Teacher Quality and Cross Country Differences in Student Achievements

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Abstract

This paper proposes and calibrates a micro-founded theoretical model of teacher selection for a panel of 22 OECD countries. Based on the model, I build time series of teacher quality (TQ) for each country, analyse its main drivers and study the importance of TQ in explaining differences in student achievements. In equilibrium the relationship between teacher salaries and TQ is non-linear and maybe non-monotonic which help us to understand the ambiguous findings in the empirical literature between teacher salaries and student achievements. My proposed measure of TQ is strongly positively correlated with cross-country dispersion in student outcomes, even controlling for student and school characteristics, family background, macroeconomic context and unobservable country fixed effects. The counterfactual exercise shows that cross-country differences in TQ explains approximately 22% of the observed cross-country variance in students outcomes, its effect is quite similar to the family background effect. Initial distributions of population skills are crucial to determine current differences in TQ across countries, while teacher salaries and labour market conditions are of lesser importance.

Keywords: Teacher selection, skills, student achievements.
JEL Classification: I20, J2, J45.

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1 Introduction

Student outcomes show high variance across countries even among OECD developed countries. Why do some countries obtain better educational results than others? This paper analyses this relevant question, focusing on the role of teacher quality.

Empirical evidence suggests that having a good teacher is probably the most important school factor for student achievements.\(^1\) Most of the empirical work states that teacher quality is not strongly correlated with observable characteristics of teachers.\(^2\) Therefore, teacher quality is difficult to approximate through observable variables. However, little is known about first, how to identify teacher quality and second, about its main determinants.

This paper try to put some light on these issues. Specifically, this paper constructs a proxy for teacher quality (TQ). It also analyses the key variables determining teacher quality and discusses its importance as a source of cross-country differences in student achievements. To this end, I propose and calibrate a theoretical model of teacher selection for a set of 22 OECD countries.

From the theoretical point of view, the model proposed here is an extension of previous theoretical models since it includes a school selection stage. Previous literature basically were focusing on the self-selection of workers between the teaching career and other occupations. And then it studies a partial equilibrium framework. The model proposed in this paper includes both stages in a general equilibrium framework instead. That is to say, the first stage is a self-selection stage which determines the supply of teachers in the economy. In the second stage, schools select some of the potential teachers to fill their vacancies. This stage analyses the demand for teachers.\(^3\)

The equilibrium of the model shows that the relationship between teacher salaries and teacher quality is non-lineal and maybe non-monotonic. Additionally, the equilibrium relationship between teacher salaries and teacher quality is quite sensitive to some key parameters of the model. Therefore, an accurate process of calibration is required to answer the research question of

\(^3\) See for instance, Nagler et al. (2015), Rothstein (2014) and Tincani (2011). My model simplifies the self-selection analyses respect to the models in those papers because I do not work in a dynamic framework, however I include the analysis of school decisions that is omitted in their models.
the paper.

Through the lens of my model, I build time series of teacher quality at the country level. With the objective to assess if my proposed measure is a good proxy of TQ, I include TQ as well as student and school characteristics, family background and educational spending as inputs in the estimation of an empirical education production function (EPF) using a panel of 22 countries for the period 2000-2015. The model highlights the importance of teacher quality as an educational input because I find that my proxy of TQ is strongly positively correlated with cross-country dispersion in student outcomes in PISA, even controlling for student and school characteristics, family background, macroeconomic context and unobservable country fixed effects. Then, based on the theoretical model and on the proposed EPF, I carry out several counter-factual exercises with the double objective of first to determine the importance of teacher quality as a source of cross-country differences in student achievements and second to discuss why teacher quality differs across countries.

I find that cross-country differences in TQ explains approximately 22% of the observed cross-country variance in observed students outcomes. Its importance is quite similar to the family background effect and clearly higher than the importance of other school inputs. When I study the main determinants of cross-country dispersion in TQ, I also find that the initial distributions of population skills are crucial to determine current dispersion in TQ across countries, since larger mean in the skills of the population derives in better teacher selection process. Moreover, teacher salaries and labour market conditions play a less important role in explaining dispersion in TQ.

The main contribution of this paper is to provide a new theoretical framework of teacher selection. The micro-founded model develop in this paper is a useful tool in several dimensions.

First, to proxy TQ at the country level. Based on the calibration of nine parameters I build a proxy of teacher quality for a large set of countries in different years. To my knowledge the only comparable proxy is the one developed in Hanushek et al. (2014) who discuss the link between teaching skills and student outcomes across countries. They measure TQ with teaching skills in literacy and numeracy using the PIAAC survey of adult skills.\footnote{\color{red}{They approximate the mean teacher skill in math (reading) with the mean cognitive skill in numeracy (literacy) of teachers measured in PIAAC.}} Although the work of Hanushek et al. (2014) is the first one in measuring

4They approximate the mean teacher skill in math (reading) with the mean cognitive skill in numeracy (literacy) of teachers measured in PIAAC.
TQ using PIAAC survey, their proxy presents several limitations: i) it includes only the cognitive skill dimension, ii) it is built for only one year since PIAAC is a cross-section data set. My proxy measured in different years help me to analyse whether teacher quality trends can explain the evolution of student achievements at a country level. To my knowledge, no previous studies include this analysis because time series of country teacher quality were not available. Having time series of teacher quality for a set of countries is important because potential reasons behind the cross-country dispersion in student outcomes could be learning culture or educational institutions. With my time series of TQ I am controlling for these unobservable country fixed effects.

Second, the model is also useful to understand the ambiguous findings in the empirical literature regarding the relationship between teacher salaries and student achievements. The evidence suggests that salaries are not a good proxy of teacher quality, since most of the studies does not find a statistically significant effect between salaries and student outcomes. On the contrary, the works of Loeb (2000) and Dolton and Marceño Gutierrez (2011) find a positive relationship between teacher salaries and student achievements. In my model, the equilibrium relationship between teacher salaries and teacher quality is non-linear and may be non-monotonic. Therefore, different countries could be in different parts of the curve. This is so because on the one hand, previous empirical analysis based on cross-section data within a country probably captures a small segment of the equilibrium curve since teacher wage dispersion is usually small within country. On the other hand, the parameters controlling the teacher selection process differ across countries, and, as I explain later on, the equilibrium curve is quite sensitive to these parameters.

Third, to understand the main variables affecting the quality of teachers. The counter-factual exercises allow me not only to identify, but also to quantify the main drivers of cross-country differences in TQ. Notice that, since the theoretical model includes supply and demand of teachers, the analysis is a general equilibrium framework that takes into account how supply and demand interacts. To my knowledge, no previous papers study this issue. Clearly, this is a first important step if we want to design economic policies oriented to improve TQ, and thus students outcomes.

The rest of this paper is organized as follows. The next section presents the theoretical model of teacher selection used to generate country measures.

\footnote{Hanushek (2003) makes an extensive review on this issue.}
of teacher quality. Section 3 includes the calibration of the model for a panel of 22 OECD countries. Section 4 presents different counter-factual exercises to analyse the importance of teacher quality as a source of cross-county differences in student achievements. Section 5 includes counter-factual exercises to discuss the determinants of cross-country differences in teacher quality. Finally, Section 6 concludes.

2 The Model

This section presents the theoretical framework to study the teacher selection process with the aim to estimate the average teacher quality for a panel of countries. My strategy relies on analysing the individual’s decision to become teachers as well as the school decision about hiring. This enable to me to estimate the general equilibrium effects of different variables affecting teachers quality.

The economy considered is inhabited by a measure one of individuals characterized by the pair \((\hat{\theta}, \hat{t})\), where \(\hat{\theta}\) is an observable skill that determines individual productivity in the non-teaching sector and \(\hat{t}\) represents the unobservable teacher skill.

For each individual, the pair of skills \((\hat{\theta}, \hat{t})\) is assumed to be distributed log-normally.

\[
\ln \left( \frac{\hat{\theta}}{\hat{t}} \right) \sim N \left( \begin{bmatrix} \mu_\theta \\ \mu_t \end{bmatrix}, \begin{bmatrix} \sigma_\theta^2 & \sigma_{\theta t} \\ \sigma_{\theta t} & \sigma_t^2 \end{bmatrix} \right).
\]

Let \(\rho_{\theta t} = \frac{\sigma_{\theta t}}{\sigma_\theta \sigma_t}\) state de correlation coefficient between both skills for the total population of potential teachers. I assume that \(\rho_{\theta t} \geq 0\), which seems consistent with recent empirical studies.\(^6\)

\(^6\)The assumption of the non-observability of the teacher skill is suggested in previous studies. See for instance Hanushek (2010) and Rothstein et al. (2014).

