Limited Participation and International Risk Sharing: Does the Nominal Exchange Rate Matter?

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Abstract

In international business cycle models with complete financial markets, the consumption growth ratio moves in tandem with the real exchange rate growth. In the data, however, their correlation is often negative. International risk sharing is quite poor in reality. This consumption-real exchange rate anomaly, also called the Backus-Smith puzzle, is one of the major puzzles in international macroeconomics. Empirical results show that the nominal exchange rate movements are the main source for this anomaly. This paper shows that an endogenously segmented asset markets model can solve the consumption-real exchange rate anomaly and reveal the role of the nominal exchange rate in international risk sharing. If the nominal exchange rate is fixed, international risk sharing improves in the simulated economy.

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1 Introduction

Backus and Smith (1993) first document the inconsistency between the theoretical and empirical results on international risk sharing. Theoretically, in complete financial markets, the correlation between the growth rate of relative consumptions and the growth rate of relative prices, i.e. real exchange rate, should be unity for any pair of countries. This result can be also stated as the full international risk sharing condition. However, empirical tests by Backus and Smith (1993) and Lewis (1996) and Chari, Kehoe, and McGrattan (2002), et al. show the cross-correlations between consumption and the real exchange rate are either close to zero or negative. Even though there is no doubt that the real economy does not yet have complete markets, and therefore no full international risk sharing, the negative correlation between the relative consumption growth and the real exchange rate growth is still puzzling. This contradiction between the theory and empirical results in international risk sharing is called the consumption-real exchange rate anomaly, also called the Backus-Smith puzzle.

So far, most of the work try to solve this puzzle mainly focus on the interaction of the real terms. However, evidence from Hess and Shin (2010), Hadzi-Vaskov (2008), Devereux and Hnatkovska (2013), et al. shows that the negative correlation between the relative consumption growth and the real exchange rate comes from the movements of the nominal exchange rate. They decompose the real exchange rate growth into the nominal exchange rate growth and the inflation differential components and find that the relative consumption growth is negatively correlated with the nominal exchange rate growth but positively correlated with the inflation differential. Moreover, if the nominal exchange rate is constant, the Backus-smith puzzle may disappear. Therefore, they argue that the fluctuation of the nominal exchange rate is likely the most important driving force behind the consumption-real exchange rate anomaly. If this is correct, the reason why the nominal exchange rate moves counter-cyclically with the international consumption ratio is a question that we cannot circumvent when we address the anomaly.

The primary contribution of this paper is to elaborate on a mechanism through which the relative consumption growth move counter-cyclically with the nominal exchange rate but cyclically with the inflation differential, and through which the fluctu-
ation of the nominal exchange rate has an adverse effect on international risk sharing. When the exchange rate is fixed, international risk sharing is improved.

The methodology I employ is the limited participation in financial markets. The model I use is closely related to the endogenous market segmentation model developed by Alvarez, Atkeson, and Kehoe (2009). In the model, households face idiosyncratic real cost when they transfer money between the goods market and the asset market.\(^1\)

The cost \(\gamma\) has a distribution \(F(\gamma)\) with density \(f(\gamma)\). At the beginning of each period, households receive the same real endowment \(y\) in the goods market. Inflation is distorting because it reduces the real endowment to be \(y/\pi\). This effect induces some households to use the real resources to pay the transfer cost for transferring money from the asset market to the goods market, therefore reducing the total amount of resources available for consumption. Households that pay the fixed cost are called active households and those that do not are called inactive households. Whether households are active or inactive in the asset market depends on the costs households must pay and the level of inflation. The higher inflation is, the more households choose to be active, so the asset market is endogenously segmented. For each inflation, there is a cost level \(\gamma_\pi\) at which the households with \(\gamma < \gamma_\pi\) choose to be active and pay the cost, while the households with \(\gamma > \gamma_\pi\) choose to be inactive and consume \(y/\pi\). It is indifferent for households to be active or inactive if their cost is equal to \(\gamma_\pi\). This cutoff rule is illustrated in Figure 1.

Domestic active households share the consumption risk with foreign active households via the international financial market, while the inactive households only consume their endowment and are inert in the financial market. They only have limited international risk sharing via international trade. Therefore, international risk sharing is incomplete at the aggregate level. The model’s results clearly indicate the existence of the inactive households, i.e. the incompleteness of international risk sharing, is the prerequisite for the negative correlation between the relative consumption growth and

\(^1\)Besides the traditional interpretation for such a cost as the brokerage fee, the bid-ask spread and the transaction tax, the literature explores more motives for this cost. Chatterjee and Corbae (1992) view the cost as a cost involved in writing enforceable private debt contracts. Reis (2006) and Alvarez, Guiso, and Lippi (2012) consider the costs of acquiring, absorbing and processing information. Gust and Lopez-Salido (2014) interpret the presence of the transfer cost as reflecting time spent on the activities of re-optimizing and responding to new information, and the human inertia of sticking to a predetermined plan. Thus one can consider the transfer cost as the aggregate effect of all kinds of frictions which can be tangible or intangible.
the nominal exchange rate. At the same time, the change of the nominal exchange rate may change the terms of trade and then change the international trade. So the nominal exchange rate may in turn affect international risk sharing.

## 2 Related Literature

The Arrow-Debreu equilibrium shows the marginal utility growth should be the same for all investors in a single-good world with complete markets,

\[
\frac{\beta_i U'(c_{i,t+1})}{U'(c_{i,t})} = \frac{\beta_j U'(c_{j,t+1})}{U'(c_{j,t})},
\]

(2.1)

where \(i\) and \(j\) refer to different investors or countries. If investors have the same homothetic utility function, then investors share the same consumption growth rate:

\[
\frac{c_{i,t+1}}{c_{i,t}} = \frac{c_{j,t+1}}{c_{j,t}}.
\]

(2.2)

Hence, regardless of relative shocks to home and foreign outputs, any shock is shared equally to investors. Only aggregate shocks should matter.

Backus and Smith (1993) investigate equation (2.2) with a stochastic exchange economy and non-tradable goods. When trade is costly, price levels are different for country \(i\) and \(j\). The risk sharing condition stated by equation 2.2 should be normalized by the difference in price levels. Given investors have the constant relative risk aversion utility function, \(U(c) = e^{1-\sigma}/(1 - \sigma)\), the equilibrium implies:

\[
\frac{c_{i,t+1}^{\sigma}/P_{i,t+1}}{c_{i,t}^{\sigma}/P_{i,t}} = \frac{c_{j,t+1}^{\sigma}/P_{j,t+1}}{c_{j,t}^{\sigma}/P_{j,t}},
\]

(2.3)

so

\[
\sigma \Delta \log \left( \frac{c_i}{c_j} \right) = \Delta \log (\varepsilon_{i,j}),
\]

(2.4)

and

\[
\rho \left[ \Delta \log \left( \frac{c_i}{c_j} \right), \Delta \log (\varepsilon_{i,j}) \right] = 1,
\]

(2.5)

where \(P_i\) and \(P_j\) denote the price levels of country \(i\) and \(j\), with both price measured in the same numeraire currency, and \(\varepsilon_{i,j}\) denotes the bilateral real exchange rate. The sign
“Δ” refers to the first difference of a variable over time. These definitions will apply throughout the paper.

Therefore, in the economy with non-tradable goods, there is a monotone relation between the bilateral real exchange rate and the consumption ratio: If the real exchange rate is higher in one state than in another, then so is the consumption ratio. Equation (2.4) or (2.5) is called the international risk sharing condition with trade frictions.

The intuition for this condition is straightforward, since households are more likely to consume more when their consumption basket is relatively cheap. However, this is at odds with the data. Backus and Smith (1993) test the cross-correlations between the growth rate of consumption ratio and the growth rate of real exchange rate with the data of eight OECD countries. Their results show there is no clear correlation between the consumption ratio and the real exchange rate. This is the origin of the Backus-Smith puzzle, i.e. the consumption-real exchange rate anomaly, one of the central puzzles in international macroeconomics. The empirical evidence showing low or negative correlations between the consumption ratio and the real exchange rate are also confirmed by Lewis (1996), Chari, Kehoe, and McGrattan (2002) and Devereux and Hnatkovska (2009), et al. Chari, Kehoe, and McGrattan (2002) test the correlations between bilateral real exchange rates and bilateral relative consumption for five countries. They report the correlations vary between -0.48 and 0.24. Corsetti, Dedola, and Leduc (2008) report that the annual real trade-weighted exchange rates between the United States and the other OECD countries move counter-cyclically with their consumption ratios.

More subtle analyses have brought some additional new insights into this anomaly. Crucini (1999) documents there is much more risk sharing across the Canadian provinces and the U.S. states than there is across the G-7 countries. Using the data of OECD countries, Hess and Shin (2010) find that nominal exchange rate movements

\(^2\)The theory implies a positive correlation between the consumption ratio and the real exchange rate and also between their growth rates, see equations (2.3) and (2.4). Empirical tests usually check the correlation between the two growth rates. This is not only because taking the first difference is useful for us to get a stationary series. Moreover, the positive correlation between the two growth rates is a deduction of the positive correlation between the two levels, consumption ratio and real exchange rate, so the rejection of the positive correlation between the growth rates can also reject the positive correlation between the levels. There are also some tests use the data of levels, such as Chari, Kehoe, and McGrattan (2002), Corsetti, Dedola, and Leduc (2008) and Bodenstein (2008). No matter the tests are based on the levels or on the growth rates, the theory are rejected without exception.

\(^3\)Obstfeld and Rogoff (2001) discuss the six major puzzles in international macroeconomics. One of the puzzles is called the international consumption correlations puzzle and the Backus-Smith puzzle is a main appearance of the international consumption correlations puzzle.
are the main source for the anomaly by decomposing the real exchange rate growth, 
\( \Delta \log(\varepsilon_{i,j}) \), into its nominal exchange rate growth, \( \Delta \log(e_{i,j}) \), and inflation differential, 
\( \Delta \log(P_j/P_i) \), components, i.e. \( \Delta \log(\varepsilon_{i,j}) = \Delta \log(e_{i,j}) + \Delta \log(P_j/P_i) \). According to their test, the average correlation between the consumption ratio growths, \( \Delta \log(c_i/c_j) \), and the real exchange rate growth is -0.098. The average correlation between the consumption ratio growth and the nominal exchange rate growth is -0.174, while the average correlation between the consumption ratio growth and inflation differential is 0.163. Moreover, they use the consumption and price for the 50 U.S. states to explore the intranational evidence on risk sharing. Their evidence suggests that if the nominal exchange rate is constant, the Backus-Smith puzzle may disappear.

