(In)Efficient Asset Trade and a rationale for a Tobin Tax

Tobias Dieler∗

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Abstract
What is the welfare effect of a Financial Transaction Tax (FTT)? I study a model which combines asset trade with real investment. In the model, the firm takes an investment decision based on its asset price. The informed trader has private information about the investment’s perspective and buys assets from uninformed traders. On every purchase, the buyer has to pay the FTT. Trade takes place for asymmetric liquidity needs. I find two types of equilibria, one in which prices reveal information, a separating equilibrium, and another in which the price is uninformative, a pooling equilibrium. Depending on the informational content of the price, the firm takes an (in)efficient investment decision. In order to compare equilibria, I show that they co-exist if both gains from liquidity asymmetry and information asymmetry are intermediate. Welfare depends on the amount of trade and information revelation. Conditions for which the separating equilibrium yields greater welfare are derived. The introduction of the tax reduces the gains of the informed trader from liquidity asymmetry by which the pooling equilibrium is more affected than the separating equilibrium.

Keywords: Tobin tax, separating equilibrium, pooling equilibrium, firm value

∗University of Bologna, Tilburg University, T.Dieler@uvt.nl, Tobias.Dieler2@unibo.it. I am indebted to my supervisors Fabio Castiglionesi and Giacomo Calzolari for their continuous and patient support. Amil Dasgupta, Giovanni Cespa, Fabio Feriozzi and Albert Menkveld gave invaluable feedback. I have also profited from discussions with Marco Ottaviani, Francesco Lippi and Xavier Vives as well as from comments of seminar participants at University of Bologna and Tilburg University.
1 Introduction

The Financial Transaction Tax (FTT), also known as Tobin tax or securities transaction tax dates back to the article of James Tobin in 1978. Since then it has been introduced in a lot of states. In some of those states it has also been abolished afterwards. Matheson (2012) gives an overview of the countries in which a FTT is currently active. Most recently, the FTT has been introduced in France (August 2012) and Italy (March 2013). In these two countries a tax between 0.1% and 0.22% is levied on purchases of stocks. In the UK, since the early 90s’, there exists a so called ”Stamp Duty” on equity purchases which amounts to 0.5%. Discussions about the introduction of a FTT have restarted in the wake of the Financial crisis in 2008, in most Western countries currently ongoing is a debate among EU-countries about the introduction of a FTT by 2016.

Most of the time, governments introduce a FTT to raise money. Tobin (1978), Stiglitz (1989) and Summers and Summers (1989) argue that the tax affects mostly short-term speculation.

In this paper, I ask a more general question. Is a FTT able to improve welfare?

Therefore, I setup the following model. There is a manager of a firm facing an investment opportunity with uncertain outcome, either good or bad. When the manager decides upon the investment, he takes into account the firm’s asset price on the secondary stock market, i.e. the manager updates his prior beliefs about the outcome of the investment opportunity.

Asset trade takes place between an informed trader and uninformed traders in a model à la Laffont and Maskin (1990). The informed trader observes either good or bad information about the investment prospect of the firm. The uninformed traders are holding the assets of the firm, i.e. they are the owners of the firm. By observing the demand of the informed trader, the uninformed traders update their beliefs about the quality of the asset and decide whether to sell. Notice, the informed trader takes into account the effect of his purchase on the asset price. Trade takes place due to asymmetric liquidity needs. The uninformed traders are

\footnote{The discussion is followed, for example, by a theme-page of the Financial Times: \url{http://www.ft.com/intl/in-depth/financial-transaction-tax}}

\footnote{The current proposition can be found here (found on May 8th 2014): \url{http://ec.europa.eu/taxation_customs/taxation/other_taxes/financial_sector/}}
more liquidity constrained than the informed trader and hence the uninformed traders want to sell the assets to the informed trader. The FTT is levied on every purchase. This is the case for most countries.

There are two types of pure strategy equilibria. First, a separating equilibrium in which the informed trader reveals private information by demanding a larger quantity when he has good information than when he has bad information. The equilibrium price hence is either high or low. In order for trade to take place, in either state, the uninformed trader has to be more liquidity constrained than the informed trader. The equilibrium asset prices depends on the liquidity needs of the uninformed trader. Or differently, when the uninformed trader needs liquidity, he is willing to decrease the price at which he sells the assets proportional to his borrowing costs. Trade occurs for an infinitesimal small difference in liquidity needs. Since the asset prices reveal available information, the firm’s manager takes an efficient investment decision and therefore the firm value is maximized given the observed information.

Second, there exists a pooling equilibrium in which the informed trader does not reveal private information by demanding the same quantity no matter whether he has good or bad information. Then the uninformed trader cannot infer information from the informed trader’s demand and hence stays with the prior beliefs. In the pooling equilibrium, the asset price reflects the expected value of the asset which is below the prospect of the informed trader with good information and above the prospect of the informed trader with bad information. Just like in the separating equilibrium, also in the pooling equilibrium, the asset price depend on the liquidity needs of the uninformed trader. Since for the informed trader with good information the pooling equilibrium price is relatively low in comparison to his prospect, he is always willing to buy. The informed trader with bad information however is only willing to buy if the negative difference between his prospect and the expected value of the asset is outweighed by the uninformed trader’s liquidity needs. In other words, in the pooling equilibrium, the uninformed trader needs to be more liquidity constrained than in the separating equilibrium for trade to take place between the uninformed trader and the informed trader with bad information. With an uninformative asset price, the firm’s manager over (under) invests

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3For an extensive discussion of asymmetric liquidity needs, refer to Dieler 2014(a).
in case of bad (good) information. Given available information, the inefficient investment leads to a lower firm value than in the separating equilibrium.

I show that separating equilibrium and pooling equilibrium co-exist if the variance of the investment’s outcome is relatively small and the difference in liquidity needs is intermediate. More generally, this characterizes a situation in which gains from asset trade for the informed trader are moderate. The welfare analysis is carried out for the set of parameters for which separating equilibrium and pooling equilibrium co-exist.

