Survival and Demise of the State: A Dynamic Theory of Secessions

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ABSTRACT

1 Introduction

“In the days of those kings the God of heaven will set up a kingdom which will never be destroyed, and that kingdom will not be left for another people; it will crush and put an end to all these kingdoms, but it will itself endure forever.” (The Bible, Daniel 2:44)

Already in pre-historic times the world has been governed by mighty kingdoms and empires, all of which have crumbled and disintegrated sooner or later, whether it may be ancient Mesopotamia with its legendary Babylon that serves as constant reminder of perishability in the Bible, or the Roman Empire, Alexander the Great’s Macedon, the Ottoman Empire, the Habsburg Empire, the British Empire or the Mongol Empire. Usually desintegration has started from the borders, with some ethnic, linguistic or religious groups seeking to split from the rest.

Leo Tolstoy’s opening claim to his book Anna Karenina of "happy families are all alike; every unhappy family is unhappy in its own way" applies as well to the survival, decline and demise of empires. The survival of historical empires varies widely in longevity – with evergreens like the Roman Empire lasting for centuries,
while Napoleon Bonaparte’s or Adolf Hitler’s attempts to world domination being (thankfully) short-lived. The way empires and nation-states die is also characterized by much variability. Most long lasting empires like the Roman one were often characterized by a very large and powerful center (Rome) and relatively small, and much fractionalized peripheral states. And even in modern nation-states minorities who are relatively small in size are usually not very vocal in wanting to break the union with the rest of the country (e.g. think of German-speaking Südtirol in Italy, Martinique and Guadeloupe in France, Galicia in Spain or the Sami people in Northern Scandinavia).

Some secessions take place in peace: The Roman Empire chose to voluntarily and peacefully split into two similarly large and similarly rich halves characterized by some salient differences in social norms (i.e. Western Rome was more and more engaging in Christianity). More recently, after the Fall of Berlin Wall, Czechoslovakia was split peacefully into two similarly large and rich halves characterized by ethnic differences.

Other secessions proceed less harmoniously: The collapses of the Soviet Empire and Yugoslavia was accompanied by a series of bloody conflicts, and disagreements on whether to splitting or staying together. In both cases the sizes of the composing regions varied a lot, and while the richest and most productive places were keen to split (i.e. Russia, the Baltic states, resp. Slovenia and Croatia) other regions opposed separation.

The existing academic literature on secessions focuses on static models, while the survival and demise of the state is an inherently dynamic phenomenon, where at the same time relative group sizes, inequalities in prosperity and heterogeneity in preferences and social norms affect the inter-temporal trade-offs of the various groups engaging in strategic interaction. To address this gap in the literature we will in the current contribution present a full-blown dynamic model of stability and break-down of states.

In particular, our model focuses on a country that is divided into two groups which vary in size, economic productivity and have different preferences over the type of public good to be supplied. The type of public goods we have in mind are among others culture, language, legislation, or sense of regional pride. There is a cost of setting up or maintaining a state. Under union the group in power selects the public good and transfers some (endogenously chosen) part of total surplus to the other group. Under secession there are two separate states where in each one
a public good is produced and the constant cost of running the state is paid. The overall trade-off is hence between the economies of scale of larger states (due to having to pay the fixed administrative cost only once) versus the cost of preference heterogeneity (the opposition group is not able to select its favored public good). In terms of timing, the group in power makes a proposal (either union, peaceful secession or conflict), which the group in opposition can either accept or reject, triggering in the latter case costly conflict. The game is infinitely repeated with secession being an absorbing state (after secession no more strategic decisions are taken).

We are able to characterize the dynamic equilibrium of this game, leading to a picture with four zones. Roughly speaking, when the opposition group is of small size and productivity differences are small, then we show that peaceful union is stable. In contrast, large imbalances of productivity lead to instability and conflict. Such conflict followed by secession is both predicted for situations when the group in power is much more productive as for situations where the opposition group is of substantially larger productivity. For some parameter values there also exists a zone of persistent non-secessionist conflict over the control for the government. Finally, we also detect a zone of peaceful secession, namely the opposition group is of intermediate or large size and of similar productivity of the group in power. In such a case both parties may find it in their interest to peacefully split. After characterizing the equilibrium of the game we discuss the predictions in the light of historical examples.

There is by now a substantial body of literature on secessions and size of nations that we summarize in the next section. The bulk of the literature focuses on the trade-off between heterogeneity of citizens on preferences and increasing returns to country size (which is also contained in our stage game). The focal point of many existing papers is then whether there are inter-group [regions] transfers that would prevent one of the groups to opt for secession and the creation of a new country. The various contributions differ in the specification of the heterogeneity in the preferences within and across groups and in the nature of the benefits of the size of the country. Our current paper makes a different point: While remaining together implies that the public decisions will have to be renegotiated at every time period in the future with the competing group with different preferences and priorities, secession implies a potential cost today but no need to bargain with the other group ever again in future. This inter-temporal argument is in our view an
essential ingredient in the arguments for or against secession, generating a very
different equilibrium characterization than in a static game with the future being
fully discounted (as we show further below). Thus, it is absolutely key to take into
account dynamic considerations when modelling secession.

A second major difference between our setting and the existing literature is that
we explicitly model conflict as integral component of the trade-off about whether to
seceede or not. As discussed further below, in the literature on conflict there a only
few dynamic frameworks, and they typically do not include the option of secession.

The reminder of the paper is organised as follows: Section ?? is devoted to
a review of the existing literature, while in Section ?? we set up the model and
characterize its equilibrium. Section  connects the model predictions to historical
examples, and Section  concludes.

2 Literature review

First and foremost, the current paper is obviously part of the literature on border
formation and secessionism.[3] One key point made by this strand of the economic lit-
erature is that the size of countries results from the trade-off between the economies
of scale of a larger country size and the costs of heterogeneity of preferences over
public goods and government policies.[4] The literature distinguishes various poten-
tial determinants of the incentives for secession such as region size (Goyal and Staal,
2004), the degree of international openness (Alesina, Spolaore and Wacziarg, 2000,
2005); the degree of democratization (Alesina and Spolaore, 1997; Arzaghi and Hen-
derson, 2005; Panizza, 1999); the optimal level of public spending (Le Breton and
Weber, 2001; Le Breton et al., 2011); the presence of mobile ethnic groups (Olof-
gård, 2003), the relative natural resource rents of different regions of a country
(Gehring and Schneider, 2017) or the presence of external threats (Alesina and
different preferences for income tax policies emerging from different regional income
distributions.

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[3] Excellent reviews of the literature on secessionism are provided in Bolton et al. (1996), Alesina
and Spolaore (2003) and Spolaore (2014).

[4] See the key contributions on this by, among others, Friedman (1977), Buchanan and Faith

when part of the population perceives secession to be economically advantageous.
Further, the literature on secessionism has studied whether there exist interregional compensation mechanisms such that potentially seceding regions are better off by staying in the union. Haimanko et al. (2005) show that in an efficient union whose citizens’ preferences are strongly polarized, a threat of secession cannot be eliminated without interregional transfers. Le Breton and Weber (2003) establish the principle of partial equalization, according to which the gap between advantageous and disadvantageous regions must be reduced, but should not be completely eliminated. Alesina and Spolaore (2003) point out the difficulties of implementing compensation transfers, such as feasibility and administrative costs, political credibility, or incompatibility with other social goals.

A few authors have explicitly introduced a conflict technology in the context of separatism. Spolaore (2008) analyzes the choice of regional conflict efforts when a peripheral (minority) region wishes to secede from the center, focusing on the trade-off between economies of scale and heterogeneity of preferences in a setting where transfers are barred. Anesi and De Donder (2013) build a static model of secessionist conflict with exogenous winning probability, showing the existence of a majority voting equilibrium with a government’s type biased in favor of the minority. Our contribution is complementary to theirs, as our dynamic setting features general transfers, and links winning probabilities to group sizes.

Also in the conflict literature there are only very few papers that have explicitly modelled incentives for secession. Morelli and Rohner (2015) have built a model allowing for both nationwide and secessionist conflict, showing that the most conflict-prone situations are those in which the mineral resources of value are mostly concentrated in the minority group region, leading to secessionist pressures. Their empirical analysis finds that indeed this constellation of most oil revenues accruing in minority regions is a major driver of civil war. One major differences between our current paper and Morelli and Rohner’s (2015) static setting is that we have a dynamic model allowing for both conflicted and peaceful secession.

In sum, ours is the first dynamic model of secession taking into account conflict

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6 Related to this, Bordignon and Brusco (2001) analyze whether constitutions should include provisions for agreed potential secessions, arguing that if peaceful secession is not foreseen, the society may incur ex-post important efficiency losses due to conflict. Yet, paradoxically, making splitting up less costly makes it more likely to happen.

7 Another article studying endogenous country borders and war is Caselli, Morelli and Rohner (2015). Contrary to our current paper, their static model focuses on interstate wars.
incentives. This framework allows us to obtain a novel equilibrium characterization of zones of peaceful union, peaceful secession, centrist conflict and secessionist conflict, which is shown to differ very substantially from the characterization that would be obtained in a static framework of the future being fully discounted.

3 The model

A country is divided into two groups, $i$ and $j$, each settled in a specific region, of population size $N_j$ and $N_i$, $N = N_j + N_i$. Let $j$ be in power, and let $n \equiv N_i/N$ denote the share of the opposition group in the population. These groups have different preferences over the type of public good to be supplied. The group in power chooses the public good they like the most and this gives each of their members a payoff differential, so that if group $h$ is in power obtains $P_h > 0$ extra utility per member of the group over the group in opposition, which gets zero by normalization. We think of language, culture, legislation, government favored religion... The cost of setting up or maintaining a state, including the provision of such public goods, is denoted by $A > 0$. We shall assume that this cost is independent of the total population.

In order to some reasonable restriction on the value of $A$, we shall assume that the expected per capita welfare gain resulting from the random choice of the initial group in power is positive. This means that

$$nP_i + (1 - n)P_j \frac{A}{N}.$$

Let $S_i$ and $S_j$ be the surplus generated by groups $i$ and $j$, $S = S_i + S_j$. This surplus can come from production as well as from non-produced rents.

When the country remains united, the distributable surplus is $S - A$ and the group in power chooses the degree of fairness in the allocation of the (net) surplus to the group in opposition. Taking the equal per capita division of the surplus as a benchmark, we say that $j$ makes the strategic choice of treating $i$ with $\lambda$ fairness if the share of surplus received by group $i$ is $\lambda n$. Group $j$ in power keeps the rest.