Hadzi-Vaskov (2008) obtains similar results based on Eurozone data. He finds that there is a clear “dichotomy” between the results for the inflation differential and nominal exchange rate changes, and concludes that nominal exchange rate behavior appears crucial for understanding the consumption-real exchange rate anomaly. Hadzi-Vaskov (2008) therefore asks why the nominal exchange rate is negatively correlated to relative consumption and why it behaves so differently from the inflation differential. Devereux and Hnatkovska (2013) investigate the consumption-real exchange rate correlation with a newly constructed multi-country and multi-regional data set. They find evidence for within-country risk sharing. However, across countries the association is reversed. They therefore define the difference in risk sharing between within countries and across counties as a border effect. They show the border effect account for 53 percent of the deviations from full risk sharing and over one-third of the border effect is due to nominal exchange rate fluctuation.

The first three columns of Table 1 show the results of the empirical test adopting the method of Hess and Shin (2010) with the data of the U.S. and other 22 OECD countries. The results are similar to those of Hess and Shin (2010). The average consumption-real exchange rate correlation is -0.135. The average consumption-nominal exchange rate correlation is -0.195, while the average consumption-inflation differential is 0.225. The appearance and disappearance of the Bretton Woods system provide us a natural experiment for exploring the role of the nominal exchange rate in the consumption-real exchange rate anomaly. I collect the related data of four Bretton Woods system members: the U.S., U.K., Australia, and South Africa, and compare the change of the
correlations between the consumption ratio growth and the real or the nominal exchange rate growth and inflation differential before and after the Bretton Woods system ended. Table 3 presents the results. The results show that the correlations between the consumption ratio growth and the real exchange rate growth become negative (or lower for the U.K.) after the Bretton Woods system ended. This change is clearly coincides with the change in the nominal exchange rate variability after the Bretton Woods system ended.

The counter-cyclical co-movements between bilateral consumption ratios and real exchange rates highlight the incompleteness of international financial risk sharing. One intuitive explanation for poor risk sharing is the existence of incomplete international financial markets and sticky goods prices. Chari, Kehoe, and McGrattan (2002) try to use sticky price model, sticky price model with incomplete financial markets and sticky price model with incomplete financial markets and habit persistence in preference to generate the negative correlation between real exchange rates and relative consumption, but none of the models succeeds. Other attempts in the track of incomplete financial markets include Corsetti, Dedola, and Leduc (2008) and Benigno and Thoenissen (2008), both of which allow the existence of only one non-contingent bond in the asset market and succeed in generating the negative correlation. One important difference between their models and that of Chari, Kehoe, and McGrattan (2002) is the existence of the non-tradable goods, so that the law of one price never holds to the aggregate prices. One weakness of the single-bond asset markets model is that the results are not robust to the introduction of a second traded asset. Benigno and Kucuk-Tuger (2008) show that quantitatively the correlation between consumption and real exchange rates is well above the one observed in the data once a second traded asset is included. Moreover, Levine and Zame (1998) show that the incompleteness of financial markets may not impede the risk sharing. Brandt, Cochrane, and Santa-Clara (2006) estimate international risk sharing with the data from financial markets and find international risk sharing is better than we used to think.

Another branch tries to address the consumption-real exchange rate anomaly within the framework of complete financial markets. Bodenstein (2008) develops a complete asset-market model with limited enforcement for international financial contracts which can address the anomaly providing that agents are not too patient. Kollman-
n (2012) examines the limited asset market participation and shows this anomaly can be explained by a simple segmentation between households. That is between a subset of households trading in complete financial markets and the remaining households having hand-to-mouth existences. Cociuba and Ramanarayanan (2011) want to endogenize the limited participation with the model in Alvarez, Atkeson, and Kehoe (2002). However, they can only reduce the correlation slightly (0.62 vs. 1).

Devereux and Hnatkovska (2013) attempt to investigate the role of nominal exchange rate in international risk sharing by combining sticky goods prices and the single-bond asset markets model of Benigno and Thoenissen (2008). Their framework generates only a slight difference in the consumption-real exchange correlation between the flexible and the fixed exchange rate regime (-0.03 vs. 0.11). The assumption of price stickiness is critical to the sign reversal of the consumption-real exchange rate correlation after nominal exchange rates are held fixed.4

It is easy to test whether the stickiness in prices can account for the consumption-real exchange rate anomaly or not. If the anomaly is caused by the sticky prices, we can imagine that the consumption-price correlation and the consumption-real exchange rate correlation should be improved when we use the prices of later periods to substitute the current prices. However, the test results do not show any evidence that the price stickiness is a reason for the consumption-real exchange rate anomaly. Column 4-7 of Table 1 show the correlations between the consumption and the 1 or 2 periods later price, the correlations between the consumption and the sticky-price corrected real exchange rate. The results show nothing improvement in the correlations. Table 2 shows the test results with the quarterly data. Again, we do not see any improvement in the consumption-price or consumption-real exchange rate correlations.

So far, the answers for the Backus-Smith puzzles, especially for the consumption-nominal exchange rate anomaly is far from thoroughly convincing. The primary motivation of the paper is to reveal the role of the nominal exchange rate movement in international risk sharing and clarify the association between the relative consumption growth and the nominal exchange rate. The core question needs to be answered is the

4Devereux and Hnatkovska (2013) show the standard sticky price model actually cannot generate a significant difference in the consumption-real exchange rate correlation between the flexible and fixed exchange rate contexts. In order to achieve a significant difference, they have to introduce a ‘rule of thumb behavior’ in price setting, as in Gali and Gertler (1999).
causality between international risk sharing and the nominal exchange rate. Does the incompleteness international risk sharing decide the negative correlation between the consumption and the nominal exchange rate or vice versa? If the negative correlation between the consumption and the nominal exchange rate is result from the incompleteness of international risk sharing, why international risk sharing would be improved when the variability of the nominal exchange rate is removed? If the causality should be reversed, what is the mechanism through which the fluctuation of nominal exchange rate may affect international risk sharing?

In the next sections, you will see the question is solved successfully with an endogenously segmented markets model.

3 Model

3.1 Outline

The model used in this paper is an extension of the model in Alvarez, Atkeson, and Kehoe (2009) with non-tradable goods and international trade. This is a cash-in-advance and endowment economy with an infinite number of periods, \( t = 0, 1, 2, \ldots \). There are two countries, the domestic country \( i \) (or country 1) and the foreign country \( j \) (or country 2). Each country is populated by infinitely lived households. There are two separate markets, an asset market and a goods market. In the asset market, households trade two currencies and two complete sets of one-period state-contingent bonds issued by the two countries’ governments respectively. At the beginning of each period, households in each country are endowed with a certain amount of tradable goods and non-tradable goods. The tradable goods of the two countries are imperfect substitutes, so there are four goods being traded in the goods market, but only the two tradable goods can be traded internationally. Each of the non-tradable goods can only be traded domestically.

Each household has to pay a cost \( \gamma \) for each transfer of cash between the asset market and the goods market. In each country, this transfer cost varies across households according to a distribution \( F(\gamma_i) \) with a density \( f(\gamma_i) \). The difference in the transfer cost separates households into active and inactive participants in the asset market.

The sources of uncertainty in the economy include the shocks to money growth
and the state of the goods endowments. In each period, the money growth and the endowment of the country determine its inflation for that period. Households in each county enter period $t$ with cash $P_{i,t-1}Y_{i,t-1}$ that they obtained by selling their endowments $Y_{i,t-1}$ in period $t - 1$ at price $P_{i,t-1}$. Households enter the goods market in period $t$ with real value $n = P_{i,t-1}Y_{i,t-1}/P_{i,t}$. Households can choose to be an active household by paying their transfer cost $\gamma$ to transfer an amount of cash $P_{i,t}x$ with real value $x$ to or from the asset market. Hence, the cash-in-advance constraint for active households is $c = n + x$, and for inactive households is $c = n$.

### 3.2 Setup and Equilibrium

The economy contains four goods: two internationally tradable goods and two non-tradable goods. Let $s^t = (s_1, ..., s_t)$ denote the history of aggregate events up to period $t$, and $g(s^t)$ denote the density of the probability distribution over such histories. Households in country $i$ are endowed with $Y^T_i(s^t)$ units of tradable goods and $Y^N_i(s^t)$ units of non-tradable goods. Let $Y(s_t) = (Y^T_i, Y^T_j, Y^N_i, Y^N_j)$ is the endowment vector in state $s_t$. Let $M_i(s^t)$ denote the money stock of country $i$ in period $t$, and then $\mu_i(s^t) = M_i(s^t)/M_i(s^{t-1})$ denote the corresponding money growth rate.

The final consumption goods $c_i$ is the aggregate of the tradable and non-tradable goods:

$$c_i(s^t) = \left[ \alpha_i c^T_i(s^t)^{\frac{\sigma - 1}{\sigma}} + (1 - \alpha_i) c^N_i(s^t)^{\frac{\sigma - 1}{\sigma}} \right]^{\frac{\sigma}{\sigma - 1}},$$

(3.1)

where $c^T_i$ is the consumption index of the aggregate of the tradable goods and $c^N_i$ is the consumption of the non-tradable goods in country $i$.

The aggregate tradable goods consumption index is determined by

$$c^T_i(s^t) = \left[ \alpha_{ii} c^{T_i}(s^t)^{\frac{\sigma - 1}{\sigma}} + \alpha_{ij} c^{T_j}(s^t)^{\frac{\sigma - 1}{\sigma}} \right]^{\frac{\sigma}{\sigma - 1}}, \quad \alpha_{ii} + \alpha_{ij} = 1,$$

(3.2)

where $c^{T_i}$ and $c^{T_j}$ denote country $i$'s consumption of the tradable goods that originate in country $i$ and $j$ respectively. If $\alpha_{ii} > \frac{1}{2}$, then there is a home-bias in consumption.

The price index for composite consumption purchases:

$$P_i(s^t) = \left[ \alpha_i \phi(P^T_i(s^t))^{1-\phi} + (1 - \alpha_i) \phi(P^N_i(s^t))^{1-\phi} \right]^{\frac{1}{1-\phi}},$$

(3.3)
where $P^T_i$ is the price index of the aggregate of the tradable goods and $P^N_i$ is the price of the non-tradable goods in country $i$.

The aggregate tradable goods price index is determined by

$$P^T_i(s^t) = \left[ \alpha_i \theta_i (P^T_i(s^t))^{1-\theta} + \alpha_j \theta_j (P^T_j(s^t))^{1-\theta} \right]^{1/\theta},$$  \hspace{1cm} (3.4)$$

where $P^T_i$ and $P^T_j$ denote the price of tradable goods that originate in country $i$ and $j$, but are consumed in country $i$.

By the law of one price, we have

$$P^T_j(i) = e_{i,j} P^T_j(s^t),$$

and

$$P^T_i(s^t) = P^T_i(s^t) e_{i,j}(s^t),$$  \hspace{1cm} (3.5)$$

where $e_{i,j}$ is the nominal exchange rate between country $i$ and $j$.