Since the firm is owned by traders, welfare in this economy is the ex-ante joint profit of traders. The expected firm value increases in information revelation. Hence, the firm owners prefer the separating equilibrium. Welfare on the asset trade market increases in quantities traded. Since expected trade in a pooling equilibrium is larger than in a separating equilibrium and agents are both traders and owners, the welfare trade-off is information vs. trade. I provide conditions on the quantities traded such that the separating equilibrium yields greater welfare.

Then, I show that if the economy is in a pooling equilibrium, there exists an optimal tax which coordinates the economy on a separating equilibrium. While the results are cast in terms of FTT, one can also interpret the tax as any other transaction cost specific to the purchaser\footnote{The same result could not be achieved through variation of the Central Bank rate. Recall from chapter 1, the interest rate difference (=liquidity difference) remains constant for changes in the FED’s rate. But what would is needed is a cost/subsidy which affects either buyer or seller.}. The mechanism works as follows. For a pooling equilibrium to exist, the gains from liquidity asymmetry must outweigh the loss from information asymmetry of the informed trader with bad information. The Pareto optimal tax reduces the gains from liquidity needs such that the loss from information asymmetry of the informed trader with bad information is no longer outweighed.

There are few analytical analysis of the FTT, most notably Subrahmanyam (1998), Dow and Rahi (2000), Dupont and Lee (2007) and Davila (2013). These models have two common shortcomings.

First, information has no social value and therefore the notion of economic welfare is restricted to the asset market. Therefore, the FTT in their models can at best mitigate inefficiencies on the asset trade market. In this paper, I extend
the definition of welfare to the real economy and can thus evaluate the FTT more holistically.

Second, the inefficiency in their models, i.e. the non-informativeness of the prices, arises by assumption. In Subrahmanyam (1998) as in Dupont and Lee (2007), there are passive noise traders ”blurring” the informational content of the prices. Dow and Rahi (2000) consider on the buying side uninformed liquidity traders in addition to the informed trader. Whether prices reveal information depends on the share of uninformed traders and is hence exogenous. Davila (2013), the closest in spirit to this analysis, adopts a different asymmetry among traders' preferences. He characterizes an optimal FTT when traders disagree in beliefs. How much information the price reveals depends on the degree of disagreement.

In order to alleviate the two drawbacks, I use a real investment function as in Dow and Rahi (2003) and introduce it in an asset trade model à la Laffont and Maskin (1990). They provide a signaling model in which non-information revelation occurs by choice. In Laffont and Maskin (1990), trade takes place for asymmetric risk aptitudes. This leads to non-linear profit functions which are hardly summable for welfare analysis. To obtain linear equilibrium profits, I introduce asymmetric liquidity needs.

The remainder of this paper is organized as follows. In section 2, I lay out the model setup. Section 3 characterizes the pooling and separating equilibrium. The welfare analysis is carried out in section 4 and section 5 concludes.

2 Model

Although I described the model setup already at length in Dieler 2014(a), I restate the setup again for better readability. The reader aware of the model setup can immediately skip to the next section where I provide the conditions for the equilibrium existence.

The model has five dates \( t \in \{0, 1, 2, 3, 4\} \) and a firm whose stock is traded in the Financial Market. There are two types of risk-neutral traders \( i \in \{I, U\} \). An informed trader \( I \) and uniformed traders \( U \) of measure \( E \). Each of the uninformed traders holds one unit of the entire stock of the asset. In line with their little asset holding, the uninformed traders are assumed to be in perfect competition and thus
price takers. Throughout the model they are treated as one representative agent with an asset holding of $E$. Informed trader and uninformed trader have different liquidity needs. Liquidity needs are modeled with discount factors $1 > \delta_i > 0$. The higher $\delta_i$ the less liquidity constrained is the trader. Assume, the informed trader is less liquidity constrained than the uninformed trader, $\delta_I > \delta_U$. The uninformed traders own assets of a firm which faces an uncertain investment opportunity $V \in \{V_H, V_L\}$.

In $t = 0$, the informed trader observes private information $\omega \in \{H, L\}$ about the profitability of the firm’s investment opportunity. With probability $0 \leq \beta \leq 1$, the firm’s investment opportunity yields a payoff $V_H$ and with probability $1 - \beta$, $V_L$. Where $V_H > V_L$. Alternatively, the informed trader can invest in a riskless asset of which the revenue is normalized to 0, i.e. both the riskless rate and the revenue of the asset are 0. In $t = 1$, the informed trader decides to buy $E \geq B \geq 0$ assets from the uninformed trader. In $t = 2$, the uninformed trader observes the informed trader’s demand and decides to sell or to keep the assets. In $t = 3$, the firm observes the asset price $P$ and takes its investment decision $k$. Eventually, in $t = 4$, either the high payoff $V_H$ or the low payoff $V_L$ realizes. The timeline is depicted in figure 1.

![Figure 1: Timeline](image)

After observing the quantity chosen by the informed trader $B$, the uninformed trader updates the prior belief and form the conditional belief $q = Pr(V_H|B)$. Similarly the firm’s manager updates his belief about the quality of the investment after observing the asset price $P$ and form the conditional belief $r = Pr(V_H|P)$. Since there is not other private or public information besides the information about the outcome of the investment opportunity, in equilibrium, the price will reflect the demand of the informed trader only, and thus $P$ conveys the same information as $B$. Therefore I can write $r = q$. For ease of notation, beliefs of both, the
uninformed trader and the firm will be denoted by \( q = Pr(V_H|B) \).

The firm value \( F \) increases in investment \( k \) at a decreasing rate \( \forall k \leq k^* \), where \( k^* \) is the optimal investment level. \( c \) is a fixed marginal cost of investment. The firm’s manager maximizes the firm value by choosing the investment level \( k \) given the price he observes on the stock market. The firm value increases in the prospect of the investment \( V_\omega \). The manager’s optimization problem is written as

\[
F(k) = kV_\omega - \frac{c}{2}k^2 \tag{1}
\]

so that the expected firm value becomes

\[
E(F|B) = kE(V_H|B) - \frac{c}{2}k^2. \tag{2}
\]

The firm value function is adopted from Dow and Rahi (2003). The concavity of the firm value function in \( k \) implies that private information has social value even ex-ante.