\(^7\)A positive correlation between teaching and non-teaching skills is implicit in the following works. Hanushek et al. (2015) study the private returns to cognitive skills using PIAAC data, and find a positive effect of cognitive skills on salaries. Hanushek et al. (2014) study the effects of teacher cognitive skills, measured also with PIAAC data, on student achievements. This work finds a positive effect of teacher skills on student outcomes. Therefore, the same skills presents positives effects in both, teaching and non-teaching sector.
I assume, that the wage in the teaching sector, $\hat{w}_0$, is the same for all teachers. In contrast, wages in the market sector, $\hat{w}_\theta$, depend on the non-teaching skill $\hat{\theta}$ (general skill in ahead) as follows:

$$\hat{w}_\theta = \hat{\theta}^\alpha,$$

where $\alpha$, which is the wage elasticity with respect to the general skill, measures the market return to the general skill $\hat{\theta}$.

The model has two stages of selection. The first one is the self-selection stage. This stage determines the supply of teachers in the economy. The second one is the school board stage and concentrates on the demand for teachers.

### 2.1 Self selection stage

In the first stage every individual $i$ makes his occupational choice to maximize his utility, $u_i$. This stage is based on a Roy model which is the most extended framework in the field of work self-selection.

The individual utility function is given by:

$$u_i = \begin{cases} \ln(\hat{w}_0) + \ln(\hat{\tau}) & \text{if be a teacher} \\ \ln(\hat{w}_\theta) & \text{otherwise} \end{cases},$$

where $\ln(\hat{\tau})$ captures the non-pecuniary utility of teaching. The parameter $\tau$ measures the elasticity of the non-pecuniary utility with respect to the individual teacher quality.

The utility obtained as a teacher depends not only on the wage, but also on the individual teacher ability. The idea behind is that teacher skills must be positively correlated with the vocation to become a teacher. Thus, the

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8This assumption captures the empirical fact that the variance between teacher salaries is very low compared to the variance of salaries in the whole economy.

9Roy models have been successfully used to study the selection process in labour market in many contexts, for instance: immigration (Borjas AER 1987), government employment (Borjas NBER 2002), manufacturing industries (Heckman JPE 1985) and entrepreneurs (Evans and Jovanovic JPE 1989).
more motivated is the teacher, the larger the non-pecuniary utility of teaching. Section 2.6 discuss the equilibrium of the model if the utility depends only on monetary compensation, $\tau = 0$.  

Given the utility function of Equation (3), individuals choose to become teachers if:

$$\ln(\hat{w}_0) + \tau \ln(\hat{t}) > \alpha \ln(\hat{\theta}).$$

(4)

Defining $\ln(\hat{x}) = x$, using equation (2) and regrouping, the condition above can be written as:

$$\alpha \theta - \tau t < w_0,$$

(5)

Additionally, define $v = \alpha \theta - \tau$ as the “self-selection function”. Since $v$ is a sum of two normal random variables, it is also distributed normally. Particularly, $v \sim N(\mu_v, \sigma_v^2)$, with mean $\mu_v = \alpha \mu_\theta - \tau \mu_t$ and variance $\sigma_v^2 = \alpha^2 \sigma_\theta^2 + \tau^2 \sigma_t^2 - 2\alpha \tau \sigma_\theta \theta$. This self-selection function will help us to identify the supply of teachers, the average quality of the teacher supply as well as the correlation of this self-selection function with the general skills.

Let define the following indicator function:

$$I = \begin{cases} 1 & \text{if the individual choose to be a teacher} \\ 0 & \text{otherwise}. \end{cases}$$

(6)

The probability that an individual chooses to work in the teaching sector, $P(I = 1)$ which is the teacher supply size, is given by:

$$P(I = 1) = P(v < w_0) = P\left(\frac{v - \mu_v}{\sigma_v} < \frac{w_0 - \mu_v}{\sigma_v}\right),$$

$$= \Phi\left(\frac{w_0 - \mu_v}{\sigma_v}\right) = \Phi(z),$$

(7)

where $\Phi$ is the cumulative distribution function of the standard normal distribution, and $z = \frac{w_0 - \mu_v}{\sigma_v}$. Since, the cumulative distribution function of

\footnote{I do not consider in the utility function the probability of not being selected for a teaching position, because I assume that individuals who are not selected as teachers can apply for a job in the market sector in the same conditions that the rest of the population.}
the standard normal is an increasing function in \( z \), and \( z \) is an increasing function of the teacher salary \( (w_0) \), the teacher supply is also an increasing function of the teacher salary.

The average quality of the teacher supply, \( E(t|I = 1) \), is computed as follows:\(^{11}\)

\[
E(t|I = 1) = E \left( t \left| \frac{v - \mu_v}{\sigma_v} < \frac{w_0 - \mu_v}{\sigma_v} \right. \right)
\]

\[
= \mu_t + \rho_{vt} \sigma_t \left( \frac{-\phi(z)}{\Phi(z)} \right), \tag{8}
\]

where \( \mu_t \) is the mean teacher skill of the total population in the economy, \( z = \frac{w_0 - \mu_v}{\sigma_v} \), \( \rho_{vt} \) is the correlation coefficient between the self-selection function and the teacher skill, and \( \phi \) and \( \Phi \) are the density function and cumulative distribution function of the standard normal distribution respectively. The expression \( \sigma_t \left( -\frac{\phi(z)}{\Phi(z)} \right) \) in Equation (8) is always negative. Therefore, the average quality among the supply of teachers, \( E(t|I = 1) \), is lower than the population mean \( (\mu_v) \) when \( \rho_{vt} \) is positive. And thus, a negative selection bias in the teacher supply arises.\(^{12}\) On the other hand, when \( \rho_{vt} \) is negative, the supply of teachers presents a positive selection bias.

The expression for \( \rho_{vt} \) is given by:

\[
\rho_{vt} = \frac{\sigma_{vt}}{\sigma_t \sigma_v} \tag{9}
\]

Since, \( \sigma_t \) and \( \sigma_v \) are always positive terms, the selection bias of the teacher supply will depend on the sign of \( \sigma_{vt} \). Moreover, \( \sigma_{vt} = \text{cov}(\alpha \theta - \tau t; t) = \alpha \sigma_{\theta t} - \tau \sigma_t^2 \). Hence, a negative selection bias arises when \( \sigma_{\theta t} > \tau \sigma_t^2 / \alpha \) while

\(^{11}\)This expression corresponds to the mean of an incidentally truncated bivariate normal distribution. Just to simplify, I present the expected mean of \( t \) instead of \( \hat{t} \). Note that, since the logarithm is a strictly increasing function, any change in the mean of \( t \) implies a change of the same sign in the mean quality of teachers (i.e. mean of \( \hat{t} \)).

\(^{12}\)A negative selection bias implies that the average teacher skill within the teacher supply is lower than in the total population. In contrast, a positive selection bias appears when the average teacher skill within the teacher supply is higher than in the total population.
a positive selection bias arises in the opposite case. Therefore, even discarding the less intuitive cases where $\sigma_{\theta t}$ is negative, a positive selection bias is possible. If the non-pecuniary utility of teaching is eliminated, $\tau = 0$, and $\rho_{\theta t} \geq 0$ holds, the model always predicts a negative selection bias in the teacher supply.

### 2.2 School selection stage

In the second stage of selection, the school board hires a fixed number of teachers, $\gamma$. Let $\Theta$ be an indicator function which states whether an individual FROM the teacher supply is selected for a teacher position.

$$\Theta = \begin{cases} 
1 & \text{if the individual is selected} \\
0 & \text{otherwise.} 
\end{cases}$$  \hspace{1cm} (10)

The probability of being selected for a teacher position is $P(\Theta = 1) = \gamma$.

I assume an educational production function in which student outcomes depends on family resources ($F$), school resources $R$, school organization$^{13}$ ($Sch$) and student characteristics ($St$). Additionally, following the empirical literature of "teacher value added", I assume that student outcomes ($y$) depends positively on the teacher ability ($TQ = E(t|\Theta = 1))$.

$$y = y(F, R, Sch, St, TQ),$$  \hspace{1cm} (11)

Schools select teachers to optimize student achievements ($y$), taking the teacher salary ($\hat{w}_0$) as given.$^{14}$ The wage bill is exogenous in the model, since it depends on the teacher staff size ($\gamma$) and the teacher salary ($\hat{w}_0$), both exogenous variables.

