Capital accumulation when consumers are tempted by others’ consumption experience

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Abstract
This paper analyzes how temptations and costly self-control influence consumer’s decisions on savings and the accumulation of wealth along life-cycle. We use an overlapping generations model where individuals are tempted to take the average consumption of agents living in the same period as their own aspiration or consumption reference. In addition, consumers also exhibit preference for self-control. Therefore, they face a self-control problem and the degree of this problem is endogenously determined by the aggregate allocation of resources. We show that temptation and costly self-control may either increase or decrease the accumulation of capital. The crucial point would be whether or not consumers take the consumption of the individuals belonging to the other living generations as a determinant of their consumption reference. This point also affects the stability and welfare properties of the competitive equilibrium. In particular, we obtain that multiple equilibrium paths may exist and that consumption externalities may generate either overaccumulation or underaccumulation of capital.

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1. Introduction

In this paper we aim at analyzing how temptation and costly self-control influence consumer’s decisions on savings and the accumulation of wealth along the life-cycle. We use an overlapping generations model where individuals are tempted to satisfy some aspirations in consumption. We specifically consider that consumers are tempted to use a reference level of consumption to compare the utility derived from their own consumption. We will assume that this reference consumption is a weighted average of the consumption of all agents living in the same period. However, consumers also exhibit a preference for self-control, so that their decisions results from accommodating two competing desires: the gratification from fully succumbing to their aspirations and the gratification derived from ex-ante ignoring their "social status" (i.e., their position in the social scale of consumption).

Several studies have investigated the empirical relevance of self-control problems by using different methodologies: (a) household-level data (see, Bucciol, 2012; and Hung et al., 2015); (b) experimental data (see, Frederick et al, 2002); and (c) survey data (see, Ameriks et al., 2007). They in general support the presence of temptation features and self-control problems. This empirical evidence have encouraged economic literature to investigate how the presence of self-control problems influence the economic decisions of individuals, as well as to study whether public authorities should intervene to alter the decentralized individuals’ response to those problems.¹

The standard macroeconomic model rests on geometric discounting of future utility. Thus, in light of the aforementioned empirical evidence, this model may be an inappropriate framework to study the intertemporal decisions on consumption. Strotz (1956) and Phelps and Pollak (1968) introduced an alternative framework with quasi-hyperbolic discounting that sets up a game between the different intertemporal selves cohabiting in an individual (i.e., a conflict between the opposite desires and aspirations that an individual may exhibit).² However, the preferences based on this quasi-hyperbolic discount are time-inconsistent and individuals does not have any commitment device. Gul and Pesendorfer (2001) introduced a class of utility functions that represent preferences displaying self-control problems and preserving time consistency. The idea is that the objective function of consumers is given by the combination of two utility functions: the commitment utility function, which corresponds to the standard utility; and the temptation utility. Thus, the actual choice is a compromise between the commitment utility and the cost of self-control (i.e., how these actual decisions depart from what the temptation would dictate).

These Gul-Pesendorfer’s preferences then reveal to be a promising fundamental to explain some economic puzzles that cannot be easily reconcile with more traditional theories. For instance, those preferences were used for dealing with the question whether or not individual save enough for their retirement and, therefore, whether or not a social security program may be socially optimal (see, for instance, Kumru and Thanopoulos, 2008 and 2011; or Bucciol, 2011). A common assumption of this literature is that the commitment and the temptation utility functions only differ in the intensity of the

¹See, for instance, Krusell et al. (2010) for a detailed discussion on this issue.
²Laibson (1997) uses this framework to prove that individuals with quasi-hyperbolic discount undersave.
trade-off between current and future consumption. These disparities in the marginal
rate of substitution between consumption at different periods are driven by: (a) either
different discount factors (see, e.g., Krusell and Smith, 2003; Krusell et al., 2007); or
different intertemporal elasticities of substitution (see, e.g., Kumru and Thanopouloos,
2011). However, all aforementioned literature assume that these two fundamentals are
dynamically constant. This has two important consequences for the predicted behavior
of the corresponding economy. On the one hand, temptation and self-control problems
have a monotonic effect on the accumulation of capital. Generally, this literature
assumes that consumers are monotonously tempted for immediate gratification, so
that it obtains that temptation and self-control problems reduce the capital stock. On
the other hand, the degree of the self-control problem remains also constant along
the process of economic growth. However, empirical evidence seems to contradict
these two results. For instance, Ameriks et al. (2007) find that self-control problems
are decreasing in scale with age and, furthermore, they lead some individuals to
underconsume and others to overconsume.

In this paper, we aim to contribute to the literature on life-cycle saving and capital
accumulation by endogenizing quite naturally the degree of the self-control problem.\(^3\)
We combine the literature on preference reversal with that on consumption externalities.
More specifically, we consider an overlapping generations model where individuals live
for two periods. Preferences are represented by the combination of two momentary
utility functions: a commitment utility function that only depends on own consumption;
and a temptation utility function that depends on both own consumption and the
weighted average consumption of all agents living in the same period. In comparing the
gratification from the two utility functions, individuals take into account the disutility
from self-control, i.e., from no fully succumbing to temptation. Consumers maximize
the discounted sum of utilities from their two periods of life, where the factor used for
discounting of commitment and temptation utility may be different.

As was mentioned before, the degree of the self-control problem faced by each
individual in our model is then endogenous as temptation depends on the resource
allocation of all agents. While the intertemporal elasticity of substitution of the
commitment utility function is constant, the temptation utility function exhibits a time-
varying intertemporal elasticity because of the presence of consumption externalities.\(^4\)
We find in this framework that temptations and costly self-control may either increase
or decrease the accumulation of capital as empirical evidence based on survey data
suggests (see, Ameriks et al., 2007). The crucial assumption would be whether or
not consumers take the consumption of the individuals belonging to the other living
generations as a determinant of their consumption reference. The level of capital will
be positively affected by the presence of endogenous self-control problems if at least one
of the following conditions holds: (a) the discount factor of temptation utility function
is sufficiently large; or (b) individuals’ aspirations depend on the consumption of other
living generations. The fact that preferences are intergenerationally dependent also
affects the stability and welfare properties of the competitive equilibrium. In particular,

\(^3\)Pavoni and Yazici (2015) assume that the individuals’ ability to self-control exogenously varies
with age, i.e, the degree of self-control problem is age-dependent.

\(^4\)Fisher and Hof (2000) discuss how consumption externalities influence the intertemporal elasticity
of substitution on consumption.
we show that multiple equilibrium paths may exist in this case, so that the aggregate equilibrium allocation is indeterminate and displays coordination failure. In addition, we prove that consumption externalities may generate either overaccumulation or underaccumulation of capital, so that the nature of the optimal policy crucially depends on how individuals form the aspirations that give rise to their temptations and self-control problem.

The paper is organized as follows. Section 2 presents the general model with the formulation of the self-control problem and interpersonally dependent preferences. We provide a detailed explanation of the novelty of this formulation with respect to the previous literature using self-control preferences. Section 3 explains how the proposed model of capital accumulation nests some well known models, which will be useful for the analysis in the rest of the paper. In Sections 4 an 5 we characterize the behavior of the capital stock at the steady-state equilibrium and along the transitional dynamics, respectively. In Section 6 we conduct the analysis of optimality by comparing the solution achieved by the social planner with the competitive solution. Section 7 contains some final comments and remarks.

2. The model

We consider an economy populated by overlapping generations of identical individuals uniformly distributed on the interval $[0, 1]$. Each of these individuals lives for two periods and has one descendant at the end of the first period of his life. Each agent supplies inelastically one unit of labor in the first period of his life and is retired in the second period. Each young individual distributes his labor income between consumption and savings. Therefore, an individual faces the following budget constraint during his first period of life:

$$c_t + s_t = w_t,$$

where $w_t$ is the wage rate per unit of time, $c_t$ is the amount of consumption of a young agent and $s_t$ is the amount saved. In the second period of their lives, individuals receive the returns on the amount of their saving, which is spent on consumption. Therefore, the budget constraint of an old individual is

$$d_{t+1} = R_{t+1}s_t,$$

where $d_{t+1}$ is the amount of consumption of an old individual and $R_{t+1}$ is the gross rate of return on saving.

Agents derive utility from the amount consumed in both periods of their life. We assume that individuals exhibit a kind of the dynamic self-control preferences due to Gul and Pesendorfer (2001, 2004). More precisely, individuals feature a temptation to take some external aspiration as a reference with respect their own consumption will be compared to. However, they are still able to exercise some level of self-control at the psychological cost of deviating from the private welfare associated with fully falling into the temptation. In particular, individual’s preferences are then represented by two functions: $u(c_t, d_{t+1})$ and $v(c_t, a_t, d_{t+1}, h_{t+1})$, where $u(.)$ is the commitment utility, $v(.)$ is the temptation utility, while $a_t$ and $h_{t+1}$ are the consumption references which
tempt the individual in his first and second period of life, respectively. Resisting this temptation gives rise to a self-control utility cost. By taking the consumption references \(a_t\) and \(h_{t+1}\) as given, the representative agent solves the following problem:

\[
\max_{\{c_t, d_{t+1}\}} [u(c_t, d_{t+1}) + v(c_t, a_t, d_{t+1}, h_{t+1})] - \max_{\{\tilde{c}_t, \tilde{d}_{t+1}\}} v(\tilde{c}_t, a_t, \tilde{d}_{t+1}, h_{t+1})
\]  

subject to the budget constraints (2.1) and (2.2), where \(c_t\) and \(d_{t+1}\) represent the commitment choice of consumptions in both periods of life, whereas \(\tilde{c}_t\) and \(\tilde{d}_{t+1}\) are the temptation alternatives to those values of consumptions (i.e., they represent the values of consumptions that individual would choose in the absence of self-control or commitment). The choice of the agent then maximizes the sum of the commitment and temptation utilities, while the cost of self-control for a given choice of \(c_t\) and \(d_{t+1}\) is given by the amount of temptation utility that the agent forgoes as a result of self-control, i.e.,

\[
\max_{\{\tilde{c}_t, \tilde{d}_{t+1}\}} v(\tilde{c}_t, a_t, \tilde{d}_{t+1}, h_{t+1}) - v(c_t, a_t, d_{t+1}, h_{t+1}).
\]