After observing information in \( t = 0 \), the informed trader decides to buy a quantity \( B \) at a price \( P \) in period \( t = 1 \). On the purchase, he pays a tax \( \tau \). When choosing \( B \), the informed trader not only conditions on his private information \( \omega \) but also takes into account the signal his choice is sending to the uninformed trader and the firm. In \( t = 4 \), when the investment value \( V_\omega \) realizes and thus the firm value \( F \), the informed trader cashes in on the assets bought. The informed trader evaluates the cash-flow from the perspective of period \( t = 1 \), i.e. when deciding on the purchase. The informed trader discounts the payoff of period \( t = 4 \) by \( \delta_l \). By how much he discounts depends on how liquidity constrained he is. If, for example, the borrowing rate is zero, the informed trader is indifferent between a payoff today and tomorrow such that \( \delta_l = 1 \). The higher the borrowing rate, the lower the discount factor and the less willing is the informed trader to give up a payoff today for a payoff tomorrow. The informed trader’s cash flow from buying the risky asset at date \( t = 1 \) is

\[
-(1 + \tau)PB + \delta_l BF. \tag{3}
\]

Instead of buying the risky asset, the informed trader can also buy the riskless
asset and obtain 0 payoff.

In $t = 2$, when selling an amount $B$ of the total endowment $E$, the uninformed trader receives a revenue $PB$ from the sale. In $t = 4$, after the realization of the investment value, just like the informed trader, the uninformed trader cashes in on the assets held. Evaluating the cash-flow from period $t = 1$, the uninformed trader discounts the payoff from period $t = 4$ by $\delta_U$. The uninformed trader’s net present value (NPV) at $t = 2$ is

$$PB + \delta_U(E - B)F$$

In order to state the expected value of the uninformed trader’s NPV, I have to specify the beliefs. Therefore, the introduction of the expected NPV is deferred to section 3. Instead of selling assets, the uninformed trader can also keep all the assets and receive a NPV at time $t = 2$ of $\delta_U EF$.

The government receives all the tax revenues, i.e.

$$\tau PB.$$  

3 Perfect Bayesian Equilibrium

The informed trader strategy is a mapping $B : \{V_\omega\} \to \mathbb{R}_0^+$ that prescribes a quantity $B(V_\omega)$ on the basis of the trader’s private information $\omega$. The uninformed trader strategy is a mapping $P : \mathbb{R}_0^+ \to \mathbb{R}_0^+$. The firm manager strategy is a mapping $k : \mathbb{R}_0^+ \to \mathbb{R}_0^+$. Conditional beliefs for the uninformed trader and the firm manager are represented by a mapping that associates to each quantity $B$ a probability function $Pr(\cdot|B)$ on $\{V_H, V_L\}$, where $Pr(V_\omega|B)$ is the probability that the uninformed trader and the firm manager attach to a value $V_\omega$ given quantity $B$.

The perfect Bayesian equilibrium is defined by a triple of strategies $(B(\cdot), P(\cdot), k(\cdot))$ and a family of conditional beliefs $Pr(\cdot|\cdot)$ such that (i) for all $B$ in the range of $B(\cdot)$, $Pr(\cdot|B)$ is the conditional probability of $V_\omega$ obtained by updating the prior $(\beta, (1-\beta))$, using $B(\cdot)$ in Bayesian fashion; (ii) for all $B(\cdot)$, $P argmax P \in E(U_U(\cdot))$, (iii) for all $B(\cdot)$ $k^* \in argmax_k E(F|B)$ and (iv) for all $\omega B \in argmax_B E(U_I(\cdot))$. 
Condition (i) stipulates that the uninformed trader and the firm’s manager have rational expectations. Conditions (ii) to (iv) require that traders be optimizing. In particular, they imply participation constraints and incentive compatibility constraints.

Market clearing takes place through the adjustment of the price $P$ to the quantity demanded $B$. I.e. the informed trader submits a market order. Observing the market order, the uninformed trader, acting as a market maker, updates the belief about the quality of the asset and sets the price. In equilibrium, there has to be a unique price-quantity-bundle $\{P, B\}$.

I derive the equilibria as in Dieler 2014(a). To avoid repetition, I remark which optimization problems remain unchanged and just state the resulting conditions as derived in Dieler 2014(a). For those optimization problems affected by the introduction of the tax $\tau$, I will briefly describe the effect of $\tau$ on the optimization problem and state the resulting condition.

3.1 Equilibrium existence

3.1.1 Separating equilibrium

In the separating equilibrium, depending on the private information $\omega \in \{H, L\}$, the informed trader buys different quantities $B^H$ and $B^L$.

The firm manager’s beliefs and the uninformed trader’s beliefs are not affected by the introduction of a tax.

$$q = Pr(H|B) = \begin{cases} 
1 & \text{if } B = B^H \\
0 & \text{if } B = B^L \\
1 & \text{if } B' \neq B^H \land B' \neq B^L
\end{cases}. \quad (6)$$

Also the firm manager’s optimal choice remains, $k^\omega = \frac{V_2}{c}$. Given the optimal choice, the firm value from the manager’s perspective (and the uninformed trader’s perspective) is $F_\omega = \frac{V_2}{\delta}$.

The tax $\tau$ is levied on purchases. The uninformed trader is only selling assets. Therefore, participation of the uninformed trader is unaffected by the tax. Prices remain hence unchanged, i.e. $P^\omega = \delta V F_\omega$. Notice, the price decreases the more
liquidity constrained the uninformed trader.

Whether the informed trader buys the risky asset or the risk-less asset is affected by the tax since he has to pay the tax on the value purchased.

The equilibrium firm value from the informed trader’s perspective after observing information $\omega$ is $F_\omega = \frac{V_\omega^2}{2c}$. With the equilibrium price $P_\omega = \delta_U F_\omega$, the participation constraint of the informed trader with information $\omega$ becomes:

$$-(1 + \tau)\delta_U F_\omega B_\omega + \delta_I B_\omega F_\omega \geq 0,$$

with the revenue of the risk-less asset normalized to 0. For any positive trade $B_\omega \geq 0$ both types’ participation constraints are satisfied if $\frac{\delta_I}{(1 + \tau)} > \delta_U$. The informed trader buys the risky asset whenever he is less liquidity constrained than the uninformed trader. The informed trader has to pay a proportion relative to $\tau$ of the purchase value to the government. For the informed trader to be willing to purchase the risky asset, the difference in liquidity needs, needs to be larger than without a tax.