When secession occurs, each group retains the own surplus and has to face the cost of running the state, $A$. In order to analyze secessions it has to be that the running cost of an independent state does not exceed the produced surplus. Hence, we shall assume that $A < \min\{S_i, S_j\}$. Each independent state produces the most preferred public good, worth $P_i$ and $P_j$, respectively.
While the group in power has the prerogative of making a proposal, the group in opposition can either accept or reject, thus triggering costly conflict. The player in power $j$ can make three types of proposals: (i) a distribution within the union under fairness $\lambda$; (ii) a peaceful secession; and (iii) trigger conflict.

The rejection of a proposal by the opposition opens a socially costly conflictual period. The power is challenged and each group has a win probability equal to its population size. The winner can either aim at conquering the power of the union and capture the entire surplus or aim at seceding and take away their own surplus forever, making the loser bear the cost of conflict $D$. We assume $D < \min\{S_i, S_j\}$.

We shall use the following normalized notation: $s = \frac{S_i}{S}, a = \frac{A}{S}, d = \frac{D}{S}, \sigma = \frac{S}{N}$.

Let us formalize our assumptions.

**Assumption 1.** We assume:

- $A < \min\{S_i, S_j\}$, equivalent to $a < \min\{s, 1 - s\}$,
- $A \leq N_i P_i + N_j P_j$, equivalent to $1 - \frac{P_i}{\sigma} n \geq \left(1 + \frac{1 - a}{(1 - n)(1 - P_j)}\right) (a - \frac{P_i}{\sigma} n)$, and
- $D < \min\{S_i, S_j\}$, equivalent to $d < \min\{s, 1 - s\}$.

Notice that $\min\{S_i, S_j\} > A$ implies that $S > A + \min\{S_i, S_j\} > A + D$. The latter inequality, or its equivalent $1 - a - d > 0$ will appear at different stages of our analysis.

Because of Assumption 2 it is immediate that in case of being victorious in a conflict, players will always opt for keeping the union. Since $\min\{S_i, S_j\} - D > 0$, there is more to grab if keeping the union. Hence, this hypothesis excludes the situation of violent conflict leading to secession. This will no longer be the case in a repeated game.

The game is a Stackelberg game: at equilibrium the group in power choses the best proposal, taking into account the reply by the group in opposition to each alternative proposal.

We shall now proceed to characterize the equilibria of the static game. To this effect, we shall examine first which is the best reply by the opposition to the different potential proposals by the group in power. Then, knowing this, we shall obtain which are the equilibrium proposals made by the group in power.
3 The model

3.1 Payoffs

Let us start by computing the payoffs to the two players in the three possible scenarios. Because of our previous point, the three scenarios are: (i) accepted union with an appropriate fairness in the distribution of the surplus; (ii) accepted secession; and (iii) conflict seizing the entire surplus.

The payoffs of each member \( i \) and \( j \) associated with the three scenarios are:

- under union with \( \lambda_j \) fairness is
  \[
  U(u, \lambda, i) = \lambda n \frac{S - A}{N_i} \quad \text{and} \quad
  U(u, \lambda, j) = (1 - \lambda n) \frac{S - A}{N_j} + P_j;
  \]

- in case of secession is
  \[
  U(s, i) = \frac{S_i - A}{N_i} + P_i \quad \text{and} \quad
  U(s, j) = \frac{S_j - A}{N_j} + P_j;
  \]

- in conflict is
  \[
  U(c, i) = n \left[ \frac{S - A - D}{N_i} + P_i \right] \quad \text{and} \quad
  U(c, j) = (1 - n) \left[ \frac{S - A - D}{N_j} + P_j \right].
  \]

We now proceed to the examination of the strategic choices by the players.

3.2 The repeated game

The game starts with one group in power. By convention this is group \( j \). Besides enacting the most preferred public good, they decide whether to announce secession or to remain together. In the latter case, the government announces the allocation of the production surplus between the two groups.

After the proposal has been made each group simultaneously decide over peace or conflict. If both choose peace the proposal is carried out. If one group at least choose conflict the outcome becomes probabilistic, with win probability equal to the
relative population size. In case of conflict there is a destruction of surplus of $D$. It is obvious that the larger is $D$ the less profitable is to challenge any proposal. Therefore we limit destruction not to exceed the surplus produced by either group, that is, $D < \min\{S_i, S_j\}$.

The winner of conflict has the option to secede. In this case, they split taking their own surplus and make the losers to bear the entire cost of conflict $D$. If they choose to keep the union, they capture the entire surplus, minus $D$ destroyed by conflict, continue as the ruler, make a proposal for the next period, and the game continues. All individuals discount the future by a factor of $\delta \in (0, 1)$.

To sum up, the group in opposition has one strategic choice only –peace or conflict– while the group in power first announces a distribution or a secession and then simultaneously decide between peace or conflict. We want to characterize the subgame perfect strategies of this game.

Let us be precise on the sequence of decisions. We describe the time line as follows:

1. *Production:* Each period starts with a group in power, say $j$, output $Y$ is produced, and the surplus $S$ is obtained.

2. *Proposal:* The group in power can decide either to continue as a united country or to secede. In the first case, group $j$ announces $\lambda$.

3. *Peace or conflict:* The two groups decide simultaneously whether to have conflict or peace. Peace prevails only if both choose peace. Group $j$ remains in power in case of peace and in case it wins the conflict, whereas group $i$ obtains power only by winning the conflict.

4. *Exercise of power.* If there has been peace, and hence $j$ remains in power the announced policies are carried out, these being either the announced distribution of the surplus or secession. If $j$ remains in power because they have been victorious in the conflict, they can opt between implementing secession or keeping the union appropriating the entire remaining surplus. Then they enter the next period as the rulers. If instead $i$ wins the conflict, they can also opt between implementing secession or keeping the union appropriating the entire remaining surplus. Then they enter the next period as the rulers.

5. *Consumption:* Consumption takes place.
We now proceed to the characterisation of the subgame perfect equilibria. To this effect it is important to bear in mind that the scenario is always the same for as long as the game lasts. The only state variable is the identity of the group in power. We want to identify all the equilibrium paths.

4 Equilibrium analysis

4.1 Preliminaries

Before examining in detail the potential equilibrium paths [in the next section] we devote this section to making some observations that will be useful for our characterization exercise.

The first point we wish to make is to underline the implication of being in a stationary environment until the game ends together with the assumption of perfect information. Full information implies that the observation of the strategic choice by the other player does not convey any new information that could influence future strategic decisions. Bearing this in mind, suppose that $h = i,j$ is in power and proposes either a level of fairness in the distribution for the entire society or secession, and suppose that the opposition group rejects it. Then every time $h$ is again in power they will make the same proposal and this proposal will again be rejected. This information can be used by the players in order to compute the payoff deriving from a strategic choice by either of the two players.

The stationarity of the environment has a useful implication for our equilibrium analysis. Consider the situation in which one of the two players, say $j$, has conquered power, makes a distribution proposal that is acceptable to $i$, and the game ends. Player $j$ would compute the minimum level of fairness making $i$ indifferent between peacefully accepting the proposal or triggering conflict and then $j$ would check whether this distribution gives to them a higher payoff than the expected payoff from making instead an unacceptable proposal, provoking conflict. It follows that in this case if $i$ had previously been in power they would also had available a level of

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8 Notice that this game bears a resemblance to the mass killings game in Esteban, Morelli and Rohner (2015). There, the decision after conquering power was whether to exterminate a share of the population of the defeated group, while here the decision is on whether to secede.
fairness permitting $i$ to propose a distribution preferable to their expected conflict payoffs from an unfair proposal and that would be acceptable to $j$. Notice that it could not pay to $i$ to ripe the possible benefits of an immediate conflict and postpone the peaceful proposal for later because, due to the stationarity of the environment, postponement would always be optimal and $i$ would always reject distributions. Hence $i$ would have made such acceptable distribution proposal when in power and the game would have never reached this branch. This observation has the following implication for SPE paths:

**Remark 2.** Any SPE path ending with a peaceful agreement on a distribution has to consist of an initial distribution proposal by the group in power $j$ which is immediately accepted by group $i$.

Remark 2 has a useful implication. Whenever a path starts with a rejection it must end either with endless conflict or with conflict followed by secession.

### 4.2 Inefficient SP paths

The group initially in opposition can influence the initial offer by the group in power by triggering conflict. But, this is a credible threat only if such one step deviation has a continuation that is itself sub-game perfect, SP. We now analyze the conditions under which such SP continuations of the rejection of the offer in the first period do exist. These paths can be used as a credible threat against any deviation by the other player. In case that there are multiple SP continuations we shall choose the one that is best for $i$.

We start by noting that this game can end after a finite number of interactions only by the opposition accepting the proposal made by the group in power or by either player seceding in case of victory. Therefore, the only possibility of the game to last indefinitely is when the two players reject each other’s proposal every period.

The question we now address is whether such an infinitely repeated sequence of conflicts can be a SP path starting with the two players playing conflict at the first iteration. Indeed, if the other player plays conflict no player would have a profitable deviation because it needs only one to provoke conflict. Notice that this would be

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9 We could add that if the one triggers conflict and the other does not, this player incurs in
the worst possible equilibrium because it involves the destruction of $D$ surplus in every period. This path consists of a sequence of strategies each rejecting the other’s proposal when in opposition and making an unacceptably unfair proposal when in power [for instance allocating zero surplus to the opposition]. This is the case in which both players permanently “punish” each other with conflict.

Accordingly with our description of the timeline of the model, at the very moment of the victory after conflict, the winner decides whether to secede or to keep the union because this choice determines the appropriation of the surplus following victory. The winner appropriates the entire remaining surplus only if they will continue in the union, irrespective of whether at the beginning of the next period they will make a distribution proposal or trigger conflict once more. When the winner decides to secede, then they simply take with them the surplus they have produced and hence make the losers to bear the full cost of conflict.

Therefore, in order to check whether permanent conflict is a SP path we need to verify whether the winner will prefer to deviate from continued conflict and opt for secession. We denote this path as path type [A]. We shall now compute the value for $i$ of being a winner and continue with conflict, $V_{cc}^i$, and compare it with the value of being a winner in conflict and deviate by choosing secession $V_{cs}^i$. We shall denote by $V_{cc}^j$ the value of being the loser in the event of conflict.