In most macroeconomic models using Gul-Pesendorfer preferences, the only difference between commitment and temptation utility is that the individual discount factor is lower in the latter. In this case, we would easily obtain that the usual time separability of the utility functions implies

\[
v'_{c_t} = \varepsilon_1 u'_{c_t},
\]

and

\[
v'_{d_{t+1}} = \varepsilon_2 u'_{d_{t+1}},
\]

where \(\varepsilon_1\) and \(\varepsilon_2\) are some particular constant, while \(u'_{c_t}\) and \(u'_{c_t}\) represents the marginal utilities for temptation and commitment, respectively, and with \(\varepsilon_1 > \varepsilon_2\). Our model is thus more general since our utility function \(v(.)\) also depends on consumption references \(a_t\) and \(h_{t+1}\) making the temptation effect endogenous, so that the two previous conditions (2.4) and (2.5) do not necessary hold. In order to formalize this feature, we derive the first order conditions of the representative consumer’s problem at period \(t\). Thus, we obtain

\[
\frac{u'_{c_t}}{u'_{d_{t+1}}} \left(1 + \frac{u'_{c_t}}{u'_{c_t}}\right) = \lambda_t,
\]

\[
\frac{u'_{d_{t+1}}}{u'_{d_{t+1}}} \left(1 + \frac{u'_{d_{t+1}}}{u'_{d_{t+1}}}\right) = \frac{\lambda_t}{R_{t+1}},
\]

where \(\lambda_t\) is the multiplier of the intertemporal budget constraint

\[
c_t + \frac{d_{t+1}}{R_{t+1}} = w_t.
\]

Combining both conditions we obtain:

\[
\frac{u'_{c_t}}{u'_{d_{t+1}}} = R_{t+1} \left(1 + \frac{u'_{d_{t+1}}}{u'_{d_{t+1}}}\right) = R_{t+1} \tilde{T}(c_t, a_t, d_{t+1}, h_{t+1}).
\]
The function $\hat{T}(\cdot)$, which we label as the \textit{temptation function}, measures how the temptation feature of preferences determines the intertemporal allocation of consumption. Note that this temptation function is equal to one in the neoclassical case (i.e., in the absence of temptation) because $v'_{ct} = v'_{dt+1} = 0$ in this case. However, if the individuals are tempted to follow the external consumption references, then the function $\hat{T}(\cdot)$ can be larger than, smaller than or equal to unity and, moreover, this value can endogenously depend on the state of the economy represented by the stock of capital. When $\hat{T}(\cdot) > 1$, for a given rate of interest, the intertemporal marginal rate of substitution must increase in order to satisfy our equilibrium condition. The agent thus decides to increase future consumption at the expense of present consumption. When $\hat{T}(\cdot) < 1$, the opposite reasoning applies implying that the marginal rate of substitution must decrease while present consumption increases. In a standard temptation model where the only difference between both utilities is the subjective discount factor, $\hat{T}(\cdot)$ is a constant lower than one because Conditions (2.4) and (2.5) hold. In this case the agent is then induced to increase present consumption. One of the main contributions of the paper is to endogenize temptation through the introduction of consumption externalities. They introduce differences between the intertemporal elasticity of substitution of the commitment utility function $u(\cdot)$ and the one of the temptation utility $v(\cdot)$. Furthermore, this gap is time-varying because of the endogenous nature of the intertemporal elasticity of substitution of the temptation utility.

In order to proceed we will now consider specific functional form for both utilities. We assume that the period utility functions $u(\cdot)$ and $v(\cdot)$ are of the CIES type:

$$
u(c_t, d_{t+1}) = \frac{c_t^{1-\sigma}}{1-\sigma} + \delta \frac{d_{t+1}^{1-\sigma}}{1-\sigma},\quad (2.7)$$

and

$$v(c_t, a_t, d_{t+1}, h_{t+1}) = \psi \left[ \frac{(c_t - \gamma_1 a_t)^{1-\sigma}}{1-\sigma} + \delta \beta \frac{(d_{t+1} - \gamma_2 h_{t+1})^{1-\sigma}}{1-\sigma} \right],\quad (2.8)$$

where $\delta > 0$ is the subjective discount factor in commitment utility; $\sigma > 0$ is the inverse of the intertemporal elasticity of substitution; $\psi > 0$ is the weight of temptation utility (i.e., it measure the strength of temptation); $\beta < 1$ is a parameter reflecting the fact that the future is more heavily discounted in temptation utility; and $\gamma_1 \in [0, 1]$ and $\gamma_2 \in [0, 1]$ measure the intensity of consumption references in the first and second period of life, respectively. If $\psi = 0$, we obtain the neoclassical model, while if $\gamma_1 = \gamma_2 = 0$, we obtain the standard temptation model where the unique deviation between the commitment and the temptation utilities is the rate at which an individual discounts the utility of his second period of life: this rate is $\delta$ in the commitment utility and $\beta\delta$ in the temptation utility. In our framework, we can assume that $\beta > 1$ and we can still obtain $\hat{T}(\cdot) < 1$ because in our economy with consumption references the value of $\hat{T}(\cdot)$ also depends on equilibrium allocations. However, we assume $\beta < 1$ to allow for the comparison of the equilibrium allocation in the proposed economy with the allocation that would emerge in the standard temptation model (i.e., when $\gamma_1 = \gamma_2 = 0$).

We assume that the temptation utility does not depend on the absolute level of consumption as in the standard temptation model, but on the comparison between this level and the consumption references. In addition, we also assume that the
consumption references depend on the current levels of consumption of the young and the old individuals. In particular, following Abel (2005) and Alonso-Carrera (2008), these consumption references are assumed to be a weighted arithmetic average of the per capita consumption of the two living generations, which we denote by $\hat{c}_t$ and $\hat{d}_t$. Preferences then display consumption externalities provided that they feature temptations (i.e., $\psi > 0$). On the one hand, we consider that

$$a_t = \frac{\hat{c}_t + \theta_1 \hat{d}_t}{1 + \theta_1},$$

(2.9)

where $\theta_1 \in [0, 1]$ is the weight given to consumption of a representative old individual in the specification of the reference for the young individuals. On the other hand, we assume that

$$ht_{t+1} = \frac{\theta_2 \hat{c}_{t+1} + \hat{d}_{t+1}}{1 + \theta_2},$$

(2.10)

where $\theta_2 \in [0, 1]$ is the weight given to consumption of a representative young individual in the specification of the reference for the old individuals. Note that the restrictions imposed on the values of the parameters imply that we are giving a larger weight to the average consumption of the individuals belonging to the same generation.

With the specification of preferences given by (2.3), (2.7), (2.8), (2.9) and (2.10), we need to ensure that the indifference curves of both commitment and temptation utility function $u(.)$ and $v(.)$ be downward sloping.\footnote{See Lahiri and Puhakka (1998) and Wendner (2002) for a detailed explanation of this requirement.} This is always the case for commitment utility but not necessarily for temptation utility. Given a value of the consumption references $a_t$ and $h_{t+1}$, the intertemporal marginal rate of substitution between consumptions at young and old ages for temptation utility is given by

$$\frac{\partial d_{t+1}}{\partial c_t} = -\frac{(c_t - \gamma_1 a_t)^{-\sigma}}{\delta \beta (d_{t+1} - \gamma_2 h_{t+1})^{-\sigma}}.$$

This expression is negative provided that effective consumption in both periods has the same sign. We rule out the possibility of negative effective consumption in order to avoid an ill-defined temptation utility function. By using expressions (2.9) and (2.10), it is possible to derive the following necessary and sufficient conditions which ensure that effective consumption is positive in the first and the second period of life along a symmetric equilibrium with $c_t = \hat{c}_t$ and $d_t = \hat{d}_t$, respectively:

$$\frac{\gamma_1 \theta_1}{1 + \theta_1 - \gamma_1} < \frac{c_t}{d_t} < \frac{1 + \theta_2 - \gamma_2}{\gamma_2 \theta_2}. $$

(2.11)

From now on, we assume that these conditions are always satisfied. Note that they restrict the domain of the initial capital stock (which is the state variable in our economy) to have a well defined equilibrium path. The strong non-linearity of the policy function relating the equilibrium values of consumptions $c_t$ and $d_t$ with the capital stock $k_t$ does not allow us to derive the explicit value of this threshold for the initial capital stock. In any case, the left-hand side of the first inequality is zero when $\theta_1 = 0$, so that this inequality always hold in this case. In addition, the second
inequality always holds when $\theta_2 = 0$ because in this case the right-hand side converges to infinite.

By particularizing Condition (2.6) with our parametrization of preferences, we obtain that the optimal plan of consumers is characterized by the following condition:

$$\frac{c_t}{d_{t+1}} = R_{t+1} \tilde{T}(c_t, a_t, d_{t+1}, h_{t+1}),$$

(2.12)

where

$$\tilde{T}(c_t, a_t, d_{t+1}, h_{t+1}) = \frac{1 + \beta \psi (1 - \gamma_2 h_{t+1}/d_{t+1})^{-\sigma}}{1 + \psi (1 - \gamma_1 a_t/c_t)^{-\sigma}},$$

(2.13)

is our parametric form of the temptation function $\tilde{T}(.)$ in (2.6). Observe that the temptation function $\tilde{T}(.)$ is constant and equal to $(1+\beta\psi)/(1+\psi) < 1$ when $\gamma_1 = \gamma_2 = 0$ (i.e., in the case of standard temptation where commitment and temptation utility functions only differ in the subjective discount factor). Otherwise, this temptation function is endogenously determined by the equilibrium value of consumptions at the first and second periods of life. In fact, we can rewrite the temptation utility function (2.8) in equivalent terms as the one used by the standard temptation case. More precisely, we can express (2.8) as

$$v(c_t, a_t, d_{t+1}, h_{t+1}) = \tilde{v}_t \left[ \frac{c_t^{1-\sigma}}{1-\sigma} + \tilde{\beta}_{t+1} d_{t+1}^{1-\sigma} \right],$$

(2.14)

with

$$\tilde{v}_t = \psi (1 - \gamma_1 a_t/c_t)^{1-\sigma},$$

(2.14)

and

$$\tilde{\beta}_{t+1} = \beta \left( \frac{1 - \gamma_2 h_{t+1}/d_{t+1}}{1 - \gamma_1 a_t/c_t} \right)^{1-\sigma}.$$ 

(2.15)

Observe that this expression of the temptation utility function only differs with the commitment one (2.7) because the presence of the variables $\tilde{v}_t$ and $\tilde{\beta}_{t+1}$. We can then assert that the self-control problem in our formulation of preferences are also fully driven by the intensity of temptation $\tilde{v}_t$ and by the difference in the way of discounting the future in the commitment utility function ($\beta$) and in the temptation utility function ($\tilde{\beta}_{t+1}$). Our temptation formulation of preferences based on interpersonal comparisons of consumption endogenizes the intensity of temptation and the discount factor. Therefore, we naturally obtain a time varying (and endogenous) self-control feature without considering any ad-hoc structural variation in the parameters defining preferences as, for instance, in Pavoni and Yazici (2015).

