Macroeconomic Effects of Gender Gaps in Entrepreneurship: frictions vs. selection*
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Abstract

This article studies the aggregate effects of the existing differences between male and female-run firms. We document that only about one fourth of the total firms are run by women and that female-run firms are about three times smaller than male-run firms. We then propose two theoretical frameworks based on Cuberes and Teignier (2016) to incorporate these facts and quantify their aggregate effects on productivity and income per capita. In the first framework, more talented women are more likely to face barriers to become firm managers, which implies a negative selection of female managers and, consequently, a smaller size of female-run firms relative to those managed by men in equilibrium. For the Latin America case, we obtain an output per capita loss due to these gender gaps is 9%, which is 1.3 times larger than the one obtained in a framework where barriers to entrepreneurship are independent of talent. In the second framework, women face frictions to access the productive inputs and have to pay a higher price to hire these inputs, which leads to smaller female-run firms relative to male-run firms.

JEL classification numbers: E2, J21, J24, O40.
Keywords: firm-size distribution, entrepreneurship talent, gender inequality, talent mis-allocation, aggregate productivity, span of control.

*Work in progress, preliminary and incomplete.
1 Introduction

Gender inequality is present in many socioeconomic indicators around the world in both developed and developing countries. These gaps can be observed in education, earnings, occupation, access to productive inputs, political representation, or bargaining power inside the household, among others. One important aspect of gender inequality in the labor market that has been largely overlooked in the literature is the low presence of women in entrepreneurial activities. In this paper we study this issue for Latin America and the Caribbean (LAC) using data from the World Bank Enterprise Survey. In that region, on average, female-run firms represent only about 25% of the total number of firms and the average firm managed by a woman is about 3 times smaller than that of man.

We develop a general equilibrium model, based on Cuberes and Teignier (2016), to quantify the effects of entrepreneurship gender gaps on aggregate productivity and income. In our theoretical framework, agents are endowed with a random entrepreneurship skill, based on which they decide to become either employers, self-employed, or workers. An employer in this model produces an homogeneous good using a span-of-control technology that combines his or her entrepreneurship skills, with capital and workers. We assume that men and women are identical in terms of their managerial skills- in the sense that both groups draw their managerial talent from identical distribution functions- but women are subject to exogenous constraints in their occupational choices, which results in a a fraction of them being excluded from employership. This generates a loss in income per worker because less able managers run firms of sub-optimal size and productivity. An important question to determine the magnitude of this aggregate loss is which type of women are excluded from employership. To determine how entrepreneurial skills and selection into entrepreneurship are related we explore some of our empirical results through the lens of our model. In our model, in the absence of gender frictions or when these frictions are independent of talent, both men and women manage firms of identical size. Since the data indicates that firm’s size is significantly smaller for men, this means, through the lens of our model, that the probability of being excluded from employership must increase with a woman’s talent. Our empirical analysis indicates that, in our set of countries, women tend to run smaller firms than men which, in our framework, implies that more talented women are less likely to become employers.

We find large effects of the employership gender gaps on aggregate and productivity. In the Latin America and Caribbean region, we estimate an 8.9% fall of aggregate output in the short run, when we keep the stock of capital fixed, and 9.4% in the long run, when this stock is also negatively affected. These losses are 1.3 times larger than the ones we would obtain

1 See the World Development Report 2012 (World Bank, 2012) for a comprehensive review of these and other gender gaps discussed in the literature.
using a model where the exclusion probability was independent of talent. The intuition behind the output loss is as follows. When a woman with very good management skills cannot become an employer, a less skilled man takes her position and becomes the manager of a firm and, as a consequence, the average firm size and output per worker in the economy fall. On top of that, if more talented women face a higher exclusion probability, the average talent of employers falls even more, which results in a larger loss.

To our knowledge, there are very few articles that quantify the macroeconomic effects of gender gaps in the labor market. The International Labor Organization provides some estimates of the output costs associated with labor gender gaps in the Middle East and Northern Africa but without proposing any specific theoretical model (ILO, 2014). One shortcoming of their exercise is that it does not allow one to shed light on the mechanisms through which gender gaps in the labor market may affect aggregate efficiency. Cavalcanti and Tavares (2016) construct a growth model based on Galor and Weil (1996) in which there is exogenous wage discrimination against women. Calibrating their model using U.S. data, they find very large effects associated with these wage gaps: a 50 percent increase in the gender wage gap in their model leads to a decrease in income per capita of a quarter of the original output. Their results also suggest that a large fraction of the actual difference in output per capita between the U.S. and other countries is indeed generated by the presence of gender inequality in wages. Hsieh et al. (2013) use a Roy model to estimate the effect of the changing occupational allocation of white women, black men, and black women between 1960 and 2008 on U.S. economic growth and find that the improved allocation of talent accounts for 17 to 20 percent of growth over this period. Finally, in the model summarized in Section 3.1 of this paper, Cuberes and Teignier (2016) calculate the macroeconomic effects of gender inequality in the labor market using data from the International Labor Organization for a large sample of countries, including several countries from the LAC region, among them, Chile, Colombia, and Peru. An important difference is that in the present paper we make use of firm-level data rather than labor aggregates to carry out our analysis. These data allows us to have implications in terms of firm size and selection that were not possible to derive in Cuberes and Teignier (2016). In terms of analyzing firm size by gender in the Latin America Region, Arellano and Peralta (2015) report significant differences in firms size and productivity in Chile but they do not use any theoretical model in their analysis.