We can now compute the values to $i$ from playing endless conflict and from seceding after a victory, respectively.

\[
V_{cc}^i = \frac{S - D - A}{N_i} + P_i + \delta \left\{ \frac{N_i}{N} V_{cc}^i + \frac{N_j}{N} V_{cc}^j \right\}, \quad \text{and} \quad V_{cs}^i = 0 + \delta \left\{ \frac{N_i}{N} V_{cc}^i + \frac{N_j}{N} V_{cc}^j \right\}.
\]

Solving, we obtain

\[
V_{cc}^i = \frac{\delta N_i}{1 - \delta N_j} V_{cc}^i,
\]

and hence

\[
V_{cs}^i = \frac{1 - \delta N_j}{1 - \delta} \left[ \frac{S - D + N_i P_i - A}{N_i} \right].
\]
Let us now compute the value of being the winner and secede $V_{cs}^s$,

$$V_{cs}^s = \frac{1}{1-\delta} \left( \frac{S_i + N_i P_i - A}{N_i} \right). \quad (5)$$

Therefore, \(i\) will prefer to continue conflict rather than deviate and secede if

$$S_j - D \geq \frac{\delta N_j}{N} (S - D + N_i P_i - A). \quad (6)$$

*Mutatis mutandis* the condition for \(j\) to continue to play conflict rather than deviate and secede is:

$$S_i - D \geq \frac{\delta N_i}{N} (S - D + N_j P_j - A). \quad (7)$$

Clearly, permanent conflict is a SP path following \(i\)'s rejection of a proposal by \(j\), whenever (??) and (??) are both satisfied. These two conditions can be rewritten as

$$s \leq (1 - d) [1 - \delta(1 - n)] - \frac{\delta}{\sigma}(1 - n)(nP_i - a\sigma)$$

and

$$s \geq d - \frac{\delta}{\sigma} n [(n - 1) P_j - (1 - a - d)\sigma]$$

These expressions are constraints on the value of the share of surplus produced by the opposition \(i\), \(s\), relative to their population size, \(n\). Group \(i\) in opposition prefers conflict to secession if the share of their surplus is sufficiently small, that is, if the surplus they will grab from the defeated group, \(1 - s\), is sufficiently large. Furthermore, the larger is the population size of group \(n\) the larger is the threshold share on the surplus to make this group prefer secession. Similarly for group \(j\): the size of the surplus produced by the opposition has to be sufficiently large to make them to prefer conflict over secession.

We now characterize the set \([A]\) of parameter values for which a continuation path of type \(A\) is a SPE.

**Lemma 3.** Let player \(i\) play conflict after the first victory. Then the necessary and sufficient condition for the sequence of endless conflict be a SPE is that

$$s \leq (1 - d) [1 - \delta(1 - n)] - \frac{\delta}{\sigma}(1 - n)(nP_i - a\sigma) \quad (8)$$

and

$$s \geq d - \frac{\delta}{\sigma} n [(n - 1) P_j - (1 - a - d)\sigma] \quad (9)$$
Furthermore, \( a < d \) is the necessary and sufficient condition for there always be \( \delta \) large enough so that the set \([A]\) be empty. The precise threshold on \( \delta \) is

\[
\delta > \frac{(1-2d)\sigma}{(1-n)n(P_i + P_j) + (1-a-d)\sigma} \equiv \delta_A
\]  

(10)

We have obtained that the conditions for the most destructive path be a SPE are rather stringent. We are interested in identifying the worst SPE continuation path starting with conflict that a player can precipitate. This will give us the minimum payoff that any SPE has to grant to this player. We have obtained the conditions by which the two players will play permanent conflict and hence the conditions under which one of the players will deviate and secede after a victory. But observe that in this case we need to verify that the other player will continue to play conflict even knowing that the opponent will secede after the first victory.

Let us consider in the first place the case in which player \( i \) chooses secession after being victorious while \( j \) continues to play indefinite conflict. For player \( i \) the payoff from seceding after victory is exactly equal as we computed in (??) and this payoff should be larger than continuing conflict as in (??). Therefore, player \( i \) will trigger conflict and secede after the first victory knowing that \( j \) will always play conflict iff

\[
S_j - D \leq \frac{\delta N_j}{N}(S - D + N_i P_i - A).
\]  

(11)

Using the same notation as before, this condition can be rewritten as

\[
s > (1-d)[1-(1-n)] - \frac{\delta}{\sigma}(1-n)(nP_i - a\sigma)
\]

We now have to check the conditions under which player \( j \) would continue to play conflict even knowing that \( i \) will eventually secede. The value after a victory of continuing with conflict is

\[
V_{j}^{cc} = \frac{S - D + N_j P_j - A}{N_j} + \delta \left[ \frac{N_j}{N} V_j^{cc} + \frac{N_i}{N} \left( \frac{1}{1-\delta} \frac{S_j + N_j P_j - A}{N_j} - \frac{D}{N_j} \right) \right].
\]

Therefore,

\[
V_j^{cc} = \frac{1}{1-\delta \frac{N_j}{N}} \left[ \frac{S - D + N_j P_j - A}{N_j} + \delta \frac{N_i}{N} \frac{S_j + N_j P_j - A - (1-\delta)D}{N_j} \right].
\]

The value \( V_j^{cc} \) has to be larger than the value of opting for secession instead after the first victory. That is

\[
V_j^{cc} \geq \frac{1}{1-\delta} \frac{S_j + N_j P_j - A}{N_j}.
\]
Operating we obtain that this inequality simplifies to

\[ S_i \geq D \left( 1 + \delta \frac{N_i}{N} \right). \]  

(12)

Bringing together inequalities (??) and (??) we completely characterize the set of parameter values for which, the path starting with \( i \) triggering conflict followed by permanent conflict by \( j \) and secession by \( i \) is a SPE. We shall denote this set by \([B_i]\). Using the same simplifying notation as above we have the following result.

**Lemma 4.** Let the opposition player start by triggering conflict. Then the continuation path with \( j \) playing conflict at every iteration and \( i \) seceding after the first victory is a SPE iff the following two inequalities are satisfied:

\[ s > (1 - d) \left[ 1 - \delta (1 - n) \right] - \frac{\delta}{\sigma} (1 - n)(nP_i - a\sigma) \]  

and

\[ s > d + \delta dn \]  

(13)

(14)

Furthermore, the set \([B_i]\) is always non-empty

At \( n = 0 \), the first constraint is below the second one if and only if \( \delta > \frac{1 - 2d}{1 - a - d} \), while at \( n = 1 \) the first constraint is always above the second one since \( 1 - d > (1 + \delta)d \).

We now turn to the case in which group \( j \) seeks to secede at the first victory while the opposition group \( i \) chooses indefinite conflict. Group \( j \) prefers secession to conflict when the opposing group plays conflict whenever inequality (??) is reversed, that is, when

\[ S_i - D < \frac{\delta N_i}{N} (S - D + N_j P_j - A). \]

In our simplified notation this inequality can be written as

\[ s < d - \frac{\delta}{\sigma} n [(n - 1)P_j - (1 - a - d)\sigma] \]  

(15)

Following the same steps as before we can obtain that the condition for \( i \) to prefer continued conflict knowing that \( j \) is seeking to secede is

\[ S_j \geq D \left( 1 + \delta \frac{N_j}{N} \right), \]
that is

\[ s < [1 - (1 + \delta)d] + \delta dn. \]  \hspace{1cm} (16)

Inequalities (16) and (17) completely characterize the set \([B_j]\) of all the parameter values for which after \(i\) has rejected the starting proposal the continuation with \(i\) playing conflict and \(j\) seceding is a SPE. We now formally introduce the following result.

**Lemma 5.** Let the opposition player start by triggering conflict. Then the continuation path with \(i\) playing conflict at every iteration and \(j\) seceding after the first victory is a SPE iff the following two inequalities are satisfied:

\[ s < d + \frac{\delta}{\sigma} n [(1 - n)P_j + (1 - a - d)\sigma] \] \hspace{1cm} (17)

and

\[ s < [1 - (1 + \delta)d] + \delta dn \] \hspace{1cm} (18)

The set \([B_j]\) is always non-empty.

At \(n = 0\), the first constraint is below the second one since \(d < 1 - (1 + \delta)d\), while at \(n = 1\) the first restriction is above the second one if and only if \(\delta > \frac{1 - 2d}{1 - a - d}\).

Considering the sets \(B_i\) and \(B_j\) together, observe that our assumption that \(\delta > \delta_A\) at \(n = 0\) simplifies precisely to \(\delta > \frac{1 - 2d}{1 - a - d}\), hence we have the situation depicted in the figure:

![Figure](source)

Consider again the case in which the opposition \(i\) rejects the proposal made by \(j\) and hence triggers conflict and both players prefer secession after victory rather than continuing with the conflict strategy. By following this strategy there will be one initial conflict and whoever is the winner the group will secede.

We now obtain the parameter values for which such continuation of the initial rejection by \(i\) is SP. Specifically we shall obtain the set of parameter values for which after being victorious that player prefers to secede rather than to continue with a new conflict, knowing that the other player will secede after the first victory.
Let us consider the opposition player $i$. Once victorious, the payoff from secession for player $i$ is

$$V_{is} = \frac{S_i + P_iN_i - A}{(1 - \delta)N_i}.$$  

The payoff from triggering a new conflict $V_{ic}$ is

$$V_{ic} = \frac{S - D + P_iN_i - A}{N_i} + \delta \left[ \frac{N_i}{N}V_{is} + \frac{N_j}{N} \left( \frac{S_i + P_iN_i - A}{(1 - \delta)N_i} - \frac{D}{N_i} \right) \right].$$

Simplifying one can easily obtain that $V_{is} \geq V_{ic}$ iff

$$S_j \leq \left( 1 + \delta \frac{N_j}{N} \right) D$$

Using our simplified notation, this inequality can be rewritten as

$$s \geq [1 - (1 + \delta)d] + \delta dn.$$  

Doing the same calculations for player $j$ one can easily obtain that $V_{js} \geq V_{jc}$ iff

$$S_i \leq \left( 1 + \delta \frac{N_i}{N} \right) D$$

Using our simplified notation, this inequality can be rewritten as

$$s \leq d + \delta dn.$$  

Inequalities (eq:ssi) and (eq:ssj) completely characterize the set $[C]$ of all the parameter values for which after $i$ has rejected the starting proposal the SPE continuation is that whoever is the winner of this first period conflict chooses to secede.

We now formally introduce the following result.

**Lemma 6.** Let the opposition player start by triggering conflict. Then the continuation path with whoever is the winner decides to secede is a SPE iff the following two inequalities are satisfied:

$$s \geq [1 - (1 + \delta)d] + \delta dn.$$  

and

$$s \leq d + \delta dn.$$  

The set $[C]$ is empty whenever $d < \frac{1}{3}$. 


Combining the results in previous Lemmas we obtain the characterization of the worst SP paths. These paths tell us of the minimum payoff that each player can guarantee for themselves.

For the sake of clarity and empirical relevance we shall focus on the case in which the loss of conflict relative to the surplus is not large and the time discount factor is high. Specifically, for the rest of the paper we shall assume that \( d < 1/3 \) and that \( a < d \) so that there exists \( \delta \geq \delta_A \) for all \( n \).

