Modeling Forex Returns Volatility: A Random Level Shift Model with Varying Jump Probability and Mean Reversion. The Case of Latin America

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Abstract

We use a basic random level shifts (RLS) with time varying probabilities and mean reversion mechanism to model the Forex returns volatility in six Latin American Forex Markets: Argentina, Brazil, Chile, Colombia, Mexico and Peru. The results reveal that since the level shifts are taken into account, the long-memory presence dissapears in the autocorrelation function of the volatility series. Furthermore, evidence of conditional hetercedasticity is also removed.

KeyWords: Random Level Shifts, Long memory, Forex Returns Volatility, Latin american Forex Markets, Time Varying probability, Mean-Reversion.

JEl Classification: C22, C51, C53

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1 Introduction

There is a concise conclusion saying that financial asset returns in high frequency (as daily or monthly) are almost unpredictable. However, their volatilities sure they aren’t. This implies great disposition to acquire great knowledge for financial economics, risk management (Bollerslev, Engle, and Nelson (1994)) and for the pricing of primary and derivative assets (Engle and Ng (1993)). We know, evidently, that volatility is unobservable and the literature has worked to get this variable either by fitting parametric econometric models such as GARCH, by studying volatility implied by option prices in conjunction with option pricing models such as Black and Scholes (1976), by modeling a stochastic volatility model or by extracting it with absolute or squared returns. Each path has its weaknesses. However, we are going to take the last form, extracting the volatility with absolute returns and form a model with this variable.

Volatility, simply defining, measures the dispersion of an asset price returns. Forex returns volatility is important for any economic agent in the world, such as for academics, policymakers, regulators, market practitioners, etc. Volatility is the one that could tell us about how news affects asset prices, what information is considerably important, and finally how markets process that information.

Policymakers are concerned in measuring volatility of asset prices to get an idea of how markets respond to uncertainty and form its expectations. Latin American countries are now concerned with Forex volatilities because of the tremendous impact of the quantitative easing applied from the U.S. Others, as well as regulators or even traders and foreign investors, must take into account not only the expected return of their trading (portfolio, for investors) strategy, but also the exposure of their transactions to risk during periods of high volatility. Traders’ negotiations will have a good performance depending upon the accuracy of their volatility predictions. In summary, economic agents use volatility predictions as an important instrument for risk management.

In financial econometrics, the literature has grown considerably through many years and many authors have tried to debate constantly about which model describes better this variable and, obviously, making good forecast performance.

First of all, we have to go back to 1982, in economics literature when scientific research done by Nelson and Plosser (1982) showed that GDP had in its autocorrelation function a slow decay, which they interpreted, with econometric results, as an conclusive evidence of a unit root. However, Perron (1989) proposed an intriguing argument saying that unit roots are confused with structural breaks. This implied that models explaining long memory are not necessarily the best ones to capture the real Data Generating Process. Even though this conclusion had a tremendous impact in econometric literature, it was much later when this concern was placed in financial econometrics.

The field of econometrics has had an evolution that could be divided in three stages. In the first stage, between the 80’s and 90’s, attention was focused between the ARMA model, then the ARFIMA (Hosking (1981)) and finally we got into the FIGARCH model (Baillie et al (1996)) where the volatility was modeled with a fractional integrated GARCH. All these models imply long memory in the series autocorrelation functions and the forecasts were made according to this fact.

The second stage, between 1997 and 2009, a timid but a great concern arised starting with Teverovsky and Taqqu (1997) who argued something that Perron (1989) questioned in economic variables: structural changes could be confused with long memory. These years saw the concern rised also with Grourieroux and Jasiak (2001), Diebold and Inoue (2001), Granger and Hyung (2004) and others.
Finally, we arrived at the current and last stage, the third, with Lu and Perron (2010), who, following the examination of Perron and Qu (2010), proposed a "Random Level Shift" model in which there are two components: a short memory and level shift component. The latter will make the returns volatility (they analyzed U.S. indices) be modeled with random regime changes which will make the autocorrelation functions present no evidence of long memory. This, obviously, proposes an alternative way of dealing with a better model to apply to volatility series and confirms the confusion between level shifts and long memory.

This is the path of research that is proposed in this investigation, following the new extensions proposed to this model by Xu and Perron (2013): A random level shift model with time-varying jump probability and mean reversion. This model is applied to the Forex returns volatilities of six Latin American countries which are: Argentina, Brazil, Chile, Colombia, Mexico and Peru. We highlight the fact that Perron’s (1989) hypothesis about Nelson and Plosser (1982), done in economic variables, is also confirmed in our investigation: once the level shifts are taken into account, the long-memory presence dissapears in the autocorrelation function of the volatility series. This is something that is also confirmed in recent evidence as in Perron and Qu (2010), Lu and Perron (2010), Li and Perron (2013), Herrera and Rodríguez (2013), Ojeda and Rodríguez (2014) and Rodríguez and Tramontana (2014).

The investigation proceeds straightforwardly. Section 2 takes us briefly to the literature review. Section 3 has the data and summary statistics. Section 4 goes then with methodology of the Basic Random Level Shift model and and the algorithm of Bai and Perron (2003). Section 5 discusses the extensions of the basic model in which section 5.1 explains the specification of the time varying jump and the mean-reverting process. This part will end with section 5.2 where it presents the estimation methodology with these extensions included. Section 6 presents the full-sample estimates obtained from the four models and the estimates with the GARCH models applied to compare results.

2 Literature Review

The discussion concerning financial volatility series has involved theoretical and empirical investigations through the last 35 years. The three stages are our main structure to study the literature. Firstly, the early model was an ARFIMA(p,d,q) proposed by Granger and Joyeux (1980) in which the principal specification was $(1 - L)^d y_t = \varepsilon_t$. The parameter $d$ could take a value different to 1. This model provided the properties of long memory and also good long-run forecast performance. A year later, Hosking (1981), analyzed the same model but he covered the feature of the model when $0 < d < 0.5$. The value of $d$ will be important to model the persistence of the autocorrelation function. When we observed this condition, then the process has the property of long memory. This model resulted with better long-run forecasts.

Then, Geweke and Porter-Hudak (1983) applied a linear regression of the log-periodogram on a deterministic regressor in which the estimator is proposed as the estimated $d$ using the lowest frequency ordinates. With this, they proved that the Gaussian noise and the integrated series are the same. Additionally, their forecasts are much better than the early models, however, the estimation is consistent only with $d < 0$.

At the same time, Engle (1982) applied one of the most influential models of volatility called ARCH (Autoregressive Conditional Heteroskedasticity) which models the conditional variance according to past squared innovations. Since then, a great number of models have appeared. Following this article, there is Bollerslev (1986) applied a GARCH (Generalized ARCH) where it not onl in-
corporated past squared innovations, but also past conditional variances. Then Baillie et al. (1996)
applied a FIGARCH (fractionally integrated GARCH) in which the conditional variance of the
process is assumed to have a slow hyperbolic rate of decay for the participation of the influence of
lagged squared innovations. They used their model to German exchange rate and concluded their
forecasts are well in long horizons and the estimators are $T^{1/2}$ consistent. The main characteristic
of all this models was the assumption of long memory.

Later, Bollerslev and Mikkelsen (1996) suggested a FIEGARCH (fractionally integrated EGARCH)
in which a mean-reverting process applied describes better the long dependence in stock volatility
of the S&P 500 index returns. Next, Ding et al. (1993) investigated also the S&P 500 stock returns
and analyzed different modelations of volatility and their autocorrelations. They concluded
ARCH models for the squared and absolute returns have long memories. Besides, they proposed
the APARCH model (Asymmetric Power ARCH) in which they allowed the series to be affected
by asymmetric impacts in the variable.

After that, Lobato and Savin (1998) had two issues to confront. First, they conducted a
semiparametric test in order to check the long memory of the S&P 500 index in absolute and
squared returns. They found it in squared returns. Second, they investigated why it has long
memory and they affirmed it was due to nonstationarity.

The second stage then arrived with Teverovsky and Taqqu (1997). Following the Perron’s (1989)
concern of many years ago, Gourieroux and Jasiak (2001) focused on studying the autocorrelogram
instead of the fractional integrated estimator. They found that the long memory properties are
present in non-linear series with infrequent breaks so they determined that the hyperbolic decay is
just due to the breaks.

At the same time, Diebold and Inoue (2001) argued that, using a simple-mixture model, a
“Permanent Stochastic Break” model of Engle and Smith (1999) or a “Markov-Switching” model
of Hamilton (1989), they concluded that long memory is highly confused with regime changes.
Their Monte Carlo evidence showed that this is the case, even with little probability of regime
changes.