In period 0, all households in both countries are identical. They start with the same endowments, $Y_i(s_0) = Y_j(s_0)$ with $Y_i^T(s_0) = Y_j^T(s_0)$ and $Y_i^N(s_0) = Y_j^N(s_0)$, an identical money stock, $M_i(s_0) = M_j(s_0)$, and the same amount of government bonds, $B_i(s_0) = B_j(s_0)$. Thus both countries have the same price level, $P_i(s_0) = P_j(s_0)$, and the nominal exchange rate $e_{i,j}(s_0)$ equals 1.

In each period, the government of each country issues one-period bonds contingent on the aggregate state $s^t$. In the case of country $i$, the government pays off outstanding bonds $B_i(s^t)$ with currency $i$ and issues new bonds $B_i(s^t, s_{t+1})$ at a price $q_i(s^t, s_{t+1})$. Hence, the country $i$ government budget constraint at $s^t$ with $t \geq 1$ is

$$B_i(s^t) = M_i(s^t) - M_i(s^{t-1}) + \int_{s_{t+1}} q_i(s^t, s_{t+1}) B_i(s^t, s_{t+1}) ds_{t+1}$$  \hspace{1cm} (3.6)$$

In period 0, we have $B_i(s_0) = \int_{s_1} q_i(s_0, s_1) B_i(s_0, s_1) ds_1$ and $M_i(s_0)$ is given. No arbitrage between the bonds of country $i$ and $j$ implies that

$$q_i(s^t, s_{t+1}) = q_j(s^t, s_{t+1}) \frac{e_{i,j}(s^t)}{e_{i,j}(s^t)}$$  \hspace{1cm} (3.7)$$

Households have to pay $\gamma(s^t)$ units of tradable goods as a price, if they want to transfer cash between the asset market and the goods market. The transfer cost varies across households in each country according to a distribution $F(\gamma(s^t))$ with density
Since the asset market is complete, the competitive equilibrium allocation and asset zero if \( x \) budget constraint for the goods market is \( x \) asset market, otherwise, \( x \) is negative. The indicator variable \( z(s^t, \gamma_i(s^t)) \) is equal to zero if \( x(s^t, \gamma_i(s^t)) \) is equal to zero, otherwise, \( x(s^t, \gamma_i(s^t)) \) is equal to one.

In period 0, the asset market budget constraint for households is given by

\[
B_i^t(s^0) + e_{i,j}(s^0)B_j^t(s^0) = \int_{s_{t+1}} [q_i(s^t, s_{t+1})B_i^t(s^t, s_{t+1}) + e_{i,j}(s^t)q_j(s^t, s_{t+1})B_j^t(s^t, s_{t+1})] ds_{t+1}
+ z_i(s^t, \gamma_i(s^t)) [P_i(s^t)x(s^t, \gamma_i(s^t)) + P_i^T(s^t)\gamma_i(s^t)]
\]  

(3.8)

where \( x(s^t, \gamma_i(s^t)) \) is the real value that households choose to transfer between the goods market and the asset market. \( P_i(s^t)x(s^t, \gamma_i(s^t)) \) is the nominal value of the transfer. A positive value for \( x \) means that households have transferred money out of the asset market, otherwise, \( x \) is negative. The indicator variable \( z(s^t, \gamma_i(s^t)) \) is equal to zero if \( x(s^t, \gamma_i(s^t)) \) is equal to zero, otherwise, \( x(s^t, \gamma_i(s^t)) \) is equal to one.

Household consumption is subject to the cash-in-advance constraint. The nominal budget constraint for the goods market is

\[
P_i(s^t)c_i(s^t, \gamma_i(s^t)) \leq P_i^{Ti}(s^{t-1})y_{i,t-1} + P_i^N(s^{t-1})Y_{i,t-1}^N + z_i(s^t, \gamma_i(s^t))P_i(s^t)x(s^t, \gamma_i(s^t)),
\]  

(3.9)

The nominal resource constraint is given by

\[
\int [P_i(s^t)c_i(s^t, \gamma_i(s^t)) + z_i(s^t, \gamma_i(s^t))P_i^T(s^t)\gamma_i(s^t)] f_i(\gamma_i(s^t))d\gamma_i(s^t)
= P_i^{Ti}(s^t)y_{i,t} + P_i^N(s^t)Y_{i,t}^N
\]  

(3.10)

Since the asset market is complete, the competitive equilibrium allocation and asset
prices can be determined from the solution to the following planning problem:

$$\max \sum_{t=1}^{\infty} \beta^t \int_{s^t} U \left( c_i(s^t, \gamma_i(s^t)) \right) f(\gamma_i(s^t)) g(s^t) d\gamma_i(s^t) ds^t$$ \hspace{1cm} (3.12)

subject to the resource constraint (3.11) and the following additional constraint

$$c_i(s^t, \gamma_i(s^t)) = z(s^t, \gamma_i(s^t)) c_{iA}(s^t, \gamma_i(s^t)) + \left[ 1 - z(s^t, \gamma_i(s^t)) \right] c_{i\bar{A}}(s^t)$$ \hspace{1cm} (3.13)

where $c_{iA}$ denotes the consumption of active households and $c_{i\bar{A}}$ denotes the consumption of inactive households. The budget constraint (3.10) indicates that the consumption of inactive households is independent of $\gamma_i(s^t)$ and is equal to

$$c_{i\bar{A}}(s^t) = \frac{P^{Ti} Y_{i,t-1} + P^{Ni} Y_{N,i,t-1}}{P_i(s^t)}$$ \hspace{1cm} (3.14)

The first-order condition for the active household consumption $c_{iA}$ is reduced to

$$\beta^t U' \left[ c_{iA}(s^t, \gamma_i(s^t)) \right] g(s^t) = \lambda_i(s^t) P_i(s^t)$$ \hspace{1cm} (3.15)

$\lambda_i(s^t)$ is the multiplier on the resource constraint of country $i$. This first-order condition clearly implies that all households that pay the fixed cost choose the same consumption level, which means that $c_{iA}(s^t, \gamma_i(s^t))$ is independent of $\gamma_i(s^t)$. So the active household consumption can be denoted as $c_{iA}(s^t)$.

Combining this first order condition for both the home and the foreign counties, we get the international risk sharing condition:

$$\frac{\lambda_i(s^t) P_i(s^t)}{\lambda_j(s^t) P_j(s^t)} = \frac{U'_i(c_{iA}(s^t))}{U'_j(c_{jA}(s^t))}$$ \hspace{1cm} (3.16)

Now, we know there are two consumption profiles in each country, the active household consumption $c_{A}(s^t)$ and the inactive household consumption $c_{\bar{A}}(s^t)$. Both are independent of the transfer cost $\gamma(s^t)$. Given the state, the only factor that determines households being active or inactive is the level of their transfer cost. So it is clear that for any state, there is a threshold transfer cost $\bar{\gamma}(s^t)$ at which the households with $\gamma(s^t) \leq \bar{\gamma}(s^t)$ pay the cost and consume $c_{A}(s^t)$. All the households with $\gamma(s^t) > \bar{\gamma}(s^t)$ choose to be inactive in the asset market and consume $c_{\bar{A}}(s^t)$.
The planner’s problem reduces to

\[
\max_{c_iA(s^t), \bar{\gamma}_i(s^t)} U(c_iA(s^t))F(\bar{\gamma}_i(s^t)) + U(c_iA(s^t))[1 - F(\bar{\gamma}_i(s^t))] \tag{3.17}
\]

subject to

\[
P_i(s^t)c_iA(s^t)F(\bar{\gamma}_i(s^t)) + \int_0^{\bar{\gamma}_i(s^t)} P_i^{Ti}(s^t)\gamma_i(s^t)f(\gamma_i(s^t))d\gamma_i(s^t) + P_i(s^t)c_i\bar{A}(s^t)[1 - F(\bar{\gamma}_i(s^t))] = 0 \tag{3.18}
\]

The first order condition is

\[
U(c_iA(s^t)) - U(c_iA(s^t)) - \frac{U'(c_iA(s^t))}{P_i(s^t)} [P_i(s^t)c_iA(s^t) + P_i^{Ti}(s^t)\bar{\gamma}_i(s^t) - P_i(s^t)c_i\bar{A}(s^t)] = 0 \tag{3.19}
\]

As Alvarez, Atkeson, and Kehoe (2009) claim: “In the decentralized economy corresponding to the planning problem, asset prices are given by the multipliers on the resource constraints for the planning problem.” According to (3.15), these multipliers are equal to the marginal utility of active households.

Hence, the pricing kernels for the domestic and the foreign country are

\[
m_i(s^t, s_{t+1}) = \beta U'(c_iA(s^{t+1})) \frac{1}{\pi_{i,t+1}} \tag{3.20}
\]

\[
m_j(s^t, s_{t+1}) = \beta U'(c_jA(s^{t+1})) \frac{1}{\pi_{j,t+1}} \tag{3.21}
\]

In complete asset markets, the pricing kernels for domestic and foreign assets are related:

\[
m_{j,t+1} = m_{i,t+1} \frac{e_{i,j,t+1}}{e_{i,i,t}} \tag{3.22}
\]

Since the two countries are identical in period 0, \(\lambda_i(s_0)\) equals \(\lambda_j(s_0)\) and \(e(s_0)\) equals 1. Given this assumption, combining equations (3.16), (3.20), (3.21), and (3.22), we can solve the international risk sharing condition recursively:

\[
\frac{U'_i(c_iA(s^t))}{U'_j(c_jA(s^t))} = \frac{P_i(s^t)}{e_{i,j}(s^t)P_j(s^t)} \tag{3.23}
\]
Goods market-clearing conditions are

\[ c_{1A}^{T1} F(\gamma_1) + \int_{0}^{\gamma_1} \gamma_1 f(\gamma_1) d\gamma + c_{1A}^{T1} [1 - F(\gamma_1)] + c_{2A}^{T1} F(\gamma_2) + c_{2A}^{T1} [1 - F(\gamma_2)] = Y_1^T \] (3.24)

\[ c_{2A}^{T2} F(\gamma_2) + \int_{0}^{\gamma_2} \gamma_2 f(\gamma_2) d\gamma + c_{2A}^{T2} [1 - F(\gamma_2)] + c_{1A}^{T2} F(\gamma_1) + c_{1A}^{T2} [1 - F(\gamma_1)] = Y_2^T \] (3.25)

\[ c_{1AN}(s^t) F(\gamma_1(s^t)) + c_{1AN}(s^t) [1 - F(\gamma_1(s^t))] = Y_{1,t}^N \] (3.26)

\[ c_{2AN}(s^t) F(\gamma_2(s^t)) + c_{2AN}(s^t) [1 - F(\gamma_2(s^t))] = Y_{2,t}^N \] (3.27)

Money market-clearing conditions are

\[ P_{1}^{T1}(s^t) Y_{1,t}^T + P_{1}^{N}(s^t) Y_{1,t}^N = M_1(s^t) \] (3.28)

\[ P_{2}^{T2}(s^t) Y_{2,t}^T + P_{2}^{N}(s^t) Y_{2,t}^N = M_2(s^t) \] (3.29)

For each period, I solve the four market-clearing conditions and the risk sharing condition (3.23) and the first order condition (3.19) for the active and inactive consumption, the threshold transfer cost and the prices for tradable and non-tradable goods.