We know that if \( d < 1/3 \) and \( \delta \geq \delta_A \) there is no combination of parameter values for which either both playing conflict indefinitely or both seceding at the first iteration can be SP equilibria. Hence, these threats are not credible and cannot be used to condition the first proposal by group \( j \) initially in power.

**Proposition 7.** Let us assume that \( 0 < a < d < 1/3 \) and that \( \delta_A < \delta < 1 \). Then:

- Under the above assumptions the sets \([A]\) and \([C]\) are empty.
- The union of the sets \([B_i]\) and \([B_j]\) contains all the pairs \( n, s \) with \( n, s \in [0,1] \). The intersection of the two sets is non empty.

The sets \([B_i]\) and \([B_j]\) and their intersection are depicted in Figure ???. At the intersection we have two possible SP continuations in which one secedes as soon as possible and the other fights indefinitely until the game ends with the secession by the other. Let us examine whether there is a reasonable criterion permitting us to chose one of the two potential SP continuations.

In order to have an intuition for the sets \([B_i]\) and \([B_j]\) let us imagine first the diagonal line going from the origin to the point \((1,1)\). In all points above this line player \( i \) will produce a share of the surplus above their population size and hence will display a higher “productivity” than the group in power. The reverse happens below this line. Observe that most of the set \([B_i]\) corresponds to parameter values in which \( i \) is more productive than \( j \), while \([B_j]\) is below the diagonal except for very high values of \( n \).

Let us take a fix \( n \) and starting from a low productivity we increase \( s \). Since we are keeping the population size constant, the per capita cost of independence and the win probabilities remain the same. Increasing \( s \) simply increases the per capita surplus obtained under secession. If \( s \) is low the opposition has more to gain

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10 On the cost of civil wars see the summary of the results in Ray and Esteban (2017).
with conflict because this gives this group access to the large aggregate surplus, with probability \( n \). The larger is \( s \) the lower is the extra payoff from conflict and the higher the payoff from secession. Above a threshold level group \( i \) will prefer to secede rather than to continue with conflict.

If we now do the complementary exercise, fixing \( s \) and increasing \( n \), the intuition is a bit more complex. An increase in \( n \) makes the win probability higher but the per capita conquered surplus smaller. If \( n \) and \( s \) are very low, even if \( i \) is the most productive, the per capita gain obtained with a victory in conflict is very large and the cost of keeping an independent state is very large too. Hence, group \( i \) will prefer indefinite conflict and postpone the secession of \( j \) as much as possible. If \( s \) is very large and we start with a small \( n \) the per capita surplus after secession will be very large and definitely preferable than the uncertain (small) gain in conflict. As \( n \) becomes larger the probability of victory in conflict goes up but the surplus produced by the opposition will continue to be small and hence group \( i \) will continue to prefer secession over indefinite conflict. For intermediate values of \( s \), the size of the captured surplus in case of victory is larger and thus makes conflict more attractive. As we consider larger \( n \) the win probability will be sufficiently high and group \( i \) will eventually start preferring conflict and delaying the secession of \( j \) as much as possible.

Let us briefly pause and examine in greater detail the strategic behavior when the parameters belong to this intersection. At the first period \( i \) rejects \( j \)'s proposal and we are interested in the strategies following such a decision bearing in mind that the payoff associated to the SP continuation path will be the minimum \( j \) has to concede in order to avoid rejection.

At this overlap set, player \( i \) has two options: either secede after the first victory [with best reply by \( j \) consisting of permanent conflict] or initiate a sequence of conflicts until \( j \) wins and secedes [with this being the best reply by \( j \)]. We assume that the relevant SP is the one giving the highest payoff to \( i \). This is consistent with using the threat of rejection as a lower bound for the initial proposal by \( j \).

Notice that we derived our previous results using the value of being victorious in a conflict followed by the defined sequence of strategies. From now on we shall compute the value \( V \) of playing rejection in the first stage followed by the SP continuations.

We start with the value to \( i \) of rejecting the first proposal followed by a SP path of type \([B_i] \), which we denote \( V_{iB_i}^B \). Since we are on a path type \([B_i] \), if in any
iteration player \( i \) win they secede and the game ends, and if the lose they get a period pay of zero and enter the new period with \( j \) in power playing conflict. Hence we can easily compute that

\[
V_{i}^{B_i} = \frac{N_i S_i + N_i P_i - A}{N(1 - \delta)N_i} + \frac{N_j}{N} \delta V_{i}^{B_i}.
\]

Solving for \( V_{i}^{B_i} \) and using our compact notation we obtain

\[
V_{i}^{B_i} = \frac{n P_i + \sigma(s - a)}{(1 - \delta)[1 - \delta(1 - n)]}
\] (23)

Let us now compute the value to \( i \) of rejection followed by a path of type \([B_j]\). In this case, whenever \( i \) win they capture the entire surplus (minus destruction \( D \)) and trigger a new conflict in the next iteration. When \( j \) win they secede, \( i \) also becomes an independent country (bearing the full cost of conflict) and the game ends.

The value \( V_{i}^{B_j} \) is

\[
V_{i}^{B_j} = \frac{N_i}{N} \left[ \frac{S + N_i P_i - A - D}{N_i} + \delta V_{i}^{B_j} \right] + \frac{N_j}{N} \left[ \frac{S_i + N_i P_i - A - D}{N_i} - \frac{D}{N_i} \right].
\]

Solving now for \( V_{i}^{B_j} \) we obtain

\[
V_{i}^{B_j} = \frac{n P_i - a\sigma}{n(1 - \delta)} + \frac{\sigma(n - d)}{n(1 - \delta n)} + \frac{s\sigma(1 - n)}{(1 - \delta)n(1 - \delta n)}
\] (24)

Solving for \( V_{i}^{B_i} = V_{i}^{B_j} \) we obtain the threshold \( s = \psi(n) \):

\[
s = \frac{[1 - \delta(1 - n)](n - d) + (1 - n)(1 - \delta n)\left(\frac{n}{\sigma} P_i - a\right)}{2n - 1}
\]

As it turns out the implicit function of \( s \) in terms of \( n \) is discontinuous at \( n = 1/2 \) and it is easy to show that as \( n \to 1/2 \) from below, \( s \to -\infty \) and the opposite from above. We can also compute that \( s(n = 0) = a + (1 - \delta)d < d \) given our previous assumptions (i.e., \( \delta > \frac{1 - 2d}{1 - d} \)) and \( s(n = 1) = 1 - d \). Therefore, it holds that \( V_{i}^{B_i} < V_{i}^{B_j} \) everywhere in \( B_i \cap B_j \).

We now proceed to characterize the SPE of the game. We simply have to identify the initial proposals that payoff dominate the payoff under the relevant path following rejection. In (??) and (??) we have computed the payoffs to player \( i \) from rejection of the initial proposal.
We now compute the equivalent payoffs for $j$, $V_{j}^{B_i}$ and $V_{j}^{B_j}$. Following the same steps as above we obtain

$$V_{j}^{B_i} = \frac{P_j}{1 - \delta} + \frac{\sigma}{1 - n} \left[ \frac{1 - d}{1 - \delta(1 - n)} - \frac{a}{1 - \delta} + \frac{ns - \delta n}{(1 - \delta)[1 - \delta(1 - n)]} \right], \text{ and } (25)$$

$$V_{j}^{B_j} = \frac{P_j(1 - n) + (1 - a - s)\sigma}{(1 - \delta)(1 - \delta n)} \quad (26)$$

### 4.3 Subgame Perfect Equilibria

After having obtained the payoffs from rejecting the initial proposal we can now examine the full path and characterize the SPE.

The potential SPE can be of the following types: $[U]$ agreement on a distribution of the surplus within the union, $[S]$ agreement on secession, $[B]$ conflict followed by secession. We have excluded permanent conflict because we have already obtained that there are no parameter values for which the two players indefinitely playing conflict is a SPE. We have also excluded conflict followed by an agreement on a distribution within the union. From Remark ?? we know that a SPE path ending with a peaceful distribution within the union cannot result from previous conflict. Finally, notice that SPE of type $[B]$ exist if and only if there are no SPE of type $[U]$ or $[S]$. Therefore we just need to compare the payoffs associated with the different outcomes.

We start by computing the value of keeping the union. For individuals of group $i$, $V_{i}^{U}$ is:

$$V_{i}^{U} = \lambda \frac{N_i S - A}{N (1 - \delta)N_i} = \lambda \frac{S - A}{(1 - \delta)N} = \lambda \frac{\sigma}{1 - \delta} (1 - a), \quad (27)$$

where $\lambda > 0$ captures the degree of fairness of the allocation of the surplus. Clearly, $\lambda = 1$ corresponds to full equality in the distribution of the monetary surplus.

For individuals of the group in power $j$ the value of union $V_{j}^{U}$ is

$$V_{j}^{U} = \left( 1 - \lambda \frac{N_i}{N} \right) \frac{S - A + N_j P_j}{(1 - \delta)N_j} = \frac{(1 - \lambda n)[(1 - n)P_j + (1 - a)\sigma]}{(1 - \delta)(1 - n)} \quad (28)$$

The value of a peaceful secession $V_{i}^{S}$ and $V_{j}^{S}$ is

$$V_{i}^{S} = \frac{S_i + N_i P_i - A}{(1 - \delta)N_i} = \frac{n P_i + \sigma(s - a)}{n(1 - \delta)} \quad \text{and} \quad (29)$$

$$V_{j}^{S} = \frac{S_j + N_j P_j - A}{(1 - \delta)N_j} = \frac{P_j(1 - n) + (1 - a - s)\sigma}{(1 - \delta)(1 - n)} \quad (30)$$
Let us start by comparing the value of a proposal of peaceful secession $V_i^S$ with either $V_i^{B_i}$ or $V_i^{B_j}$. Using (??), (??), and (??), we can immediately obtain that $V_i^S > V_i^{B_i}$ for all the parameter values. Hence a necessary condition for the rejection of secession proposal is that the parameters belong to the set $[B_j]$, that is, it has to be player $j$ the one that will secede after the first victory. In fact, it is easy to obtain that rejection can only happen if in addition the parameters satisfy that $s < 1 - \frac{d}{n}$.

We shall denote by $[R]$ the set of parameters satisfying the inequality $s < 1 - \frac{d}{n}$.

**Lemma 8.** Let $j$ start by proposing secession. Then the peaceful secession will be rejected by $i$ if and only if the parameter values belong to the set $[R]$, otherwise the peaceful secession will be accepted by $i$. The set $[R]$ is a subset of $[B_j]$.