Contributing with this line of investigation there is Granger and Hyung (2004) who analyzed
this relationship between regime changes and long memory processes. They concluded that it is
difficult to distinguish between long memory processes and structural breaks to determine whether
the time series has long memory or not. However, using the S&P 500 index, they showed that a
regime-changing model has a better fit than a fractional integrated one (proposed by Geweke and
Porter-Hudak (1983)).

Following these articles there is Mikosh and Stărică (2004a) who mentioned that the long-range
dependence and the IGARCH process could be represented as non-stationary models. This is due
to the fact that it is not possible to distinguish sufficiently whether the process is stationary or it
has long memory. Thus, some changes in the conditional variance could take us to get confused
with IGARCH models. The same authors, in Mikosh and Stărică (2004b), offered a goodness-of-fit
test to verify the resemblance of the spectral density of a GARCH process to the logarithm of
absolute stock index returns. With Monte Carlo evidence, it was showed that the GARCH process
is rejected and this is due to changes in the unconditional variance. To prove their hypothesis, they
studied the S&P 500 index returns since 1953 and determined that a GARCH(1,1) is not a good
process to model the series. Nonetheless, if four years of observations since 1973 are omitted (the
oil shock period), then it is possible to assert that the process has short memory.

On their behalf, Stărică and Granger (2005) relinquished to the assumption of stationarity for
the S&P 500 returns volatility. They treated the non-stationary series with stationary models and showed that the behaviour of the volatility is concentrated in shifts of the unconditional variance. Their forecasts with non-stationary unconditional variance modeling are greater than the stationary long-memory processes and the GARCH(1,1).

However, our last third stage had something important to contribute since its beginning. It was with Lu and Perron (2010) that a new path of investigation has grown with them. They proposed a simple-mixture process with a short memory component and with a random level shift component. The latter contains an occurrence variable with a Bernoulli distribution. They applied a modified Kalman filter with a re-colapsing procedure done by Harrison and Stevens (1976). With these features, they used their model to S&P 500, AMEX, Dow Jones and NASDAQ index returns volatility. Their results got as a conclusion that once the level shifts are taken into account, no presence of long memory is seen in their autocorrelations.

Peruvian econometric literature hasn’t been outside of contributions. Herrera and Rodríguez (2013) followed as a base the work of Perron and Qu (2010) and applied some tests to evaluate if volatility returns of the peruvian index and exchange rate could be modeled as a process with short memory and level shift. Their results, even though through graphics and by analysis they proved the series showed no long memory when level shift are taken into account, didn’t conclude the same because the presented estimators didn’t agree with the previous conclusion.

However, Ojeda and Rodríguez (2014) applied the random level shift model as well as in Lu and Perron (2010) with the same volatility series of Herrera and Rodríguez (2013) and their results proved satisfactory saying that long memory in their autocorrelations disappear once the level shifts are considered, reinforcing the principal conclusion of the latter authors. Then, Rodríguez and Tramontana (2014) applied the same estimation methodology but in this case they did it with the stock returns volatility series of six latin american countries. Their results, once again, demonstrated all the conclusions of this authors since Perron and Qu (2010) and their forecasts (as well as in Ojeda and Rodríguez (2014), and Lu and Perron (2010)) resulted superior to the other long-memory models that have tried to accomplished the same goal.

Li and Perron (2013) applied the same method, but this time to the logarithm of the absolute exchange rate returns of Germany and Japan. They arrived at the same conclusion as the last authors and even their forecasts have better results in some indeces and depending on the horizon.

Finally, Xu and Perron (2013) followed the same direction of investigation and used the same indeces as in Perron and Qu (2010). However, now they extended the random level shift model with time-varying probability and mean reversion. The first one is an extension that allows the jump probability to change according to volatility clusters that are present in the series. This is done because it changes if there is a special event as well as a financial crisis. The other extension, the mean reversion expresses the notion that volatility tends to follow a downward path if there is an upward change which implies definitely a process in which the volatility follows its mean of past realizations. We apply four models as in Xu and Perron (2013): A basic random level shift (basic RLS), a basic RLS with time-varying jump probability, a basic RLS with mean reversion and finally a RLS with time-varying probability and mean reversion.
3 The Models

3.1 The Basic Random Level Shift Model (Basic RLS Model)

3.1.1 Model and Estimation Methodology

The first model applied is the following:

\[ y_t = a + \tau_t + c_t \]  
(1)

where \( a \) is a constant, \( \tau_t \) is the random level shift component and \( c_t \) is a short memory process. The second component follows the path:

\[ \tau_t = \tau_{t-1} + \delta_t \]  
(2)

where

\[ \delta_t = \pi_t \eta_t. \]  
(3)

The component \( \pi_t \), has a Bernoulli distribution that takes value 1 with probability \( \alpha \) and value 0 with probability \( 1 - \alpha \). If the variable takes the first value, then the level shift \( \eta_t \) occurs with distribution \( N(0, \sigma_\eta^2) \). Generally proposed, the short-component is modelled as \( c_t = C(L)e_t \), with \( e_t \sim \text{i.i.d.} \, N(0, \sigma_e^2) \) and \( E|e_t|^r < \infty \) for \( r > 2 \). The polynomial \( C(L) \) satisfies \( C(L) = \sum_{i=0}^{\infty} c_i L^i \), \( \sum_{i=0}^{\infty} |c_i| < \infty \) and \( C(1) \neq 1 \).

Although Lu and Perron (2010) and Li and Perron (2013) considered “\( c_t = e_t \)”, we follow the work of Ojeda and Rodríguez (2014) in which this short memory process is modeled as: \( c_t = \phi c_{t-1} + e_t \). As shown in Lu and Perron (2010), once the level shifts are accounted for, almost nothing of the serial correlation remains.

In order to develop the study, the error term is going to be a mixture of two normal distributions. Thus, we could write: \( \delta_t = \pi_t \eta_{t1} + (1 - \pi_t) \eta_{t2} \) with \( \eta_{t1} \sim \text{i.i.d.} \, N(0, \sigma_{\eta_1}^2) \) and \( \sigma_{\eta_1}^2 = \sigma_{\eta_2}^2 = \sigma_{\eta_2}^2 = 0 \). Since \( \sigma_{\eta_2}^2 = 0 \), we get into account that when \( \pi_t \) takes value 0 with probability \( (1 - \alpha) \), then this has no effect in the level shift component \( \tau_t \). To form the state-space form, we will put the model in first differences:

\[ \Delta y = \tau_t - \tau_{t-1} + c_t - c_{t-1} = \delta_t + c_t - c_{t-1} \]  
(4)

Knowing that \( c_t = \phi c_{t-1} + e_t \), we get the state-space representation as follows:

\[ \Delta y = H X_t + \delta_t \]  
(5)

\[ X_t = F X_{t-1} + U_t \]

where \( X_t = [c_t, c_{t-1}]' \), \( H = [1, -1] \),

\[ F = \begin{pmatrix} \phi & 0 \\ 1 & 0 \end{pmatrix}, \]  
(6)
and $U_t$ is a 2-dimensional normally distributed random vector with mean zero and covariance matrix with the form:

$$Q = \begin{pmatrix} \sigma^2_x & 0 \\ 0 & 0 \end{pmatrix}. \quad (7)$$

In what follows, we apply a special variant of the models proposed by Wada and Perron (2007). They used their model to analyse a trend-cycle decomposition of macroeconomic time series with a mixture of normal distributions for the shocks which affect the slope, level and cyclical components. Our model already imply this mixture, however there is one variance that is zero. Consequently, we define the observation data vector in time $t$ as $Y_t = (\Delta y_1, ..., \Delta y_T)$ and the parameter vector as $\theta = [\sigma^2_{\eta}, \alpha, \sigma^2_{e}, \phi]$. Having these, the log-likelihood function is:

$$\ln(L) : \sum_{t=1}^{T} \ln f(\Delta y_t|Y_{t-1}; \theta), \quad (8)$$

where:

$$f(\Delta y_t|Y_{t-1}; \theta) = \sum_{i=1}^{2} \sum_{j=1}^{2} f(\Delta y_t|s_{t-1} = i, s_t = j, Y_{t-1}, \theta) \Pr(s_{t-1} = i, s_t = j|Y_{t-1}, \theta). \quad (9)$$

In this last equation, $s_t$ is a variable that indicates the level shift occurrence. Hence, when $s_t = 1 (= 2)$, then $\pi_t = 1 (= 0)$ and there will be a level shift (will not be a level shift). Now we define $\xi_{ij}^{t} = f(\Delta y_t|s_{t-1} = i, s_t = j, Y_{t-1}, \theta), \forall i, j \in 1, 2$ so applying the rule of Bayes and of the conditional probability, and knowing that $s_t$ is independent of past realizations, we have:

$$\tilde{\xi}_{ij}^{t-1} = \Pr(s_{t-1} = i, s_t = j, Y_{t-1}, \theta) \quad (10)$$

$$= \Pr(s_t = j) \sum_{k=1}^{2} \Pr(s_{t-2} = k, s_{t-1} = i|Y_{t-1}, \theta).$$

Additionally:

$$\tilde{\xi}_{ik}^{t-1} = \Pr(s_{t-2} = k, s_{t-1} = i|Y_{t-1}, \theta) \quad (11)$$

$$= \frac{f(\Delta y_{t-1}|s_{t-2} = k, s_{t-1} = i, Y_{t-2}, \theta) \Pr(s_{t-2} = k, s_{t-1} = i|Y_{t-2}, \theta)}{f(\Delta y_{t-1}|Y_{t-2}; \theta)}$$