The rest of the paper is organized as follows. Section 2 presents the empirical facts and Section 3 sketches the general equilibrium occupational choice model based on Cuberes and Teignier (2016). The numerical results of the paper are presented and discussed in Section 4, while Section 5 contains the sensitivity analyses. Finally, Section 5 concludes.

\footnote{See Cuberes and Teignier (2014) for a critical literature review of the two-directional link between gender inequality and economic growth.}
2 Empirical facts

We start the analysis documenting the existing differences in employership between men and women in the LAC region. First, using data for the latest available year on all the countries in this region from the International Labour Organization (KILM, 8th edition) on employers by gender, we find that women are clearly underrepresented in employership. As we can see in Table 1, the share of female employers in the working-age population is less than one third the male one.

Table 1: Share of employers in working-age population by gender (Data source: ILO KILM, 8th edition)

<table>
<thead>
<tr>
<th>(%)</th>
<th>Females</th>
<th>Males</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>LAC</td>
<td>1.75</td>
<td>5.66</td>
<td>0.31</td>
</tr>
</tbody>
</table>

We then compute the size distribution of firms with male and female top managers in LAC using from the World Bank Enterprise Survey for the year 2010. The World Bank Enterprise Survey is a firm-level survey of a representative sample of an economy’s private sector conducted for a large set of countries. For the Latin America and the Caribbean region, it contains information for 13,855 firms, with sizes between 1 and 21,955 employees, and from 26 different countries.

As we can see in Table 2, female-run firms are not only outnumbered by male-run firms, but they are also significantly smaller in terms of employees. The median size of firms with a female manager, for example, is 16 workers, while it is 30 workers for firms with a male manager.

3 http://kilm.ilo.org/KILMnetBeta/default2.asp
4 http://www.enterprisesurveys.org/
5 The Enterprise Survey is answered, in most cases, by business owners and top managers. Typically 1200-1800 interviews are conducted in larger economies, 360 interviews are conducted in medium-sized economies and for smaller economies 150 interviews take place. The manufacturing and services sectors are the primary business sectors of interest. This corresponds to firms classified with ISIC codes 15-37, 45, 50-52, 55, 60-64, and 72. Formal (registered) companies with 5 or more employees are targeted for interview. Firms with 100% government/state ownership are not eligible to participate in an Enterprise Survey. In each country, businesses in the cities/regions of major economic activity are interviewed. The sampling methodology for Enterprise Surveys is stratified random sampling, that is, all population units are grouped within homogeneous groups and simple random samples are selected within each group.
6 The countries are Argentina (1054 firms), The Bahamas (150 firms), Barbados (150 firms), Bolivia (362 firms), Brazil (1802 firms), Chile (1033 firms), Colombia (942 firms), Costa Rica (538 firms), Dominican Republic (360 firms), Ecuador (360 firms), El Salvador (360 firms), Grenada (153 firms), Guatemala (590 firms), Guyana (165 firms), Honduras (360 firms), Jamaica (376 firms), Mexico (1480 firms), Nicaragua (336 firms), Panama (365 firms), Paraguay (361 firms), Peru (1000 firms), St. Kitts and Nevis (150 firms), St. Vincent and the Grenadines (154 firms), Trinidad and Tobago (370 firms), Uruguay (607 firms), and Venezuela (320 firms).
Table 2: Percentiles of the firm-size distribution by gender (Data source: WB Enterprise Survey)

<table>
<thead>
<tr>
<th></th>
<th>10 pct</th>
<th>25 pct</th>
<th>50 pct</th>
<th>75 pct</th>
<th>90 pct</th>
</tr>
</thead>
<tbody>
<tr>
<td>Females</td>
<td>5</td>
<td>8</td>
<td>16</td>
<td>45</td>
<td>119</td>
</tr>
<tr>
<td>Males</td>
<td>6</td>
<td>12</td>
<td>30</td>
<td>100</td>
<td>279</td>
</tr>
</tbody>
</table>