### 3.3 Model Implications

With the assumption of a CRRA utility, and taking logarithms on both sides of equation (3.23), we get:

\[ \sigma \log(c_{A_i}/c_{A_j}) = \log(\varepsilon_{i,j}) \]

\[ = \log(\varepsilon_{i,j}) + \log(P_j/P_i) \] (3.30)

Equation (3.30) indicates that the log of the consumption ratio between the domestic and foreign active households correlates perfectly with the log of the real exchange rate, i.e. \( \rho[\log(c_{A_i}/c_{A_j}), \log(\varepsilon_{i,j})] = 1 \). Therefore, the behavior of active households cannot explain the consumption-real exchange rate anomaly. It is proved that the log of the consumption of active households is strictly increasing in the log of inflation, when inflation is around 1, see Proposition 3 of Alvarez, Atkeson, and Kehoe (2009). Therefore, the log of the active household consumption ratio should negatively correlate with the log of the price ratio, i.e. \( \rho[\log(c_{A_i}/c_{A_j}), \log(P_j/P_i)] < 0 \). If this holds, we can
immediately deduce that $\rho[\log(c_{Ai}/c_{Aj}), \log(e_{i,j})] > 0$ and $\rho[\log(e_{i,j}), \log(P_{j}/P_{i})] < 0$, if $\rho[\log(c_{Ai}/c_{Aj}), \log(e_{i,j})] = 1$.

The features of the model’s equilibrium refer to active households can be characterized in the following proposition:

**Proposition 1.** The active households of any two counties have full international risk sharing. The correlation between their consumption ratio and the real exchange rate is unity. The consumption ratio is negatively correlated with the price ratio but positively correlated with the nominal exchange rate of the two counties.

It is not easy to obtain the data for the consumption of active households in each country, so it is difficult to check the value of $\rho[\log(c_{Ai}/c_{Aj}), \log(P_{j}/P_{i})]$ and the value of $\rho[\log(c_{Ai}/c_{Aj}), \log(e_{i,j})]$ empirically. However, we can use the data of the nominal exchange rate and the prices to check whether the nominal exchange rate and the price ratio negatively correlated with each other. With the U.S. as country $j$, the first column of Table 4 shows the correlations between the nominal exchange rate growth rates and the price ratio growth rates for some of the OECD countries.$^5$ We can see the correlations for all of the countries are negative. Equations (3.4) and (3.5) show why the nominal exchange rate and the price ratio should be negatively correlated. When substituting equation (3.5) into equation (3.4), we can see that, given $P_{Ti}^{T_{i}}$ and $P_{Tj}^{T_{j}}$, the higher the nominal exchange rate, then the lower is the tradable goods price ratio $P_{j}^{T}/P_{i}^{T}$, so that given the non-tradable goods price $P_{i}^{N}$ and $P_{j}^{N}$, the higher the nominal exchange rate, then the lower is the price ratio $P_{j}/P_{i}$. Hence, the nominal exchange rate negatively correlates with the price ratio.

The correlation between the inactive household consumption ratio, $\log(c_{\bar{Ai}}/c_{\bar{Aj}})$, and the real exchange rate is implied in equations (3.14), (3.4) and (3.5). According to equation (3.14), the consumption of inactive households is proportional to their nominal endowment, $P_{i}^{T_{i}}(s^{t-1})Y_{i_{t-1}}^{T_{t}} + P_{i}^{N}(s^{t-1})Y_{i_{t-1}}^{N}$, but inversely proportional to the price level $P_{i}(s^{t})$, so we get $\rho[\log(c_{\bar{Ai}}/c_{\bar{Aj}}), \log(P_{j}/P_{i})] > 0$. By equations (3.4) and (3.5), the nominal exchange rate negatively correlates with the price ratio, so we immediately get $\rho[\log(c_{\bar{Ai}}/c_{\bar{Aj}}), \log(e_{i,j})] < 0$.

---

$^5$I choose to test the growth rates instead of the levels so that this test is consistent with other tests in this paper.
The features of the model’s equilibrium refer to inactive households can be characterized in the following proposition:

**Proposition 2.** The consumption ratio of inactive households between any two countries is positively correlated with the price ratio but negatively correlated with the nominal exchange rate of the two counties.

So the empirical tests are consistent with the behavior of inactive households. It is worth noting that Proposition 2 is independent of the utility function of households.

The model implies that the consumption of the inactive households decreases with inflation, but increases with the appreciation of the nominal exchange rate since this improves the terms of trade. When inflation is moderate, the consumption of the active households increases with inflation, but decreases with the appreciation of the nominal exchange rate since it can reduce inflation. Thus the consumption patterns of the two types of households in response to inflation and the nominal exchange rate variability are opposite. The correlations between the aggregate consumption ratio and the real exchange rate and the price ratio and the nominal exchange rate are the aggregate results of these two types of consumption. If the consumption of the inactive households dominates the consumption of the active households, then the correlation between the aggregate consumption ratio and the price ratio is positive. The correlation between the aggregate consumption ratio and the nominal exchange rate, however, is negative. In case the consumption of active households dominates the consumption of inactive households, these results are reversed.6

Now the causality between international risk sharing and the nominal exchange rate has been revealed. The existence of inactive households reduces international risk sharing and causes the negative correlation between the aggregate consumption and the nominal exchange rate. However, the variability of the nominal exchange rate can in turn influence international risk sharing by changing inflation. Therefore, when the variability of the nominal exchange rate is removed, international risk sharing may improve. We will see more details with the simulated economy in the next section.

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6Mathematically, \( \rho \left[ \log \left( \frac{c_i}{c_j} \right), \log(\varepsilon_{i,j}) \right] = \rho \left[ \log \left( \frac{F(\bar{\gamma}_i) \bar{c}_i A + (1 - F(\bar{\gamma}_i)) \bar{c}_i \bar{A}}{F(\bar{\gamma}_i) \bar{c}_j A + (1 - F(\bar{\gamma}_i)) \bar{c}_j \bar{A}} \right), \log(\varepsilon_{i,j}) \right] \). If \( (1 - F(\bar{\gamma}_i)) \bar{c}_i \bar{A} \gg F(\bar{\gamma}_i) \bar{c}_i A \) and \( (1 - F(\bar{\gamma}_i)) \bar{c}_j \bar{A} \gg F(\bar{\gamma}_i) \bar{c}_j A \), then \( \rho \left[ \log \left( \frac{1 - F(\bar{\gamma}_i)}{1 - F(\bar{\gamma}_i)} \bar{c}_j \bar{A} \right), \log(\varepsilon_{i,j}) \right] \approx \rho \left[ \log \left( \frac{(1 - F(\bar{\gamma}_i)) \bar{c}_i \bar{A}}{(1 - F(\bar{\gamma}_i)) \bar{c}_j \bar{A}} \right), \log(\varepsilon_{i,j}) \right] \), and vice versa.
4 Calibration and results

4.1 Calibration

Table 5 shows the main parameter values. Preference is represented by the CRRA preference of the form $U(c) = c^{1-\sigma}/(1 - \sigma)$. The coefficient of relative risk aversion $\sigma$ is set to 2. The discount factor $\beta$ is set to 0.95. The elasticity of the substitution between tradable and non-tradable goods, $\phi$, is set to 0.44, and the elasticity of the substitution between home and foreign-originated tradable goods, $\theta$, is set to 2, following the suggestion of Stockman and Tesar (1995). In line with the calibrations of Corsetti, Dedola, and Leduc (2008) and Benigno and Thoenissen (2008), the value of tradable goods share in the final consumption goods, $\alpha$, is set to 0.55, while the weight of the domestic tradable goods, $\alpha_{ii}$, is set to 0.72.

The state for the household endowments of the two countries follows a Markov chain. The endowment vector and the transition matrix are taken from Bodenstein (2008), who calibrates the parameters based on the GDP of G7 countries.

The transfer cost $\gamma$ is not directly observable. I have chosen the level of the transfer cost based on the fraction of the active and inactive households we observe in the real world. Based on the U.S. data, empirical results of Mankiw and Zeldes (1991), Haliassos and Bertau (1995), Vissing-Jorgensen (2002), and Christelis and Georgarakos (2011) show that most U.S. households do not hold stocks. The fraction of households that possess stocks in United States is around 25%. However, Bucks, Kennickell, Mach, and Moore (2009) in a later study show that about 50% of U.S. households own stock directly or indirectly. Moreover, Bonaparte and Cooper (2009) find that only 8.6% of households owning stocks actually adjust their portfolio of common stocks on a monthly basis and the fraction is no more than 71.0% on an annual basis. Based on the above empirical results, I have chosen the level of the transfer cost so that the active households are no more than 50% of the total number of households. I assume $\gamma$ has a uniform distribution with 0 as the lower bound, so the fraction of the active households depends on the largest value of the transfer cost. The greater the value is of the transfer cost, then the lower the fraction is of active households.

I have chosen for inflation the annual mean, $\bar{\pi}$, as 3%. The inflation target has a volatility of 0.0115. This is based on the empirical results of de Vries and Wang (2013).
I assume that the governments of the two countries make their decisions regarding each period’s money stock using the following rule: at the beginning of each period, the governments observe the realization of the endowment \( Y_{i,t} \) and \( Y_{j,t} \), and then they decide the money stock of this period based on the endowment and the target inflation. So the period \( t \) money stock can be summarized in the following equation:

\[
M_i(s^t) = \left[ p_i^T Y_{i,t} + p_i^N Y_{i,t} \right] (\bar{\pi} + \varepsilon_t) \tag{4.1}
\]

\[
M_j(s^t) = \left[ p_j^T Y_{j,t} + p_j^N Y_{j,t} \right] (\bar{\pi} + \varepsilon_t) \tag{4.2}
\]

### 4.2 Numerical results

The economy is simulated 50 times over 300 periods for each set of transfer cost. Table 6 shows the main results of the economy. The reported numbers in Table 6 are the average over the 50 simulations. The results are highly consistent with the analysis and prediction in Section 3.3. The fraction of active households continues to shrink with the increase of the transfer cost. The model succeeds in generating the negative correlations between the consumption ratio and the real exchange rate. More importantly, it generates negative correlations between the consumption ratio and the nominal exchange rate and it generates positive correlations between the consumption ratio and the price ratio.\(^7\) This is fully consistent with the empirical results of Hess and Shin (2010) and Hadzi-Vaskov (2008) and this paper, see Table 1. Therefore, the extension of the segmented asset markets model from Alvarez, Atkeson, and Kehoe (2009) helps to solve the Backus-Smith puzzle at both the real and the nominal level. As the model predicts, the negative correlation between the consumption ratio and the real or the nominal exchange rates are caused by the existence of inactive households in the asset market. So these correlations become more and more negative as the fraction of the inactive households increases. It is reasonable to think that the difference in the fraction of the active and inactive households in different countries is one of the reasons for the difference in the consumption-exchange rate correlation. The model also predicts the negative correlation between the price ratio.