We close the presentation of the model’s fundamentals by showing the features of the production-side of the economy. There is a single commodity $y_t$ that can be used for either consumption or investment. We assume that this good is produced by means of the Cobb-Douglas production function

$$y_t = A k_t^{\alpha},$$

(2.16)

where $A$ is the constant total factor productivity, $k_t$ is the stock of capital and $\alpha \in (0, 1)$ is the share of capital income in output. We also suppose full depreciation of the capital
stock after one period. Perfect competition among firms leads the rental prices of the two inputs, capital and labor, to equal their marginal productivities, i.e.,

\[ R_t = \alpha A k_t^\alpha - 1, \]  

and

\[ w_t = (1 - \alpha) A k_t^\alpha. \]  

By imposing the consistency equilibrium conditions \( c_t = \hat{c}_t \) and \( d_t = \hat{d}_t \), we define the competitive equilibrium of this economy as a path \( \{k_t\}_{t=0}^\infty \) that, for a given initial value of the capital stock \( k_0 \) satisfying Condition (2.11), solves the difference equation (2.12), together with (2.1), (2.2), (2.9), (2.10), (2.17), (2.18), the market clearing conditions for capital markets

\[ k_{t+1} = s_t, \]  

(i.e., the capital stock installed in period \( t+1 \) is equal to the aggregate saving in period \( t \)) and the aggregate resource constraint

\[ y_t = c_t + d_t + k_{t+1}, \]  

(i.e., aggregate output in period \( t \) is distributed between aggregate consumption of young individuals, consumption of old individuals and investment). By combining these equilibrium conditions, we obtain that, given an initial capital stock \( k_0 \), the equilibrium paths are fully defined by the following second-order difference equation in capital stock:

\[ G(k_t, k_{t+1}) = \delta(\alpha A)^{1-\sigma} T(k_t, k_{t+1}, k_{t+2}), \]  

where

\[ G(k_t, k_{t+1}) = \frac{k_{t+1}^{1-\alpha(1-\sigma)}}{[(1 - \alpha) A k_t^\alpha - k_{t+1}]^{1-\sigma}}, \]  

and \( T(.) \) is the temptation function (2.13) at the equilibrium, i.e.,

\[ T(k_t, k_{t+1}, k_{t+2}) = \frac{1 + \beta \psi(1 + \theta_2)\left[1 + \theta_2 - \gamma_2 - \frac{\gamma_2 \theta_2 [1 - \alpha] A k_{t+1}^\alpha - k_{t+2}}{\alpha A k_{t+1}^\alpha} \right]^{-\sigma}}{1 + \psi(1 + \theta_1)\left[1 + \theta_1 - \gamma_1 - \frac{\gamma_1 \theta_1 A k_t^\alpha}{[1 - \alpha] A k_t^\alpha - k_{t+1}} \right]^{-\sigma}}. \]  

Observe that Equation (2.21) reduces to a first-order difference equation if either \( \gamma_2 = 0 \) or \( \theta_2 = 0 \). Therefore, a second-order difference equation arises if the old individuals use the average consumption of the young individuals as a reference (i.e., \( \theta_2 > 0 \)). Finally, we also directly conclude that the variables \( \tilde{\psi}_t \) and \( \tilde{\beta}_{t+1} \), which determine the degree of the self-control problem, and which were defined in (2.14) and (2.15), are endogenously determined by the dynamic evolution of the capital stock. Obviously, this only happens when preferences are intergenerationally dependent: individuals take the consumption experience of the individuals belonging to the other living generation as a consumption reference, i.e., either \( \theta_1 > 0 \) or \( \theta_2 > 0 \). Otherwise, the degree of the self-control problem is exogenous, although it still depend on the intensity at which individuals compare their consumption experience with those of the individuals belonging to their own generation (i.e., \( \gamma_1 \) and \( \gamma_2 \)).

\(^6\)One can assert that one period of the model consists on about 30 years for the empirical evaluation. Hence, full depreciation of capital stock is not an unrealistic assumption.
3. Some special cases

Our model with self-control and interpersonally dependent preferences nests some other models that have largely been studied by the literature on capital and wealth accumulation. This property is very useful to know how these proposed preferences affect the dynamic behavior of capital and wealth. Thus we next present how our model particularizes in these well known models.

**Standard neoclassical model.** We obtain this particular model if we eliminate the temptation feature from our model, i.e., we impose \( \psi = 0 \). In this way preferences are only characterized by the commitment utility function. In this case, our model then simply reduces to the standard overlapping generations model proposed by Diamond (1965). From now on, we will denote the capital stock of this model without self-control problems by \( k_t^n \). Therefore, the equilibrium path of this particular economy is given by the path \( \{k_t^n\}_{t=0}^{\infty} \) that solves the dynamic equation (2.21) with the temptation function \( T(k_t, k_{t+1}, k_{t+2}) = 1 \).

**Standard temptation model.** We eliminate the interpersonal dependence feature from our model, i.e., we impose \( \gamma_1 = \gamma_2 = 0 \). In this case, preferences are still characterized by the interaction between the commitment and the temptation utility functions. However, the self-control problem only arises because the subjective discount factors in both utility functions are different: this factor is \( \delta \) for the commitment utility function, whereas it is \( \beta \) for temptation utility function. In this case, our model reduces to the OLG version of the self-control model introduced by the seminal papers from Gul and Pesendorfer (2001, 2004) and Krussell et al. (2010). The degree of the self-control problem in this standard temptation model is exogenous (and constant). Effectively, we directly obtain from (2.14) and (2.15) that \( \tilde{\psi}_t = \psi \) and \( \tilde{\beta}_{t+1} = \beta \) for all generations born at period \( t \). From now on, we will denote the capital stock of this standard temptation model by \( k_t^s \). Therefore, the equilibrium path of this particular economy is given by the path \( \{k_t^s\}_{t=0}^{\infty} \) that solves the dynamic equation (2.21) with the temptation function \( T(k_t, k_{t+1}, k_{t+2}) = (1 + \beta \psi) / (1 + \psi) < 1 \).

**Model with consumption externalities.** Consider now that preferences are only characterized by the temptation utility function. This happens in our economy when \( \psi \) tends to infinite. This particular economy then corresponds to an economy where the individual fully succumbs to temptation. Hence, the intertemporal consumption plan chosen by each young consumer in this case is given by the solution of the following problem:

\[
\max_{\{c_t, d_{t+1}\}} v(c_t, a_t, d_{t+1}, h_{t+1}),
\]

subject to the budget constraints (2.1) and (2.2), with the consumption references (2.9) and (2.10) taken exogenously, and where the utility function \( v(\cdot) \) is given by (2.8).\(^8\) Observe that this particular model is therefore fully equivalent to the standard model.

\(^7\)Note that in this case the constants \( \psi_1 \) and \( \psi_2 \) in (2.4) and (2.5), respectively, reduce to \( \psi_1 = \psi \) and \( \psi_2 = \psi \beta \).

\(^8\)Obviously, in this case there is no self-control cost because of the absence of commitment utility function.
with consumption externalities largely studied by the literature. In fact, our model reduces in this case to the model without self-control problems but with consumption externalities analyzed by Abel (2005) and Alonso-Carrera et al. (2008). From now on, we will denote the capital stock of this model with consumption externalities (i.e., with the absence of commitment or without any self-control device) by $k_t^T$. Therefore, the equilibrium path of this particular economy is given by the path $\{k_t^T\}_{t=0}^\infty$ that solves the dynamic equation

$$G(k_t, k_{t+1}) = \beta \delta (\alpha A)^{1-\sigma} T(k_t, k_{t+1}, k_{t+2}),$$

(3.1)

where

$$T(k_t, k_{t+1}, k_{t+2}) = \frac{(1 + \theta_2)^{\sigma} \left\{ 1 + \theta_2 - \gamma_2 - \frac{\gamma_2 \theta_2 [(1-\alpha)Ak_{t+1}^\gamma - k_{t+2}]}{\alpha Ak_{t+1}^\gamma} \right\}^{-\sigma}}{(1 + \theta_1)^{\sigma} \left\{ 1 + \theta_1 - \gamma_1 - \frac{\gamma_1 \theta_1 \alpha Ak_{t}^\gamma}{(1-\alpha)Ak_{t}^\gamma - k_{t+1}} \right\}^{-\sigma}},$$

which is derived from solving the representative consumers’ problem and imposing the market clearing conditions (2.19) and (2.20), as well as the consistency equilibrium conditions $c_t = \tilde{c}_t$ and $d_t = \tilde{d}_t$. Obviously, this equilibrium allocation is equal to the hypothetical temptation allocation $\{\tilde{k}_t, \tilde{c}_t, \tilde{d}_t\}_{t=0}^\infty$ of our model with the general specification of self-control, interpersonally dependent preferences. Note finally that the dynamic equation (3.1) differs from the dynamic equation (2.21) of our general model in: (a) the discount factor, which is now $\delta \beta$ instead of $\delta$; and (b) the function $T(.)$, although we should not properly label it as temptation function.

In order to analyze how the endogenous temptation affects the capital and wealth accumulation, we will next compare the equilibrium paths of our model with those corresponding to the previous special cases, mainly the standard neoclassical model.