Since the individual teacher skill is unobservable, and $\rho_{\theta t} \geq 0$ holds, schools seek to screen the true teaching ability of candidates based on its observable general skills $\theta$ and then select the fraction $\gamma$ of individuals with higher level of $\theta$. The school strategy follows from the fact that individuals

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$^{13}$The typical variables included in the educational production function related with school organization are the student teacher ratio, levels of school autonomy and school size.

$^{14}$I assume that school budget is enough to hire $\gamma$ teachers at the exogenous teacher salary, $\hat{w}_0$. 

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with higher level of $\theta$ also have a higher expected teacher ability $E(t)$. The conditional expectation of interest is then the expected average teacher skill of teachers, that is $TQ = E(t|\Theta = 1)$. Indeed, maximizing $TQ$ is equivalent to maximize student outputs, given the educational production function of equation (11).

Let define $\theta^*$ as the minimum level of the general skill required to obtain a teacher position such that a fraction $\gamma$ of individuals within the teacher supply has a level of general skill larger than $\theta^*$, that is $\int_{\theta^*}^{\infty} f(\theta/I = 1)\partial \theta = \gamma$ holds. The value of $\theta^*$ is endogenously determined by the the teacher supply size $(\int_{\theta^*}^{\infty} f(\theta/\Theta = 1)\partial \theta)$ and by the distribution of $\theta$ within the teacher supply. A rise on $\theta^*$ implies a rise on the expected general skill among teachers $E(\theta/\Theta = 1)$, and therefore, a rise in the expected ability of teachers, $TQ$. Summarizing, the optimization problem of the School board is expressed as follows:

\[
\begin{align*}
\max_{\theta^*} TQ &= f(\theta^*) \\
\text{st} \quad y &= y \left( \frac{TQ}{+} \right) \\
\theta^* &\in f(\theta/I = 1) \\
\int_{\theta^*}^{\infty} f(\theta/\Theta = 1)\partial \theta &= \gamma,
\end{align*}
\]

Since the conditional expectation $TQ = E(t|\Theta = 1)$ does not have a closed expression, in section 3.4 I will obtain them by simulation. Before doing this, the next section discuss under which circumstances the school strategy attracts good teachers.

### 2.3 The effectiveness of the school strategy

We will show that the effectiveness of the school strategy depends on a key parameter which is endogenously determined in the model, the correlation between the teacher skill and the general skill within the teacher supply, i.e. $\tilde{\rho}_{\theta t}$. This is so because schools use the general skill as a proxy for the unobservable teacher skill. And the larger the correlation between the two within the teacher supply the more precise the proxy is, meaning a more
effective school strategy. On the contrary, when this correlation is weak, the signal will be much less precise.

I denote this correlation as \( \tilde{\rho}_{\theta t} = E(\rho_{\theta t} | I = 1) \), which is given by the following expression:

\[
\tilde{\rho}_{\theta t} = \frac{\tilde{\sigma}_{\theta t}}{\tilde{\sigma}_t \tilde{\sigma}_\theta},
\]

(13)

where \( \tilde{\sigma}_{\theta t} \) represents the covariance between the teacher skill and the general skill restricted to the teacher supply, and \( \tilde{\sigma}_t \) and \( \tilde{\sigma}_\theta \) are the restricted standard deviations of the teacher and general skills respectively.\(^{15}\) In what follows, I study its quantitative value and how teacher wages, \( \hat{w}_0 \) affect its magnitude.

The correlation between skills within the teacher supply could be higher, lower or equal than the correlation in the total population. This is so because first, the restricted standard deviations \( \tilde{\sigma}_t, \tilde{\sigma}_\theta \) are always lower than the population ones (\( \sigma_t \) and \( \sigma_\theta \)). And second, the restricted covariance \( \tilde{\sigma}_{\theta t} \) could be higher or lower than the population value (\( \sigma_{\theta t} \)).\(^{16}\)

If \( \rho_{\theta t} \) is relatively low, \( \tilde{\rho}_{\theta t} > \rho_{\theta t} \) holds for any teacher salary. On the other hand, if \( \rho_{\theta t} \) is relatively high, the inequality \( \rho_{\theta t} > \tilde{\rho}_{\theta t} \) holds for any teacher salary. Additionally, when \( \tilde{\rho}_{\theta t} \neq \rho_{\theta t} \), the distance between the restricted and unrestricted correlation coefficients decreases with the supply of teachers, which in turn, depends positively on wages. Therefore, a rise in teacher wages determines the convergence between the restricted and the unrestricted correlation.\(^{17}\)

Since the convergence between the restricted and the unrestricted correlation could be from above or from below, the sign of the relationship between teacher salary \( \hat{w}_0 \) and the effectiveness of the proxy \( \tilde{\rho}_{\theta t} \) is not theoretically defined. Specifically, if \( \tilde{\rho}_{\theta t} < \rho_0 \) holds for any level of \( \hat{w}_0 \), a positive relationship between the teacher salary and the quality of the proxy, \( \tilde{\rho}_{\theta t} \), is obtained. In this case, when schools fix a higher salary, the proxy to identify the best teachers becomes powerful. On the contrary, when \( \tilde{\rho}_{\theta t} < \rho_0 \) holds for any \( \hat{w}_0 \), a negative relationship between \( \tilde{\rho}_{\theta t} \) and \( \hat{w}_0 \) is obtained. In this

\(^{15}\)See Rao (1989) for details regarding the computation of the restricted variances and covariance.

\(^{16}\)Particularly, \( \sigma_{\theta t} \leq \tilde{\sigma}_{\theta t} \) if an only if \( \rho_{\theta t} \leq \tau / \alpha \).

\(^{17}\)In the extreme, when \( w_0 \) tends to infinite and thus, the teacher supply tends to the total population, i.e. \( \tilde{\rho}_{\theta t} \) tends to \( \rho_{\theta t} \).
case, when schools fix a higher salary, the proxy to identify the best teachers becomes weaker.

2.4 The Equilibrium in the market for teachers

The equilibrium of the model is defined as a minimum level of general skill required to be selected for a teacher position and the associated distribution of the teacher skill among teachers \((\theta^*, f(t/\Theta = 1))\), such that:

- No individuals wishes to change occupation from the teacher sector to the market sector, given the market wages.
- School board maximizes student outcomes given the teacher salary.
- School board hires the required number of teachers. That is, \(\theta^*\) holds that:
  \[
  \int_{\theta^*}^{\infty} f(t/\Theta = 1) d\theta = \gamma
  \]

Note that some individuals that choose to apply for a teacher position in the self selection stage could not become teachers, so that and excess of supply in the teacher market could be observed in equilibrium.\(^\text{18}\) Nevertheless, no agent has incentives to change her decisions.

2.5 Equilibrium representation

Even though the model generates the complete equilibrium distribution of the teacher ability among teachers, in this section I concentrate only on the mean of the distribution of TQ.

For each level of teacher salary and each parameter, the model states a TQ in equilibrium. Since TQ is a key input for student outcomes, it is relevant to study its relationship with teacher wages.

The "Teacher Wage-Teacher Quality" curve (in advance TW-TQ curve) shows for each level of teacher salary, \(w_0\), and a given vector for the rest of model parameters, the expected teacher ability of the teacher staff, \(E(t/\Theta =

\(^{18}\)Recall that, individuals who are not selected for a teacher position can apply for a job in the market sector as any other individual.
1), that holds the equilibrium conditions of the model. Figure 1 represents TW-TQ curve. Parameters in this figure were chosen in order to: i) work with standard normal marginal distributions in both skills, $\mu_t = 0$, $\sigma_t = 1$, $\mu_\theta = 0$, $\sigma_\theta = 1$; ii) fix an elasticity of the individual utility respect to the general skill higher than the elasticity respect to the teacher skill, $\alpha = 1$, $\tau = 0.1$; iii) Work with a positive, but not very strong, correlation between skills, $\rho_{\theta t} = 0.5$.

Figure 1 shows an increase and concave TW-TQ equilibrium curve. Teacher salaries increase TQ but at a decreasing rate. That is, when salaries are relatively low, an increase in salaries have a strong impact on the quality of the teacher staff. When salaries are relatively high instead (i.e. more than two times the median wage on the non-teaching sector), the effects of teacher salaries on the TQ is smaller and becomes almost zero for higher levels of teacher wages. The shape of the curve is the consequence of the interaction of three different mechanisms of transmission which are discussed in the next section 2.6.