\(^7\)Throughout this paper, the tests for the correlations between the consumption and the real exchange rate, and the nominal exchange rate or the price ratio are always refer to the correlations between their growth rates. The words ‘growth rate’ are often omitted deliberately for the purpose of conciseness.
In the standard business cycle models with a representative agent and complete markets, the cross-country correlation for consumption growth rate is usually much higher than the data. Backus, Kehoe, and Kydland (1992) report that the correlations between Japan and the major European countries lie between 0.22 and 0.48. The correlation between the U.S. and the European aggregate is 0.46. Brandt, Cochrane, and Santa-Clara (2006) test the annual consumption growth correlations between the U.S. and three other countries, the U.K., Germany and Japan. They report correlations ranging from 0.24 to 0.42. I test the annual correlations for consumption growth between the U.S. and some OECD counties. The results are shown in the second column of Table 4. The correlations range from -0.376 (Sweden vs. U.S.) to 0.605 (Canada vs. U.S.) with an average of 0.289. Standard business cycle models usually predict a very high correlation. In my model, the cross country correlation for consumption growth is 0.820 when there is no limited participation ($\gamma = 0$). However, the cross-country consumption correlation is reduced to the level that we see in the data in the presence of inactive households. The results clearly show that the correlations continue to decrease and can even be negative as the fraction of inactive households increases.

The model clearly demonstrates the mechanisms that influence how the nominal exchange rate affects international risk sharing. The negative correlation between the aggregate consumption ratio and the nominal exchange rate is due to the negative correlation between the nominal exchange rate and inactive household consumption, while the reason for their negative correlation is that the nominal exchange rate may dampen the effect of inflation on the consumption of the inactive households by changing the terms of trade. The appreciation of the nominal exchange rate may increase the consumption of the inactive households. Therefore, the effect of the nominal exchange rate on international risk sharing depends on the existence of the inactive households in the economy. The first column of Table 6 shows that the correlation between the consumption ratio and the real exchange rate is unity and the correlation between the consumption ratio and the nominal exchange rate is no longer negative when there are no inactive households in the economy.

\footnote{When there is no transfer cost between the goods market and the financial market, the model degenerates to the standard international business model with a representative agent who is active in the financial market.}
4.3 Risk sharing under fixed regime

In order to explore more details regarding the causality between the nominal exchange rate and international risk sharing, I will now assume that the government of one country decides to fix the exchange rate of its currency to that of another country in order for us to see how the results might change when the variability of the nominal exchange rate is removed.

According to the classic open-economy trilemma, a country cannot simultaneously achieve exchange rate stability, capital market openness, and monetary policy autonomy. Therefore, any country that wants to peg their currency to the currency of another country has to make a trade-off between the freedom of financial flows and monetary sovereignty. For example, within the Euro area, all the Member States have adopted the Euro as their common currency, i.e. a fixed 1:1 exchange rate, and all have open capital markets, so they do not have monetary sovereignty. They have to follow the edicts of the European Central Bank. In contrast to the Euro area, the Chinese government has chosen to maintain their monetary sovereignty although they peg the Renminbi to the U.S. dollar. As a result, the Chinese financial market is closed to foreign capital.

In my simulated economies under fixed exchange rate, e=1, both conditions are considered, i.e. whether the international financial market is either open or closed. Each economy is simulated with or without transfer costs. When there is a transfer cost, I assume the transfer cost has a uniform distribution with the largest value of 2% of \( Y^T \). Table 7 shows the main results of the fixed exchange rate economies. The results show that when the exchange rate is fixed, the fraction of the active households is larger than that of the flexible exchange rate under the same inflation and transfer cost. This is because when the exchange rate is fixed, the distorting effect of inflation on the consumption of the inactive households is more significant, so more households are willing to pay the transfer cost and become active households. If the exchange rate is fixed and there are open international financial markets, the correlation between the aggregate consumption and the real exchange rate is positive (0.488), while this correlation is negative when the exchange rate is flexible (-0.534). Moreover, the aggre-

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From 1995 to 2005, the Chinese government pegged the Renminbi to the U.S. dollar with a target exchange rate of 8.28 RMB for each US$ and only allowed variations in the exchange rate within a very narrow margin. After 2005, the Chinese government announced the reform of the exchange rate. They have allowed the RMB to appreciate gradually and have loosened the constraints on the exchange rate variation.
gate consumption growth correlation between the two counties increases to 0.758 from 0.356 under the flexible exchange rate. The change in the exchange rate regime mainly affects the inactive households. The correlation between the inactive household consumption and the real exchange rate increases to 0.929 under the fixed exchange rate and open international financial markets from -0.586 under the flexible exchange rate. The consumption growth correlation for active households also increase somewhat, it rise from 0.816 to 0.986. When there are only active households in the economy, this correlation increase from 0.804 to 0.828. Therefore, we see the elimination of the variability in the exchange rate may increase the consumption growth correlation for both the active and inactive households. However, the effect is more significant on inactive households than on active households.

The results also indicate that the freedom of international financial flows is also necessary for international risk sharing. If the international financial markets are closed, then the consumption-real exchange rate correlation is -0.550 when all the households are active and -0.442 when some of the households are inactive, and the consumption correlations are both -0.075. Specifically, the consumption-real exchange rate correlation for inactive households increases to 0.367 from -0.586 and their consumption growth correlation increases to 0.007 from -0.152, when compared with the results under the flexible exchange rate. However, blocking the international financial flows is a disaster for international risk sharing of active households. Comparing with the results under the flexible exchange rate, the consumption-real exchange rate correlation for active households decreases to -0.333 from 1.00 and their consumption growth correlation decrease to 0.015 from 0.931. As a result, international risk sharing is poor when we look at the aggregate household consumption once international financial flows are blocked.

China is the only main economy in the world that does not open its financial market to the foreign capital. Let us use the example of China to see the realistic effect on international risk sharing as international financial flows are blocked. Table 8 shows the correlations between China and several main economies in the world, Australian, Japan, the U.K. and U.S. We can see not only the real and the nominal exchange rates have negative correlations with the consumption, but the price ratios are also negatively correlated with consumption. China’s consumption growth is negatively correlated
with the consumption growths of all the countries in the table except for Japan. The consumption growth correlation between China and Japan is a tiny 0.062. The empirical results for China are consistent with the model results for a closed economy.

Besides looking at the correlations, I also use the international risk sharing index developed by Brandt, Cochrane, and Santa-Clara (2006) to measure the level of international risk sharing. This index is labeled as the BCSW index in Table 7.\(^\text{10}\) The value of this index lies between 1 and -1; with 1 indicating perfect international risk sharing and 0 indicating that the two countries’ marginal utility growths are uncorrelated. If the index is -1, then the marginal utility growth of one country is just the opposite of the other. In other words, the higher the index value is, then the higher is the level of international risk sharing.

From the results in Table 7, we see that when there is an open international financial market, then the index value for active households is 1 regardless whether the exchange rate is fixed or flexible. The difference in the index values between the fixed and flexible exchange economies shows that the adverse effects of the exchange rate movement on international risk sharing mainly affect the inactive households. The BCSW index value is -0.202 for inactive households when the exchange rate is flexible, while the index value is 0.848 which causes the index values for all households to increase from 0.155 to 0.802. We see again the importance of openness in international financial markets. When there is no financial flow between the two countries, international risk sharing is almost 0 for the active households of the two countries (BCSW index: -0.018). And while, removing the exchange rate variability may slightly improve international

\(^{10}\)The original definition for the index of international risk sharing in Brandt, Cochrane, and Santa-Clara (2006) is:

\[
1 - \frac{\text{var}(\log m_{j,t+1} - \log m_{i,t+1})}{\text{var}(\log m_{j,t+1}) - \text{var}(\log m_{i,t+1})} = 1 - \frac{\text{var} \left( \log \frac{m_{j,t+1}}{m_{i,t+1}} \right)}{\text{var}(\log m_{j,t+1}) - \text{var}(\log m_{i,t+1})}.
\]

I have some disagreement with this definition. In an open economy with variability in the exchange rate, full international risk sharing is equivalent to the condition of \(m_{j,t+1} = m_{i,t+1} \frac{e_{t+1}}{e_t}\), not \(m_{j,t+1} = m_{i,t+1}\). However, when we calculate the index for international risk sharing following the definition of Brandt, Cochrane, and Santa-Clara (2006), we can not get the highest value of 1 even if the full international risk sharing condition is satisfied as long as there is variability in the exchange rate. Therefore, I make the following amendment to the definition for the index of international risk sharing:

\[
1 - \frac{\text{var}(\log m_{j,t+1} - \log m_{i,t+1} - \log \frac{e_{t+1}}{e_t})}{\text{var}(\log m_{j,t+1}) - \text{var}(\log m_{i,t+1})}.
\]

I refer to the new index as the BCSW index. This amendment is particularly important for studies which are designed to investigate the effect of the exchange rate on international risk sharing, because the value of the new index is controlled for variability in the exchange rate.
risk sharing for inactive households (-0.202 vs. -0.007), this effect is too small however to prevent aggregate international risk sharing from slipping into the negative (0.155 vs. -0.153).

The results from closed economy reveal the importance of financial openness to international risk sharing. They also help us to understand why the nominal exchange rate cannot account for all of the border effect. As Popper, Mandilaras, and Bird (2013) report that there is no absolute financial openness for any economy. Even if there is no any obstruction from the government, the home bias of investors may impede international risk sharing to a certain extent. Hence, the nominal exchange rate can only explain a part of the border effect.

5 Conclusions and policy implications

Empirical studies, such as Hess and Shin (2010) and Devereux and Hnatkovska (2013), show the nominal exchange rate variability is an important source of the consumption-real exchange rate anomaly. This is confirmed by the empirical tests in this paper. The tests in this paper do not support that the price stickiness a reason for the anomaly. Moreover, the change of the correlations for the consumption and real exchange rate anomaly before and after the ending of the Bretton Woods system (the fixed exchange rate regime) provides new evidence that it is the nominal exchange rate fluctuation, not the price ratio, that causes the consumption-real exchange rate anomaly.