The latter statement follows from the strict increasingness and concavity of $s(n) = 1 - \frac{d}{n}$ with respect to $n$ and the fact that $s(1) = 1 - d$. Observe that the constraint will overlap with the intersection between $B_i$ and $B_j$, which as we saw is part of $B_j$ (it is the path $B_j$ giving the highest payoff to $i$ in the intersection). $[R]$ does not overlap with $[B_i]$ above this intersection. We denote by $[K]$ the complement of set $[R]$.

In view of this result player $j$ knows that for all the parameter values in $[K]$ they can obtain for sure the maximum between secession and the associated conflict path. Therefore, this will be the minimum payoff that player $j$ has to obtain from a distribution within the union. If the parameter values belong to $[R]$, so that $i$ would not accept a secession, a distribution within the union is a SPE if the two players obtain a payoff at least as high as the one they obtain by following the conflict path of type $B_j$. Otherwise, the SPE starts with conflict followed by the secession of $j$.

We start with this latter case with the parameters in $[R]$, in which $i$ would reject a peaceful secession. We check whether $j$ can find a distribution within the union that for the two players it is weakly preferable to conflict of type $B_j$. If such a distribution exists, this will be the unique SPE. If no such distribution exists, the sole SPE consists of permanent conflict by player $i$ and secession by $j$ with their first victory. We now obtain the degree of fairness that would make each player indifferent.
to conflict and then verify whether they are mutually compatible. To this effect, denote by $\lambda^B_j$ and $\lambda^B_i$ the $\lambda$s of indifference with respect to the conflict payoff for $i$ and $j$ respectively, $V^B_j$ and $V^B_i$. Then, a distribution within the union can be a SPE iff $\lambda^B_j \geq \lambda^B_i$, that is, the fairness required to make $i$ accept the distribution is less than the one necessary make $j$ prefer that distribution over conflict along the path $B_j$. By the same argument, if the parameters belong to $[R]$ and $\lambda^B_j < \lambda^B_i$, the unique SPE consists of conflict after the initial proposal and $j$ seceding.

Using (??) and (??), the $\lambda^B_j$ that equates the two payoffs is

$$\lambda^B_j = \frac{1}{n(1-\delta n)} \left[ \frac{\sigma(1-n)s}{P_j(1-n) + \sigma(1-a)} + (1-\delta)n \right]$$

(31)

Repeating the same exercise with player $i$ we obtain $\lambda^B_i$ to be

$$\lambda^B_i = \frac{1}{(1-a)n} \left[ \frac{nP_i}{\sigma} - a + \frac{s(1-n) + (1-\delta)(n-d)}{1-\delta n} \right].$$

(32)

For the distribution to be feasible it must hold that $\lambda^B_i < \frac{1}{n}$. Let us now analyze the case in which player $i$ would accept a secession if proposed. More specifically, let us start restricting to parameters in the set $[B_i] \subset [K]$. We are interested in checking whether $j$ would indeed propose a peaceful secession knowing that $i$ would accept it.

We start by verifying whether $j$ would prefer conflict along the path $B_i$ rather than propose a peaceful secession that would be accepted. From (??) and (??), with some manipulation, we can easily obtain that

$$V^S_j - V^B_j = \sigma \frac{d - (1-n)s}{(1-n)[1-\delta(1-n)]}$$

which is positive if and only if $s < \frac{d}{(1-n)}$. Suppose this is true and thus $j$ prefers peaceful secession to conflict. We still need to verify that there is no peaceful distribution acceptable to $i$ and that $j$ prefers to a peaceful secession. We have that a distribution within the union is a SPE if and only if $\lambda^S_j \geq \lambda^B_i$. Otherwise, the SPE consists in $j$ proposing a secession and $i$ accepting it. Using (??), (??), (??), and (??) we can easily obtain:

$$\lambda^B_i = \frac{\sigma(s-a) + nP_i}{\sigma(1-a)[1-\delta(1-n)]}$$

(33)
and
\[ \lambda^S_j = \frac{s\sigma}{n[(1-n)P_j + (1-a)\sigma]} \] (34)

Suppose now that \( s > \frac{d}{1-n} \) and so \( j \) prefers conflict to peaceful secession. We can easily obtain that
\[ \lambda^{B_i}_j = \frac{\sigma[ns + (1-\delta)d]}{n[1-\delta(1-n)][(1-n)P_j + (1-a)\sigma]} \] (35)

Finally, suppose we are in \([K \cap B_j]\) where \( i \) would accept a secession proposed by \( j \), but we are in the part of \([K]\) that coincides with \( B_j \). Within this set we know that a peaceful secession proposal would be accepted. Notice first that \( j \) could not trigger conflict following a path \( B_j \) because they would end up deciding to secede rather that to continue conflict. Hence this must also be true in the first iteration and \( j \) would prefer peaceful secession to conflict followed by a path \( B_j \), knowing that \( i \) would accept a peaceful secession proposal.

Now it only remain to be checked whether \( j \) would propose a peaceful secession or a distribution within the union. Following the same steps as before we shall obtain the degree of fairness that make \( i \) indifferent to a path \( B_j \), \( \lambda^{B_i}_j \), and \( j \) indifferent to secession, \( \lambda^S_j \). Once again a distribution within the union will be a SPE iff \( \lambda^S_j > \lambda^{B_i}_j \). Otherwise, \( j \) proposes a secession and \( i \) accepts it. This completes our characterization of SPE.

Summing up, our characterization strategy has been as follows. We have first partitioned the set of feasible parameter values into the set \([R]\) and its complement \([K]\). The set \([R]\) is the one for which player \( i \) would reject a peaceful secession if proposed by \( j \). We have obtained the type of SPE that can arise under this set of parameter values. We have also partitioned the set of parameter values into the set \( B_i \) and its complement \( B_j \). The set \( B_i \) consists of all parameter values for which the SP deviation is conflict followed by \( i \) seceding after the first victory. In the complement set \( B_j \) it is group \( j \) the one that secedes after the first victory. We have then given separate examination of the intersection of set \([K]\) with sets \([B_i]\) and \([B_j]\). For \([K \cap B_i]\) we have also split it into two subsets depending on whether \( s \geq \frac{d}{1-n} \).

We provide our comprehensive characterization of the SPE in the following Proposition.
Proposition 9. For every array of feasible parameter values there is a unique SPE. The types of SPE are as follows:

- **Peaceful Union**: \( j \) proposes a distribution with \( \lambda_j \) fairness and \( i \) accepts it when:
  
  \[- (n, s) \in R \text{ and } \lambda_i^{B_j} \leq \lambda_j^{B_j} \text{ and } \lambda_i^{B_j} \leq \frac{1}{n}\]
  
  \[- (n, s) \in K \cap B_i, s \leq \frac{d}{1-n} \text{ and } \lambda_i^{B_i} \leq \lambda_j^{S} \text{ and } \lambda_i^{B_i} \leq \frac{1}{n}\]
  
  \[- (n, s) \in K \cap B_i, s > \frac{d}{1-n} \text{ and } \lambda_i^{B_i} \leq \lambda_j^{B_i} \text{ and } \lambda_i^{B_i} \leq \frac{1}{n}\]
  
  \[- (n, s) \in K \cap B_j \text{ and } \lambda_i^{B_j} \leq \lambda_j^{S} \text{ and } \lambda_i^{B_j} \leq \frac{1}{n}\]

- **Peaceful Secession**: \( j \) proposes secession and \( i \) accepts it when:
  
  \[- (n, s) \in K \cap B_i, s \leq \frac{d}{1-n} \text{ and } \lambda_j^{S} < \lambda_i^{B_i}\]
  
  \[- (n, s) \in K \cap B_j \text{ and } \lambda_j^{S} < \lambda_i^{B_j}\]

- **Conflict Secession**: \( j \)'s proposal is rejected and either \( i \) or \( j \) secedes after the first victory when:
  
  \[- (n, s) \in R \text{ and } \lambda_i^{B_i} > \lambda_j^{B_i} \text{ or } \lambda_i^{B_j} > \frac{1}{n}\]
  
  \[- (n, s) \in K \cap B_i, s > \frac{d}{1-n}, \text{ and } \lambda_i^{B_i} > \lambda_j^{B_i} \text{ or } \lambda_i^{B_i} > \frac{1}{n}\]

5 Comparing Equilibria

We shall now compare equilibria in greater detail in order to explicitly relate the exogenous parameters to the different SPE. One of the specificities of our approach is that we have modelled the decision to secede or not in an inter-temporal model. How large is the time discount factor \( \delta \) will tell us the importance of future in secession decisions. Hence, we will examine the two extreme cases with \( \delta = 0 \) and \( \delta \to 1 \). We shall also discuss how the set of equilibria changes for intermediate values of the time discount factor.
5.1 Equilibria with far fetched players

We shall start by concentrating on the case of arbitrarily large time discount factor. Specifically we shall look at the limit case when $\delta \to 1$.

Let us start by computing the limit values for the key $\lambda$ permitting to identify the different equilibria.

\[ \lambda_j \equiv \lambda^B_j = \lambda^S_j = \frac{\sigma s}{n \left[ P_j (1 - n) + \sigma (1 - a) \right]} \]  \hspace{1cm} (36)

and

\[ \lambda_i \equiv \lambda^B_i = \lambda^S_i = \frac{n P_i + \sigma (s - a)}{n \sigma (1 - a)} \] \hspace{1cm} (37)

Since we are focusing on the case in which groups assign the maximum value to future payoffs, for the case of threats consisting of conflict until one of the two secedes, it becomes undistinguishable which of the two provokes secession after the first victory. After all, in both cases the two players will have the secession payoff forever. It also becomes undistinguishable from the case in which secession starts in the first period.

We have shown that the characterization of the different equilibria depends on the relative value of the degrees of fairness that make each of the groups indifferent to the pairs of alternatives considered. We have now just one critical fairness level for each group.

We can easily obtain

\textbf{Lemma 10.} The degrees of fairness \((\lambda_i, \lambda_j)\) satisfy that

\[ \lambda_i < \lambda_j \iff s < s^U \equiv \frac{[(1 - n) \frac{P_i}{\sigma} + (1 - a)][a - n \frac{P_i}{\sigma}]}{(1 - n) \frac{P_j}{\sigma}} \] \hspace{1cm} (38)

Further, the feasibility of transfers implies that

\[ \lambda_i \leq \frac{1}{n} \iff s \leq s^\lambda \equiv 1 - \frac{n P_i}{\sigma} \] \hspace{1cm} (39)

Using this information we can characterize the SPE in terms of the parameter values. Indeed, we know that unless the group in power prefers secession, $\lambda_j \geq \lambda_i$ is a necessary and sufficient condition for a peaceful union be a SPE in which the group in power will offer $\lambda_i$ to the opposition. Here we give a complete characterization of the SPE.
Proposition 11. We have the following SPE:

- **Peaceful Union** iff \( s \leq s^U \)

- **Peaceful Secession** iff \( s > s^U \), and either \( s < \frac{d}{1-n} \) and \( (n, s) \in B_i \) or \( s > 1 - \frac{d}{n} \) and \( (n, s) \in B_j \),

- **Conflict Secession** iff \( s > s^U \), and either \( s \geq \frac{d}{1-n} \) and \( (n, s) \in B_i \) or \( s \leq 1 - \frac{d}{n} \) and \( (n, s) \in B_j \).

