Understanding US Export Dynamics: Does endogenising the export share help?

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Abstract

In this paper we investigate whether a model which endogenises the share of exported varieties can help us understand aggregate export fluctuations over the business cycle. For that purpose, we estimate two similar models with heterogeneous firms and endogenous entry, one where the share of exported varieties varies endogenously, and one where that share is constant, using data for the US and the euro area. We compare the two models’ overall performance as well as their ability to help us understand US export dynamics. Different from previous evaluations, we take into account that business cycle fluctuations arise not only from productivity shocks but also from monetary policy shocks, preference shocks and shocks to the uncovered interest rate parity condition. We show that, while allowing for heterogeneous firms and endogenous entry helps match US export dynamics, endogenising the share of exported varieties does not significantly progress our understanding of aggregate export dynamics.

Keywords: Export dynamics, heterogeneous firms, extensive margin of trade, international business cycles.

JEL codes: F41; F44; E32.

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1 Introduction

A vast literature on export dynamics - both empirical and theoretical - testifies of the gaps in our understanding of these dynamics. In particular, as pointed out by Engel and Wang (2011), standard open economy models cannot replicate the observed behaviour of exports. In this paper we investigate whether a model which endogenises the share of exported varieties can help us understand aggregate export fluctuations over the business cycle. We do so by comparing the performance of such a model to the performance of a similar model where the share of exported varieties does not fluctuate in response to business cycle movements.

The framework we use in our investigation is motivated by empirical evidence pointing out that the number of varieties available within an economy as well as the share of those varieties which is exported change over the business cycle. For instance, Broda and Weinstein (2010) find that the number of new varieties introduced by US firms is positively correlated to their sales and consumption. And Ghironi and Melitz (2007) show that, in the US, the number of exported varieties is positively correlated to exports, while Devereux and Hnatkovska (2012) show that changes in the share of traded relative to non-traded output is positively correlated to GDP growth in a range of OECD countries including the US. Our framework therefore endogenises, first, the firms’ choice whether to introduce new varieties and, second, whether to export or only supply those varieties to the domestic market.

We build on an important strand of literature which seeks to introduce these two features into standard open economy general equilibrium models. The seminal paper within that literature is Ghironi and Melitz (2005) who develop an open economy general equilibrium model in which firms are heterogeneous with respect to their individual levels of productivity. In this model both product creation and exporting is endogenously determined. Firms have to pay a sunk entry cost to start producing their product variety, hence, varieties available within an economy fluctuates over the business cycle. Moreover, because firms face fixed and variable export costs, only the most productive firms will find it profitable to export their product variety. As a result, not only the set of varieties available within an economy changes with the economic conditions, but also the share of those varieties that are exported. We introduce nominal rigidities in the form of staggered wage setting and monetary policy into the model developed by Ghironi and Melitz (2005).

We evaluate the model’s ability - and in particular the implications of an endogenous share of exported goods (endogenous tradability) - to improve our understanding of US business cycles and of export dynamics. Different from previous evaluations of this framework, such as Ghironi and Melitz (2007), we take into account that business cycle fluctuations arise not only from productivity shocks but also from monetary policy shocks, preference shocks and shocks to the uncovered interest rate parity (UIP) condition. We estimate two similar models, one where the share of exported varieties varies endogenously, and one where that share is constant. We calibrate the structural parameters of the models using empirical evidence provided in the literature. We then estimate the parameters related to the stochastic shock processes using data for the US and the euro area with Bayesian estimation methods. We compare the two models’ overall performance as well as their ability to help us understand US export dynamics.

We find that both models perform reasonably well in accounting for US export dynamics. They generate substantial volatility and account for the pro-cyclical behaviour of exports and imports documented in Engel and Wang (2011). While they generally predict exports fairly well compared to other standard open economy models, they cannot explain the entire great trade collapse in the aftermath of the great recession. Our analysis shows that a framework with heterogenous firms and endogenous entry and exports in the spirit of Ghironi and Melitz (2005) helps our understanding of US export dynamics. We then check the contribution of endogenous tradability to the model performance by comparing the two models. The model comparison shows that allowing for the share of exported varieties to vary en-
dogenously in a way consistent with Ghironi and Melitz (2005) does not significantly improve the models’ fit to US business cycle fluctuations. And moreover, it is not clear that it helps us understand aggregate US export dynamics. In other words, the impact of endogenizing the share of traded varieties is marginal within this framework.

Our results imply that endogenizing the share of traded goods may be costly. In the Ghironi and Melitz (2005) model, the productivity threshold for exporting requires that the marginal firm makes zero export profits. The productivity threshold at which firms can export depends on productivity and demand conditions, such that the share of firms exporting varies with economic conditions. Once we fix share of exported varieties, the zero profit export cut-off condition becomes redundant. Our analysis show that this condition is hard to fit the data in some cases.

There is a vast international macroeconomics literature that builds on Ghironi and Melitz (2005), as do we. Within this framework, the authors analyse the implications of their model for international prices, focusing in particular on the Harrod-Balassa-Samuelson effect. This framework has been used to study a number of different topics, ranging from the elasticity of exports and imports to relative prices (Ruhl (2008)) to the consequences of trade integration (Cacciatore and Ghironi (2013)) and optimal monetary policy (Cooke (2016)). Our paper is more closely related to the strand of literature that studies the frameworks ability to improve our understanding of business cycles and of export dynamics. Ghironi and Melitz (2007) find that their model can replicate the link between the number of varieties traded across countries and aggregate trade levels, as well as some cross-correlations of trade with GDP. Devereux and Hnatkovska (2011) find that including an extensive margin for trade allows to better replicate observed changes in the share of traded output over the business cycle and helps explain the consumption risk sharing puzzle. However, the model with extensive margin of trade does not out-perform a similar model without an extensive margin when it comes to matching moments and does mixed at explaining trade dynamics and their relation to GDP. Both those papers rely on productivity shocks being the source of business cycle fluctuations. To the best of our knowledge, the framework has not been used to evaluate model performance in matching business cycle properties and more specifically trade fluctuations in an environment where shocks other than to productivity can also cause business cycle fluctuations.

Our findings are important as we show that endogenizing the share of exported goods as in Ghironi and Melitz (2005) is not sufficient to make significant progress in our understanding of aggregate export dynamics. Future research should aim at improving both the modelling of the extensive margin of trade and the intensive margin of trade.

In the next section, we present the two versions of the Ghironi and Melitz (2005) framework we use to understand whether endogenizing the share of exported varieties improves our understanding of business cycles and export dynamics. In Section 3, we present our estimation strategy and the data used for the estimation exercises. We then discuss how well the models fit the data and in particular exports in Section 4. In Section 5 we present some robustness and sensitivity analysis to clarify that our results are robust.

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1 Note that our model identifies business cycles to be driven by a number of types of shocks, and while an extensive margin of trade leads to an improvement in matching some data in the face of productivity shocks, it does not do so in the face of other shocks. Our results are robust to different model assumptions, calibrations and data periods.

2 Note that in their model extensive margin of trade only changes through the movements in the share of non-traded goods. Their model excludes firm entry.
2 Models

2.1 Benchmark Model: Model-1

The benchmark model we present here closely follows Ghironi and Melitz (2005), but expands that model by introducing a role for monetary policy. It is a two country general equilibrium model in which firms are heterogeneous with respect to their relative productivity levels. Firms face fixed and variable costs of exporting so only more productive firms are able to export. This structure allows variations both at extensive and intensive margins of trade. Financial markets are incomplete at the international level. Different from Ghironi and Melitz (2005), we introduce nominal rigidities in labour markets such that there is a role for monetary policy in our framework. In particular, wages are staggered a la Calvo (1983) in both countries. And monetary policy is conducted according to a Taylor type rule. We will denote the foreign country variables with an asterisk (*).

2.1.1 Households

The representative home household’s lifetime utility function can be expressed as a function of consumption ($C_t$) and labour ($L_t$):

$$U_t = E_t \sum_{t=0}^{\infty} \beta_t \chi_t \left[ \frac{C_t^{1-\gamma} - 1}{1-\gamma} - \kappa \frac{L_t^{1+\eta}}{1+\eta} \right]$$

(1)

where $E_t$ denotes the expectations at time $t$, $\beta$ is the discount factor, $1/\gamma$ is the intertemporal elasticity of substitution and $\eta$ is the inverse of the Frisch elasticity of labour supply. The parameter $\chi$ is the preference shock which affects the discount factor. It follows an AR(1) process:

$$\ln(\chi_t) = \rho_{\chi} \ln(\chi_{t-1}) + \varepsilon_{\chi,t}$$

(2)

where $0 \leq \rho_{\chi} < 1$ and $\varepsilon_{\chi,t} \sim N(0, \sigma_{\chi}^2)$.

Consumers can consume differentiated goods ($\omega$) defined over a continuum set of goods, $\Omega$:

$$C_t = \left[ \int_{\omega \in \Omega} c_t(\omega)^{\frac{1}{\theta+1}} d\omega \right]^\frac{\theta+1}{\theta}$$

such that $\theta$ denotes the elasticity of substitution between the differentiated goods. Each period only a subset of varieties are actually available for consumption and that subset is allowed to be different across countries and vary over time.

The corresponding consumer price index for the Home economy is:

$$P_t = \left[ \int_{\omega \in \Omega} p_t(\omega)^{1-\theta} d\omega \right]^\frac{1}{1-\theta}$$

The optimal allocation of nominal expenditure of the representative household in the Home country for each differentiated good $\omega$ yields the following demand function:

$$c_t(\omega) = \left( \frac{p_t(\omega)}{P_t} \right)^{-\theta} C_t$$
2.1.2 Firms

A continuum of monopolistically competitive firms in each country produce differentiated goods $\omega \in \Omega$. To start producing i.e. to enter the market, firms need to pay a sunk cost ($f_{E,t}$) in the form of a labour requirement which is equal to $w_t f_{E,t}/Z_t$ in real terms. $w_t$ is the real wage and $Z_t$ is the aggregate productivity in the Home country. Once a firm has entered the market, it draws its productivity from a common distribution $G(z)$ where $z \in [z_{min}, \infty)$. This productivity does not vary over the life of the firm. A firm produces in every period until it is hit by an exogenous "death" shock. In every period, firms face this death shock with probability $\delta \in (0, 1)$, which is independent from their relative productivity. The production function of a firm has constant returns to scale functional form with labour being the only production input:

$$Y_t(\omega) = Z_t z(\omega) l_t(\omega)$$

where $Y_t(\omega)$ denote the production of variety $\omega$ and $l_t$ is the amount of labour required to produce that. $Z_t$ is the country-specific productivity level, and $z(\omega)$ denotes the firm-specific productivity level. The aggregate technology shock, $Z_t$, has the following stochastic form:

$$\ln(Z_t) = \rho_s \ln(Z_{t-1}) + \varepsilon_{z,t}$$

where $0 \leq \rho_s < 1$ and $\varepsilon_{z,t} \sim N(0, \sigma_z^2)$.

Once firms enter, they produce and sell in the domestic market. Firms can also export, and every period firms decide whether they will do so. To export in a given period, firms need to pay a fixed export cost ($\tau_X$) in effective labour units as well as an iceberg cost, $\tau_d$. Firms decide to export if they extract positive profits from exporting, and this depends on their relative productivity and demand conditions. Only the most productive firms - who can afford this fixed cost as well as the variable iceberg cost - will export.