In the separating equilibrium, I have to show, given beliefs $q$ as specified in 6, prices $P_\omega = \delta_U F_\omega$ and firm value $F_\omega = \frac{V_\omega^2}{2c}$, that $B^H$ and $B^L$ are optimal choices for the respective type of informed trader. In particular that the informed trader with good information $H$ does not want to mimic the informed trader with bad information $L$ and vice versa:

$$-(1 + \tau)\delta_U F_\omega B_\omega + \delta_I B_\omega F_\omega \geq -(1 + \tau)\delta_U F_{-\omega} B^{-\omega} + \delta_I B^{-\omega} F'_\omega$$

where $-\omega \neq \omega$.

If an informed trader with information $\omega$ chooses $B^{-\omega}$, from his perspective the firm’s value $F'_\omega = \frac{V_{-\omega}}{c}(V_\omega - \frac{V_{-\omega}}{2})$. Moreover, I have to show that the informed trader with information $\omega$ chooses $B^\omega$ and not any other quantity $B' \neq B^\omega \forall \omega$, i.e.

$$(-P^H + \delta_I F^H_B)B^H \geq (-P' + \delta_I F^H_B)B' \forall B'$$

and

$$(-P^L + \delta_I F^L_B)B^L \geq (-P' + \delta_I F^L_B)B' \forall B'.$$
With the off-equilibrium price \( P' = \delta_U F_H \). The reasoning for finding \( B^H \) and \( B^L \) is identical to Dieler 2014(a) and yields the following conditions:

\[
B^H = E, \quad \frac{-(1 + \tau)\delta_U + \delta_I}{-(1 + \tau)\delta_U F_L + \delta_I F_H} E \geq B^L \geq \max\{0, \frac{-(1 + \tau)\delta_U F_H + \delta_I F_L}{-(1 + \tau)\delta_U + \delta_I F_L} E\}. \tag{12}
\]

In the following proposition, I report for which parameters of the model, there exists a separating equilibrium.

**Proposition 1.** Separating equilibrium. There exist separating equilibria with price \( P^\omega = \delta_U F_H, \omega \in \{H, L\} \) and quantities as specified in (11) and (12) if

- \( \frac{\delta_U F_H - F'_H}{F_H - F_H'} > \frac{\delta_I}{(1 + \tau)} > \delta_U \) and \( V_H > \frac{1}{2}(1 + \sqrt{5})V_L \) or
- \( \frac{\delta_U F_H - F'_H}{F_H - F_H'} > \frac{\delta_I}{(1 + \tau)} > \delta_U \) and \( \frac{1}{2}(1 + \sqrt{5})V_L > V_H > V_L \).

In the separating equilibrium prices reveal available information. Trade is maximal given good information. For the informed trader with good information not to mimic the informed trader with bad information, trade must be less than maximal in the case of bad information. Moreover, the maximal amount of trade, refer to the left hand side of inequality (12) in the presence of bad information is decreasing the higher the tax \( \tau \). The intuition is that the tax increases the potential gain of the informed trader with bad information from mimicking the informed trader with bad information and thus pay a lower price.

Also, existence of the separating equilibrium depends on the tax \( \tau \). An increase in the tax reduces the difference in liquidity needs and hence the scope for trade. In the separating equilibrium, trade takes place due to asymmetric liquidity needs, i.e. the informed trader buys assets from the uninformed trader if the uninformed trader is asking a lower price than the informational value in order to satisfy liquidity needs. If however the informed trader has to pay a tax on his purchases, the gain from the liquidity difference shrinks.

### 3.1.2 Pooling equilibrium

In the pooling equilibrium, the informed trader buys the same quantity \( B^P \) in either state.
Again, beliefs of the manager and the uninformed trader are unaffected by the tax \( \tau \): 

\[
q = \Pr(H|B) = \begin{cases} 
\beta & \text{if } B = B^P \\
1 & B' \neq B^P 
\end{cases} .
\] (13)

Also the firm manager’s optimal choice remains, 

\[
k^P = \frac{E(V)}{c} \quad \text{with} \quad E(V) = \beta V_H + (1 - \beta) V_L .
\]

Given the optimal choice, the firm value from the manager’s perspective (and the uninformed trader’s perspective) is 

\[
F_P = \frac{E(V)^2}{2c} .
\]

Participation of the uninformed trader is unaffected by the tax. Prices remain hence unchanged, i.e. \( P = \delta_U F_P \).

The informed trader’s participation however is affected by the tax. Given the price \( P = \delta_U F_P \), the informed trader decides whether to buy \( B^P \) of the risky asset or the riskless asset which gives a return of 0. His participation constraints in either state is

\[-(1 + \tau) \delta_U F_P B^P + \delta_I B^P F_P^r \geq 0 .\] (14)

The firm value from the perspective of the informed trader takes into account the investment decision of the manager, 

\[
k^P = \frac{E(V)}{c} \quad \text{given the privately observed information } \omega .
\]

Therefore, the firm value from the informed trader’s perspective are 

\[
F^\omega_P = \frac{E(V)}{c}(V_\omega - \frac{E(V)}{2}) \quad \text{with } F^H_P > 0 \quad \text{if and only if } \frac{V_L}{V_H - V_L} > \beta .
\]

Observe, 

\[
F^H_P > F_P > F^L_P .
\]

Implying that the informed trader with good information faces a higher return from the risky asset than the informed trader with bad information, given the uninformed choice of the manager. Both, the informed trader with good information and the informed trader with bad information however faces a lower payoff than if the manager made an informed choice, 

\[
F^\omega_P < F^\omega .
\]

The participation constraint for the informed trader with bad information is more binding. In fact, if \( \omega = L \), the left hand side of the inequality is smaller than if \( \omega = H \). Solving the participation of the informed trader with bad information for \( \delta_I \) yields for any \( B^P \geq 0 \): 

\[
\frac{\delta_I}{1 + \tau} \geq \delta_U \frac{F_P}{F^H_P} .
\]

Since \( \frac{F_P}{F^H_P} > 1 \), the informed trader needs to be considerably less liquidity constrained than the uninformed trader, and in particular less than in the separating equilibrium where participation was ensured if \( \delta_I > \delta_U \). Observe that \( \frac{F_P}{F^H_P} \) increases in \( \beta \). That is the more likely the good outcome, the larger
needs to be the difference between the informed trader’s liquidity needs and the uninformed trader’s liquidity needs.