Figure 1 shows graphically that the firm-size distribution of female-run firms is shifted to the left relative to the distribution of firms managed by men or, in other words, the distribution of female-run firms has a thicker density at low numbers of employees.\(^7\)

Figure 1: Firm-size distribution by gender (Data source: WB Enterprise Survey)

3 Theoretical framework

3.1 Setup description

In this section, we briefly describe the general equilibrium framework presented in Cuerbes and Teignier (2016), which is based on the span-of-control model of Lucas (1978). It consists on an occupational choice model where agents are endowed with a random entrepreneurship skill, based on which they decide to work as either employers, self-employed, or workers. We also consider a fourth employment category, namely the out-of-necessity entrepreneurs, who choose this occupation because they had no other occupational choices apart from running their own business.\(^8\) An employer in this model can produce goods using a span-of-control technology that combines his or her entrepreneurship skills, with

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\(^7\)We plot the natural logarithm of the number of employees to help visualize the plots. Without taking logs, the large number of small firms in the data make it hard to appreciate the shape of the entire distribution.

\(^8\)Using data from the Global Entrepreneurship Monitor survey, Poschke (2013) finds that out-of-necessity entrepreneurs represent almost 50% of all entrepreneurs in non-OECD countries.
capital, and workers:

\[ y(x) = x \left( k^{\alpha}n^{1-\alpha} \right)^{\eta}, \]  

(1)

where \( y \) denotes the amount of output, \( x \) the employer’s managerial talent, \( k \) the units of capital rented, and \( n \) the units of labor. The parameter \( \eta \) takes a value between 0 and 1, which implies that firms with different productivity levels coexist in equilibrium and that more talented agents manage larger firms. On the other hand, a self-employed agent can produce goods using a similar technology - adjusted by the productivity parameter \( \tau \) - but without hiring any workers:

\[ \tilde{y}(x) = \tau x k^{\alpha \eta}. \]

The parameter \( \tau \) is a self-employed productivity parameter, which can be interpreted as one minus the fraction of time a self-employed agent spends on management tasks.

Figure 2 displays the payoff of the three occupations at each talent level. It shows that in this model agents with the highest entrepreneurship skill (those with a talent equal or larger than \( z_2 \)) optimally choose to become employers, whereas those with the least skill (with a talent lower than \( z_1 \)) become workers, leaving the self-employed occupation to agents with intermediate skill levels. Specifically, denoting the employers’ profits by \( \pi_e(x) \), the self-employed profits by \( \pi_s(x) \), the equilibrium wage rate \( w \), and the capital rental rate by \( r \), the payoff functions are

\[
\pi_e(x) \equiv \max_{w,r} \left\{ y(x) - rk - wn \right\}, \\
\pi_s(x) \equiv \max_{r} \left\{ \tilde{y}(x) - rk \right\}.
\]

**Figure 2:** The occupational map

In this economy, aggregate production per capita is the sum of output by employers and
self-employed:

\[ y \equiv \frac{Y}{N} = \left[ \int_{z_2}^{\infty} y(x)d\Gamma(x) + \int_{z_1}^{z_2} \tilde{y}(x)d\Gamma(x) + \frac{1 - \theta_f}{2} \int_{1}^{z_2} \tilde{y}(x)d\Gamma(x) + \frac{1 - \theta_m}{2} \int_{1}^{z_1} \tilde{y}(x)d\Gamma(x) \right] , \]

where \( \theta_m \) and \( \theta_f \) are fraction of men and women who cannot get job offers as workers and choose self-employment even if their managerial talent is \( z_1 \). \( \Gamma \) denotes the cumulative density function of managerial talent, which is assumed to be Pareto:

\[ \Gamma \left( x \right) = 1 - x^{-\rho} , \quad x \geq 0 , \tag{2} \]

where \( \rho > 0 \).

As Appendix B shows, using the talent distribution defined in equation (2) and the optimal number of employees, \( n \left( x \right) = \left[ x\eta(1 - \alpha) \left( \frac{\alpha}{1 - \alpha} \right) ^{\alpha \eta} \left( \frac{w}{w_{\eta}} \right) ^{1/(1 - \eta)} \right] \), we obtain the following cumulative density function of firms’ size \( S(n) \) in equilibrium:

\[ S \left( n \right) = 1 - n^{-\rho(1 - \eta)} \left( \eta \left( \frac{\alpha}{r} \right) ^{\alpha \eta} \left( \frac{1 - \alpha}{w} \right) ^{1-\alpha \eta} \right) ^{\rho} , \]

where \( n \) is the number of workers, our measure of firm’s size. We can then calculate the statistic \( \Psi \):

\[ \Psi \equiv \frac{\partial \ln \left( 1 - S \left( n \right) \right)}{\partial \ln (n)} = -\rho \left( 1 - \eta \right) , \tag{3} \]

which corresponds to the log-log slope of the function \( 1 - S \left( n \right) \).

