The Marginal Product of Capital: New Facts and Interpretation

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Convergence in aggregate MPKs across countries

Mean-reversion

Fall in dispersion

Notes: (a) Geometric growth rate and MPK at the beginning of the sample are calculated relative to the world. The OLS coefficient is -.5377, significant at the 1% level. (b) The standard deviation of MPKs reduces on average by .0005 each year.
No correlation with capital inflows

Capital Inflows and Changes in MPKs

Capital Inflows (relative to initial output)

Change in MPK 1981-2007
What drives the convergence in MPKs?

1. New Facts
   - Convergence in MPKs
   - Pos. correlation with capital shares
   - Neg. correlation with capital-labor ratio
   - No correlation with capital inflows
   - Increase in North-South trade
What drives the convergence in MPKs?

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2. Interpretation: Trade integration leads to convergence
   - Multi-country, dynamic Heckscher-Ohlin-Krugman model with costly trade
   - Two channels:
     - Sectoral reallocation
     - Aggregate savings
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     - Aggregate savings

3. Quantification
Literature

- **Efficiency of capital allocation**
  - Caselli & Feyrer (2007)
  - Heathcote & Perri (2013)
  - Chirinko & Mallick (2008), Lowe et al. (2012)
  - David et al. (2014), Monge-Naranjo et al. (2015)

- **Allocation puzzle**
  - Gourinchas & Jeanne (2013), Aizenmann et al. (2007), Alfaro et al. (2014)

- **International Trade**

- **Labor share & CES**
How do I measure the MPK?

Following Caselli & Feyrer:

\[
\text{Return to capital}_{c,t} = (1 - \alpha_{L_{c,t}} - \alpha_{N_{c,t}}) \frac{P^Y_{c,t} Y_{t,i}}{P^K_{c,t} K_{c,t}} + (1 - \delta_{c,t}) \frac{P_{K_{c,t+1}}}{P_{K_{c,t}}}
\]

**Four adjustments:**

1. Natural capital
2. Relative prices
3. Depreciation rate
4. Capital gain (future price of capital goods)

Convergence Return
Evidence across countries

### OLS Regression for Avg. Return 1970-2012

<table>
<thead>
<tr>
<th>Explanatory variable</th>
<th>Slope coefficient</th>
<th>Constant</th>
<th>$R^2$</th>
<th>Obs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Avg. $\frac{Y}{L}$</td>
<td>$-0.011^*$</td>
<td>0.041***</td>
<td>0.063</td>
<td>60</td>
</tr>
<tr>
<td></td>
<td>(0.054)</td>
<td>(0.000)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Avg. $\frac{K}{L}$</td>
<td>$-0.013^{***}$</td>
<td>0.045***</td>
<td>0.133</td>
<td>60</td>
</tr>
<tr>
<td></td>
<td>(0.004)</td>
<td>(0.000)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Avg. Capital Share</td>
<td>0.426***</td>
<td>0.005</td>
<td>0.624</td>
<td>60</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.182)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$; p-values in parentheses. Averages are computed over the deviation from the cross-country mean weighted by capital.
Evidence over time
Channels of convergence

Capital accumulation

Sectoral Reallocation

Notes: Geometric growth rates are calculated relative to the world. (a) OLS coefficient of -.137 is significant at the 1% level. (b) OLS coefficient of .587 is significant at the 1% level.
Additional Facts

- North-South Trade has increased
- No capital inflows to high MPK economies
Model description

- **Set-up**
  - World of $N_c$ countries
  - Labor and capital as factors of production
  - Country $c$ defined by $\left( \beta, A^L, A^K, L, \{\tau_{c,c'}\}^{N_c}_{c'=1} \right)$

- **Consumption**
  - Overlapping generations model
  - Closed international financial markets

- **Production**
  - Two intermediates - labor & capital-intensive
  - Tradable inputs with monopolistic competition
Consumers

Set-up

\[
\max_{C_{c,t}, C_{c,t+1}} U_{c,t} = \frac{(C_{c,t})^{1-\rho}}{1-\rho} + \beta_c \frac{(C_{c,t+1})^{1-\rho}}{1-\rho}
\]

s.t. budget constraints:

\[
P_{c,t} C_{c,t} = w_{c,t} - s_{c,t}
\]
\[
P_{c,t+1} C_{c,t+1} = \left( \frac{r_{c,t+1}}{P_{c,t}} + (1 - \delta) \frac{P_{c,t+1}}{P_{c,t}} \right) s_{c,t}
\]
Aggregate savings determine next period’s capital stock:

\[ K_{c,t+1} = \frac{S_{c,t}}{P_{c,t}} L_{c,t} \]

where

\[ S_{c,t} = \frac{1}{\beta_c^{\frac{1}{\rho}}} \frac{\beta^{1/\rho} W_{c,t}}{\beta_c^{1/\rho} + \left( \frac{r_{c,t+1}}{P_{c,t+1}} + (1 - \delta) \right)^{\rho - 1/\rho}} \]
Producers