Each firm produces one variety $\omega$ with associated productivity level $z$ and maximises profits subject to a downward sloping demand curve. Profits obtained by selling to the domestic ($\Pi^D_t$) and the export market ($\Pi^X_t$) respectively are:

$$\Pi^D_t (\omega) = \frac{p_{D,t}(\omega)}{P_t} y_{D,t}(\omega) - w_t l_{D,t}(\omega)$$

$$= \frac{p_{D,t}(\omega)}{P_t} Z_t z(\omega) l_{D,t}(\omega) - w_t l_{D,t}(\omega)$$

$$= \frac{p_{D,t}(\omega)}{P_t} y_{D,t}(\omega) - \frac{w_t}{Z_t z(\omega)} y_{D,t}(\omega)$$

$$\Pi^X_t (\omega) = \frac{p_{X,t}(\omega)}{P_t} y_{X,t}(\omega) - w_t l_{X,t}(\omega) - \frac{w_t f_{X,t}}{Z_t}$$

$$= \frac{p_{X,t}(\omega)}{P_t} Z_t z(\omega) l_{X,t}(\omega) - w_t l_{X,t}(\omega) - \frac{w_t f_{X,t}}{Z_t}$$

$$= \frac{p_{X,t}(\omega)}{P_t} y_{X,t}(\omega) - \frac{\tau_t w_t}{Z_t z(\omega)} y_{X,t}(\omega) - \frac{w_t f_{X,t}}{Z_t}$$

where $y_{D,t}(\omega)$ and $y_{X,t}(\omega)$ denote the production of variety $\omega$ for the domestic and the exports market respectively, $l_{D,t}(\omega)$ and $l_{X,t}(\omega)$ denote the amount of labour required to produce it, $Q_t = \frac{S_t P_t}{P_t}$ is the real exchange rate (RER) and $S_t$ is the nominal exchange rate defined as the home currency price of buying one unit of foreign currency. $p_{D,t}(\omega)$ denotes the price charged in the domestic market for variety $\omega$. 

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\(\omega\), and \(p_{X,t}(\omega)\) denotes the corresponding price charged in the export market.

Because each firm produces a single variety associated with a level of productivity \(z\), such that \(z\) is a good summary statistics for a firm, we index variables by \(z\) in the rest of the paper rather than by \(\omega\). Prices are flexible, and therefore set as a mark-up over marginal costs. The firm producing variety \(\omega\) with associated productivity level \(z\) sets the following prices:

\[
\rho_{D,t}(z) \equiv \frac{p_{D,t}(z)}{P_t} = \frac{\theta}{(\theta - 1)} w_t w_t, \quad \rho_{X,t}(z) \equiv \frac{p_{X,t}(z)}{P^*_t} = \frac{\theta}{(\theta - 1)} \tau_t Z_t \rho_t \tag{3}
\]

Plugging in the prices into the profit equations yield:

\[
\Pi^D_t(\omega) = \frac{1}{\theta - 1} w_t l_{D,t}(z), \quad \Pi^X_t(\omega) = \frac{1}{\theta - 1} w_t l_{X,t}(z) - \frac{w_t f_{X,t}}{\tau_t} \tag{4}
\]

Alternatively, by plugging in the prices and the demand functions, we can express the profits as:

\[
\Pi^D_t(z) = \left(\frac{(\rho_{D,t}(z))^{(1 - \theta)} C_t}{\theta}\right), \quad \Pi^X_t(z) = \left(\frac{Q_t(p_{X,t}(z))^{(1 - \theta)} C^*_t}{\theta}\right) - \frac{w_t f_{X,t}}{\tau_t} \tag{5}
\]

All firms have the choice to export, but only those with a relative productivity \(z\) above a cutoff level \(z_{X,t} = \inf\{z : \Pi^X_t(z) > 0\}\) ensuring non-negative profits from exporting, will do so. The exporting firms sell their goods both in local and foreign markets. So, while all firms can export, a firm with productivity between \(z_{\text{min}}\) and the export cutoff level, \(z_{X,t}\), will decide to serve only the local market\(^3\).

The export productivity cutoff level \(z_{X,t}\) varies with economic conditions, and therefore so does the number of exporting firms. The size of the non-traded sector is thus determined endogenously. As a result, given a mass \(N_{D,t}\) of firms in the home country, \(N_{X,t} = [1 - G(z_{X,t})]N_{D,t}\) of them also export. This structure is symmetric in the foreign country; \(z_{X,t}\) fluctuates endogenously in an isomorphic way.

### 2.1.3 Firm Averages

Average productivity for all domestic firms and for only exporting firms respectively are:

\[
\tilde{z}_D \equiv \left[\int_{z_{\text{min}}}^{\infty} z_{\theta - 1} dG(z)\right]^{\frac{1}{\theta - 1}}, \quad \tilde{z}_{X,t} \equiv \left[\frac{1}{1 - G(z_{X,t})}\int_{z_{X,t}}^{\infty} z_{\theta - 1} dG(z)\right]^{\frac{1}{\theta - 1}} \tag{6}
\]

The consumption in each country is a combination of the domestically produced goods and imported goods. Among the firms in the home country; \(N_{D,t}\) of local firms charge an average nominal price of \(\tilde{p}_{D,t} = p_{D,t}(\tilde{z}_D)\) and \(N_{X,t}\) of exporters charge \(\tilde{p}_{X,t} = p_{X,t}(\tilde{z}_X)\) in the foreign country. Similarly, in the foreign country, the \(N^*_t\) of foreign firms charge an average nominal price of \(\tilde{p}^*_t = \tilde{p}_{D,t}(\tilde{z}_D)\) and \(N^*_{X,t}\) of the exporters charge \(\tilde{p}^*_{X,t} = \tilde{p}_{X,t}(\tilde{z}_X)\) on average. Hence, the share of domestically produced goods for home consumers is \(N_{D,t}(\tilde{p}_{D,t})^{1 - \theta}\) and the share of imports is \(N_{X,t}(\tilde{p}^*_{X,t})^{1 - \theta}\). The consumer price index of the home country can be written as then:

\[
P_t = [N_{D,t}(\tilde{p}_{D,t})^{1 - \theta} + N^*_{X,t}(\tilde{p}^*_{X,t})^{1 - \theta}]^{1/1 - \theta} \tag{7}
\]

Equivalently:

\[
1 = N_{D,t}(\tilde{p}_{D,t})^{1 - \theta} + N^*_{X,t}(\tilde{p}^*_{X,t})^{1 - \theta} \tag{8}
\]

\(^3\)The lower bound for idiosyncratic productivity, \(z_{\text{min}}\), is below \(z_{X,t}\).
The average total profits in the Home country are:

$$\Pi_t = \Pi_t^D + \left[ \frac{N_{X,t}}{N_{D,t}} \right] \Pi_t^X = \Pi_t^D + [1 - G(z_{X,t})] \Pi_t^X$$  \hspace{1cm} (9)

Following Ghironi and Melitz (2005), we assume that relative productivity is drawn from a Pareto distribution with lower bound $z_{\text{min}}$ and shape parameter $k$ which is higher than $\theta - 1$:

$$G(z) = 1 - \left( \frac{z_{\text{min}}}{z} \right)^k.$$  \hspace{1cm} (6)

By defining $\phi \equiv \left\{ k / (k - (\theta - 1)) \right\}^{1/(\theta - 1)}$ and integrating (6), one can obtain: $\bar{z}_D = \phi z_{\text{min}}$ and $\bar{z}_{X,t} = \phi z_{X,t}$. So the share of home exporting firms, the extensive margin of trade, can be written as:

$$N_{X,t} = [1 - G(z_{X,t})]N_{D,t} = \left( \frac{z_{\text{min}}}{z_{X,t}} \right)^k N_{D,t}$$

Using productivity averages we can rewrite the share of exporters as:

$$\frac{N_{X,t}}{N_{D,t}} = \left( \frac{z_{\text{min}} \phi}{\bar{z}_{X,t}} \right)^k$$  \hspace{1cm} (10)

The marginal exporting firm will make zero profit, so for the cut-off firm $\Pi_t^X(\bar{z}_{X,t}) = 0$. Using Equation (5) the zero export condition will imply:

$$Q_t \left( \rho_{X,t} \bar{z}_{X,t} \right)^{(1-\theta)} \frac{C_t^*}{\theta} = \frac{w_t f_{X,t}}{Z_t}$$

Plugging in the optimal prices from Equation (3) yields:

$$\frac{\theta}{\theta - 1} \frac{\tau_t w_t}{Q_t Z_t z_{X,t}} = \left( \frac{w_t f_{X,t} \theta}{Q_t Z_t C_t^*} \right)^{1/\theta - 1}$$

Solving for $z_{X,t}$ yields:

$$z_{X,t} = \left( \frac{\tau_t}{\theta - 1} \right) \left( \frac{w_t \theta}{Z_t Q_t} \right)^{\theta/\theta - 1} \left( \frac{f_{X,t}}{C_t^*} \right)^{1/\theta - 1}$$

The average profits of exporters will satisfy the following:

$$\bar{\Pi}_t^X(\bar{z}_{X,t}) = \frac{Q_t (\bar{\rho}_{X,t} \bar{z}_{X,t})^{(1-\theta)} C_t^*}{\theta} - \frac{w_t f_{X,t}}{Z_t}$$

Now we plug in the solution for $z_{X,t}$ as $z_{X,t} = \phi z_{X,t}$ and use the zero-profit condition to obtain:

$$\bar{\Pi}_t^X = \left( \frac{w_t f_{X,t} \theta}{Z_t} \right) (\phi^{\theta - 1} - 1)$$

Since $(\phi^{\theta - 1} - 1) \equiv \left[ k / (k - (\theta - 1)) \right]^{(\theta - 1)/(\theta - 1)} - 1$, we can express the above equation as:

$$\bar{\Pi}_t^X = (\theta - 1) \left( \frac{w_t f_{X,t}}{Z_t} \right) \left( \phi^{\theta - 1} \right)$$  \hspace{1cm} (11)

This is the average profits from exports. The results of the foreign country counterpart are analogous.

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4Ghironi and Melitz (2005) assume Pareto distribution for firm productivity as this distribution fits firm level data quite well. A Pareto distribution is a skewed and heavy-tailed distribution. As it is heavy-tailed for a finite mean and variance the shape parameter needs to be sufficiently high: $k > 1$ ensures a finite mean and $k > 2$ ensures a finite variance. In our case the mean firm size/sale will be finite when $k/\theta - 1 > 1$. 

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2.1.4 Free Entry

Firms enter to the market at time $t$ and start their production at $t + 1$, so some of these new entrants will die (with probability $\delta$) before starting the production at the end of period $t$. The total number of firms at period $t$ in home country will be equal to the new entrants and established firms who survived from the previous period:

$$N_{D,t} = (1 - \delta)(N_{D,t-1} + N_{E,t-1})$$

(12)

Households will decide to start a business by calculating the expected present discounted value of future profits which gives simply the post entry value:

$$\tilde{v}_t = \mathbb{E}_t \sum_{s=t+1}^{\infty} [\beta(1 - \delta)]^{s-t} \left( \frac{C_{t+s}}{C_t} \right)^{-\gamma} \Pi_s$$

(13)

The free entry condition implies that, firms will enter until the average firm value is equal to the entry cost; $\tilde{v}_t = w_t f_{E,t}/Z_t$.

2.1.5 Budget Constraint

Households finance their expenditure through labour income and holdings of home and foreign denominated bonds. In addition households buy the shares from a mutual fund and receive dividend in return, $x_t$. Households can trade these shares domestically. Each period that mutual fund pays the entry costs, collects the profits and distributes them to the owners of the shares. We assume that labour income is subsidised at a constant rate, $\sigma$. International asset markets are incomplete in the sense that households are able to trade only nominal bonds. We follow Benigno (2001) in modelling the incomplete asset market structure. Households in the Home country can hold two kinds of nominal bonds; one is denominated in units of the home currency and the other is denominated in the foreign currency. However, the bonds issued by the Home country are not traded internationally for simplicity. Households in the home country face an additional cost when they take a position in the foreign asset market. As discussed in Schmitt-Grohe and Uribe (2003), we thus avoid non-stationarity in the model, as the cost function $\Theta(\cdot)$ ensures a stationary distribution of wealth across countries. The budget constraint in real terms is:

$$C_t + B_{H,t+1} + \tilde{v}_t (N_{D,t} + N_{E,t}) x_{t+1} + \frac{Q_t B_{F,t+1}}{\Theta(Q_t B_{F,t+1})} + TX_t = (1 + r_t)B_{H,t} + (1 + \sigma)w_t L_t$$

$$+ Q_t (1 + r^*_t) B_{F,t} + (\Pi_t + \tilde{v}_t) N_{D,t} x_t$$

where $B_{H,t}$ and $B_{F,t}$ are home and foreign currency nominated bonds, $(1 + r_t)$ and $(1 + r^*_t)$ are the gross real interest rate at home and foreign country, respectively. $TX_t$ denotes the lump sum taxes paid by the household.

