An underdevelopment trap:
Informality, human capital and firm innovation

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Abstract

Developing countries are characterized by a large informal sector, poorly skilled labor force, and less dynamic formal firms in terms of innovation and employment growth. We argue that these salient characteristics of developing economies reinforce each other leading to an underdevelopment trap. To understand the nature of this phenomenon, we set up a quantitative model of firm dynamics and calibrate it to Mexico, which has a large informal sector as a “typical” developing economy. Unlike standard firm dynamics models, we allow worker’s skills to depreciate while unemployed or involved in informal activities. We also incorporate firm innovation leading to skill biased technological progress. We found that skills depreciation is a quantitatively important channel that amplifies the impact of distortions and public policies on firm innovation, the distribution of firms, and aggregate total factor productivity.

*This is work in progress. This draft is very preliminary and incomplete. Please, do not cite without authors’ permission. Álvarez-Parra is affiliated with CAF, Development Bank of Latin America, and UCV. Toledo is affiliated with the University of Kent.
1 Introduction

Informality is a pervasive characteristics for developing economies. The “shadow economy” is estimated to be about 38% of GDP in Sub-Saharan Africa whereas it is 13% in high-income OECD countries (see Schneider et al., 2010). Since informal sector is predominantly dominated by self-employed low-skilled workers, a substantial fraction of the labor force in developing economies works in low-productivity or subsistence economic units, and —as a reflection— there is a relatively small fraction of salaried workers (CAF, 2013).

In terms of firms dynamics, underdevelopment is characterized by firms’ resistance to grow and to increase productivity over their life cycle. In the U.S., the average 40-year-old plant employs almost eight times as many workers as the typical five-year-old plant or younger. In contrast, surviving plants in India exhibit almost no growth, and in Mexico the average 40-year-old plant employs barely twice as many workers as an average young plant (Hsieh and Klenow, 2012). This dynamic translates into a firm distribution biased toward small firms.

From a general equilibrium perspective, these two characteristic of developing economies are related and may reinforce each other through two channels: i) workers’ skill dynamic, and ii) firms’ innovation. On the one hand, when technological progress is skilled biased, scarcity of skilled workers reduces firms’ incentives to innovate or adopt new technologies. On the other hand, lack of dynamism of formal firms reduces the creation of formal jobs and pushes a large fraction of the labor force toward informality. This, in turn, has an adverse effect on human capital accumulation not only because informal jobs may erode workers skills, but also because the relative low return of skills under informality discourage investing in human capital.

To the best of our knowledge, there is no research allowing to understand and quantify the economic mechanisms behind the interaction between informality, human capital accumulation, and firm innovation\(^1\). In this paper we fill this gap by providing a firm dynamic general

\(^1\)Some ideas in this regard are suggested in a recent non-technical report by CAF (2013).
equilibrium model that introduces endogenous skills dynamic and firm innovation/technology adoption.

We calibrate our model economy to a “typical” developing country with a large informal sector as Mexico. We start by evaluating the ability of our framework to reproduce salient characteristics of developing economies. We then use our quantitative framework to investigate to what extent certain policies (e.g. taxes/subsidies), and institutional characteristics of the economy (e.g., search frictions), interact with workers’ human capital accumulation and firm innovation, to explain the size of informal sector, the skill composition of workers, firm growth and size distribution, aggregate TFP and other salient features of underdevelopment.

The rest of the paper is organized as follows. Section 2 provides some facts that describe the productive landscape of developing economies and motivate the main ingredients of our framework. Section 3 presents the model and describe the stationary equilibrium. In Section 4 we calibrate our model economy and discuss the quantitative results. Finally, Section 6 concludes.

2 The productive landscape of developing economies

Underdevelopment is characterized by excessive entrepreneurial activity and scarcity of salaried workers. Poschke (2014) documents that entrepreneurship increases with income per-capita. Likewise, CAF (2013) reports that entrepreneurship rates in Latin America and in the ”Pacific and East Asia” region are three times larger than in OECD countries.2

However, as Table 1 suggests, this excess of entrepreneurship is explained by the large fraction of self-employed workers. When merging employers and self-employed, the fraction of entrepreneurs in Latin America is more than 3 times higher than in the U.S. However, the fraction of employers is around 4% in both regions. In contrast, almost 30% of the labor force

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2Both studies use the Global Entrepreneurship Monitor (GEM) database. This rate is measured as the fraction of population between 18 and 65 years old that are starting or running a business younger than 42 months.
in Latin America is self-employed compared to only around 6% in the U.S. As a result, Latin America has a lower fraction of salaried workers. In the U.S., salaried workers account for more than 80% of the labor force while in Latin America this fraction is around 50%.