Three stages of production

\[ B_{c,t}^{\text{Final}} = \left( P_{c,t}^K (1-\sigma) + P_{c,t}^L (1-\sigma) \right)^{\frac{1}{1-\sigma}} \]
Producers

Three stages of production

- \( B_{c,t}^{Final} = \left( P_{c,t}^K 1 - \sigma + P_{c,t}^L 1 - \sigma \right)^{1 \over 1 - \sigma} \)

- \( B_{c,t}^{Assembly} (i) = \left( \int_0^{N_i} p_{c,t}(\omega_i)^{1 - \epsilon} d\omega_i \right)^{1 \over 1 - \epsilon} \)
Producers

Three stages of production

- \( B_{c,t}^{Final} = \left( P_{c,t}^K 1^{1-\sigma} + P_{c,t}^L 1^{1-\sigma} \right)^{\frac{1}{1-\sigma}} \)
- \( B_{c,t}^{Assembly} (i) = \left( \int_0^{N_i} p_{c,t}(\omega_i)^{1-\epsilon} d\omega_i \right)^{\frac{1}{1-\epsilon}} \)
- \( p_{c,t}(\omega_i) = \frac{\epsilon}{\epsilon - 1} \tau_{c,t} \left( \frac{r_{c,t}}{A_{c,t}^L} \right)^{z_i} \left( \frac{w_{c,t}}{A_{c,t}^L} \right)^{1-z_i} \)

Assume \( z_i = 1 \) in K-sector & \( z_i = 0 \) in L-sector
Three stages of production

\[ B_{c,t}^{\text{Final}} = \left( P_{c,t}^K 1 - \sigma + P_{c,t}^L 1 - \sigma \right)^{\frac{1}{1 - \sigma}} \]

\[ B_{c,t}^{\text{Assembly}}(i) = \left( \int_0^{N_i} p_{c,t}(\omega_i)^{1-\epsilon} d\omega_i \right)^{\frac{1}{1-\epsilon}} \]

\[ p_{\tilde{c},t}(\omega_i) = \frac{\epsilon}{\epsilon - 1} \tau_{c,\tilde{c}} \left( \frac{r_{c,t}}{A_{c,t}^K} \right)^{z_i} \left( \frac{w_{c,t}}{A_{c,t}^L} \right)^{1-z_i} \]

Assume \( z = 1 \) in K-sector & \( z = 0 \) in L-sector
Market clearing

Factor market clearing gives:

\[ N_{K,t} = \frac{1}{f \epsilon} \sum_c A^K_{c,t} K_{c,t} \]  

\[ n^K_{\bar{c},t} = A^K_{\bar{c},t} K_{\bar{c},t} \left( \sum_c A^K_{c,t} K_{c,t} \right)^{-1} \]  

Goods market clearing requires:

\[ \left( \frac{r_{\bar{c},t}}{A^K_{\bar{c},t}} \right)^{-\epsilon} \sum_c \tau^{1-\epsilon}_{\bar{c},c} \left( P^K_{c,t} \right)^{\epsilon} Q^K_{c,t} = (\epsilon - 1)^{1-\epsilon} \epsilon^\epsilon f \]
Steady state

Combined with market clearing conditions and a normalization, steady state depends on following system of $N_c$ equations:

$$K_c^* = \frac{\frac{1}{\beta_c^{\frac{1}{\rho}}}}{\beta_c^{\frac{1}{\rho}}} \left(\frac{r_c^*}{P_c^*} + (1 - \delta)\right)^{\frac{\rho - 1}{\rho}} \frac{w_c^*}{P_c^*} L_c$$

(4)

Details: What determines $K^*$?  
Pure HO Properties
A pure Heckscher Ohlin Model

- $\epsilon \to \infty$, $f \to 0$
- Collapse input and intermediate stage of production
- Three groups: North, South and Autarky → sort depending on export-sector

$$(r_c, w_c) = \begin{cases} 
(A_c^K p_w^K, \tau A_c^L p_w^L) & \text{if } c \in N \\
(\tau A_c^K p_w^K, A_c^L p_w^L) & \text{if } c \in S
\end{cases}$$
Market clearing (I)

Aggregate price level:

\[
P_{N,t} = \left( (p_W^K)^{1-\sigma} + (\tau p_W^L)^{1-\sigma} \right)^{\frac{1}{1-\sigma}}
\]

\[
P_{S,t} = \left( (\tau p_W^K)^{1-\sigma} + (p_W^L)^{1-\sigma} \right)^{\frac{1}{1-\sigma}}
\]