### 4. Long-run effects of temptations

In this section we analyze how the self-control problems affect the stationary level of the capital stock. For that purpose, we first characterize the existence properties of the steady-state equilibrium. The steady state of this economy is characterized by the fix points of the dynamic equation (2.21). Let us denote the stationary value of the capital stock by $k$. To proceed with the study of the existence of this fix point, we decompose Equation (2.21) into the following two functions:

$$G(k) = \frac{k^{1-\alpha(1-\sigma)}}{\delta(\alpha A)^{1-\sigma} (w - k)^{\sigma}},$$

(4.1)

and

$$\tilde{T}(k) = \frac{1 + \beta \psi \left\{ 1 + \theta_2 - \gamma_2 - \frac{\gamma_2 \theta_2 [(1-\alpha)(w-k)]}{\alpha w} \right\}^{-\sigma} (1 + \theta_2)^{\sigma}}{1 + \psi \left\{ 1 + \theta_1 - \gamma_1 - \frac{\gamma_1 \theta_1 \alpha k}{(1-\alpha)(w-k)} \right\}^{-\sigma} (1 + \theta_1)^{\sigma}},$$

(4.2)

(Alonso-Carrera et al. (2008) basically combines the model with consumption externalities by Abel (2005) and the model with dynastic altruism due to Barro (1974)).
where \( w \) represents the stationary value of the wage rate, i.e., \( w = (1 - \alpha) Ak^\alpha \). Note that function (4.2) corresponds to our temptation function (2.23) evaluated at the state state, i.e., \( \tilde{T}(k) = T(k, k, k) \). We should also constrain the domain of the capital stock to ensure that the stationary value of the effective consumptions (i.e., the arguments of the temptation function) are positive. We find that the stationary stock of capital has to satisfy three restrictions. Firstly, we have to impose that \( w - k > 0 \) to insure that real consumption at young age is positive. Thus, we obtain the following condition:

\[
k < \k_1 = [(1 - \alpha) A]^{1/\alpha}.
\]

Secondly, we also have to impose that \( 1 - \gamma_1 a/c > 0 \), which happens when

\[
k < \k_2 = [(1 - \alpha) A - \frac{\alpha \gamma_1 \theta_1 A}{1 + \theta_1 - \gamma_1}]^{1/\alpha}.
\]

Observe that \( \k_1 > \k_2 \) unless \( \gamma_1 \theta_1 = 0 \) in which case they are equal. Finally, we also have to impose that \( 1 - \gamma_2 h/d > 0 \), which happens when

\[
k > \k_3 = [(1 - \alpha) A - \frac{\alpha (1 + \theta_2 - \gamma_2) A}{\gamma_2 \theta_2}]^{1/\alpha}.
\]

Note that \( \k_3 \) can be either positive or negative depending on the parameter values. If \( \k_3 < 0 \), then the stationary stock of capital has to satisfy \( k > 0 \). In fact, \( \k_3 \) converges to minus infinite when \( \gamma_1 \theta_1 \) tends to zero. Condition (2.11) guarantees that \( \k_2 > \k_3 \).

**Proposition 4.1.** In this economy, there is always a unique steady-state equilibrium with \( k > 0 \).

**Proof.** We proceed with the following three steps:

(a) We compute the derivates of functions \( G(k) \) and \( \tilde{T}(k) \). We obtain

\[
G'(k) = \frac{(1 - \alpha) [w - (1 - \sigma)k] k^{\alpha(\sigma - 1)}}{\delta(\alpha A)^{1-\sigma}(w - k)^{1+\sigma}} > 0,
\]

and

\[
\tilde{T}'(k) = -\left\{ \frac{\sigma \psi}{1 + \psi [\zeta_1 - \eta_1 \phi(k)]^{-\sigma}} \right\} \beta \eta_2 \left[ \zeta_2 - \eta_2 / \phi(k) \right]^{-\sigma - 1} \left[ \frac{\phi'(k)}{\phi(k)} \right]
\]

\[
+ \frac{\eta_1 \phi'(k) [\zeta_1 - \eta_1 \phi(k)]^{-\sigma - 1} (1 + \beta \psi [\zeta_1 - \eta_1 \phi(k)]^{-\sigma})}{[1 + \psi (\zeta_1 - \eta_1 \phi(k))]^{-\sigma} [\phi(k)]},
\]

with

\[
\zeta_1 = \frac{1 + \theta_1 - \gamma_1}{1 + \theta_1},
\]

\[
\eta_1 = \frac{\alpha \gamma_1 \theta_1}{(1 - \alpha)(1 + \theta_1)},
\]

\[
\zeta_2 = \frac{1 + \theta_2 - \gamma_2}{1 + \theta_2}.
\]
\[ \eta_2 = \frac{(1 - \alpha)\gamma_2\theta_2}{\alpha(1 + \theta_2)}, \]

and

\[ \phi(k) = \frac{w}{w - k}. \]

Since effective consumption is positive for the young and the old and given that \( \phi'(k) = \frac{(1-\alpha)^2 A k^\alpha}{(w-k)^2} > 0, \) \( T'(k) < 0. \) Given the behavior of our functions \( G(\cdot) \) and \( \tilde{T}(\cdot), \) they will cross at most once.

(b) We characterize the limits of functions \( G(\cdot) \) and \( \tilde{T}(\cdot) \) at the upper bound of the domain for \( k. \) We obtain that

\[ \lim_{k \to \min\{\kappa_1, \kappa_2\}} \left[ G(k) - \tilde{T}(k) \right] > 0, \]

because

\[ \lim_{k \to \min\{\kappa_1, \kappa_2\}} G(k) = \begin{cases} \infty & \text{if } \theta_1 = 0 \text{ (i.e., } \kappa_1 = \kappa_2) \\ M & \text{otherwise} \end{cases}, \]

and

\[ \lim_{k \to \min\{\kappa_1, \kappa_2\}} \tilde{T}(k) = \begin{cases} \frac{1 + \beta \psi (1 + \theta_2 - \gamma_2)^{-\sigma} (1 + \theta_2)^\sigma}{1 + \psi (1 - \gamma_1)^{-\sigma}} & \text{if } \theta_1 = 0 \text{ (i.e., } \kappa_1 = \kappa_2) \\ 0 & \text{otherwise} \end{cases}. \]

The previous limits derive from the fact that \( w - k = 0 \) at \( k = \kappa_1. \)

(c) We characterize the limits of functions \( G(\cdot) \) and \( \tilde{T}(\cdot) \) at the lower bound of the domain for \( k. \) We obtain that

\[ \lim_{k \to \max\{0, \kappa_3\}} \left[ G(k) - \tilde{T}(k) \right] < 0, \]

because

\[ \lim_{k \to \max\{0, \kappa_3\}} G(k) = \begin{cases} 0 & \text{if } \theta_2 = 0 \text{ (i.e., } \kappa_3 < 0) \\ M & \text{otherwise} \end{cases}, \]

and

\[ \lim_{k \to \max\{0, \kappa_3\}} \tilde{T}(k) = \begin{cases} \frac{1 + \beta \psi (1 - \gamma_2)^{-\sigma}}{1 + \psi \left[ (1 + \theta_1 - \gamma_1 - \gamma_2)^{-\sigma} (1 + \theta_2)^\sigma \right]} & \text{if } \theta_2 = 0 \text{ (i.e., } \kappa_3 < 0) \\ \infty & \text{otherwise} \end{cases}. \]
The proposition directly follows from the previous three statements.

Given that there is always a unique steady-state in our temptation economy, it will be easier to make comparative static analysis and a steady-state comparison with the standard neoclassical model. It should be noticed that since \( \tilde{T}'(k) < 0 \), the following inequality always applies: \( \min \{ \tilde{T}(0), \tilde{T}(\tilde{k}_3) \} > \max \{ \tilde{T}(\tilde{k}_1), \tilde{T}(\tilde{k}_2) \} \).

Before proceeding we also define \( \epsilon^T_k = -\tilde{T}'(k)k/\tilde{T}(k) > 0 \) which is the elasticity of the temptation function with respect to the stationary stock of capital.

**Proposition 4.2.** The stationary value of the capital stock \( k \) increases with \( \beta, \gamma_2 \) and \( \theta_2 \), whereas it decreases with \( \gamma_1 \) and \( \theta_1 \). However, the response of \( k \) to changes in \( \psi \) is ambiguous.

**Proof.** Define \( \pi = \{ \psi, \beta, \gamma_1, \gamma_2, \theta_1, \theta_2 \} \). Applying the implicit function theorem to (2.21) evaluated at the steady-state equilibrium, we obtain that

\[
\frac{\partial k}{\partial \pi} = \frac{T'_\pi}{G'_k - T'_k},
\]

where \( G'_k \) is the derivative of Function (4.1) with respect to \( k_1 \); and \( T'_\pi \) and \( T'_k \) denote the derivative of Function (4.2) with respect to \( k \) and \( \pi \), respectively. We know from the proof of Proposition 4.1 that \( G'_k > 0 \) and \( T'_k < 0 \). Hence, the sign of \( \partial k / \partial \pi \) coincides with the sign of \( T'_\pi \). We directly obtain from Function (4.2) that \( T'_\beta > 0, T'_1 < 0, T'_2 > 0, T'_\theta_1 < 0, \) and \( T'_\theta_2 > 0 \). In addition, we get

\[
T'_\psi = \frac{\beta [1 - \gamma_2 (h/d)]^{-\sigma} - [1 - \gamma_1 (a/c)]^{-\sigma}}{1 + \psi [1 - \gamma_1 (a/c)]^{-\sigma}},
\]

whose sign cannot be established. The proposition then follows.

We must remark that the stationary value of the capital stock can either increase or decrease with the parameter \( \psi \) driving the intensity of the temptation and self-control problem. This response will be crucial for the influence of these phenomena on the equilibrium dynamics of the economy. We will illustrate this point in the next section.

Finally, we now compare the stationary levels of capital in our model of temptation with the standard neoclassical one. The first thing to notice is that since \( G'(k) > 0 \), when \( \tilde{T}(k) > 1 \), the steady-state capital stock of our economy \( k \) is higher than the one of the neoclassical economy \( k^n \). In this case, the agent wishes to consume more in the future and tends to accumulate more capital. On the contrary, when \( \tilde{T}(k) < 1 \), the neoclassical economy delivers a higher steady-state capital stock. It should be noticed that in the standard model of temptation where the only difference between commitment and temptation utility is the discount factor (i.e., \( \gamma_1 = \gamma_2 = 0 \)) \( \tilde{T}(k) < 1 \) and the neoclassical economy accumulates more capital. This discussion is summarized in the following proposition.

**Proposition 4.3.** At the steady-state equilibrium, the capital stock \( k \) of our economy with temptation and self-control problems is larger than the capital stock \( k^n \) of the standard neoclassical economy if and only \( \tilde{T}(k^n) > 1 \).
Since \( k^n \) is implicitly given by the equation \( G(k^n) = 1 \), we cannot derive a general condition on parameters driving the comparison between the steady-state of our model with temptation and self-control problems and the standard neoclassical model. The next result provides this condition for some particular cases.

**Proposition 4.4.** The following statements hold:

(a) When \( \theta_1 = \theta_2 = 0 \), then \( k > k^n \) if and only if

\[
\beta^{1/\sigma} > \frac{1 - \gamma_2}{1 - \gamma_1}.
\]

(b) When \( \theta_1 > 0 \) and \( \theta_2 = 0 \), a sufficient condition for \( k < k^n \) is

\[
\beta^{1/\sigma} < \frac{(1 + \theta_1)(1 - \gamma_2)}{1 + \theta_1 - \gamma_1 - \frac{\gamma_2(1 - \alpha)}{(1 - \sigma)}}.
\]

(c) When \( \theta_1 = 0 \) and \( \theta_2 > 0 \), a sufficient condition for \( k > k^n \) is:

\[
\beta^{1/\sigma} > \frac{1 + \theta_2 - \gamma_2}{(1 - \gamma_1)(1 - \theta_2)}.
\]

**Proof.** We separately prove the three statements.

(a) When \( \theta_1 = \theta_2 = 0 \), the \( \tilde{T}(k) \) is constant and equal to

\[
\tilde{T}(k) = \frac{1 + \beta \psi (1 - \gamma_2)^{-\sigma}}{1 + \psi (1 - \gamma_1)^{-\sigma}}.
\]

Therefore, using Proposition 4.3, we directly obtain the first statement in the corollary.

