Health, taxation, and fresh starts

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Abstract

Forgiveness is a fairness ideal that defends that a fresh start should be conferred on those who regret their past choices. Despite the ethical debate that such an ideal has generated, too few models include either a full axiomatic justification of it, or additional source of unfairness. Moreover, the principle of forgiveness has been virtually neglected in relation to health, which is a very particular good that has attracted a lot of attention nowadays. In a model where individuals differ in both their health care needs and their lifestyle preferences, we first axiomatically present a social ordering function that allows us to bring the forgiveness ideal and the compensation problem together. The social preferences that we derive give top priority to that individual with the maximum level of what we call regret, which is the distance, in terms of consumption, between the individual's final bundle and an ideal choice without any regret or disability whatsoever. Next, we characterise an incentive-compatible fresh start policy which satisfies that social preferences, and we describe some features of the optimal tax-redistribution scheme that it generates. The policy advocates balancing both health care treatments and reductions to non-medical consumption. Results are mainly driven by incentive-compatibility and the health care needs heterogeneity. The full egalitarian ideal can only be achieved in a very limited and specific form.

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1. Introduction

In this paper we deal with the standard compensation problem in which society is concerned for those inequalities (outcome differences) that individuals are not responsible for. Additionally, we want to put that ideal together with the less studied principle of forgiveness. Our aim is to deal with this problem in a model with health, which is a very particular good that cannot be treated as alternative outcomes such as consumption or income.

Forgiveness is an ethical principle that aims to deal with those who regret their previous choices and need a fresh start (see Arneson 1989; Dworkin 2000, 2002; and Fleurbaey 1995, 2002, 2005a, 2008). This is a controversial ideal that some authors are reluctant to endorse as they argue that it may entail delicate moral and incentive issues (e.g., Arneson 1989; and Dworkin 2002). The argument is twofold. On the one hand, it seems to be unfair to help those who following their own preferences have mismanaged their resources. On the other hand, individuals may fake regret in order to get extra resources that they do not deserve, that is ‘eat the cake and have it too’. Fleurbaey (2005a, 2008) challenges these viewpoints. First, he argues that if the possibility of granting individuals with a fresh start were free, basically none would be against the principle of forgiveness. Therefore, the moral argument seems to be only valid in cases in which fresh starts entail a cost to other agents. However, Fleurbaey defends that the idea of charging costs to other individuals is the whole spirit of any redistribution policy. Second, in terms of efficiency, Fleurbaey shows...
that an incentive-compatible fresh start policy controls the incentives to misreport the real preferences, but moreover it increases freedom as individuals are no longer forced to bear the consequences of their early choices.

Besides the forgiveness ideal, we want social preferences to account for the compensation problem as well. Theories of fairness and responsibility defend that, when evaluating inequalities, society should only try to reduce outcome differences that are a result of illegitimate sources of inequality (e.g., Rawls 1971; Dworkin 1981a,b; Arneson 1989; Cohen 1989; and Roemer 1998). These sources refer to the circumstances for which the individual cannot be held responsible for.

The main aim of this paper is to bring these two general ethical ideals together in a framework in which health is considered as one of the components of the individual’s well-being. There is a large consensus that health is a very specific and crucial outcome that cannot be analysed as alternative goods like consumption. In terms of forgiveness, it is not so infrequent to observe the implementation of such a principle to real-life situations in which health is involved. For instance, public health services tend to treat all individuals who are in a bad health condition, regardless of their previous lifestyle. As regards the compensation problem, there are also legitimate and illegitimate sources of inequality in health and health care (see Fleurbaey and Schokkaert 2009). Individual preferences are one of the main factors that explain the need for health care since many of our illnesses are caused by lifestyle choices. Although it is a controversial issue, especially in relation to health, we endorse the view that considers that individual preferences are a legitimate source of inequality (see Fleurbaey 2008; Fleurbaey and Schokkaert 2011; and Fleurbaey and Maniquet 2011).

Previous papers have individually dealt with these two ethical principles. In a model in which health is not considered, Fleurbaey (2005a, 2008) proposes to measure the level of regret that any individual may experience as the difference between her initial endowment and what he calls the Equivalent Initial Share (EIS). This value is the amount of resources that the individual would need to buy a bundle that yields exactly the same level of utility as her current choice. In order to deal with the compensation for illegitimate sources of inequality when individuals differ in their preferences about health and consumption, Fleurbaey (2005b) derives social preferences that give top priority to that agent with the smallest Full-Health Equivalent Consumption (FHEC). Such an equivalent is defined as the minimum level of consumption that the individual would accept to exchange her actual choice for one in which she had perfect health. Fleurbaey and Schokkaert (2011) justify detailed why one should focus on the state of perfect health as the reference value. The main reason is that it makes that interpersonal comparisons boil down to assess levels of consumption, but moreover it avoids conflicts between redistribution and Pareto conditions (see Fleurbaey 2005b).

Since the EIS and the FHEC aim to solve very different problems, it is clear that these two measures are incompatible. An additional problem that they present is that, as Calo-Blanco (2014) shows, neither of them is a suitable measure for a health model with forgiveness. On the one hand, the FHEC is crucially affected by the direction in which individual preferences may change. On the other hand, the EIS is only valid in scenarios in which the trade-off between goods, like health and consumption, is the same for all agents. Clearly, this is not the case if we allow for differences in health care needs. Therefore, if we want to accommodate the ideal of forgiveness to a standard model of fairness and responsibility with health, we should use other normative criterion than the FHEC or the EIS.

Allowing for differences in both individual preferences and health care disposition, we present a social ordering function that singles out a specific way of ranking different allocations when it is acceptable to compensate those individuals who regret their initial choices, and/or have high health care needs. Such an ordering is obtained by means of efficiency and fairness properties, and it is the lexicographic extension of the result obtained in Calo-Blanco (2014). According to this social ordering function, society should grant the highest priority to that individual with the highest level of what we call regret. This value is the difference, in terms of consumption, that exists between the individual’s current situation and the hypothetical choice she would have made with her final preferences if she had the most favourable health disposition.

After characterising the social ordering function that maximises welfare, we next analyse the fresh start policy that is in line with it. In a second-best scenario, we describe some features of the optimal tax-redistribution scheme that the policy generates. The planner designs a tax scheme that allows it to collect monetary resources, and the aim of such a scheme is twofold. On the one hand, it induces individuals to limit the amount of resources that they spend on non-medical consumption. On the other hand, it provides the planner with resources to treat those who are in a bad health condition. We obtain that the goal of perfect equality can only be achieved in very limited scenarios, and that the cost of both giving those agents who regret their previous choice a fresh start and compensating individuals with
a poor health disposition, is shared by all members of society. The main effect of including additional informational constraints in the model is that it makes even more difficult to get the full egalitarian ideal.

The paper is organised as follows. Section 2 introduces the model and the notation. Section 3 first presents the ethical requirements that society wants to satisfy, and next it shows the axiomatic derivation of the social preferences. Section 4 analyses the optimal incentive-compatible fresh start policy that maximises social welfare. Finally, it also displays the numerical computation for a particular parameter configuration of the model. Section 5 offers the conclusions of this study. All proofs are contained in an appendix.

2. The model

Let us consider a population with a finite set of individuals \( N = \{1, \ldots, n\}. \) Health is a variable that ranges from full ill-health to perfect health, that is, \( h \in H = [0, 1]. \) Let \( h^* := 1 \) denote the state of perfect health. Consumption is the aggregation of the whole expenditure on non-medical goods, with \( c \in \mathbb{R}_+. \) A health-consumption bundle for individual \( i \in N \) is a pair \( (c_i, h_i) \in Z = \mathbb{R}_+ \times H, \) where \( c_i \) denotes consumption and \( h_i \) the individual’s chronic health state. An allocation defines a collection \( z_N = \{z_i\}_{i \in N}. \)

Every individual \( i \in N \) is assumed to have well-defined preferences \( R_i \) over the health-consumption space \( Z, \) which are described by a complete preorder. Moreover, preferences must also be continuous, convex, and strictly monotonic. Let \( R \) denote the set of such preferences. For any individual \( i \in N, \) let us denote that \( (c, h)R(c', h') \) (respectively, \( P_i \) and \( I_i \)) when health-consumption bundle \((c, h)\) is weakly preferred (respectively, strictly preferred and indifferent) to bundle \((c', h').\)

In order to model the principle of forgiveness, that is, the concern for those individuals who regret their previous decisions, we distinguish between two sorts of preferences. Agents use an ex ante profile \( R^e_N = (R^e_i)_{i \in N} \in \mathcal{R}^N \) to make their initial choices. Next, they get utility from an ex post profile \( R^p_N = (R^p_i)_{i \in N} \in \mathcal{R}^N \) that may differ from the ex ante preferences.