Market clearing world prices:

\[
\left( \frac{p_{W,t}^K}{p_{W,t}^L} \right)^\sigma = \left( \frac{P_{N,t}}{P_{S,t}} \right)^{1-\sigma} \frac{\sum_{c \in S} A_c L_c + \tau \sigma \sum_{c \in N} A_c L_c}{\sum_{c \in N} A_c K_c + \tau \sigma \left( \frac{P_N}{P_S} \right)^{1-\sigma} \sum_{c \in S} A_c K_c}
\]
Market clearing (II)

\[ \tilde{k}_{\text{South}}^* \leq \left( \frac{p^L_W}{\tau p^K_W} \right)^{\sigma} < \tilde{k}_{\text{Autarky}}^* < \left( \frac{\tau p^L_W}{p^K_W} \right)^{\sigma} \leq \tilde{k}_{\text{North}}^* \]
Properties of Pure HO model

1. Capital-abundant countries have lower MPKs
2. Reduction in trade cost leads to convergence in MPKs
3. Positive correlation between MPKs and capital-shares
4. Positive correlation between changes in MPKs and changes in capital-shares
5. Negative correlation between changes in MPKs and capital-labor ratios
Property 1 & 2

1. Capital-abundant countries have lower MPKs
2. Reduction in trade cost leads to convergence in MPKs

\[
\frac{r_N}{P_N} \frac{A^K_N}{A^K_S} \frac{\left( (p_K^W)^{1-\sigma} + \tau^{1-\sigma} (p_L^W)^{1-\sigma} \right)^{1\over \sigma-1}}{\left( (p_K^W)^{1-\sigma} + \tau^{\sigma-1} (p_L^W)^{1-\sigma} \right)^{1\over \sigma-1}} < 1
\]
Short comings:

- Only North-South trade
- Same conditional factor prices within block
- Move in and out of autarky as endowments change
- No decreasing returns
Full model

\[
\frac{A^K_K}{A^K_S} = \frac{\left( (\hat{p}_i^{-\sigma} - \tau^{1-e} \hat{p}_i^{e-\sigma}) \frac{P_N^\sigma Y_N}{P_S^\sigma Y_S} \right)^{\frac{\epsilon-1}{\epsilon-\sigma}} - \tau^{1-e} \left( \hat{p}_i^{e-\sigma} - \hat{p}_i^{-\sigma} \tau^{1-e} \right)^{\frac{\epsilon-1}{\epsilon-\sigma}}}{\left( (1 - \hat{p}_i^{-e} \tau^{1-e}) \frac{\epsilon-1}{\epsilon-\sigma} - \left( (\hat{p}_i^{-e} - \tau^{1-e}) \frac{P_N^\sigma Y_N}{P_S^\sigma Y_S} \right)^{\frac{\epsilon-1}{\epsilon-\sigma}} \right) (\tau)^{1-e}}
\]

where \( \hat{p}_K = \frac{r_N A^K_S}{r_S A^K_N} \) and \( \hat{p}_L = \frac{w_N A^L_S}{w_S A^L_N} \).
Quantitative Exercise

- **Baseline**
  - Match Output and bilateral imports with $A^L$ and $\tau_{i,j}$ (asymmetric)
  - $\epsilon - 1 = 14$ - trade elasticity/ determines inc. returns to scale
  - $\sigma < 1$ & $A^K$ chosen to match US elasticity between K and L and capital share
  - fixed cost assumed equal across countries

- **Counterfactual Exercises**
  - What would have happened to returns in 1970 if the trade cost had been as low as 2000?
  - What would have happened to returns in 1970 if the distribution of capital had been like the 2000s?
Model Fit 1970

Y/K

Bilateral Imports

Capital share

MPK
Model Fit 2000

Y/K

Bilateral Imports

Capital share

MPK
Sources of convergence

$A^K K/A^L L$

$log(K/L)$

Avg. Trade cost

Betas
Counterfactual exercise

**Fit MPK - $\Delta \tau$**

- **Total effect**
- **No $\Delta K$**

**Change MPK - $\Delta \tau$**
Quantify contribution

\[ \text{Index} = 1 - \frac{\sum_i |\Delta MPK^{cf} - \Delta MPK|}{\sum_i |\Delta MPK^{cf}|} \]

- Only fall in \( \tau \) - index = 0.326
- Only change in endowment - index = 0.737
Conclusion