(b) When \( \theta_1 > 0 \) and \( \theta_2 = 0 \), the domain for the capital stock is \((0, \bar{k}_2)\) and, moreover, we know from the proof of Proposition 4.1 that

\[
\lim_{k \to 0} \tilde{T}(k) = \frac{1 + \beta \psi (1 - \gamma_2)^{-\sigma}}{1 + \psi \left[ 1 + \theta_1 - \gamma_1 - \frac{\gamma_2(1 - \alpha)}{(1 - \sigma)} \right]^{-\sigma} (1 + \theta_1)^{\sigma}},
\]

and

\[
\lim_{k \to \bar{k}_2} \tilde{T}(k) = 0.
\]

Therefore, if \( \lim_{k \to 0} \tilde{T}(k) < 1 \), then \( \tilde{T}(k) < 1 \) for all \( k_t \in (0, \bar{k}_2) \). The second statement of the corollary then directly follows from using Proposition 4.1.

(c) When \( \theta_1 = 0 \) and \( \theta_2 > 0 \), the domain for the capital stock is \((\bar{k}_3, \bar{k}_2)\) and, moreover, we know from the proof of Proposition 4.1 that

\[
\lim_{k \to \bar{k}_3} \tilde{T}(k) = \infty,
\]

15
and
\[
\lim_{k \to \bar{k}_2} \tilde{T}(k) = \frac{1 + \beta \psi (1 + \theta_2 - \gamma_2)^{-\sigma} (1 + \theta_2)^\sigma}{1 + \psi (1 - \gamma_1)^{-\sigma}}.
\]

Therefore, if \( \lim_{k \to \bar{k}_2} \tilde{T}(k) > 1 \), then \( \tilde{T}(k) > 1 \) for all \( k \in (\bar{k}_3, \bar{k}_2) \). The third statement of the corollary then directly follows from using Proposition 4.1.

We can also derive from Proposition 4.3 the stationary property of the capital stock \( k^* \) of the standard temptation model, where the self-control problem arises from the fact that consumers are only tempted toward immediate gratification (i.e., \( \beta < 1 \)). The next result states this property.

**Corollary 4.5.** If \( \gamma_1 = \gamma_2 = 0 \), then \( k = k^* < k^n \).

**Proof.** When \( \gamma_1 = \gamma_2 = 0 \), we derive from (2.23)
\[
\tilde{T}(k) = \frac{1 + \beta \psi}{1 + \psi} < 1.
\]

Therefore, using Proposition 4.3, we directly obtain the corollary.

The previous three results illustrate the importance of temptation and self-control problems based on interpersonal comparison of consumption concerning the steady-state equilibrium. The main conclusion is that the introduction of consumption externalities in the temptation utility function \( v(.) \) may reverse the result of the standard temptation model where the presence of \( \beta < 1 \) implies that the stationary capital stock is always smaller than the one of the neoclassical model. Effectively, Corollary 4.5) remarks the crucial role of \( \gamma_1 \) and \( \gamma_2 \) for the consumption-saving decision. However, the two kind of consumption externalities driving the temptation have an opposite effect. On the one hand, the presence of consumption externalities in the temptation of young individuals (i.e., \( \gamma_1 > 0 \)) encourage their consumption. Conversely, the presence of consumption externalities in the temptation of old individuals (i.e., \( \gamma_2 > 0 \)) encourage young individuals to save in order to finance the competition for social status when old.

The results in this section also suggest that the fact of whether or not individuals consider the consumption of the other living generation as a determinant of temptation has a second order effect on the value of the stationary capital stock. Effectively, both \( \theta_1 \) and \( \theta_2 \) can either enforce or partially compensate the aforementioned effects of \( \gamma_1 \) and \( \gamma_2 \). When \( \theta_1 > 0 \), the young agent compares his current consumption level with the one of other young agents as well as with the one of older agents. This fact encourages capital accumulation if the consumption externality in the first period of life decreases with \( \theta_1 \). Similarly, we can explain the effect of \( \theta_2 \) on the stationary value of capital stock. When \( \theta_2 > 0 \), old individuals compare their current level of consumption with the one of other old agents as well as with the one of younger agents. This fact encourages capital accumulation if the second period consumption externality in the first period of life increases with \( \theta_2 \).
In any case, the presence of temptation based on the intergenerational comparison of consumption are very important for the behavior of the capital stock along the transition dynamics. Observe that in this case the temptation function (2.23) is endogenous. In particular, we observe from the proof of Proposition 4.1 that the previous function is decreasing in capital, so that the degree of temptation also decreases when individuals accumulate capital. As will be shown in the next section, this will be crucial to the dynamic properties of the model.

5. Comparative dynamic analysis of temptations

Concerning the dynamics, we will study different cases. We first focus on the case where $\theta_1 = \theta_2 = 0$ which implies that the temptation function is independent of the capital stock. We then study the case where $\theta_1 > 0$ and $\theta_2 = 0$ making the temptation function dependent of $k_t$ and $k_{t+1}$ but independent of $k_{t+2}$. In this case, an analytical analysis is still possible. Finally, we study numerically the case where $\theta_2 > 0$, where the temptation function depends on the capital stock in three different periods.

5.1. The case with $\theta_1 = \theta_2 = 0$

In this case, we are able to rank the capital stocks of three economies which differ in terms of the temptation weight ($\psi$): the neoclassical economy ($k_t^0$) in which $\psi = 0$, our temptation economy ($k_t^0$) where $0 < \psi < \infty$ and finally the economy where the individual succumbs fully to temptation ($k_t^T$) in which $\psi \to \infty$. The only difference between these economies is the form of the temptation function $T(\cdot)$ as was shown in Section 3. In the three economies, the temptation function does not depend on the capital stock which greatly simplifies the analysis. However, since we are outside the steady-state equilibrium, we should first characterize the stability properties of this equilibrium. From the equilibrium condition (2.21), we implicitly obtain the policy function relating the equilibrium value of $k_{t+1}$ with respect to $k_t$. Denote this policy function by

$$k_{t+1} = \Gamma(k_t).$$

(5.1)

**Proposition 5.1.** When $\theta_1 = \theta_2 = 0$, the steady-state equilibrium is locally stable.

**Proof.** Applying the implicit function theorem to (2.21), we obtain that

$$\frac{\partial k_{t+1}}{\partial k_t} = -\frac{G'_{k_t}}{G'_{k_t+1}},$$

where

$$G'_{k_t} \equiv \frac{\partial G(k_t, k_{t+1})}{\partial k_{t+1}} = -\frac{\sigma \alpha \omega_t k_t^{1-\alpha} (1-\sigma)}{k_t (w_t - k_{t+1})^{\sigma-1}} < 0$$

and

$$G'_{k_{t+1}} \equiv \frac{\partial G(k_t, k_{t+1})}{\partial k_t} = \left[1 - \alpha (1 - \sigma) \right] k_t^{\alpha (1-\sigma)} \left( \frac{k_t}{\alpha \omega_t} \right)^{\alpha} G'_{k_t} > 0.$$
We also obtain that $G_{k_{t+1}} + G_{k_t} > 0$ when evaluated at the steady-state provided that $w > k$ which is always the case in order to obtain positive consumption in the first period. Therefore, we conclude that $\partial k_{t+1}/\partial k_t \in (0,1)$. ■

We now proceed with the comparative dynamic analysis to determine the impact of the self-control problem. The next result states the dynamic response of the capital stock to marginal variations in the parameters driving the self-control problem: $\psi$, $\beta$, $\gamma_1$ and $\gamma_2$.

**Proposition 5.2.** If $\theta_1 = \theta_2 = 0$, then the capital stock $k_t$ at any period $t$:

(a) increases with $\beta$ and $\gamma_2$;
(b) decreases with $\gamma_1$; and
(c) increases with $\psi$ if and only if $\beta^{1/\sigma} > \frac{1-\gamma_2}{1-\gamma_1}$.

**Proof.** Define $\pi = \{\psi, \beta, \gamma_1, \gamma_2\}$. Applying the implicit function theorem to (2.21), and using the fact that

$$T(k_t, k_{t+1}, k_{t+2}) = \frac{1 + \beta \psi (1 - \gamma_2)^{-\sigma}}{1 + \psi (1 - \gamma_1)^{-\sigma}},$$

in this case, we obtain

$$\frac{\partial k_{t+1}}{\partial \pi} = \frac{T'_\pi}{G'_{k_{t+1}}},$$

where $T'_\pi$ is the derivative of $T$ with respect to the corresponding parameter in $\pi$. Since $G'_{k_{t+1}} > 0$ as was proved in Proposition 5.1, the sign of $\partial k_{t+1}/\partial \pi$ coincides with the sign of $T'_\pi$. We obtain that

$$T'_\beta = \frac{\partial T}{\partial \beta} = \frac{\psi (1 - \gamma_2)^{-\sigma}}{1 + \psi (1 - \gamma_1)^{-\sigma^2}} > 0,$$

$$T'_\gamma_1 = \frac{\partial T}{\partial \gamma_1} = -\frac{\sigma \psi [1 + \beta \psi (1 - \gamma_2)^{-\sigma}]}{[(1 - \gamma_1)^{1+\sigma}] [1 + \psi (1 - \gamma_1)^{-\sigma}]^2} < 0,$$

$$T'_\gamma_2 = \frac{\partial T}{\partial \gamma_2} = \frac{\sigma \beta \psi}{[(1 - \gamma_2)^{1+\sigma}] [1 + \psi (1 - \gamma_1)^{-\sigma}]} > 0,$$

and

$$T'_\psi = \frac{\partial T}{\partial \psi} = \frac{\beta (1 - \gamma_2)^{-\sigma} - (1 - \gamma_1)^{-\sigma}}{[1 + \psi (1 - \gamma_1)^{-\sigma}]^2}.$$  

The proposition then directly follows. ■

Finally, we next compare the equilibrium path of the capital stock in the three aforementioned economies: our economy with self-control problems arising from interpersonally dependent preferences, the neoclassical economy and the economy where consumers fully succumb to temptation.
Proposition 5.3. The following statements hold when $\theta_1 = \theta_2 = 0$:

(a) If $\beta^{1/\sigma} > \frac{1-\gamma_2}{1-\gamma_1}$, $k_t^p < k_t < k_t^T$.