Therefore we have:

$$\tilde{\xi}_{ij}^{t+1} = \Pr(s_{t+1} = i, s_t = k, Y_t, \theta) = \Pr(s_{t-1} = i) \sum_{j=1}^{2} \tilde{\xi}_{jk}^{t} \quad (12)$$

which in matrix form is as follows:
Additionally, the conditional log-likelihood function for $\Delta y_t$ follows a normal density which is:

$$
\varpi = f(\Delta y_t | s_{t-1} = i, s_t = j, Y_{t-1}, \theta) = \frac{1}{\sqrt{2\pi} f_t^{ij} \sqrt{2}} \exp \left( -\frac{v_t^{ij} (f_t^{ij})^{-1/2} v_t^{ij}}{2} \right), \tag{13}
$$

where $v_t^{ij}$ is the prediction error and $f_t^{ij}$ is its variance. These two are defined as:

$$
v_t^{ij} = \Delta y_t - \Delta y_{t|t-1} = \Delta y_t - E [\Delta y_t | s_t = i, s_{t-1} = j, Y_{t-1}, \theta], \tag{14}
$$

$$
f_t^{ij} = E (v_t^{ij} v_t^{ij}).
$$

We should say that $\Delta y_{t|t-1}^{ij}$ depends only up to "t - 1" information. The basic tools are predictions for the state variables and their variances, which are given by:

$$
\begin{align*}
X_{t|t-1}^i &= F X_{t-1|t-1}^i, \\
P_{t|t-1}^i &= FP_{t-1|t-1}^i F' + Q.
\end{align*}
\tag{15}
$$

The prediction error is $v_t^{ij} = \Delta y_t - H X_{t|t-1}^i$, so that $f_t^{ij} = H P_{t|t-1}^i H' + R_j$, where $R_j$ is the variance of the error term, which takes values $R_j = \sigma^2_j (= 0)$ when $\pi_t = 1 (= 0)$ with probability $\alpha$ and $(1 - \alpha)$ respectively. Now, using these, we have the updating formulas:

$$
\begin{align*}
X_{t|t}^{ij} &= X_{t|t-1}^i + P_{t|t-1}^i H' \left( H P_{t|t-1}^i H' + R_j \right)^{-1} (\Delta y_t - H X_{t|t-1}^i), \tag{16} \\
P_{t|t}^{ij} &= P_{t|t-1}^i - P_{t|t-1}^i H' \left( H P_{t|t-1}^i H' + R_j \right)^{-1} H P_{t|t-1}^i.
\end{align*}
$$

As in Perron and Wada (2009), we will reduce the dimension of the estimation method by adopting the re-collapsing procedure made by Harrison and Stevens (1976), given by:

$$
X_{t|t}^j = \frac{\sum_{i=1}^2 \Pr(s_{t-1} = i, s_t = j | Y_t, \theta) X_{t|t}^{ij}}{\Pr(s_t = j | Y_t, \theta)}, \tag{17}
$$

and

$$
P_{t|t}^j = \frac{\sum_{i=1}^2 \Pr(s_{t-1} = i, s_t = j | Y_t, \theta) \left( P_{t|t}^{ij} + \left( X_{t|t}^j - X_{t|t}^{ij} \right) \left( X_{t|t}^j - X_{t|t}^{ij} \right)^T \right)}{\Pr(s_t = j | Y_t, \theta)}. \tag{18}
$$

8
3.1.2 Alternative Level Shift Estimate (Bai-Perron Algorithm)

Since we get the smoothed level shift component, this really performs very poorly in the presence of multiple regime changes. This is the reason we are going to use additionally the algorithm of Bai and Perron (2003) to obtain a better level shift component.

It all starts with the following multiple linear regression with m breaks \( (m + 1) \) regimes:

\[
y_t = x_t'\beta + z_t'\delta_j + u_t,
\]

where \( t = T_{j+1} + 1, ..., T_j \) for \( j = 1, ..., m + 1 \).

In this model, \( y_t \) is going to be the dependent variable, \( x_t \) \((p \times 1)\) and \( z_t \) \((q \times 1)\) are vectors with \( j \) as vector parameters with \( j = 1, ..., m + 1 \) and \( u_t \) is the perturbation error. The break points, given by the \( (T_1, ..., T_m) \) indices are treated as unknown variables (with the convention \( T_0 \) and \( T_{m+1} = T \)). The main purpose is to estimate the unknown regression coefficients together with the break points. This is a parcial structural break model due to the fact that the parameter vector \( \beta \) is not subject to changes. When \( p = 0 \), we obtain a pure structural break model. The variance \( u_t \) need not be constant.

Thus, the multiple linear regression system could be formulated in matrix form as:

\[
Y = X\beta + Z\delta + U
\]

where \( Y = (y_t, ..., y_T)' \), \( X = (x_t, ..., x_T) \), \( U = (u_t, ..., u_T) \) and \( Z \) is the matrix which diagonal divides \( Z \) in \( (T_1, ..., T_m) \), \( Z = diag(Z_{t_1}, ..., Z_{t_{m+1}}) \) with \( Z_t = (z_{T_{t-1}+1}, ..., z_{t_1}) \). Besides, \( \delta^0 = (\delta^0_1, ..., \delta^0_{m+1})' \) and \( (T^0_1, ..., T^0_{m+1}) \) are used to denote the true values of \( \delta \) and the break points, respectively. The matrix \( Z^0 \) is the one that diagonally partitions \( Z \) in \( (T^0_1, ..., T^0_{m+1}) \). Consequently, we assume the data generating process is modeled as:

\[
Y = X\beta^0 + Z^0\delta^0 + U
\]

Least squares is the estimation method. For each partition \( m \) \((T_1, ..., T_m)\), and minimizing the sum of squared residuals:

\[
S_T(.) = (Y - X\beta - Z\delta)(Y - X\beta - Z\delta) = \sum_{i=1}^{m+1} \sum_{t=T_{i-1}+1}^{T_i} [y_t - x_t'\beta - z_t'\delta_j]^2,
\]

we obtain \( \hat{\beta}(\{T_j\}) \) and \( \hat{\delta}(\{T_j\}) \). Then, substituting these into \( S_T \), we proceed to find the break date estimators \( (T^0_1, ..., T^0_{m+1}) \):

\[
(T^0_1, ..., T^0_{m+1}) = \arg \min_{T_1, ..., T_m} S_T(T_1, ..., T_m)
\]

3.2 Extensions of the Basic RLS Model

As well as in real variables, financial variables tend to have periods in which its behavior tends to have a particular fluctuations or reactions such as in crisis or due to changes in government policy. It is well noticed in financial volatilities, that volatility jumps tend to cluster during financial turbulences. This takes us to the conclusion that level shifts are not necessarilly \( i.i.d. \), which is
what the first model assumed. Instead, jump probability should change depending on financial stress, policy announcements or real economic changes that make this jump changes accordingly.

Another important aspect is the mean reversion process. It is more appropriate to say that volatility tends to follow a mean, which makes the variable revert downward if volatility goes up or vice versa. This extensions should take us to assert that we could boost forecasting performance. In the next part, we follow the Xu and Perron (2013) notation.

The specification of the Vaying Jump Probability is as follows:

\[ p_t = f(p, x_{t-1}) \]

where \( p \) is a constant and \( x_{t-1} \) are covariates that would give us better predictions of probability shifts, and \( f \) is as function that bounds \( p_t \) to \([0,1]\). It should be clear that \( x_{t-1} \) is set at time \( t \), so it is useful for forecasting.

Related to this, Martens et al. (2004) mention that there is a strong relationship between current volatility and lagged returns, something called “The leverage effect”. To model this, we follow Engle and Ng (1993) statement called “The news impact curve”, which is:

\[
\log(\sigma^2_t) = \beta_0 + \beta_1 r_{t-1} + \beta_2 I(r_{t-1} < 0) + \beta_3 |r_{t-1}| I(r_{t-1} < 0)
\]

where \( \sigma^2_t \) is volatility and \( I(\mathcal{A}) \) is an indicator function of the event \( \mathcal{A} \). Following Martens et al. (2004), \( \beta_1 \) is not significant, then it is ignored. We assume, to our purposes, large negative returns beyond a threshold \( a \), stated in relation to the probability that a return exceeds \( a \). We use \( a \) as three bottom values: 1%, 2.5% and 5%. Therefore, the functional form is the the next one:

\[
f(p, x_{t-1}) = \begin{cases} 
\Phi(p + \gamma_1 \{x_{t-1} < 0\} + \gamma_2 \{x_{t-1} < 0\}|x_{t-1}|) & \text{for } |x_{t-1}| > a \\
\Phi(p) & \text{otherwise}
\end{cases} \quad (19)
\]

where \( \Phi(.) \) is a normal cdf function, so that \( f(p, x_{t-1}) \) is bounded between 0 and 1, as required.