### 3.2 Introducing gender gaps into the framework

As explained above, the model assumes that men and women are identical in terms of their managerial skills, in the sense that they draw their talent from the same distribution function. However, in our framework, women are subject to an exogenous constraint in their occupational choice, namely a fraction of women who would like to be employers are excluded from this occupation. This distorts the efficient talent allocation and reduces the average talent of employers. Specifically, females face a probability \( 1 - \tilde{\mu} \left( x \right) \) of being excluded from employership, which depends on their ability in the following way:

\[ \tilde{\mu} \left( x \right) = \mu x^\gamma , \tag{4} \]
where $\mu$ is a positive constant while $\gamma$ can be positive or negative. When $\gamma < 0$, the slope of this exclusion function is negative, i.e. $\tilde{\mu}'(x) = \gamma \mu x^{\gamma - 1} < 0$, which implies that more talented women are more likely to be excluded from employership.

The parameter $\mu$ determines the fraction of women who are excluded from employership when $x = 1$, while the parameter $\gamma$ determines the change in this exclusion probability when $x$ rises. The more negative is $\gamma$, the larger the output loss, since it implies that more talented women face a higher probability of being excluded from employership. Our analysis in section 2 shows that female-run firms are significantly smaller in all the cases, which we interpret as evidence that $\gamma < 0$ in the LAC region.

The value of $\gamma$ can be estimated comparing the cumulative density functions of male and female-run firms. Under the exclusion probability in equation (4), the expected number of workers of a firm run by a female manager with talent $x$ becomes

$$n_f(x) = \mu x^{\gamma + \frac{1}{1-\eta}} \left[ \eta (1 - \alpha) \left( \frac{\alpha}{1 - \alpha} \right)^{\alpha \eta} \frac{w^{\alpha \eta - 1}}{r^{\alpha \eta}} \right]^{1/(1-\eta)},$$

which is simply the product of the probability that a woman is not excluded from employership, $\tilde{\mu}(x)$, and the firms size at given talent level $x$, $n(x)$. Replacing this into equation (2), we obtain the cumulative density function of female-run firms

$$S_f(n) = 1 - n - \frac{\rho (1 - \eta)}{1 + \gamma (1 - \eta)} \left( \mu^{1-\eta} \eta \left( \frac{\alpha}{w} \right)^{\alpha \eta} \left( \frac{1 - \alpha}{r} \right)^{1-\alpha \eta} \right)^{1/(1-\eta)},$$

where $S_f(n)$ denotes the density function of female-run firms. This leads to the statistic $\Psi_f$, which is the female-equivalent of the one defined in equation (3):

$$\Psi_f \equiv \frac{\partial \ln (1 - S_f(n))}{\partial \ln (n)} = -\frac{\rho (1 - \eta)}{1 + \gamma (1 - \eta)},$$

Taking the ratio between equations (3) and (5), we get

$$\frac{\Psi_f}{\Psi} = \frac{1}{1 + \gamma (1 - \eta)},$$

which can be estimated using the firm-size distributions of men and women.

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9 Since they do not have firm-level information, Cuberes and Teignier (2016) assume that $\gamma = 0$, which means that the probability that a woman is banned from becoming an employer is independent of her managerial talent.
4 Numerical results

4.1 Model Parametrization

To simulate the model we first need to give values to its different parameters, namely the technology parameters \([\alpha, \eta, \tau, \rho]\), the out-of-necessity entrepreneurs parameter \(\theta\), as well as the female exclusion probability parameters \([\mu, \gamma]\). In this section, we describe the procedure and the data moments used to parametrize the model. Table 3 summarizes these values.

Table 3: Parameter values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\tau)</td>
<td>0.71</td>
<td>To match the share of male self-employed.</td>
</tr>
<tr>
<td>(\eta)</td>
<td>0.72</td>
<td>To match the share of male employers.</td>
</tr>
<tr>
<td>(\rho)</td>
<td>4.41</td>
<td>To match the males’ firms size distribution.</td>
</tr>
<tr>
<td>(\alpha)</td>
<td>0.08</td>
<td>To match the aggregate capital income share.</td>
</tr>
<tr>
<td>(\theta^f)</td>
<td>0.88</td>
<td>To match the share of male necessity entrepreneurs.</td>
</tr>
<tr>
<td>(\theta^m)</td>
<td>0.87</td>
<td>To match the share of female necessity entrepreneurs.</td>
</tr>
<tr>
<td>(\gamma)</td>
<td>0.90</td>
<td>To match the females’ firms size distribution.</td>
</tr>
<tr>
<td>(\mu)</td>
<td>0.67</td>
<td>To match the share of female employers.</td>
</tr>
</tbody>
</table>

