A Small Open Economy Analysis* of the Eurobonds

Alejandro Forcades Pujol†

December 5, 2014

Abstract

This paper aims to study what would have been the effects of implementing the so-called Eurobonds to the European sovereign debt crisis (2009-12). Any country can issue eurobonds up to 60% of its GDP, any debt beyond this threshold must be issued as a purely national debt. The Euro area is divided in four groups according to the initial government debt and its respective bond yields during the sovereign debt crisis; a small open economy model is set up for each one (baseline model); and then, a Eurobonds scheme is implemented in order to study the overall and heterogeneous effects of this coordinated fiscal policy. Three scenarios are considered for the policy: lower, middle and upper bound for the eurobonds yields. These represent the quality of the Euro-area institutions, which is defined in this context of public debt as an addition of the degree of integration and the degree of automaticity of fiscal mechanisms. GIIPS (groups 1 and 2) are able to reduce the crowding-out effect on productive investment, taxes, and debt in all scenarios as well as increase GDP and welfare. The rest of countries (groups 3 and 4) lose in terms of GDP, welfare and debt in the middle and upper bound. In the lower, all the groups are better off, increasing GDP and welfare, while reducing taxes and debt. In the middle, overall final debt-to-GDP ratio significantly falls, overall GDP notably grows and welfare gains are high, pointing out that it is not a zero-sum game for this scenario, the benefits of GIIPS surpass the costs of the rest of countries. On the other hand, overall results are the opposite for the upper bound yields, although less significant. Therefore, the key message is that the eurobonds offer a good opportunity to address times of soaring sovereign spreads, but the degree of success depends on the political-institutional reforms at the European Monetary Union level.

---

*I would like to thank Albert Marcet [my supervisor] for all the help.
†Universitat Autònoma de Barcelona, e-mail: alejandroforcades@uab.eu.
1 Introduction

The European sovereign debt crisis (2009-12) is a period characterized by two features, a continuation of the GDP fall started in 2007 with the international financial crisis and a soar in sovereign bond spreads in GIIPS (Greece, Ireland, Italy, Portugal and Spain) but not in the rest of the Euro area, what is called the fiscal fragmentation. The first feature has been somehow general in all countries of the Monetary Union, the second has highlighted specific economic consequences for GIIPS that have not taken place in the rest of Euro-area members. Figure 1 shows how these countries (GIIPS) experienced a high increase on its government borrowing rates compared to the rest of the Euro-area members.

Figure 1. 10-year government bond yields (Eurostat).

In this context, several unified fiscal policies have been proposed to address the fiscal fragmentation, most of them around a new idea of creating a eurobond. The idea is that countries pool their debts as eurobonds in order to avoid self-fulfilling debt crisis in a context of over-indebtedness but, at the same time, it should face a moral hazard problem, because it must incentive fiscal discipline to all participants in the policy. The first eurobonds proposal that takes into account both sides was written by Jacques Delpla and Jakob von Weizsäcker (2010)\textsuperscript{1}.

Before going to the details of the policy and the analysis, this paper highlights the economic consequences of this fragmented increase in sovereign risks. In this line, a summary of the main consequences follows.

Main economic consequences of the sovereign debt crisis:

1. Fiscal performance. In some countries, specially GIIPS, the inertia of an unaffordable government expenditure was not compensated by tax collection since the financial crisis and posterior recession in the private economy (2007-?). Despite that a countercyclical fiscal policy was necessary in order to return to a sustainable primary deficits, the fiscal effort of taxpayers has been much stronger due to the debt burden because sovereign debt crisis has soared the interest payments of these countries. In previous years, tax collection financed the primary government expenditure, nowadays, it must also repay the higher interests. This is very problematic when debt is very high, as it is the case nowadays in the Euro area, because although a country had reached a fiscal discipline, the debt service would not allow debt level reductions. Figure 2 evidences that GIIPS have not been able to reduce the growth rate of debt in spite of the austerity measures since 2010, in fact growth rates are even increasing since 2009.

