2\right)(x_t - \bar{x})}} - 2
\]

where \( S'' \) is a parameter that determines the curvature of the cost function, and where \( x_t \equiv \frac{\bar{K}_t}{i_{t-1}} \zeta_{c,t} \) is the growth rate of investment multiplied by a shock on the marginal
efficiency of investment in producing capital. In stationarized terms, this is

\[ S(x_t) = e^{\sqrt{S''/2}}(\zeta_i\mu^*_z\tau_{it-1} - \mu^*_z\tau) + e^{-\sqrt{S''/2}}(\zeta_i\mu^*_z\tau_{it-1} - \mu^*_z\tau) - 2 \]  

(A.18)

with first-order derivative

\[ S'(x_t) = \sqrt{S''/2}(e^{\sqrt{S''/2}}(\zeta_i\mu^*_z\tau_{it-1} - \mu^*_z\tau) + e^{-\sqrt{S''/2}}(\zeta_i\mu^*_z\tau_{it-1} - \mu^*_z\tau) - 2) \]  

(A.19)

### A.3.2 Asset pricing

From (A.15), the (real) stochastic discount factor which satisfies \( E_t(\beta^{t}_{t+1}) = \frac{\pi_{t+1}}{R_t} \) is

\[ \beta^*_{t,t+1} = \beta \frac{1}{\mu^*_{z,t+1}} \frac{\zeta_t\lambda_{z,t+1}}{\zeta_t\lambda_{z,t}} \]  

(A.20)

Therefore, the return on raw capital (to the households) can be defined as

\[ E_t(\beta^*_{t,t+1}R^*_{t+1}^{raw,real}) = 1 \]

which, by the first-order condition on investment (A.16), is

\[ R^*_{t+1}^{raw,real} = \frac{\frac{q_{t+1}}{Y}S'(\frac{\zeta_{t+1}\mu^*_{z,t+1}\tau_{t+1}}{\mu^*_{z,t+1}})\zeta_{t+1}\left(\frac{\mu^*_{z,t+1}\tau_{t+1}}{\mu^*_{z,t+1}}\right)^2}{\frac{1}{\mu^*_{z,t+1}} - q_t \left[ 1 - S\left(\frac{\zeta_t\mu^*_{z,t}\tau_{t}}{\mu^*_{z,t}}\right) - S'\left(\frac{\zeta_t\mu^*_{z,t}\tau_{t}}{\mu^*_{z,t}}\right)\frac{\zeta_t\mu^*_{z,t}\tau}{\mu^*_{z,t}}\right]} \]

or, calculating an alternative first-order condition on capital as

\[ E_t\left( \beta \frac{1}{\mu^*_{z,t+1}} \frac{\zeta_t\lambda_{z,t+1}}{\zeta_t\lambda_{z,t}} \frac{q_{t+1}1 - \delta}{q_t} \right) = 1, \]

can be also written as

\[ R^*_{t+1}^{raw,real} = \frac{q_{t+1}1 - \delta}{q_t} \]

42
A.3.3 Entrepreneurs’ rent of capital

Entrepreneurs rent the effective capital to the monopolistic competition firms. This defines a (nominal) rate of return on effective capital, in detrended terms, as

$$R_k^t \equiv \frac{(1 - \tau^k)(u_t r^k_t - a(u_t)) + (1 - \delta)q_t}{\Upsilon q_{t-1}} + \tau^k \delta$$  \hspace{1cm} (A.21)

where \(u\) is the variable utilization rate of effective capital, \(r^k\) is the rental rate of effective capital, and \(a(u_t)\) an utilization rate cost function as

$$a_t = \frac{r^k_t}{\sigma_a} (e^{\sigma_a(u_t - 1)} - 1)$$  \hspace{1cm} (A.22)

with \(\sigma_a\) a parameter. In steady-state, note that \(a = 0\) regardless the value of \(\sigma_a\).\(^{20}\)

Entrepreneurs chose the optimal utilization rate of capital so as to maximize (A.21), i.e so as to satisfy the first-order condition

$$r^k_t = r^k_{ss} \exp (\sigma_a (u_t - 1))$$  \hspace{1cm} (A.23)

with \(r^k_{ss}\) the steady-state value of the rental rate of capital.