Households make the intertemporal decision by maximising (1) subject to (14). This yields the following Euler equations for bonds and share holdings respectively and, combined with the analogous

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In order to have a well-behaved steady state in the model, we impose the following restrictions on the cost function: $\Theta(\cdot)$ is a differentiable decreasing function in the neighbourhood of steady state level of net foreign assets and when the net foreign assets are in the steady state level ($B_{F,t} = 0$), the cost function is equal to 1 ($\Theta(0) = 1$). See Benigno (2001) for a more detailed explanation.
foreign conditions, the UIP condition:

\[ 1 = E_t \left[ \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} \left( \frac{X_{t+1}}{X_t} \right) (1 + r_{t+1}) \right] \]  

(15)

\[ \tilde{v}_t = E_t \left[ \beta (1 - \delta) \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} \left( \frac{X_{t+1}}{X_t} \right) (\tilde{v}_{t+1} + \tilde{\Pi}_{t+1}) \right] \]  

(16)

\[ 1 = \beta (1 + r^*_{t+1}) \Theta(Q_t B_{F,t+1}) E_t \left[ \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} \left( \frac{X_{t+1}}{X_t} \right) \frac{Q_{t+1}}{Q_t} \right] \]  

(17)

Given the rejection of the UIP condition at empirical level\(^6\), we introduce an exogenous shock to Equation 17:

\[ 1 = \beta \psi_t (1 + r^*_{t+1}) \Theta(Q_t B_{F,t+1}) E_t \left[ \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} \left( \frac{X_{t+1}}{X_t} \right) \frac{Q_{t+1}}{Q_t} \right] \]  

(18)

where \( \psi_t \) has the following autoregressive process:

\[ \ln(\psi_t) = \rho_{\psi} \ln(\psi_{t-1}) + \varepsilon_{\psi,t} \]

with \( 0 \leq \rho_{\psi} < 1 \) and \( \varepsilon_{\psi,t} \sim N(0, \sigma^2_{\psi}) \).

The situation of foreign households is analogous.

2.1.6 Labour Supply and Wage Setting

Expanding on Ghironi and Melitz (2005), we introduce nominal rigidities through labour market frictions. There is monopolistic competition among households in the labour market, in the sense that households offer differentiated labour services. As described in Erceg et al. (2000), an “employment agency” combines individual household’s supply in the following Dixit-Stiglitz form:

\[ L_t = \left[ \int_0^1 L_t(i) \frac{\theta_w - 1}{\theta_w - 1} di \right]^{\frac{\theta_w - 1}{\theta_w - 1}} \]

where \( \theta_w > 1 \) is the elasticity of substitution between labour inputs.

The aggregate nominal wage index in home country can be defined as:

\[ W_t = \left[ \int_0^1 W_t(i) \left[ 1 - \theta_w \right] di \right]^{\frac{1}{1-\theta_w}} \]

The cost minimisation problem of producers gives a downward sloping labour demand curve. The total demand for household \( i \)'s labour services by all firms is:

\[ L_t(i) = \left[ \frac{W_t(i)}{W_t} \right]^{\frac{1}{1-\theta_w}} L_t \]

(19)

Wages are staggered á la Calvo (1983); in a given period \( (1 - \xi) \) of households are able to adjust their wages. To choose the optimum wage \( \tilde{W}_t(i) \), households maximise the expected lifetime utility (1)

\(^6\)See, for instance, Lewis (1995).
subject to the budget constraint and the labour demand curve (19). The first order condition for this nominal wage setting problem is:

$$
\sum_{k=0}^{\infty} (\beta \xi)^k E_t \left[ L_{t+k}(i) U_C(C_{t,t+k}) \left( (1 + \sigma) \frac{W_i(t)}{P_{t+k}} - \frac{\theta_w}{\theta_w - 1} MRS_{t,t+k} \right) \right] = 0
$$

(20)

where $MRS_{t,t+k}$ is the marginal rate of substitution between consumption and labour in period $t + k$ for the household resetting the wage in period $t$, i.e. $MRS_{t,t+k} = -\frac{U_L(L_{t,t+k})}{U_C(C_{t,t+k})}$.

When wages are flexible ($\xi \to 0$), the real wage multiplied by the subsidy will be equal to the mark-up over the marginal rate of substitution:

$$(1 + \sigma) \frac{W_i}{P_t^i} = \frac{\theta_w}{\theta_w - 1} MRS_{t,t+k}
$$

(21)

In order to ensure a flexible wage equilibrium in steady state, we assume that the subsidy cancels out the monopolistic distortion in steady state, implying that $(1 + \sigma) = \frac{\theta_w}{\theta_w - 1}$. The structure is symmetric across countries.

### 2.1.7 Monetary Policy

The monetary policy instrument is the nominal interest rate paid on bonds. We assume that monetary policy is conducted using a Taylor type rule which targets the domestic inflation rate and the growth rate of the output, and that monetary policy is subject to shocks denoted $\varepsilon_m$ and $\varepsilon_m^*$.

$$
\frac{\dot{i}}{i} = (\frac{i_t - 1}{i})^{1-\Gamma_i} \left( \frac{\pi_t \Gamma_{\pi}}{\pi_t^{\Gamma_{\pi}}} \right) \left[ \frac{\gamma_l}{\gamma_{-1}} \right]^{1-\Gamma_i} \exp(\varepsilon_{m,t})
$$

(22)

$$
\frac{\dot{i}^*}{i^*} = (\frac{i_t^* - 1}{i^*})^{1-\Gamma_{i^*}} \left( \frac{\pi_t^* \Gamma_{\pi^*}}{\pi_t^{\Gamma_{\pi^*}}} \right) \left[ \frac{\gamma_l^*}{\gamma_{-1}^*} \right]^{1-\Gamma_{i^*}} \exp(\varepsilon_{m^*,t})
$$

(23)

where $\varepsilon_{m,t} \sim N(0, \sigma_m^2)$ and $\varepsilon_{m^*,t} \sim N(0, \sigma_m^{**2})$.

### 2.1.8 Market Clearing and the Definition of Some International Variables

Aggregating across household budget constraints, shows that revenue from production (labour income and profits) and bond holdings is invested in new firms as well as used for consumption of domestic and imported goods and bond purchases:

$$
w_t L_t + N_{D,t} \bar{\Pi}_t = C_t + N_{E,t} \bar{\varepsilon}_t + \frac{Q_t B_{F,t+1}}{\Theta(Q_t B_{F,t+1})} - Q_t (1 + \tau_t) B_{F,t}
$$

For the labour market clearing condition we calculate the labour demand in each market. We use Equation (4) to obtain the average amount of labour demanded by a firm with average productivity: $\tilde{I}_{D,t} = (\theta - 1) \bar{\Pi}_t^D / w_t$ and $\tilde{I}_{X,t} = (\theta - 1) \bar{\Pi}_t^X / w_t + (\theta - 1) f_{X,t} / Z_t$. To calculate the amount of labour hired in total economy we multiply these by $N_{D,t}$ and $N_{X,t}$ respectively. Also new entrants need to hire $f_{E,t} / Z_t$ of labour to pay the entry cost. Then the labour market clearing condition in home country is given by:

$$
L_t = \frac{\theta - 1}{w_t} N_{D,t} \bar{\Pi}_t^D + \frac{\theta - 1}{w_t} N_{X,t} \bar{\Pi}_t^X + \frac{\theta}{Z_t} N_{X,t} f_{X,t} + \frac{1}{Z_t} N_{E,t} f_{E,t}
$$

(24)
We define the current account as the change in claims on foreign agents:
\[ CA_t = \frac{Q_t B_{E,t}}{(1 + r_t^G)} \Theta(Q_t B_{E,t}) - Q_{t-1} B_{E,t-1} \]  
(25)

The total output produced in the Home country is equal to the sum of home consumption, investment plus net exports:
\[ Y_t = C_t + C_{X,t} - C^*_{X,t} + N_{E,t} v_t \]  
(26)

Given our interest in trade dynamics, we also define total exports and imports of home country. We measure them as a ratio of output:
\[ \frac{X_t}{Y_t} = \frac{Q_t \tilde{\rho}_{X,t} N_{X,t} (\tilde{\rho}_{X,t})^{1-\theta} C_t^*}{Y_t} \]  
(27)
\[ \frac{M_t}{Y_t} = \frac{\tilde{\rho}_{X,t} N_{X,t} (\tilde{\rho}_{X,t})^{1-\theta} C_t^*}{Y_t} \]  
(28)

where \( X_t \) and \( M_t \) are the exports and imports of home country, respectively. Both of them are expressed in the home currency.

The RER we have been using so far is a welfare based RER, \( Q_t = \frac{s_t^p P^C}{P_t} \) which includes the variety effect. We construct the CPI based RER to be consistent with the data as the statisticians report only the weighted averages. The CPI based transformation of \( Q_t \) discounts the impact arising from the changes in the variety. This can be done by simply using the average prices \( \tilde{P}_t \), \( \tilde{P}_t^* \), as they are simple weighted averages and correspond to the data much closer: \( P_t = N_t^{1/1-\theta} \tilde{P}_t \) and \( P_t^* = (N_t^*)^{1/1-\theta} \tilde{P}_t^* \). Hence the CPI based RER will be: \( \tilde{Q}_t = \frac{s_t^p P^C}{\tilde{P}_t} \).

Following Ghironi and Melitz (2005), we will now re-write the RER to be able understand the predictions of the model about deviations from PPP. We will use the price index equation (Equation (7)) since we know that \( P_t = N_t^{1/1-\theta} \tilde{P}_t ^* \):\footnote{The total variety available at home is reflected by: \( N_t = N_{D,t} + N_{X,t}^* \)}
\[ \tilde{Q}_t^{1-\theta} = (N_t^*)^{-1} S_t^{1-\theta} \left[ \frac{N_{D,t} (\tilde{P}_{D,t})^{1-\theta} + N_{X,t} (\tilde{P}_{X,t})^{1-\theta}}{(N_t)^{-1} \left[ N_{D,t} (\tilde{P}_{D,t})^{1-\theta} + N_{X,t} (\tilde{P}_{X,t})^{1-\theta}] \right) \right] \]

Plug in the optimal prices from Equation (3) and their foreign counterparts:
\[ \tilde{Q}_t^{1-\theta} = (N_t^*)^{-1} S_t^{1-\theta} \left[ \frac{N_{D,t} \left( \frac{\theta}{\theta-1} \frac{W_t^*}{Z_t^{1/2} x_{D,t}} \right)^{1-\theta} + N_{X,t} \left( \frac{\theta}{\theta-1} \frac{W_t}{Z_t^{1/2} x_{X,t}} \right)^{1-\theta}}{(N_t)^{-1} \left[ N_{D,t} \left( \frac{\theta}{\theta-1} \frac{W_t^*}{Z_t^{1/2} x_{D,t}} \right)^{1-\theta} + N_{X,t} \left( \frac{\theta}{\theta-1} \frac{W_t}{Z_t^{1/2} x_{X,t}} \right)^{1-\theta}] \right) \right] \]

Note that, for this expression we are not using the consumption goods as units of measure any more, hence, we are using nominal wages now, \( W_t \), not the real wages, \( w_t \).

Dividing the above expression by \( \tilde{P}_{D,t} = \frac{\theta}{(\theta-1)} \frac{W_t^*}{Z_t^{1/2} x_{D,t}} \) and simplifying yields:
\[ \tilde{Q}_t^{1-\theta} = (N_t^*)^{-1} S_t^{1-\theta} \left[ \frac{N_{D,t} \left( \frac{W_t^*}{Z_t^{1/2} x_{D,t}} \right)^{1-\theta} \left( \frac{Z_t^{1/2} x_{D,t}}{W_t} \right)^{1-\theta} + N_{X,t} \left( \frac{Z_t^{1/2} x_{X,t}}{W_t} \right)^{1-\theta} \right]}{(N_t)^{-1} \left[ N_{D,t} \left( \frac{W_t}{Z_t^{1/2} x_{D,t}} \right)^{1-\theta} \left( \frac{Z_t^{1/2} x_{D,t}}{W_t} \right)^{1-\theta} + N_{X,t} \left( \frac{Z_t^{1/2} x_{X,t}}{W_t} \right)^{1-\theta}] \right) } \]
Note that an increase in real exchange rate, both for $\tilde{Q}$ and $Q$, means a depreciation.