In the pooling equilibrium, I have to show that there exists a single $B^P$ for which both types of informed traders are willing to buy the risky asset. In particular, that there is no $B' \neq B^P$ that either of the two types would prefer over buying $B^P$. This is formalized in the following incentive compatibility constraint:

$$(-1 + \tau)\delta U F_P + \delta I F_H \geq \left(-1 + \tau\right)B' \quad \forall B'$$

$$(-1 + \tau)\delta U F_P + \delta I F_L \geq \left(-1 + \tau\right)B' \quad \forall B'$$

with $P' = \delta U F_H$. The off-equilibrium price follows with the off-equilibrium beliefs and the uninformed trader breaking even. Incentive compatibility is satisfied if $B^P = E$. The reasoning for this result

From 15 we observe, the informed trader with good information is indifferent between the equilibrium payoff and the off-equilibrium payoff if $E > B^P = \frac{(-1 + \tau)\delta U + \delta I F_H}{-1 + \tau}$. The equilibrium payoff however is maximized for $B^P = E$. This is a sufficient condition for incentive compatibility of the informed trader with bad information.

The lower bound for $B^P$, $\frac{(-1 + \tau)\delta U + \delta I F_H}{-1 + \tau}$ is decreasing in $\tau$. $B^P = E$ hence remains an optimal choice for the informed trader for any $1 > \tau > 0$.

The existence of a pooling equilibrium is summarized in the following proposition.

**Proposition 2.** Pooling equilibrium. If $\delta U F_H - F_P > \frac{\delta I}{1 + \tau} > \delta U F_P$ and $\beta < \min\{\frac{V_L}{V_H - V_L}, \frac{V_I}{(V_H - V_L)^2}\}$, there exists a pooling equilibrium with a price $P = \delta U F_P$ and equilibrium trade $B^P = E$.

The pooling equilibrium depends on the tax through the participation constraint of the informed trader and the incentive compatibility constrained of the informed trader. As for the separating equilibrium, the gain from trade due to the difference in liquidity needs decreases and thus the range of the existence of the pooling equilibrium. The optimal amount of trade is not affected by the tax. This is a crucial observation for the welfare analysis.
3.2 Equilibrium characterization

After stating the existence conditions for each type of equilibrium, I can now characterize all possible equilibria given the beliefs in 6 and 13. The objective is to characterize equilibria depending on the liquidity difference, $\delta_I - \delta_U$. Observe from the two previous propositions that the existence of equilibria depends on the following three thresholds which characterize the liquidity difference: $F_H - F_P$, $F_P$ and $F_L$. In order to rank them, I need to derive conditions on $\beta$ and the difference $V_H - V_L$, the parameters on which the firm values $F$ depend. The derivation of the condition is identical to Dieler 2014(a). The equilibrium characterization is a preparatory step for the welfare analysis. It shows when the parameter areas for which separating equilibrium and pooling equilibrium exist respectively, overlap and when they do not.

Lemma 1. Separating equilibrium only. For $1 \geq \beta > \min \{ \frac{V_L}{V_H - V_L}, \frac{V_L^2}{(V_H - V_L)^2} \}$, there exists a separating equilibrium only (if $\frac{F_H - F_L}{F_H - F_P} > \frac{\delta_I}{1 + \tau} > \max \{ \delta_U, \frac{F_H}{F_L} \}$).

Lemma 1 tells, if the difference between the high outcome and the low outcome is very small, only the separating occurs.

Lemma 2. Separating equilibrium and pooling equilibrium do not overlap. For $\min \{ 1, \frac{V_L}{V_H - V_L}, \frac{V_L^2}{(V_H - V_L)^2} \} > \beta > \frac{V_L^2}{(V_H - V_L)V_H}$, there exists

- a separating equilibrium only (if $\frac{F_H - F_L}{F_H - F_P} > \frac{\delta_I}{1 + \tau} > \max \{ \delta_U, \frac{F_H}{F_L} \}$) and
- a pooling equilibrium only (if $\frac{F_H - F_L}{F_H - F_P} > \frac{\delta_I}{1 + \tau} > \delta_U \frac{F_H}{F_P}$).

For a given $V_H - V_L$ and $\beta$, the separating equilibrium exists for a small liquidity difference and the pooling equilibrium for a large liquidity difference since $\frac{F_P}{F_P} > \frac{F_H - F_L}{F_H - F_H}$.

Lemma 3. Separating equilibrium and pooling equilibrium overlap. For $\min \{ 1, \frac{V_L^2}{(V_H - V_L)V_H} \} > \beta$, there exists

- a pooling equilibrium only if $\delta_U \frac{F_H - F_P}{F_H - F_P} > \frac{\delta_I}{1 + \tau} > \delta_U \frac{F_H - F_L}{F_H - F_P}$,
- both a pooling equilibrium and a separating equilibrium if $\delta_U \frac{F_H - F_P}{F_H - F_P} > \frac{\delta_I}{1 + \tau} > \max \{ \delta_U \frac{F_P}{F_L}, \delta_U \frac{F_H}{F_L} \}$ and
a separating equilibrium only if \( \frac{\delta_U}{F_P} > \frac{\delta_I}{1+\tau} > \max\{\delta_U, \delta_U \frac{F_H}{F_L}\} \).