A.3.4 Monopolistic producers

- The cost minimization problem

Monopolistic producers, indexed by \(j\), demand effective capital and labor so as to maximize their cost of production, i.e

$$\min P_t r^k_t K_{j,t} + W_t l_{j,t}$$

\(^{20}\)Note that the return on effective capital (to entrepreneurs) can easily be related to the return on raw capital (to households) from (A.3.2), expressed in nominal terms, as

$$R^t_{t+1} = R^{raw}_{t+1} \left[ \frac{(1 - \tau^k)(u_{t+1} r^k_{t+1} - a(u_{t+1}))}{(1 - \delta)q_{t+1}} + 1 \right] + \tau^k \delta.$$
\[
\text{s.t } Y_{j,t} = \varepsilon_t (K_{j,t})^\alpha (z_{j,t})^{1-\alpha} - \varphi z_t^* 
\]

with \( \varphi z_t^* \) a fixed cost of production and \( \varepsilon_t \) a stationary production technology shock. The (stationarized) first-order conditions are

\[
r_t^k = \alpha \varepsilon_t \left( \frac{Y_{t} \mu_{z,t} h_t (w_t^*)^{\frac{\lambda_w}{\lambda_f-1}}}{u_t k_t} \right)^{1-\alpha} s_t \tag{A.24}
\]

with respect to \( K_{j,t} \), and

\[
w_t = (1 - \alpha) \varepsilon_t \left( \frac{h_t (w_t^*)^{\frac{\lambda_w}{\lambda_f-1}} \mu_{z,t} Y}{k_t u_t} \right)^{-\alpha} s_t 
\]

with respect to \( l_{j,t} \), substituting \( l_t = h_t (w_t^*)^{\frac{\lambda_w}{\lambda_f-1}} \), where \( s_t \) are the real marginal costs as

\[
s_t = \frac{1}{\varepsilon_t} \left( \frac{r_t^k}{\alpha} \right)^{\alpha} \left( \frac{w_t}{1 - \alpha} \right)^{1-\alpha} \tag{A.25}
\]

The stationarized production function finally reads as

\[
y_{z,t} = (p_t^*)^{\frac{\lambda_f}{\lambda_f-1}} \varepsilon_t \left[ \left( \frac{u_t k_t}{\mu_{z,t} Y} \right)^\alpha (h_t (w_t^*)^{\frac{\lambda_w}{\lambda_f-1}})^{1-\alpha} - \varphi \right] \tag{A.26}
\]

- Price maximization and the aggregate price index

The Dixit-Stiglitz demand for the good of producer \( j \) is

\[
Y_{i,t} = Y_t \left( \frac{p_{j,t}}{p_t} \right)^{-\frac{\lambda_f}{\lambda_f-1}}
\]

where \( \lambda_f \) is the elasticity of substitution among intermediate goods and is stochastic to allow for a price markup shock.
The price index $p_t^*$ evolves as

$$p_t^* = \left[ (1 - \xi_p) \left( \frac{K_{p,t}}{F_{p,t}} \right)^{1-\lambda_f} + \xi_p \left( \frac{\tilde{\pi}_t}{\pi_{t-1}} \right)^{1-\lambda_f} \right]^{1-\lambda_f} \tag{A.27}$$

where a fraction $\xi_p$ of monopolistic intermediate producers cannot re-optimize their price in period $t$ and adjust it according to a rule of thumb

$$\tilde{\pi}_t = \left( \frac{\pi_{t-1}^{\text{target}}}{{\pi_t}^{\text{target}}} \right)^{1-t} \tag{A.28}$$

where $\pi_{t-1}^{\text{target}}$ is the target inflation rate of the monetary authority, while a fraction $(1 - \xi_p)$ re-set their price in period $t$ to the optimal level

$$\tilde{p}_t = \frac{K_{p,t}}{F_{p,t}} = \left( \frac{1 - \xi_p \left( \frac{\tilde{\pi}_t}{\pi_{t-1}} \right)^{1-\lambda_f}}{1 - \xi_p} \right)^{1-\lambda_f} \tag{A.29}$$

with $F_{p,t}$ and $K_{p,t}$ auxiliary recursive variables solving

$$F_{p,t} = \zeta_c \lambda z_t y z t + \beta \xi_p E_t \left[ \left( \frac{\tilde{\pi}_{t+1}}{\pi_{t+1}} \right)^{1-\lambda_f} F_{p,t+1} \right] \tag{A.30}$$

and

$$K_{p,t} = \zeta_c \lambda z_t \lambda f, t y z, t s_t + \beta \xi_p E_t \left[ \left( \frac{\tilde{\pi}_{t+1}}{\pi_{t+1}} \right)^{1-\lambda_f} K_{p,t+1} \right] \tag{A.31}$$