As in Ghironi and Melitz (2005), we define terms of labour which measures the cost of effective labour across countries: $ToL_t \equiv S_t(W^*_t/Z^*_t)/(W_t/Z_t)$. We now re-write what we obtained previously:

$$\tilde{Q}_t^{1-\theta} = \frac{N_{D,t}(ToL_t)^{1-\theta} \left(\frac{\tau^*_t \tilde{z}_D}{\tilde{z}_X}\right)^{1-\theta}}{N_{D,t} + N_{X,t}(ToL_t)^{1-\theta} \left(\frac{\tau_t \tilde{z}_D}{\tilde{z}_X}\right)^{1-\theta}}$$  \hspace{1cm} (29)

### 2.2 Model With Fixed Share of Exporters: Model-2

One of the key features of our benchmark model presented previously is that exports vary both at the intensive and extensive margin. In this model, the extensive margin of trade fluctuates through two key channels: First, through the endogenous entry of firms producing new varieties, and second, through the endogenous determination of the share of traded varieties. Instead, in the second model, which we present here, we remove the endogenous tradedness assumption i.e. the share of exported varieties within an economy is fixed. In our empirical analysis below, this model will allow us to evaluate the importance of fluctuations in the share of exported goods for our understanding of export dynamics over the business cycle.

In this second model the productivity threshold does not vary with economic conditions. Having a fixed the productivity threshold implies that $\tilde{z}_X$ is not a variable anymore but a parameter. The pricing decisions will change accordingly:

$$\tilde{\rho}_{D,t} = \frac{\theta}{(\theta - 1)} \frac{w_t}{Z_t \tilde{z}_min \phi}, \quad \tilde{\rho}_{X,t} = \frac{\theta}{(\theta - 1)} \frac{\tau_t w_t}{Q_t Z_t \tilde{z}_X}$$

where $\phi \equiv \{k/(k - (\theta - 1))\}^{1/(\theta - 1)}$.

The share of exporting firms is fixed now:

$$N_{X,t} = \left(\frac{z_{min} \phi}{\tilde{z}_X}\right)^k N_{D,t}$$  \hspace{1cm} (30)

This equation shows that, in this second model, trade varies at the extensive margin only as a consequence of new varieties being introduced into the economy, not through changes in the share of varieties exported.

With fixed share of exporters the zero profit export cut-off condition (Equation (26)) disappears. For the remaining equilibrium conditions see Model-1.

### 3 Estimation

#### 3.1 Estimation Method and Data

We estimate the persistence and variance of shocks by using Bayesian estimation techniques\(^8\). We calibrate the rest of the parameters as most of these parameters affect the steady state of the model which is solved externally by using a non-linear system solver. Since Metropolis Hastings procedure updates the value of the estimated parameter at each iteration, this would change the steady state for each one of the draws, potentially leading to instability and convergence issues.

\(^8\)See, An and Schorfheide (2007) for a detailed description of the Bayesian estimation method.
The log-linearised system of equations that we use for the estimation can be found in Appendix C for Model-1 and in Appendix F for Model-2. To obtain the posterior distributions, we run four parallel chains of 300000 replications of the Metropolis Hastings algorithm with an acceptance rate around 30% in both estimations. We discard the first half of the draws. We observe the convergence through Brooks and Gelman (1998) statistics.

We estimate the models using the data for the US and Euro Area over the period 1984:Q1-2014:Q4. We assume that the US is the home country and the Euro Area is the foreign country. We use data on output, consumption and inflation per country as well as data on the real exchange rate. We plot the observables in Appendix H where we present additional tables and figures (see, Figure 13). To be consistent with the model specification, we take the log of the data variables, and we demean all variables except for output and consumption that we de-trend using the HP-filter. We compute the real exchange rate such that an increase in that variable corresponds to a depreciation, as is the case in the model. The details on data sources can be found in Appendix G.

3.2 Priors and Calibration

We present the calibrated parameters in Table 1. We assume that the deep parameters are symmetric between the US and Euro Area. We set the discount factor to 0.99 so that the steady state interest rate is 4% per year. The cost of intermediation in the international bond markets is calibrated following Ghironi and Melitz (2005). The value of the coefficient of risk aversion is set to 1.5 and the inverse Frisch elasticity is set to 2. The choice of the value of these parameters are standard in the DSGE literature. The value of the elasticity of substitution between differentiated labour inputs is in line with Erceg et al. (2000) and imply a 33% mark-up. We assume that the average duration of wage contracts is four quarters.

We calibrate the Taylor Rule parameters following Rabanal and Tuesta (2010). We rely on their estimation as their model is estimated on the US-Euro Area data and our policy rule has the same functional form. The interest rate smoothing parameter is 0.79, the reaction coefficient to inflation is 1.3 and to output growth is 0.75.

We follow Ghironi and Melitz (2005) for the calibration of the parameters relating to the productivity distribution across firms and the extensive margin of trade. Differently from Ghironi and Melitz (2005) however we assume that the costs related to entry and exporting do not vary with time; we calibrate these parameters to the steady state calibration of Ghironi and Melitz (2005). The value chosen for elasticity of substitution parameter (θ = 3.8) implies a dispersion parameter (k) of 3.4. The value of the dispersion parameter of the productivity distribution ensures that the standard deviation log plant sales is equal to 1.67 as reported in Bernard et al. (2003). We fix the value of the iceberg cost, τ, to 1.3, and fixed exporting cost, \( f_x \), to 0.0085. We normalise \( z_{\text{min}} \) and \( f_E \) to 1. In Model-2, the productivity threshold is fixed. We set the value of \( z_x \) to its steady state value in Model-1. Hence the share of exporters in Model-2 matches the steady state share of exporters in Model-1.

We assume that the shock processes are different across two countries. Table 2 summarises the prior distributions of the estimated parameters. As the persistence of the shock processes are bounded between 0 and 1, the prior assumed to have a beta distribution. We set the prior mean of these parameters to 0.85 with a standard deviation of 0.1. The standard deviation of the shocks assumed to follow an inverse

---

9 We used the MATLAB version of Dynare to implement our empirical analysis (see, Adjemian et al. (2011)).
10 Note that the model counterpart of the real exchange rate data is the log-linearised CPI-based real exchange rate (\( \hat{q}_t \)) that we derived in Section 2.1.8.
11 As we are working with quarterly data, we use 1600 for the smoothing parameter of the HP-filter.
12 This then implies: 1.67 = 1/(k + 1 - θ).
Table 1: Parameter Values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.99</td>
<td>Discount factor</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>1.5</td>
<td>Degree of risk aversion</td>
</tr>
<tr>
<td>$\eta$</td>
<td>2</td>
<td>Inverse Frisch elasticity of labour supply</td>
</tr>
<tr>
<td>$\theta$</td>
<td>3.8</td>
<td>Elasticity of substitution between goods</td>
</tr>
<tr>
<td>$\omega$</td>
<td>0.0025</td>
<td>Cost of international financial intermediation</td>
</tr>
<tr>
<td>$\theta_w$</td>
<td>4</td>
<td>Elasticity of substitution between labour inputs</td>
</tr>
<tr>
<td>$\xi = \xi^*$</td>
<td>0.75</td>
<td>Duration of wages</td>
</tr>
<tr>
<td>$\Gamma_i = \Gamma_i^*$</td>
<td>0.79</td>
<td>Interest rate smoothing parameter</td>
</tr>
<tr>
<td>$\Gamma_p = \Gamma_p^*$</td>
<td>1.3</td>
<td>Interest rate sensitivity to inflation</td>
</tr>
<tr>
<td>$\Gamma_y = \Gamma_y^*$</td>
<td>0.75</td>
<td>Interest rate sensitivity to output growth</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.025</td>
<td>Probability of firm death</td>
</tr>
<tr>
<td>$k$</td>
<td>3.4</td>
<td>Dispersion of the productivity distribution</td>
</tr>
<tr>
<td>$f_e = f_e^*$</td>
<td>1</td>
<td>Entry cost</td>
</tr>
<tr>
<td>$fx = fx^*$</td>
<td>0.0085</td>
<td>Fixed export cost</td>
</tr>
<tr>
<td>$\tau = \tau^*$</td>
<td>1.3</td>
<td>S.S. value of per unit export cost</td>
</tr>
<tr>
<td>$z_{min} = z_{min}^*$</td>
<td>1</td>
<td>Lower productivity bound</td>
</tr>
<tr>
<td>$z_x = z_x^*$</td>
<td>2.94</td>
<td>Fixed productivity threshold (Model-2)</td>
</tr>
</tbody>
</table>

gamma distribution with a mean of 0.1 and a standard deviation of 2\(^{13}\).

### 3.3 Posterials

The estimation results are reported in Table 2. We present the posterior mean and standard deviation as well as the mode and the 5th and 95th percentiles of the posterior distribution. In Appendix H, we report the posterior distribution of the estimated parameters for both models (see, Figure 14 and Figure 15).

\(^{13}\)Our prior choice is quite standard among the estimated DSGE models. See for instance, Smets and Wouters (2004) for the persistence parameters and see Jacob and Peersman (2013) for the shock variances.
## Table 2: Prior and Posterior Distributions

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Density</th>
<th>Mean</th>
<th>Std.</th>
<th>Mode</th>
<th>Std.</th>
<th>Mean</th>
<th>[5, 95]</th>
<th>Mode</th>
<th>Std.</th>
<th>Mean</th>
<th>[5, 95]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_z$</td>
<td>TFP-US</td>
<td>B</td>
<td>0.85</td>
<td>0.10</td>
<td>0.97</td>
<td>0.011</td>
<td>[0.95, 0.98]</td>
<td>0.97</td>
<td>0.010</td>
<td>0.97</td>
<td>[0.95, 0.98]</td>
</tr>
<tr>
<td>$\rho_z^*$</td>
<td>TFP-EA</td>
<td>B</td>
<td>0.85</td>
<td>0.10</td>
<td>0.96</td>
<td>0.015</td>
<td>[0.94, 0.98]</td>
<td>0.96</td>
<td>0.015</td>
<td>0.96</td>
<td>[0.93, 0.98]</td>
</tr>
<tr>
<td>$\rho_x$</td>
<td>Preference-US</td>
<td>B</td>
<td>0.85</td>
<td>0.10</td>
<td>0.72</td>
<td>0.031</td>
<td>[0.66, 0.77]</td>
<td>0.72</td>
<td>0.031</td>
<td>0.72</td>
<td>[0.67, 0.77]</td>
</tr>
<tr>
<td>$\rho_x^*$</td>
<td>Preference-EA</td>
<td>B</td>
<td>0.85</td>
<td>0.10</td>
<td>0.74</td>
<td>0.035</td>
<td>[0.67, 0.79]</td>
<td>0.73</td>
<td>0.036</td>
<td>0.73</td>
<td>[0.67, 0.79]</td>
</tr>
<tr>
<td>$\rho_\psi$</td>
<td>UIP</td>
<td>B</td>
<td>0.85</td>
<td>0.10</td>
<td>0.89</td>
<td>0.021</td>
<td>[0.85, 0.92]</td>
<td>0.88</td>
<td>0.020</td>
<td>0.88</td>
<td>[0.85, 0.91]</td>
</tr>
<tr>
<td>$\epsilon_z$</td>
<td>TFP-US</td>
<td>IG</td>
<td>0.10</td>
<td>2.00</td>
<td>0.117</td>
<td>0.007</td>
<td>[0.105, 0.131]</td>
<td>0.121</td>
<td>0.007</td>
<td>0.122</td>
<td>[0.109, 0.135]</td>
</tr>
<tr>
<td>$\epsilon_z^*$</td>
<td>TFP-EA</td>
<td>IG</td>
<td>0.10</td>
<td>2.00</td>
<td>0.113</td>
<td>0.007</td>
<td>[0.101, 0.126]</td>
<td>0.114</td>
<td>0.007</td>
<td>0.115</td>
<td>[0.103, 0.127]</td>
</tr>
<tr>
<td>$\epsilon_x$</td>
<td>Preference-US</td>
<td>IG</td>
<td>0.10</td>
<td>2.00</td>
<td>0.707</td>
<td>0.044</td>
<td>[0.642, 0.793]</td>
<td>0.702</td>
<td>0.044</td>
<td>0.712</td>
<td>[0.638, 0.788]</td>
</tr>
<tr>
<td>$\epsilon_x^*$</td>
<td>Preference-EA</td>
<td>IG</td>
<td>0.10</td>
<td>2.00</td>
<td>0.647</td>
<td>0.042</td>
<td>[0.583, 0.725]</td>
<td>0.640</td>
<td>0.041</td>
<td>0.648</td>
<td>[0.577, 0.717]</td>
</tr>
<tr>
<td>$\epsilon_\psi$</td>
<td>UIP shock</td>
<td>IG</td>
<td>0.10</td>
<td>2.00</td>
<td>0.019</td>
<td>0.001</td>
<td>[0.016, 0.023]</td>
<td>0.024</td>
<td>0.002</td>
<td>0.025</td>
<td>[0.021, 0.029]</td>
</tr>
<tr>
<td>$\epsilon_m$</td>
<td>Mon. Policy-US</td>
<td>IG</td>
<td>0.10</td>
<td>2.00</td>
<td>0.032</td>
<td>0.002</td>
<td>[0.029, 0.036]</td>
<td>0.032</td>
<td>0.002</td>
<td>0.033</td>
<td>[0.029, 0.036]</td>
</tr>
<tr>
<td>$\epsilon_m^*$</td>
<td>Mon. Policy-EA</td>
<td>IG</td>
<td>0.10</td>
<td>2.00</td>
<td>0.033</td>
<td>0.002</td>
<td>[0.030, 0.037]</td>
<td>0.033</td>
<td>0.002</td>
<td>0.033</td>
<td>[0.030, 0.037]</td>
</tr>
</tbody>
</table>