Pooling equilibrium and separating equilibrium overlap if for an increasing difference in the investment’s outcome \( V_H - V_L \), the probability of observing the high outcome \( \beta \) decreases and the liquidity difference is intermediate. This characterizes a situation in which the informed trader can make moderate gains from trade since both the gain from the liquidity asymmetry and the gain from information asymmetry are moderate.

4 Welfare

The firm representing the real economy is entirely owned by the traders. Therefore, welfare is captured by the sum of the traders’ profits and the government’s revenue from the tax. Consider ex-ante profits as in Dow and Rahi (2003) and Laffont and Maskin (1990).

4.1 Profits

Denote by \( \psi \in \{S, P\} \) either type of equilibrium, i.e. separating equilibrium or pooling equilibrium. The equilibrium profit of an informed trader \( I \) in a separating equilibrium with \( P^\omega = \delta_U F_\omega \) and \( B^\omega \) is

\[
\Pi^{\omega SI} = (-1 + \tau)\delta_U F_\omega B^\omega.
\] (17)

For the uninformed trader \( U \) the equilibrium profit becomes

\[
\Pi^{\omega SU} = \delta_U E F_\omega.
\] (18)

The government’s revenue is

\[
\Pi^{\omega SG} = \tau \delta_U F_\omega B^\omega.
\] (19)

Analogously, I obtain the profit of the informed trader \( I \) in a pooling equilib-
rium with the equilibrium price \( P = \delta_U F_P \) and the equilibrium quantity \( B^P \)

\[
\Pi^{\omega PI} = -(1 + \tau)\delta_U F_P + \delta_I F_P^\omega)B^P. \tag{20}
\]

For the uninformed trader \( U \) the equilibrium profit becomes

\[
\Pi^{\omega PU} = \delta_U F_P E. \tag{21}
\]

And the government’s revenue is

\[
\Pi^{\omega PG} = \tau\delta_U F_P B^P. \tag{22}
\]

Observe that the profit of the informed trader is increasing in the quantities \( B^\omega \) and \( B^P \). Furthermore, profits are increasing in the firm value \( F \). These two observations will guide the following welfare analysis.

### 4.2 Welfare

The sum of the expected profits in the separating equilibrium is:

\[
W_S = \delta_U E(\beta F_H + (1 - \beta)F_L) + (\delta_I - \delta_U)(\beta F_H B^H + (1 - \beta)F_L B^L). \tag{23}
\]

And welfare in the pooling equilibrium becomes:

\[
W_P = \delta_U E F_P + (\delta_I(\beta F_P^H + (1 - \beta)F_P^L) - \delta_U F_P)B^P. \tag{24}
\]

Observe, the tax \( \tau \) does not enter in either of the two welfare functions, which implies there is no primary tax effect. This is due to the fact that all agents, traders and government, are risk neutral and thus have a linear utility function. Therefore, the tax payment of the trader and the tax revenue of the government cancel out. The tax affects welfare through the participation constraints and incentive compatibility constraints of the informed trader.
4.3 Welfare analysis

The objective of the analysis is to find a tax \( \tau \) such that it improves welfare. Therefore I first Pareto rank separating equilibrium and pooling equilibrium and then show for which tax the Pareto optimal equilibrium is attained.

It makes sense to compare equilibrium if they exist for the same values of parameters. From lemma 3, we know pooling equilibrium and separating equilibrium overlap if

\[
\min \{1, \frac{V^2}{(V_H - V_L)(V_H)}\} > \beta \quad \text{and} \quad \delta_U \frac{F_H - F_P}{F_H - F_P} > \frac{\delta_I}{1 + \tau} > \max \{\delta_U \frac{F_P}{F_P}, \delta_U \frac{F_H}{F_H}\}.
\]

Moreover, observe adjacent to the area of multiple equilibria, there is the area

\[
\delta_U \frac{F_P}{F_P} > \frac{\delta_I}{1 + \tau} > \max \{\delta_U, \delta_U \frac{F_H}{F_H}\}\] in which only the separating equilibrium exists.

Consider the following situation. The economy is in a pooling equilibrium with

\[
\delta_U \frac{F_H - F_L}{F_H - F_L} > \delta_I > \max \{\delta_U \frac{F_P}{F_P}, \delta_U \frac{F_H}{F_H}\}. \quad \text{The introduction of a tax} \ \tau \ \text{changes the liquidity difference to} \ \delta_U \frac{F_P}{F_P} > \frac{\delta_I}{1 + \tau} > \max \{\delta_U, \delta_U \frac{F_H}{F_H}\} \ \text{such that only a separating equilibrium exists. Therefore I have to show that welfare in the separating equilibrium is larger when equilibria overlap and that welfare in the separating equilibrium is still larger when only the separating equilibrium exists and the informed trader has to pay a tax.}
\]

Given the equilibrium quantities \( B^P = B^H = E \), welfare in the separating equilibrium is larger than in the pooling equilibrium if

\[
\Delta W = W_S - W_P = \delta_U E (\beta F_H + (1 - \beta) F_L) + (\delta_I - \delta_U)(\beta F_H E + (1 - \beta) F_L B^L) - \delta_I (\beta F_P^H + (1 - \beta) F_P^L) E > 0. \tag{25}
\]

If separating equilibrium and pooling equilibrium co-exist, from Dieler 2014(a), we know for which conditions the separating equilibrium yields larger welfare than the pooling equilibrium. For readability, the proposition is reported here again 5

**Proposition 3. Welfare comparison.** The separating equilibrium yields greater welfare than the pooling equilibrium if \( \delta_U \frac{F_H - F_P}{F_H - F_P} > \delta_I > \delta_U \frac{F_P}{F_P}, \frac{V^2}{(V_H - V_L)} > \beta \) and

- either
  \[
  \frac{(\delta_I - \delta_U)F_H}{\delta_I F_H - \delta_U F_L} E > B^L \geq 0, \quad \frac{V^2}{V_H(V_H - V_L)} > \beta > \frac{3V_L - V_H}{(V_H - V_L)^2} \quad \text{and} \quad V_H > (1 + \sqrt{2}) V_L
  \]

- or

\[
\text{For the proof, refer to Dieler 2014(a).}
\]
\[ B^L(V_H) = \frac{23\delta_I(V_H V_L - V_L^2)}{(\delta_I - \delta_U)V_L^2} E, \min\{\delta_U F_H - F_p, \delta_U \frac{1}{1-\beta}\} > \delta_I, \beta > \frac{2V_L - 3V_H}{V_H - V_L} \text{ and either } \frac{3}{2}V_L > V_H > V_L \text{ or } 2V_L > V_H > \frac{1}{4}(3 + \sqrt{17})V_L. \]

After the introduction of the tax, the separating equilibrium yields larger welfare for the following tax.