The next procedure involves creating a mean reverting mechanism to the level shift model. We do this because volatility, after a turbulence period where it goes up, tend to decrease to its long run mean. Hence, we apply the next specification:

\[
\eta_{it} = \beta(\tau_{it-1} - \tilde{\tau}_t) + \tilde{\eta}_{it}
\]

where \( \tilde{\eta}_{it} \sim N(0, \sigma^2_\eta) \), \( \tau_{it-1} \) is the filtered estimate of the jump component at time \( t \) and \( \tilde{\tau}_t \) is the mean of all the filtered estimates of the jump component from the first observation up to time \( t \). As a result, the mean reverting process is given by the condition: \( \beta < 0 \). It should be clear that the magnitude of \( \beta \) gives us the speed of reversion. This parameter with the mean are obtained up to time \( t \). Additionally, it will have a great impact on forecasts because high or low volatility will imply in the future lower or higher behavior respectively.

### 3.2.1 Estimation Methodology

Continuing with the notation, we also use the notation of Hamilton (1994). Assume \( Y_t = (\Delta y_1, ..., \Delta y_t) \) is the vector of data up to time \( t \) and the parameter vector as \( \theta = [\sigma^2_n, p, \sigma^2_\eta, \gamma_1, \gamma_2, \beta, \phi] \). Also let 1 represent a (4x1) vector of ones, the symbol \( \odot \) is an element-by-element multiplication, \( \tilde{\xi}_{it-1} = \)
$\text{vec}(\hat{\xi}_{t|t-1})$ with the $(i, j)^{th}$ element of $\hat{\xi}_{t|t-1}$ ($\hat{\omega}_t$) being $\Pr(s_{t-1} = i, s_{t} = j, Y_{t-1}, \theta)$ ($f(\Delta y_t|s_{t-1} = i, s_{t} = j, Y_{t-1}, \theta)$) for $i, j \in \{1, 2\}$ and we have $s_{t} = 1(= 2)$ if $\pi_{t} = 1 (= 0)$. Having this, the log-likelihood function is:

$$\ln(L) : \sum_{t=1}^{T} \ln f(\Delta y_t|Y_{t-1}; \theta)$$

where

$$f(\Delta y_t|Y_{t-1}; \theta) = \sum_{i=1}^{2} \sum_{j=1}^{2} f(\Delta y_t|s_{t-1} = i, s_{t} = j, Y_{t-1}, \theta) Pr(s_{t-1} = i, s_{t} = j, Y_{t-1}, \theta)$$

$$= 1'(\tilde{\xi}_{t|t-1} \odot \hat{\omega}_t)$$

Now we center our attention in the first vector. Following the basic model:

$$\tilde{\xi}_{t|t-1}^{ij} = \Pr(s_{t-1} = i, s_{t} = j, Y_{t-1}, \theta)$$

$$= \Pr(s_{t} = j) \sum_{k=1}^{2} \Pr(s_{t-2} = k, s_{t-1} = i|Y_{t-1}, \theta).$$

and

$$\tilde{\xi}_{t|t-1}^{ki} = \Pr(s_{t-2} = k, s_{t-1} = i|Y_{t-1}, \theta)$$

$$= \frac{f(\Delta y_{t-1}|s_{t-2} = k, s_{t-1} = i, Y_{t-2}, \theta) Pr(s_{t-2} = k, s_{t-1} = i|Y_{t-2}, \theta)}{f(\Delta y_{t-1}|Y_{t-2}; \theta)}.$$

Consequently, the evolution of $\tilde{\xi}_{t|t-1}$ is given by:

$$\begin{bmatrix}
\tilde{\xi}_{t+1|t}^{11} \\
\tilde{\xi}_{t+1|t}^{21} \\
\tilde{\xi}_{t+1|t}^{12} \\
\tilde{\xi}_{t+1|t}^{22}
\end{bmatrix} = 
\begin{bmatrix}
p_{t+1} & p_{t+1} & 0 & 0 \\
0 & 0 & p_{t+1} & p_{t+1} \\
(1 - p_{t+1}) & (1 - p_{t+1}) & 0 & 0 \\
0 & 0 & (1 - p_{t+1}) & (1 - p_{t+1})
\end{bmatrix}
\begin{bmatrix}
\tilde{\xi}_{t|t}^{11} \\
\tilde{\xi}_{t|t}^{21} \\
\tilde{\xi}_{t|t}^{12} \\
\tilde{\xi}_{t|t}^{22}
\end{bmatrix}.$$

which compactly is:

$$\tilde{\xi}_{t+1|t} = \Pi \tilde{\xi}_{t|t}$$

with

$$\tilde{\xi}_{t|t} = \frac{\tilde{\xi}_{t|t-1} \odot \hat{\omega}_t}{1'(\tilde{\xi}_{t|t-1} \odot \hat{\omega}_t)}$$

(21)
The conditional likelihood for $\Delta y_t$, the prediction equations (for the state variable and its variance), the updating formulas and the recollapsing procedure equations are given by the same equations: (13), (15), (16) and (17, 18) respectively. Now, the extensions must take us to make some variations in the prediction error. Since we have now the mean reverting process, we specify the model as:

$$y_t = a + \tau_t + c_t$$

$$\Delta y = \tau_t - \tau_{t-1} + c_t - c_{t-1}$$

$$\tau_t - \tau_{t-1} = \pi_t(\tau_{t|t-1} - \bar{\tau}_t) + (1 - \pi_t)\eta_{2t}$$

When $\pi_t = 1$ at time $t$, then we need to extract the mean reversion term, which is independent of $\pi_t$. Hence,

$$\varpi = f(\Delta y_t | s_{t-1} = i, s_t = j, Y_{t-1}, \theta) = \frac{1}{\sqrt{2\pi}} |f_{ij}^t|^{-1/2} \exp \left( -\frac{\bar{v}_{ij}^t (f_{ij}^t - 1/2)v_{ij}^t}{2} \right)$$

$$\bar{v}_{ij}^t = \begin{cases} v_{ij}^{11} = \beta(\tau_{ij|t-1} - \bar{\tau}_{ij}) \\ v_{ij}^{21} = \beta(\tau_{ij|t-1} - \bar{\tau}_{ij}) \\ v_{ij}^{12} = \beta(\tau_{ij|t-1} - \bar{\tau}_{ij}) \\ v_{ij}^{22} = \beta(\tau_{ij|t-1} - \bar{\tau}_{ij}) \end{cases}$$

$$f_{ij}^t = E\left( \bar{v}_{ij}^t \bar{v}_{ij}^t \right) = \text{HP}_{ij|t-1}^t H_t + R_t$$

Our principal model states: $y_t = a + \tau_t + c_t$, then $\tau_{ij|t-1} = y_t - a - c_{ij|t-1} = y_t - a - [0 \ 1] X_{t|t-1}^{ij}$, with $X_{t|t-1}^{ij}$ being a state variable that can be updated every time period. Thus, $\tau_{t|t-1} - \bar{\tau}_t$ is known at time $t$. Also $R_t = \sigma_{\eta_t}^2 (= 0)$ has probability $"p_t" \ (1 - p_t)$, respectively.

4 Full Sample Estimation Results

We use daily data for our models. We extract the volatility series proxied by log absolute returns. Our basic data comes from the daily closing prices, say $P_t$, and we extract the returns as $r_t = \ln(P_t) - \ln(P_{t-1})$. Knowing that we could probably have extreme negative volatility, we bound absolute returns away from zero by adding a small constant 0.001, so our volatility comes from the computation of $y_t = \ln(|r_t| + 0.001)$.

Our forex data consists of six countries with different time spans: Argentina (01/02/2002 - 07/02/2014; 2958 observations), Brazil (04/01/1999 - 07/02/2014; 3785 observations), Chile (04/01/1993 - 07/02/2014; 5282 observations), Colombia (20/08/1992 - 07/02/2014; 5259 observations), Mexico (02/01/1992 - 07/02/2014; 5636 observations) and Peru (03/01/1997 - 07/02/2014; 4251 observations).