A.3.5 Wage maximization and the aggregate wage index

A fraction $(1 - \xi_w)$ of monopoly unions can reoptimize their wage at time $t$ while others cannot and get a wage inflation indexation $\tilde{\pi}_{w,t}$. Therefore, the aggregate wage index $w_t^*$
evolves as

\[ w_t^* = \left(1 - \xi_w \right) \left(1 - \xi_w \right) \left( \frac{\pi_{w,t}}{\pi_{w,t}} \right) \left( \frac{\mu_z^*}{\mu_z^*} \right)^{1-t_{t_h}} \left( \frac{1}{\lambda_w} \right)^{1-\lambda_w} + \xi_w \left( \frac{\pi_{w,t}}{\pi_{w,t}} \right) \left( \frac{\mu_z^*}{\mu_z^*} \right)^{1-t_{t_h}} w_{t-1} \]

(A.32)

where \( 1/(1 - \lambda_w) \) is the elasticity of substitution among labor inputs and \( h_t \) is the aggregate labor input used by intermediate good producers. In the indexing process of wages, the trend of wages is the weighted average of the steady-state value of technological growth rate, \( \mu_z^* \), with a weight \( (1 - \iota_{t_h}) \), and of the growth rate as of time \( t \), \( \mu_z^* \) with a weight \( \iota_{t_h} \). As for the price index, the variables \( F_{w,t} \) and \( K_{w,t} \) are introduced to characterize the dynamics of nominal wages. They satisfy the following laws of motion

\[
F_{w,t} = \zeta_{c,t} \lambda_{z,t} \frac{1-t_{t_h}}{\lambda_w} h_t \left( w_t^* \right)^{\lambda_w / \lambda_w - 1} + \beta \xi_w E_t \left[ \left( \mu_z^* \right)^{1-t_{t_h}} \left( \frac{\mu_z^*}{\mu_z^*} \right)^{1-t_{t_h}} \right] \]

(A.33)

and

\[
K_{w,t} = \zeta_{c,t} \zeta_t \left( w_t^* \right)^{\lambda_w / \lambda_w - 1} h_t \left[ \left( \frac{\pi_{w,t+1}}{\pi_{w,t+1}} \right)^{1-\sigma_L} \pi_{w,t+1} \pi_{w,t+1} \right] + \beta \xi_w E_t \left[ \left( \frac{\pi_{w,t+1}}{\pi_{w,t+1}} \right)^{1-\sigma_L} \pi_{w,t+1} \right] \]

(A.34)

A.3.6 Monetary policy rule

B Definition and Properties of Functions \( F(\bar{\omega}) \), \( G(\bar{\omega}) \), and \( \Gamma(\bar{\omega}) \)

This sections is based on the Appendix A of BGG. \( F(\omega) \) is the continuous probability distribution of \( \omega \in [0, \infty) \) with \( F(0) = 0 \) and \( \int_0^\infty \omega dF(\omega) = 1 \). The power density
function of $\omega$ is $f(\omega) = F(\omega)$. The gross share of profits going to the lender is denoted $\Gamma(\omega)$ and defined as follows

$$\Gamma(\omega) \equiv \int_0^\omega \omega f(\omega) \, d\omega + \int_\omega^\infty f(\omega) \, d\omega \quad (\text{B.35})$$

with

$$\Gamma'(\omega) = 1 - F(\omega) > 0 \quad (\text{B.36})$$

and

$$\Gamma''(\omega) = -f(\omega) < 0 \quad (\text{B.37})$$

The expected monitoring costs are

$$\mu G(\omega) \equiv \mu \int_0^\omega \omega f(\omega) \, d\omega \quad (\text{B.38})$$

with

$$\mu G'(\omega) = \mu \omega f(\omega) \quad (\text{B.39})$$

The net share of profits going to the lender is

$$\Gamma(\omega) - \mu G(\omega) \quad (\text{B.40})$$

with

$$\lim_{\omega \to 0} \Gamma(\omega) - \mu G(\omega) = 0 \quad (\text{B.41})$$

and

$$\lim_{\omega \to \infty} \Gamma(\omega) - \mu G(\omega) = 1 - \mu \quad (\text{B.42})$$

and

$$\Gamma'(\omega) - \mu G'(\omega) = [1 - F(\omega)] \left( 1 - \mu \frac{f(\omega)}{1 - F(\omega)} \right) \gtrless 0 \text{ for } \omega \gtrless \omega^* \quad (\text{B.43})$$
We add the following properties

\[
\lim_{\omega \to 0} G(\omega) = 0 \quad \text{(B.44)}
\]

and,

\[
\lim_{\omega \to 0} \Gamma(\omega) = 0 \quad \text{(B.45)}
\]

and,

\[
\lim_{\omega \to 0} \Gamma'(\omega) = 1 \quad \text{(B.46)}
\]

and,

\[
\lim_{\omega \to 0} \left[ \Gamma'(\omega) - \mu G'(\omega) \right] = 1 \quad \text{(B.47)}
\]

and, following CMR, we notice that the two functions are related as follows

\[
\Gamma(\omega) = \omega \left[ 1 - F(\omega) \right] + G(\omega) \quad \text{(B.48)}
\]