Model-1 | Model-2
---|---
Log-Marginal Density | 83.94 | 93.53

1 B stands for beta and IG stands for inverse gamma distributions.
2 Standard deviation of shocks are in percentages.
An evident result in Table 2 is that the estimation results of Model-1 and Model-2 are remarkably close to each other; in fact posterior mean and the mode of some parameters are equivalent in the two models. In addition, as also found in Smets and Wouters (2004) or in Lubik and Schorfheide (2006) the parameter values are fairly similar across two countries. The estimated values of the AR(1) coefficient is high for all shocks but especially productivity shocks are much persistent in both countries. The high persistence of TFP shocks is consistent with the values that are used in the DSGE literature. The standard deviation of the disturbances are also almost symmetric between the two countries with preference shocks being the most volatile.

4 Model Fit to Export Dynamics

In this section, we analyse how well a model that incorporates extensive margin of trade both through firm entry and endogenous tradability predicts export fluctuations over the business cycle. We further compare this performance with a model where the extensive margin only varies through entry. Hence, we evaluate whether having endogenous share of traded goods improves our understanding on US export dynamics.

We first assess the overall fit of each model by comparing the marginal likelihood (see, Table 2). The model with fixed share of exporters (Model-2) has a higher marginal likelihood (93.53) than the Model-1 in which the share of exporters are variable (83.94). This implies an improvement in the model performance once we fix the share of traded goods. Even though in Model-1 we allow for more variation in trade dynamics through the endogenous determination of non-traded goods, the model comparison shows that this variation is costly. In Model-1, the productivity threshold for exporting is determined such that the marginal firm has zero export profits (see, Equation 11). The productivity threshold at which firms can export depends on productivity and demand conditions, therefore the share of firms exporting varies with economic conditions. Once we fix share of exported varieties, the zero profit export cut-off condition becomes redundant. The model comparison analysis imply that removing the zero profit export cut-offs from the model improves the fit.

On the other hand, it may still be that trade patterns are better matched when the proportion of traded goods allowed to vary over time alongside the number of varieties available in the economy. A model with endogenous share of non-traded goods may replicate export dynamics better, although it does not improve the overall fit of the model. To investigate this, we present one-step-ahead forecasts (one-sided predicted values) of exports to GDP ratio along with the US exports to GDP ratio data in Figure 1.

Figure 1 shows that both models fit exports data reasonably well. Expectedly both models fail to account for the significant increase in exports prior to the recent financial crisis. Nevertheless models are able to replicate export patterns fairly well in the aftermath of the great trade collapse period; i.e. after mid-2009. The similarity between the predictive performance of the two models over the sample period implies that allowing for a variation in the share of exported goods does not improve the empirical fit of exports. In fact, the root mean square error (RMSE) statistics, which illustrates the deviations between

14 As an alternative we present the smoothed estimates (two-sided predicted values), which are also obtained through the Kalman filter, in Appendix H (see Figure 16).

15 The bilateral exports and imports data are only available in nominal terms. What we observe in the data is then:

\[
\frac{X_t}{Y_t} = \frac{S_t\tilde{p}_{X,t} N_{X,t}(\tilde{p}_{X,t})^{1-θ} C_t^*}{P_t Y_t}, \quad \frac{M_t}{Y_t} = \frac{\tilde{p}_{X,t} N_{X,t}(\tilde{p}_{X,t})^{1-θ} C_t}{P_t Y_t}
\]

This could cause measurement problems since in the model we present all variables relative to the CPI. But we evade this issue as a result of the observational equivalence between the above equations and the Equation 27 and Equation 28.
the predicted and observed values, show that Model 2 is preferable to Model 1 with a very small margin as the magnitude of the deviation is lower (RMSE Model 1: 64%, RMSE Model 2: 62%).

To elaborate on our comparison of fit to export dynamics, we now present the second moments of selected variables and compare them with the ones obtained from the models. Similar to our previous analysis, the statistics obtained from Model-1 and Model-2 are very similar to each other. There are differences at a very small margin. Both models successfully account for the quantitative properties of exports. Engel and Wang (2011) documents two key empirical regularity of exports: First, exports are highly volatile, second exports are pro-cyclical. Our models successfully account for both empirical facts. We also report the correlation between exports and the real exchange rate. Models are able to generate the positive relationship between the two variables. Although Model-1 and Model-2 underpredict the standard deviation of the real exchange rate and consumption, values are still fairly close to the data. Importantly, they produce a consumption volatility that is lower than output which is a well known business cycle feature of advanced economies. Both models match the persistence of exports and the real exchange rate reasonably well. In our model structure, nominal rigidities in the labour market combined with the endogenous persistence arising from the time lag in the number of firms help models to generate substantial persistence. However, they fail to match the persistence of output and consumption. These variables have higher persistence in the data than what Model-1 and Model-2 produce. Models are able to replicate the counter-cyclical behaviour of the real exchange rate. Despite matching the correlation of international variables, both models fail to account for the cross-country correlation of output and consumption. In fact, both models predict a negative correlation between the home and foreign output.
which is a well-known failure of international real business cycle models (international co-movement puzzle).

Overall, the analysis based on the second order properties of the data confirms our previous results. Both models do a fairly good job in accounting for the quantitative properties of exports. However, it remains difficult to distinguish between the model performances in terms of fit to the data. They produce very similar quantitative results. While Model-1 performs slightly better in matching some moments, the Model-2 does better in others.

<table>
<thead>
<tr>
<th>Table 3: Selected Second Moments</th>
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<tr>
<td></td>
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<tr>
<td>Std.dev.s (σ)</td>
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<tr>
<td>Exports</td>
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<tr>
<td>RER</td>
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<tr>
<td>σ(Cons.)/σ(GDP)</td>
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<tr>
<td>Autocorrelations</td>
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<td>Exports</td>
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<tr>
<td>GDP</td>
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<tr>
<td>RER</td>
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<td>Consumption</td>
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<td>Cross-Correlations</td>
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<td>Exports-GDP</td>
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<tr>
<td>Exports-RER</td>
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<tr>
<td>RER-GDP</td>
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<tr>
<td>GDP-GDP*</td>
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<tr>
<td>GDP-Cons.</td>
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<tr>
<td>Cons.-Cons.*</td>
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</tbody>
</table>

Note: All series are HP-filtered and in logarithms except for the exports data. Exports refer to the HP-filtred ratio of nominal exports to nominal US GDP. The moments are calculated for the US (home) and Euro Area (foreign) data for the period 1984:Q1-2014:Q4 with an exception. Bilateral exports data is available only after 1999. The statistics related to the exports -including the GDP and the real exchange rate for calculating their correlation- are calculated for the period 1999:Q1-2014:Q4. The model counterpart of the statics are obtained from the HP-filtered posterior mean.

So far, our results do not present satisfactory evidence supporting the existence of endogenous selection of exports for the model performance. We further test how these two models replicate the US import dynamics. Figure 2 shows the US HP-filtered imports (from the Euro Area) to GDP ratio data compared to the HP-filtered one-step-ahead forecast series\(^16\). Both models replicate the fluctuations of imports over the business cycle fairly well except for the period of the 'great trade collapse' (2007:Q3-2009:Q2). Importantly, when we compare the RMSE statistics, we find that the model with endogenous set of traded goods (Model-1) fits the data better (RMSE Model 1: 63%, RMSE Model 2: 66%). As in exports case, the difference between the predictive performance of the two models is very minor. Our investigation shows that incorporating fluctuations in the subset of exported varieties into the model does not present

\(^{16}\)We also plot the two sided predicted values of imports in Appendix H.
a significant improvement for the match with the trade dynamics. When there is an improvement, it is at a very small margin.

Before concluding this section, we repeat our analysis with the data until the recent financial crisis. As the trade collapsed during the crisis period, it is difficult to match the trade fluctuations. This may be the reason of having mixed results on the importance of endogenous tradability of varieties. We re-estimate our models for the period 1984:Q1-2006:Q4 with the same estimation method. The one-step-ahead forecast of the exports and imports are presented in Figure 3 and Figure 4, respectively. For space concerns, we do not report the posterior results we obtain from this estimation, but we show the marginal likelihood statistics in Figure 3 and Figure 4. The marginal likelihood comparison shows that the overall fit of Model-2 is better than Model-1 as in our benchmark estimations. In terms of matching the export and import dynamics, both models perform much better. Once we remove the crisis period from the estimations, for both imports and exports the RMSE statistics are lower than the benchmark analysis. Unlike our previous results, for both exports and imports Model-1 has a better prediction performance (a lower RMSE statistics). Although, this implies that, during ‘normal times’, a model that incorporates endogenous selection of traded varieties replicates trade patterns better, it is hard to argue that this feature is essential since the improvement is at a very small margin.

Our findings show that both Model-1 and Model-2 do reasonably well in matching export and import fluctuations between the US and the Euro Area except for the great trade collapse period. This analysis is important considering the relatively small literature on the ability of the international real business cycle models in accounting for the behaviour of exports and imports (see, Engel and Wang (2011)). Our results also confirm the findings presented by Ghironi and Melitz (2007). By using the model of Ghironi
and Melitz (2005), they show that this model can replicate the cyclical properties of the US trade over the business cycle. In our empirical analysis, we confirm this result, but, more importantly, we add to this literature by showing that this performance is mostly due to the firm entry, i.e. introduction of new varieties, rather than endogenous selection of the share of those varieties traded. When we remove the endogenous tradability assumption from the model (Model-2), we still find that the model replicates export fluctuations very closely to the one with endogenous tradability. In fact, removing this feature improves the model performance in terms of fit to the overall data and in some cases there is an improvement in predicting the export fluctuations.

5 Robustness

In this section, we check the sensitivity of our results on some key parameters of the model. We repeat our estimations and statistical analysis by varying the values of the duration of the wage contracts, elasticity of substitution between goods as well as iceberg costs. We use the same priors and the data as the benchmark analysis for the robustness estimations.

Wage Stickiness
It can be argued that the labour market in the US is more flexible than the Euro Area. To see the role of the rigidities in the Euro Area labour market, we increase the duration of wages ($\xi^*$) to 0.9 in the foreign country. Hence, while in the US the duration of wage contracts is 4 quarters, it is 10 quarters in the Euro Area.

Figure 5 and Figure 6 show the one-step-ahead forecast of exports and imports under asymmetric labour market structure. We also note the marginal likelihood statistics at the bottom of the figures along with the RMSE statistics. This exercise confirms our results from our benchmark analysis. In terms of overall fit to the data, Model-2 performs better as it has a higher marginal likelihood. But, the marginal likelihood statistics of the two models are somewhat close to each other; more than the benchmark case. Both models do a satisfactory job in reproducing export and import fluctuations except for the period of great trade collapse. As previously found, there is a slight improvement in export prediction of Model-2 and import prediction of Model-1. We conclude from this investigation that, cross-country differences in the labour market structure does not have a significant impact for the analysis of this paper.