This paper explores the mechanisms which underpin the co-movement between consumption, the nominal exchange rate and inflation within the framework of limited participation in the asset markets. By introducing non-tradable goods, international trade and home bias into the endogenously segmented asset markets model developed by Alvarez, Atkeson, and Kehoe (2009), I show that this framework can solve the consumption-real exchange rate anomaly at both the real and the nominal level. Just with the standard calibration, it generates correlations between the relative consumption and real exchange rate, nominal exchange rate and the price ratio consistent with the data under either the flexible or fixed exchange rate regime. The model shows that the existence of inactive households in the financial markets is a key reason for the poor
international risk sharing we have observed. The nominal exchange rate influences international risk sharing by affecting the terms of trade and then the consumption of inactive households. The segmentation between active and inactive households also helps us to understand why international risk sharing estimated with financial market data is better than that estimated with aggregate consumption.

The model implies the negative correlations between the nominal exchange rate and the price ratio, which is confirmed by the data of OECD counties. The presence of inactive households also reduces the cross-country correlations of consumption growth to the level of the data. The role of inactive households in reducing the cross-country consumption correlations are indirectly confirmed by Zhang (2013), who uses micro-level household consumption data in the U.S. and U.K. and shows that the stockholders’ consumption correlation is considerably higher than that of the aggregate consumption growth.

The results in this paper have obviously policy implications. Cochrane (2005) (p. 56) states: “better risk sharing is much of the force behind financial innovation. Many successful new securities can be understood as devices to share risks more widely.” This paper points out that the limited participation in financial markets is another important factor to impede risk sharing besides the incompleteness of the financial markets, therefore it tells us that try to reduce the tangible and intangible cost for participating in financial markets is also an important way for improving risk sharing. Moreover, financial openness is necessary for international risk sharing. Given the incompleteness of financial markets and free international financial flows, the fixed exchange rate may improve international risk sharing. This is a strong support for the one international currency regime, such as the use of euro.
A variables and equations

A.1 Exogenous variables

$M_1$: Money supply in domestic country;
$M_2$: Money supply in foreign country;
$Y^T_1$: Tradable good endowment in domestic country;
$Y^N_1$: Non-tradable good endowment in domestic country;
$Y^T_2$: Tradable good endowment in foreign country;
$Y^N_2$: Non-tradable good endowment in foreign country;
$e(s_0) = 1$: The nominal exchange rate at period 0;
$P^T_1(s_0) = 1$: The tradable goods price in domestic country at period 0;
$P^T_2(s_0) = 1$: The tradable goods price in foreign country at period 0;

A.2 Endogenous variables need to be solved

1. $c^T_{1A}$: Tradable good consumption of active households in domestic country;
2. $P^T_1$: Tradable good price of domestic country;
3. $P_1$: Price level of domestic country;
4. $c_{1A}$: Aggregate consumption of active households in domestic country;
5. $c^N_{1A}$: Non-tradable good consumption of active households in domestic country;
6. $P^N_1$: Non-tradable good price of domestic country;
7. $c^T_{1A}$: Tradable good consumption of inactive households in domestic country;
8. $c_{1A}$: Aggregate consumption of inactive households in domestic country;
9. $c^N_{1A}$: Non-tradable good consumption of inactive households in domestic country;
10. $c^T_{2A}$: Tradable good consumption of active households in foreign country;
11. $P^T_2$: Tradable good price of foreign country;
12. $P_2$: Price level of foreign country;
13. $c_{2A}$: Aggregate consumption of active households in foreign country;
14. $c^N_{2A}$: Non-tradable good consumption of active households in foreign country;
15. $P^N_2$: Non-tradable good price of foreign country;
16. $c^T_{2A}$: Tradable good consumption of inactive households in foreign country;
17. $c_{2A}$: Aggregate consumption of inactive households in foreign country;
18. $c_{2A}^N$: Non-tradable good consumption of inactive households in foreign country;
19. $c$: Exchange rate between domestic and foreign currencies;
20. $\bar{\gamma}_1$: Cutoff level of transfer cost in domestic country;
21. $\bar{\gamma}_2$: Cutoff level of transfer cost in foreign country;
22. $c_{T11}^A$: Tradable good originated in domestic country and consumed by inactive households in domestic country;
23. $c_{T12}^A$: Tradable good originated in foreign country but consumed by inactive households in domestic country;
24. $c_{T21}^A$: Tradable good originated in foreign country and consumed by inactive households in foreign country;
25. $c_{T11}^A$: Tradable good originated in domestic country but consumed by inactive households in foreign country;
26. $P_{T1}^1$: The price of tradable good originated in domestic country and consumed in domestic country;
27. $P_{T1}^2$: The price of tradable good originated in foreign country but consumed in domestic country;
28. $P_{T2}^2$: The price of tradable good originated in foreign country and consumed in foreign country;
29. $P_{T2}^2$: The price of tradable good originated in foreign country but consumed in foreign country;
30. $c_{T1}^A$: Tradable good originated in domestic country and consumed by active households in domestic country;
31. $c_{T2}^A$: Tradable good originated in foreign country but consumed by active households in domestic country;
32. $c_{T2}^A$: Tradable good originated in foreign country and consumed by active households in foreign country;
33. $c_{T1}^A$: Tradable good originated in domestic country but consumed by active households in foreign country;
34. $c_1$: Total consumption in domestic country;
35. $c_2$: Total consumption in foreign country;
A.3 Model equations

\[ c_{1A}^T = \left( \frac{\alpha}{P_1^T} P_1 \right)^\phi c_{1A} \]  
(A.1)

\[ c_{1A}^N = \left( \frac{1 - \alpha}{P_1^N} P_1 \right)^\phi c_{1A} \]  
(A.2)

\[ c_{1A}^T = \left( \frac{\alpha}{P_1^T} P_1 \right)^\phi c_{1A} \]  
(A.3)

\[ c_{1A}^N = \left( \frac{1 - \alpha}{P_1^N} P_1 \right)^\phi c_{1A} \]  
(A.4)

\[ c_{2A}^T = \left( \frac{\alpha}{P_2^T} P_2 \right)^\phi c_{2A} \]  
(A.5)

\[ c_{2A}^N = \left( \frac{1 - \alpha}{P_2^N} P_2 \right)^\phi c_{2A} \]  
(A.6)

\[ c_{2A}^T = \left( \frac{\alpha}{P_2^T} P_2 \right)^\phi c_{2A} \]  
(A.7)

\[ c_{2A}^N = \left( \frac{1 - \alpha}{P_2^N} P_2 \right)^\phi c_{2A} \]  
(A.8)

\[ c_{1A} = \left[ \frac{\alpha(c_{1A}^T)^{\frac{\phi-1}{\sigma}} + (1 - \alpha)(c_{1A}^N)^{\frac{\phi-1}{\sigma}}} {c_{1A}^T} \right]^{\frac{\phi}{\phi-1}} \]  
(A.9)

\[ c_{2A} = \left[ \frac{\alpha(c_{2A}^T)^{\frac{\phi-1}{\sigma}} + (1 - \alpha)(c_{2A}^N)^{\frac{\phi-1}{\sigma}}} {c_{2A}^T} \right]^{\frac{\phi}{\phi-1}} \]  
(A.10)

\[ c_{1A}^T = \left[ \frac{\alpha_{11}(c_{1A}^{T1})^{\frac{\phi-1}{\sigma}} + (1 - \alpha_{11})(c_{1A}^{T2})^{\frac{\phi-1}{\sigma}}} {c_{1A}^T} \right]^{\frac{\phi}{\phi-1}} \]  
(A.11)

\[ c_{2A}^T = \left[ \frac{\alpha_{22}(c_{2A}^{T2})^{\frac{\phi-1}{\sigma}} + (1 - \alpha_{22})(c_{2A}^{T1})^{\frac{\phi-1}{\sigma}}} {c_{2A}^T} \right]^{\frac{\phi}{\phi-1}} \]  
(A.12)

\[ c_{1A}^{T1} = \left( \frac{\alpha_{11}}{P_1^T} P_1^T \right)^\theta c_{1A} \]  
(A.13)

\[ c_{1A}^{T2} = \left( \frac{1 - \alpha_{11}}{P_1^T} P_1^T \right)^\theta c_{1A} \]  
(A.14)

\[ c_{2A}^{T2} = \left( \frac{\alpha_{22}}{P_2^T} P_2^T \right)^\theta c_{2A} \]  
(A.15)
\[ c_{2A}^T = \left( \frac{1 - \alpha_{22}}{P_{21}^T} P_2^T \right)^\theta c_{2A}^T \]  
(A.16)

\[ c_{1A}^T = \left( \frac{\alpha_{11}}{P_{11}^T} P_1^T \right)^\theta c_{1A}^T \]  
(A.17)

\[ c_{1A}^T = \left( \frac{1 - \alpha_{11}}{P_{11}^T} P_1^T \right)^\theta c_{1A}^T \]  
(A.18)

\[ c_{2A}^T = \left( \frac{\alpha_{22}}{P_{22}^T} P_2^T \right)^\theta c_{2A}^T \]  
(A.19)

\[ c_{2A}^T = \left( \frac{1 - \alpha_{22}}{P_{21}^T} P_2^T \right)^\theta c_{2A}^T \]  
(A.20)

\[ P_{1}^{T_2} = e P_{2}^{T_2} \]  
(A.21)

\[ P_{2}^{T_1} = \frac{1}{e} P_{1}^{T_1} \]  
(A.22)

\[ c_{1A} = \frac{P_{1}^{T_1} Y_{1,-1} + P_{1}^{N} Y_{1,-1}^{N}}{P_{1}} \]  
(A.23)

\[ c_{2A} = \frac{P_{2}^{T_2} Y_{2,-1} + P_{2}^{N} Y_{2,-1}^{N}}{P_{2}} \]  
(A.24)

\[ \frac{c_{1A}^{-\sigma}}{1 - \sigma} - \frac{c_{1A}^{-\sigma}}{1 - \sigma} - \frac{c_{1A}^{-\sigma}}{P_{1}} (P_{1} c_{1A} + P_{1}^{T_1} \gamma_{1} - P_{1} c_{1A}) = 0 \]  
(A.25)

\[ \frac{c_{2A}^{-\sigma}}{1 - \sigma} - \frac{c_{2A}^{-\sigma}}{1 - \sigma} - \frac{c_{2A}^{-\sigma}}{P_{2}} (P_{2} c_{2A} + P_{2}^{T_2} \gamma_{2} - P_{2} c_{2A}) = 0 \]  
(A.26)

\[ \frac{P_{1}}{e P_{2}} = \frac{c_{1A}^{-\sigma}}{c_{2A}^{-\sigma}} \]  
(A.27)

\[ c_{1A}^{T_1} F(\gamma_{1}) + \int_{0}^{\gamma_{1}} \gamma_{1} f(\gamma_{1}) d\gamma + c_{1A}^{T_1} [1 - F(\gamma_{1})] + c_{2A}^{T_2} F(\gamma_{2}) + c_{2A}^{T_2} [1 - F(\gamma_{2})] = Y_{1}^{T} \]  
(A.28)

\[ c_{2A}^{T_2} F(\gamma_{2}) + \int_{0}^{\gamma_{2}} \gamma_{2} f(\gamma_{2}) d\gamma + c_{2A}^{T_2} [1 - F(\gamma_{2})] + c_{1A}^{T_1} F(\gamma_{1}) + c_{1A}^{T_1} [1 - F(\gamma_{1})] = Y_{2}^{T} \]  
(A.29)

\[ c_{1A}^{N} F(\gamma_{1}) + c_{1A}^{N} [1 - F(\gamma_{1})] = Y_{1}^{N} \]  
(A.30)

\[ c_{2A}^{N} F(\gamma_{2}) + c_{2A}^{N} [1 - F(\gamma_{2})] = Y_{2}^{N} \]  
(A.31)

\[ P_{1}^{T_1} Y_{1}^{T} + P_{1}^{N} Y_{1}^{N} = M_{1} \]  
(A.32)

\[ P_{2}^{T_2} Y_{2}^{T} + P_{2}^{N} Y_{2}^{N} = M_{2} \]  
(A.33)

\[ c_{1} = F_{1}(\gamma_{1}) c_{1A} + (1 - F_{1}(\gamma_{1})) c_{1A} \]  
(A.34)