### Table 1: Occupational structure and firm size

<table>
<thead>
<tr>
<th>Country</th>
<th>Year</th>
<th>Labor Force(^a) (percentage)</th>
<th>Employers by firm size (%)</th>
<th>Salaried workers by firm size (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Employer</td>
<td>Self-employed</td>
<td>Salaried worker</td>
</tr>
<tr>
<td>Argentina</td>
<td>2010</td>
<td>4.1</td>
<td>16.5</td>
<td>71.3</td>
</tr>
<tr>
<td>Bolivia</td>
<td>2008</td>
<td>5.9</td>
<td>33.9</td>
<td>37.2</td>
</tr>
<tr>
<td>Brazil</td>
<td>2009</td>
<td>4.0</td>
<td>18.9</td>
<td>61.3</td>
</tr>
<tr>
<td>Chile (^b)</td>
<td>2009</td>
<td>2.8</td>
<td>18.1</td>
<td>68.5</td>
</tr>
<tr>
<td>Colombia</td>
<td>2010</td>
<td>4.5</td>
<td>39.1</td>
<td>41.7</td>
</tr>
<tr>
<td>Costa Rica (^b)</td>
<td>2010</td>
<td>3.1</td>
<td>17.5</td>
<td>70.6</td>
</tr>
<tr>
<td>Ecuador</td>
<td>2010</td>
<td>3.3</td>
<td>30.0</td>
<td>52.4</td>
</tr>
<tr>
<td>El Salvador</td>
<td>2010</td>
<td>3.8</td>
<td>28.3</td>
<td>53.7</td>
</tr>
<tr>
<td>Guatemala</td>
<td>2006</td>
<td>3.8</td>
<td>30.6</td>
<td>50.8</td>
</tr>
<tr>
<td>Honduras</td>
<td>2009</td>
<td>2.4</td>
<td>42.9</td>
<td>42.9</td>
</tr>
<tr>
<td>Mexico</td>
<td>2006</td>
<td>3.9</td>
<td>21.8</td>
<td>64.9</td>
</tr>
<tr>
<td>Nicaragua</td>
<td>2005</td>
<td>4.4</td>
<td>29.8</td>
<td>46.9</td>
</tr>
<tr>
<td>Panama</td>
<td>2010</td>
<td>3.0</td>
<td>24.9</td>
<td>61.9</td>
</tr>
<tr>
<td>Paraguay</td>
<td>2010</td>
<td>5.0</td>
<td>32.2</td>
<td>49.6</td>
</tr>
<tr>
<td>Peru</td>
<td>2010</td>
<td>5.7</td>
<td>35.9</td>
<td>41.5</td>
</tr>
<tr>
<td>Dominican Republic</td>
<td>2010</td>
<td>3.6</td>
<td>42.0</td>
<td>49.0</td>
</tr>
<tr>
<td>Uruguay (^b)</td>
<td>2010</td>
<td>4.5</td>
<td>21.0</td>
<td>68.5</td>
</tr>
<tr>
<td>Venezuela, RB</td>
<td>2007</td>
<td>3.9</td>
<td>32.9</td>
<td>54.9</td>
</tr>
<tr>
<td>United States (^b)</td>
<td>2011</td>
<td>3.3</td>
<td>6.1</td>
<td>80.4</td>
</tr>
<tr>
<td>Latin America</td>
<td></td>
<td>4.0</td>
<td>28.7</td>
<td>54.8</td>
</tr>
</tbody>
</table>

\(^a\) The difference between the sum of employer, self-employed, and salaried participation in the active population and the 100% is explained by the participation of family workers without remuneration and of unemployed workers.

\(^b\) In these countries, the distribution by size is made between companies of up to nine workers and of more than nine.

n.a.: not available

Source: CAF 2013.

In addition to this particular occupational structure, developing economies exhibit, relative to high income countries, a firm size distribution biased toward smaller economic units. For instance, Poschke (2014) reports that the average firm size increases with income per-capita.\(^3\) This is true even when excluding self-employed workers. In Latin America, less than 10% of employers have more that 10 workers whereas in the U.S. more than 30% of employers do.

\(^3\) Poschke also documents that the dispersion and the skewness of the firm size distribution increases with per-capita income.
Likewise, the fraction of salaried workers in firms with 10 or more employees is only about 50% in Latin America and 85% in the U.S. (see Table 1).

A large fraction of small firms in developing economies is a reflection of the lack of dynamism of plants and firms over their life cycle, as documented by Hsieh and Klenow (2012) for the case of Mexico, India and the U.S. Figure 1 provides further evidence in this regard. In particular, this figure plots per-capita GDP against the average ratio of current size to initial size for plants over 10 years old. These variables display a correlation of about 0.4.⁴

**Figure 1: Average plant relative-employment for old plants vs GDP**

![Figure 1: Average plant relative-employment for old plants vs GDP](image)

Source: Own elaboration based on WBES.

When it comes to innovation, firms in developing economies spend little as Figure 2 shows. While Latin American firms devote around 2.5% of their sales to innovation on average, European firms spend almost 4%. Differences in spending in R&D are even greater. Among other factors, the quality of labor force contributes to this low innovation intensity in Latin America. The fraction of firms that mention “lack of qualified personnel” as an obstacle to innovation is

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We first control for the effects of age, sector, foreign ownership and whether the plant is part of a bigger firm. In particular, we linearly regress the size ratio against these observables and use the residuals.
37% in Argentina, 35% in Chile, 28% in Costa Rica and almost 27% in Uruguay (IDB, 2010).

Figure 2: Investment in innovation by firms

<table>
<thead>
<tr>
<th>Country</th>
<th>R&amp;D Intensity (%)</th>
<th>Innovation Investment Intensity (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sweden</td>
<td>4.3</td>
<td>3.5</td>
</tr>
<tr>
<td>France</td>
<td>2.8</td>
<td>2.2</td>
</tr>
<tr>
<td>Denmark</td>
<td>3.1</td>
<td>2.2</td>
</tr>
<tr>
<td>Germany</td>
<td>3.0</td>
<td>2.3</td>
</tr>
<tr>
<td>Luxembourg</td>
<td>5.3</td>
<td>3.3</td>
</tr>
<tr>
<td>Netherlands</td>
<td>5.1</td>
<td>4.1</td>
</tr>
<tr>
<td>Belgium</td>
<td>5.0</td>
<td>4.1</td>
</tr>
<tr>
<td>Austria</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>Italy</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>3.0</td>
<td>3.3</td>
</tr>
<tr>
<td>Spain</td>
<td>3.0</td>
<td>3.0</td>
</tr>
<tr>
<td>Norway</td>
<td>1.2</td>
<td>1.2</td>
</tr>
<tr>
<td>Chile</td>
<td>1.2</td>
<td>1.2</td>
</tr>
<tr>
<td>Brazil</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>Costa Rica</td>
<td>2.0</td>
<td>2.0</td>
</tr>
<tr>
<td>Argentina</td>
<td>2.0</td>
<td>2.0</td>
</tr>
<tr>
<td>Uruguay</td>
<td>2.0</td>
<td>2.0</td>
</tr>
<tr>
<td>Panama</td>
<td>1.2</td>
<td>1.2</td>
</tr>
<tr>
<td>Colombia</td>
<td>2.0</td>
<td>2.0</td>
</tr>
</tbody>
</table>


Notes: Indicators refer to the Manufacturing Industry. Weighted shares are reported only in the case of OECD countries and Brazil. The indicators reported are averages in the total sample of companies (except for Chile, Spain, and Italy, whose averages correspond to shares of the total number of innovating companies).