**Proposition 4. Pareto optimal FTT.** There is a Pareto optimal tax

- With \( B^L \geq 0 \), \( \min\{\gamma_L^2(V_H V_L - V_L^2) - 1, \frac{\delta_U}{\delta_U} - 1\} > \tau > \frac{\delta_I}{\delta_U} F_p - 1 \) if
  \[ \min\{\delta_U F_p F_p^{-1}, \delta_U F_H F_H^{-1}\} > \delta_I > \delta_U F_p F_p^{-1}. \]
- And a Pareto optimal tax with \( B^L > 0 \), \( \min\{\frac{\delta_U}{\delta_U} - 1, \frac{\beta}{1-\beta}\} > \tau > 0 \), if
  \[ \frac{\delta_U}{1-\beta} > \delta_I > \delta_U F_p F_p^{-1} \text{ and } \beta > \frac{2V_H - 3V_L}{V_H - V_L}. \]

The proof is relegated to appendix A.

As we know from lemma 3, pooling equilibrium and separating equilibrium overlap if gains for the informed trader from both information asymmetry and liquidity asymmetry are intermediate. For given information asymmetry, for a small liquidity asymmetry there exists a separating equilibrium only. The idea of proposition 4 is to decrease liquidity asymmetry such that the pooling equilibrium ceases to exist and the separating equilibrium exists only.

In the following, I try to shed some light on the underlying mechanism. Recall, prices have two components, the informational component, \( E(V_\omega | B) \) and the liquidity component, \( \delta_U \). In a pooling equilibrium, the informational component of the price, \( E(V) \), is higher than private information of the informed trader with bad information \( V_\omega \). In order for him to buy the asset nevertheless, the uninformed trader must be sufficiently liquidity constrained such that the informational loss is at least outweighed by the liquidity gain. When pooling equilibrium and separating equilibrium co-exist, liquidity gains outweigh informational loss by few. The introduction of the tax decreases further the liquidity gain up to the point that the informed trader’s informational loss is no longer outweighed by the liquidity gain and thus the informed trader with bad information no longer wants to buy the asset.

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*a*For a given \( \beta \), information asymmetry is large if \( V_H - V_L \) is large. Or, for a given \( V_H - V_L \), information asymmetry is large if \( \beta = \frac{1}{2} \).
5 Conclusion

I identify a "case" in which asset trade leads to economic inefficiency. I show that in this "case" the inefficiency can be alleviated by the introduction of a FTT. The "case" is an economy consisting of an asset trade market and a firm representing the real economy. The firm has an investment opportunity with an uncertain return. Before deciding on the investment level, the firm’s manager consults the firm’s asset price. It does so because the informed trader has superior information about the investment’s return. The asset trade market is modeled in a simple signaling setup in which a monopolistically informed trader, with either good or bad information, buys assets from uninformed traders. On every purchase, a FTT is levied. The uninformed trader sells the assets for liquidity needs. I find a separating equilibrium in which the informed trader chooses different quantities depending on the information observed. Hence, prices reveal available information and the firm takes the efficient investment decision. There exists also a pooling equilibrium in which the informed trader chooses the same quantity regardless of his information. Therefore the price does not reveal available information and the firm takes an inefficient investment decision. The welfare ranking of the two types of equilibria depends on both the amount of trade and information revelation. For the trader(s) who own(s) the firm, information revelation (separating equilibrium) is always better. The pooling equilibrium features more trade than the separating equilibrium and thus increases welfare of the pooling equilibrium. The trade-off for total welfare is hence information revelation vs. trade. I provide conditions on the traded quantities for which the separating equilibrium yields greater welfare. If the economy is in a pooling equilibrium, the government can introduce a tax which coordinates the economy on the Pareto optimal separating equilibrium.

The main contribution of this article is to identify an endogenous inefficiency which can be corrected by a FTT.

It remains a partial equilibrium model and hence intrinsically features the typical shortcomings. First, it still does not allow to study the effect of the tax on a more complex economy with different countries for example. The paper also does not provide any insights on inter-temporal effects of the tax.
A Proof of proposition 4

Using the terms for $F_H$, $F_L$, $F_P^H$ and $F_P^L$, the welfare comparison of inequality 25 can be rewritten as

$$\Delta W = \frac{(1 - \beta)(\beta \delta I E(V_H - V_L)^2 + (\delta I - \delta U)(B^L - E)V_L^2)}{2c}$$

$$\Rightarrow (\delta I(\beta(V_H - V_L)^2 - V_L^2) + \delta U V_L^2) + (\delta I - \delta U)V_L^2B^L > 0$$

Recall that $\delta U \frac{F_H - F_P}{F_H - F_P^H} > \delta I > max\{\delta U \frac{F_P}{F_P^H}, \delta U \frac{F_H}{F_P^H}\}$ and that $\delta U \frac{F_P}{F_P^H} > \frac{\delta I}{\delta U} > max\{\delta U, \delta U \frac{F_H}{F_P^H}\}$. The last inequality can be rewritten in terms of the tax $\tau$, $\frac{\delta I}{\delta U} - 1 > \tau > \frac{\delta I}{\delta U} - 1$ if $V_H < 2V_L$.