29
\[ c_2 = F_2(\bar{\gamma}_2)c_{2A} + (1 - F_2(\bar{\gamma}_2))c_{2\bar{A}} \]  

(A.35)

For each period, we have 35 equations to solve 35 variables. Given the state of the first period (period 0), we can solve these variables at any period numerically.

### A.4 Model equations at period 0

\[ c_1^T = Y_1^T \]  

(A.36)

\[ c_1^N = Y_1^N \]  

(A.37)

\[ c_2^T = Y_2^T \]  

(A.38)

\[ c_2^N = Y_2^N \]  

(A.39)

\[ c_1 = \left[ \alpha_1(c_1^T)^{\frac{\phi - 1}{\phi}} + (1 - \alpha_1)(c_1^N)^{\frac{\phi - 1}{\phi}} \right]^{\frac{\phi}{\phi - 1}}, \]  

(A.40)

\[ c_2 = \left[ \alpha_2(c_2^T)^{\frac{\phi - 1}{\phi}} + (1 - \alpha_2)(c_2^N)^{\frac{\phi - 1}{\phi}} \right]^{\frac{\phi}{\phi - 1}}, \]  

(A.41)

\[ c_1^T = (\alpha P_1)^\phi c_1 \]  

(A.42)

\[ c_1^N = \left( \frac{1 - \alpha}{P_1^N} P_1 \right)^\phi c_1 \]  

(A.43)

\[ c_2^T = (\alpha P_2)^\phi c_2 \]  

(A.44)

\[ c_2^N = \left( \frac{1 - \alpha}{P_2^N} P_2 \right)^\phi c_2 \]  

(A.45)

\[ P_1 = \left[ \alpha^\phi + (1 - \alpha)^\phi (P_1^N)^{1 - \phi} \right]^{\phi \over \phi - 1} \]  

(A.46)

\[ P_2 = \left[ \alpha^\phi + (1 - \alpha)^\phi (P_2^N)^{1 - \phi} \right]^{\phi \over \phi - 1} \]  

(A.47)
References


Given the state, the cutoff rule function $h$ is a function of $\gamma$, which measures the net gain to a household from switching from being an inactive household with consumption $y/\pi$ to an active household with consumption $c_A$. The expression of $h$ is just the left side of equation (3.19). The first two terms on the left side of equation (3.19) measure the utility gain from the household’s switch from inactivity to activity, and the third term measures the utility cost of such a switch. When the transfer cost of the household is at $\bar{\gamma}$, the utility gain is equal to the utility cost, and the household is indifference to being active or inactive. For more detail of the cutoff rule function, please refer to Alvarez, Atkeson, and Kehoe (2002).
Table 1: Annual consumption-exchange rate correlation between U.S. and the OECD countries

<table>
<thead>
<tr>
<th>Country</th>
<th>$\rho [\sim, \Delta \tilde{e}_{i,j}]$</th>
<th>$\rho [\sim, \Delta \tilde{\epsilon}_{i,j}]$</th>
<th>$\rho [\sim, \Delta \tilde{p}_{ji}]$</th>
<th>$\rho [\sim, \Delta (\tilde{e}<em>{i,j}\tilde{p}</em>{ji})]$</th>
<th>$\rho [\sim, \Delta \tilde{p}_{ji+1}]$</th>
<th>$\rho [\sim, \Delta (\tilde{e}<em>{i,j}\tilde{p}</em>{ji+1})]$</th>
<th>$\rho [\sim, \Delta (\tilde{e}<em>{i,j}\tilde{p}</em>{ji+2})]$</th>
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<td>0.008</td>
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Notes: The annual data are from the World Bank, 1971-2012. The sign “~” represents the consumption growth differential between U.S. and the other countries, $\Delta \log \left( \frac{c_i}{c_j} \right)$. The notation $\Delta \tilde{e}_{i,j}$ represents the change of the log of real exchange rate, $\Delta \tilde{e}_{i,j} = \Delta \log (\tilde{e}_{i,j}) = \Delta \log \left( \frac{e_{i,j} p_j}{p_i} \right)$; $\Delta \tilde{\epsilon}_{i,j}$ represents the change of nominal exchange rate, $\Delta \tilde{\epsilon}_{i,j} = \Delta \log (\epsilon_{i,j})$, $\Delta \tilde{p}_{ji}$ represents the inflation differential between U.S. and the other countries, $\Delta \tilde{p}_{ji} = \Delta \log \left( \frac{p_j}{p_i} \right)$, and $\Delta \tilde{p}_{ji+1}$ represents the inflation differential at time $t+1$, $\Delta \tilde{p}_{ji+2}$ represents the inflation differential at time $t+2$, $\Delta (\tilde{e}_{i,j}\tilde{p}_{ji+1}) = \Delta \log \left( \frac{e_{i,j} p_{j+1}}{p_{i+1}} \right)$, $\Delta (\tilde{e}_{i,j}\tilde{p}_{ji+2}) = \Delta \log \left( \frac{e_{i,j} p_{j+2}}{p_{i+2}} \right)$. 
Table 2: Quarterly Consumption-exchange rate correlation between U.S. and some other countries & regions

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<th>Country</th>
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</tr>
<tr>
<td>UK</td>
<td>-0.068</td>
<td>-0.136</td>
<td>0.266</td>
<td>0.019</td>
<td>-0.132</td>
<td>0.028</td>
<td>-0.130</td>
<td>0.145</td>
<td>-0.097</td>
<td></td>
</tr>
<tr>
<td>AVE</td>
<td>-0.126</td>
<td>-0.143</td>
<td>0.046</td>
<td>-0.113</td>
<td>-0.162</td>
<td>-0.033</td>
<td>-0.156</td>
<td>0.061</td>
<td>-0.138</td>
<td></td>
</tr>
</tbody>
</table>

Notes: The quarterly data are from Datastream. The country code “SA” represents South Africa, and “TW” represents Taiwan. All the notations have the same meanings as in Table 1. Due to the availability for the quarterly data, the data for different countries or regions has different start time. The data periods for different countries or regions are as following: Q4 1971-Q2 2013 for AUS and UK; Q1 1991-Q2 2013 for CAN, DNK, DEU, and NZW; Q1 1975-Q2 2013 for FIN; Q1 1990-Q2 2013 for FRA; Q1 1980-Q2 2013 for NOR; Q2 1982-Q2 2013 for SA; Q1 1993-Q2 2013 for SWE; Q1 1992-Q2 2013 for TW.
Table 3: Correlations before and after the end of the Bretton Woods system

<table>
<thead>
<tr>
<th>Country pair</th>
<th>$\rho \left[ \sim, \Delta \log(e_{i,j}) \right]$</th>
<th>$\rho \left[ \sim, \Delta \log(e_{i,j}) \right]$</th>
<th>$\rho \left[ \sim, \Delta \log \left( \frac{p_j}{p_i} \right) \right]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>before</td>
<td>after</td>
<td>before</td>
<td>after</td>
</tr>
<tr>
<td>US-AUS</td>
<td>0.070</td>
<td>-0.034</td>
<td>0</td>
</tr>
<tr>
<td>US-SA</td>
<td>0.061</td>
<td>-0.105</td>
<td>-0.010</td>
</tr>
<tr>
<td>US-UK</td>
<td>-0.005</td>
<td>-0.068</td>
<td>0.113</td>
</tr>
</tbody>
</table>

Notes: In order to obtain enough observations, the quarterly data are used for this analysis. All the data are from Datastream. The starting period is the first quarter of 1959, and the end period is the second quarter of 2013. The word “before” represents the periods before the end of Breton Woods system from Q1 1959-Q3 1971. The word “after” represents the periods after the end of Breton Woods system from Q4 1971-Q2 2013. The periods “before” and “after” for SA is Q1 1959-Q1 1982 and Q2 1982-Q2 2013, since South Africa rand is not un-pegged to U.S. dollar until 1982 Q1. Again, the sign “∼” represents the consumption growth differential between the U.S.A and the other countries, $\Delta \log \left( \frac{c_i}{c_j} \right)$.