Figure 2. Debt-to-GDP ratio (Eurostat)

2. Sovereign debt has crowded out or displaced productive investment. Broner, Erce, Martin and Ventura (2014) pointed out that as sovereign spreads rose, domestic residents, above all
banks, increased significantly their exposure to their own sovereign debt in GIIPS and, at the same time, domestic credit shifted from the private to the public sector in GIIPS. They collected information on the lending by domestic banks, through loans and bonds. Figure 3 shows the ratio of public credit (government of the country) over private credit (non-financial corporations and households), it gives us evidence of the credit reallocation from the private to the public sector that took place in GIIPS coinciding with the increase in sovereign spreads. The figure shows that banks significantly reduced the loans to the private sector, firms and households, and raised holdings of government bonds since 2009.

Figure 3. Credit allocation in GIIPS, Broner, Erce, Martin, and Ventura (2014).

On the other hand, the sovereign debt spreads of the rest of Euro area were low (Figure 1). Figure 4 takes France and Germany as a counterpart to illustrate that no credit reallocation took place in the rest of countries, comparatively, we can see that the ratio was much more constant during the sovereign debt crisis (2009-2012).
Figure 4. Credit allocation in the rest of countries, same source.

![Credit Allocation Diagrams](image)

3. From a fragmented national debt market (fiscal fragmentation) to a financial fragmentation of the Euro-area credit markets. Figure 5 evidences that the transmission of ECB’s policy to borrowing in the real economy was broken in GIIPS during the sovereign debt crisis, it shows that borrowing rate of non-financial corporations (NFC) are in line with sovereign bond yields not with the ECB base rate, which we can see that in previous periods it was the case. On the other hand, borrowing rates evolve according to the reference interest rate, before and after the debt crisis, in the rest of countries (Finland, France, Germany and Netherlands). In the words of Broner, Eorce, Martin and Ventura (2014), this suggests that the credit reallocation from the private to the public sectors pointed out above led to crowding out and more difficult access to credit for domestic firms and consumers in GIIPS. Since a high percentage of jobs are in small and medium-sized firms (SMES), US (50%), France (60%), Spain (67%), and Italy (80%)

2 Data from the Organisation for Economic Co-operation and Development (OECD).

5
4. Political and institutional crisis, at European and country level. The monetary union is not accompanied by a political union, the European sovereign debt crisis has pointed out that the status quo was not able to handle this kind of crisis. For this motive, the European Financial Stabilisation Mechanism (EFSM) and European Financial Stability Facility (EFSF) were created, that together with the IMF provided rescue packages to some governments: Greece (2010 and 2012), Portugal (2011) and Ireland (2010). Later, they were integrated in the European Stability Mechanism (ESM), which is the permanent crisis resolution mechanism for the countries of the Euro area, it gave financial aid to banking systems of Spain and Cyprus (2012). Another source of discussion has been the disagreement about the role of ECB, for example: Germany defended that its only purpose was to control inflation and a lot of countries wanted the ECB to intervene in sovereign debt market.

This political scenario, lately reactive (only corrective) measures and disagreement in very important points of the Euro area, feed back sovereign risks, it added fear of systemic consequences on top of the current increasing uncertainty of unsustainable fiscal paths in some countries. Even, doubts around the sustainability of the euro and the functioning of the Euro area as a system have been largely extended.

Therefore, Euro-area institutions lack of proactive (automatic) mechanisms and more integration that ensures that if any country struggles the system has a proper plan to address this event. We are referring to the implementation of eurobonds and the creation of a banking union for the unique money in order to avoid another financial fragmentation in the future.
These consequences have represented a second hit to the GDP after the international financial crisis of 2007, deepening the recession of GIIPS.

This study develops a theory (baseline model) in order to capture the first two consequences, fiscal performance and crowding-out effect of public debt on productive investment, we want to see whether the lending behavior of banks in different countries can be explained by the differences in bond yields. Therefore, I focus on constructing a theory that includes two key ingredients: fiscal policy with government debt bought by resident and foreign investors and a central role of credit or investment on real economy with financial frictions. Financial frictions impede that firms and investors could borrow credit from abroad. There is no money and default risk is treated as an exogenous interest rate in the baseline model.