(b) If $\beta^{1/\sigma} < \frac{1-\gamma_2}{1-\gamma_1}$, $k_t^p < k_t < k_t^a$.

Proof. The result directly follows from Proposition 5.2 by noting that the neoclassical model arises when $\psi = 0$, the model where consumers fully succumbs to temptation emerges when $\psi$ tends to infinite, and the model with self-control problems from interpersonally dependent preferences is given by $\psi \in (0, \infty)$.

When $\beta$ and $\gamma_2$ are sufficiently large compared to $\gamma_1$, the economies with temptation will generate a capital stock higher than the neoclassical economy. In this case, the temptation degree is given by $T > 1$ and, therefore, the young agent wishes to increase future consumption at a given rate of interest. Since temptation induces the agent to accumulate more capital, an economy in which the latter fully succumbs to temptation will generate the highest capital stock.

When $\gamma_1$ is sufficiently high, it is the neoclassical economy that will generate the highest capital stock. In this case, the economies with temptations generate a function $T < 1$ implying that the agent wishes to increase present consumption. Since temptation induces now the agent to accumulate less capital, it is the economy where the agent does not fully succumbs to temptation that delivers the highest capital stock among the two temptation economies. It is interesting to notice that when $\gamma_1 \geq \gamma_2$, case (a) always applies and a temptation economy cannot generate more capital than a neoclassical one. A sufficiently high value of $\gamma_2$ (i.e., when temptation is more intense in the second period of life) is thus necessary in order to observe a temptation economy with a higher capital.

The comparison between the neoclassical case and our economy including temptation and externalities is valid in a case without distortionary taxes. If a government imposes a set of taxes on the temptation economy, our equilibrium condition becomes:

$$G(k_{t+1}, k_t) = \delta(\alpha A)^{1-\sigma} \frac{(1 + \tau^*_t)^{(1 + \tau^*_t)}(1 + \tau^p_{t+1})[1 + \beta \psi(1 - \gamma_2)]}{(1 + \tau^d_{t+1})[1 + \psi(1 - \gamma_1)]},$$

where $\tau^*_t$ is a tax on present consumption, $\tau^p_{t+1}$ is a subsidy on the capital stock and $\tau^d_{t+1}$ is a tax on second period consumption. When the value of $T$ without taxes is larger than one, a combination of taxes such that $(1 + \tau^*_t)(1 + \tau^p_{t+1}) < 1 + \tau^d_{t+1}$ generates a temptation economy closer to the neoclassical solution (compared to a temptation economy without taxation). Clearly such a policy imposes higher taxes on future consumption to reduce the impact of temptation on the marginal rate of substitution between current and future consumption. Obviously, when the value of $T$ without taxes is smaller than one, a similar reasoning can be made with a combination of taxes such that $(1 + \tau^*_t)(1 + \tau^p_{t+1}) > 1 + \tau^d_{t+1}$, i.e., by imposing higher taxes on current consumption. It is even possible that the economy with temptation and the neoclassical economy exhibit the same capital stock level (i.e., $k_t = k_t^p$) if

$$\frac{(1 + \tau^*_t)(1 + \tau^p_{t+1})}{1 + \tau^d_{t+1}} = \frac{1 + \psi(1 - \gamma_1)}{1 + \beta \psi(1 - \gamma_2)}.$$
5.2. The case with \( \theta_1 > 0 \) and \( \theta_2 = 0 \)

We will now focus on the case where \( T \) is a function of the capital stock. However, when \( \theta_2 > 0 \), the function depends on \( k_{t+2} \), so that the dynamics are characterized by a second order difference equation, which does not allow us to obtain general analytical results. Setting \( \theta_2 = 0 \) greatly simplifies the analysis and we should follow this path in the following. In this case, we also proceed by analyzing the response of the equilibrium value of \( k_{t+1} \) with respect to the weight of the temptation utility \( \psi \) for a given value of \( k_t \). To this end, we start by characterizing the stability properties of the steady state equilibrium.

**Proposition 5.4.** When \( \theta_1 > 0 \) and \( \theta_2 = 0 \), the steady-state equilibrium is locally stable.

**Proof.** See Appendix A. ■

We next state the results from the comparative dynamic analysis to determine the impact of the self-control problem.

**Proposition 5.5.** If \( \theta_1 > 0 \) and \( \theta_2 = 0 \), then the capital stock \( k_t \) at any period \( t \):

(a) increases with \( \beta \) and \( \gamma_2 \); and
(b) decreases with \( \gamma_1 \) and \( \theta_1 \); and
(c) decreases with \( \psi \) when

\[
\beta^{1/\sigma} < \frac{(1 + \theta_1)(1 - \gamma_2)}{1 + \theta_1 - \gamma_1 - \gamma_1 \theta_1 \alpha \frac{k_t^\sigma}{(1 - \sigma)k_t^{\sigma} - k_{t+1}^{\sigma}}},
\]

(5.2)

**Proof.** Define \( \pi = \{\psi, \beta, \gamma_1, \gamma_2, \theta_1, \theta_2\} \). Applying the implicit function theorem to (2.21), and using the fact that

\[
T(k_t, k_{t+1}) = \frac{1 + \beta \psi (1 - \gamma_2)^{-\sigma}}{1 + \psi(1 + \theta_1)^\sigma \left(1 + \theta_1 - \gamma_1 - \frac{\gamma_1 \theta_1 \alpha k_t^\sigma}{(1 - \sigma)k_t^{\sigma} - k_{t+1}^{\sigma}}\right)^{-\sigma}},
\]

in this case, we obtain

\[
\frac{\partial k_{t+1}}{\partial \pi} = \frac{\delta(\alpha \cdot \beta)^{1-\sigma} T_{\pi}'}{G'_{k_{t+1}} - T_{k_{t+1}'}},
\]

(5.3)

where \( G'_{k_{t+1}} \) is the derivative of function \( G(.) \) with respect to \( k_{t+1} \); and \( T_{k_{t+1}'} \) and \( T_{\pi}' \) denote the derivative of function \( T(.) \) with respect to \( k_{t+1} \) and \( \pi \), respectively. We already know from the proof of Proposition 5.1 that \( G'_{k_{t+1}} > 0 \), and it is straightforward to check that \( T_{k_{t+1}'} > 0 \) when \( \theta_2 = 0 \). Thus, the sign of \( \partial k_{t+1}/\partial \pi \) coincides with the sign of \( T_{\pi}' \). We easily obtain that \( T_{\beta} > 0 \), \( T_{\gamma_1} < 0 \), \( T_{\gamma_2} > 0 \), \( T_{\theta_1} < 0 \), and \( T_{\theta_2} > 0 \). In addition, we get

\[
T_{\psi}' = \frac{\beta(1 - \gamma_2)^{-\sigma} \left[1 + \theta_1 - \gamma_1 - \frac{\gamma_1 \theta_1 \alpha k_t^\sigma}{(1 - \sigma)k_t^{\sigma} - k_{t+1}^{\sigma}}\right]^{-\sigma} (1 + \theta_1)^\sigma}{1 + \psi \left[1 + \theta_1 - \gamma_1 - \frac{\gamma_1 \theta_1 \alpha k_t^\sigma}{(1 - \sigma)k_t^{\sigma} - k_{t+1}^{\sigma}}\right]^{-\sigma} (1 + \theta_1)^\sigma}.
\]

(5.4)
We proceed in several steps to characterize the sign of $T'_\psi$.

(a) Since consumptions $c_t$ and $d_{t+1}$ are normal goods, we can assert that $\partial s_t/\partial w_t \in (0, 1)$ and, therefore, we conclude that

$$\frac{w_t}{(w_t - k_{t+1})}, \quad (5.5)$$

is an increasing function of $k_t$. Obviously, this implies that the term

$$1 + \theta_1 \gamma_1 \frac{\gamma_1 \theta_1 \alpha w_t}{(1 - \alpha)(w_t - k_{t+1})} \gamma^{-\sigma},$$

also increases with $k_t$.

(b) Notice that the ratio $k_{t+1}/k_t$ converges to one as $k_t$ tends to zero. Therefore, we directly obtain

$$\lim_{k_t \to 0} T'_\psi = \frac{\beta (1 - \gamma_2)^{-\sigma} - [1 + \theta_1 - \gamma_1 (1 + \theta_1 \alpha / (1 - \alpha))]^{-\sigma} (1 + \theta_1)^\sigma}{\left[1 + \gamma_1 (1 + \theta_1 \alpha / (1 - \alpha))^{-\sigma} (1 + \theta_1)^\sigma\right]^2}.$$

This limit is positive if and only if Condition (5.2) holds.

(c) Note that the effective consumption in the first period of life decreases with $k_t$ because Expression (5.5) is an increasing function of $k_t$. Therefore, there exists a threshold $\bar{k}$ such that this effective consumption would be negative for a $k_t > \bar{k}$. In fact, the value of $\bar{k}$ is implicitly given by the first condition in (2.11). This threshold $\bar{k}$ is then the upper bound of the domain of the capital stock for which equilibrium is defined. Furthermore, we also obtain

$$\lim_{k_t \to \bar{k}} T'_\psi = 0.$$

Combining these different results we can conclude that $T'_\psi < 0$ on the interval $k_t \in (0, \bar{k})$ when Condition (5.2) holds. Therefore, the proposition directly follows.

Proposition 5.6 does not exclude the capital stock of our temptation economy to be larger than the one of the neoclassical model. In fact, we know from Proposition 5.5 that $T'_\psi > 0$ for sufficiently small values of the capital stock when Condition 5.2 does not hold. Hence, we conclude that $k_t > k^n_1$ for this range of the capital stock. Furthermore,
in that case, we cannot discard that the sign of $T'_0$ becomes negative as the capital stock increases. If this happens, then the paths of the capital stocks corresponding to the two aforementioned economies will cross at some level of the capital stock. At this point we assert that the later property of the equilibrium dynamics is very unlikely or it occurs for very large values of the capital stock. Observe from the proof of Proposition 5.5 that $T'_0 < 0$ requires either a large value of the capital stock or a small value of $\beta (1 - \gamma_2)^{-\sigma}$. However, small values of the later expression leads Condition 5.2 to hold and, therefore the derivative $T'_0$ is negative for the entire domain of the capital stock.

In fact, we did not find a numerical example where this derivative becomes negative when Condition 5.2 does not hold.