We start by choosing the values of \(\theta^f\) and \(\theta^m\) to match the fraction of necessity entrepreneurs, which is 25% in the case of males and 35% in the case of females.\(^{10}\) We next write the parameters \([\alpha, \rho, \gamma]\) as a function of \(\eta\). In particular, to get a capital income share equal to 1/3, a value that is generally accepted, the value of \(\alpha\) must satisfy \(\alpha\eta + (1 - \eta) = 1/3\) provided that profits are considered capital income. Similarly, using equations (3) and (5) together with the fact that \(\Psi\) is equal to -1.2 and \(\Psi^f\) is equal to -1.6 (see Figure 3), we get that \(\rho (1 - \eta) = -1.2\) and \(1 + \gamma (1 - \eta) = 0.75\). Finally, we jointly estimate the rest of the parameters \([\eta, \tau, \mu]\) to match the shares of male employers and self-employed, as well as the ratio of female-to-male employers in the working-age population.

Figure 3 shows the ratios of the slopes of the male and female distribution functions in the data. As we can see in this figure, the absolute value of the slope is larger for females than for males, with the female-to-male ratio equal to 0.75.\(^{11}\)

\(^{10}\)See the 2015-16 Global Entrepreneurship Report, page 128 for the LAC region.

\(^{11}\)We do the estimation using the sample of firms with size between 10 or more workers and the largest female-run firm.
4.2 Economic losses due to the entrepreneurship gender gaps

The introduction of the employership gender gaps leads to an important fall of aggregate output, both in the short run —when the capital stock is kept fixed— and the long run — when the capital stock is adjusted to its new steady-state level. As explained in Section 3.1., the explanation for this output loss is that aggregate productivity decreases due to the drop in the average talent of employers caused by the exclusion of some talented female employers. As Table 4 shows, we estimate a fall of aggregate output of about 9% in the short run and 9.4% in the long run. These losses are almost one third larger than the ones we would obtain using a model where the exclusion probability was independent of talent, i.e. where $\gamma = 0$, since the fact that $\gamma < 0$ implies that there is a negative selection of women into employership.

| Table 4: Output losses due to employership gender gaps in LAC |
|--------------|----------------|--------------|----------------|
| Short run    | Long run       |
| Baseline sim. | Ratio to $\gamma = 0$ | Baseline sim. | Ratio to $\gamma = 0$ |
| (%)          | (%)            | (%)          | (%)            |
| 8.9          | 1.3            | 9.4          | 1.3            |

When a woman with good management skills happens to be barred from employership, the demand of labor and capital decreases. As a result, the equilibrium wage and interest rate decreases, which makes it more profitable for less skilled agents to become entrepreneurs. In other words, both occupational thresholds $z_1$ and $z_2$ fall. On top of that, if more talented women face a higher exclusion probability, the average talent of employers...
falls even more, which results in a larger fall of total output. Table 5 illustrates this intuition: the introduction of the gender gaps reduces the equilibrium wage rate as well as the talent thresholds $z_1$ and $z_2$. If $\gamma < 0$ the fall is even larger and, moreover, the gender gap in the average earnings of employers becomes positive because now the average talent of female managers is lower than the male managers one. Specifically, the talent thresholds and the wage rate fall by almost 10%, while the fall would be 7.5% if we did not take into account the difference in the size distribution (i.e. if we assumed that $\gamma = 0$).

Table 5: Short-run results under different scenarios

<table>
<thead>
<tr>
<th></th>
<th>No gender gaps (normalized at 100)</th>
<th>Gender gaps with $\gamma &lt; 0$</th>
<th>Gender gaps with $\gamma = 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Workers’ wage</td>
<td>100</td>
<td>90.8</td>
<td>92.5</td>
</tr>
<tr>
<td>Interest rate</td>
<td>100</td>
<td>91.1</td>
<td>93.1</td>
</tr>
<tr>
<td>Talent threshold $z_1$</td>
<td>100</td>
<td>90.8</td>
<td>92.5</td>
</tr>
<tr>
<td>Talent threshold $z_2$</td>
<td>100</td>
<td>90.8</td>
<td>92.5</td>
</tr>
<tr>
<td>Employers earnings gender gap (%)</td>
<td>0</td>
<td>44.0</td>
<td>0</td>
</tr>
</tbody>
</table>

5 Sensitivity analysis

In this section we present our main results for other parameter values in order to see the sensitivity of the results to different assumptions. The first row of table 6 shows the results under the benchmark simulation, as in Table 4, while the rest of rows show the short and long-run losses under different scenarios.