Every individual is also characterised by the amount of medical expenses that she needs to reach any health state. We denote the individual \( i \)'s health disposition by \( m_i \in \mathbb{R}_+, \) which describes how much health expenditure is needed to bring this individual to health state \( h \in H, \) that is \( m_i h. \) Individual \( j \in N \) is said to have a strictly better health disposition than any other individual \( k \in N \) if \( m_j < m_k. \) Let \( M = [m, \bar{m}] \) denote the set of all the possible health dispositions, with \( m \) and \( \bar{m} \) determining, respectively, the best and the worst health care needs. Such limits are assumed to be fixed for all possible allocations. Let \( M_N = (m_i)_{i \in N} \in \mathcal{M}^N \) denote population’s profile of health dispositions.

Additionally, all individuals are endowed with an equal level of income \( \omega \in \mathbb{R}_+ \) that they have to allocate between consumption and medical expenditure.

An economy is described by a list \( e = (R^e_N, M_N, \omega) \in \mathcal{E}, \) where \( \mathcal{E} \) denotes the set of all the economies satisfying the assumptions presented above. Social preferences allow us to compare allocations in terms of forgiveness and responsibility. They are denoted by \( \mathcal{R}(e), \) and let us assume that they are described by a complete preorder. \( z \mathcal{R}(e) z' \) means that allocation \( z \) is at least as good as \( z'. \) The corresponding strict social preference and social indifference relations are denoted by \( \mathcal{P}(e) \) and by \( \mathcal{I}(e), \) respectively. It is important to remark that the definition of the economy entails that the viewpoint of the ex ante preferences is discarded from the social evaluation (e.g., Fleurbaey 2005a).

Let us turn now to introduce the set of definitions that are needed to present the ethical principles that we will use to single out a particular social ordering function. We start by introducing the set of all the bundles that any individual can afford in the laissez-faire scenario:

\[ B(\omega, m_i) = \{(c, h) \in Z \mid c + m_i h \leq \omega\}. \]

Next, we formally define the concept of full-health equivalent consumption. Such a value is the smallest level of consumption which one individual would accept, according to her ex post preferences, to exchange her present bundle for one in which she has perfect health (see Fleurbaey 2005b).

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1 A group of objects \( a_N = (a_i)_{i \in N} \) denotes a list such as \((a_1, \ldots, a_i, \ldots, a_n).\)
Definition 2. For all \( i \in N, R'_i \in R \) and \( z_i \in Z \), the individual \( i \)'s full-health equivalent consumption is the value \( c'_i(z_i) \) that satisfies:
\[
c'_i(z_i) = \min \{ c' \in \mathbb{R}_+ \mid (c', h') R'_i(c_i, h_i) \}.
\]

Finally, we define an individual \( i \)'s ideal situation in which she would be maximising her true preferences in the case she had the best health disposition possible (see Calo-Blanco 2014):

Definition 3. For all \( i \in N \) and \( R'_i \in R \), the individual \( i \)'s most preferred bundle is the health-consumption pair \( z_i \in Z \) that satisfies:
\[
\bar{z}_i = \max_{z_i \in \mathbb{R}} B(\omega, m).
\]

This particular reference point is extremely useful at the time of evaluating the individual’s actual situation in terms of both regret and health care needs. Let us denote \( \bar{z}_N = (\bar{z}_i)_{i \in N} \in Z^n \) as the most preferred allocation.

3. Fairness requirements and social preferences

In this section we introduce the ethical requirements that we impose on our social preferences, and next we derive the social ordering function that, while respecting the ideal of responsibility, aims to give those who regret their previous choices a fresh start.

We first introduce two basic axioms commonly used in the literature (e.g., Fleurbaey and Maniquet 2011).

Axiom 1. (Strong Pareto): For all \( e \in E \) and \( z_N, z'_N \in Z^n \), if \( z_i = z'_i \) for all \( i \in N \), then \( z_N R(e) z'_N \). If moreover, \( z_1 R' \) for some \( j \in N \), then \( z_N P(e) z'_N \).

Axiom 2. (Separation): For all \( e \in E \) and \( z_N, z'_N \in Z^n \), if there exists \( i \in N \) such that \( z_i = z'_i \), then \( z_N R(e) z'_N \Leftrightarrow z_{N\setminus\{i\}} R(\varepsilon_i) z_{N\setminus\{i\}} \), where \( z_{N\setminus\{i\}} = (z_1, \ldots, z_{i-1}, z_{i+1}, \ldots, z_n) \), and \( \varepsilon_i \) is the economy with reduced population \( \{1, \ldots, i-1, i+1, \ldots, n\} \).

Strong Pareto rules out inefficient allocations, whereas Separation is a robustness property that states that when one agent has the same bundle in two different allocations, the ranking over these two allocations remains the same if we remove that agent from the economy (see d’Aspremont and Gevers 1977 and Fleurbaey and Maniquet 2008).

In order to obtain our characterisation results, we combine such basic requirements with axioms modeling transfers. More precisely:

Axiom 3. (Equal Preferences Priority): For all \( e \in E \) and \( z_N, z'_N \in Z^n \), if there exist \( j, k \in N \) with \( R'_j = R'_k \) such that:
\[
z'_j \succ P'_j \succ P'_k z_k \succ z_k',
\]
with \( z_i = z'_i \) for all \( i \neq j, k \), then \( z_N P(e) z'_N \); if otherwise \( z_j' = z_k \) and \( z_j = z_k' \), then \( z_N I(e) z'_N \).

Axiom 4. (Linearity): For all \( e \in E \), \( z_N, z'_N \in Z^n \) and \( \beta > 0 \),
\[
[(h^*, c_i)_{i \in N}] R(e) [(h^*, c_i + \beta)_{i \in N}] \Rightarrow [(h^*, c_i + \beta)_{i \in N}] R(e) [(h^*, c_i)_{i \in N}].
\]

Axiom 5. (Minimal Solidarity) For all \( e \in E \) and \( z_N, z'_N \in Z^n \), if there exist \( j, k \in N \) and \( e > 0 \) such that \( h_j = h_k = 0 \), \( h'_j = h'_k = e \), with \( z'_j P'_j z'_k P'_k z'_j \) and \( z_k P'_k z'_k P'_k z'_k \), where,
\[
\begin{align*}
    c_j &= c'_j - e, \\
    c_k &= c'_k + e,
\end{align*}
\]
with \( z_i = z'_i \) for all \( i \neq j, k \), then \( z_N P(e) z'_N \); if otherwise \( c'_j(z_j) - c'_k(z_k) = c'_k(z_k) - c'_j(z_j) < 0 \) and \( c'_j(z_j) - c'_j(z_j') = c'_k(z_k) - c'_k(z_k') > 0 \), then \( z_N I(e) z'_N \).
Let us briefly explain these redistribution principles. *Equal Preferences Priority* considers that transfers reducing inequality between individuals with the same preferences are always desirable, regardless of how much wealth the ‘rich’ loses (e.g., Fleurbaey and Maniquet 2006). Note that such a criterion implies an infinite aversion to inequality. *Linearity* is a consumption invariance at perfect health which states that if the consumption of all individuals in two different allocations increases in the same amount, provided that all of them have perfect health, social preferences should not be reversed. This property is similar to the equal social gains axioms proposed by Østerdal (2005) and Hougaard et al. (2013). Finally, *Minimal Solidarity* is a requirement of solidarity from individuals who have an ‘excessive’ level of welfare. It entails, from those agents, a monetary compensation to those who are worst-off than in their most preferred bundle (see Moulin 1987; and Maniquet and Sprumont 2005).

After having presented the ethical requirements that we impose on our social ordering function, we turn now to characterise our social ordering. In order to execute such a task, let us first introduce one additional definition:

**Definition 4.** For all \( e \in E, i \in N \) and \( z_i \in Z \), the individual \( i \)'s regret function is:

\[
\rho_i(z_i) = c_i^*(z_i) - c_i^*(z_i).
\]

Such a definition provides us with a specific (monetary) measure of the individual’s utility loss as a result of not being in her most preferred bundle, and hence it considers both changes in the individual’s preferences and/or by a poor health disposition. We use function \( \rho_i(z_i) \) to define the social ordering function:

**Social Ordering Function 1.** \((p\text{-healthy leximinmax})\) For all \( e \in E \) and \( z_N, z'_N \in Z^n \),

\[
z_N R^{ex}_p(e) z'_N \Leftrightarrow (\rho_i(z'_i))_{i \in N} \geq_L (\rho_i(z_i))_{i \in N}.
\]

This social ordering function ranks first, in lexicographic terms, that allocation in which the highest value of the regret function across the population is smaller.