In summary, the productive face of underdevelopment is characterized by a large fraction of the labor force working in subsistence or low productivity economic units in the informal sector. As a result, a relatively small proportion of workers have formal jobs. From the perspective of formal firms, there exists a lack of dynamism in terms of innovation and employment growth, which implies smaller firms and relatively few formal jobs.

This distorted occupational structure may induce deterioration of skills that can harm productivity.\(^5\) Indeed, there is a large literature regarding human capital depreciation due to unemployment (Addison and Portugal, 1989, Jacobson et al., 1993, and Neal, 1995). In general,

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\(^5\)This idea seems to be a concern for policy makers in the U.S. as stated by Ben Bernake in a Q&A session of the Senate Banking Committee hearing in 2012 when he claimed that...one concern we do have, of course, is the fact that more than 40% of the unemployed have been unemployed for six months or more. Those folks are either leaving the labor force or having their skills eroded. Although we have not seen much sign of it yet, if that situation persists for much longer then that will reduce the human capital that is part of our growth process going forward. (see Laureys, 2012).
this literature shows that time out of work implies wage losses larger than it can be explained by forgone experience alone, a loss interpreted as human capital depreciation. Moreover, a Edin and Gustavsson (2008) directly test this hypothesis based on a Swedish survey that contains individual test scores from literacy test. They find that a full year in non-employment is “associated with a 5-percentile move down in the skill distribution.”

Recent evidence also suggests that self-employment and under-employment harm human capital and employment prospects (Williams, 2000, Mavromaras et al., 2013). For the case of Australia, Mavromaras et al. (2013) find evidence that underutilization of skills does influence negatively future employment prospects. They find this effect for both men and women and for all education levels. In our view, self-employment and informal jobs in developing countries is associated with a underutilization of skills as it typically involves subsistence activities and low productivity jobs. Moreover, indirect evidence of human capital depreciation from non-employment is also suggested by the fast drop in the exit rate from unemployment and self-employment as the time spent in these states increases (Narita, 2011).

3 The economy

We model an economy populated by a continuum of workers of measure 1 who belong to a representative household, and a continuum of competitive firms of mass $M$. The economy is made of two sectors or islands: formal and informal. Firms are owned by the household and are located in the formal island of the economy. In contrast, workers may live in any of the two islands.

Workers in the formal island face a competitive labor market, and can move to the informal island any time. In contrast, moving from the informal to the formal island is subject to search frictions. Workers in the informal island can participate in informal productive activities.

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6 The survey is the International Adult Literacy Survey, and studies have found that the skills captured by it have effects on wages.
3.1 Technology and firm behavior

Firms have a decreasing returns to scale neoclassical production function that takes two labor inputs: skilled \((h)\) and unskilled \((l)\). Firms are characterized by an idiosyncratic productivity shock \(z\) as well as by a skill-biased productivity level \(\Gamma \in \{\Gamma_0, \Gamma_1, \ldots, \Gamma_{N-1}\}\). We denote a firm’s type by \(x = (z, \Gamma)\), and the distribution of firms over \(x\) by \(\mu(x)\). We assume that \(z \in \{z_0, z_1, \ldots, z_{N-1}\}\) follows a stationary Markov process with transition probability matrix \(\Psi\).

Firms choose how much to produce each period by hiring labor of each type. Each firm’s output is given by the following technology: \(y = zf(l, \Gamma h)\). The function \(f\) is strictly concave, increasing in both arguments, and satisfies the Inada conditions.

Firms also choose how much to invest in innovation each period, which defines how their \(\Gamma\) evolves over time. In particular, if a firm with productivity level \(\Gamma = \Gamma_m\) invests an amount \(a \geq 0\) in innovation, next period it will have \(\Gamma' = \Gamma_{m+1}\) with probability \(\phi(a)\). With probability \((1 - \phi(a))\), \(\Gamma\) remains unaltered (i.e., \(\Gamma' = \Gamma_m\)). Function \(\phi\) is increasing and satisfies \(\phi(0) = 0\).

After production takes place, firms may die exogenously with probability \((1 - \zeta)\). New firms can also enter the market at the beginning of each period. New firms come from a large number of available projects. These projects are endowed with \(\Gamma = \Gamma_0\), and have to pay an entry fixed cost \(\kappa\). Each new firm draws its idiosyncratic productivity \(z\) from distribution \(\Psi_0(z)\) after incurring in cost \(\kappa\). In equilibrium, free entry implies that

\[
\sum_{i=0}^{N-1} \Pi(z_i, \Gamma_0)\Psi_0(z_i) - \kappa \leq 0, \tag{1}
\]

where \(\Pi(z, \Gamma)\) is the value of a firm with productivity levels \((z, \Gamma)\). This condition is satisfies with equality in an equilibrium with entry \((M^0 > 0)\).

Firms also pay proportional taxes on gross revenues \(zf(l, \Gamma h)\). The tax rate each firm faces
may depend on its employment level \((h + l)\). Let us denote this tax rate function as \(\tau(h + l)\).

The firm’s problem is to maximize the expected discounted value of dividends 
\[
d(h, l, a, x) = (1 - \tau(h + l))zf(l, \Gamma h) - w_h h - w_l l - a,
\]
where \(w_h\) and \(w_l\) denote wages in the formal island. This problem can be represented by the following Bellman equation:

\[
\Pi(x) = \max_{\{l, h, a\} \geq 0} \left\{ d(h, l, a, x) + \beta \zeta E[\Pi(x')|x, a] \right\},
\]

s.t. 
\[
d(h, l, a, x) \geq 0,
\]
where \(\beta\) is the individuals’ discount rate.

### 3.2 Household and individual behavior

Each worker is endowed with an unit of time and can be skilled or unskilled. She can also be employed in a formal job or non-employed. We denote by \(\lambda = \{\lambda_j\}\) the distribution of workers over skills \(j = \{h, l\}\) and employment states \(i = \{e, n\}\).