Inequality (??) tells us whether the degree of fairness that \( j \) has to display to make the opposition accept union rather than conflict is not as high as the maximum \( j \) would tolerate before preferring any other option. Therefore, the necessary and sufficient condition for there to exist a level of fairness that makes union a SPE is that the pair \((n, s)\) satisfies that \( s \leq s^U(n) \). Let us examine the properties of the \( s^U(n) \) function. We start by noting that \( s^U \) is strictly decreasing and concave function. Since we are assuming that \( \min\{P_i, P_j\} > a \), we have that \( s^U(0) = a + (1-a)a\frac{P_j}{P_i} < 1 \) and \( \lim_{n \to 1} s^U = -\infty \) when \( \frac{P_i}{P_j} > a \). Therefore the larger \( P_i \) and \( P_j \) relative to the economic surplus \( \sigma \), the smaller is the set of parameter values for which a peaceful union is a SPE. Finally, an increase in the cost of running an independent state \( a \) enlarges the set of parameter values for which union is a SPE.

Summing up the analysis of the SPE with peaceful union, we find that the opposition should not be too powerful either in population size or in the share of surplus they produce. The size of the threshold values for the population and economic power yielding union as a SPE increases with \( a \) and decreases with the size of \( P_i \) and \( P_j \). Note that these threshold values are independent of \( d \).

When a peaceful union is not possible we have that secession is a SPE, either by a peaceful agreement or after a preceding conflict.

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11 Note that because of Assumption ?? we have that \( s^U \leq s^\lambda \) for all \( n \) and hence all the associated SPE equilibrium \( \lambda \) are feasible. Since we want the Assumption to hold for all \( n \), this amounts to effectively assume that \( \min\{P_i, P_j\} \geq \frac{a \sigma}{N} = a \sigma \).

Derivation: Assumption ?? establishes that \( N_iP_i + N_jP_j > A \). Dividing both sides and with some manipulation we obtain \( nP_i + (1-n)P_j > a \sigma \). Taking the second term of the sum to the other side of the inequality and multiplying both sides by \( \frac{1-a}{(1-n)\frac{P_j}{P_i}} \) and rearranging we finally obtain that

\[
1 - \frac{P_i}{\sigma} n > \left( 1 + \frac{1-a}{(1-n)\frac{P_j}{P_i}} \right) (a - \frac{P_i}{\sigma} n).
\]

Hence \( s^\lambda > s^U \).
There are two areas of parameter values in which the sole SPE entails conflict followed by secession. One area corresponds to the case in which the opposition produces a high share of the surplus and they are not very large in population. They are the most “productive” group. Secession is profitable to the opposition group because they will control a large surplus. The group in power find unacceptable the size of the transfer to the opposition necessary to make them to accept the union. Since the group in power is the largest, and hence have a high probability of winning in conflict, they prefer to postpone secession as much as possible by triggering conflict until the opposition wins and secedes.

The second area consists of the SPE paths in which the opposition triggers conflict in every period until group \( j \) wins and secedes. Because the opposition is large in population size, they have an advantage in conflict, supplemented with the grabbing of a large surplus. To see this clearly, think of the group in power being a tiny minority producing almost the entire surplus. It is immediate that it pays the super-majoritarian opposition to trigger conflict indefinitely with the near certainty of victory. In view of this, the group in power finds it optimal to separate from the large and poor group.

In both cases, not surprisingly, an increase in the cost of conflict reduces the set of parameter values for which conflict followed by secession is a SPE. Also aligned with intuition, the value of the public good payoffs relative to the economic surplus is not relevant here. The reason is that we are comparing two paths both yielding secession and that hence differ on whether to reach it either by agreement or after conflict. And this essentially depends on \( d \).

Peaceful secession is a SPE when the opposition group is large and the differences in productivity with the other group are small. In this case, the opposition has a significant chance of winning a conflict. Since the productivities of the two groups are not very different, the main advantage of conquering power or of seceding is the possibility of producing the most preferred public good. In order to keep the union the group in power would have to compensate economically the opposition for giving up on their preferred public good. Since the productivity of the two is similar, and given that the the opposition is “too” large to be compensated from the perspective of the group in power, they prefer both to bear the cost of a separate state and enjoy the public good most preferred by each group.

The next Figure depicts the different equilibria on the \((n, s)\) 1 × 1 square for a
specification of parameters consistent with our assumptions. Notice that for the set $A$ of permanent conflict to be empty (i.e., $\delta > \delta_A$), we need that $a < d$.

Figure

In sum, the model predicts that the union will stay when the group out of power is not too mighty either in population size or in economic surplus. The critical level of might depends on the cost of setting up a state and on the importance on the non-economic gains relative to the monetary gains. Else we shall have secession.

Secessions can be mutually agreed or resulting from costly conflict. When the opposition is powerful but differences in productivity are moderate we shall see secession by agreement. Conflict will take place only when there is a substantial imbalance between population size and share of surplus, that is when there are important differences in “productivity”.

5.2 Equilibria with short-sighted players

In the previous section we have evaluated the equilibrium conditions at $\delta = 1$, hence giving maximum weight to the future payoffs relative to present benefits and costs. In order to have an sense of the role of future, and hence, of the modelling of this problem as a repeated game rather than a one-shot game, we shall now perform the same exercise evaluating the conditions at $\delta = 0$. In contrast, now we shall give no weight to the future.

Fundamental aspects of the model change when only the present costs and benefits count. Challenging a proposal leads to conflict with a value that solely depends on the one period cost and potential benefits from grabbing surplus. And clearly, as long as $D < \min S_j, S_i$, the winner of this conflict will take the power of the entire country and expropriate the entire surplus, rather than just keep their own surplus and secede. How the game continues is immaterial as seen at the beginning of the game.

In the previous sections we have obtained that the type of SPE depends on the payoff that the opposition can grant to herself by rejecting the proposal and triggering conflict. We obtained four types of threats that we denoted by $C, B_i, B_j$ and $A$. Type $C$ was the case in which group $i$ challenges the proposal and whoever wins the conflict chooses secession. Types $B_i$ and $B_j$ correspond to the opposition
initiating conflict and then only $i$ or only $j$ willing to secede after victory, while the other party prefers to continue conflict. Finally, type $A$ consist of the cases in which both players prefer to fight indefinitely.

As we have informally argued above, we cannot have threats of type either $B$ or $C$, because in case of victory players will prefer to retain the power over the full union and capture the entire surplus. If players are short-sighted, they will never choose to secede after a conflict. On the other hand, we have already seen that the set $A$ of threats was empty for a time discount factor sufficiently high. Therefore, what happens is that this is the only non-empty set when $\delta = 0$. Indeed, as we vary the weight of the future we are also changing the kind of threats that are SPE deviations and hence the kind of SPE that we will have at the beginning of the game.

The game is a Stackelberg game: at equilibrium the group in power choses the best proposal, taking into account the reply by the group in opposition to each alternative proposal.

We shall now proceed to characterize the equilibria of the static game. To this effect, we shall examine first which is the best reply by the opposition to the different potential proposals by the group in power. Then, knowing this, we shall obtain which are the equilibrium proposals made by the group in power.

5.2.1 Best Replies by the Opposition

The choices by the opposition are either to trigger conflict or to accept the proposal if it yields at least as much utility as conflict.

We start by underlining that the payoff from a distribution of the surplus within the union depends on the degree of fairness chosen by the group in power. Since the payoff from conflict is given by the parameter values, there is always a level of fairness which we denote by $\lambda_i^A$— for which the opposition $i$ weakly prefers the union over conflict (of type $A$). However, the transfer associated with $\lambda_i^A$ might be unfeasible or unacceptable to the group $j$ in power. We shall examine this issue when we discuss the choices by the group in power.

Another scenario consists of the case in which the group in power triggers conflict. In that case there is nothing player $i$ can do, but to play conflict as well.

Finally, let us examine the conditions under which $i$ will accept or reject a
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peaceful secession proposal. In view of (??) and (??) player $i$ accepts a secession proposal iff

$$\frac{S_i + N_i P_i - A}{N} \geq \frac{S + N_i P_i - A - D}{N}.$$ 

Using our normalized notation, the condition can be rewritten as follows

**Lemma 12.** Player $i$ accepts secession if proposed by $j$ iff

$$s \geq \zeta(n) = a + \left(1 - a - d - \frac{P_i}{\sigma}\right)n + \frac{P_i}{\sigma}n^2.$$ (40)

In the $[0, 1] \times [0, 1]$ space with $n$ horizontal and $s$ vertical, the indifference line, $s = \zeta(n)$, is convex and starts at $\zeta(0) = a$ and ends at $\zeta(1) = 1 - d$. For all the points above this $\zeta(n)$ curve player $i$ will accept secession if they were proposed and below that curve player $i$ would reject a proposed secession and initiate conflict.

Figure: Accepted secession

Summing up on the choices by the opposition, we have the following points: (i) conflict is responded with conflict; (ii) proposed secession is responded with acceptance if $s \geq \zeta(n)$ and with conflict otherwise; and finally (iii) the union will be accepted provided $\lambda \geq \lambda_i^A$, otherwise it will be rejected.

### 5.2.2 Proposals by the Group in Power

We now examine the preferred proposals by the group in power, knowing what will be the reply by the opposition. We shall examine the cases in which (i) the group in power proposes a fair distribution within the union because this is preferred to proposing secession or conflict and (ii) the group in power proposes secession because it is preferable to the other alternatives. The complement to this analysis is when conflict is the preferred option.

We first check when the group in power will prefer union to conflict and to secession. As before, we compute the level of fairness in a union necessary to make the group in power $j$ indifferent to secession $\lambda_j^S$ and to conflict $\lambda_j^A$. Then union is preferable to either secession or conflict whenever $\lambda_i^A < \lambda_j^S$ and $\lambda_i^A < \lambda_j^A$, respectively.