On top of that, I evaluate the implementation of the eurobonds into the baseline model. Concretely, taking the exact definition of Delpla and von Weizsäcker (2010), from now on “the eurobonds proposal”. In their own words, sovereign debt of Euro-area countries should be split as follows, 

"Blue Bonds: EU countries should pool up to 60 percent of GDP of their national debt under joint and several liability as senior sovereign debt, thereby reducing the borrowing cost for that part of the debt. Red debt: any national debt beyond a country’s Blue Bond allocation should be issued as national and junior debt with sound procedures for an orderly default, thus increasing the marginal cost of public borrowing and helping to enhance fiscal discipline. Independent Stability Council (ISC): Blue Bond allocations to member states are to be proposed by an ISC and voted on by member states parliaments in order to safeguard fiscal responsibility."

Therefore, the eurobonds is a policy for weak and strong fiscal countries because the fact of pooling debt will reduce the possibility of self-fulfilling debt crisis, pooled debt will be a very liquid and safe asset, and an orderly default will impose fiscal discipline, avoiding the moral hazard problem. For these reasons, I abstract from modeling moral hazard and the perceived default risk is treated as an exogenous eurobonds yields for the pooled debt and as part of an endogenous interest-rate premium for the national debt (debt larger than the 60% of GDP) when the eurobonds are implemented to the baseline model. In addition, the eurobonds yields represent the quality of Euro-area institutions.

Their analysis is qualitative, focuses on defining the instrument, developing the institutional setup in order to guarantee a suitable implementation and guessing the possible general fiscal policy implications and the heterogeneity of fiscal benefits and costs on different countries. On the other hand, this paper develops a theory to contrast their fiscal hypothesis, given the assumptions of the
proposal and later debates, and besides testing the idea, it also studies the effects of the policy on fiscal performance and productive credit or investment. Related to the latter, although Delpla and von Weizsäcker (2011)\(^3\) propose that the national bonds should be kept out of the banking system, the investors are not restricted to buy them in this paper.

Concretely, model is used (i) to see what would be the fiscal implications of this coordinated policy in order to contrast the eurobonds proposal, (ii) to see what would be the fiscal performance with and without the policy, (iii) to analyse whether eurobonds would be a good solution to the crowding-out effect on GIIPS, (iv) to evaluate the income change and welfare, measured by output and utility, respectively, implied by eurobonds and (v) to evaluate the overall outcomes of the policy. All of this considering that there is another group of countries of the monetary union that may have costs, that is taking into account the heterogeneous effects of the policy.

In Section 2, I present the baseline model which is based on small open economy models. Then, I introduce theoretically the eurobonds to the baseline model.

In Section 3, I firstly evaluate the replication capability of the baseline model with respect to government debt, bond holdings of agents and productive investment for our period of interest (2009-2012). Secondly, as I said above, I use the model to study the effects of implementing the eurobonds for the same period, simulations are performed and the results are discussed.

Section 4 highlights the main conclusions, it discusses the validity of results and also evaluates the policy from a general perspective considering the overall results.

Now, a brief literature review follows.

### 1.1 Literature review

As it is explained in the introduction, I use the exact definition of the eurobonds of Jakob von Weizsäcker and Jacques Delpla (2010) along the paper. Their proposal suggests that sovereign debt in Euro-area countries should be split into two parts. The first part, the senior ‘Blue’ tranche of up to 60 percent of GDP, would be pooled among participating countries and jointly and severally guaranteed. The second part, the junior ‘Red’ tranche, would keep debt in excess of 60 percent of GDP as a purely national responsibility, leading to a differentiation in interest rates. The disciplining effect of the higher marginal cost of borrowing is the most important distinction between The Blue Bond proposal and the first generation of proposals to pool the debt of EU countries in a eurobond, Bonnevay (2010), Leterme (2010) or the concerns voiced by Issing (2009).

model with credit frictions. Their paper develops a theory in order to explain why domestic residents buy their own sovereign debt in turbulent times and why foreigners sell their holdings of bonds, among other things. They argue that the cause is the credit discrimination, the expected return of sovereign bonds is higher for residents than foreigners because the probability of repayment in a scenario of default is higher for the former. In addition, financial frictions provoke that these purchases displace productive investment. The present article, assumes that residents and foreigners have the same expected return because the purpose of the paper is completely different and must take into account the rest of Euro-area members, besides GIIPS, where these domestic (foreign) purchases (sales) did not take place, or were very low comparatively. But, this paper also assumes the presence of financial frictions because firms can only receive capital from domestic agents, which are credit constrained because they can not borrow from abroad.