The main conclusion from these first two subsections is that the sign of the comparison between the capital stock in our model with self-control problems and the one in the neoclassical model is likely to be monotone along the entire transition path provided that $\theta_2 = 0$. Along the entire transition path, the former capital is always either larger or smaller than the neoclassical one. However, this is not necessarily the case when $\theta_2 > 0$, i.e., when old individuals include the living young individuals in their reference set. In this case, the paths of these two aforementioned capital stocks might cross, so that the sign of the comparison between them can change. We will numerically illustrate this point in the next subsection.

5.3. The case with $\theta_2 > 0$

We finally study the case with $\theta_2 > 0$, which implies that our function $T$ now depends on $k_t$, $k_{t+1}$ as well as $k_{t+2}$. This property does not allow us to give an analytical characterization of the comparative dynamic analysis. In this subsection, we numerically compare the equilibrium behavior of capital in the model with self-control problems and in the standard neoclassical model. To this end, we set the values of parameter as follows. We consider that each model period corresponds to 30 years. We first calibrate the neoclassical model as in de la Croix and Michel (2002). We set the scale parameter $A = 6.75$ to obtain a stationary capital stock around unity in the neoclassical version of the model. Following the RBC literature, we consider $\alpha = 1/3$ and $\delta = 0.3$ to replicate the share of labor income in aggregate output and a quarterly subjective discount factor of 0.99, respectively. We consider $\sigma = 1.1$ to replicate a saving rate of 15.36%, which is in line with the evidence reported by Maddison (1992). We finally fix the rest of the parameters, $\{\psi, \beta, \gamma_1, \gamma_2, \theta_1, \theta_2\}$, which characterize equilibrium dynamics of our model with self-control problems. To this end, we should observe that the dynamic behavior of the capital stock crucially depends on the values of these parameters. For instance, consider the following values: $\psi = 1$, $\beta = 0.85$, $\gamma_1 = 0.5$, $\gamma_2 = 0.8$ and $\theta_1 = 0.5$. We will next illustrate how the dynamic properties of the model are driven by the values of $\theta_2$, i.e., the weight that the consumption of young individuals has on the consumption reference of old individuals.

Firstly, the fact that old individuals are tempted by the consumption of living young individuals might have a significant influence on the stability properties of the steady-state equilibrium. This is a clear consequence of the second-order nature of the dynamic equation (2.21) defining the equilibrium path in this case. In fact, we cannot exclude the steady-state equilibrium to be locally indeterminate. We next discuss the logical
of this stability property.

**Proposition 5.7.** Consider that \( \theta_2 > 0 \). The steady state equilibrium is locally indeterminate if and only if one of the following two pair of conditions holds:

(a) When \( G'_{k_{t+1}} - \delta (\alpha A)^{1-\sigma} T'_{k_{t+1}} > 0 \), then

\[
T'_{k_{t+2}} < \frac{G'_{k_t} - G'_{k_{t+1}} + \delta (\alpha A)^{1-\sigma} (T'_{k_{t+1}} - T'_k)}{\delta (\alpha A)^{1-\sigma}},
\]

(5.6)

(b) When \( G'_{k_{t+1}} - \delta (\alpha A)^{1-\sigma} T'_{k_{t+1}} < 0 \), then

\[
T'_{k_{t+2}} < \frac{G'_{k_{t+1}} + G'_{k_t} - \delta (\alpha A)^{1-\sigma} (T'_{k_{t+1}} + T'_k)}{\delta (\alpha A)^{1-\sigma}},
\]

(5.7)

where \( T'_{k_{t+2}} \), \( T'_{k_{t+1}} \) and \( T'_k \) are the derivatives of the temptation function (2.23) with respect to \( k_{t+2} \), \( k_{t+1} \) and \( k_t \) at the steady state, respectively; and \( G'_{k_t} \) and \( G'_{k_{t+1}} \) are the derivative of the function (2.22) with respect to \( k_t \) and \( k_{t+1} \) at the steady state, respectively.

**Proof.** See Appendix B. ■

Given a value for \( k_0 \) multiple equilibrium paths converging to the stationary value \( k \) might then exist when \( \theta_2 > 0 \). Unfortunately, we cannot find a threshold value of \( \theta_2 \) determining the existence of local indeterminacy. However, we can still numerically illustrate that this result is feasible for reasonable values of the parameters, and, moreover, we can also provide some economic intuition of the mechanism behind this result. In fact, we obtain local indeterminacy in our numerical example for values of \( \theta_2 \) sufficiently large. For the proposed numerical example, indeterminacy emerges for values of \( \theta_2 \) larger than 0.84. This numerical result is independent of the value of \( \theta_1 \), but it requires a sufficiently large value of \( \gamma_1 \).\(^\text{10}\)

The mechanism behind this result is based on the existence of strong complementarities in the saving decisions. In our model with temptation, the returns on saving have two components: (a) the standard market return given by the marginal productivity of capital; and (b) the one given by the fact that savings would allow individuals to satisfy temptation when old. Hence, indeterminacy .

Consider that we have an equilibrium path with a given level of savings. We can obtain another equilibrium path in the case that one of the condition sets in Proposition 5.7 holds as follows. Current young individuals will increase savings if they expect that the next generation will consume a lot when young. In this case, the consumption reference when old will be large and the net returns on augmenting savings will increase even when the market return decreases. This expectation will be self-fulfilling if \( \theta_2 \) is sufficiently large. The current increase in savings leads to a high production in the next period, which will result in an increase in the consumption of the young individuals. This

\(^{10}\)For this numerical example, local indeterminacy emerges even with \( \theta_1 = 0 \) provided that \( \gamma_1 \) is not smaller than 0.5.
translates into an increase in the consumption reference of the old individuals, which will be larger the higher the value of $\theta_2$. Observe that indeterminacy will require the consumption of future young individuals to increase a lot with the current increase in savings. Our numerical simulations suggest that this is only possible if $\gamma_1$ is sufficiently large. In this case, the weight of the consumption reference of young individuals is large and, therefore, the marginal utility of their consumption is also large.

In addition, the comparison of the equilibrium dynamics of the capital stock also crucially depends on the values of these parameters. In fact, starting from the same initial value of the capital stock $k_0$, the capital stock of our model with self-control problems may be larger or smaller than the one corresponding to the neoclassical model along the entire equilibrium path. More interesting, for some parameter values, the two paths of the capital stock corresponding to the two aforementioned models cross once. For instance, by considering $\theta_2 = 0.5$, Figure 1 illustrates the dynamics of the capital stock corresponding to the two models. Panel (a) gives the policy functions relating the present and the future capital stock, whereas Panel (b) provides the equilibrium paths when $k_0$ is equal to 25% of the stationary capital stock of the neoclassical model (i.e., when $\psi = 0$). By starting at the same initial value, the neoclassical capital stock is larger (smaller) than the one in our model with self-control problems in the first (last) part of the transition dynamics.

With slight changes in the benchmark values of the parameters, we can illustrate how important is the assumption that temptation is based on intergenerational consumption comparisons. We numerically obtain that $k_t > k_t^n$ for all $t$ if we reset $\theta_2 = 0$, whereas if we instead fix $\theta_1 = 0$ we get a comparison between $k_t$ and $k_t^n$ qualitatively identical to the one in Figure 1. In addition, we get $k_t < k_t^n$ by modifying the benchmark numerical example and setting $\theta_2 = 0$ and $\gamma_2 = 0.5$. However, the comparison in Figure 1 qualitatively maintains in the case where $\theta_2 = 0.5$ and $\gamma_2 = 0.5$. Therefore, our numerical simulations suggest that a non-monotonic relationship between the capital stock of our model with self-control problems and the one of the neoclassical model requires the old individuals to be tempted by the consumption of the living young individuals (i.e., $\theta_2 > 0$).

The previous numerical results confirm that the existence of and endogenous self-control problems has large consequences for the intertemporal decisions on consumption and saving. This is specially true when old individuals take the consumption of living young individuals as an important determinant of the consumption reference driving their temptation.

6. Optimal allocation

Since we are in an OLG framework, the neoclassical solution is not necessarily the optimal one. Hence, it is necessary to analyze how self-control problems affect the

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\[11\] To facilitate the illustration, we have plotted the path of capital stock by years. To this end, we have assumed that the value corresponding to one model period is generated in each of the 30 years at the same proportion. We have then plotted the accumulated stock at each of these years.
welfare properties of the equilibrium path. We should start by establishing what is a socially optimal allocation in the proposed model. Following Drugeon and Wigniolle (2017), we consider that the social planner omits the cost arising from the self-control problem. Therefore, the social planner solve the following problem:

\[
\max_{\{c_t, d_t, k_{t+1}\}} \sum_{t=0}^{\infty} \zeta^t \left( \frac{c_t^{1-\sigma}}{1-\sigma} + \delta \frac{d_{t+1}^{1-\sigma}}{1-\sigma} \right),
\]

subject to the aggregate resource constraint

\[
Ak_t^* = c_t + d_t + k_{t+1},
\]

where \(\zeta\) is the weight that the social planner assigns to each generation. This approach will allow us to compare the optimal allocation with the one of the temptation economy. By following the standard procedure, we obtain the first order conditions and, after a simple manipulation, we obtain the following conditions characterizing the socially planned solution:

\[
\frac{\zeta}{\delta} \left( \frac{c_t^*}{d_t^*} \right)^{-\sigma} = 1, \quad (6.1)
\]

\[
\frac{1}{\zeta} \left( \frac{c_t^*}{c_{t+1}^*} \right)^{-\sigma} = \alpha A(k_{t+1}^*)^{\alpha-1}, \quad (6.2)
\]

where starred variables denote optimal outcomes. The first condition determines the optimal allocation of aggregate consumption between the two living generation in any period \(t\). In addition, Condition (6.2) characterizes the optimal intertemporal allocation of consumption for a generation born in period \(t\).