Notice that the non-linear shape of the curve could help us to understand the non-conclusive relationship of previous empirical works between teacher salaries and student outcomes. Hanushek (2003) makes an extensive review of several studies analysing the relationship between teacher salary and student performance. He shows that most of the studies do not find a statistically significant positive correlation. As exceptions, Loeb (2000) and Dolton and Marcenaro Gutierrez (2011) find a positive relationship between teacher salaries and student achievements.

One possible explanation for this ambiguous findings is that different countries could be in different parts of the curve. This is so because on the one hand, empirical analysis based on cross-section data within a country probably captures only a small segment of the equilibrium curve since teacher wage dispersion is usually small within country. Note that a linear regression over different segments of the TW-TQ curve of Figure (1) could generate very different results. In fact, an estimation over a sample of low teacher wages probably suggests a significant relationship between teacher salaries and student outcomes. However with a sample of high teacher salaries it would be more difficult to find a significant relationship. On the other hand, the parameters controlling the teacher selection process differ across-countries, and as it is explained in the section 2.7, the equilibrium curve is very sensitive to the value of these parameters.
2.6 Teacher salary and teacher quality: Three channels of transmission

Teacher salaries impact the quality of teachers through three different channels, the quantity channel, the quality channel and the signal channel. Furthermore, the sign of last two channels depends on $\rho_{\theta t}$. I also will discuss how each channel operates for different values of $\rho_{\theta t}$.

The quantity channel affects the teacher supply size ($\gamma$). A rise in the teacher wages increase the incentives to becoming a teacher. Therefore, a higher teacher salary determines a larger teacher supply and more possibilities for schools to select a teacher. Panel A of Figure 2 shows the relationship between teacher salaries and the teacher supply size for different values of $\rho_{\theta t}$. This channel is almost insensitive to different values of $\rho_{\theta t}$.

The quality channel affects the average teacher skill of individuals within the teacher supply, $E(t/I = 1)$. The sign of the quality channel effect depends on $\rho_{\theta t}$. With a high correlation between general and teacher skills, a rise on teacher salary determines a rise on the average teacher ability within the teacher supply. Note that a higher quality of teacher supply improves the space of school selection. In contrast, the quality of the teacher supply is almost independent of the teacher wage in case of a small correlation among skills (see the case of $\rho_{\theta t} = 0.1$).\(^{19}\)

\(^{19}\)Indeed, assuming $\rho_{\theta t} < 0.1$ the sign of this channel becomes negative. Since, empirical evidence is in favour of $\rho_{\theta t} > 0$ we don not concentrate in this case.
Finally, panel C of Figure 2 shows the signal channel (i.e. $\tilde{\rho}_t$) for different values of $\rho_t$. As was explained in Section 2.3, the larger the correlation between the teaching and the general skill within the teacher supply ($\tilde{\rho}_t$) the more powerful is the signal, and thus the more efficient is the selection strategy of schools.

The relationship between the teacher salary $\hat{w}_0$ and $\tilde{\rho}_t$ depends on the correlation between both skills in the total population, $\rho_t$. Indeed, as we can see from the graph, $\tilde{\rho}_t$ and $\hat{w}_0$ are increasing (decreasing) in teacher wages when the correlation between skills in the total population is strong (weak). Therefore, when $\rho_t$ is high, paying high wages make the signal in the selection process more accurate. On the contrary, when $\rho_t$ is low, paying higher salaries determines a worse signal in the selection process.

In summary, for economies with a strong correlation between skills, a rise in teacher salaries drives to a more efficient way of selecting teachers. This is so because with a larger wages i) the space of selection is larger, ii) those agents are of better quality and iii) the signal is powerful (the restricted correlation is larger). On the contrary, economies with a weak correlation between skills (see for instance, the case of $\rho_t = 0.1$), a rise in the teacher salary drives to a less efficient way of selecting teachers. With a larger wages i) the space of selection is larger but ii) the average quality of the teachers does not increase and iii) the proxy of the unobservable teacher skill is less
accurate. As consequence of these opposing effects, the marginal effect of teacher salary on TQ is weaker (or even negative).\textsuperscript{20}

2.7 Equilibrium sensitivity

Figure 3 shows how sensitive is the equilibrium TW-TQ curve to changes in the exogenous parameters of the population skills.

Panels A and B of Figure 3 show the sensitivity of the equilibrium curve to changes on the mean ($\mu_t$) and variance ($\sigma^2_t$) of the teacher skill distribution. A rise in the mean $\mu_t$ shift the equilibrium curve up without changing significantly its shape. This result is quite intuitive. A population characterized by high levels of teacher skill drives to better teachers. On the other hand, a rise in the variance of teachers skills $\sigma^2_t$ modifies the slope of the curve since it becomes steeper, specially when salaries are relatively low.

Panels C and D of Figure 3 show the sensitivity of the equilibrium curve to changes in the mean ($\mu_\theta$) and variance ($\sigma^2_\theta$) of the general skill distribution. A higher mean of the general skill distribution $\mu_\theta$ determines a fall on the average quality of teachers and a steeper curve.

Panel D shows that the larger the variance flatter the TW-TQ equilibrium curve. If $\sigma^2_\theta$ and salaries are low, the average TQ is low. However, if the variance is low and salaries are high the average TQ is high.

Finally, panel E shows the effects on the equilibrium results derived from changing the correlation between the two skills of the model ($\rho_{\theta t}$). Indeed, for a high correlation between both skills the curve TW-TQ is strictly increasing and nearly linear. In contrast, for smaller values of $\rho_{\theta t}$ it becomes more concave. In the extreme, i.e. $\rho_{\theta t} = 0.1$, the equilibrium relationship becomes decreasing at the end, and therefore, we obtain a non-monotonic equilibrium curve.

Figure 4 shows the sensitivity of the TW-TQ curve to changes in the rest of model parameters ($\alpha, \tau$ and $\gamma$). Panel A shows that with a rise on the market return of the general skill ($\alpha$) the TW-TQ curve becomes flatter. Panel B shows the sensitivity of the TW-TQ curve to changes in the non-pecuniary returns to the individual teacher skill ($\tau$). Like other parameters,\textsuperscript{20}See Appendix I for a graphical analysis of the three channels for different values of $\rho_{\theta t}$.\textsuperscript{16}
changes on $\tau$ have two different effects. First, a rise on $\tau$ moves up the TW-TQ curve and simultaneously, makes the TW-TQ equilibrium curve more concave. For the extreme case of zero non-pecuniary return (the blue line) the TW-TQ curve is nearly linear. On the other hand, higher values of $\tau$ implies an increase in the slope when teacher salaries are low together with a marginal effect of almost zero when teacher salaries are larger than one time the median wage in the market sector (see for instance the violet line corresponding with $\tau = 0.4$).

**Figure 3 - Impact of changes in the skill distribution parameters on the TW-TQ equilibrium curve**

Finally, panel C shows the sensitivity of the equilibrium curve to changes in the share of teachers on the labour force (parameter $\gamma$). An increase on teacher, i.e. on $\gamma$, moves down the TW-TQ equilibrium curve without changing significantly its shape.

Two key conclusions are derived from previous analysis. First, the equilibrium relationship between teacher salary and the average quality of teachers could be non-linear and maybe non-monotonic. Therefore, the marginal effect of the teacher salary on the TQ is non-constant and depends on the level of teacher salaries and the parameters values of the model.

Second, the model rationalize very different different shapes of the TW-TQ curve, which are derived from the interactions of three different channels operating in the model discussed in previous section.
3 Calibration

The sensitivity of the equilibrium curve TW-TQ to some parameter values introduces difficulties to use the model in order to identify the optimal allocation of school resources and to evaluate the effectiveness of different policies. Therefore, using the model to evaluate alternatives policies requires an accurate process of calibration. The model only will be useful if the correct scenario is identified.

The model has nine parameters to determine. Five parameters are associated to the skill distribution among the population: the mean and variance of the marginal distribution of the unobservable teacher skills, $\mu_t$ and $\sigma_t$, mean and variance of the marginal distribution of the observable general skill, $\mu_\theta$ and $\sigma_\theta$ and the correlation between both skills $\rho_{\theta t}$. Three parameters are related with the labour market conditions: $\hat{w}_0$ measures the teacher salary relative to the median wage in the market sector; $\alpha$ states the elasticity of market wages to the observable general skill and $\gamma$ indicates the share of teachers among the total labour force. Finally, parameter $\tau$ represents the elasticity of the non-pecuniary utility to the individual teacher skill.