We can present now the first result of this paper, which establishes that a society that wants to satisfy all the ethical requirements presented above should rank allocations according to the \( R^{ex}_p \) criterion:

**Theorem 1.** On the domain \( E \), a social ordering function satisfies Strong Pareto, Separation, Equal Preferences Priority, Linearity and Minimal Solidarity if and only if it coincides with \( R^{ex}_p \).

That is, society should give absolute priority to that agent with the highest value of the regret function in the population. This outcome extends the result obtained in Calo-Blanco (2014) to meet the lexicographic criterion. Figure 1 illustrates this social ordering.

Let us consider an economy with just two individuals, \( j \) and \( k \), and let us initially focus on allocation \( z'_N \). Individual \( k \) chooses her initial bundle with the best health disposition possible and she does not regret her choice. Therefore she does not experience any regret whatsoever. However, individual \( j \) does not have the best health disposition, but moreover she picks her initial bundle with the wrong preferences. According to Theorem 1, any redistribution policy that moves individuals to an alternative allocation like \( z_N \) would improve social welfare. That is, \( z_N R^{ex}_p(e) z'_N \) since the highest level of regret is smaller in this new allocation \( z_N \) (see Figure 1).

So far we have established a specific rule that permits us to make welfare assessments in the present scenario. However, we have not yet mentioned the way in which compensations should be paid. This task will be undertaken in the next section.

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2Where \( \geq_L \) denotes the *leximax* criterion as proposed by Bossert et al. (1994). According to this criterion, allocation \( a_N \) is said to be socially better than any other allocation \( a'_N \) “if the highest value in \( a_N \) is higher than the highest value in \( a'_N \). If the highest values are identical, then society eliminates that value from the two allocations and compare the highest values in the reduced allocations, and so on.”
4. Implementing the fresh start policy

We now turn to analyse an incentive-compatible redistribution scheme that satisfies the social ordering function derived in the previous section. For ease of exposition, let us introduce the following simplifications.

First, individual preferences are required to satisfy the single-crossing property, condition that, interestingly enough is not necessary to obtain Theorem 1. More precisely, the property entails that any two indifference curves of two different preferences cross no more than once. We can make use of such a condition to assert that for any \((c, h) = R_j \in \mathcal{R}\), individual preferences present a higher concern for health than those of \(R_k \in \mathcal{R}\), if they satisfy the following relations:

\[
\begin{align*}
&c' > c \text{ and } (c, h)I_k(c', h') \Rightarrow (c, h)P_j(c', h') \\
&c' < c \text{ and } (c, h)I_j(c', h') \Rightarrow (c, h)P_k(c', h').
\end{align*}
\]

In other words, individual \(j\) presents a higher concern for health than any other individual \(k\) if, for the same health disposition, the former devotes less resources to non-medical consumption than the latter. Let us assume that any individual who changes her preferences becomes more prudent, that is, for all \(i \in N\) either \(R_t \equiv R_a\) or \(R_t \succ h R_a\). We also assume that there exists a finite number of preferences \(\mathcal{R} = \{R_1, \ldots, R_f, \ldots, R_F\}\), that are ranked according to their level of concern for health, with \(R_f \succ R_{f+1} \succ h R_f\) for all \(R_f \in \mathcal{R} \setminus \mathcal{R}_F\).

Second, following Valletta (2013), let us consider that there are only two types of health care needs, so that individuals have either a good or a bad health disposition, that is \(M = \{m^g, m^b\}\), with \(m^g < m^b\). Additionally, let us assume that for all \(R_f \in \mathcal{R}\) there exist \(j, k \in N\) with \(R_j = R_k = R_f\) such that \(m_j = m^g\) and \(m_k = m^b\).

Let us introduce now a social planner that aims to minimise the value of the regret function as it is established in Theorem 1. The public policy that is used to maximise social welfare is twofold. On the one hand, all agents are taxed in order to collect monetary resources that will be subsequently used to fund additional health treatments. Actually, this tax can be understood as a limitation of the share of the initial endowment that can be devoted to non-medical consumption. On the other hand, the planner designs a health-treatment policy that aims to treat all those individuals who are not in good health.

Interestingly enough, such general and common policies can be related to measures that have been recently implemented in western societies. For instance, some countries are mandating their citizens to increase their health insurance coverage (e.g., Doonan and Tull 2010; and Paccagnella et al. 2013), but moreover, many governments have recently passed pieces of legislation that try to limit or ban certain consumption habits (e.g., Poutvaara and Siemers 2008). Note that the need to raise money to fund the health treatment affects all the individuals of society. Some agents may find difficult to expend a big share of their resources on non-medical consumption, while others are being forced to fund a public service that, on average, they are less likely to use. The social planner should aim to find a fair balance between these two effects.

In order to characterise this optimal tax-treatment policy, we first need to define the pieces of information that are available for the social planner. As in Valletta (2013), let us consider that health expenditure is observable.
planner sets the tax system as a function of both the initial endowment and the individual’s health expenditure. Since the value of $\omega$ is the same for all agents, let us denote the tax paid by any individual $i \in N$ as $\tau(m_i) : [0, \omega] \to \mathbb{R}_+$. Regarding the \textit{ex post} public health treatment that any individual may receive, we will consider two different scenarios according to the available pieces of information. In the first one we will assume that the level of health prior to the treatment, $h^o_i \in H$, is observable. In this case, it is possible to define the \textit{ex post} public health treatment that each individual may receive as a function of both, her health expenditure and her health state, that is, $s(m_i, h^o_i) : [0, \omega] \times [0, 1] \to \mathbb{R}_+$. Albeit to consider that health is observable is a reasonable assumption, it makes the design of the redistribution policy too direct, despite the fact that preferences and health dispositions are not observable. Therefore, in the second scenario that we consider, we will assume that the level of health is no longer observable. This implies that any additional treatment must be designed according to the individual’s health expenditure alone, that is, $s(m_i, h) : [0, \omega] \to \mathbb{R}_+$.

For both possible scenarios, let us have that the individual $i$’s level of health after that public additional treatment, $h^m_i \in H$, is given by the whole health expenditure that has been devoted to her, conditional on her health disposition, that is:

$$h^m_i = \frac{\omega - \tau(m_i) + s(\cdot)}{m_i}.$$  

That is, the final level of health is determined by the sum of both private and public resources, where $s(\cdot)$ is the value of the health treatment received by the individual in the corresponding scenario. Note that this is a transfer that must be exclusively used to subsidise the health expenditure, and hence it cannot be used to increase the non-medical consumption. Let us, then, define the \textit{net tax} paid by agent $i$ as the difference between the tax that she is paying and the value of the health treatment that she may receive from the planner, that is, $\tau^i(z_i) = \tau(m_i) - s(\cdot)$.

Let us define now, for any of the two redistribution schemes that we are going to consider, the individual $i$’s health-consumption feasible set in the presence of a social planner as:

**Definition 5.** For all $e \in E$ and $i \in N$, the individual $i$’s health-consumption feasible set under the planner’s intervention is:

$$B^i(\omega, m_i) = \{(c, h) \in Z \mid c + m_i h \leq \omega - \tau(m_i) + s(\cdot)\}.$$  

Additionally, we also require that any allocation induced by the planner must satisfy the following two conditions.

First, it has to be feasible, that is, $\sum_{i} (c_i + m_i h^m_i) \leq n \omega$. Second, the allocation must be incentive-compatible, which means that, when possible, agents have no \textit{ex ante} incentives to impersonate any other individual. As the pieces of information may vary, this idea must be specifically defined for each possible scenario. When health is observable, this property implies that for any $j, k \in N$ it has to be the case that $m_j \leq m_k \Rightarrow z_j R^m_j z_k$.

That is, \textit{ex ante}, no individual envies the final bundle that any other individual who has not a better health disposition may get, while the opposite is not true. This property implicitly entails that for all $j, k \in N$ such that $R^m_j = R^m_k$ and $m_j = m_k$, it must be the case that $z_j = z_k$, whatever their \textit{ex post} preferences are. Such a condition stems from the fact that, as both have made the same initial choice, the planner cannot distinguish between them, and one would always apply for any compensation addressed to the other.