1. New facts on the aggregate MPK - convergence and its channels
2. Trade integration as an important driver is consistent with the data
3. Quantify contribution of fall in trade cost as \(\approx 30\%\)

\[\Rightarrow\] Equalized MPKs are the result of sectoral reallocation and (domestic) changes in factor endowments and do not indicate the absence of international financial frictions
Thank you!
Conditions for a high steady-state capital

In the absence of trade costs when input prices are equalized, the steady state capital is higher for patient countries whose capital-productivity is not too high, such that:

\[
\frac{k^*_c}{k^*_c'} > 1 \quad \text{if:} \quad \begin{cases} 
(1 - \delta) \left( \left( \frac{\beta_c}{\beta_{c'}} \right)^{\frac{1}{\rho - 1}} - 1 \right) + \frac{r}{A^K P} \left( \left( \frac{\beta_c}{\beta_{c'}} \right)^{\frac{1}{\rho - 1}} A^K_{c'} - A^K_c \right) < 0 \quad \text{for } \rho < 1 \\
\beta_c - \beta_{c'} > 0 \quad \text{for } \rho = 1 \\
(1 - \delta) \left( \left( \frac{\beta_c}{\beta_{c'}} \right)^{\frac{1}{\rho - 1}} - 1 \right) + \frac{r}{A^K P} \left( \left( \frac{\beta_c}{\beta_{c'}} \right)^{\frac{1}{\rho - 1}} A^K_{c'} - A^K_c \right) > 0 \quad \text{for } \rho > 1
\end{cases}
\]
Property 3
Positive correlation between MPKs and capital-shares

\[
\frac{r_N K_N}{P_N Q_N} < \frac{r_S K_S}{P_S Q_S} < \frac{Q_N^K S}{Q_N^K D} < \frac{Q_S^K S}{Q_S^K D} \frac{P_N^{1-\sigma}}{\tau^{\sigma-1} P_S^{1-\sigma}} < 1
\]

Necessary condition for the North to have a lower capital-share:

\[
\frac{P_N^{1-\sigma}}{\tau^{\sigma-1} P_S^{1-\sigma}} > 1
\]

\[
(\tau^2)^{1-\sigma} > 1
\]
Property 4
Positive correlation between changes in MPKs and changes in capital-shares

\[ S_c^K = \frac{r_c K_c^*}{w_c L_c + r_c K_c^*} \]

\[ = \left( 1 + \frac{P_c}{r_c} + \beta_c \frac{1}{\rho} \right)^{\frac{1}{1-\rho}} \left( \left( \frac{r_c}{P_c} \right)^{\frac{\rho}{1-\rho}} + (1 - \delta) \left( \frac{r_c}{P_c} \right)^{\frac{\rho-1}{\rho}} \right) \]
Trade growth

Level of Trade

Share of Trade

Notes: Total trade is calculated as the sum of the value of exports of goods as reported the IMF Direction of Trade statistics (DOTS) of the 60 countries in the sample. The sample captures ≈ 70% of world exports. The division into “North” and “South” follows the IMF classification into Advanced and Developing Economies.
No correlation with capital inflows

Capital Inflows and Changes in MPKs

Capital Inflows (relative to initial output) vs. Change in MPK 1981-2007

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Convergence in return as well as both components

Mean-reversion

Fall in dispersion

Notes: (a) Geometric growth rates and return at the beginning of the sample are calculated relative to the world. OLS coefficient of -1.491 significant at the 1% level. A level of MPK twice the world MPK is associated with a 1.491 percentage points lower growth rate (b) The standard deviation in the return reduces on average by 0.0005 each year or .021 over the whole sample.

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How do my results differ from Caselli & Feyrer?

- Variation with new data is about $2 \times$ higher
- Poor countries are disproportionately affected
- $\approx 75\%$ due to better data on labor share and natural resources, $\approx 24\%$ due to rel. prices
- No significant corr. with income for both samples

Difference between measures by income (1996)
Growth decomposition
Which margin of adjustment drives convergence?

- 3/4 of reduction relative to $MPK^W$ is explained by reduction in $\frac{Y}{K}$ - which is driven by increase in $\frac{K}{L}$
- 1/4 of reduction relative to $MPK^W$ comes from adjustments in the capital share
- Rel. price has no explanatory power
Fact 1: Convergence - Return to capital

\[ \frac{\alpha P^Y Y}{PK^K} + (1 - \delta) \frac{PK_t}{PK_{t+1}} \]

Percentile difference
S.D. of capital gain (3-y moving average)

\[(1 - \delta) \frac{P_{t+1}^K}{P_t^K}\]
World return

World MPK

Real World Return

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Correlation with GDP per worker over time

Correlation log(GDP p.w.) with 95% CI
Correlation with Capital per worker over time

Correlation log(K/L) with 95% CI

Year