Non-employed individuals are located in the informal island and split their time among the following activities: searching for a formal job, working in an informal activity and, if unskilled, participating in a training program. They become employed (i.e., move to the formal island) next period with probability \(\pi_j(s_j), j = \{h, l\}\), where \(s_j \in [0, 1]\) is the time devoted to search for a formal job. It is worth noting that we associate informal work with self-employment.\(^8\) For simplicity, the model does not differentiate between a self-employed and an informal salaried worker. Implicitly, we make two assumptions. First, finding a salaried job in the informal sector do not require searching or, alternatively, that there are virtually no 

\(^7\)In a broader sense, \(\tau\) should be interpreted as a firm specific distortion instead of a formal tax. A recent literature has established that size-dependent distortions have important implication for aggregate TFP (see, for example, Guner et al., 2008, Restuccia and Rogerson, 2008, and Hsieh and Klenow, 2009).

\(^8\)This is consistent with informality in developing economies as most of informal employment is self-employment as opposed to salaried work. In Mexico, for instance, about 60% of informal workers are self-employed according to our own calculations using the ENOE 2013. Of the remaining 40%, the vast majority works in very small firms or establishments.
search frictions in the informal labor market. Second, given a worker’s skill level, her earnings profile is the same whether self-employed or salaried in an informal job.

Self-employed workers earn \( \omega_j^nx_j \) units of the consumption good, where \( x_j \) represent the fraction of time spent in the informal activity. Unskilled non-employed individuals also choose how much time \( t \in [0, 1] \) to spend on education/training at cost \( \eta \) per unit of time. They will be skilled next period with probability \( \rho(t) \), with \( \rho'(t) > 0 \) and \( \rho(0) = 0 \). Finally, skilled non-employed workers may lose their skills with probability \( \delta_h \).

Individuals in the formal island decide each period whether to work and earn a competitive skill-dependent wage \( w_j \), or quit and become non-employed (i.e., moving to the informal island). At the end of each period, formal workers of skill level \( j = \{h, l\} \) may exogenously become non-employed with probability \( 1 - \xi_j \), which we interpret as layoffs.

We assume that individuals share risk perfectly within the representative household. The representative household has time-separable preferences only over consumption \( c \) and discounts future utility with factor \( \beta \). As all relevant decisions are made by individual workers, the problem of the representative household is trivial. Given individual allocations \((s_h, s_l, t)\) and the distribution of workers \( \lambda \), household consumption is simply defined by its budget constraint:

\[
c = w_h \lambda_h^c + w_l \lambda_l^c + \omega_h^n (1 - s_h) \lambda_h^n + \omega_l^n (1 - s_l - t) \lambda_l^n - \eta t \lambda_l^n + D - kM^0 + B + T, \tag{3}
\]

where \( D = M \sum_{x \in X} d(x) \mu(x) \) is the total dividends of the firms, \( B = \lambda_l^n b_l + \lambda_h^n b_h \) is total unemployment benefit payments, and \( T \) is a government lump-sum transfer.

The problem of skilled and unskilled non-employed workers is given by the following Bellman equations:

\[
U_h = \max_{s_h} \left\{ \omega_h^n (1 - s_h) + b_h^n + \beta [\pi_h(s_h)W_h + (1 - \pi_h(s_h)) (\delta U_t + (1 - \delta_h)U_h)] \right\}, \tag{4}
\]

\[
U_l = \max_{s_l, t} \left\{ \omega_l^n (1 - s_l - t) + b_l^n - \eta t + \beta [\rho(t) (\pi_l(s_l)W_h + (1 - \pi_l(s_l))U_h) + (1 - \rho(t)) (\pi_l(s_l)W_l + (1 - \pi_l(s_l))U_l)] \right\}. \tag{5}
\]
where $b^n_j$ represents unemployment benefits, and $W_j$ is the value of being employed given by

$$W_j = \max \left\{ U_j; w_j + \beta \left[ \xi_j W_j + (1 - \xi_j)U_j \right] \right\}. \quad (6)$$

### 3.3 Equilibrium

We focus on a stationary equilibrium where the distributions of workers $\lambda$ and firms $\mu$ are invariant. Before defining the equilibrium, let us introduce some simplifying notation. Let us denote $x_{im} = (z_i, \Gamma_m)$, and $\mu_{im} = \mu(x_{im})$. Also, let us represent the fraction of new entrant firms of type $x_{i0}$ as $\mu_{i0}^0$.

**Definition 1** *A stationary equilibrium with tax/distortion profile $\tau$ is described by distributions $\{\lambda, \mu\}$, a mass of firms $\{M^0, M\}$, allocations $\{c, h(x), l(x), a(x), s_h, s_l, t\}$, prices $\{w_h, w_l\}$, value functions $\{V, U_h, U_l, W_h, W_l, \Pi\}$, and government transfers $T$ such that:

- Distributions $\{\lambda, \mu\}$ are invariant, which implies:

  - The distribution of firms $\mu$ satisfies the following conditions:

    $$\mu_{im} = \zeta \sum_{j=0}^{N-1} \mu_{jm} \psi_{ji} [1 - \phi(a(x_{jm}), \Gamma_m)]$$

    $$+ \xi \sum_{j=0}^{N-1} \mu_{j,m-1} \psi_{ji} \phi(a(x_{j,m-1}), \Gamma_{m-1}), \quad \forall m > 0, \quad (7)$$

    $$\mu_{i0} = \zeta \sum_{j=0}^{N-1} \mu_{j0} \psi_{ji} [1 - \phi(a(x_{j0}), \Gamma_0)] + \mu_{i0}^0, \quad (8)$$

    where $\mu_{i0}^0 = (1 - \zeta) \Psi_0(z_i)$, and $\psi_{ji}$ is the transition probability from $z_j$ to $z_i$.\"
The distribution of workers $\lambda$ satisfies the following conditions:

$$\lambda_t^e = \xi_t \lambda_t^e + \pi(s_t)(1 - \rho(e))\lambda_t^n; \quad (9)$$

$$\lambda_h^e = \xi_h \lambda_h^e + \pi(s_h)\lambda_h^n + \pi(s_t)\rho(e)\lambda_t^n; \quad (10)$$

$$\lambda_t^n = (1 - \pi(s_t))(1 - \rho(e))\lambda_t^n + (1 - \pi(s_h))\delta_h \lambda_h^n + (1 - \xi_t)\lambda_t^e; \quad (11)$$

$$\lambda_h^n = 1 - (\lambda_t^e + \lambda_h^e + \lambda_t^n). \quad (12)$$

- The mass of new entrant firms satisfies $M^0 = (1 - \xi)M$.