However, the incentive-compatible constraint cannot be established in such a direct way when health is not observable. In this case one has to focus on observable bundles that consists of consumption and total medical expenditure. Consequently, we define the agent $i$’s \textit{ex ante} preferences over such bundles as $R^o_i$, which can be derived from the ordinary \textit{ex ante} preferences as follows (see Fleurbaey and Maniquet 2006; and Valletta 2013):

$$(c, mh) R^o_i (c', mh') \iff (c, h) R^o_i (c', h').$$

\footnote{Individual $j$ might consider reporting a level of health lower than the one that she could obtain if, with the additional treatment not initially designed for her, she would end up better-off than with the scheme that has been actually designed for her. For instance, as individual $j$ presents a better health disposition than agent $k$, she might take advantage of this fact in order to get, with the scheme designed for $k$, a level of health that is higher than the one initially expected from her. However, since health is observable, the planner can always limit the increase in health to what was expected from someone with the health care needs that the policy is defined to, and hence individual $j$ would have no incentives to deliberately diminish her own level of health. Therefore when establishing the incentive-compatible constraints we can perfectly focus on health-consumption bundles.}
Therefore, an allocation is said to be incentive-compatible when health is not observable if and only if no individual envies the bundle of any other agent which is feasible for her, that is:

\[
\text{for all } j,k \in N, (c_j, mh_j) R^m_j (c_k, mh_k) \text{ or } (c_k, h_k) \notin B'(\omega, m_j).
\]

The implications of the planner’s policy in the case in which the level of health is observable can be summarised in the following theorem:

**Theorem 2.** For any economy \(e \in E\), the redistribution scheme \((\tau(m;h_i), s(m;h_i,h^0))_{i \in N}\) that induces a feasible and incentive-compatible allocation \(z \in \mathbb{Z}^n\) which minimises the highest regret \(\rho(z)\) across individuals is such that:

i) If perfect equality in terms of regret is achieved, all bundles, but those chosen with preferences \(R_F\), are equal. Moreover, they are all located in one single indifference curve belonging to preferences type \(R_F\).

ii) The highest level of regret in society is determined by an agent endowed with ex ante preferences \(R_1\) and a bad health disposition, who may or may not regret her initial choice, that is, \(\rho_f(z^*_f)\), where \(R_f \in \mathcal{R}\).

iii) The best-off in terms of regret is a steady good health individual who has preferences \(R_f \in \mathcal{R}\).

iv) The highest level of consumption is determined by a bad health disposition agent who chooses with preferences \(R_1\), whereas the highest health state is defined by an agent with preferences \(R_F\) who does not necessarily have a good health disposition.

v) The highest net tax is paid by someone who has ex ante preferences \(R_F\).

The first result of Theorem 2 states that the full egalitarian ideal is not always achievable. This is an important difference with respect to Fleurbaey’s (2005a) framework, in which when the variety of preferences is limited to only two sets it is possible to obtain the perfect equality outcome. This shows that differences in health disposition play a key role in the model. The second idea that we can extract from the initial point is that, when possible, perfect equality is characterised in a very specific way in which all individuals, but those belonging to type \(R_F\), end up with the same health-consumption bundle. On top of that, such a bundle must be in the indifference curve of type \(R_F\) in which the bundles of all those individuals with that particular preferences are located. Therefore, the aim of any social planner should be to try to make equal the value of the individual well-being measure across the population, albeit it must be aware that the higher the number of types, the more difficult to reach that objective.

The second statement of the theorem characterises the worst-off individual. Clearly, it has to be someone who presents any of the features that define the regret function. Because of incentive-compatibility, the highest level of regret would never be determined by an agent with a good health disposition, since it is always possible to find a similar agent with a bad health disposition who is not better-off. Interestingly enough, the line of reasoning is not the same with regard to the regretted choices, as it can be the case that for the same bundle the steady individual is worse-off than the regretful one. This is due to the fact that the regret function is defined for each type of preferences, so its final value depends on the specific shape of the indifference curves that pass through the most preferred bundle and the individual’s actual choice, and not on the ‘size’ of her regret. Moreover, it does not need to be that the worst-off is defined by an agent who presents the two issues at the same time. That would be the case when the reference individual is someone who sticks to preferences \(R_1\), that is \(\max_{i \in N} \rho_i = \rho_1(z^*_1)\).

The intuition behind the third statement of the theorem is rather clear. A steady individual who has good health, whatever the type she belongs to, neither regrets her choice nor has a bad health disposition. Therefore, this individual is not suffering any of the features that directly define the regret function. By incentive-compatibility she always has the opportunity of mimicking any other agent with the same ex post type of preferences if the latter would be better-off. Additionally, we find that it is not possible to identify the set of preferences, or the health-consumption bundle, related to that individual who is determining the lowest level of regret in society. Once again, this is due to the fact that, opposite to the framework developed by Fleurbaey (2005a), the measure we have defined to evaluate well-being is not unique but a function of the individual’s own ex post traits.

Regarding the extreme values in the optimal allocation, because of the incentive-compatible and single-crossing properties, the largest levels of health and consumption are given, respectively, by individuals who present the highest and the lowest concern for health. The most interest part of this fourth point is that such a result is not symmetric.

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When dealing with the health state, the individual with the highest value is someone who can be endowed with any possible health disposition. However, this is not the case when dealing with the largest level of consumption, which is always determined by someone with a bad health disposition. Apart from the fact that improving the utility of this individual by means of health is costlier, such an asymmetry is due to the fact that $R_1$ determines the lowest concern for health, and hence there is no agent who regrets her choice and \textit{ex post} wants to adopt such preferences.

Finally, as regards the last statement of Theorem 2, we have that the one paying the highest net tax is someone with \textit{ex ante} preferences $R_f$. The reason is that such an agent has an advantage with respect to other individuals as she will never regret her choice. Interestingly enough, this individual does not need to be endowed with a good health disposition. Once again, the existence of different health care needs may block further redistribution between agents.

Let us proceed now to analyse the case in which the level of health is no longer observable. The implications of the planner’s policy in this second redistribution scenario can be summarised in the following theorem:

\textbf{Theorem 3.} For any economy $e \in E$, the redistribution scheme $(\tau(m_i h_i), s(m_i h_i))_{i \in \mathbb{N}}$ that induces a feasible and incentive-compatible allocation $z \in Z^n$ which minimises the highest $\rho_i(z_i)$ across individuals is such that:

i) \textbf{Perfect equality in terms of regret cannot be achieved, unless all individuals have perfect health and} $c^*_i(\xi_j) = c^*_i(\xi_k)$ \textbf{for all} $j, k \in \mathbb{N}$.

ii) \textbf{The best-off in terms of regret is a steady good health individual who has preferences} $R_f \in \mathcal{R}$, whereas the highest level of regret is determined by $\rho_f(\xi^*_j)$, where $R_f \in \mathcal{R}$.

The first result clearly limits the findings obtained in Theorem 2. Apart from the difficulties that incentive-compatibility entails at the time of achieving the full egalitarian ideal, the restriction on the available pieces of information makes it virtually impossible to achieve that perfect egalitarian goal, except in rather simple scenarios in which differences in preferences would play no role whatsoever. Since the planner must provide equal treatment to those who present the same health expenditure, any good health individual would always apply for additional resources that had been designed for an agent with a bad health disposition. As the former has a good health disposition, she would end up better-off than the former.

The result presented in the second point of the Theorem 3 replicates the one obtained in the previous theorem. That is, it shows that the factor which is mainly driving how inequality is distributed is not the lack of information or the incentive-compatibility constraint, as in Fleurbaey (2005a), but the differences in the individuals’ health care needs.

We conclude our analysis by presenting a numerical example of how the incentive-compatible policy may be designed in the two presented redistribution schemes. The utility function we use is taken from Fleurbaey (2005b). Let us assume that individuals’ preferences are represented by the function:

$u(c_i, h_i) = c_i h_i^{25}.$

Let us assume that there are just two types of preferences, $\mathcal{R} = \{R_1, R_2\}$, with $R_2$ exhibiting a higher concern for health than preferences type $R_1$, more precisely $\delta_2 = 1.5 > 0.5 = \delta_1$. Additionally, each individual is also characterised by her health disposition, which can be either good, $m^e = 1$, or bad, $m^b = 1.25$. The initial endowment is assumed to be equal for all agents; more precisely, $\omega_i = 1$ for all $i \in \mathbb{N}$.

Let us consider that there are only four agents (or four kinds of agents who are equally distributed), namely $N = \{P, H, S, G\}$. Individuals $P$ and $H$ have the preferences associated with the highest concern for health, that is $R_2$, but $H$ is considered to have a worse health disposition than individual $P$, which means that $m_P = m^e$ and $m_H = m^b$. Agents $S$ and $G$ are endowed with the best health disposition, $m_S = m_G = m^e$, and they make their initial choice with the preferences that present the lowest concern for health, $R_1$. However, after having decided their bundle, individual $G$ regrets her previous decision and wishes she had behaved as agent $P$, and hence her \textit{ex post} preferences would be $R_2$.