I introduce country-specific spreads in a very similar way as Schmitt-Grohé and Uribe (2003). They proposed a small open economy model with a debt-elastic interest-rate premium, based on the empirical evidence of Akiotby and Stratmann (2008), with a technical purpose, to induce stationarity of the equilibrium dynamics. Under their formulation the deterministic steady state of the small open economy model is independent of the initial net foreign asset position of the economy, concretely external debt and capital, which allows to apply approximation techniques to get a solution of the model. I use a variation of this kind of spread in order to represent the eurobonds policy, a spread that is zero if a country government debt does not go beyond the 60% of GDP but generates positive and increasing spreads when government debt is larger than this threshold. I introduce it because reflects the paradigm that eurobonds want to impose, shifting from a situation where spread drivers are uncertain to more objective causes based on perceived default risk where debt level would play a center role. That is, I argue that this formulation serves as a good theoretical representation of an increasing borrowing rates at the margin.

2 Model

The theoretical analysis is based on four small open economies (SOE) corresponding to the following classification based on the debt-to-GDP ratio at the beginning of the sovereign debt crisis and on the two patterns of bond yields described in the introduction (Figure 1):

<table>
<thead>
<tr>
<th>Bonds yields (2009-2012)</th>
<th>Debt-to-GDP &lt; 60% [low]</th>
<th>Debt-to-GDP &gt; 60% [high]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sovereign crisis, GIIPS. (high)</td>
<td>I: Spain, Ireland</td>
<td>II: Italy, Portugal, Greece</td>
</tr>
<tr>
<td>No sovereign crisis (low)</td>
<td>III: Finland, Luxembourg</td>
<td>IV: Germany, France, Austria, Netherlands</td>
</tr>
</tbody>
</table>

These two dimensions, initial government debt level and actual bond yields, have been chosen because are crucial in the posterior policy evaluation.
This division is done because these four groups of countries are a good simplification of the whole Euro area, in the rest of the paper letter “j” will refer to each I, II, III and IV groups or economies. Hereafter, “GIIPS” will refer to groups I and II together, and “the rest of countries” to groups III and IV.

In every Small Open Economy $j$ there are the following agents: representative agent (resident investors or households), risk-neutral international investors, firms and government. Since households are the last owner of banks in the real world, when a household invests in a firm in the model it is equivalent to a bank giving credit to a non-financial firm or a self-employed.

I assume that in each period the government have the ability to borrow in a risk-free internationally traded real bond which can be bought by resident and foreign investors. The equilibrium condition of international financial market, or rest of the world, determines a law of motion of interest rate (bond yields) which is exogenous to every economy $j$.

In every SOE, I consider two different cases: baseline model and eurobonds scheme. Therefore, these are the descriptions of both cases for all economies:

### 2.1 Baseline model

The behaviours of the agents are the following:

The households choose consumption, $c_{jt}$, labor, $l_{jt}$, investment, $i_{jt}$, and bond holdings, $b_{jt}$, in order to maximize the discounted utility, $\sum_{t=0}^{\infty} \beta_j^t U(c_j^t, g_j^t)$, where $U(c_j^t, g_j^t) = \log(c_j^t) + \log(g_j^t)$, subject to the budget constraint,

$$
(1 + \tau_{t}^j)c_{jt}^t + i_{jt}^t + b_{j,t+1}^t = (1 - \tau_{t}^j)k_{j,t}^t + (1 - \tau_{t}^{w,j})w_{jt}^t l_{jt}^t + b_{jt}^t(1 + r_{jt}^t),
$$