Table 1 gives summary statistics of these volatilities proxies and shows their unconditional distribution characteristics. The six forex return volatility series have similar characteristics: mean, standard deviation and extreme values. All of them show a positive skewness, i.e., a right-tailed
distribution and Argentina clearly has a high value compared to the other series. The kurtosis of Brazil, Chile and Colombia are less than 3 of the normal distribution. However for the other countries, the results are greater.

Figure 1 provides the Forex returns and we could say that Argentina and Mexico had less turbulence in contrast to other countries, watching the series globally. A main characteristic shared for all the countries is the great variation of the returns in the financial crisis occurred in 2008. Peru, in contrast to others, do not show big extreme values (negative or positive) as well as the other countries. Figure 2 shows the well-known fact about the autocorrelation functions of financial volatility series: the long memory. All the countries show little accommodation to the confidence bands, which means all the values of the autocorrelations are significant. This stylized fact has been the base for all the models that have been developed previously. Now we pass to the next section to prove the contrary: the long-memory is disappears once the level shifts are considered.

4.1 Estimation Results from the Models

First of all, we present the results from estimating the basic random level shift model using forex return volatility series. These are reported in Table 2. In all the series all our parameters are significant even at 1% level of significance. Argentina and Mexico shows clearly a great dispersion from the mean in the probability shift, \( \sigma_\eta \), in contrast to the other countries. The standard deviation of the short memory component, \( \sigma_e \), have great resemblance between all the countries, but no for Argentina and Peru, which have lower estimates. The estimates of the AR(1) for the same component is significant just for Colombia, Mexico and Peru. The estimates of the jump probability is close to 1.5% for all the countries except for Mexico. Given this number, we could get the number of breaks for each country volatility: 44 for Argentina, 62 for Brazil, 45 for Chile, 99 for Colombia, 17 for Mexico and 71 for Peru. For the last country, we could compare the results with Ojeda and Rodríguez (2014) and we could say that, even though we have more observations, the results are quite the same for each estimate, even in the standard deviation of each one.

Figure 3 shows the volatility series together with the smoothed and Bai-Perron level shift component. Given the number of breaks calculated with our model, we use these numbers to get our results of the Bai-Perron level shift component. The two level shifts estimates behave almost in the same way, which affirms the robustness of the former. Figure 4 and 5 present the short memory autocorrelation functions with the smoothed and Bai-Perron \( \tau_t \). The two functions do not show any persistence. The hypothesis about long-memory is proved: it disappears once the level shifts are taken into account.

In Table 3, we present the estimation results when a time varying probability is incorporated into the RLS model. For each series, we consider three different threshold levels in order to evaluate the robustness of our results. We have as threshold levels: 1, 2.5 and 5% which correspond to the returns that are below these levels. The estimate of \( \sigma_\eta \) clearly shows great similarities for all the countries with respect to the Basic RLS model and between the thresholds too. It is also the case for \( \sigma_e \).

The estimate of \( \gamma_1 \) and \( \gamma_2 \), that corresponds to the components \( 1\{x_{t-1} < 0\}, 1\{x_{t-1} < 0\}|x_{t-1} \) (respectively) in the specification of the time-varying probability, is positive in all cases. At 5% of threshold, \( \gamma_1 (\gamma_2) \) results significant for all the countries except for Argentina (Argentina, Brazil and Peru) so Argentina is the only one that presents a clear evidence of no varying-jump probability. At 2.5% of threshold, \( \gamma_1 (\gamma_2) \) shows significance for all the countries except for Colombia (Argentina).
At 1% of threshold, which will be of interest for the fourth model, the results present $\gamma_1$ as significant for all the countries except for Argentina, Colombia and Peru. Concerning $\gamma_2$, all the estimates are significant.

Besides, we should say that a positive estimate $\gamma_1$ is consistent with the assertion of the “news impact” effect. Since our return data has negative values, a positive $\gamma_2$ is consistent with the well-known evidence that large negative returns correspond with higher volatility and this is approximated with a higher probability of the jumps, according to our model. Large negative returns then has asymmetric effects.

Additionally, we should point out that the estimate of $p$ is negative since we use a normal cdf functional form of $p_t$. Besides, the greater the threshold, the more significant becomes $\gamma_1$ and the less significant becomes $\gamma_2$; see, in particular, the results of Brazil, Colombia and Peru. Figure 6 presents the smoothed estimates of the level shift component for a threshold at 1% for each country. It reveals the clear resemblance to the evolution of its volatility series. Figure 7 presents the autocorrelation functions of the short memory component. As it is appreciated, once again the long-memory disappears.

In Table 4, we show the results of our estimation when we incorporate only a mean reversion component in the model. This is done like this because we want to evaluate the effect of this component into the model. As they show, all the estimates of $\beta$ are significantly negative. This clearly indicates the mean-reverting process is surely present in the volatility series. About $\sigma_q$, it is highly significant for all the Argentina, Chile, Mexico and Peru. Comparing the results with the Basic RLS (Table 2), we can find the value remains almost the same for Argentina and Mexico, however, for the other countries, the estimate value goes down. This could be explained with the fact that the standard error does not incorporate so much to the total variation to the jump probability as well as the mean reversion part. Finally the estimate values of the short memory component, $\sigma_e$, remain the same, implying robustness about the this feature in the model. As another fact, we should say that the level shift probability increases in this models for Brazil, Colombia and Peru. There is obviously a clear change that the mean reversion process has in the model.

Figure 8 shows the smoothed estimates of the level shift component, $\tau_{itT}$, for each country. We can assure that it presents more short-term variability against the basic RLS model, which can explain why jumps in the model with mean reversion occur more frequently. Figure 9 now shows the autocorrelation function of the short memory component of this model and it clearly doesn’t have any presence of long memory as shown in the last two models.

Table 5 presents the estimates of the modified RLS combining both time varying probabilities and mean reversion, using a threshold value of 1%. First of all, the estimate $\beta$ is again significantly negative, which tell us this variable is present for the level shift component. Besides, this variable in both tables 4 and 5 has similar results, which confirm us the robustness of our findings. About the estimates of $\gamma_1$ and $\gamma_2$, they are all positive.

In Table 3, even though our results were not satisfactory corresponding to the time-varying probabilities for Argentina, Colombia and Peru (for $\gamma_1$), in Table 5 our results show a clear evidence the model has all these variables significant even in some countries at 1% significant. Figure 10 presents the smoothed estimates of the level shift component with this model and it shows, as the earlier figures, almost the same variability. Finally, we have with Figure 11 the short memory component with this last model incorporating all the extensions. The hypothesis is also confirmed as with the other models discussed: The remaining noise is uncorrelated, then it justifies the
specification of the model and its conclusion: Once the level shifts are taken into account, the long-memory of the volatility autocorrelation function is no longer present.

4.2 Long-memory, level shifts and Conditional Heteroskedasticity

Given the conclusion above, it would be interesting to analyze the effect of level shifts within the presence of conditional heteroskedasticity. There is concise consensus saying that stock and forex returns exhibit conditional heteroskedasticity. For that reason, the GARCH(1,1) model, introduced by Bollerslev (1986), who followed Engle (1982), has been extensively used to model these returns and is one of the best models available to make forecasting. Although Lamoureux and Lastrapes (1990) proposed the conclusion that structural changes in the level of variance can magnify the evidence of conditional heteroskedasticity, it was until Lu and Perron (2010) who finally presented an assess to prove whether this regime changes can completely eliminate all traces of conditional heteroskedasticity. We use the model proposed of the latter authors to present evidence of this effect. In order to compare the estimates with the CGARCH model, we include to this the dummy variables associated with the level shift detected by Bai and Perron (2003). We have then four models to apply: A GARCH, a CGARCH, a CGARCH with Bai-Perron (2003) level shift $\tau_t$ and a CGARCH with the smoothed level shift component.

Having the fact that our returns present a Student-t distribution, we apply the first model, a GARCH(1,1) with this feature for the demeaned returns process $\tilde{r}_t$, 

$$\tilde{r}_t = \sigma_t \varepsilon_t$$  \hspace{1cm} (25)

$$\sigma_t^2 = \mu + \beta_r \tilde{r}_{t-1}^2 + \beta_\sigma \sigma_{t-1}^2,$$  \hspace{1cm} (26)

where $\varepsilon_t$ is i.i.d Student-t distributed with mean 0 and variance 1. We make the degrees of freedom be estimated from our model.