Table 1 presents the \textit{laissez-faire} and the two optimal allocations for this particular example. Without the planner’s intervention, allocation $z_0 = (P_0, H_0, S_0, G_0)$, both individuals $P$ and $H$ choose a lower level of consumption than the rest of the agents, and they get a higher level of health. In terms of the regret function, this value is 0 for $P$ and $S$. 

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to any health treatment off. P depends, once more, on the individual’s health expenditure. However, the announced ex post from the rest of the individuals in order to treat agents S not care about health at all. In this case, any limitation in consumption induced by the tax has such a high utility cost H that in this particular example all individuals will finish with the same loss in terms of regret; that is to say, the burden more than P also experiences a positive utility loss. Finally, agent S in this simplified scenario with only four agents. In this particular case the best-o-

<table>
<thead>
<tr>
<th>Individual P</th>
<th>Individual H</th>
<th>Individual S</th>
<th>Individual G</th>
</tr>
</thead>
<tbody>
<tr>
<td>$z_{P_0} = (0.4, 0.6)$</td>
<td>$z_{H_0} = (0.4, 0.48)$</td>
<td>$z_{S_0} = (0.67, 0.33)$</td>
<td>$z_{G_0} = (0.67, 0.33)$</td>
</tr>
<tr>
<td>$p_{P_0} = 0$</td>
<td>$p_{H_0} = 0.053$</td>
<td>$p_{S_0} = 0$</td>
<td>$p_{G_0} = 0.058$</td>
</tr>
<tr>
<td>$z_{P^*} = (0.38, 0.57)$</td>
<td>$z_{H^*} = (0.43, 0.52)$</td>
<td>$z_{S^*} = (0.54, 0.45)$</td>
<td>$z_{G^*} = (0.54, 0.45)$</td>
</tr>
<tr>
<td>$p_{P^*} = 0.024$</td>
<td>$p_{H^*} = 0.024$</td>
<td>$p_{S^*} = 0.024$</td>
<td>$p_{G^*} = 0.024$</td>
</tr>
<tr>
<td>$\tau_{P^*} = 0.055$</td>
<td>$\tau_{H^*} = -0.081$</td>
<td>$\tau_{S^*} = 0.013$</td>
<td>$\tau_{G^*} = 0.013$</td>
</tr>
<tr>
<td>$z_{P^{**}} = (0.42, 0.63)$</td>
<td>$z_{H^{**}} = (0.42, 0.50)$</td>
<td>$z_{S^{**}} = (0.53, 0.43)$</td>
<td>$z_{G^{**}} = (0.53, 0.43)$</td>
</tr>
<tr>
<td>$p_{P^{**}} = -0.02$</td>
<td>$p_{H^{**}} = 0.038$</td>
<td>$p_{S^{**}} = 0.038$</td>
<td>$p_{G^{**}} = 0.038$</td>
</tr>
<tr>
<td>$\tau_{P^{**}} = -0.043$</td>
<td>$\tau_{H^{**}} = -0.043$</td>
<td>$\tau_{S^{**}} = 0.043$</td>
<td>$\tau_{G^{**}} = 0.043$</td>
</tr>
</tbody>
</table>

Table 1: Results of the numerical example

because both are properly maximising their utilities and have the best health disposition, something that is not the case for H and G.

In the first scheme, when the planner is able to observe the individuals’ level of health, the optimal redistribution scheme is given by $z^* = (P^*, H^*, S^*, G^*)$ in Table 1. To compensate agents H and G, the public authority taxes all individuals but H, but at the same time induces S to reduce her level of consumption down to a 54% of the initial endowment. Note that the highest net tax is paid by agent P, and that individual H is being subsidised.

As a result of all this, agent P ends up with a lower level of both health and consumption, and hence her well-being decreases. Regarding agents S and G, this approach means that they consume less than they would prefer, but their health state will anyway be increased. However, since individual S has stronger preferences for consumption, she also experiences a positive utility loss. Finally, agent H gets both more health and consumption; in fact, she consumes more than P because the trade-off between health and consumption is costlier for the former than for the latter. Note that in this particular example all individuals will finish with the same loss in terms of regret; that is to say, the burden of both H’s poor health disposition and the mistake made by G is equally borne by all members of this particular society. The full egalitarian goal is achieved here because the number of agents is very limited, as for instance, we are not considering regretful individuals with a bad health disposition.

The optimal policy in the first redistribution scheme is affected by changes in the exogenous parameters. Figure 2 shows how it varies according to individual valuation of health. The solution becomes radical when individual S does not care about health at all. In this case, any limitation in consumption induced by the tax has such a high utility cost for agent S that the policy cannot be implemented. Consequently, the social planner is forced to raise more money from the rest of the individuals in order to treat agents S and G for their very poor health state. The social planner’s intervention tends to focus on individual H when both attitudes towards health, $\delta_P$ and $\delta_S$, converge. In the limit, since agents P and S would have the same preferences for health, both would be equally treated by the social planner.

The results of the second redistribution scheme, when it is not possible to observe the individuals’ level of health, is given by allocation $z^{**} = (P^{**}, H^{**}, S^{**}, G^{**})$ in Table 1. In such a case the planner designs a tax scheme that depends, once more, on the individual’s health expenditure. However, the announced ex post treatment would not depend on the observed health, but on such an expenditure alone. This implies that, for instance, agent P could apply to any health treatment offered to individual H as long as she chooses the same health expenditure than H.

Just as expected, in the optimal allocation perfect equality in terms of regret can no longer be achieved, not even in this simplified scenario with only four agents. In this particular case the best-off individual is P. Actually, she is even better-off than in her most preferred bundle. This is because, for her, it is worth impersonating individual H. Therefore, if we want to increase the welfare of the latter, we have to extract more resources from S and G. Both of them pay more taxes, but they are also induced to reduce their level of consumption down to a 53%) of the initial income.
5. Concluding remarks

Forgiveness is an ethical ideal that advocates compensating those individuals who regret their past choices. Standard models of fairness and responsibility are not fit to include such a principle in a health model. In this paper we have derived a social ordering function that brings both approaches together. Such a social ordering function has been obtained by means of basic ethical principles, and it entails minimising, in lexicographic terms, the highest level of regret in society. This level of regret is the distance, in full-health equivalent terms, between the individual’s actual choice and a hypothetical best bundle.

We have next characterised a fresh start policy that, by means of collecting taxes and subsidising additional health treatments, aims to minimise the highest level of regret in society. In a scenario in which health is assumed to be observable, we have obtained that the full egalitarian goal can only be achieved in a very particular form. This is due to the interaction between incentive-compatibility and the existence of different health care needs. Moreover, extreme values in the optimal allocation are significantly determined by the asymmetry in the change of preferences, that is, in the fact that any regretful individual always becomes more health concerned. Characterisation results are more limited when the planner cannot observe the individuals’ health state. In that particular scenario we have obtained that the full equity ideal is almost impossible to reach.

Finally, we would like to discuss two issues of the assumptions we have made. First, we have dealt with health from a one-dimensional viewpoint. In reality, it is a multidimensional good that includes other different aspects such as heart condition, motor function, mental condition, etc. However, when evaluating health state overall, many economists prefer to use measures that aggregate all these conditions in a single variable, such as the popular quality-adjusted life year (QALY). Therefore, we can understand our definition of health as the final value of such a variable. Second, we have considered any level of health as acceptable, as long as it maximises individual well-being. This might be controversial in the field of health, as it implies that any agent could freely choose her level of health. In real life, we find that the public authorities normally guarantee all individuals a certain minimum level of health, whatever their preferences. This requirement can be easily introduced in the model, without altering its main message, by assuming that all individuals must have a certain minimum health state.

Appendix A. Proof of Theorem 1

The proof of the result that for all $z_N, z'_N \in \mathbb{Z}^n$ such that $\max_i \rho_i(z_i) < \max_i \rho_i(z'_i) \Rightarrow z_N P(e)z'_N$ is provided in Calo-Blanco (2014). What we do here is to extend that result to meet the lexicographic criterion (see Hammond 1976). We split such a characterisation in two steps.

Step 1: For any economy $e \in \mathcal{E}$, let us consider two individuals $j, k \in N$ and two allocations $z_N, z'_N \in \mathbb{Z}^n$ such that, without loss of generality, $\rho_j(z'_j) = \rho_k(z_k) > \rho_k(z'_k) = \rho_j(z'_j)$, and $z_i = z'_i$ for all $i \neq j, k$. Moreover, let us also assume that $\rho_k(z_k) > 0$. When the signs of the regret functions are different to the one proposed here the proof is analogous. We need to prove that it must be the case that $z_N I(e)z'_N$. Opposite to the desired result, let us assume that $z_N P(e)z'_N$. 