I calibrate the model for a set of 22 OECD countries in the years 2000,
2003, 2006, 2009, 2012 and 2015.\textsuperscript{21} I assume that all the parameters of the population distribution of skills and parameters $\tau$ and $\gamma$ are time invariant \textsuperscript{22} and parameters $\alpha$ and $\hat{w}_0$ varies across years. Furthermore, I assume that parameters $\mu_t$, $\sigma_t$, $\mu_\theta$, $\sigma_\theta$, $\hat{w}_0$, $\alpha$ and $\gamma$ varies across countries while parameters $\rho_\theta$ and $\tau$ are constant across countries.

Observable parameters are approximated with country data. Unobservable parameters are approximated indirectly. Next section describes the strategy applied in each case.

\subsection*{3.1 Observable Parameters}

The individual general skill and the labour market condition are assumed as observable variables in the model. Thus, these variables are approximated from country statistics.

Parameters corresponding to the moments of the country general skill distribution ($\mu_\theta$ and $\sigma_\theta$) are approximated with the corresponding moments of an average of skills included in the PIAAC survey for adult population. The PIAAC survey for adult population is used to have a comparable measure of skills across countries. This survey measures adults’ skills in literacy, numeracy and problem solving in technology-rich environments - and gathers information and data on how adults use their skills at home, at work and in the wider community.\textsuperscript{23}

Hanushek et al. (2015) studies the wage returns to the cognitive skills measured in the PIAAC survey. In line with the assumption of equation (2) regarding the general skill, the paper finds a positive effect on salaries from all three cognitive skills measured in the survey. Based on these result, the general skill of our model ($\theta$) is approximated as an average of the test scores in the three skills measured in PIAAC.\textsuperscript{24} Test scores are standardized with

\begin{footnotesize}
\begin{itemize}
\item \textsuperscript{21}Years and countries were selected for reasons of data availability. The database includes the years where PISA test is available. The set of countries was restricted by the information required to the calibration of parameters.
\item \textsuperscript{22}The assumption of a fixed distribution of skills in the adult population for a period of 12 years is not so strong. For instance, the PIAAC survey measures skills for population between 16 and 66 years (50 cohorts). So, in a period of 12 years only a quarter of the cohorts changes. On the other hand, assuming a time invariant utility function for a reduced period seems quite reasonable.
\item \textsuperscript{23}See more details about PIAAC survey in OECD(2013).
\item \textsuperscript{24}Since the wage returns to each skill included in PIAAC is different, our general skill is a weighted average of these skills. The weight of each skills follows from the returns
\end{itemize}
\end{footnotesize}
mean $\mu_\theta$ zero and standard deviation of one across countries $\sigma_\theta$.

The wage returns to the general skill (parameter $\alpha$) is approximated by the returns to education. Since the individual general skill $\theta$ was calibrated based on the cognitive skills of the PIAAC survey, the best option to approximate $\alpha$ would be the average returns to these skills. However, this option is available only for a single year (2012). Then, in order to consider the time dynamic of parameter $\alpha$, it was approximated with the wage returns to education, available on Education at a Glance-OECD (EG, OECD) for all the years included in the analysis.\(^{25}\) In line with the model, the wage return to tertiary education captures a premium associated to an observable approximation of individual skills.

Parameter $\hat{w}_0$, indicates the relative teacher salary respect to the median wage in the market sector. It is approximated with the ratio of the teacher salary in the lower secondary education (available at Education at Glance, OECD 2001) and the GDP per habitant (from IMF) both in PPP dollars.

Finally, parameter $\gamma$ is approximated with the share of teachers in the total employment, measured by Education at a Glance, OECD, 2001.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
<th>Source</th>
</tr>
</thead>
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<td>PIAAC</td>
</tr>
<tr>
<td>$\sigma^2_\theta$</td>
<td>variance of the general skill distribution</td>
<td>PIAAC</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>share of teacher into the labour force</td>
<td>EG</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>elasticity of wages with respec to the general skill</td>
<td>EG</td>
</tr>
<tr>
<td>$\hat{w}_0$</td>
<td>teacher salary relative to the median wage in the market sector</td>
<td>EG and IMF</td>
</tr>
</tbody>
</table>

### 3.2 Unobservable parameters

Teacher skills are unobservable and thus parameters $\mu_t$, $\sigma_t$, and $\rho_{\theta t}$ can not be approximated directly by country data. Similarly, since individual pref-

\(^{25}\)The wage returns to education present a high positive correlation of 0.68 across countries with the returns to the cognitive skills in PIAAC estimated in Hanushek et al (2015).
erences are unobservable, also there is not information for parameter \( \tau \). All these unobservable parameters are approximated indirectly.

We assume that the variance of the teacher skill distribution, \( \sigma_t \), is equal to the variance of the general skill, \( \sigma_\theta \). In practice, this decision implies to work with only eight, instead of nine parameters. The correlation between variances in observable skills is strong across countries.\(^{26}\) And then, I assume that countries with a high variance in some observable skill usually exhibit large variance in unobservable skills too.

Meanwhile, parameters \( \mu_t \), \( \rho_\theta \) and \( \tau \) are calibrated together through an iterative process explained in Figure 7.

I start approximating the average teacher quality \( \mu_t/\Theta = 1 \) for each country in 2015, based on PISA results for that year.

According to equation (11) in the theoretical model, the following educational production function is assumed:

\[
y_{is} = \beta_0 TQ_{cts} + \beta_1 S_i + \beta_2 F_i + \beta_3 SCH_i + \beta_4 M_{ct} + T_t + C_c + u_{is}, \quad (14)
\]

where \( y_{is} \) states the average student outcome for individual \( i \) in subject \( s \); \( TQ_{cts} \) is the simulated TQ for country \( c \), in period \( t \) for subject \( s \), \( S_i \) represents a set of individual characteristics; \( F_i \) states the family background for individual \( i \); \( SCH_i \) states the school characteristics for individual \( i \); \( M_{ct} \) represents a set of macro variables for country \( c \) in period \( t \), \( C_c \) is a fixed effect associated to country \( c \) and \( u_{is} \) is the residual of the equation.

Taking expectations on equation 14, assuming \( E(u_{is}) = 0 \), and regrouping, the expected teacher quality for each country, subject and period can be computed.

\[
E(TQ_{is}) = E(y_{is} - B_1 S_i + B_2 F_i + B_3 SCH_i + B_4 M_{ct} + T_t + C_c) \quad (15)
\]

The result of equation (15) is used to obtain the average teacher quality, \( \mu_t/\Theta = 1 \), for each country and subject in 2015. Note that, this value is an

\(^{26}\)Cross-country standard deviations in numeracy and literacy skills of PIAAC survey show a correlation of 0.84. Moreover, cross-country correlation among the cognitive skills measured in PISA are of the order of 0.97.
endogenous result of the model and it differs from parameter $\mu_t$ which states the average teacher skill of the total population.

To operate with equation (15), first at all, I need an approximation to each component of the right side of the equation. Student characteristics includes gender, age, migration status, and dummies regarding repetition and difference between the language of the test and language at home. Family background is approximated with the index of economic social and cultural status of PISA. This index was created on the basis of the highest levels of education and occupational status of the student's parents; the number of books at home; and an index of other home resources. School characteristics includes index of autonomy developed by PISA, school size and the student teacher ratio. Finally, I control by the educational spent in primary and secondary education as percentage of the GDP, the PIB per capita in PPP dollars, and the growth rate.

Finally, the vectors of parameters $B_i$ are estimated by running the OLS regression with the objective to obtain an empirical estimation of the TW-TQ curve. The empirical TW-TQ is estimated based on our sample of 22 OECD countries between 2000 and 2015. The estimation includes all the controls of equation (14) with the exception of TQ which is replaced by the relative teacher salaries. In concrete, the following regression is estimated:

$$y_{is} = b_1 \hat{w}_{ct} + b_2 \hat{w}_{ct}^2 + B_1 S_i + B_2 F_i + B_3 SCH_i + B_4 M_{ct} + T_t + C_c + u_{is}, \quad (16)$$

where $\hat{w}_{ct}$ indicate the ratio among the teacher salary and the GDP per habitant (both in PPP dollars) for country $i$ in period $t$. Consistent with our theoretical equilibrium curve TW-TQ, the empirical specification in Equation 16 need to include a quadratic term to captures non-linear effects of salaries.