**Elasticity of Substitution Between Differentiated Goods**

17 Even though, generally US labour markets are considered to be more flexible than they are in the Euro Area, the estimates of the Calvo parameter for wages in some cases found to be higher in the US than in the Euro Area. See, for instance, Smets and Wouters (2004).

18 We do not report the posterior statistics and the two-sided prediction results in the robustness section for space concerns. They are available on request from the authors.
Figure 5: One-sided predicted export values and actual exports: Wage Stickiness

RMSE Model-1: 0.67
RMSE Model-2: 0.63
Marginal Likelihood Model-1: 202.70
Marginal Likelihood Model-2: 204.503

Note: Data is the HP-filtered ratio of nominal exports of US to Euro Area to nominal US GDP. We also HP-filtered the one-sided predicted export values that we obtain from our estimations. The bilateral exports data is only available since the launch of the Euro. To match the periods between the data and Kalman filtered estimates, we drop the data points prior to 1999.
Figure 6: One-sided predicted import values and actual imports: Wage Stickiness

RMSE Model-1: 0.62
RMSE Model-2: 0.66
Marginal Likelihood Model-1: 202.70
Marginal Likelihood Model-2: 204.503

Note: Data is the HP-filtered ratio of nominal imports of US from Euro Area to nominal US GDP. We also HP-filtered the two-sided predicted import values that we obtain from our estimations.
In our benchmark calibration, we follow Ghironi and Melitz (2005) and set the value for the intratemporal elasticity of substitution between goods ($\theta$) to 3.8. This value yields a relatively high mark-up (around 35%) compared with the calibrations in the business cycle literature\(^{19}\). To test the sensitivity of the results to our parametrisation, we re-estimate the models by setting the value of the elasticity of substitution between differentiated goods to 6\(^{20}\). This value implies a 20% mark-up which is more standard in the literature.

Figure 7: One-sided predicted export values and actual exports: Elasticity of Substitution

We plot the exports and imports one-step-ahead forecasts resulting from this sensitivity analysis in Figure 7 and Figure 8. As in our previous results, Model-2 has a better overall fit to the data. It has a larger likelihood although the differences are small. The increase in the value of elasticity of substitution parameter slightly lowers the exports and imports predictive performance of both models as RMSE statistics higher than the benchmark analysis.

**Per Unit Export Cost**

Existence of trade costs in this model, differentiates the structure from a standard open economy

\(^{19}\)Their choice is result of the fixed costs in the model. Even though the mark-up over marginal cost is quite high, the mark-up over average cost is quite reasonable. See, Ghironi and Melitz (2005) for further details.

\(^{20}\)In our benchmark calibration, we ensure that the standard deviation of the log plant sales is consistent with the findings of Bernard et al. (2003). Consistently, here, we adjust the value of the dispersion parameter ($\ell$) so that the standard deviation remains in its initial value.
Figure 8: One-sided predicted import values and actual imports: Elasticity of Substitution

RMSE Model-1: 0.70
RMSE Model-2: 0.72
Marginal Likelihood Model-1: 98.342
Marginal Likelihood Model-2: 99.503

Note: Data is the HP-filtered ratio of nominal imports of US from Euro Area to nominal US GDP. We also HP-filtered the two-sided predicted import values that we obtain from our estimations.
DSGE model. The dynamics of the model are conditional on the parameterisation of these costs. To see the implications of the value of iceberg costs \((\tau, \tau^*)\) on our results, we decrease the value to 1.15 from 1.3.

In Figure 9 we report the one-step-ahead forecasts of exports when we reduce the value of iceberg costs in both countries. Unlike our previous estimations, the marginal likelihood analysis shows that Model-1 performs better than Model-2 with a lower per unit export cost. In terms of exports prediction, the predictive performance of both models are very close to the benchmark case. They both predict export fluctuations reasonably well over the business cycle -except for the crisis period- with a RMSE statistics around 60%. Similar results emerge for imports prediction (see, Figure 10). Although two model specifications generate very similar import forecasts, Model-1 has a marginally smaller RMSE.

Figure 9: One-sided predicted export values and actual exports: Iceberg Cost

<table>
<thead>
<tr>
<th></th>
<th>RMSE Model-1</th>
<th>RMSE Model-2</th>
<th>Marginal Likelihood Model-1</th>
<th>Marginal Likelihood Model-2</th>
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<tbody>
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<td>Data</td>
<td></td>
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<tr>
<td>Model-1</td>
<td>0.63</td>
<td>0.62</td>
<td>62.598</td>
<td>53.541</td>
</tr>
<tr>
<td>Model-2</td>
<td></td>
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Note: Data is the HP-filtered ratio of nominal exports of US to Euro Area to nominal US GDP. We also HP-filtered the one-sided predicted export values that we obtain from our estimations. The bilateral exports data is only available since the launch of the Euro. To match the periods between the data and Kalman filtered estimates, we drop the data points prior to 1999.

To sum up, in our sensitivity checks we find that the changing the values of the key parameters of the models do not significantly change the export -and also import- prediction performance. The robustness exercise further confirm that allowing for variations on the share of exported varieties either does not improve the model performance or it does so at a very small margin.

**Fixed Export Cost**

One possible reason of having non-clear results on the importance of endogenous tradability for
Figure 10: One-sided predicted import values and actual imports: Iceberg Cost

RMSE Model-1: 0.64
RMSE Model-2: 0.68
Marginal Likelihood Model-1: 62.598
Marginal Likelihood Model-2: 53.541

Note: Data is the HP-filtered ratio of nominal imports of US from Euro Area to nominal US GDP. We also HP-filtered the two-sided predicted import values that we obtain from our estimations.
understanding trade fluctuations can be related to trade costs. In the model set-up, to start producing its variety a firm needs to pay a one time fixed entry cost, to export that variety on the other hand, costs have to be paid each and every period. Hence, it may be that exporting is too costly and consequently cut-off point does not vary significantly in Model-1. To test for the implications of trade costs, we repeat our analysis by reducing the fixed exporting cost substantially. We lower the value of fixed exporting cost to 0.004 from 0.0085.

Figure 11: One-sided predicted export values and actual exports: Fixed Export Cost

![Graph showing predicted export values and actual exports](image)

- RMSE Model-1: 0.57
- RMSE Model-2: 0.51
- Marginal Likelihood Model-1: 46.250
- Marginal Likelihood Model-2: 85.282

Note: Data is the HP-filtered ratio of nominal exports of US to Euro Area to nominal US GDP. We also HP-filtered the one-sided predicted export values that we obtain from our estimations. The bilateral exports data is only available since the launch of the Euro. To match the periods between the data and Kalman filtered estimates, we drop the data points prior to 1999.

We show the one-step-ahead forecast of exports and imports in Figures 11 and 12 together with the RMSE and marginal likelihood statistics. Interestingly, the results confirm our benchmark analysis. Model-2 matches the overall business cycle dynamics of the US and the Euro Area better than the Model-1. Both models predict the exports and imports fluctuations reasonably well except for the period of great trade collapse. But it is not clear whether having endogenous number of exporters, i.e. varieties, is critical for our understanding of US dynamics or not. This exercise shows that our results are not a consequence of relatively high trade costs. Our results rather imply that variations in the share of exporters does not have a significant impact on aggregate trade dynamics.
Figure 12: One-sided predicted import values and actual imports: Fixed Export Cost

RMSE Model-1: 0.62
RMSE Model-2: 0.67
Marginal Likelihood Model-1: 46.250
Marginal Likelihood Model-2: 85.282

Note: Data is the HP-filtered ratio of nominal imports of US from Euro Area to nominal US GDP. We also HP-filtered the two-sided predicted import values that we obtain from our estimations.
6 Conclusion

In this paper, we estimate two similar models with heterogeneous firms and endogenous entry, one where the share of exported varieties varies endogenously, and one where that share is constant, using data for the US and the euro area. We investigate whether the model which endogenises the share of exported varieties can help us understand aggregate export fluctuations over the business cycle. Different from previous evaluations, we take into account that business cycle fluctuations arise not only from productivity shocks but also from monetary policy shocks, preference shocks and shocks to the uncovered interest rate parity condition. A comparison of the two models’ overall performance as well as their ability to help us understand US export dynamics shows that, while allowing for heterogeneous firms and endogenous entry helps match US export dynamics, endogenising the share of exported varieties does not significantly progress our understanding of aggregate export dynamics.

The conclusion from our paper is not that firms’ endogenous decisions on whether to export or not is unimportant for our understanding of overall export dynamics. Instead, we conclude that in order to improve our understanding of export dynamics, further research on the ways in which the share of exported varieties and the extensive margin of trade respond to different shocks over the business cycle is necessary. We thus encourage a research agenda focusing on exporting firms’ behaviour in response to business cycle shocks, so that we can improve our understanding of the determinants of changes in the intensive as well as the extensive margin of trade.
Appendices

Appendix A  Steady State: Model-1

In this section we list the steady state of the system. The steady state of the level variables will be indicated by overbars \( \bar{\cdot} \). There are no technological improvements in steady state: \( \bar{Z} = \bar{Z}^* = 1 \). We further assume: \( \bar{\tau} = \bar{\tau}^*, \bar{f}_E = \bar{f}_E^*, \bar{C} = \bar{C}^* \) and \( \bar{L} = \bar{L}^* \). These assumptions imply a symmetric steady state: \( \bar{Q} = \bar{Q} = \frac{\bar{C}}{\bar{C}^*} = 1 \).

We first pin down the export cut-off, \( \bar{z}_x \), and import prices, \( \bar{\rho}_X \), then solve for the other endogenous variables.

In steady state, the free entry equation implies that \( \bar{v} = \bar{w} \bar{f}_E \). We can re-write the Euler equation for share holdings (13) as:

\[
\bar{v} = \bar{w} \bar{f}_E = \frac{\beta(1-\delta)}{1-\beta(1-\delta)} \bar{\Pi} \quad (A.1)
\]

We use Equation (A.1) to get the steady state average profit:

\[
\bar{\Pi}_X = (\theta - 1) \left( \bar{w} \bar{f}_X \right) \left( \frac{\theta^{\theta-1}}{k} \right) \quad (A.2)
\]

Using equations (5), the average profits of home firms in steady state can be written as:

\[
\bar{\Pi}_D = \left( \bar{\rho}_D \right)^{(1-\theta)} C, \quad \bar{\Pi}_X = \left( \bar{\rho}_X \right)^{(1-\theta)} \bar{C} - \bar{w} \bar{f}_X \quad (A.3)
\]

This implies:

\[
\bar{\Pi}_D = \left( \frac{\bar{\rho}_X}{\bar{\rho}_D} \right)^{(\theta-1)} \left( \bar{\Pi}_X + \bar{w} \bar{f}_X \right)
\]

We can replace \( \bar{\Pi}_X \) by Equation (A.2) and plug in the steady state prices, \( \bar{\rho}_D = \frac{\theta}{(\theta-1) \bar{z}_X} \), \( \bar{\rho}_X = \frac{\theta}{(\theta-1) \bar{z}_X} \):

\[
\bar{\Pi}_D = \left( \frac{\bar{\rho}_D}{\bar{\rho}_X} \right)^{(\theta-1)} \left( (\theta-1) \bar{w} \bar{f}_X \left( \frac{\theta^{\theta-1}}{k} \right) + \bar{w} \bar{f}_X \right) \quad (A.4)
\]

The steady state share of exporting firms among all domestic firms is:

\[
\frac{\bar{N}_X}{\bar{N}_D} = \left( \frac{z_{min} \phi}{\bar{z}_X} \right)^k \quad (A.5)
\]

We know that, the average total dividends in home country are:

\[
\bar{\Pi} = 1 - \frac{\beta(1-\delta)}{\beta(1-\delta)} \bar{w} \bar{f}_E = \bar{\Pi}_D + \left[ \frac{\bar{N}_X}{\bar{N}_D} \right] \bar{\Pi}_X \quad (A.6)
\]

\(^{21}\)This section follows the online appendix provided by the authors for the Ghironi and Melitz (2005) paper.