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Appendix B. Proof of Theorem 2

Let us now introduce two additional individuals \(b, c\) such that \(R'_b = R'_c \in \mathcal{R}\) and \(R'_c = R'_c \in \mathcal{R}\). Let us also assume that there exist \(z_b, z'_b, z_c, z'_c \in Z\) with \(h_b = h'_b = h_c = h'_c = h^*\) such that (see Figure A.3a):

\[
\rho_b(z'_b) = \rho_c(z_c) = \rho_f(z_b) > \rho_b(z_b) = \rho_c(z'_c) = \rho_a(z_b).
\]

According to the initial assumptions, if we apply Separation we can add identical individuals in both allocations without altering social preferences over them, that is, \((Z_N, z_b, z_c, z'_b, z'_c, z_c)\). If we apply Strong Pareto (which implies Pareto Indifference) and Equal Preferences Priority twice, we can induce the following relations: \((z'_N, z'_b, z'_c, z'_b, z'_c, z_c)\). Using Transitivity and Strong Pareto we have that \((z'_N, z'_b, z'_c, z'_b, z'_c, z_c)\), and according to Separation we can state that \((z'_N, z'_b, z'_c, z'_b, z'_c, z_c)\). Let us introduce now four additional bundles \(z''_b, z''_b, z''_c, z''_c \in Z\), with \(h''_b = h''_b = h''_c = h''_c = h^*\) and \(\rho_b(z''_b) = \rho_c(z''_c) = \rho_b(z''_b) + \beta\) and \(\rho_b(z''_b) = \rho_c(z''_c) = \rho_b(z''_b) + \beta\), such that \(\rho_b(z''_b) < 0\) and \(\rho_b(z''_b) > 0\). Finally, using Linearity we get that \((z''_b, z''_c, z'_b, z'_c, z_b)\). However, if we apply the Minimal Solidarity axiom we obtain that \((z''_b, z''_c, z'_b, z'_c, z_b)\), which yields the desired contradiction.

**Step 2:** In this last step of the proof we extend the result in order to meet the lexicographic criterion, that is, we have to show that \((\rho(z'_c))_{i \in N} \succeq_L (\rho(z'_c))_{i \in N} \Rightarrow z_N \mathcal{P}(e)z'_N\). Without loss of generality, let us assume that there exist \(j, k \in N\) such that \(\max_{i \in N} \rho_i(z'_c) = \rho_j(z'_c) = \rho_k(z'_c) = \max_{i \in N} \rho_j(z'_c)\) (see Figure A.3b). Additionally, let us assume that \(\max_{i \in N} \rho_i(x_i) < \max_{i \in N} \rho_i(x'_i)\), where \(x_{N-1}, x'_{N-1} \in Z^{n-1}\) are allocations that are constructed as the original ones but removing, in each one of them, the individual with the highest level of regret. By Strong Pareto we have that \(z''_N \mathcal{I}(e)z''_N\), where \(z''_N\) is the allocation in which for all individual \(i\) we have that \(z''_N \not\subset z_i\), and moreover \(h_i = h^*\). Likewise, we have that \(z''_N \mathcal{I}(e)z''_N\). Let us consider now one additional agent \(b\) who shares ex post preferences with individual \(j\), that is \(R'_b = R'_c \in \mathcal{R}\). Such an individual has a bundle at \(z''_N \not\subset z_i\). According to Minimal Solidarity we can establish that \((z''_N, z''_b, z''_c, z''_b, z''_c, z''_c)\), where \(z''_b, z''_c \in Z\) are the bundles that satisfy, respectively, \(c''_j(z''_b) - c''_j(z''_b) < 0\) and \(c''_j(z''_c) - c''_j(z''_c) > 0\). Now, if we apply Equal Preferences Priority we get that \((z''_N, z''_b, z''_c, z''_b, z''_c, z''_c)\), where \(z''_b \not\subset z''_c\). If we apply Minimal Solidarity once more we have that \((z''_N, z''_b, z''_c, z''_b, z''_c, z''_c)\), where \(z''_b \not\subset z''_c\). We may remove, in each allocation, the individual with the highest level of regret, using the Separation axiom, and based on the initial assumptions, we know from step 2 that \((z''_N, z''_b, z''_c, z''_b, z''_c, z''_c)\). By Separation and Transitivity we get that \((z''_N, z''_b, z''_c, z''_b, z''_c, z''_c)\). Finally, if we apply Separation and Strong Pareto we reach the desired result \(z_N \mathcal{P}(e)z''_N\).

**Appendix B. Proof of Theorem 2**

We know from the results in Fleuraeby (2005a) that it must be the case that in any final optimal allocation \(z_N = (c^*_N, h^*_N)_{i \in N} \in Z^*\) all resources are exhausted, that is, \(\sum_{i \in N} (c^*_i + m_i h^*_i) = n_o\), where \(c^*_i\) and \(h^*_i\) are, respectively, the individual’s \(i\) levels of consumption and health after the tax-health-treatment scheme has been applied. To show this result, let us assume an incentive-compatible allocation \(x_N \in Z^*\) such that \(\sum_{i \in N} (c^*_i + m_i h^*_i) < n_o\).
If $x_j = x_k$ for all $j, k \in N$, we can find $\epsilon > 0$, such that if we replace the original allocation $x$ by $(c_i^+ + \epsilon, h_i^+)$ for all $i \in N$, we would obtain a new feasible incentive-compatible allocation in which, because of strict monotonicity, all individuals would be better-off.

Let us consider now that $F = 2$. Let $z_i^m$ denote the bundle associated to an agent with ex ante preferences $R_f \in \mathcal{R}$ who has a health disposition $m \in M = \{g, h\}$. Because of incentive-compatibility for all $f = 1, 2$, we have that $z_i^m R_f z_i'^m$. Suppose that the relation is that of indifference for both types. In that case, if $z_i^m \not\sim z_i'^m$, because of the single crossing-property there exists at least one type for which $z_i^m P_f z_i'^m$, with $R_f \not= R_{f'}$. If we focus on $R_f$, and moreover $z_i^m P_f z_i'^m$, we can increase the consumption of both individuals type $R_f$ by small quantities $\epsilon^g > 0$ and $\epsilon^h > 0$, and moving to a new feasible and incentive-compatible allocation in which both are better-off. If, on the other hand, $z_i^m R_f z_i'^m$, once more we can increase the consumption of both individuals type $R_f$ by small quantities $\epsilon^g > 0$ and $\epsilon^h > 0$. Additionally, and in order to satisfy incentive-compatibility, because of the single-crossing property a quantity $\delta$ of the total health expenditure should be removed from $\delta^m$. Such additional resources could be used to increase the level of consumption of individuals type $R_f$, and everybody in society would be better-off (see Figure B.4a). Alternative scenarios are proved in a similar fashion.

When $F > 2$, we know from Fleurbaey (2005a) that for the bad health disposition there exists, at least one, $R_f \in \mathcal{R}$ such that for all $R_{f'} \not= R_f$ we have that $z_i^m P_f z_i'^m$. Therefore, because of incentive-compatibility it has to be the case that for all $R_{f'} \not= R_f$, $z_i^m P_f z_i'^m$. Moreover, if we have that $z_i^m P_f z_i'^m$, then, if we slightly increase the level of consumption in bundle $z_i^m$, we have a new feasible and incentive-compatible allocation that yields a higher level of social welfare. If, on the contrary, $z_i^m R_f z_i'^m$, for all $R_{f'} \not= R_f$ with bad health disposition we have that because of incentive-compatibility $z_i^m R_f z_i'^m$. Suppose that we slightly increase the bundle of all individuals with preferences $R_f$. We know that $z_i^m P_f z_i'^m$, so individual $R_f$ with bad health disposition is better-off and incentive-compatibility is not violated. Regarding the other type $R_{f'}$ agent, for those $R_{f'}$ who $z_i^m P_f z_i'^m$, $z_i^m$ was not feasible, so nor is it the new one. With respect to that (single) type $R_{f'}$ who presents $z_i^m R_f z_i'^m$, as we have done previously, we can define a bundle in which agent $R_{f'}$ is better-off and agent $R_{f'}$ is equal-off (see Figure B.4b). Therefore, this new allocation is both feasible and incentive-compatible, but moreover it entails a higher level of social welfare.

We can proceed now to prove the five different points of Theorem 2.

i) Let us consider an allocation with $F$ different types of preferences. If the level of regret is the same for all individuals we have that $z_i^m P_f z_i'^m$, for all $R_f \in \mathcal{R}$ and $m, m' \in M$. Moreover, because of incentive-compatibility, it has to be the case that $z_i^m = z_i'^m$, for all $R_f, R_{f'} \in \mathcal{R} \setminus \{R_f\}$ and $m, m' \in M$. All these effects can be easily shown in Figure B.5a.