- $U_h$ and $U_l$ solve the non-employed individuals problems (4) and (5) with associated time allocations $s_h$, $s_l$ and $t$.

- $W_h$ and $W_l$ are given by equations (6).

- $\Pi$ solves the firm’s dynamic programming problem (2) with associated allocation $\{h(x), l(x), a(x)\}$.

- The free-entry condition (1) is satisfied with equality: $E_z \Pi(z, \Gamma_0) = \kappa$.

- Labor market clears: $\lambda_h^e = M \sum_{i,m} h(x_{im}) \mu_{im}$ and $\lambda_t^e = M \sum_{i,m} l(x_{im}) \mu_{im}$.

- Government balances its budget: $M \sum_{i,m} \tau_{im} y_{im} \mu_{im} = B + T$, where $\tau_{im} = \tau(h(x_{im}) + l(x_{im}))$, $y_{im} = z_i f(l(x_{im}), \Gamma_m h(x_{im}), k(x_{im}))$, and $B = \lambda_t^n b_t + \lambda_h^n b_h$.

- Allocations are feasible: $c = M \sum_{i,m} (y_{im} - a_{im}) \mu_{im} + \omega_h^n (1 - s_h) \lambda_h^n + \omega_t^n (1 - s_t - t) \lambda_t^n - \kappa M^0 - \eta t \lambda_t^n \geq 0$.

4 Quantitative analysis

We first calibrate the model to Mexico, an emerging economy with a large informal sector.\textsuperscript{9}

Then, we study to what extent certain policies and institutional characteristics of the economy,

\textsuperscript{9}In the calibration, we use the Mexican labor force survey ENOE extensively.
interacting with workers skill dynamics and firm innovation, to explain some salient features, such as the size of informal sector, the skill composition of workers, firm size distribution, among others, of developing economies.

4.1 Calibration

To provide a quantitative framework, we need first to choose functional forms for the production function $f$, the probability functions $\phi$, $\pi$ and $\rho$, and the size-dependent distortion $\tau$.

We assume a CES technology. Thus, $f$ takes the following form:

$$f(l, h) = \left[ \alpha l^\chi + (1 - \alpha)(\Gamma h)^\chi \right]^{\upsilon/\chi},$$  \hspace{1cm} (13)

where $\alpha \in (0, 1)$ is the share parameter, and $\chi \leq 1$ defines the elasticity of substitution between skilled and unskilled labor equal to $\frac{1}{1-\chi}$. Finally, $\upsilon \in (0, 1)$ defines the (decreasing) returns to scale of the production function.

The probability functions take the general form $g(x) = 1 - exp(-\theta x)$. This function has some convenient properties: it is increasing, bounded below 1, and satisfies $g(0) = 0$. We denote the parameters that describe function $\phi$, $\pi_j$, $j = \{h, l\}$, and $\rho$ as $\theta_a$, $\theta_{sj}$ and $\theta_l$, respectively.

Finally, the function $\tau$ is defined as

$$\tau(x) = \frac{\bar{\tau} + a}{1 + exp\left(\frac{\bar{n} - x}{\mu_n}\right)} - a,$$  \hspace{1cm} (14)

where $\mu_n$ represent the mean firm size, and $a = \bar{\tau}/exp(\bar{n}/b)$.\footnote{Size-dependent frictions have been found to be an important source of productivity losses, especially when they increase with size. They are usually modelled either as a 2-step discrete function (as in Restuccia and Rogerson, 2008) or as a smoothly increasing function on firm productivity (Hsieh and Klenow, 2012). Our proposed function, is flexible enough to approximate both approaches. We define the friction as a function of the relative size as a convenient normalization.} This function is increasing in $x$ and satisfies the following conditions: $\tau(0) = 0$ and $\lim_{x \to +\infty} \tau(x) = \bar{\tau}$.\footnote{10}
We also need to define the transition matrix $\Psi$ of the firm’s productivity level $z$, and the distribution $\Psi_0$, where new entrant firms draw their initial productivity.

In order to calibrate the productivity process $z$, we first define the following first-order autoregressive process for the auxiliary variable $\hat{z}$:

$$\hat{z}' = (1 - \psi_z)\mu_z + \psi_z \hat{z} + \epsilon_z \quad (15)$$

with $\epsilon_z \sim N(0, \sigma_\epsilon)$.

Using Tauchen’s (1986) method, we approximate this autoregressive process with a $N_z$-state Markov chain with transition matrix $\Psi$, and define $z_i = \exp(\hat{z}_i)$. Notice that the stochastic process for $z$ is completely described by $(\mu_z, \psi_z, \sigma_\epsilon)$ and $N_z$, which we arbitrarily set to 21.$^{11}$

The distribution $\Psi_0$ is assumed to be a discrete approximation of a Weibull distribution with parameters $\psi_W$ (scale) and $\sigma_W$ (shape). We calibrate these parameters in order to match the unconditional mean of the productivity process $z$ given by $E(z) = \exp(\mu_z + \sigma_z^2/2)$.\(^{12}\) So, in practice, we only need to choose either $\psi_W$ or $\sigma_W$, as the other is pinned down by $E(z)$ above.

As for the grid of the skill-biased productivity level $\Gamma$, we set $N_\Gamma = 6$, and normalize its lowest value to one (i.e., $\Gamma_0 = 1$). We also set $\Gamma_{N_\Gamma-1} = 2$, which seems to be large enough for our purposes.

We have 25 parameters to calibrate. A subset of 13 parameters,

$$(\beta, \mu_z, \nu, \chi, \zeta, \xi_l, \xi_h, \bar{\tau}, \bar{n}, \theta_l, b_{l}^h, b_{l}^l),$$

are set a priori as follows.

We consider a quarterly frequency, and accordingly set $\beta = 0.99$ as it is standard. We set $\mu_z = 0$, and abstract from unemployment benefits as these do not exist in Mexico (i.e., $b_{l}^n = 0$).