Combining (6.1) and (6.2) we obtain

\[
\frac{1}{\delta} \left( \frac{c_t^*}{d_{t+1}^*} \right)^{-\sigma} = \alpha A(k_{t+1}^*)^{\alpha-1}, \quad (6.3)
\]

which can be compared to a similar condition from the competitive equilibrium:

\[
\frac{1}{\delta} \left( \frac{c_t}{d_{t+1}} \right)^{-\sigma} = \alpha A(k_{t+1}^{*})^{\alpha-1} T(k_{t+2}, k_{t+1}, k_t). \quad (6.4)
\]

Comparing both expressions, we should be able to determine which economy will generate the highest capital stock. There are two sources of inefficiency. On the one hand, the presence of consumption externalities in temptation utility distorts the marginal rate of substitution between present and future consumption. This inefficiency is given by the value of the temptation function \(T(k_{t+2}, k_{t+1}, k_t)\). Obviously, this distortion does not emerge in the neoclassical economy where \(T(k_{t+2}, k_{t+1}, k_t) = 1\). On the other hand, in our temptation economy and in the neoclassical one, there is the standard distortion in the marginal rate of transformation that determines the intergenerational allocation of consumption. To show the consequences of these two sources of inefficiency, we focus on the steady-state allocations. By combining (6.1) and (6.3), we obtain the modified golden rule in our economy as follows

\[
\alpha A(k^*)^{\alpha-1} = \frac{1}{\zeta}.
\]
Since $\bar{T}(k) = T(k, k, k)$ is decreasing (see Proposition 4.1), there is over-accumulation (under-accumulation) of capital at the steady-state equilibrium if $\zeta \alpha k^{\alpha - 1} T(k) < (>) 1$. By using this fact, the next proposition establishes the dynamic efficiency of our model with temptation.

**Proposition 6.1.** If $\zeta \alpha k^{\alpha - 1} T(k) > 1$ the competitive equilibrium is dynamically efficient, whereas this equilibrium is dynamically inefficient when $\zeta \alpha k^{\alpha - 1} T(k) < 1$.

Observe that dynamic efficiency of the competitive equilibrium crucially depends on whether the temptation function $T(k)$ is larger or smaller than one. In particular, we can obtain that the dynamic efficiency property of our temptation economy can be the opposite of the one of the neoclassical economy. The reason of this conclusion is that the temptation function $T(k)$ is equal to one at the neoclassical economy but it can be different than one.

Regarding the policies to decentralize the optimal outcome, we should note that "the sole replication of the optimal consumption sequence does not suffice when there are temptation motives" (Drugeon and Wigniolle, 2017). In defining the optimal allocation the social planner does not take into account that individuals suffer a disutility from not fully succumbing to temptation. Therefore, the optimal policies consist in restricting the menu of actions that individuals have access to. As Gul and Pesendorfer (2004) point out, one should not allow individuals to face those activities or goods generating self-control problems. In our framework, this policy reduces to impose that the maximum level of consumption in each period has to be the optimal one as in Drugeon and Wigniolle (2017).

### 7. Concluding remarks

We have analyzed how temptation and costly self-control influence consumer’s decisions on savings and the accumulation of wealth along the life-cycle. We have used an overlapping generations model where individuals were tempted to take the average consumption of agents living in the same period as a consumption reference. Their decisions result from the compromise between two competing desires: the gratification from fully succumbing to temptation and the gratification derived from ignoring how their choices will determine their position in the social scale of consumption. The presence of consumption externalities generates a time-varying and endogenous gap between the intertemporal elasticity of substitution for commitment consumption and the one for temptation consumption. Therefore, the degree of the self-control problem is endogenously determined by the aggregate allocation of resources. We have showed that the influence of temptation and costly self-control on consumption-savings decision crucially depends on whether or not consumers take the consumption of the individuals belonging to the other living generation as a determinant of their consumption reference. In particular, we have obtained that consumption externalities may generate either overaccumulation or underaccumulation of capital.

The main conclusion from the results of the paper is that our preferences displaying endogenous temptation and self-control problems may have important implications for the dynamics of income and wealth inequality. Since the degree of the self-control...
problem is driven by the consumption externalities, the aggregate performance of the economy would significantly depend on the income inequality and wealth distribution. In this case, the intensity of the self-control problem faced up by each individual is crucially determined by his position in the social scale of consumption. Hence, our framework contains an interesting propagation mechanism for income inequality and structural shocks.

Finally, future research may also focus on extending our analysis to consider other forms of consumption references for temptation utility function. More precisely, we may include phenomena as, for instance, habit formation. In contrast with what happens in our framework, individuals internalize in that alternative case that their current decisions affect the degree of their future self-control problem. This makes the problem more complex and, therefore, deserves some effort to investigate its consequences for economic decisions.
References


Appendix

A. Proof of Proposition 5.4

We prove that the steady-state equilibrium is stable when \( \theta_1 > 0 \) and \( \theta_2 = 0 \). From our equilibrium condition (2.21), we obtain the following implicit function:

\[
k_{t+1} = \Omega(k_t).
\]

Taking the derivative of this expression and evaluating it at the steady-state we can assess the stability of the dynamical system in this case. In particular, we obtain

\[
\frac{\partial k_{t+1}}{\partial k_t} = \frac{G'_{k_{t+1}} - T'_{k_t}}{G'_{k_t} - T'_{k_{t+1}}}.
\] (A.1)

We first prove that this derivative is always positive. To this end, we write our function \( T(k_t, k_{t+1}) \) as

\[
T(k_t, k_{t+1}) = \frac{1 + \beta(1 - \gamma_2)^{-\sigma}}{1 + \psi[\zeta_1 - \eta_1 \phi(k_t, k_{t+1})]^{-\sigma}},
\]

where

\[
\phi(k_t, k_{t+1}) = \frac{w_t}{w_t - k_{t+1}},
\]

with

\[
\phi'_{k_{t+1}} = \frac{w_t}{(w_t - k_{t+1})^2},
\]

and

\[
\phi'_{k_t} = -\frac{w'_{k_t} k_{t+1}}{(w_t - k_{t+1})^2}.
\]

We then obtain:

\[
G'_{k_{t+1}} = \frac{[1 - \alpha(1 - \sigma)]k_{t+1}^{-\alpha(1 - \sigma)} + \sigma k_{t+1}^{1-\alpha(1-\sigma)}(w_t - k_{t+1})^{-1}}{(w_t - k_{t+1})^\sigma} < 0,
\]

\[
G'_{k_t} = -\frac{\sigma w'_{k_t} k_{t+1}^{1-\alpha(1-\sigma)}}{(w_t - k_{t+1})^{1+\sigma}} > 0,
\]

\[
T'_{k_{t+1}} = -\frac{(1 + \beta(1 - \gamma_2)^{-\sigma})\psi[\zeta_1 - \eta_1 \phi(k_t, k_{t+1})]^{-\sigma - 1} \eta_1 \phi'_{k_{t+1}}}{\{1 + \psi[\zeta_1 - \eta_1 \phi(k_t, k_{t+1})]^{-\sigma}\}^2} > 0,
\]

and

\[
T'_{k_t} = -\frac{(1 + \beta(1 - \gamma_2)^{-\sigma})\psi[\zeta_1 - \eta_1 \phi(k_t, k_{t+1})]^{-\sigma - 1} \eta_1 \phi'_{k_t}}{\{1 + \psi[\zeta_1 - \eta_1 \phi(k_t, k_{t+1})]^{-\sigma}\}^2} < 0.
\]

For our dynamical system to be locally stable, we need that the derivative (A.1) evaluated at the steady state be lower than one. The following condition should then be satisfied:

\[
-G'_{k_t} - G'_{k_{t+1}} < -\delta(\alpha A)^{1-\sigma}(T'_{k_{t+1}} + T'_{k_t})
\] (A.2)
We first prove that the term on the left hand side is always negative at the steady state. Comparing the expressions of $G_k^t$ and $G_{k+1}^t$, and using the fact that $w_k' = \alpha(w/k)$, we can conclude that $G_k^{t+1} + G_k^t > 0$ at the steady state provided that $w > k$, which is always the case in order to obtain positive consumption in the first period. With this result in hand, we know from inequality (A.2) that a sufficient condition for local stability is that the right hand side evaluated at the steady-state be positive. By using the fact that $w_0 k = (w/k)$, we can conclude that $G_k^{t+1} + G_k^t > 0$ at the steady state provided that $w > k$, which is always the case in order to obtain positive consumption in the first period.

With this result in hand, we know from inequality (A.2) that a sufficient condition for local stability is that the right hand side evaluated at the steady-state be positive. By using the fact that $w_0 k = (w/k)$, we obtain that $T_k^{t+1} + T_k^t < 0$ at the steady state. This result implies that condition (A.2) is always satisfied and our dynamical system is locally stable.

**B. Proof of Proposition 5.7**

Given the second-order nature of the dynamic equation (2.21) characterizing the equilibrium, we make use of the following variable transformation to study the stability:

$$k_{t+1} = x_t.$$  \hspace{1cm} (B.1)

Hence, the equilibrium dynamics is given by (B.1) and the implicit function derived from (2.21):

$$x_{t+1} = \Phi(x_t, k_t).$$  \hspace{1cm} (B.2)

The determinant and the trace of the Jacobian of the dynamic system (B.1)-(B.2) are respectively given by

$$Det = -\frac{\partial x_{t+1}}{\partial k_t} = -\frac{G'_k - \delta(\alpha A)^{1-\sigma} T'_k}{\delta(\alpha A)^{1-\sigma} T'_k},$$

and

$$Tr = \frac{\partial x_{t+1}}{\partial x_t} = \frac{G'_k - \delta(\alpha A)^{1-\sigma} T'_k}{\delta(\alpha A)^{1-\sigma} T'_k},$$

where $T'_k$, $T'_k$ and $T'_k$ are the derivatives of the temptation function (2.23) with respect to $k_{t+2}$, $k_{t+1}$ and $k_t$ at the steady state, respectively; and $G'_k$ and $G'_k$ are the derivative of the function (2.22) with respect to $k_t$ and $k_{t+1}$ at the steady state, respectively. From the proof of Proposition 5.4 we know that $G'_k > 0$ and $T'_k < 0$ and, moreover, we can directly obtain that $T'_k < 0$, so that $Det < 0$. However, the sign of the trace is ambiguous since we cannot determine the sign of the derivative $T'_k$.

Hence, we must consider two cases.

(a) If $G'_k - \delta(\alpha A)^{1-\sigma} T'_k > 0$, then the trace is negative since $T'_k < 0$. In this case, the Jacobian of the dynamic system (B.1)-(B.2) has two eigenvalues in the interval $(-1, 1)$ if and only if $1 + Tr + Det > 0$. Hence, we conclude that the steady state is locally indeterminate when Condition (5.6) holds.

(b) If $G'_k - \delta(\alpha A)^{1-\sigma} T'_k < 0$, then the trace is positive since $T'_k < 0$. In this case, the Jacobian of the dynamic system (B.1)-(B.2) has two eigenvalues in the interval $(-1, 1)$ if and only if $1 - Tr + Det > 0$. Hence, we conclude that the steady state is locally indeterminate when Condition (5.7) holds.
Figure 1. Comparative dynamics ($\theta_1 > 0$ and $\theta_2 > 0$)

(a) Policy function of $k_{t+1}$ on $k_t$

(b) Equilibrium path as $k_0 < k$