\(^{22}\)Note, we are using the symmetric steady state assumption: \( C = C^* \)
We can now plug in equations (A.2), (A.4) and (A.5) into Equation (A.6) and solve for $\tilde{z}_X$:

$$\frac{1 - \beta(1 - \delta)}{\beta(1 - \delta)} \bar{f}_X = (\tilde{z}_X)^{1 - \delta} (\bar{T}_{z_{\min}})_{\delta - 1} \phi^{(\theta - 1)} + (\tilde{z}_X)^{-k} (z_{\min})^k \phi^k \left( \frac{\theta - 1}{k - (\theta - 1)} \right)$$

where $\phi \equiv \{k/(k - (\theta - 1))\}^{1/(\theta - 1)}$. We can solve the steady state value of $\tilde{z}_X$ numerically by plugging in the value of parameters.

We will now solve for the steady state value of $\tilde{\rho}_X$.

The steady state market clearing condition is:

$$\bar{C} = \bar{w} \bar{L} + \bar{N}_D \bar{P} - \bar{N}_E \bar{v}$$

The total number of firms in steady state becomes:

$$\bar{N}_E = \frac{\delta}{1 - \delta} \bar{N}_D$$

By using this, we can re-write the market clearing as:

$$\frac{\bar{C}}{\bar{w}} = \bar{L} + \bar{N}_D \bar{f}_E \frac{1 - \beta}{\beta(1 - \delta)}$$

Also from $\bar{P}_X = (\bar{\rho}_X)^{(1 - \theta)} \bar{C} - \bar{w} \bar{f}_X = (\bar{w} \bar{f}_X) \left( \frac{\phi^{\theta - 1}}{k} \right)$ as:

$$\frac{\bar{C}}{\bar{w}} = (\bar{\rho}_X)^{(\theta - 1)} \theta \phi^{(\theta - 1)} \bar{f}_X$$

Combining equations (A.8) and (A.9) yields:

$$(\bar{\rho}_X)^{(\theta - 1)} \theta \phi^{(\theta - 1)} \bar{f}_X = \bar{L} + \bar{N}_D \bar{f}_E \frac{1 - \beta}{\beta(1 - \delta)}$$

We will use the price index in symmetric steady state: $1 = \bar{N}_D (\bar{\rho}_D)^{1 - \theta} + \bar{N}_X (\bar{\rho}_X)^{1 - \theta}$. When we multiply this expression with $(\bar{\rho}_X)^{\theta - 1}/\bar{N}_D$, we obtain:

$$\frac{(\bar{\rho}_X)^{\theta - 1}}{\bar{N}_D} = \left( \frac{\bar{\rho}_X}{\bar{\rho}_D} \right)^{\theta - 1} + \frac{\bar{N}_X}{\bar{N}_D}$$

Plugging in Equation (A.5) and using $\bar{\rho}_X/\bar{\rho}_D = \bar{T}_{z_{\min}} \phi/\bar{z}_x$ gives:

$$\frac{(\bar{\rho}_X)^{\theta - 1}}{\bar{N}_D} = \left( \bar{T}_{z_{\min}} \phi \right)^{\theta - 1} + \left( \frac{z_{\min}}{\bar{z}_x} \right)^k \phi^k$$

We can now solve for $\bar{\rho}_X$ by plugging in Equation (A.12) into the Equation (A.10):

$$(\bar{\rho}_X)^{1 - \theta} = \left( \frac{\theta \phi^{\theta - 1} \bar{f}_X - A^{-1} \bar{f}_E \frac{1 - \beta}{\beta(1 - \delta)}}{\bar{L}} \right)$$

where A is the right hand side of the Equation (A.12).

The list of all the steady state relationships is as follows:
• $1 + r = 1/\beta \rightarrow$ Consumption Euler equation

$$\frac{1 - \beta(1 - \delta)}{\beta(1 - \delta)} \bar{f}_E = (\ddot{z}_X)^{1-\theta} (\ddot{\tau} z_{min})^{\theta-1} \phi^{2(\theta-1)} + (\ddot{z}_X)^{-k} (z_{min})^k \phi^k \left( \frac{\phi^{\theta-1}}{k} \right)$$

$$\bar{\rho}_X^{1-\theta} = \frac{\left\{ \theta \phi^{\theta-1} \ddot{f}_X - A^{-1} \bar{f}_E \frac{1 - \beta}{\beta(1 - \delta)} \right\}}{L}$$

• $\bar{N}_D = A^{-1}(\bar{\rho}_X)^{\theta-1} \rightarrow$ Equation (A.12)

$\bar{N}_E = \frac{\theta}{1-\theta} \bar{N}_D$

$\bar{\rho}_D = \bar{\rho}_X \ddot{z}_X \bar{\tau}_{\min} \phi \rightarrow \bar{\rho}_D = \bar{\tau} z_{\min} \phi / \ddot{z}_X$

$\bar{w} = \frac{\theta}{1-\theta} \bar{\rho}_X \ddot{z}_X \rightarrow \bar{\rho}_X = \frac{\theta}{(\theta-1)} \frac{\ddot{w}}{\ddot{z}_X}$

• $\bar{C} = \bar{w} \left( \bar{L} + \bar{N}_D \bar{f}_E \frac{1 - \beta}{\beta(1 - \delta)} \right) \rightarrow$ Equation (A.9)

$\bar{v} = \bar{w} \bar{f}_E \rightarrow$ free entry condition

$\ddot{\Pi} = \frac{1 - \beta(1 - \delta)}{\beta(1 - \delta)} \bar{w} \bar{f}_E \rightarrow$ Share holdings Euler equation

$\ddot{\Pi}_D = \frac{(\bar{\rho}_D)^{(1-\theta)}}{\theta} \bar{C}$

$\ddot{\Pi}_X = \frac{(\bar{\rho}_X)^{(1-\theta)}}{\theta} - \bar{w} \ddot{f}_X$

$\bar{N}_X = \left( \frac{\ddot{z}_X}{\ddot{z}_X} \right)^k \bar{N}_D \rightarrow$ Equation (A.5)

### Appendix B  List of Equilibrium Conditions: Model-1

• Price Index: $1 = N_{D,t}(\bar{\rho}_{D,t})^{1-\theta} + N_{X,t}(\bar{\rho}_{X,t})^{1-\theta}$

• Prices: $\bar{\rho}_{D,t} = \frac{\theta}{(\theta-1)} Z_t w_t / z_{\min} \phi$ and $\bar{\rho}_{X,t} = \frac{\theta}{(\theta-1)} Q_t \tau_t w_t / z_{X,t}$

where $\phi \equiv \{k/(k-(\theta-1))\}^{1/(\theta-1)}$

• Free Entry: $\ddot{v}_t = w_t f_{E,t}/Z_t$

• Number of Firms: $N_{D,t} = (1 - \delta)(N_{D,t-1} + N_{E,t-1})$

• Total Profits: $\ddot{\Pi}_t = \ddot{\Pi}_D^\phi + \left[ \frac{N_{X,t}}{N_{D,t}} \right] \ddot{\Pi}_X^\phi$
• Profits: \( \Pi^D_t = \frac{(\hat{\rho}_D)_{t}\left(1-\theta \right) C_t}{\theta} \), \( \Pi^X_t = \frac{Q_t(\hat{\rho}_X)_{t}\left(1-\theta \right) C^*_t}{\theta} - \frac{w_t f_{X,t}}{Z_t} \)

• Bond Euler Equation:

\[
1 = E_t \left[ \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} \left( \frac{X_{t+1}}{X_t} \right) (1 + r_{t+1}) \right]
\]

• Share holdings Euler Equation:

\[
\tilde{v}_t = E_t \left[ \beta(1 - \delta) \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} \left( \frac{X_{t+1}}{X_t} \right) (\tilde{v}_{t+1} + \Pi_t) \right]
\]

• UIP: \( 1 = \beta \psi_t (1 + r_{t+1}) \Theta(1) Q_t B_{F,t+1} E_t \left[ \beta(1 - \delta) \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} \left( \frac{X_{t+1}}{X_t} \right) \frac{Q_{t+1}}{Q_t} \right] \)

• Share of exporting firms: \( \frac{N_{X,t}}{N_{D,t}} = \left( z_{\min}\phi \right)^k \)

• Zero profit export cut-off: \( \tilde{\Pi}^X_t = (\theta - 1) \left( \frac{w_t f_{X,t}}{Z_t} \right) \left( \frac{\phi^{\theta-1}}{k} \right) \)

• Wage Phillips curve:

\[
\sum_{k=0}^{\infty} (\beta \xi)^k E_t \left[ L_{t+k}(i) U_C(C_{t+k}) \left( 1 + \sigma \right) \frac{\tilde{W}_t(i)}{P_{t+k}} - \frac{\theta w}{\theta w - 1} MRS_{t+k} \right] = 0
\]

• Labour Market Clearing:

\[
L_t = \frac{\theta - 1}{w_t} N^D_t \tilde{\Pi}^d_t + \frac{\theta - 1}{w_t} N^X_t \tilde{\Pi}^X_t + \frac{\theta}{Z_t} N^X_t f_{X,t} + \frac{1}{Z_t} N^E_t f_{E,t}
\]

• Current Account:

\[
\frac{Q_t B_{F,t}}{(1 + r_t) \Theta(1)} = Q_t B_{F,t-1} - w_t L_t + N_{D,t} \tilde{\Pi}_t - C_t - N_{E,t} \tilde{v}_t
\]

• GDP: \( Y_t = C_t + C_{X,t} - C^*_t + N_{E,t} \tilde{v}_t \)

• Exports: \( \frac{X_t}{Y_t} = \frac{Q_t \hat{\rho}_X N_{X,t}(\hat{\rho}_X)_{t}^{1-\theta} C^*_t}{Y_t} \)

• Imports: \( \frac{M_t}{Y_t} = \frac{\hat{\rho}_X N_{X,t}(\hat{\rho}_X)_{t}^{1-\theta} C_t}{Y_t} \)

The equilibrium conditions for the foreign country are analogous.

### Appendix C  Log-Linearised System of Equations: Model-1

In this section, we present the log-linear approximation of the model around the non-stochastic steady state. We use lower case variables to denote the deviations of a variable from its steady state value\(^{23}\).
Price Indices: $(\tilde{p}_D)^{1-\theta} \tilde{N}_D ((1-\theta) \tilde{p}_{D,t} + n_{D,t}) = -(\tilde{p}_X)^{1-\theta} \tilde{N}_X ((1-\theta) \tilde{p}_{X,t} + n_{X,t})$

Prices: $\tilde{p}_{D,t} = \bar{w}_t - \bar{z}_t$ and $\tilde{p}_{X,t} = \bar{w}_t - q_t - \bar{z}_t - \tilde{z}_X,t$

Free Entry: $\tilde{v}_t = \bar{w}_t - \bar{z}_t$

Number of Firms: $n_{D,t} = (1-\delta) \left( n_{D,t-1} + \frac{\bar{N}_E}{\bar{N}_D} n_{E,t-1} \right)$

Total Profits: $\pi_t = \frac{\bar{N}_D}{\bar{N}} \tilde{p}_{D,t} + \frac{\bar{N}_X}{\bar{N}_D} (\tilde{p}_{X,t} + n_{X,t} - n_{D,t})$

Profits: $\tilde{\pi}_{D,t} = (1-\theta) \tilde{p}_{D,t} + c_t$ and

$$\tilde{\pi}_{X,t} = \frac{1}{\bar{N}_X} \left[ \frac{1}{\theta} \bar{C}^*(\tilde{p}_X)^{1-\theta} (q_t + (1-\theta) \tilde{p}_{X,t} + c_t^*) - \bar{w}_t (w_t - z_t) \right]$$

Bond Euler Equation: $c_t = E_t c_{t+1} - \frac{1}{\gamma} (\bar{t}_t - (E_t \bar{p}_{t+1} - p_t)) - \frac{1}{\gamma} (\chi_{t+1} - \chi_t)$

Share holdings Euler Equation: $\tilde{v}_t = \gamma (c_t - E_t c_{t+1}) + (E_t \chi_{t+1} - \chi_t) + \frac{\bar{v}}{\bar{v} + \bar{N}} E_t \tilde{v}_{t+1} + \frac{\bar{N}}{\bar{v} + \bar{N}} E_t \tilde{\pi}_{t+1}$

UIP Condition: $E_t q_{t+1} - q_t = (i_t - E_t (p_{t+1} - p_t)) - (i_t^* - E_t (p_{t+1}^* - p_t^*)) + \omega b_t + \psi_t$