---

$^{11}$In the discretization, we set the elements of the support of $\hat{z}$ evenly spaced in the range $[\hat{z}_0, \hat{z}_{N_z-1}]$. The upper and lower bound for $\hat{z}$ are set to $+/- \lambda_z$ unconditional standard deviations from the unconditional mean of $\hat{z}$. That is, $\hat{z}_0 = \mu_z - \lambda_z \frac{\sigma_z}{\sqrt{1 - \psi_z}}$ and $\hat{z}_{N_z-1} = \mu_z + \lambda_z \frac{\sigma_z}{\sqrt{1 - \psi_z}}$. We choose $\lambda_z = 2.5$.

$^{12}$Since $\hat{z}$ is distributed $N(\mu_z, \sigma_z)$, with $\sigma_z = \sigma_\epsilon / \sqrt{1 - \psi_z^2}$, the stochastic variable $z$ is distributed $LN(\mu_z, \sigma_z)$. 

---

14
We choose $\nu = 0.9$ which is close to constant return to scale. For the case of Mexico, Parro (2013) uses an elasticity of substitution between unskilled labor and a compounded factor including capital and skilled labor of 1.67. We take this value as reference, and accordingly set $\chi = 0.4$.

For the 1990s, Bartelsman et al. (2009) find firm annual exit rate for Mexico ranging from 4% (for enterprises with more than 20 employees) to around 11% (for enterprises with more than 1 employee). We then set $(1 - \zeta) = 0.02$.

Transitions from non-employment (unemployed and informal workers) to skilled and unskilled formal employment, obtained from ENOE, pin down probabilities $\xi_h$ and $\xi_l$. In the stationary equilibrium the mass of workers leaving formal employment must be equal to the mass going into it. Therefore, for each type of skill $j$

$$(1 - \xi_j) \lambda^e_j = \text{fraction of workers from NE to E}_j.$$  

Since we also calculate $\lambda^e_j$ from ENOE, we obtain $(\xi_h, \xi_l) = (0.93, 0.84)$.

The parameter $\theta_t$ of the function $\rho(t)$ is set to 0.0645. We choose this value in order to match the median years of educations of 16 (i.e., 4-year college degree) of skilled workers in ENOE 2013. In contrast, the vast majority of workers with no more than a high-school education (12 years) are unskilled. Let us assume that, on average, a full-time college student becomes skilled after 4 years (16 quarters). Since $\rho(1)$ is the probability of becoming skilled of a full-time college student each quarter, we can model the number of quarters (trials) until becoming skilled (success) as a Geometric random variable with Bernoulli probability $\rho(1)$. Thus, the average time until success is $1/\rho(1)$. Therefore, $16 = (1 - \exp(-\theta_t))^{-1}$, which gives

13A workers is defined as informal if she (i) is non-professional self-employed, (ii) is a domestic worker, (iii) is a subsistence agricultural worker or (iii) works in a economic unit with no establishment and it does not use formal accounting. To classify formal workers as skilled they must be working in an occupation with a code smaller than 3000 in the Mexican 4-digit occupational classification SINCO 2011. Notice that we assume observability of skills only for workers in formal jobs. For informal workers, last formal occupation may not represent current skill level due to skill depreciation and training.
us the value for $\theta_t$ above.

Regarding the parameters of the function $\tau$, we proceed as follows. First, following Ordonez (2014), we set the legal tax burden for formal firms equivalent to 25% of their income. We assume, however, partial tax compliance. We approximate the evasion/elusion rate according to firm size by the elusion/evasion rate of labor regulations and payroll taxes based on the fraction of workers in an establishment not registered with the IMSS (computed from ENOE). We compute the effective tax as the product of the legal tax and the compliance rate. Finally, we obtain $(\bar{\tau}, \bar{n}, \bar{b})$ to minimize the sum of squared differences between function $\tau$ and an interpolated version of this effective tax scheme. By this procedure we get $(\bar{\tau}, \bar{n}, \bar{b}) = (0.25, 1.48, 1.33)$. In the Appendix, we show the compliance rate and effective tax profile as well as a figure plotting $\tau$ together with the (interpolated version of the) effective tax rate.

The remaining 12 parameters $(\kappa, \alpha, \theta_s, \sigma_z, \psi_z, \sigma_W, \theta_{sh}, \theta_{st}, \eta, \omega_{lh}^n, \omega_{lh}^n)$ are calibrated such that the model reproduces the 12 moments shown in Table 2. It is worth mentioning that we take advantage of the structure of our model which—given the mass of skilled and unskilled formal workers $(\lambda_{eh}, \lambda_{el})$, which we calculate from ENOE—allows a block-wise calibration. In particular, in a first phase we impose $(\lambda_{eh}, \lambda_{el})$ and solve for equilibrium in the (formal) labor market together with parameters $(\kappa, \alpha, \theta_s, \sigma_z, \psi_z)$ in order to match the six moments in the top panel of the table. In this block, we attempt to match the skill wage premium and five moments describing the firm-size distribution. Afterwards, we impose the equilibrium wages found in the previous step into the household problem and calibrate $(\theta_s, \theta_t, \eta, \omega_{lh}^n, \omega_{lh}^n)$ targeting the remaining 5 moments. This strategy greatly simplifies the calibration procedure.\(^{15}\)

\(^{14}\) At the moment, we only minimize the distance between the model and data moments using a relatively coarse grid search.