The first alternative scenario, presented in the second row, shows the loss in per-capita output due to the introduction of the employership gender gaps in a context where the self-employment gender gap is also present. To be precise, we infer from the data that about 16% of women barred from employership can become self-employed while the rest become workers. Taking this into account, we find that introducing the gender gaps in employers generates significantly larger losses, both in the short run and the long run. Intuitively, the output loss is now larger because there is a larger fall in the share of entrepreneurs and because there is larger fall in equilibrium wage and, consequently, on the average talent of entrepreneurs.

The third row displays the losses under the parameter values used in Cuberes and Teignier (2016), namely $[\eta, \rho, \tau] = [0.79, 6.5, 0.7]$, which were estimated using data for the OECD countries. In this case, the model predicts significantly smaller losses (about one
third smaller) as well as a lower difference with respect to the $\gamma = 0$ framework. The main difference with respect to the benchmark parametrization are a somewhat larger value for $\eta$, which reduces the contribution of managerial talent on the firm productivity, and especially a larger value for $\rho$, which increases the slope of the pdf and reduces the thickness of the talent distribution tail. As a result, the fall in the talent threshold due to the introduction of the employership gaps is smaller, which implies a lower loss of aggregate income.

The fourth row, on the other hand, shows that the income losses would also be somewhat smaller if the firm profits (or compensation to managerial talent ) were considered labor income instead of capital income, i.e. if $\alpha \eta = 1/3$ and $\alpha = 0.41$. The long-run results are not affected because the capital stock adjusts to its steady-state value, which makes the value of $\alpha$ irrelevant to determine the income fall. In the short-run, the income fall is lower because the importance of the fixed factor —namely, capital— is lager. Intuitively, in this case, there is larger difference between the short and long run results because the long-run capital adjustment increases with $\alpha$.

Finally, the last row shows the results under an alternative setup where self-employment is never an optimal occupational choice but only a necessity occupation for those unable to find a job as worker. In this setup, the loss in income would be somewhat lower than in the benchmark simulation, especially in the short run, although the ratio to the $\gamma = 0$ case are the same. The explanation is that the parametrization of this alternative setup leads larger values for $\eta$ and $\rho$ which, as discusses above, implies smaller output losses.\textsuperscript{12}

\textsuperscript{12}In particular, under this alternative setup, matching the fraction of employers as well as the firm-size distribution implies a value of $\eta = 7.24$ and $\rho = 0.83$. 
Table 6: Output losses due to employership gender gaps in LAC

<table>
<thead>
<tr>
<th></th>
<th>Short run</th>
<th></th>
<th>Long run</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Output loss (%)</td>
<td>Ratio to $\gamma = 0$</td>
<td>Output loss (%)</td>
<td>Ratio to $\gamma = 0$</td>
</tr>
<tr>
<td><strong>Benchmark simulation</strong></td>
<td>8.9</td>
<td>1.30</td>
<td>9.4</td>
<td>1.29</td>
</tr>
<tr>
<td><strong>Self-employment gender gap present</strong></td>
<td>11.8</td>
<td>1.23</td>
<td>12.5</td>
<td>1.23</td>
</tr>
<tr>
<td><strong>OECD parameters</strong> ($\eta, \rho, \tau$) = (0.79, 0.5, 0.7)</td>
<td>5.56</td>
<td>1.23</td>
<td>6.32</td>
<td>1.23</td>
</tr>
<tr>
<td><strong>Profits as labor inc. ($\alpha = 0.41$)</strong></td>
<td>6.37</td>
<td>1.30</td>
<td>9.4</td>
<td>1.29</td>
</tr>
<tr>
<td><strong>Only necessity self-employment</strong></td>
<td>6.83</td>
<td>1.30</td>
<td>8.13</td>
<td>1.30</td>
</tr>
</tbody>
</table>

6 Conclusions

In this paper we document that in the LAC region only about one fourth of the total firms are run by women and that female-run firms are around three times smaller than male-run firms. We quantify the output losses caused by these gender gaps using an occupational choice model where women face frictions to become firm managers which depend positively on their managerial talent. In our benchmark simulation, we find that the aggregate long-run output loss due to these gender gaps is almost 9% in the short run and 9.5% in the long run, which is due to the drop in average managerial talent and the resulting fall in aggregate productivity. This loss is about 1.3 larger than the ones estimated under a framework with frictions that are independent of talent. Our sensitivity analyses show that the income losses may take values between 6.4% and 11.8% in the short run, and between 8.1% and 12.5% in the long run.