A union proposal will be accepted by the opposition provided the degree of implicit fairness is not less than $\lambda_i^A$. Using (??) and (??), we can easily compute
the value of \( \lambda_i^A \) that makes \( i \) indifferent between accepting or rejecting the sharing of the surplus within the union as

\[
\lambda_i^A = 1 + \frac{N_i P_i - D}{S - A} = \frac{1 - a - d + \frac{P_i}{1-a} n}{1-a}.
\]  

(41)

Since \( a + d < 1 \) by Assumption ??, it follows that \( \lambda_i^u > 0 \) for all parameter values satisfying this Assumption.

Let us now check for the constraint that \( \lambda_i^A \leq \frac{1}{n} \). Define \( \kappa(n) = \frac{1}{n} - \lambda_i^A(n) = \frac{1}{n} - \frac{1-a-d+n}{1-a} \). This is a *decreasing* function of \( n \). Hence, if we implicitly define \( n^c \) as the solution to \( \kappa(n^c) = 0 \), we shall have that \( \lambda_i^A \) will be feasible for all \( n \leq n^c \).

Note now that \( \kappa(1) = \frac{d-P_i}{1-a} \). Therefore, whenever \( d \geq \frac{P_i}{\sigma} \), that is \( D > P_i N \), we shall have that \( n^c \geq 1 \) and hence this constraint will not be binding. Fairness \( \lambda_i^A \) will be feasible for all \( n \in (0, 1) \). When \( d < \frac{P_i}{\sigma} \) we have that \( n^c < 1 \) and hence for all \( n \in (n^c, 1) \) the opposition will not be offered a fair distribution within the union: even surrendering the entire surplus this would not be enough to appease the opposition.

Solving \( \kappa(n^c) = 0 \) we obtain

\[
n^c \equiv \frac{\sigma}{2P_i} \left[ - (1 - a - d) + \sqrt{(1-a-d)^2 + 4(1-a)\frac{P_i}{\sigma}} \right].
\]  

(42)

Intuitively, if the cost of conflict is very low relative to the gain in the public good and the win probability of the opposition is very high –larger than \( n^c \)– the group in power will be unable to buy off the opposition and hence will have to choose conflict.

We can now compare the payoff for \( j \) of conflict and secession.

We start with a fair distribution of the surplus within the union be preferred to conflict, that is \( \lambda_i^A \leq \lambda_j^A \).

Using (??) and (??) we can easily obtain \( \lambda_j^A \)

\[
\lambda_j^A = 1 + \frac{1-n}{n} \frac{d}{1-a} + \frac{1-n}{(1-a)\sigma} P_j.
\]  

(43)

Comparing \( \lambda_j^A \) with \( \lambda_i^A \) it is immediate that the necessary and sufficient condition for \( \lambda_i^A \leq \lambda_j^A \) is that

\[
\psi(n) = \sigma d + nP_j - (P - i + P_j)n^2 \geq 0.
\]

Let \( n^{cc} \) be the positive solution to \( \psi(n^{cc}) = 0 \). Then
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\[ n^{cc} = \frac{P_j}{2(P_i + P_j)} + \sqrt{\left(\frac{P_j}{2(P_i + P_j)}\right)^2 + \frac{P_j}{P_i + P_j} \frac{D}{P_j} N}. \]  (44)

It can be readily obtained that \( \psi(n) \geq 0 \) as \( n \leq n^{cc} \). Furthermore, \( n^{cc} \geq 1 \) as \( d \geq \frac{P_i}{\sigma} \).

Putting together the previous two observations we have the following result.

**Lemma 13.** Union with fairness \( \lambda_i^A \) is preferred to conflict by the group in power \( j \)

- when \( d \geq \frac{P_i}{\sigma} \) for all consistent parameter values,
- when \( d < \frac{P_i}{\sigma} \) for \( n \leq \min\{n^c, n^{cc}\}\)\(^{12}\) otherwise, group \( j \) prefers conflict to union.

Let us stress that the group in power \( j \) will prefer conflict to the most favorable fairness in a union only when the cost of conflict is small relative to the benefits of the public good, \( d < \frac{P_i}{\sigma} \), and the population size of the opposition is large enough, \( n > \min\{n^c, n^{cc}\} \). The intuition is simple. If the cost of conflict is large relative to the benefits of the independent public good, the opposition will be ready to accept moderate appeasing transfers. As the cost becomes lower, large opposition groups with high win probability will find conflict so profitable that there is no acceptable transfer capable to appease them.

We now examine the conditions under which \( j \) prefers a fair distribution within the union over peaceful secession, that is, \( \lambda_i^A \leq \lambda_j^S \). Using (??) and (??) we obtain the \( \lambda_j^S \) that makes group \( j \) indifferent between the two options:

\[ \lambda_j^S = \frac{s}{(1 - a)n}. \]  (45)

Comparing (??) with (??) we obtain the condition for group \( j \) prefer union to secession.

**Lemma 14.** Player \( j \) prefers a union with fairness \( \lambda_i^A \) to secession if and only if

\[ s \geq \vartheta(n) = (1 - a - d)n + \frac{P_j}{\sigma} n^2. \]  (46)

\(^{12}\) By computing the derivative of \( n^c \) and \( n^{cc} \) evaluated at \( d = \frac{P_i}{\sigma} \), that is, at \( n^c = n^{cc} = 1 \) we can obtain that for \( d \) close to \( \frac{P_i}{\sigma} \), \( \min\{n^c, n^{cc}\} = n^{cc} \). At the other end, we can show that as \( d \) approaches zero \( n^c < \sqrt{\frac{\sigma(1-a)}{P_i}} \) and \( n^{cc} \to \frac{P_j}{P_i + P_j} \). Which of the two is lowest cannot be established without further restrictions on the parameter values.
\( \vartheta \) is a strictly increasing, convex function with \( \vartheta(0) = 0 \) and \( \vartheta(1) = 1-a-d+\frac{P_i}{\sigma} \). Note that \( \vartheta(1) = 1-a-d+\frac{P_i}{\sigma} \geq 1-a \) as \( d \leq \frac{P_i}{\sigma} \).

The condition of Lemma ?? is straightforward. When the opposition has little surplus to contribute the group in power prefers to secede and keep the own surplus.

The conditions under which the group in power will prefer secession over a union with fairness \( \lambda^i_A \) precisely is the complement of the conditions in Lemma ??, just discussed.

Let us finish the analysis of the choices by \( j \) by considering when secession will be preferable to conflict. From (??) and (??) we obtain the conditions under which player \( j \) prefers secession to conflict.

\[ s \leq \phi(n) = d + \left[ 1-a-d+\frac{P_j}{\sigma} \right] n - \frac{P_j}{\sigma} n^2. \] (47)

\( \phi \) is a concave function with \( \phi(0) = d \) and \( \phi(1) = 1-a \).

The threshold lines on the preferences by group \( j \) are depicted in Figure ?? in the Figure we have assumed that \( d > \frac{P_i}{\sigma} \). If we reverse the inequality, the vertical line is located at \( n < 1 \).

### 5.2.3 Equilibria

For the sake of an orderly discussion let us first consider the case in which conflict is relatively costly, that is, \( d > \frac{P_i}{\sigma} \). This implies that \( \min\{n^c, n^{cc}\} > 1 \) and hence for all parameter values satisfying this restriction a fair distribution is acceptable to both groups.

From Lemma ?? we know that for all \( s \geq \vartheta(n) \) group \( j \) prefers union to secession. Furthermore, by Lemma ?? we know that union is also preferred to conflict. Since, union has been evaluated at the fairness level that makes it acceptable to \( i \), we can conclude that for all \( s \geq \vartheta(n) \) the unique equilibrium is a fair distribution of the surplus within the union.

Again from Lemma ?? for all \( s < \vartheta(n) \) group \( j \) prefers secession over a fair union. Furthermore, since also \( s < \phi(n) \), by Lemma ??, we add that \( j \) also prefers secession.
to conflict. In spite that \( j \) prefers secession overall, by Lemma ?? player \( i \) will accept it only if \( s \geq \zeta(n) \). Therefore, for all \( s \in [\zeta(n), \phi(n)] \) the unique equilibrium is peaceful secession. When \( s \leq \min\{\zeta(n), \phi(n)\} \) then any secession proposal would be rejected by \( i \). Since by Lemma ?? \( j \) prefer a fair union to conflict, the unique equilibrium is a fair union.

Let us now examine the changes to be made in our characterization of equilibria when \( d < \frac{P_i}{\sigma} \), that is, when \( \min\{n^c, n^{cc}\} < 1 \). In order to simplify the analysis of this case of low cost of conflict we shall assume that \( d \) cannot be arbitrarily small. Specifically we will concentrate on the case in which \( \frac{P_i}{\sigma} - a \leq d < \frac{P_i}{\sigma} \).

By Lemma ?? we know that for all \( n < 1 \) group \( j \) will choose conflict rather than union and this will modify our analysis for these values of \( n \). But note too that a higher \( d \) also shifts upwards the function \( \vartheta(\cdot) \). In particular, this function will now intersect the function \( \phi(\cdot) \). It can be readily verified that this intersection takes place precisely at \( n = n^{cc} \).

Notice first, that for all \( n \leq \min\{n^c, n^{cc}\} \) the analysis is exactly the same as before. Hence, for \( n \leq \min\{n^c, n^{cc}\} \) and for all \( s \in [\zeta(n), \phi(n)] \) the unique equilibrium is peaceful secession, and otherwise the equilibrium is a fair distribution within the union.

From Lemma ?? we know that for all \( s \geq \vartheta(n) \) group \( j \) prefers union to secession and, by Lemma ??, that conflict is preferred to union. Hence, for all \( s \geq \vartheta(n) \) the unique equilibrium is conflict.

We now examine the parameter values such that \( \phi(n) < s < \vartheta(n) \). Notice that since we are in the set \( n \geq n^{cc} \) a fair distribution is excluded as an option. The choices by \( j \) are either secession or conflict. By Lemma ?? whenever \( \phi(n) < s \) player \( j \) prefers conflict to secession and by Lemma ?? whenever \( s < \vartheta(n) \) player \( j \) prefers secession to union. Therefore, we can conclude that the equilibrium chosen by player \( j \) will again be conflict.

Let us now consider the case in which \( \zeta(n) \leq s \leq \phi(n) \). By Lemma ?? \( j \) prefers secession to conflict and by Lemma ?? player \( i \) would accept the proposed secession. Therefore, for these parameter values the equilibrium will be secession.

Finally, for \( s < \zeta(n) \) we know that the options for player \( j \) are either secession or conflict. But, by Lemma ?? for these parameter values \( i \) will reject secession. Hence, for this set of parameters the only equilibrium is conflict.

This completes the characterization of equilibria. We summarize our findings in
the following Proposition.