We turn now to show that a situation like the one described above can be stable, that is, incentive-compatible and with overall resources exhausted. If $z_i^m \not\sim z_i^m$ it has to be the case that $z_i^m$ is in the tangency of the line defined by $\tau^m(z_i^m)$. Let us assume that for any other type $R_{f'} \not= \{R_f, R_{f'}\}$ the bundle $z_i^m$ is in the tangency of $\tau^m(z_i^m)$, and moreover $\rho_{f'}(z_i^m)$, for all $R_f, R_{f'} \in \mathcal{R}$ (see Figure B.5b). In this case one could think of moving $z_i^m \to x_i^m$ in order to save
some resources, however, such a movement would increase the highest level of regret, for instance for the case of $\rho_F(z_1^g)$, and hence it would not be optimal. If we have the case that some resources could be saved when moving any other bundles upwards along its own indifference curve, in this case the new allocation would violate incentive-compatibility, since individuals $z_1^g$ would also apply for that bundle. Therefore, if by chance society ends up in such a point, being all resources exhausted, the situation may be blocked and hence it would be optimal.

Finally, in order to conclude the proof of this initial point, let us focus on the scenario in which just two types of preferences exist, that is $F = 2$. In Figure B.6a we have the case in which perfect equality in terms of regret is achieved. If resources are exhausted we have that this allocation is optimal. Any movement that implies no increase in the highest level of regret and it is incentive compatible entails an increase in total expenditure, so it would not be feasible. Finally, it is worth remembering that perfect equality may not be achieved, not even when the variety of preferences is limited. Figure B.6b depicts a case of inequality with just two types of preferences. In this particular case we have that $\max_{i \in N} \rho_1 = \rho_2(z_2^g) = \rho_2(z_1^g) = \max(\rho_1(z_1^g), \rho_1(z_2^g), \rho_2(z_1^g), \rho_2(z_2^g))$, and moreover all resources are being used. In order to minimise the highest level of regret we need to save some resources from bundles other than $z_1^g$ and $z_2^g$. For instance, one could extract resources from $z_2^g$ who has a strictly higher level of regret. Since she is in the tangency of its net tax, it has to be the case that she is strictly worse-off. But since $z_2^g z_1^g$ and the fact that $z_1^g$ is also in the tangency of her net tax, it has to be the case that this agent must also be worse-off in order to satisfy incentive-compatibility. But since $z_2^g z_1^g$ we also have that $z_1^g$ is worse-off. However, we have to move this bundle to the left, along the indifference curve of preferences $R_2$ in order not to increase the maximum level of regret, and moreover bundle $z_2^g$ must be moved in the same way as well in order to keep incentive-compatibility. However, since both are in the tangency between the type 2 preferences and the net tax paid, it has to be that this movement entails an increase in expenditure, since we are distorting the bundles to the left with respect to $R_2$ and $\overline{R}$. However, depending on the values of the health dispositions and the number of agents, such a movement may not be feasible. Likewise, in order to satisfy the lexicographic part of the social ordering, if for instance $\rho_2(z_2^g) > \rho_1(z_1^g)$, it is not possible to transfer resources between them since either feasibility or incentive-compatibility or both may not be satisfied.

ii) Between steady individuals, due to incentive-compatibility, one with a good health disposition can never be worse-off than any other bad health disposition agent with whom the former shares preferences and steadiness. According to both incentive-compatibility and the single-crossing property, for every bad health disposition agent with preferences $R_f \in R$ there is always another type $R_f' \in R$ with $R_f' > h R_f$ and bad health disposition agent who $ex post$ adopts the preferences of the former and is not better-off than her. Note that this is true for all preferences but $R_1$, since for this specific type there is no individual to her right who may be worse-off by making the mistake of using the wrong preferences. Once more, because of incentive-compatibility and the single-crossing property, for each type of preferences $R_f \in R$ there is another type $R_f' \in R$ with $R_f' > h R_f$ such that both change preferences to a third set $R_{f'} \in R$ with $R_{f'} > h R_f$. In this case we have that, when both have the same health disposition, regretful agent $R_f$ cannot be better-off than regretful agent $R_f$. Again, this is true for all types of preferences but $R_1$. In order to finish the proof of this second point we have to show that the highest level of regret across the population is defined by one individual with $ex ante$ preferences $R_1$ who is endowed with a bad health disposition. According to what we have
already shown we have to focus on three specific cases.

The first case, the one in which the highest level of regret is defined by one individual with bad health disposition that changes from preferences $R_1$ to any other set $R_f$ with $R_f \neq R_1$, that is $\max_{i \in N} \rho_i = \rho_f(z_{1}^b)$, is already depicted in Figure B.6b. The second scenario we analyse is the case in which the highest level of regret is defined by one steady individual who uses preferences $R_1$, that is $\max_{i \in N} \rho_i = \rho_1(z_{1}^b)$. Once more, we show such a result with a picture. As we can see in Figure B.7a $\max_{i \in N} \rho_i = \rho_1(z_{1}^b) = \rho_1(z_{1}^g)$. The depicted allocation is incentive-compatible, and let us assume that all resources are exhausted. In this case there is no way to extract resources while keeping the highest level of regret. Any attempt to get resources from these bundles would imply increasing the expenditure in bundles $z_{1}^g$ and $z_{1}^b$ in order to keep incentive-compatible. Moreover, it is not possible to keep $z_{1}^b$ equal-off without increasing expenditure on her.

With these two particular examples for a $F = 2$ scenario we have shown that bad health disposition agents with ex ante preferences $R_1$, regardless they regret or not their choice, may define the highest level of regret in society. The point that is left to prove is the fact that whenever $\max_{i \in N} \rho_i = \rho_f(z_{1}^b)$ with $R_f \neq R_1$ it must be the case that there exists $R_f^* \in \mathcal{R}$ such that $\rho_f(z_{1}^b) = \rho_f(z_{1}^b)$. Let us consider then that $\max_{i \in N} \rho_i = \rho_f(z_{1}^b)$, and that there is no other individual with the same level of regret. The idea is to show that one can always save some resources without increasing the highest level of regret until $\rho_f(z_{1}^b) = \rho_f(z_{1}^b)$. For instance, as it can be seen in Figure B.7b, one can subtract resources from $z_{1}^b$. If there is no incentive-compatibility problems we can keep doing this until we reach a new incentive-compatible allocation in which there exists one type of preferences $R_f^* \in \mathcal{R}$ such that $\max_{i \in N} \rho_i = \rho_f(z_{1}^b) = \rho_f(z_{1}^b)$. However, it may be the case that when changing $z_{1}^b$, because of incentive-compatibility, ex ante individuals with preferences $R_1$ opt to buy another bundle $z_{1}^b$, with $R_f^* \in \mathcal{R}$. In that case we would also have to reduce all those bundles that may provide agents type $R_1$ with incentive to misreport their real ex ante preferences. By doing so we are increasing the level of regret of both $\rho_f(z_{1}^b)$ and $\rho_f(z_{1}^b)$, with $R_f^* \succ^h R_f$. As we already know, it has to be the case that $\rho_f(z_{1}^b) \geq \rho_f(z_{1}^b)$ and moreover $\rho_f(z_{1}^b) \geq \rho_f(z_{1}^b)$, and hence they cannot be determining the highest level of regret. We can then keep moving the bundles down until $\max_{i \in N} \rho_i = \rho_f(z_{1}^b) = \rho_f(z_{1}^b)$ for any $R_f^* \in \mathcal{R}$.

iii) The proof of the first part of this third case is rather straightforward. Due to a direct application of the incentive-compatibility requirement, we have that for any steady bad health disposition individual with preferences $R_f \in \mathcal{R}$ there exists another good health disposition agent with the same preferences who, by definition, cannot be worse-off. With respect to regretful good health agents, because of incentive-compatibility, we know that for all $R_f, R_f' \in \mathcal{R}$ it must be the case that $\rho_f(z_{1}^f) \geq \rho_f(z_{1}^f)$. Therefore, when one individual with a good health disposition changes preferences from $R_f$ to $R_f'$, with $R_f' \succ^h R_f$, it cannot be the case that she ends up better-off than the steady good health disposition agent who belongs to type $R_f$. Finally, using a combination of the arguments that we have just laid out one can easily show that regretful bad health disposition agents cannot be determining the lowest level of regret in society.