the law of motion of capital,

$$
i_{jt}^t = k_{j,t+1}^t - (1 - \delta^j)k_{jt}^t,
$$

the appropriate non-negativity constraints, $c_{jt}^t, k_{j,t+1}^t \geq 0$, and the market clearing condition of labor,

$$
l_{jt}^t \in [0, 1],
$$

where $b_{j,t+1}^t$ are the government non-contingent real bonds bought at the end of period $t$ yielding a interest rate, $r_{t+1}$, in period $t + 1$. $i_{jt}^t$ denotes investment in new units of physical capital $(k_{jt}^t)$ in period $t$, $c_{jt}^t$ the consumption, $l_{jt}^t$ the labor choice, $\tau_{jt}^j$ the consumption tax rate, $\tau_{jt}^k$ the capital tax rate and $\tau_{jt}^{w,j}$ the labour tax rate.

The agent is also subject to a No-Ponzi-game condition,
\[
\lim_{T \to \infty} \frac{b^T_{t+1}}{\prod_{s=0}^{T}(1 + r^s_t)} \geq 0. \tag{3}
\]

The government balances its budget by selling bonds,
\[
g^T_t - d^T_t = \tau^k_t r^k_t k^T_t + \tau^c_t c^T_t + \tau^w_t w^T_t l^T_t - d^T_t (1 + r^T_t), \tag{4}
\]
where \(d^T_t\) and \(d^T_{t+1}\) denotes the debt position assumed at the beginning and end of period \(t\), respectively. The primary government expenditure \((g^T_t)\) is consumed or produces a public good that enters in the agent’s utility function separably. The fiscal policy instruments \(\{g^T_t, \tau^k_t, \tau^w_t, \tau^c_t\}_{t=0}^{\infty}\) are exogenous.

The government also satisfies the No-Ponzi-game condition,
\[
\lim_{T \to \infty} \frac{d^T_{t+1}}{\prod_{s=0}^{T}(1 + r^s_t)} \leq 0. \tag{5}
\]

The government debt is bought by residents and foreigners,
\[
d^T_t = b^T_t + b^{f,j}_t, \tag{6}
\]
where \(b^{f,j}_t\) is the quantity of government bonds bought by international investors.

Firms take the prices as given, so we have perfect competition. The firms choose capital, \(k^j_t\), and labor, \(l^j_t\), to maximize profits,
\[
\Pi^j_t(k^j_t, l^j_t) = y^j_t - r^k_t k^j_t - w^j_t l^j_t, \tag{7}
\]
subject to a Cobb-Douglas technology,
\[
y^j_t = \left(k^j_t\right)\alpha^j \left(l^j_t\right)^{1-\alpha^j},
\]
where \(y^j_t\) is the output produced by the factors: \(k^j_t\), initial physical capital in period \(t\), and \(l^j_t\), quantity of labor at \(t\). In the side of costs, \(r^k_t\) is the return of physical capital and \(w^j_t\) is the real wage.

The feasibility constraint is the following equation:
\[
c^j_t + g^j_t + k^j_{t+1} - (1 - \delta^j)k^j_t - b^{f,j}_t = y^j_t - b^{f,j}_t (1 + r^j_t). \tag{8}
\]
Computation of equilibrium

**Definition.** Given \( k^0, d^0, b^0, b^{j,0} \), and \( \{ g^j, \tau^{kj, j}, \tau^{wj, j}, r^j \}_{t=0}^{\infty} \), the competitive equilibrium is a sequence of quantities, \( \{ c^j_t, k^j_{t+1}, l^j_t, d^j_{t+1}, b^j_{t+1}, b^{j+1} \}_{t=0}^{\infty} \), and prices, \( \{ r^k_j, w^j \}_{t=0}^{\infty} \), such that

- Given \( \{ r^k_j, w^j \}_{t=0}^{\infty} \), households, choosing \( \{ c^j_t, k^j_{t+1}, b^j_{t+1} \}_{t=0}^{\infty} \), maximize their utility subject to the budget constraint (1) and No-Ponzi-game condition (3).
- Given \( \{ r^k_j, w^j \}_{t=0}^{\infty} \), firms, choosing \( \{ k^j_t, l^j_t \}_{t=0}^{\infty} \), maximize profits (7).
- Public sovereign debt, \( \{ d^j_{t+1} \}_{t=0}^{\infty} \), satisfies the government budget constraint (4) and the No-Ponzi-game condition (5).