Table 2 reports the values for parameters $b_1$ and $b_2$ from the empirical estimation of the TW-TQ curve of equation (16). The quadratic term results significant, so that the empirical TW-TQ curve is also concave.

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27 The index of other home resources was created on the basis of whether there is a quiet study space, internet access, TV, and stories and novels at home. For more details on the IESCS, see the Annex I of PISA (2015)- OECD.

28 For a complete estimation of the regression see the appendix.
Table 2. Empirical TW-TQ curve

<table>
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<th>student score</th>
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</thead>
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<td>$b_1$</td>
<td>34.771**</td>
</tr>
<tr>
<td>$b_2$</td>
<td>-8.934**</td>
</tr>
</tbody>
</table>

Average R-Squared 0.2282
Observations 577909
Sample 2000-2015

Once I have an estimation of $\mu_t/\Theta = 1$ for each country in 2015, the second step of the process involves to recuperate parameter $\mu_t$, that is, the population mean of the teacher skill distribution. This step implies to solve the model in an inverse sense. Solving the model in this way requires first, to use the values for each exogenous parameter, including $\rho_{0t}$ and $\tau$ still in process of calibration. Therefore, to run this step, I use initial values for those parameters. I call these values as $\rho_{0t}^0$ and $\tau^0$.

After this step, I obtain a preliminary calibration for $\mu_t$ conditional on the initial values $\rho_{0t}^0$ and $\tau^0$. In the third step, parameters $\rho_{0t}$ and $\tau$ are calibrated in order to minimize the distance between the theoretical and empirical TW-TQ curve. The empirical curve is building based on previous estimation. The theoretical TW-TQ is computed using the calibrated values for observable parameters and the initial values for the unobservable ones.

Finally, the four step consist on check the distance between the initial values of $\rho_{0t}$ and $\tau$ and the values obtained in previous step. Let’s define $\epsilon = 0.001$ as a convergence criteria. So, this step implies to check if the following conditions are satisfied:

1. $|\rho_{0t}^0 - \rho_{0t}| < \epsilon$
2. $|\tau^0 - \tau| < \epsilon$

If the convergence conditions are satisfied, the process of calibration concludes. If not, the iterative process back to the second step taking as inputs the new values of $\rho_{0t}$ and $\tau$ instead of the initial values $\rho_{0t}^0$ and $\tau^0$. Figure 6 shows the theoretical (blue line) and empirical (green line) curves together after the calibration. Figure 7 summarize the process of calibration for parameters $\mu_t$, $\rho_{0t}$ and $\tau$. The appendix presents in details the parameters obtained in each step of the iteration process.
Figure 6 - Theoretical vs Empirical TW-TQ curve

![Graph showing theoretical vs empirical TW-TQ curve]

Figure 7. The iterative process of calibration

- Set initial values for all parameters except $\mu_t$ including $\rho_{\theta t}$ and $\tau$
- Select $(\mu_t/\Theta = 1)$ to match with PISA 2015 scores
- $\mu_t$ is recover from $(\mu_t/\Theta = 1)$ through the theoretical model
- $\rho_{\theta t}$ and $\tau$ minimize distance between theoretical and empirical TW-TQ curves
- If not satisfied then recalculate $\mu_t$ using the new values for $\rho_{\theta t}$ and $\tau$
- Check whether $\rho_{\theta t}$ and $\tau$ satisfies convergence condition
3.3 Calibration Results

Tables 3, 4 and 5 summarize the parameters values for our set of 22 OECD countries. Table 3 presents the time invariant parameters while Tables 4 and 5 shows the time-variant parameters between 2000 and 2015. By observing these tables three results of the calibration are particularly remarkable.

First, cross-country variance between the unobservable teacher skill and the observable general skill are quite similar. So, even though the calibration determines big differences in the distribution of teacher skills across-countries, those differences are in line with cross-country differences in other measured skills. Particularly, cross-country variance in the teacher skill is similar to the variance in the average skills measured in the PIAAC survey.

Second, the calibrated correlation between the teacher and general skill ($\rho_{\theta,t}$) is positive, specifically 0.57. This correlation is clearly weaker than the individual-level correlation between the skills in the PIAAC survey (0.87 between numeracy and literacy and 0.73 between numeracy and problem solving) and in PISA results (0.85 between maths and reading and 0.9 between maths and science). This result support the idea that observable skills are not a very good proxy for teacher skills, as is shown by previous works (see Hanushek and Rivkin, 2006 and Hanushek, 2010).

Finally, the elasticity of the non-pecuniary utility respect to the individual teacher skill is relatively small (approximately a quarter) compared to the elasticity of wages respect to the general skills. This result highlights that, even though, good teachers obtain in average higher utility of teaching (so, the motivation factor matters), wages are the most relevant factor in the individual occupational choice.

3.4 Model Results

This section discusses the model results. Our model is able to generate the complete distribution of teacher quality among teachers for each country. Nevertheless, in this work, I focus on the distribution’s mean as a summary statistic.

Figure 8 shows the average teacher quality between 2000 and 2015 for each country of our panel. The model results predicts that Korea, Canada and Japan have the highest teacher quality in the considered period in math
### Table 3. Time invariant parameters

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<th></th>
<th>$\mu_{\text{Math}}$</th>
<th>$\mu_{\text{Read}}$</th>
<th>$\mu_g$</th>
<th>$\sigma_g$</th>
<th>$\gamma$</th>
<th>$\rho$</th>
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<td>0.4072</td>
<td>0.7666</td>
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### Table 4. Time-variant parameters

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Table 5. Time-variant parameters

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<td>UK</td>
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<td>1.10</td>
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<td>0.92</td>
<td>0.95</td>
<td>0.91</td>
<td>1.10</td>
</tr>
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</table>

while Estonia, Finland and Canada have the highest teacher quality in reading. In the lower bound of the distribution Slovak Republic and US shows the lower teacher quality of our sample in Math. In reading, again Slovak Republic present the lower estimations of teacher quality followed by Italy and Czech Republic. Even though we observe changes in the relative positions of countries among subjects, we have a strong cross-country correlation between teacher quality in both subjects.

Once I obtain the TQ results, a natural question arise: Are the model results a good proxy of the country teacher quality? It is difficult to answer this question because there are not quantitative measures of teacher quality at a country level. The only consensus in previous literatures states that teacher quality impact student outcomes.\(^{29}\) I evaluate my proposed measure of TQ based on its correlation with student outcomes.

A first approximation is a simple correlation between the model measure of teacher quality and student outcomes. I use the PISA results as a comparable measure of student outcomes across-countries. The correlation between TQ and student outcomes in math and reading is 0.73 in our panel\(^{29}\)See for instance Hanushek and Rivkin (2006), Hanushek (2010) and Hanushek and Rivkin (2010).
of 22 countries between 2000 and 2015. Figure 9 shows a scatter plot between the average TQ by countries and the average student achievement in PISA, both for the period 2000-2015 for the two different subjects. The correlation between the measure of country TQ and country results on PISA is strong in both, math and reading.

**Figure 8 - Teacher quality by countries.**
(Math average 2000-2015)

(Read average 2000-2015)
A second more elaborated approximation is to estimate an educational production function (EPF) including our model measure of TQ as an educational input together with other controls and PISA test scores. Then, I use the measure of teacher quality to estimate the educational production function proposed in equation (14).

The estimation of equation 14 identifies the parameter of the model measured of TQ based on the dispersion between and within countries and between subjects. Note that, having time series of teacher quality for a set of countries is important for my baseline specification. This is so because potential reasons behind the cross-country dispersion in student outcomes could be learning culture or educational institutions, which are set up at the country level. With my time series of TQ I am controlling for for these unobservable country fixed effects.

Table 4 summarizes the estimations using the panel. Column (1) presents the results of a OLS estimation without any control except time dummies and country fixed effects. Column (2) shows a regression between PISA results and the model TQ measure including controls by individual characteristics, family background, school resources and macro variables. Finally, column (3) make the same estimation of column (2) using a different sample. The results shows that the effect of our measure of TQ over student outcomes is strongly robust to introducing controls and change the estimation sample. Then, in what follows, I assume that our model measure is a good proxy of the average teacher quality at a country level.
### Table 4 - Effect of TQ on student achievements

<table>
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<th>(3)</th>
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<td>TQ</td>
<td>8.3562***</td>
<td>9.1052***</td>
<td>10.0148***</td>
</tr>
<tr>
<td>Student characteristics</td>
<td>no</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>Family background</td>
<td>no</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>School characteristics</td>
<td>no</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>Macro controls</td>
<td>no</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>Country fixed effects</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
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<tr>
<td>Time dummies</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>R-Squared</td>
<td>0.0344</td>
<td>0.2473</td>
<td>0.2509</td>
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This section discusses the main question of the paper, that is, what is the role of teacher quality to explain cross-country differences in student outcomes?