The second model is a Components GARCH (CGARCH) model with the following specification:

$$\tilde{r}_t = \sigma_t \varepsilon_t$$  \hspace{1cm} (27)

$$m_t = \mu + \rho (m_{t-1} - \mu) + \phi (\tilde{r}_{t-1}^2 + \sigma_{t-1}^2),$$

$$(\sigma_t^2 - m_t) = \beta_r (\tilde{r}_{t-1}^2 - m_{t-1}) + \beta_\sigma (\sigma_{t-1}^2 - m_{t-1}),$$

Third, we apply a CGARCH with the Bai-Perron (2003) level shift component. We incorporate this feature with the following specification:

$$\tilde{r}_t = \sigma_t \varepsilon_t$$  \hspace{1cm} (28)

$$m_t = \mu + \rho (m_{t-1} - \mu) + \phi (\tilde{r}_{t-1}^2 + \sigma_{t-1}^2) + \sum_{i=2}^{m+1} D_{i,t} \gamma_i,$$

$$(\sigma_t^2 - m_t) = \beta_r (\tilde{r}_{t-1}^2 - m_{t-1}) + \beta_\sigma (\sigma_{t-1}^2 - m_{t-1}),$$
where $D_{i,t} = 1$ if $t$ is in regime $i$, i.e., $t \in \{T_{i-1} + 1, \ldots, T_{i}\}$, and 0 otherwise, with $T_{i}$ ($i = 1, \ldots, m$) being the break dates documented in Figure 3 (again $T_0 = 0$ and $T_{m+1} = T$, the number of breaks is obtained from the point estimate of $\alpha$). The coefficients $\gamma_i$ which index the magnitude of the shifts, are parameters that are going to be estimated with the others, while the number of breaks is obtained from the point estimate of $\alpha$.

Finally, we apply a CGARCH but with the smoothed level shift component. The specification is as follows:

$$r_t = \sigma_t \varepsilon_t$$
$$m_t = \mu + \rho (m_{t-1} - \mu) + \phi (\tilde{r}_{t-1}^2 + \sigma_{t-1}^2) + \tau_t,$$

$$\sigma_t^2 = \beta_r \tilde{r}_{t-1}^2 - m_{t-1} + \beta_\sigma \sigma_{t-1}^2 - m_{t-1},$$

(29)

The results are shown in Table 6. The first model is the GARCH applied to Forex returns. As well as it is seen, the two parameters are highly significant and together sum close to 1. The second parameter $\beta_\sigma$ results really high. We also have the CGARCH model applied to the same variable and no level shifts within the permanent component. The results are quite similar from the first and $\rho$ results very close to 1. This result must not be unfamiliar given the fact the long memory is present in the volatility autocorrelation functions of all the countries.

Now we have the next model in which a level shift is incorporated: CGARCH with the level shift componentof Bai-Perron (2003). In all the countries, $\beta_\sigma$ now is not significant. The estimate of $\beta_r$ is not significant just for Peru, however the coefficient presents itself little value. Besides interestingly, the estimates of $\rho$ are now very well below one, ranging in average in 0.55. Hence, introducing the corresponding level shifts imply a completely different interpretation of the data.

The last model, the CGARCH with the smoothed level shift component instead of the term $\sum_{i=2}^{m+1} D_{i,t} \gamma_i$. The results are quite similar to the last model for $\beta_\sigma$, however it is different for $\beta_r$. It should be pointed out that, in this case, the estimates of the parameters are quite sensitive to little changes in the sample used.

5 Conclusions

The Forex returns volatilities analyzed, as well as of the asset returns, show clear evidence about long memory. This stylized fact has taken to many authors apply models that contribute to affirm the same knowledge and progress has been made according to this. The discussion about long and short memory continues growing constantly and we are not far from this path. However, our point of view takes the opposite direction and our estimates are clear to defend our hypothesis. The level shift component explains why the long memory conception appeared as a confusion between them. We argued in this investigation that our results show clear evidence about the following: The long memory disappears completely; our well-known fact about the long-memory is offset by the also well-known hypothesis of Perron (1989): Structural breaks are confused with long-memory.

References


### Table 1. Summary Statistics of the Volatility Series

<table>
<thead>
<tr>
<th>Volatility</th>
<th>Mean</th>
<th>SD</th>
<th>Max</th>
<th>Min</th>
<th>Skew</th>
<th>Kur</th>
</tr>
</thead>
<tbody>
<tr>
<td>Argentina</td>
<td>-6.026</td>
<td>0.739</td>
<td>-1.503</td>
<td>-6.908</td>
<td>1.473</td>
<td>5.838</td>
</tr>
<tr>
<td>Brazil</td>
<td>-5.145</td>
<td>0.857</td>
<td>-2.259</td>
<td>-6.908</td>
<td>0.065</td>
<td>2.569</td>
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<tr>
<td>Chile</td>
<td>-5.616</td>
<td>0.736</td>
<td>-3.044</td>
<td>-6.908</td>
<td>0.245</td>
<td>2.440</td>
</tr>
<tr>
<td>Colombia</td>
<td>-5.668</td>
<td>0.767</td>
<td>-2.564</td>
<td>-6.908</td>
<td>0.400</td>
<td>2.637</td>
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<td>Mexico</td>
<td>-5.576</td>
<td>0.817</td>
<td>-1.679</td>
<td>-6.908</td>
<td>0.440</td>
<td>3.148</td>
</tr>
<tr>
<td>Peru</td>
<td>-6.148</td>
<td>0.597</td>
<td>-3.738</td>
<td>-6.908</td>
<td>0.940</td>
<td>3.524</td>
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### Table 2. Estimates of the Basic RLS Model

<table>
<thead>
<tr>
<th></th>
<th>$\sigma_\eta$</th>
<th>$\alpha$</th>
<th>$\sigma_e$</th>
<th>$\phi$</th>
<th>Likelihood</th>
</tr>
</thead>
<tbody>
<tr>
<td>Argentina</td>
<td>1.309$^a$</td>
<td>0.015$^a$</td>
<td>0.496$^a$</td>
<td>-</td>
<td>2413.592</td>
</tr>
<tr>
<td></td>
<td>(0.198)</td>
<td>(0.003)</td>
<td>(0.008)</td>
<td></td>
<td></td>
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<tr>
<td>Brazil</td>
<td>0.535$^a$</td>
<td>0.016$^b$</td>
<td>0.745$^a$</td>
<td>-</td>
<td>4414.095</td>
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<tr>
<td></td>
<td>(0.120)</td>
<td>(0.007)</td>
<td>(0.009)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Chile</td>
<td>0.477$^a$</td>
<td>0.009$^b$</td>
<td>0.636$^a$</td>
<td>-</td>
<td>5272.241</td>
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<tr>
<td></td>
<td>(0.128)</td>
<td>(0.004)</td>
<td>(0.007)</td>
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<tr>
<td>Colombia</td>
<td>0.435$^a$</td>
<td>0.0188$^b$</td>
<td>0.647$^a$</td>
<td>0.066$^a$</td>
<td>5358.872</td>
</tr>
<tr>
<td></td>
<td>(0.110)</td>
<td>(0.009)</td>
<td>(0.007)</td>
<td>(0.016)</td>
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<tr>
<td>Mexico</td>
<td>1.072$^a$</td>
<td>0.003$^a$</td>
<td>0.670$^a$</td>
<td>0.071$^a$</td>
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<td>(0.001)</td>
<td>(0.007)</td>
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<tr>
<td>Peru</td>
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<td>0.490$^a$</td>
<td>0.104$^a$</td>
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<td></td>
<td>(0.084)</td>
<td>(0.005)</td>
<td>(0.006)</td>
<td>(0.020)</td>
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Note: Standard errors are in parentheses; estimates with a, b or c are significant at the 1%, 5%, 10% levels respectively.
Table 3. Estimates of the RLS Model with Time Varying Probabilities

<table>
<thead>
<tr>
<th></th>
<th>Argentina</th>
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<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
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<tr>
<td>Threshold</td>
<td>( \sigma_\eta )</td>
<td>( p )</td>
<td>( \sigma_\varepsilon )</td>
<td>( \gamma_1 )</td>
<td>( \gamma_2 )</td>
<td>Likelihood</td>
<td></td>
</tr>
<tr>
<td>5%</td>
<td>1.120&lt;sup&gt;a&lt;/sup&gt;</td>
<td>-2.175&lt;sup&gt;a&lt;/sup&gt;</td>
<td>0.477&lt;sup&gt;a&lt;/sup&gt;</td>
<td>0.519</td>
<td>142.698</td>
<td>2379.683</td>
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<tr>
<td></td>
<td>(0.150)</td>
<td>(0.184)</td>
<td>(0.008)</td>
<td>(0.374)</td>
<td>(10869.088)</td>
<td></td>
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</tr>
<tr>
<td>2.5%</td>
<td>1.244&lt;sup&gt;a&lt;/sup&gt;</td>
<td>-2.115&lt;sup&gt;a&lt;/sup&gt;</td>
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<td></td>
<td>(0.140)</td>
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<td>1%</td>
<td>1.192&lt;sup&gt;a&lt;/sup&gt;</td>
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<td>(0.173)</td>
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<td>(0.043)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