\(^{15}\) In the Appendix we provide further details on our calibration procedure.
### Table 2: Calibration results

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Target</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa$</td>
<td>0.1</td>
<td>% formal employment firms $&lt; 20 = 55%$</td>
<td>ENOE Q1-Q2 2013</td>
</tr>
<tr>
<td>$\theta_a$</td>
<td>40</td>
<td>% formal employment firms $&lt; 100 = 78%$</td>
<td>ENOE Q1-Q2 2013</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.6</td>
<td>Skill premium (formal sector) = 45.5%</td>
<td>ENOE Q1 2013</td>
</tr>
<tr>
<td>$\sigma_z$</td>
<td>0.25</td>
<td>Median/mean firm size = 0.22</td>
<td>ENOE Q1-Q2 2013</td>
</tr>
<tr>
<td>$\psi_z$</td>
<td>0.96</td>
<td>% Coef. of variation of firm size = 4.95</td>
<td>ENOE Q1-Q2 2013</td>
</tr>
<tr>
<td>$\sigma_W$</td>
<td>2</td>
<td>3rd quartile/mean firm size = 0.83</td>
<td>ENOE Q1-Q2 2013</td>
</tr>
<tr>
<td>$\theta_{sh}$</td>
<td>0.447</td>
<td>Transition informal to skilled formal = 1.1%</td>
<td>ENOE Q2-2013</td>
</tr>
<tr>
<td>$\theta_{sl}$</td>
<td>2.411</td>
<td>Transition informal to unskilled formal = 5.94%</td>
<td>ENOE Q2-2013</td>
</tr>
<tr>
<td>$\eta$</td>
<td>0.572</td>
<td>Fraction of time job searching = 16.7%</td>
<td>ENOE Q2-2013</td>
</tr>
<tr>
<td>$\omega_h^s$</td>
<td>1.619</td>
<td>Fraction of formal skilled workers $\lambda_h^s = 16%$</td>
<td>ENOE Q2-2013</td>
</tr>
<tr>
<td>$\omega_i^u$</td>
<td>0.679</td>
<td>Fraction of formal unskilled workers $\lambda_i^u = 44%$</td>
<td>ENOE Q2-2013</td>
</tr>
<tr>
<td>$\delta_h$</td>
<td>0.047</td>
<td>Skill premium (informal sector) = 20%</td>
<td>ENOE Q2-2013</td>
</tr>
</tbody>
</table>

#### 4.2 Discussion

The model exactly matches the five moments used to calibrate the household block. Although that is not the case for the six moments we target in the calibration of the firm block, the model does reproduce them reasonably well as the top panel of Table 3 shows.

The bottom panel shows seven more moments from our computed equilibrium in the benchmark calibration, and —when available— their data counterpart. The model reproduces quite well the size of the informal sector in terms of employment as well as in terms output. Although the model underestimates the mass of very small formal firms (under 5 workers), it does capture reasonably well the fraction of firms under 10 workers.

In our model, formal workers have higher wages. This is true for both level of skills but the formality premium is much higher for skilled workers. This is consistent with a lower return to skills in the informal sector, which at 20\% is less than half the skilled premium in the formal sector. The flatter return to skills in the informal sector reduces incentive for human capital accumulation, specially when it is very likely for workers to end up in this segment.

The model also reproduces reasonably well the firm growth dynamic in Mexico. For in-
Table 3: Results in benchmark calibration

<table>
<thead>
<tr>
<th>Moment</th>
<th>Model</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Skill Premium (formal sector)</td>
<td>.458</td>
<td>.455</td>
</tr>
<tr>
<td>Coefficient of variation of firm size</td>
<td>3.32</td>
<td>4.95</td>
</tr>
<tr>
<td>Median/mean firm size</td>
<td>.24</td>
<td>.22</td>
</tr>
<tr>
<td>3rd quartile/mean firm size</td>
<td>1.01</td>
<td>.9</td>
</tr>
<tr>
<td>Share of formal employment in firms of less than 20 workers</td>
<td>.573</td>
<td>.55</td>
</tr>
<tr>
<td>Share of formal employment in firms of less than 100 workers</td>
<td>.765</td>
<td>.78</td>
</tr>
<tr>
<td>Formality Premium (for skilled workers)</td>
<td>.535</td>
<td>N.A.</td>
</tr>
<tr>
<td>Formality Premium (for unskilled workers)</td>
<td>.276</td>
<td>N.A.</td>
</tr>
<tr>
<td>Share of formal employment in firms of less than 5 workers</td>
<td>.163</td>
<td>.30</td>
</tr>
<tr>
<td>Share of formal employment in firms of less than 10 workers</td>
<td>.395</td>
<td>.45</td>
</tr>
<tr>
<td>Fraction of time spent in training/education</td>
<td>.075</td>
<td>.12</td>
</tr>
<tr>
<td>Informal output as a fraction of GDP</td>
<td>.32</td>
<td>.30-40</td>
</tr>
<tr>
<td>Fraction of informal employment</td>
<td>.343</td>
<td>.367</td>
</tr>
</tbody>
</table>

stance, in lines with Hsieh and Klenow (2012), conditional on survival, firms barely double their initial size after 30 years.

In terms of worker’s dynamics, Figure 3 shows the expected wage losses after job displacement of a skilled worker as well as her corresponding job finding rate and her probability of being skilled as a function of the non-employment spell (i.e., time spent in the informal island). The expected wage loss is measured as the log difference between the worker’s last formal skilled wage and the current average formal wage. This is increasing in the length of the non-employment spell, as one should expect. After two quarters in non-employment, a skilled worker is expected to suffer a wage loss of more than 3%. After one and two years of job displacement, a skilled worker expects wage losses of 6.5% and 12.1%, respectively. This is clearly due to skills depreciation during non-employment. The longer this spell is, the more likely the worker becomes unskilled and earns a lower wage. Additionally, unskilled individuals have a slightly lower probability of finding a formal job, which is why we observe a decreasing job finding rate in the middle panel in Figure 3.
Figure 3: Life-cycle profile of a skilled worker after job displacement

- Expected wage loss
- Job finding rate
- Probability of being skilled
4.3 Couterfactual experiment: A lower cost of education

To highlight the importance of human capital accumulation, in this exercise we reduce the cost of education $\eta$ by 5%, while the remaining parameter values stay unchanged, we observed a drop in the size of the informal sector from about 34% to nearly 29%. This is the result of not only an increase in formal employment from 60% to 63% but also a decrease in the time devoted to informal activities by non-employed workers and more time spent in both job searching and education. Unskilled individuals now spend about twice as much time in education because it is less costly. As a result, the size of the informal sector as a fraction of GDP falls five percentage points.