In our framework, we explain all the existing differences in firm size between men and women by introducing an exogenous friction into the occupational choice of women and making this friction correlate positively with the managerial talent of women positive correlation between the managerial talent of women. This positive correlation could be the result of discrimination in the labor market but also the result of differences in preferences, like a stronger taste for family time. More work is clearly needed to determine the fundamental
causes behind the difference between male and female-run firm distributions. Alternative explanations for these gender differences could be differences in the talent distribution, maybe resulting from differences in education choices or constraints or from differences in access to credit.
References


The economy we consider has a continuum of agents indexed by their entrepreneurial talent \( x \), drawn from a cumulative distribution \( \Gamma \) that takes values between \( B \) and \( \infty \). We assume the economy is closed and that it has a workforce of size \( N \) and \( K \) units of capital. Labor and capital are inelastically supplied in the market by consumers, in exchange for a wage rate \( w \) and a capital rental rate \( r \) respectively. These inputs are then combined by firms to produce an homogeneous good. Agents decide to become either firm workers, who earn the equilibrium wage rate \( w \) —which we assume to be independent of their entrepreneurial talent—, or entrepreneurs, who earn the profits generated by the firm they manage.\(^{13}\) In the model, we also include a fourth category, namely the out-of-necessity entrepreneurs, who choose this occupation because they had no other occupational choices apart from running their own business. We denote by \( 1 - \theta \) the fraction of both males and females that are out-of-necessity entrepreneurs.

An agent with entrepreneurial talent or productivity level \( x \) who chooses to become an employer and hires \( n(x) \) units of labor and \( k(x) \) units of capital produces \( y(x) \) units of output and earns profits \( \pi(x) = y(x) - rk(x) - wn(x) \), where the price of the homogeneous good is normalized to one. As in Lucas (1978) and Buera and Shin (2011), the production function is given by

\[
y(x) = x (k(x)^{\alpha} n(x)^{1-\alpha})^{\eta},
\]

where \( \alpha \in (0, 1) \) and \( \eta \in (0, 1) \). The parameter \( \eta \) measures the span of control of entrepreneurs and, since it is smaller than one, the entrepreneurial technology involves an element of diminishing returns. On the other hand, an agent with talent \( x \) who chooses to become self-employed uses the amount of capital \( \tilde{k}(x) \), produces \( \tilde{y}(x) \) units of output and earns profits \( \tilde{\pi}(x) = \tilde{y}(x) - r \tilde{k}(x) \). The technology he or she operates is

\[
\tilde{y}(x) = \tau x \tilde{k}(x)^{\alpha \eta},
\]

where \( \tau \) is the self-employed productivity parameter.\(^{14}\) One interpretation of this parameter is that self-employed workers have to spend a fraction of their time on management tasks, which would imply that \( \tau \) is equal to the fraction of time available for work to the power \( (1 - \alpha) \eta \). As explained below, we estimate this parameter to match the average fraction of self-employed in the data.

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\(^{13}\)In what follows we will refer to an entrepreneur as someone who works as either an employer or a self-employed.

\(^{14}\)The consumption good produced by the self-employed and the capital they use is the same as the one in the employers’ problem. However, it is convenient to denote them \( \tilde{y} \) and \( \tilde{k} \) to clarify the exposition.
A.1 Agents’ optimization

A.1.1 Employers

Employers choose the units of labor and capital they hire in order to maximize their current profits $\pi$. The optimal number of workers and capital stock, $n(x)$ and $k(x)$ respectively, depend positively on the productivity level $x$, as equations (9) and (10) show:

$$n(x) = \left[ x\eta(1-\alpha) \left( \frac{\alpha}{1-\alpha} \right)^{\alpha} \frac{w^{\eta-1}}{r^{\alpha \eta}} \right]^{1/(1-\alpha)},$$

(9)

$$k(x) = \left[ x\eta \left( \frac{1-\alpha}{\alpha} \right)^{\eta(1-\alpha) \frac{\eta(1-\alpha)-1}{w^{\eta(1-\alpha)}}} \right]^{1/(1-\eta)}.$$  

(10)

A.1.2 Self-employed

When we solve for the problem of a self-employed agent with talent $x$ who wishes to maximize his or her profits, we find

$$\bar{k}(x) = \left( \frac{x \alpha \eta}{r} \right)^{\frac{1}{\eta}}.$$  

(11)

A.1.3 Occupational choice

Figure (2) displays the shape of the profit functions of employers, $\pi_e(x)$, and self-employed, $\pi_s(x)$, as well as the wage earned by workers as a function of talent $x$. The agents’ optimization determines the relevant talent cutoffs for the occupational choices. Here we present the equations that define these thresholds: the first one, $z_1$, defines the earnings such that agents are indifferent between becoming workers or self-employed and it is given by

$$w = \tau z_1 \bar{k}(z_1)^{\alpha \eta} - r \bar{k}(z_1).$$

(12)