For the log-normal distribution \( \ln(\omega) \sim N(-\sigma^2/2, \sigma^2) \), we have \( E(\omega) = 1 \) and

\[
\Gamma(\omega) = \Phi(z - \sigma) + \omega \left[ 1 - \Phi(z) \right] \quad \text{(B.49)}
\]

and

\[
\Gamma(\omega) - \mu G(\omega) = (1 - \mu) \Phi(z - \sigma) + \omega \left[ 1 - \Phi(z) \right] \quad \text{(B.50)}
\]

where \( \Phi(\cdot) \) is the c.d.f. of the standard normal and \( z \equiv (\ln(\omega) + 0.5\sigma^2)/\sigma \).

\[
F'_t = \text{normpdf} \left( \frac{\ln(\bar{\omega}) + \sigma^2_t/2}{\sigma_{t-1}} \right) \quad \text{(B.51)}
\]
C Data

C.1 Macroeconomic and Financial Series

We follow Christiano et al. (2014) for the definition of macroeconomic and financial series.

- GDP: Real Gross Domestic Product, Billions of Chained 2009 Dollars, Quarterly, Seasonally Adjusted Annual Rate (Fred series)
- Consumption: US : Real Personal Consumption Expenditures: Nondurable Goods + Real Personal Consumption Expenditures: Services, Billions of Chained 2009 Dollars, Quarterly, Seasonally Adjusted Annual Rate (Fred series1 + series2 and before 1999, BEA NIPA Table 2.3.3)
- Investment: US : Real Personal Consumption Expenditures: Durable Goods + Real Gross Private Domestic Investment, Billions of Chained 2009 Dollars, Quarterly, Seasonally Adjusted Annual Rate (Fred series1 + series2 and before 1999, BEA NIPA Table 2.3.3)
- Inflation: US : GDP Implicit Price Deflator, Index 2009=100, Quarterly, Seasonally Adjusted (Fred series), logarithmic first difference
- Price of investment: US : Gross Private Domestic Investment Implicit Price Deflator, Index 2009=100, Quarterly, Seasonally Adjusted (Fred series), divided by GDP Deflator
- Hours worked: US : Nonfarm Business Sector: Hours of All Persons, Index 2009=100, Quarterly, Seasonally Adjusted (Fred series)
- Wage: US : Nonfarm Business Sector: Compensation Per Hour, Index 2009=100, Quarterly, Seasonally Adjusted (Fred series), divided by GDP Deflator
- Short-term risk-free rates: US : Effective Federal Funds Rate, Percent, Quarterly, Not Seasonally Adjusted (Fred series)
- Credit: US : Nonfinancial Noncorporate Business; Credit Market Instruments; Liabil-
ity + Nonfinancial Corporate Business; Credit Market Instruments; Liability, Level, Billions of Dollars, Quarterly, Not Seasonally Adjusted (Fred series1 + series2), divided by GDP Deflator

- Credit spread: US : Moody’s Seasoned Baa Corporate Bond Yield, Percent, Quarterly, Not Seasonally Adjusted (Fred series), less 10-year Government Bond Yield
- Entrepreneurial net worth: US : Wilshire 5000 Total Market Index, Quarterly, Not Seasonally Adjusted (Fred series), divided by GDP Deflator

C.2 Firm Creation Series

To build the firm creation series, we combine two datasets.

- New Business Incorporations (historical series): From the monthly New Business Incorporations series, from 1948M1 to 1994M12, we construct a quarterly sample. The source of the series is the Survey of Current Business, January/February 1996 (Table 13), available on https://fraser.stlouisfed.org/.
- Number of establishments births (recent series): From the Bureau of Labor Statistics, we download the total private sector establishments births series (quarterly and seasonally adjusted), available https://www.bls.gov/.

Figure C.1 shows the two series which are then chained and divide by the population series to get a measure of firm creation per capita consistent with our model.
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<tr>
<th>Economic Parameters</th>
<th>Post. mean</th>
<th>Post. stdv</th>
<th>Post. mode without search</th>
<th>10% without search</th>
<th>90% without search</th>
<th>Post. mode with search</th>
<th>10% with search</th>
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<td>Separation rate</td>
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</table>

**Note:** Prior means and standard deviations are the same for both models.
Figure 6: Impulse response functions to a positive separation rate shock. Percentages on the vertical axis.

Figure 7: Impulse response functions to a positive separation rate shock (search variables). Percentages on the vertical axis.
Figure 8: Openings and closings of firm establishments in the US. Source: Bureau of Labor Statistics.
Figure C.1: New firm creation

*Source:* See Appendix.