Another interesting effect is on the skill composition of workers. We observe that the fraction of skilled workers increases from nearly 21% in our baseline calibration to about 23.5%. As a fraction of formal employment, skilled workers now account for 27.5%, about one percentage point increase relative to the baseline case. On the firms’ side, this change in the skill composition of workers has one important effect. Namely, firms spend slightly more in innovation, which in turn has a small positive effect on average labor productivity.

The increase in labor productivity and the changes in the mass and composition of formal employment result in a smaller wage premium. This falls from almost 0.46 to 0.43 as unskilled wages increase about 1% whereas skilled wages fall by 1.5%. This explains why unskilled non-employed workers spend more time searching for a job. The fall in skilled wages partially offsets the positive effect on human capital accumulation that the lower $\eta$ has. However, it is important to notice that if it were not for the increase in the average productivity of skilled workers due to more skill-biased innovation, the drop in the skilled wage rate would be larger and it would further dampen the effect of the lower cost of education.

In terms of the firm size distribution we also observe a shift toward larger firms. Now the share of employment in firms with less than 5 workers is 15.7% compared to 16.3%. For firms under 10, 20 and 100 workers, the shares of employment are 40%, 58% and 77%, respectively,
which are slightly larger than in the baseline case (see Figure 3).

Along the transition path from the baseline stationary equilibrium to the new one, we observe a spike in education the first few periods after the drop in $\eta$. As the mass of skilled workers rises and, consequently, the formal skill premium falls, education declines to its new steady state, which is higher than its initial level (i.e., the previous stationary equilibrium).

This exercise shows that human capital accumulation not only has a direct effect on informality but also interacts with firm innovation in such a way that strengthen the overall effect once we account for general equilibrium effects on wages.

### 4.4 Other counterfactual and policy exercises

In this section we will perform various experiments with our quantitative model in order to assess the impact of policies and frictions on different dimensions:

1. Social and productivity policies.
   - Introducing unemployment insurance
   - Subsidizing training programs
   - Subsidizing firm innovation

2. Taxation
   - Tax cuts (equivalent to reduce $\bar{\tau}$)
   - Flattening tax scheme $\tau$ (i.e., improving tax enforceability, reducing evasion).
   - Taxing informality
   - Progressive wage tax

3. Labor market: search frictions
In these exercises we will investigate further the importance of endogenous skill accumulation. We will also provide a quantitative decomposition of the importance of the different channels (firm innovation, capital accumulation, searching, etc.) through which each experiments affects the economy. Finally, we study the dynamics toward a new steady state.

We are especially interested in two policy exercises. In the first we introduce unemployment insurance. That is, we set \( b_l = b_h = rw_t \), where \( r \) is the replacement rate. In the second exercise, we introduce a flatter tax scheme as well as a tax on informal activities.

5 Conclusions

[IN PROGRESS]

References


A Appendix

A.1 ENOE: Some statistics

A.2 Calibration

Here information for calibration of $\tau$. 

<table>
<thead>
<tr>
<th>Firm size</th>
<th>Effective rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>.005</td>
</tr>
<tr>
<td>8</td>
<td>14</td>
</tr>
<tr>
<td>14</td>
<td>18</td>
</tr>
<tr>
<td>18</td>
<td>20</td>
</tr>
<tr>
<td>26</td>
<td>21</td>
</tr>
<tr>
<td>40</td>
<td>23</td>
</tr>
<tr>
<td>76</td>
<td>25</td>
</tr>
<tr>
<td>175</td>
<td>26</td>
</tr>
<tr>
<td>251</td>
<td>27</td>
</tr>
</tbody>
</table>
Figure 4: Function $\tau$

Source: Own elaboration.
We solve for the household block parameters in two steps. First —given parameters \((\xi_l, \xi_h, \delta_h)\) and the data moments \((\lambda_{el}, \lambda_{eh}, \hat{s})\) and transitions rates from an informal job to a skilled and unskilled formal jobs \((T_{hf}^{inf} \text{ and } T_{lf}^{inf}, \text{ respectively})\) — we solve for \((\lambda_{el}^e, \lambda_{eh}^e, \pi_h, \pi_l, \rho, s_h, s_l, t, \theta_s, \theta_\rho)\) using the following system of equations:

\[
\begin{align*}
\lambda_{el}^e &= \xi_l \lambda_{el}^e + \pi_l (1 - \rho) \lambda_{ln}^n, \\
\lambda_{eh}^e &= \xi_h \lambda_{eh}^e + \pi_h \lambda_{hn}^n + \pi_l \rho \lambda_{ln}^n, \\
\lambda_{ln}^n &= (1 - \pi_l) (1 - \rho) \lambda_{ln}^n + (1 - \pi_h) \delta_h \lambda_{hn}^n + (1 - \xi_l) \lambda_{el}^e, \\
\lambda_{hn}^n &= 1 - (\lambda_{el}^e + \lambda_{eh}^e + \lambda_{ln}^n) \\
\pi_h &= 1 - \exp(-\theta_s s_h), \\
\pi_l &= 1 - \exp(-\theta_s s_l), \\
\rho &= 1 - \exp(-\theta_\rho s_l), \\
\hat{s} &= \frac{1}{(\lambda_{ln}^n + \lambda_{hn}^n)} (\lambda_{hn}^n s_h + \lambda_{ln}^n s_l), \\
T_{hf}^{inf} &= \lambda_{ln}^n \rho \pi_l (1 - s_l - t) + \lambda_{hn}^n \pi_h (1 - s_h), \\
T_{lf}^{inf} &= \lambda_{ln}^n (1 - \rho) \pi_l (1 - s_l - t).
\end{align*}
\]

Secondly, we solve for \((\eta, w_{ln}^n, w_{hn}^n)\) such that \((s_h, s_l, t)\) are consistent with individuals’ optimality conditions. In this step we need equilibrium wages \((w_h, w_n)\) and use the parameters obtained in first step. Notice that this step requires iteration over value functions \((W_j, U_j)\). We start with an initial guess for \((W_j, U_j)\), then we get \((\eta, w_{ln}^n, w_{hn}^n)\) from the 3 FOCs from the individual problem. Then we update \((W_j, U_j)\) using their Bellman equations. We do that until convergence.