|                | Brazil                     |            |            |        |        |            |            |
| Threshold      | \( \sigma_\eta \)         | \( p \)    | \( \sigma_\varepsilon \) | \( \gamma_1 \) | \( \gamma_2 \) | Likelihood |
| 5%             | 0.449<sup>a</sup>         | -2.120<sup>a</sup> | 0.745<sup>a</sup> | 0.253<sup>a</sup> | 24.541  | 4404.834   |
|                | (0.106)                   | (0.418)    | (0.009)    | (0.095)| (310.601) |
| 2.5%           | 0.462<sup>a</sup>         | -2.113<sup>a</sup> | 0.745<sup>a</sup> | 1.218<sup>b</sup> | 0.076<sup>a</sup> | 4405.906   |
|                | (0.105)                   | (0.414)    | (0.009)    | (0.488)| (0.012) |
| 1%             | 0.465<sup>a</sup>         | -2.121<sup>a</sup> | 0.745<sup>a</sup> | 1.784<sup>c</sup> | 0.118<sup>a</sup> | 4404.861   |
|                | (0.104)                   | (0.408)    | (0.009)    | (0.979)| (0.031) |

|                | Chile                      |            |            |        |        |            |            |
| Threshold      | \( \sigma_\eta \)         | \( p \)    | \( \sigma_\varepsilon \) | \( \gamma_1 \) | \( \gamma_2 \) | Likelihood |
| 5%             | 0.450<sup>a</sup>         | -2.399<sup>a</sup> | 0.636<sup>a</sup> | 0.656<sup>b</sup> | 0.398<sup>a</sup> | 5270.988   |
|                | (0.114)                   | (0.433)    | (0.007)    | (0.278)| (0.135) |
| 2.5%           | 0.452<sup>a</sup>         | -2.419<sup>a</sup> | 0.635<sup>a</sup> | 1.193<sup>a</sup> | 0.382<sup>b</sup> | 5268.813   |
|                | (0.107)                   | (0.417)    | (0.007)    | (0.429)| (0.184) |
| 1%             | 0.444<sup>a</sup>         | -2.373<sup>a</sup> | 0.635<sup>a</sup> | 1.689<sup>b</sup> | 0.433<sup>b</sup> | 5269.273   |
|                | (0.120)                   | (0.419)    | (0.007)    | (0.852)| (0.190) |

Note: Standard errors are in parentheses; estimates with a, b or c are significant at the 1%, 5%, 10% levels respectively.
### Table 3. (cont.) Estimates of the RLS Model with Time Varying Probabilities

**Colombia**

<table>
<thead>
<tr>
<th>Threshold</th>
<th>$\sigma_{\eta}$</th>
<th>$p$</th>
<th>$\sigma_{\epsilon}$</th>
<th>$\phi$</th>
<th>$\gamma_1$</th>
<th>$\gamma_2$</th>
<th>Likelihood</th>
</tr>
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<tbody>
<tr>
<td>5%</td>
<td>0.337$^a$</td>
<td>-2.018$^a$</td>
<td>0.640$^a$</td>
<td>0.057$^a$</td>
<td>1.722$^b$</td>
<td>0.276$^b$</td>
<td>5345.784</td>
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<tr>
<td></td>
<td>(0.092)</td>
<td>(0.474)</td>
<td>(0.007)</td>
<td>(0.016)</td>
<td>(0.737)</td>
<td>(0.134)</td>
<td></td>
</tr>
<tr>
<td>2.5%</td>
<td>0.311$^a$</td>
<td>-1.904$^a$</td>
<td>0.640$^a$</td>
<td>0.057$^a$</td>
<td>2.647</td>
<td>0.371$^a$</td>
<td>5345.966</td>
</tr>
<tr>
<td></td>
<td>(0.0974)</td>
<td>(0.506)</td>
<td>(0.007)</td>
<td>(0.016)</td>
<td>(3.661)</td>
<td>(0.137)</td>
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</tr>
<tr>
<td>1%</td>
<td>0.335$^a$</td>
<td>-1.897$^a$</td>
<td>0.641$^a$</td>
<td>0.062$^a$</td>
<td>3.195</td>
<td>0.135$^a$</td>
<td>5351.935</td>
</tr>
<tr>
<td></td>
<td>(0.125)</td>
<td>(0.587)</td>
<td>(0.007)</td>
<td>(0.017)</td>
<td>(15.966)</td>
<td>(0.036)</td>
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**Mexico**

<table>
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<th>$p$</th>
<th>$\sigma_{\epsilon}$</th>
<th>$\phi$</th>
<th>$\gamma_1$</th>
<th>$\gamma_2$</th>
<th>Likelihood</th>
</tr>
</thead>
<tbody>
<tr>
<td>5%</td>
<td>0.993$^a$</td>
<td>-2.909$^a$</td>
<td>0.670$^a$</td>
<td>0.072$^a$</td>
<td>1.116$^a$</td>
<td>0.706$^a$</td>
<td>5886.841</td>
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<tr>
<td></td>
<td>(0.011)</td>
<td>(0.340)</td>
<td>(0.007)</td>
<td>(0.015)</td>
<td>(0.288)</td>
<td>(0.242)</td>
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</tr>
<tr>
<td>2.5%</td>
<td>0.924$^a$</td>
<td>-2.859$^a$</td>
<td>0.668$^a$</td>
<td>0.068$^a$</td>
<td>1.660$^a$</td>
<td>0.088$^a$</td>
<td>5885.390</td>
</tr>
<tr>
<td></td>
<td>(0.005)</td>
<td>(0.304)</td>
<td>(0.007)</td>
<td>(0.015)</td>
<td>(0.413)</td>
<td>(0.011)</td>
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<tr>
<td>1%</td>
<td>1.009$^a$</td>
<td>-2.800$^a$</td>
<td>0.669$^a$</td>
<td>0.070$^a$</td>
<td>1.678$^a$</td>
<td>0.532$^b$</td>
<td>5889.191</td>
</tr>
<tr>
<td></td>
<td>(0.012)</td>
<td>(0.250)</td>
<td>(0.007)</td>
<td>(0.015)</td>
<td>(0.642)</td>
<td>(0.266)</td>
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</tr>
</tbody>
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**Peru**

<table>
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<th>Threshold</th>
<th>$\sigma_{\eta}$</th>
<th>$p$</th>
<th>$\sigma_{\epsilon}$</th>
<th>$\phi$</th>
<th>$\gamma_1$</th>
<th>$\gamma_2$</th>
<th>Likelihood</th>
</tr>
</thead>
<tbody>
<tr>
<td>5%</td>
<td>0.553$^a$</td>
<td>-2.148$^a$</td>
<td>0.476$^a$</td>
<td>0.062$^a$</td>
<td>0.015$^a$</td>
<td>258.096</td>
<td>3206.984</td>
</tr>
<tr>
<td></td>
<td>(0.074)</td>
<td>(0.230)</td>
<td>(0.007)</td>
<td>(0.021)</td>
<td>(0.0004)</td>
<td>(10428.507)</td>
<td></td>
</tr>
<tr>
<td>2.5%</td>
<td>0.516$^a$</td>
<td>-2.076$^a$</td>
<td>0.478$^a$</td>
<td>0.071$^a$</td>
<td>2.242$^b$</td>
<td>0.212$^b$</td>
<td>3215.909</td>
</tr>
<tr>
<td></td>
<td>(0.075)</td>
<td>(0.241)</td>
<td>(0.007)</td>
<td>(0.022)</td>
<td>(0.934)</td>
<td>(0.083)</td>
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</tr>
<tr>
<td>1%</td>
<td>0.586$^a$</td>
<td>-2.096$^a$</td>
<td>0.477$^a$</td>
<td>0.067$^a$</td>
<td>5.631</td>
<td>0.110$^a$</td>
<td>3211.692</td>
</tr>
<tr>
<td></td>
<td>(0.072)</td>
<td>(0.193)</td>
<td>(0.006)</td>
<td>(0.020)</td>
<td>(118.490)</td>
<td>(0.005)</td>
<td></td>
</tr>
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</table>

Note: Standard errors are in parentheses; estimates with $a$, $b$ or $c$ are significant at the 1%, 5%, 10% levels respectively.
Table 4. Estimates of the RLS Model with Mean Reversion