We can easily prove the second part of the statement by means of a simple example. For instance, in the proof of the second point of Theorem 2 we have presented some examples in which the allocation is incentive-compatible, resources are exhausted, and the level of regret is not the same for all individuals. In order to show the result one just...
need to bend the indifference curves until either individual with preferences either \( R_1 \) or the one with the set \( R_2 \) is the best-off.

\( iv) \) The fact that in the optimal incentive-compatible allocation the extreme values of consumption and health are determined, respectively, by preferences \( R_1 \) and \( R_F \) is due to the joint application of the incentive-compatible and single-crossing properties.

First, we turn to show that the bundle defining the highest level of health can be associated with any possible health care needs. The scenario in which the highest value is determined by \( z_f^1 \) has already been shown (see, for instance, Figure B.6b). The case in which such a value is defined by someone with a bad health disposition is shown in Figure B.8a. The depicted allocation is incentive-compatible and all resources are exhausted. The way of improving welfare would be to take resources from \( z_f^2 \), but doing so we would have to move \( z_f^1 \) as well. Changing the latter would imply either a possible not feasible allocation or an increase in the highest level of regret.

Finally, we show that the bundle \( z_f^1 \) cannot be determining the largest level of consumption in society. Let us assume, opposite to the desired outcome, that this value is indeed defined by \( z_f^1 \). In that case we would have that \( c_f^1 \) would be the largest level of consumption in society, and moreover, that any bundle \( z_f^j \) would be "left-upwards" with respect to the indifference curve of \( R_1 \) that goes through \( z_f^1 \). If there is no incentive-compatible problems with any other bundle \( z_f^j \), one just have to move \( z_f^j \) until the point in which \( z_f^j R_1 z_f^{j'} \) is reached. Because of the slopes of the budget sets we would obtain that \( c_f^1 \leq c_f^j \), and moreover we would not be facing any difficulties with regretful individuals. However, problems may arise if there is any \( z_f^j \) that may prevent us from applying the strategy laid out previously. Without loss of generality, let us consider the example depicted in Figure B.8b, in which \( z_f^1 R_1 z_f^j \) and \( \max_{i \in N} \rho_i(z_f^j) = \rho_f(z_f^j) = \rho_f(z_f^1) \). There exist three different alternatives in such a scenario. First, if \( z_f^1 \) is distorted to the left with respect to \( R_F \), we can move the bundle to the right along the indifference curve of \( R_F \), while saving some resources and keeping the highest level of regret in society constant. Second, if \( z_f^1 \) is neutral with respect to \( R_F \), it must be the case that \( z_f^1 \) is distorted to the left with respect to \( R_F \). If we move the bundle \( z_f^1 \) to the right along the indifference curve of \( R_F \) we would be saving some money without increasing the highest level of regret. We should not face incentive-compatibility problems with other good health disposition agents since either \( z_f^j \) is already, for all of them, a better option than bundles along the involved indifference curve, or they are not interested in an increase of consumption at the expense of reducing the health state. Third, we have the case in which \( z_f^1 \) is distorted to the right with respect to \( R_F \). In this situation we can slightly move the bundle backwards along the indifference curve of \( R_F \). Next, we could also reduce the value of \( z_f^j \), or any other bundle that is preventing \( z_f^j \) from accepting the new bundle designed for her. This movement is acceptable since any bundle \( z_f^j \) with \( R_F \neq R_1 \) cannot be directly determining the highest level of regret, and moreover any possible bundle associated to a bad health disposition must be in the upper counter set of \( \rho_F(z_f^i) \). Additionally, it must be the case that \( \rho_F(z_f^i) < \rho_F(z_f^j) \), for any \( R_F \in \mathcal{F} \) and \( R_F \neq \mathcal{F} \).

\( v) \) Let us initially focus on individuals with a bad health disposition. Let us assume that, opposite to the desired result, the highest net tax is not defined by bundle \( x_F^p \), and hence there exists \( R_F \in \mathcal{R} \setminus \{ R_F \} \) such that \( x^*(z_F^p) > x^*(z_F^q) \). This bundle cannot be distorted to the right because one could save resources by moving it backwards along its own
the highest level of regret, which is given by $z^g_1 + f^n_1$, that can be obtained with a smaller cost than that of $z^g_f$. Therefore, we can move all bundles $z^b_{f+1}$, such that $z^b_{f+1} R_{f+1} z^b_f$. This movement saves resources, respects incentive-compatibility and does not increase the highest level of regret, which is given by $\rho_f(z^b_f)$, with $R_f \in \mathcal{R}$. Moreover, because of the single-crossing property we have that $z^b_{f+1}$ entails a lower cost than $z^b_f$. Note that the movements described here would never increase the highest level of regret, value that is defined by bundle $z^b_f$ and a given set of preferences $R_f \in \mathcal{R}$.

Let us introduce now the agents who have a good health disposition. Since they do not interact with the bad health agents in terms of incentive-compatibility, the previous result remains true. Let us assume an incentive-compatible allocation in which there exists $R_f \in \mathcal{R} \setminus \{R_F\}$ such that $\tau^n(z^f_f) > \tau^n(z^m_m)$ for all $R_f \neq R_f$ and $m \in M$. Once again, we have that the bundle, $z^f_f$, is neutral with respect to $\tau^n(z^f_f)$. The line of reasoning is the one proposed above, and the fact that because of the incentive-compatibility there is no problem with the bundles of the bad health disposition agents. Since it is in the tangency of the highest expenditure, $\tau^n(z^f_f)$, one can next move bundle $z^f_{f+1}$ until we reach that $z^f_{f+1} R_{f+1} z^f_f$. Because of the initial conditions and the single-crossing property we have, as long incentive-compatible is satisfied, that $z^f_{f+1}$ can be obtained with a lower expenditure than that needed to obtain $z^f_f$. If there exist problems with additional bundles $z^f_{f+1} R_{f+1} z^m_m$, one can concatenate them as we did in the first part of the proof of the present point. If, moreover, there exists $z^b_{f+1}$ that, because of incentive-compatibility, may prevent us from moving $z^g_{f+1}$, we also have to reduce the health expenditure in the former bundle until we get that $z^g_{f+1} R_{f+1} z^b_f$. If this movement generates some tension with any other bundle $z^g_{f+1} R_{f+1} z^m_m$ we have to reduce the treatment of this one as well. Additionally, because of incentive-compatibility there is no problem with other bundles $z^g_{f+1}$. Once again, this whole strategy does not increase the highest level of regret.

Notice that, as Figure B.7a shows, it can be the case that an individual with a bad health disposition is the one paying the highest net tax.

**Appendix C. Proof of Theorem 3**

Following the line of reasoning used at the beginning of the previous proof, it is straightforward to show that in this new scenario all resources are exhausted in the optimal allocation.

Let us proceed now to prove the three different points of Theorem 3.

i) Let us assume that the level of regret is the same for all agents, and that individuals with a bad health disposition are not in the state of perfect health. We know from Theorem 2 that in order to have perfect equality all bundles but those associated to preferences $R_F$ must be equal, so in the present case we have that $z^f_f = z^g_f$ for any $R_f \in \mathcal{R} \setminus \{R_F\}$ (figure B.6a depicts this sort of scenarios). However, because of incentive-compatibility, agents who have a good health disposition would also receive the additional treatment that was originally designed for those with a bad health disposition.
disposition, and hence such an allocation would not longer be optimal. This effect is due to the fact that, because health is no longer observable, the planner cannot discriminate between agents who present the same health expenditure.

So far we have assumed that health can always be increased, which implicitly implies that \( h < h' \) for that particular individual. Let us show now that when individuals are in the state of perfect health full equality cannot be achieved either. Let us consider a simple scenario with just two different preferences, \( R = \{R_1, R_2\} \), in which bad health disposition agents have perfect health, and moreover they are located in the same indifference curve as her good health disposition counterpart. If \( z_{1b}^{\prime} \neq z_{2b}^{\prime} \) we would have that either \( z_{2b}^{\prime}P_{z_{1b}^{\prime}} \) or \( z_{2b}^{\prime}P_{z_{2b}^{\prime}} \), and hence incentive-compatibility would be not satisfied. Then it should be the case that \( z_{1b}^{\prime} = z_{2b}^{\prime} \), but because of the single-crossing property we would have that \( z_{2b}^{\prime}P_{z_{2b}^{\prime}} \). When all individuals have the same bundle, with perfect health, unless it is the case that \( c_j^*(\tau_j) = c_k^*(\tau_k) \) for all \( j, k \in N \), the level of regret cannot be the same for all agents. Finally, it is straightforward to depict an example in which perfect equality can be achieved when there is just one single type of preferences.

ii) The proof is similar to that of Theorem 2.

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References