If $x \leq z_1$ agents choose to become workers, while if $x > z_1$ they become self-employed or employers. The second cutoff, $z_2$, determines the choice between being a self-employed or an employer and it is given by

$$\tau z_2 \bar{k}(z_2)^{\alpha \eta} - r \bar{k}(z_2) = z_2 x \left( k(z_2)^{\alpha} n(z_2)^{1-\alpha} \right)^{\eta} - r k(z_2) - w n(z_2)$$

(13)

so that if $x > z_2$ an agent wants to become an employer.
A.2 Competitive Equilibrium

We assume that women represent half of the population in the economy and that there is no unemployment. Moreover, any agent in the economy can potentially participate in the labor market, except for the restrictions on women described above. Under these assumptions, in equilibrium, the total demand of capital by employers and self-employed must be equal to the aggregate capital endowment (in per capita terms) $k$:

$$k = \frac{1}{2} \left[ \int_{z_2}^{\infty} k(x) d\Gamma(x) + \int_{z_1}^{z_2} \tilde{k}(x) d\Gamma(x) + (1 - \theta) \int_{B}^{z_1} \tilde{k}(x) d\Gamma(x) \right]$$

$$+ \frac{1}{2} \left[ \int_{z_2}^{\infty} \mu(x) k(x) d\Gamma(x) + \int_{z_1}^{z_2} \tilde{k}(x) d\Gamma(x) + \int_{B}^{\infty} (1 - \mu(x)) \tilde{k}(x) d\Gamma(x) + (1 - \theta) \int_{B}^{z_1} \tilde{k}(x) d\Gamma(x) \right].$$

The upper term is the demand for capital by men and the two lower terms are the women’s demand for capital. The demand for capital by male-run firms has three components: the first one represents the capital demand by employers, while the second and third terms represent the demand by self-employed. i.e. those who have the right ability to be self-employed plus the capital demand by those who become self-employed because they do could not find a job as workers. These \textit{out-of-necessity} self-employed demand the optimal amount of capital given their talent or ability.

The demand of capital by female-run firms has four components. The first one represents the capital demand by female employers, i.e. those with enough ability to be employers and who are allowed to be so, while the second term represent the capital demand by women who have the right ability to be self-employed and are allowed to work. The third term shows the capital demand by women who become self-employed because they are excluded from employership and, finally, the last term shows the fraction of females who would like to be workers but, since they are “excluded” from this occupation, they choose to become \textit{out-of-necessity} self-employed if they are not excluded from entrepreneurship.

Similarly, the labor market-clearing condition is given by

$$\frac{1}{2} \left[ \int_{z_2}^{\infty} n(x) d\Gamma(x) \right] + \frac{1}{2} \left[ \int_{z_2}^{\infty} \mu(x) n(x) d\Gamma(x) \right] = \theta \Gamma(z_1),$$

which shows that, in equilibrium the aggregate labor demand is equal the aggregate labor

\footnote{As explained in section 3, a fraction $(1 - \theta)$ of both males and females with ability below $z_1$ become self-employed because they would like to be workers but are not allowed to do so and choose their second-best option.}
supply. The first term is the labor demand by male employers and the second one corresponds to the labor demand by female employers, i.e. those women with enough ability to be employers who are allowed to choose their occupation freely. The labor supply shows the fraction of men of women who choose to become workers and are not forced to be necessity self-employed.

A competitive equilibrium in this economy is a pair of cutoff levels \((z_1, z_2)\), a set of quantities \(\left[n(x), k(x), \bar{k}(x)\right] \forall x\), and prices \((w, r)\) such that entrepreneurs choose the amount of capital and labor to maximize their profits, and labor and capital markets clear.

### B APPENDIX B: Details on size-distribution calculations

Given a level of talent \(x\), firms choose to have \(n(x) = \left[ x\eta(1 - \alpha) \left( \frac{\alpha}{1 - \alpha} \right)^{\eta(\alpha - 1)} \frac{w^{\alpha \eta - 1}}{r^{\alpha \eta}} \right]^{1/(1-\eta)}\) workers. We can invert this function to find

\[
x = n^{1-\eta} \left[ \eta(1 - \alpha) \left( \frac{\alpha}{1 - \alpha} \right)^{\eta} \frac{w^{\alpha \eta - 1}}{r^{\alpha \eta}} \right]^{-1}
\]

Substituting this expression into the cumulative density function \(2\) gives us

\[
S(n) = 1 - n^{-\rho(1-\eta)}B^\rho \left( \eta \left( \frac{\alpha}{r} \right)^{\eta} \left( \frac{1 - \alpha}{w} \right)^{1-\alpha \eta} \right)^\rho.
\]