Our educational production function states that cross-country dispersion in student outcomes could arise basically from differences in: teacher quality, student characteristics, family background, school organization like school autonomy, school size and the student-teacher ratio, educational expenditures, and unobservable country fixed effects.\(^{30}\) To my knowledge, this is the first exercise that makes an integrated analysis of several inputs in the production function of student achievement to evaluate the relevance of each one as sources of cross-country dispersion in student outcomes. To this end, using my theoretical model and the estimated EPF, I study several counter-factual exercises.

Each exercise consists on removing the country differences in all sources of dispersion except to one—the selected input. I repeat this exercise for each one of the school inputs included in my educational production function. In this way, I identify the individual contribution of each source to cross-country differences in student outcomes. Then, I compute how much of the observed variance on test scores in the data could be explained by the variance of the

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\(^{30}\)This fixed effect captures for instance differences in the characteristics of the educational systems or the importance attached to education by the society.
specific source. Table 5 summarize the results.\textsuperscript{31}

In line with Hanushek (2003, I also find that TQ is single school input with highest impact on student outcomes. Nearly a of a quarter of the cross-country variance in student achievements observed in test scores is obtained by considering only differences in TQ. However, in order to contextualize the importance of TQ, note that the TQ effect is quite similar to the family background effect.

The others school inputs have a weaker impact. Indeed, a scenario in which all country differences are removing except the variables related with school organization (school autonomy and school size) exhibit a cross-country variance in student achievements of approximately a 5% of the observed variance in our panel of countries. Taking into account only differences in the student teacher ratio produces a variance on PISA of a 1% of the observed variance in the panel. Finally, differences in educational spend generate a cross-country dispersion of student outcomes equivalent to an 8% of the total variance of the panel. That is, with the exception of TQ, differences in school inputs play a minor role to explain cross-country differences in student achievements.

\begin{table}[h]
\centering
\begin{tabular}{l|c}
\hline
 & \% of the observed variance explained by each component \\
\hline
Teacher Quality & 22\% \\
School organization & 5\% \\
Student Teacher ratio & 1\% \\
Educational spent (\% of GDP) & 8\% \\
Family Background & 20\% \\
\hline
\end{tabular}
\caption{Effect of Educational Inputs on cross-country variance in student achievements}
\end{table}

4 Why TQ is different across-countries?

If TQ is so important at explaining cross-country differences in student outcomes, we need to go further into the analysis and ask why TQ differs across

\textsuperscript{31}Individual contributions do not sum 100 because the sources covariate among them and I include only variables related with the education system.
countries. The theoretical model of this paper helps us to put a light into the answer of this question.

The model allows me to study the importance of different sources of TQ dispersion across countries. In particular, I classify three different sources: i) country differences in teacher salaries, \( \hat{w}_0 \); ii) country differences in labour market conditions (summarized with the parameters \( \alpha \) and \( \gamma \)); and; iii) country differences in initial distributions of skills, (summarized with the parameters \( \mu_\theta, \sigma_\theta, \mu_t \) and \( \sigma_t \)). Of course, these different source of variance covariate among them. So, removing one source of TQ differences not necessarily reduce the cross-country variance of TQ.

I use counter-factual exercises to evaluate the importance of each determinant. Each exercise consist on modify one of the model parameters in order to obtain new estimations of TQ for each country and year. Table 6 summarized the results.

In a first exercise, to evaluate the role of teacher salaries, I compute a new vector of country measures of TQ under the assumption that each country pay the same relative teacher wage, which I assume to be equal to the average wage \( \hat{w}_0 \) across all countries of the panel and across years. In this new set up, without differences teacher wages, the variance of TQ in the panel falls by 6.6%. The decrease in TQ dispersion is because the simulated TQ after removing differences in teacher wages and the observed vector of teacher salaries is positively correlated (i.e. 0.24). That is, between 2000 and 2015, countries with better estimations of TQ after removing differences in wages, also paid in average better salaries. Consequently, during this period teacher salaries amplify the TQ dispersion across countries. Nevertheless, the effect of removing differences in \( \hat{w}_0 \) is relatively weak.

A second source of TQ differences across countries are the labour market conditions which are captured in the model by parameters \( \alpha \) and \( \gamma \). I simulate alternative scenarios removing differences in labour market parameters by imposing to all countries the mean of the panel.

In a second exercise, I remove differences in the wage returns to the observable skill (\( \alpha \)). In this scenario the variance of TQ remains almost constant (move down 0.2%). The almost null effect of \( \alpha \) over TQ dispersion is because the observed values of (\( \alpha \)) across countries in average covariate negatively with other sources of dispersion in our panel. Then, even though the exercise remove the variance of one element with impact on the TQ variance, the overall variance of TQ remains almost constant. This result
imply that, current differences in the labour market returns does not play a key role to explain differences in TQ across countries.

In the same line, I build a third scenario removing differences in the share of teachers in the labour force (\(\gamma\)). In this case, the variance of TQ fails in 3.5\%. Then, the share of the teacher staff is a relevant amplifier of TQ variance even though it effect is weak.

Finally, I evaluate the role of the general skills of the population. For that, I simulate a fourth scenario without differences in the population distribution of skills. In this case I impose to all countries the same values (panel means) for parameters \(\mu_\theta, \sigma_\theta, \mu_t\) and \(\sigma_t\). Under this assumption, cross-country differences in TQ follows entirely from the fact that teacher salaries and labour market conditions determine that the hired teachers belongs to different percentiles of the skills distribution. In this scenario, the cross-country variance of TQ fails by 91\%. This scenario highlights the importance of initial distributions of population skills to determine the quality of teachers.

To understand the implication of this result, we can think in an scenario in which teachers in all countries belongs to the same percentile of the population skill distribution. In this case, all the cross-country differences in TQ would follow from the initial distribution of the population skills. Of course, countries with an initial distribution located at the right (higher mean) will hire teachers with higher skills. So, in order to converge in terms of TQ, countries countries where the initial distribution of skills is characterized by a smaller mean would need hire their teachers from a higher percentile of the population distribution respect to the average.

Summarizing, previous counter-factual exercises suggest that initial distribution of skills determines most of current TQ differences. The teacher selection process derives in better results in countries with a high quality space of selection. In this sense, the history of educational results and its implications on current populations skills plays a key role to explain current dispersion in student achievements.
Table 5 - Sources of TQ differences across countries

<table>
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<th>Modified parameters</th>
<th>Imputed value</th>
<th>Variance in TQ</th>
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<tr>
<td>Ex. 1 $\tilde{w}_0$</td>
<td>panel mean</td>
<td>-6.6%</td>
</tr>
<tr>
<td>Ex. 2 $\alpha$</td>
<td>panel mean</td>
<td>-0.2%</td>
</tr>
<tr>
<td>Ex. 3 $\gamma$</td>
<td>panel mean</td>
<td>-3.5%</td>
</tr>
<tr>
<td>Ex. 4 $\mu_t, \mu_\theta, \sigma_t, \sigma_\theta$</td>
<td>panel mean</td>
<td>-91%</td>
</tr>
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</table>

Additionally, observed differences in teacher salaries and labour market conditions act as amplifiers of initial differences of populations skills. For instance, countries with better initial conditions in terms of the teacher ability distribution also pay, in average, higher teacher salaries. Therefore, a different configuration of labour market conditions, and particularly of teacher salaries would be required to break down the hysteresis introduced by initial differences in population skills.

5 Conclusions

This paper evaluates to what extent teacher quality explains cross-country differences in student achievements. Additionally, using counter-factual exercises the paper analyses the main drivers of teacher quality variation across countries.

To do this, I propose and calibrate a micro-founded model to study the teacher selection process. Based on the calibrated model, I build time series of teacher quality at a country level for 22 OECD countries and use them as input in the estimation of an education production function (EPF). As far as I know, no previous studies provide time series of teacher quality. These time series allows me to analyse whether teacher quality trends can explain the evolution of student achievements at a country level. Finally, using the theoretical model and the estimated EPF, I carry out several counter-factual exercises to evaluate the contribution of different variables for explaining cross-country differences in student outcomes.