**Proposition 16.** The static secession game always yields unique equilibria that are as follows:

- Let $n \leq \min\{n^c, n^\infty\}$. Then, for the parameter values such that $s \in [\zeta(n), \phi(n)]$ the equilibrium is peaceful secession and otherwise the equilibrium is a fair union.

- Let $\min\{n^c, n^\infty\} \geq n < 1$ so that $d < \frac{p_i}{\sigma}$. Then,
  - for all $s \geq \vartheta(n)$, the unique equilibrium is conflict;
  - for all $\phi(n) < s < \vartheta(n)$, the unique equilibrium is conflict.

The following Figure depicts the different equilibria.

The general picture is as follows. Let us start with the case $d > \frac{p_i}{\sigma}$. Since the cost of conflict is high relative to the gain of implementing the most preferred good, conflict is never an equilibrium. As we have seen, it is always feasible to the group in power to buy off the opposition. Hence the only possible equilibria are a fair distribution within the union and agreed secession.

For agreed secession be an equilibrium the population size of the opposition has to be above a minimum. If their size is low, they have a low probability of winning in a conflict and hence cannot impose a large transfer under the threat of conflict. Hence, the group in power will prefer to keep the union rather than secession.

For larger population size, secession is an option as long as the productivity of the opposition is low, that is, $s < n$. In this case, there are of a sufficient size to exert pressure for a substantial transfer. But since their contribution to the total surplus is small, the group in power prefers secession.

\[13\] Note that when $d > \frac{p_i}{\sigma}$ this inequality entails no effective restriction on the values of $n$. 

Figure: Equilibria with $d < \frac{p_i}{\sigma}$
Finally, is the size of the opposition is sufficiently large and their productivity low. Their expected payoff from conflict might be higher than the payoff from becoming independent. Think of the limit case of a large opposition generating no surplus. Then, the group in power prefers to buy them off through a sufficiently large transfer rather than facing a high probability of losing their high surplus.

Let us consider again the case of a large opposition with a high probability of winning in conflict. The lower is the cost the higher will be for them the payoff from conflict $d < \frac{P_i}{\sigma}$ and hence the higher the transfer necessary to buy them off. Clearly, there is a threshold on the cost below which the appeasing transfer would be excessive for the group in power and they would rather prefer to have conflict.

The fundamental comparative statics are as follows. Increases in $A$ and decreases in $D$ shrink the set of parameter values for which there is agreed secession to the benefit of union. It is rather obvious that as the cost of setting up a new state goes up, the less profitable will be to secede to produce one's own preferred public good. The effect of decreases in the cost of conflict $D$ is slightly more subtle. With a higher cost of conflict, the cost of the rejection of proposals by the opposition goes up. Knowing this, the group in power can buy off the opposition for less when faced between accepting this distribution within the union of playing conflict. Therefore, the group in power will be able to impose union for a broader set of parameter values.

Let us now consider the effect of an "increase in nationalism" within the i group in opposition, that is, an increase in $P_i$. Let us start with a low $P_i$, for instance $P_i < \sigma d = \frac{D}{N}$. An increase in $P_i$ has the effect of broadening the set of parameter values for which agreed secession is the equilibrium behavior. Opposition with smaller and larger sizes and larger surplus will be in the agreed secession set. As soon as $P_i > \frac{D}{N}$, large opposition, poor groups will start preferring conflict because the group in power will not be ready to offer a large enough transfer to make union acceptable to the opposition. Since $P_i$ can be manipulated by propaganda, this seems to be a particularly relevant point.

The bottom line is that when the opposition is small in numbers, we will observe union. As the population size becomes larger we shall see agreed secession for the cases in which the productivity of the seceded group is low. If the opposition becomes a very large group, then we risk conflict.

Why we don’t see agreed secessions when the opposition has high productivity? Because the group in power will in this case be ready to agree with higher transfers
in order not to lose access to the entire surplus, mostly produced by the opposition.

6 Discussion

What are we learning from the dynamic game relative to the static version?

We wish to compare here the predictions of our dynamic model with the ones derived from the static version of this model. To be precise, we compare the results under a sufficiently high time discount factor versus making $\delta = 0$ and hence valuing the payoffs at the current period only.

A first observation is that while secession seems exceptional in the static model, it is the equilibrium outcome in the dynamic game for a wide range of parameter values. Furthermore, in the dynamic game we can also have equilibria in which secession is reached after a sequence of conflict periods. In sum, in the dynamic model we have fewer instances of peaceful distributions within the union and also equilibria in which either of the groups secedes after winning the conflict against the other party.

Let us discuss the rationale of this difference in predictions and how this deeply depends on the dynamic nature of the model. Indeed one of the arguments we have raised in the introduction in support of the need for a dynamic model has been that secessions also had the important effect of stopping any future interaction with the other party.

In the static game we have either peaceful equilibria, within the union or with secession, and conflict. The terms of a peaceful distribution are determined by the payoff resulting from conflict: neither player will settle for less.

There is a clear analog between the peaceful equilibria in the dynamic game and the corresponding situations in the static game. One simply needs to multiply by the discount factor. Of course, allocations that are an equilibrium in one scenario might not be equilibria in the other. The case of one shot conflict can be paralleled by the indefinite conflict in the dynamic game [although such conflict path is a SPE only if the time discount factor is low enough].

Yet, the dynamic game is not the mere scaling of the static game. This is so precisely because the indefinite future leads to strategies that are not chosen in the static game. In particular notice that in the dynamic game, a player may decide to secede after the first victory in case the other player starts a conflict path. Consider the parameter values in $B_j$. There, the SP continuation consists of the group in
power seceding as soon as possible, precisely to avoid the repeated costly conflict. It is the threat of future conflict what makes \( j \) decide to secede. Furthermore, the fact that \( j \) chooses to secede after conflict means that this path yields him a higher payoff than to continue along the conflict path. Therefore, the payoff that \( j \) will obtain if \( i \) rejects the proposal and initiates conflict and \( j \) secedes is higher than repeated conflict.

The prospect of eventually seceding —which is increasingly valuable for \( j \) as the discount factor increases— implies that \( j \) wants to retain a higher share of the surplus in case of peace than what they would aspire to in case the threat was continuous conflict. Consequently, the set of equilibria with a peaceful union has to be smaller in the dynamic than in the static game. And the higher is the discount factor the greater the difference between the predictions of the two models.

The same argument holds true for the case in which the opposition plans to secede after the first victory, e.g. we are in \( B_i \). The time discounted payoff from this strategy is larger than the continued conflict path, and the more so the higher is the time discount factor. Therefore, their demands to accept a peaceful distribution in the union will be higher and hence harder to satisfy.

It is the threat capacity of strategic choices to be made in future what determines the equilibria today. In particular, as we have discussed, the strategic choices in a dynamic model [with a sufficiently high discount factor] make unions harder to survive than what a static model would predict.

7 Empirics

The equilibrium characterization of our model contains clear-cut predictions on in what zones of parameter values union or secession should prevail, and under what conditions splitting up is peaceful or conflicted. There are two zones of secessionist conflict, the first of which is in the zone where the opposition group is small in size yet highly prosperous. Let us start with taking stock of the existing empirical evidence on this.
7.1 Existing evidence

7.1.1 Evidence on the presence of particularly rich or particularly poor minorities being associated with secessionist conflict

There are manifold examples of secessionist conflicts with resource-rich ethnic minority group wanting to split. Examples include the armed separatist movement in the now independent Timor-Leste, historical fighting in Nigeria’s Biafra region and recent fighting in the Niger Delta regions of Nigeria, Katanga’s attempt to secede from the Congo from 1960 to 1963, Basque country’s armed struggle for independence from Spain, the rebellion of the Aceh Freedom Movement in Indonesia starting in 1976 and the armed fight of the Sudan People’s Liberation Army beginning in 1983. Other ethnically divided countries with separatism linked to large local natural resources include Angola, Myanmar, Democratic Republic of Congo, Morocco and Papua New Guinea.

While in the examples above the prosperity of separatist regions has been linked to natural resource abundance, there are many more cases where prosperous regions aim to secede – even if the source of wealth are not spoils from nature. Conflicted secessions from regions that were substantially richer than the country average include Slovenia and Croatia’s bids to split from Yugoslavia, or Eritrea’s war of independence from Ethiopia. In all of these cases the secessionist region is prosperous relative to the rest of the country and is relatively small in size. In line with our model, this creates an explosive blend.

There are also various historical examples of substantially below average income regions engaging in violent separatism. Spare a thought for example on the conflict in Chechnya.

Various anecdotal and case study accounts also show that indeed both the poorest and the richest regions tend to develop grievances against the central state and build nationalist movements (see Gourevitch, 1979; Horowitz, 1985; Bookman, 1992). There is also more systematic econometric evidence consistent with this model prediction. Drawing on a sample of 31 federal states, Deiwiks, Ceder-

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14 This draws on the more detailed accounts of Ross (2004), Collier and Hoeffler (2006) and Morelli and Rohner (2015).
15 According to Collier and Hoeffler (2006) also Bangladesh’s split from Pakistan and the Southern states attempt to secede from the US Confederacy can be seen as partially motivated to escape trade policies that were perceived as harmful for the concerned separatist regions.
man and Gleditsch (2012) show that secessionist conflict takes place in regions that are either substantially below country average or substantially above average, with average-income regions being the most peaceful. This is exactly in line with the predictions of our model.

7.1.2 Evidence on separatism being less violent when the groups involved are of intermediate or large size and of similar prosperity

According to our equilibrium characterisation there is peaceful secession when the opposition group is of intermediate or large size and is similarly rich than the governing group. For example the split between Czech Republic and Slovakia – two places of comparable size and prosperity – has been peaceful, as has the split of ancient Rome in two similar halves – West and East Rome. Britain is of similar GDP per capita as the EU average and of large size. Its split from the EU has been – so far – carried out within the boundaries of the law. Other examples of peaceful secessions between similarly sized and prosperous partners include Singapore-Malaysia, Austria-Hungary and Norway-Sweden (see Young, 1994).

7.1.3 Evidence on peaceful union when minority groups are small

Many long-lasting states are either characterised by ethnic homogeneity or extreme ethnic fractionalisation, while ethnically polarized countries are less likely to experience persistent peaceful union (Montalvo and Reynal-Querol, 2005; Esteban, Mayoral and Ray, 2012). As predicted by our model, when potential separatist groups are very small in size, setting up their own state would be very costly, and peaceful union can be more easily sustained. One can think for example about the cases of German-speaking Südtirol in Italy, Martinique and Guadeloupe in France, Galicia in Spain or the Sami people in Northern Scandinavia.

8 Conclusion

TBD
References


8 Conclusion


LeBreton, Michel and Shlomo Weber (2003) ??????