<table>
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<tr>
<th></th>
<th>$\sigma_\eta$</th>
<th>$\alpha$</th>
<th>$\sigma_e$</th>
<th>$\phi$</th>
<th>$\beta$</th>
<th>Likelihood</th>
</tr>
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<tbody>
<tr>
<td>Argentina</td>
<td>0.965</td>
<td>0.016</td>
<td>0.496</td>
<td>-</td>
<td>-0.738</td>
<td>2403.952</td>
</tr>
<tr>
<td></td>
<td>(0.067)</td>
<td>(0.004)</td>
<td>(0.008)</td>
<td>-</td>
<td>(0.123)</td>
<td></td>
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<tr>
<td>Brazil</td>
<td>0.106</td>
<td>0.065</td>
<td>0.744</td>
<td>-</td>
<td>-0.223</td>
<td>4403.7421</td>
</tr>
<tr>
<td></td>
<td>(0.072)</td>
<td>(0.030)</td>
<td>(0.009)</td>
<td>-</td>
<td>(0.015)</td>
<td></td>
</tr>
<tr>
<td>Chile</td>
<td>0.156</td>
<td>0.035</td>
<td>0.633</td>
<td>-</td>
<td>-0.209</td>
<td>5264.194</td>
</tr>
<tr>
<td></td>
<td>(0.054)</td>
<td>(0.020)</td>
<td>(0.007)</td>
<td>-</td>
<td>(0.015)</td>
<td></td>
</tr>
<tr>
<td>Colombia</td>
<td>0.052</td>
<td>0.084</td>
<td>0.632</td>
<td>-</td>
<td>-0.288</td>
<td>5341.479</td>
</tr>
<tr>
<td></td>
<td>(0.126)</td>
<td>(0.021)</td>
<td>(0.007)</td>
<td>-</td>
<td>(0.016)</td>
<td></td>
</tr>
<tr>
<td>Mexico</td>
<td>0.907</td>
<td>0.003</td>
<td>0.669</td>
<td>0.069</td>
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<td>5892.114</td>
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<tr>
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<td>(0.128)</td>
<td>(0.001)</td>
<td>(0.007)</td>
<td>(0.015)</td>
<td>(0.062)</td>
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<tr>
<td>Peru</td>
<td>0.141</td>
<td>0.109</td>
<td>0.473</td>
<td>-</td>
<td>-0.310</td>
<td>3213.794</td>
</tr>
<tr>
<td></td>
<td>(0.025)</td>
<td>(0.023)</td>
<td>(0.006)</td>
<td>-</td>
<td>(0.014)</td>
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</tr>
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</table>

Note: Standard errors are in parentheses; estimates with a, b or c are significant at the 1%, 5%, 10% levels respectively.
Table 5. Estimates of the RLS Model with a Time Varying Probability of Shifts and Mean Reversion, Threshold: 1%

<table>
<thead>
<tr>
<th>Country</th>
<th>$\sigma_\eta$</th>
<th>$p$</th>
<th>$\sigma_e$</th>
<th>$\phi$</th>
<th>$\gamma_1$</th>
<th>$\gamma_2$</th>
<th>$\beta$</th>
<th>Likelihood</th>
</tr>
</thead>
<tbody>
<tr>
<td>Argentina</td>
<td>0.912$^a$</td>
<td>-2.120$^a$</td>
<td>0.491$^a$</td>
<td>-</td>
<td>2.271</td>
<td>0.050$^a$</td>
<td>-0.598$^a$</td>
<td>2394.529</td>
</tr>
<tr>
<td>Brazil</td>
<td>0.134$^a$</td>
<td>-1.447$^a$</td>
<td>0.743$^a$</td>
<td>-</td>
<td>0.507$^c$</td>
<td>0.344$^a$</td>
<td>-0.184$^a$</td>
<td>4397.301</td>
</tr>
<tr>
<td>Chile</td>
<td>0.173$^a$</td>
<td>-1.859$^a$</td>
<td>0.633$^a$</td>
<td>-</td>
<td>0.455$^b$</td>
<td>0.245$^a$</td>
<td>-0.207$^a$</td>
<td>5263.683</td>
</tr>
<tr>
<td>Colombia</td>
<td>0.060$^a$</td>
<td>-1.400$^a$</td>
<td>0.632$^a$</td>
<td>-</td>
<td>0.228$^a$</td>
<td>0.992$^b$</td>
<td>-0.292$^a$</td>
<td>5339.754</td>
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<tr>
<td>Mexico</td>
<td>0.925$^a$</td>
<td>-2.770$^a$</td>
<td>0.669$^a$</td>
<td>0.068$^a$</td>
<td>1.396$^b$</td>
<td>0.091$^a$</td>
<td>-0.292$^a$</td>
<td>5887.014</td>
</tr>
<tr>
<td>Peru</td>
<td>0.166$^a$</td>
<td>-1.346$^a$</td>
<td>0.471$^a$</td>
<td>-</td>
<td>1.276$^a$</td>
<td>0.502$^c$</td>
<td>-0.340$^a$</td>
<td>3205.012</td>
</tr>
</tbody>
</table>

Note: Standard errors are in parentheses; estimates with a, b or c are significant at the 1%, 5%, 10% levels respectively.
<table>
<thead>
<tr>
<th></th>
<th>Argentina</th>
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<th>Brazil</th>
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<th>Chile</th>
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<tr>
<td></td>
<td>$\beta_r$</td>
<td>$\beta_{\sigma}$</td>
<td>$\rho$</td>
<td>$\varphi$</td>
<td>$\beta_r$</td>
<td>$\beta_{\sigma}$</td>
<td>$\rho$</td>
<td>$\varphi$</td>
<td>$\beta_r$</td>
<td>$\beta_{\sigma}$</td>
<td>$\rho$</td>
<td>$\varphi$</td>
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<tr>
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<td>0.837</td>
<td>-</td>
<td>-</td>
<td>0.140</td>
<td>0.861</td>
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<td></td>
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<td>-</td>
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<tr>
<td>CGARCH</td>
<td>0.288</td>
<td>0.632</td>
<td>0.998</td>
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<td>0.061</td>
<td>0.792</td>
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<td>(0.000)</td>
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<tr>
<td>CGARCH (BP)</td>
<td>0.044</td>
<td>0.016</td>
<td>0.509</td>
<td>0.048</td>
<td>0.042</td>
<td>0.017</td>
<td>0.510</td>
<td>0.056</td>
<td>-0.092</td>
<td>0.073</td>
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<td>(0.236)</td>
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<td>(0.992)</td>
<td>(0.994)</td>
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<td>CGARCH (SM)</td>
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<td>0.169</td>
<td>0.034</td>
<td>0.018</td>
<td>0.512</td>
<td>0.080</td>
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<tr>
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<td>Mexico</td>
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<td>Peru</td>
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<td>(0.000)</td>
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<td>CGARCH</td>
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<td>0.782</td>
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<td>0.020</td>
<td>0.065</td>
<td>0.818</td>
<td>1.000</td>
<td>0.101</td>
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<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
</tr>
<tr>
<td>CGARCH (BP)</td>
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<td>0.020</td>
<td>0.475</td>
<td>0.077</td>
<td>-1.027</td>
<td>1.787</td>
<td>0.776</td>
<td>1.205</td>
<td>0.040</td>
<td>0.016</td>
<td>0.500</td>
<td>0.040</td>
</tr>
<tr>
<td></td>
<td>(0.961)</td>
<td>(0.999)</td>
<td>(0.000)</td>
<td>(0.039)</td>
<td>(0.675)</td>
<td>(0.470)</td>
<td>(0.000)</td>
<td>(0.623)</td>
<td>(0.000)</td>
<td>(0.884)</td>
<td>(0.000)</td>
<td>(0.000)</td>
</tr>
<tr>
<td>CGARCH (SM)</td>
<td>0.048</td>
<td>0.017</td>
<td>0.503</td>
<td>0.073</td>
<td>0.043</td>
<td>0.016</td>
<td>0.502</td>
<td>0.047</td>
<td>0.041</td>
<td>0.016</td>
<td>0.501</td>
<td>0.043</td>
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<tr>
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<td>(0.913)</td>
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<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.853)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.008)</td>
<td>(0.928)</td>
<td>(0.000)</td>
<td>(0.003)</td>
</tr>
</tbody>
</table>

Note: BP: with Bai-Perron $\tau_G$; SM: with smoothed $\tau_G$; p value in parenthesis
Figure 1. Forex Returns
Figure 2. Volatility Autocorrelation Functions
Figure 3. Volatility and the two Level Shift Components (Smoothed and Bai-Perron $\tau_\ell$)
Figure 4. Short Memory Component Autocorrelation Functions from Basic RLS (Volatility minus smoothed level shift component)
Figure 5. Short Memory Component Autocorrelation Functions (Volatility minus Bai-Perron Level Shift Component from Basic RLS)
Figure 6. Smoothed Level Shift Component from RLS model with Time-Varying Jump Probability: Threshold: 1%
Figure 7. Short Memory Component Autocorrelation Functions from RLS model with Time-Varying Jump Probability: Threshold: 1%
Figure 8. Smoothed Level Shift Component from RLS model with Mean Reversion)
Figure 9. Short Memory Component Autocorrelation Functions from RLS model with Mean Reversion
Figure 10. Smoothed Level Shift Component from RLS model with Time-Varying Jump Probability and Mean Reversion: Threshold: 1%
Figure 11. Short Memory Component Autocorrelation Functions from RLS model with Time-Varying Jump Probability and Mean Reversion: Threshold: 1%