The paper has three key results. First, the model measure of teacher quality shows a significant positive correlation with student outcomes in
PISA across-countries, even controlling for country fixed effects. This result suggests that our measure of teacher quality at the country level seems a reasonable one. Second, TQ is the single educational input with the largest impact on cross-country variance in student achievements explaining approximately 22% of it. Although, this result is in line with previous literature that highlights the importance of teacher quality as an educational input, I go further by comparing how important is TQ with respect to other inputs of the educational production function. I found that the impact of TQ is quite similar to the impact of family inputs and clearly higher than the effect of other school inputs. Third, initial distributions of population skills determine most of current differences in TQ across countries. The effect of initial distributions results is broadly larger than the effect of teacher salaries and labour market conditions.

Changing labour market conditions and particularly teacher salaries is a key factor to break down the hysteresis introduced by initial differences in the skills of the population. This is a big challenge for policy makers in countries with poor educational outcomes.

The micro-founded model could be a useful tool to analyse the effectiveness of different educational policies and to evaluate reallocations within the educational budget. We leave this for future research.

6 References


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7 Appendix I. Transmission channels from teacher salaries to teacher quality

In this section, I explore graphically the three channels through which teacher salaries impact on the quality of teachers in two different scenarios. First assuming that correlation between the two skills are strong $\rho_{\theta t} = 0.7$, and a second scenario under a weaker correlation of $\rho_{\theta t} = 0.1$.

Recall that three characteristics of the individuals that choose to apply for a teacher position (that is the supply of teachers), are relevant to study the school selection stage:

Firstly, supply size matters (i.e. the quantity channel). A higher number of individuals in the teacher supply increases the selection possibilities of schools. That is, schools are restricted to a larger subset when it solves its optimization problem. Just as example, if teacher supply is equal to the number of teacher required by the education system, the stage of selection developed by schools is not relevant. In fact, schools just hire all the individuals into the teacher supply. Therefore, supply size is the variable that determines the quantity channel.

Secondly, supply quality matters (i.e. the quality channel). Since schools select teachers from the teacher supply, the average teacher skill of this sub-
group of the population is a key variable too. Independently of the selection strategy, the selection process will be very inefficient if all the individuals in the teacher supply are bad teachers. A higher average teacher skill within the teacher supply is a signal of a better quality in the selection space of schools. Therefore, the average teacher skill of the subset of individuals integrating the teacher supply illustrates our quality channel.

Finally, the correlation between teacher skills and general skills restricted to the teacher supply matters (the signal channel). Since schools use the general skill as signal to hire teachers (teacher skill is unobservable in the model), the correlation among both skills within the teacher supply determines the power of the signal used by schools.

Figure 14 shows a simulated population of 1,000,000 individuals for which skills follows a log-normal distribution (see equation 1). In the picture, each individual - characterized by a pair of skills $(t, \theta)$ - is identified as a point in the plane. The upper panel of the figure presents the total population while the lower panel focuses on the individuals who apply for a teacher position (i.e. the teacher supply).

The blue line of the upper panel is the indifference line for a different teacher salary\(^{32}\). Individuals below the indifference line choose to apply for a job in the education sector. On the other hand, individuals above the indifference line choose to work in the market sector.

The scatter plot in Figure 14 shows a strong correlation among the skills. As a consequence of this correlation, people with a high level of general skill, usually have a high level of teacher skill.

Note that the sub-population below the indifference curve has a different distribution of skills compared to the distribution of skill in the total population. Therefore, even though I assume a fixed mean, variances and correlation among skills for the total population, these moments are endogenous within the teacher supply. The lower panel of the Figure focuses on the individuals that choose to work in the teacher sector for the following relative teacher salaries (0.3; 1; 2.7).

A rise in the relative teacher salary moves up the indifference line. More individuals are located under the indifference line as a consequence of the

\[^{32}\text{Remember that each individual chooses the sector to work in order to maximize his utility (see equation 4. Indifference line has the following expression: } \theta = \frac{w_0 + c + r t}{a}. \text{ The indifference curve is computed assuming that } \tau = 0.1 \text{; and } c = 0 \text{; and } w_0 = 0 (or } \hat{w}_0 = 1)\]
rise in the teacher salary. That is, more individuals apply for a job in the teacher sector. A second effect of a rise in teacher salary is a higher quality of the teacher supply. A higher teacher salary induces to individuals with higher general skill (and therefore higher potential market wage) to choose the teacher sector. But, given the strong correlation between skills, these individuals, on average, also have a higher teacher skill. Finally, the figure shows that correlation between skills within the teacher supply, measured by the slope of the regression line -red line- in the lower panel, is stronger as higher is the teacher salary. Therefore, in this context a rise in the teacher salary increase the power of the signal used by schools in the second stage of selection.

In the second example, I simulate a population of 1,000,000 individuals for which the correlation between the two skills is very weak ($\rho_{\theta t} = 0.1$). Figure 15 presents the total population of individuals together with indifference curves for different levels of teacher salaries. Again, the upper panel of the figure presents the total population while the lower panel focuses on the individuals who apply for a teacher position (teacher supply).

Figure 14 - Teacher salary and teacher quality under a strong correlation between skills

In this case, the points in the plane do not follow a clear pattern. Therefore, individuals with a high level of $\ln(\theta)$ show a high dispersion in terms of $\ln(t)$. As before, a higher teacher salary moves up the indifference line.
Like in the strong correlation population, a higher teacher salary increase the number of individuals in the teacher supply. That is, a higher teacher salary enlarge the space of selection of schools (quantity channel). However, in this case the average quality of teacher supply does not change when teacher wage is higher. As before, a rise in the teacher supply induces to workers with higher general skill to choose the teacher sector, but in this case individuals with higher \( \ln(\theta) \) do not necessarily has a higher \( \ln(t) \). Therefore in this scenario, the quality channel does not operates or is very weak compared to the previous case. Finally, the picture show that in this case, as the small is the teacher supply and the higher is the correlation between skills, the higher is the power of the signal used by schools.

**Figure 15 - Teacher salary and teacher quality under a weak correlation between skills**

![Graph showing teacher salary and teacher quality](image)

8 Appendix II. The Iteration process of calibration

The process starts with the approximation of the average teacher quality \( (\mu_t/\Theta = 1) \) for each country in 2012, using PISA test scores for this year. The normalized values obtained operating with equation (15) are the following:
Table 7. Teacher quality by country 2012

<table>
<thead>
<tr>
<th>Country</th>
<th>$\mu_t / \Theta = 1_{(math)}$</th>
<th>$\mu_t / \Theta = 1_{(read)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Austria</td>
<td>0.1336</td>
<td>-0.687</td>
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<td>0.567</td>
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<tr>
<td>Czech. R</td>
<td>0.0203</td>
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<td>0.782</td>
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<td>France</td>
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<td>0.417</td>
</tr>
<tr>
<td>Germany</td>
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<td>0.249</td>
</tr>
<tr>
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<td>0.544</td>
</tr>
<tr>
<td>Italy</td>
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<td>1.829</td>
</tr>
<tr>
<td>Korea</td>
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<td>1.417</td>
</tr>
<tr>
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<td>Slovak, R.</td>
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<td>-2.118</td>
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<tr>
<td>Spain</td>
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<td>-0.796</td>
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<td>Sweden</td>
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<td>-1.107</td>
</tr>
<tr>
<td>UK</td>
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<td>-0.272</td>
</tr>
<tr>
<td>US</td>
<td>-0.7802</td>
<td>-0.081</td>
</tr>
</tbody>
</table>

The second step of the process involves to recuperate parameter $\mu_t$. For that, I need to solve the model in an inverse sense.

Parameters to solve the model in the first iteration are:

Table 8. Parameter values.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>values</th>
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<tbody>
<tr>
<td>$\mu_\theta$</td>
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<tr>
<td>$\sigma_t$</td>
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</tr>
<tr>
<td>$\sigma_\theta$</td>
<td>1</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>1</td>
</tr>
<tr>
<td>$\gamma$</td>
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<tr>
<td>$\rho_{\theta t}$</td>
<td>0.5</td>
</tr>
<tr>
<td>$\tau^0$</td>
<td>0.1</td>
</tr>
</tbody>
</table>

The third step implies to estimate parameters $\rho_{\theta t}$ and $\tau$ in order to minimize the distance between the theoretical and empirical TW-TQ. The process converge to $\rho_{\theta t} = 0.57$ and $\tau = 0.24$ starting from different initial values of $\rho_{\theta t}$ and $\tau$. 41