Consider $\Delta W = (\delta I(\beta(V_H - V_L)^2 - V_L^2) + \delta U V_L^2)E + (\delta I - \delta U)V_L^2B^L$. If $(\delta I(\beta(V_H - V_L)^2 - V_L^2) + \delta U V_L^2) \geq 0$, $B^L$ can be as small as 0. $(\delta I(\beta(V_H - V_L)^2 - V_L^2) + \delta U V_L^2) \geq 0$ if $\delta I < \delta U \frac{V_L^2}{V_L^2 - \beta(V_H - V_L)^2}$. For a separating equilibrium to exist, $\delta I > \delta U(1 + \tau)$ if $V_H < 2V_L$. For $1 > \delta I > 0$, $\delta U \frac{V_L^2}{V_L^2 - \beta(V_H - V_L)^2} > \delta I > \delta U(1 + \tau)$. The condition on $\delta I$ is satisfied if $\tau < \frac{V_L^2}{V_L^2 - \beta(V_H - V_L)^2} - 1$. From the existence condition of the separating equilibrium, $\tau > \frac{\delta I}{\delta U} F_P^H - 1$. In order for a positive $\tau$ to exist, $\frac{V_L^2}{V_L^2 - \beta(V_H - V_L)^2} - 1 > \frac{\delta I}{\delta U} F_P^H - 1$. This is satisfied for $\delta I < \delta U \frac{F_P}{F_P^H} \frac{V_L^2}{V_L^2 - \beta(V_H - V_L)^2}$. The latter condition does not violate the co-existence condition $\delta I > \delta U \frac{F_P}{F_P^H}$. Therefore, there exist a Pareto improving tax $min\{\frac{V_L^2}{V_L^2 - \beta(V_H - V_L)^2} - 1, \frac{\delta I}{\delta U} - 1\} > \tau > \frac{\delta I}{\delta U} F_P^H - 1$ if

$$min\{\delta U \frac{F_P}{F_P^H} \frac{V_L^2}{V_L^2 - \beta(V_H - V_L)^2}, \delta U \frac{F_H - F_P}{F_H - F_P^H}\} > \delta I > \delta U \frac{F_P}{F_P^H}. \quad (26)$$

Otherwise, $B^L$ has to be strictly larger than 0. In the following, a such $B^L$ is derived. Therefore, I characterize a $B^L(V_H)$, that yields a non-negative difference $W_S - W_P$ for any $V_H$, given $V_L$. Conjecture, the larger $V_H$, the smaller can be $B^L$.

A change in $V_H$ affects the welfare difference

$$\frac{\partial \Delta W}{\partial V_H} = \frac{(1 - \beta)(2\beta \delta I E(V_H - V_L)^2 + (\delta I - \delta U)\frac{\partial B^L(V_H)}{\partial V_H}V_L^2)}{2c}$$

In order to derive the conditions for which $\Delta W > 0$, I study the minimum of $\Delta W$. A candidate minimum of $\Delta W$ will have to satisfy $\frac{\partial \Delta W}{\partial V_H} = 0$ and $\frac{\partial^2 \Delta W}{\partial V_H^2} \geq 0$. From
the necessary condition, an ordinary differential equation is obtained

\[
\frac{\partial B^L(V_H)}{\partial V_H} = -\frac{2\beta \delta_I(V_H - V_L)}{(\delta_I - \delta_U)V_L^2} E
\]

The function for \(B^L(V_H)\) satisfying the necessary condition is

\[
B^L(V_H) = \frac{2\beta \delta_I(V_H - V_L)}{(\delta_I - \delta_U)V_L^2} E
\]

In order for \(B^L(V_H) > 0\), \(V_H < 2V_L\). From the existence condition of the separating equilibrium, \(\frac{\delta_I - (1+\tau)\delta_U}{\delta_I - (1+\tau)\delta_U} F_H E > B^L\). In order to show that there exists a \(B^L > 0\) satisfying both the existence condition of the separating equilibrium and the minimum condition of the welfare difference, \(\frac{\delta_I - (1+\tau)\delta_U}{\delta_I - (1+\tau)\delta_U} F_H E > 2\beta \delta_I(V_H - V_L)\).

The latter inequality is satisfied if

\[
\frac{\delta_I}{\delta_U} - 1 > \tau, \quad \text{(27)}
\]

\[
\delta_I > \delta_U(1 + \tau) \quad \leftrightarrow \quad \frac{\delta_I}{\delta_U} - 1 > \tau \quad \text{(28)}
\]

\[
\frac{\delta_I}{\delta_U} V_H - V_L > 1 \quad \text{(29)}
\]

\(\frac{\delta_I}{\delta_U} - 1 > \tau\) coincides with the existence condition and is hence always satisfied. Since \(\frac{\delta_I}{\delta_U} V_H > \frac{\delta_I}{\delta_U} V_L\), condition (29) is satisfied as well.

With \(B^L(V_H)\),

\[
\Delta W = \frac{(1 - \beta)(\delta_U - (1 - \beta)\delta_I)V_L^2 E}{2c}
\]

\(\Delta W > 0\) if \((\delta_U - (1 - \beta)\delta_I) > 0\), i.e. \(\frac{\delta_I}{1 - \beta} > \delta_I\). Remains to be shown that there exists a positive \(\delta_I\) which satisfied both the non-negativity condition of \(\Delta W\) and the existence condition of the separating equilibrium: \(\frac{\delta_I}{1 - \beta} > \delta_I\). This condition poses restrictions on (i) \(\frac{\delta_U}{1 - \beta} > \delta_I > \delta_U(1 + \tau)\) if \(\tau < \frac{\beta}{1 - \beta}\) and (ii) \(\frac{\delta_U}{1 - \beta} > \delta_I > \delta_U \frac{F_H}{F_P}\).

\[
\frac{1}{1 - \beta} > \frac{F_H}{F_P} \quad \text{if} \quad \beta > \frac{2V_H - 3V_L}{V_H - V_L}.
\]
There exists a Pareto improving tax $\min\{\delta_L - 1, \frac{\beta}{1-\beta}\} > \tau > 0$, if

$$B^L = \frac{2\beta\delta_L(V_HV_L - \frac{V_H^2}{2})}{(\delta_I - \delta_U)V_L^2}E,$$

(31)

$$2V_L > V_H > V_L,$$

(32)

$$\frac{\delta_U}{1-\beta} > \delta_I > \frac{\delta_U}{F_P} F_P$$

and

(33)

$$\beta > \frac{2V_H - 3V_L}{V_H - V_L}.$$

(34)

References