Table 4: Exchange rate-price correlation and consumption growth correlation between U.S. and other OECD countries

<table>
<thead>
<tr>
<th>Country</th>
<th>$\rho [\Delta \log(e_{i,j}), \Delta \log(P_j/P_i)]$</th>
<th>$\rho [\Delta \log(c_i), \Delta \log(c_j)]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>AUS</td>
<td>-0.088</td>
<td>0.090</td>
</tr>
<tr>
<td>AUT</td>
<td>-0.069</td>
<td>0.227</td>
</tr>
<tr>
<td>BEL</td>
<td>-0.211</td>
<td>0.366</td>
</tr>
<tr>
<td>CAN</td>
<td>-0.083</td>
<td>0.605</td>
</tr>
<tr>
<td>DDK</td>
<td>-0.154</td>
<td>0.518</td>
</tr>
<tr>
<td>FIN</td>
<td>-0.090</td>
<td>0.401</td>
</tr>
<tr>
<td>FRA</td>
<td>-0.357</td>
<td>0.453</td>
</tr>
<tr>
<td>GRC</td>
<td>-0.493</td>
<td>0.290</td>
</tr>
<tr>
<td>ISL</td>
<td>-0.818</td>
<td>0.321</td>
</tr>
<tr>
<td>IRL</td>
<td>-0.438</td>
<td>0.589</td>
</tr>
<tr>
<td>ITA</td>
<td>-0.448</td>
<td>0.254</td>
</tr>
<tr>
<td>JPN</td>
<td>0.006</td>
<td>0.441</td>
</tr>
<tr>
<td>KOR</td>
<td>-0.156</td>
<td>0.157</td>
</tr>
<tr>
<td>LUX</td>
<td>-0.266</td>
<td>0.268</td>
</tr>
<tr>
<td>MEX</td>
<td>-0.883</td>
<td>-0.219</td>
</tr>
<tr>
<td>NLD</td>
<td>-0.089</td>
<td>0.454</td>
</tr>
<tr>
<td>NEL</td>
<td>0.043</td>
<td>0.401</td>
</tr>
<tr>
<td>NOR</td>
<td>-0.087</td>
<td>0.319</td>
</tr>
<tr>
<td>PRT</td>
<td>-0.498</td>
<td>0.063</td>
</tr>
<tr>
<td>ESP</td>
<td>-0.278</td>
<td>0.449</td>
</tr>
<tr>
<td>SWE</td>
<td>-0.014</td>
<td>0.304</td>
</tr>
<tr>
<td>SWZ</td>
<td>-0.041</td>
<td>-0.376</td>
</tr>
<tr>
<td>TUR</td>
<td>-0.898</td>
<td>0.131</td>
</tr>
<tr>
<td>AVE</td>
<td>-0.279</td>
<td>0.283</td>
</tr>
</tbody>
</table>

Notes: All the annual data are from the World Bank, from 1971-2012.
<table>
<thead>
<tr>
<th>Parameter Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk aversion ($\sigma$)</td>
<td>2</td>
</tr>
<tr>
<td>Discount factor ($\beta$)</td>
<td>0.95</td>
</tr>
<tr>
<td>Elasticity of substitution:</td>
<td></td>
</tr>
<tr>
<td>Domestic and foreign tradables ($\theta$)</td>
<td>2</td>
</tr>
<tr>
<td>Tradable and non-tradables ($\phi$)</td>
<td>0.44</td>
</tr>
<tr>
<td>Share of tradables ($\alpha$)</td>
<td>0.55</td>
</tr>
<tr>
<td>Share of domestic tradables</td>
<td>$\alpha_{11} = \alpha_{22} = 0.72$</td>
</tr>
<tr>
<td>Endowment vector:</td>
<td></td>
</tr>
<tr>
<td>$y_T^T(h)$</td>
<td>1.0257;</td>
</tr>
<tr>
<td>$y_T^T(l)$</td>
<td>0.9743;</td>
</tr>
<tr>
<td>$y_N^T(h)$</td>
<td>2.4684;</td>
</tr>
<tr>
<td>$y_N^T(l)$</td>
<td>2.4316;</td>
</tr>
</tbody>
</table>

Table 5: Parameter values
Table 6: Correlations under limited participation

<table>
<thead>
<tr>
<th>Active households (%)</th>
<th>100</th>
<th>42.2</th>
<th>33.0</th>
<th>22.9</th>
<th>16.9</th>
<th>12.2</th>
</tr>
</thead>
</table>

a) Correlations for all households:

\[ \rho \left[ \Delta \log \left( \frac{c_i}{c_j} \right), \Delta \log(\varepsilon_{i,j}) \right] \]

<table>
<thead>
<tr>
<th>0</th>
<th>1.00</th>
<th>-0.457</th>
<th>-0.502</th>
<th>-0.591</th>
<th>-0.669</th>
<th>-0.731</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>-0.422</td>
<td>-0.470</td>
<td>-0.565</td>
<td>-0.646</td>
<td>-0.709</td>
<td></td>
</tr>
<tr>
<td>-0.402</td>
<td>0.293</td>
<td>0.353</td>
<td>0.463</td>
<td>0.553</td>
<td>0.620</td>
<td></td>
</tr>
<tr>
<td>0.804</td>
<td>0.346</td>
<td>0.356</td>
<td>0.304</td>
<td>0.160</td>
<td>-0.012</td>
<td></td>
</tr>
<tr>
<td>-0.141</td>
<td>-0.151</td>
<td>-0.176</td>
<td>-0.190</td>
<td>-0.183</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b) Correlations for inactive households:

\[ \rho \left[ \Delta \log \left( \frac{c_i}{c_j} \right), \Delta \log(\varepsilon_{i,j}) \right] \]

<table>
<thead>
<tr>
<th>0</th>
<th>1.00</th>
<th>-0.521</th>
<th>-0.586</th>
<th>-0.671</th>
<th>-0.729</th>
<th>-0.764</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.863</td>
<td>-0.521</td>
<td>-0.581</td>
<td>-0.660</td>
<td>-0.714</td>
<td>-0.747</td>
<td></td>
</tr>
<tr>
<td>-0.402</td>
<td>0.293</td>
<td>0.353</td>
<td>0.463</td>
<td>0.553</td>
<td>0.620</td>
<td></td>
</tr>
<tr>
<td>0.512</td>
<td>0.555</td>
<td>0.611</td>
<td>0.652</td>
<td>0.676</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-0.141</td>
<td>-0.151</td>
<td>-0.176</td>
<td>-0.190</td>
<td>-0.183</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

C) Correlations for active households:

\[ \rho \left[ \Delta \log \left( \frac{c_i}{c_j} \right), \Delta \log(\varepsilon_{i,j}) \right] \]

<table>
<thead>
<tr>
<th>0</th>
<th>1.00</th>
<th>1.00</th>
<th>1.00</th>
<th>1.00</th>
<th>1.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.863</td>
<td>0.999</td>
<td>0.999</td>
<td>0.999</td>
<td>0.999</td>
<td></td>
</tr>
<tr>
<td>-0.402</td>
<td>-0.979</td>
<td>-0.981</td>
<td>-0.984</td>
<td>-0.985</td>
<td>-0.984</td>
</tr>
<tr>
<td>0.512</td>
<td>0.747</td>
<td>0.816</td>
<td>0.999</td>
<td>0.922</td>
<td>0.946</td>
</tr>
</tbody>
</table>

Notes: The first row of the table shows the largest values in the distribution of the transfer cost, which is shown as the percentage of the tradable goods endowment. The second row shows the fractions of the active households corresponding to the largest values of the transfer cost. Panels a, b, and c report the correlations for the aggregate consumption, inactive household consumption, and active household consumption. For each panel, the first row shows the correlations between the consumption ratio and the real exchange rate; the second row shows the correlations between consumption ratio and the nominal exchange rate; the third row shows the correlations between consumption ratio and price ratio; the fourth row shows the consumption growth correlations between the two countries. The last row of the table shows the correlations between the nominal exchange rate and the price ratio.
Table 7: Model results under fixed exchange rate

<table>
<thead>
<tr>
<th>( \gamma_{max.} )</th>
<th>Flexible Ex.</th>
<th>Fixed Ex. &amp; Open</th>
<th>Fixed Ex. &amp; Closed</th>
</tr>
</thead>
<tbody>
<tr>
<td>([% Y^T(s_t)])</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Active households (%)</td>
<td>100</td>
<td>33.0</td>
<td>100</td>
</tr>
</tbody>
</table>

a) Correlations for all households:

| \( \rho[\Delta \log \left( \frac{c_i}{c_j} \right), \Delta \log (\varepsilon_{i,j})] \) | 1.00 | -0.534 | 1.00 | 0.488 | -0.550 | -0.442 |
| \( \rho[\Delta \log (c_i), \Delta \log (c_j)] \) | 0.804 | 0.356 | 0.828 | 0.758 | -0.075 | -0.075 |

b) Correlations for inactive households:

| \( \rho[\Delta \log \left( \frac{c_i}{\bar{A}c_j} \right), \Delta \log (\varepsilon_{i,j})] \) | - | -0.586 | - | 0.929 | | 0.367 |
| \( \rho[\Delta \log (\bar{A}c_i), \Delta \log (\bar{A}c_j)] \) | - | -0.151 | - | 0.833 | | 0.007 |

c) Correlations for active households:

| \( \rho[\Delta \log \left( \frac{\bar{A}c_i}{\bar{A}c_j} \right), \Delta \log (\varepsilon_{i,j})] \) | - | 1.00 | - | 1.00 | - | -0.333 |
| \( \rho[\Delta \log (\bar{A}c_i), \Delta \log (\bar{A}c_j)] \) | - | 0.816 | - | 0.986 | - | 0.015 |

d) BCSW international risk sharing index:

| For all households | 1.00 | 0.155 | 1.00 | 0.802 | -0.148 | -0.153 |
| For inactive households | - | -0.202 | - | 0.848 | - | -0.007 |
| For active households | - | 1.00 | - | 1.00 | - | -0.018 |

Notes: The columns labeled as “Flexible Ex.” include the results under the flexible exchange rate, the columns labeled as “Fixed Ex. & Open” include the results under the fixed exchange rate with an open international financial market, and the columns labeled as “Fixed Ex. & Closed” include the results under the fixed exchange rate with a closed international financial market. The first row of the table shows the largest values in the distribution of the transfer cost, which is shown as the percentage of the tradable goods endowment. The second row shows the fractions of the active households corresponding to the largest values of the transfer cost. Panels a, b, and c report the correlations for the aggregate consumption, inactive household consumption, and active household consumption. For each panel, the first row shows the correlations between the consumption ratio and the real exchange rate; the second row shows the consumption growth correlations between the two countries. Panel d reports the BCSW international risk sharing index for all households, inactive households and active households, respectively.
<table>
<thead>
<tr>
<th>Country</th>
<th>$\rho[\sim, \Delta \log(e_{i,j})]$</th>
<th>$\rho[\sim, \Delta \log(e_{i,j})]$</th>
<th>$\rho[\sim, \Delta \log\left(\frac{P_j}{P_i}\right)]$</th>
<th>$\rho[\Delta \log(c_i), \Delta \log(c_j)]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>AUS</td>
<td>-0.342</td>
<td>-0.395</td>
<td>-0.213</td>
<td>-0.106</td>
</tr>
<tr>
<td>JPN</td>
<td>-0.560</td>
<td>-0.651</td>
<td>-0.360</td>
<td>0.062</td>
</tr>
<tr>
<td>UK</td>
<td>-0.443</td>
<td>-0.666</td>
<td>-0.160</td>
<td>-0.518</td>
</tr>
<tr>
<td>US</td>
<td>-0.673</td>
<td>-0.676</td>
<td>-0.492</td>
<td>-0.329</td>
</tr>
</tbody>
</table>

Notes: The annual data are from World Bank, 1994-2012, where the sign “∼” represents the consumption growth differential between China and the other countries, $\Delta \log\left(\frac{c_i}{c_j}\right)$. 

42