Share of Exporting Firms: $n_{X,t} = n_{D,t} - k \tilde{z}_X,t$

Zero Profit Export Cut-off: $\tilde{\pi}_{X,t} = \bar{w}_t - \bar{z}_t$

Wage Phillips Curve: $\pi_t^\varphi = \beta E_t \pi_{t+1}^\varphi - \lambda (w_t - \eta \bar{t}_t - c_t)$

where $\lambda = \frac{(1-\beta \xi)(1-\xi)}{(1+\eta \theta_w)\xi}$

Market Clearing:

$$l_t = (\theta - 1) \left[ \frac{\bar{N}_D}{\bar{W} \bar{L}} (n_{D,t} + \pi_{D,t} - w_t) + \frac{\bar{N}_X}{\bar{W} \bar{L}} (n_{X,t} + \pi_{X,t} - w_t) \right]$$

$$+ \theta \left[ \frac{\bar{N}_X}{\bar{W} \bar{L}} (n_{X,t} - z_t) + \frac{\bar{N}_E}{\bar{W} \bar{L}} (n_{E,t} - z_t) \right]$$

Current Account: $\beta c_t - b_{t-1} = \bar{W} \bar{L} (w_t + l_t) + \bar{N}_D \pi_{D,t} + \bar{C} c_t - \bar{N}_E \bar{v} (n_{E,t} + v_t)$

Exports: $x_t = \tilde{p}_{X,t} + q_t + n_{X,t} - \theta \tilde{p}_{X,t} + c_t^*$

Imports: $m_t = \tilde{p}_{X,t} + n_{X,t} - \theta \tilde{p}_{X,t} + c_t$

Output:

$$y_t = \bar{C} \frac{C_t}{Y} c_t - \frac{\bar{N}_X}{\bar{Y}} (\tilde{p}_X)^{-\theta \bar{C}} \left( n_{X,t} - \theta \tilde{p}_{X,t} + c_t \right) + \frac{\bar{N}_X}{\bar{Y}} (\tilde{p}_X)^{-\theta \bar{C}} \left( n_{X,t} - \theta \tilde{p}_{X,t} + c_t^* \right) + \frac{\bar{N}_E}{\bar{Y}} (n_{E,t} + v_t)$$
RER:

\[
\tilde{q}_t = \left(2ND\tilde{\rho}_D^{1-\theta} - 1 \right) tol_t + \left(1 - ND\tilde{\rho}_D^{1-\theta} \right) (\tilde{z}_{X,t} - \tilde{z}_{X,t}) -
\]

\[
+ \frac{1}{\theta - 1} \left( \frac{ND}{ND + NX} - \frac{ND\tilde{\rho}_D^{1-\theta}}{\tilde{\rho}_D} \right) \left((n^*_D,t - n_{X,t}) - (n_D,t - n^*_X,t)) \right)
\]

where \( tol_t = q_t + (w^*_t - z^*_t) - (w_t - z_t) \) corresponds to the terms of labour.

Taylor Rule: \( i_t = \Gamma_{i_{t-1}} - \left(1 - \Gamma_{i_{t-1}}\right) \Gamma_{\epsilon}(p_t - p_{t-1}) + \epsilon_t \)

Stochastic Processes:

\[ z_t = \rho_z z_{t-1} + \epsilon_{z,t} \]
\[ \chi_t = \rho_{\chi} \chi_{t-1} + \epsilon_{\chi,t} \]
\[ \psi_t = \rho_{\psi} \psi_{t-1} + \epsilon_{\psi,t} \]

Appendix D  List of Equilibrium Conditions: Model-2

The equilibrium conditions in Model-2 are different from Model-1 since we do not allow the share of exporters to vary any more.

The equilibrium conditions that are different from Model-1 are:

- Prices: \( \tilde{\rho}_{D,t} = \frac{\theta}{(\theta - 1)} \tilde{w}_t \frac{\phi}{z_t \tilde{z}_{min,\theta}} \) and \( \tilde{\rho}_{X,t} = \left( \frac{\theta}{(\theta - 1)} \tilde{w}_t \tilde{z}_{X} \frac{\phi}{z_t} \right) \frac{\phi}{z_t \tilde{z}_{min,\theta}} \)

where \( \phi \equiv \left( \frac{k}{k - (\theta - 1)} \right)^{1/(\theta - 1)} \)

- Share of exporting firms: \( \frac{N_{X,t}}{N_{D,t}} = \left( \frac{\tilde{z}_{min,\theta}}{z_X} \right)^{k} \)

The rest of the equilibrium conditions are same as Model-1 with only one exception. Given the fixed productivity threshold in this model, the zero export cut-off condition disappears. See, Appendix B for the remaining equilibrium conditions.

Appendix E  Steady State: Model-2

Here, we list the steady state of the model with fixed export productivity threshold. As in our benchmark model, the steady state of the level variables will be indicated by over bars. There are no technological improvements in steady state: \( \tilde{Z} = \tilde{Z}^* = 1 \). We further assume: \( \tau = \tilde{\tau}^* \), \( f_E = \tilde{f}_E \), \( C = \tilde{C}^* \) and \( L = \tilde{L}^* \).

We first pin down the steady state value of total number of firms, \( \bar{N}_D \) then solve for the other endogenous variables.

From the steady state derivations of the benchmark model, we know that:

\[
\bar{v} = w\tilde{f}_E = \frac{\beta(1 - \delta)}{1 - \beta(1 - \delta)} \bar{\Pi}
\]
The prices in steady state: \( \tilde{\rho}_D = \frac{\theta}{(\theta - 1)} \tilde{w}_D = \frac{\theta}{(\theta - 1)} \frac{\omega}{\phi z_{min}} \), \( \tilde{\rho}_X = \frac{\theta}{(\theta - 1)} \omega \). We will re-write the price index by using this relationship:

\[
1 = N_D \tilde{\rho}_D^{1-\theta} \left[ 1 + \left( \frac{\phi z_{min}}{z_X} \right)^k \left( \frac{\tau \phi z_{min}}{z_X} \right)^{1-\theta} \right] \tag{E.2}
\]

We have previously derived the steady state value of the average total dividends:

\[
\bar{\Pi} = 1 - \beta(1 - \delta) \bar{\omega} \bar{f}_E = \bar{\Pi}_D + \left[ \frac{\bar{N}_X}{N_D} \right] \bar{\Pi}_X \tag{E.3}
\]

Recall equation A.3:

\[
\bar{\Pi}_D = \left( \tilde{\rho}_D \right)^{(1-\theta)} \frac{C}{\theta}, \quad \bar{\Pi}_X = \left( \tilde{\rho}_X \right)^{(1-\theta)} \frac{C}{\theta} - \bar{w} \bar{f}_X
\]

We can re-write \( \bar{\Pi}_X \) as:

\[
\bar{\Pi}_X = \left( \tilde{\rho}_D \right)^{(1-\theta)} \frac{C}{\theta} \left( \frac{\tau \phi z_{min}}{z_X} \right)^{1-\theta} - \bar{w} \bar{f}_X
\]

The steady state share of exporting firms is:

\[
\frac{\bar{N}_X}{N_D} = \left( \frac{z_{min} \phi}{z_X} \right)^k \tag{E.4}
\]

We will plug this into E.3 along with the profit equations:

\[
\frac{1 - \beta(1 - \delta)}{\beta(1 - \delta)} \bar{w} \bar{f}_E = \left( \tilde{\rho}_D \right)^{(1-\theta)} \frac{C}{\theta} + \left( \frac{z_{min} \phi}{z_X} \right)^k \left( \frac{\tau \phi z_{min}}{z_X} \right)^{1-\theta} - \bar{w} \bar{f}_X
\]

We can re-write this as:

\[
\frac{1 - \beta(1 - \delta)}{\beta(1 - \delta)} \bar{f}_E + \left( \frac{z_{min} \phi}{z_X} \right)^k \bar{f}_X = \left( \tilde{\rho}_D \right)^{(1-\theta)} \frac{C}{\theta} \left( \frac{\tau \phi z_{min}}{z_X} \right)^{1-\theta} + \left( \frac{z_{min} \phi}{z_X} \right)^k \left( \frac{\tau \phi z_{min}}{z_X} \right)^{1-\theta}
\]

We use the equation E.2 to replace \( \left( \tilde{\rho}_D \right)^{(1-\theta)} \) and the steady state market clearing condition (equation A.8) to replace \( \bar{C} / \bar{w} \). We then solve it for \( N_D \):

\[
N_D = \left[ \theta \left( \frac{1 - \beta(1 - \delta)}{\beta(1 - \delta)} \right) \bar{f}_E + \theta \left( \frac{z_{min} \phi}{z_X} \right)^k \bar{f}_X - \left( \frac{1 - \beta}{\beta(1 - \delta)} \right) \bar{f}_E \right]^{-1} \tag{E.5}
\]

The derivation of the rest of the steady state relationships is trivial.

**Appendix F  Log-Linearised System of Equations: Model-2**

In this section, we present log-linearised system of equations of Model-2. We only show the equations that are different from the ones listed in Appendix C for Model-1:

**Prices**: \( \tilde{\rho}_{D,t} = w_t - z_t \) and \( \tilde{\rho}_{X,t} = w_t - q_t - z_t + \tau_t \)

**Share of Exporting Firms**: \( n_{x,t} = n_{D,t} \)

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RER:
\[
\tilde{q}_t = \left(2N_D\tilde{\rho}^{1-\theta}_D - 1\right)\ t_{0t} + \left(1 - N_D\tilde{\rho}^{1-\theta}_D\right)\left(\tau_t - \tau_t^*\right) \\
+ \frac{1}{\theta - 1}\left(\frac{N_D}{N_D + N_X} - N_D\tilde{\rho}^{1-\theta}_D\right)\left((n_{D,t} - n_X,t) - (n_{D,t} - \dot{n}_X,t)\right)
\]
Remember that, zero profit export cut-off equation drops off in this model.

Appendix G Data

For the US, we take the quarterly real GDP and real consumption data from the Bureau of Economic Analysis (BEA). Both series are seasonally adjusted and expressed at annual rates in chained 2009 prices. We obtain the US CPI series from the OECD Main Economic Indicators database to calculate the inflation rate. All data for the Euro Area as well as the bilateral nominal exchange rate come from the Area Wide model (AWM) (see, Fagan et al. (2001)). The nominal exchange rate is defined as Euro per US dollar in the AWM database. Since in the model nominal exchange rate is the home currency price of the foreign currency and the US is the home country, we take the inverse of the series. The population data for the age between 15 and 64 for the US are taken from the OECD Historical population data and projections and for the Euro Area from the EUROSTAT. Both population series are available at annual frequency; we transform them into quarterly data by using linear interpolation. To express output and consumption in per capita terms, we divide them by population. We multiplied output and consumption series by 100 after taking logs and de-trending.

We use exports and imports data for our analysis. Quarterly bilateral export and import data are taken from the US Census Bureau. The series are reported in the US dollar. We seasonally adjust them by using the X-13 ARIMA method. Since these series are only available in nominal terms, we divide them by the nominal US GDP and measure them as a ratio of US nominal GDP. The data for the US nominal GDP is taken from the OECD Quarterly National Accounts.
Figure 13: Data Series

Note: All series are in natural logarithms. $c$ is the consumption of the US, $c^*$ is the consumption of the Euro Area, $y$ is the output of the US, $y^*$ is the output of the Euro Area, $\pi$ and $\pi^*$ are the inflation of the US and Euro Area respectively, finally, $\tilde{q}$ is the bilateral real exchange rate. Consumption and output for both countries are HP-filtered and expressed in percentages. Inflation and real exchange rate series are demeaned.
Figure 14: Model-1: Prior and Posterior Distributions
Figure 15: Model-2: Prior and Posterior Distributions
Figure 16: Two-sided predicted export values and actual exports

Data
Model−1
Model−2

RMSE Model-1: 0.80
RMSE Model-2: 0.75

Note: Data is the HP-filtered ratio of nominal exports of US to Euro Area to nominal US GDP. We also HP-filtered the two-sided predicted export values that we obtain from our estimations.
Figure 17: Two-sided predicted import values and actual imports

RMSE Model-1: 0.63
RMSE Model-2: 0.67

Note: Data is the HP-filtered ratio of nominal imports of US from Euro Area to nominal US GDP. We also HP-filtered the two-sided predicted import values that we obtain from our estimations.
References