\( \{ c^j_t, k^j_{t+1}, l^j_t, b^j_{t+1} \}_{t=0}^{\infty} \) satisfy the market clearing conditions of goods (8), \( \{ l^j_t \}_{t=0}^{\infty} \) of labor (2) and \( \{ d^j_{t+1}, b^{j+1} \}_{t=0}^{\infty} \) of assets (6).

**Equilibrium conditions**

The first-order conditions (FOC) of households’ maximization problem are the marginal utility is equal to Lagrange multiplier times consumption taxes,

\[ \beta^j_t u_c(c^j_t, g^j_t) = \lambda^j_t (1 + \tau^{wj, j}_t), \]  

the Euler equation of bonds,

\[ \frac{u_c(c^j_t, g^j_t)}{(1 + \tau^{wj, j}_t)} = \beta^j_t \left(1 + r^k_{t+1}\right) \frac{u_v(c^j_{t+1}, g^j_{t+1})}{(1 + \tau^{wj, j}_{t+1})}, \]  

the Euler equation of capital,

\[ \frac{u_v(c^j_t, g^j_t)}{(1 + \tau^{wj, j}_t)} = \beta^j_t \left(1 + (1 - \tau^{k, j}_t)r^k_t - \delta^j_t\right) \frac{u_v(c^j_{t+1}, g^j_{t+1})}{(1 + \tau^{wj, j}_{t+1})}, \]

and the Transversality conditions,

\[ \lim_{T \to \infty} \frac{b^{j+1}}{\prod_{s=1}^{T}(1 + r^j_s)} = 0 \]

12
\[ \lim_{T \to \infty} \lambda^j_T k^j_{T+1} = 0. \]  
(13)

Since resident investors have no disutility from labor, the optimal labor choice is \( l^j_T = 1 \).

From the firms’ maximization problem we find that the competitive prices of factors are equal to the marginal productivity,

\[ r^k_j = \alpha^j y^j_l \]  
(14)

\[ w^j_l = (1 - \alpha^j) \left( \frac{1}{k^j_l} \right)^{\alpha^j}. \]  
(15)

Combining (10) and (11) we get the no-arbitrage condition,

\[ (1 + r^j_{t+1}) = \left( 1 + (1 - r^k_j) r^k_{t+1} - \delta^j \right). \]  
(16)

Using the perfect-competition price of capital (14), the capital next period is:

\[ k^j_{t+1} = \left( \frac{\alpha^j (1 - r^k_{t+1})}{r^j_{t+1} + \delta^j} \right)^{\frac{1}{\gamma - \delta^j}}. \]  
(17)

Using the household budget constraint (1) and prices of productive factors in equilibrium (14)-(15), I solve for \( c^j_t, r^k_j \) and \( w^j_l \), respectively, in the Euler equation of bonds (10) which can be rewritten as

\[ \frac{(1 - \tau^k_j) \alpha^j k^j_{t+1} (1 - \tau^w_j)(1 - \alpha^j) k^j_{t+1} + h^j_{t+1} (1 + r^j_{t+1})(1 - \delta^j) k^j_{t+1} - k^j_{t+2} - b^j_{t+1}}{(1 - \tau^k_j)\alpha^j k^j_{t+1} + (1 - \tau^w_j)(1 - \alpha^j) k^j_{t+1} + h^j_{t+1} (1 + r^j_{t+1}) + (1 - \delta^j) k^j_{t+1} - k^j_{t+2} - b^j_{t+2}} = \beta^j \left( 1 + r^j_{t+1} \right) \]  
(18)

In addition, from the government budget constraint (4) substituting forward, and using the transversality condition,

\[ \lim_{T \to \infty} \frac{d^j_{T+1}}{\prod_{t=0}^{T} (1 + r^j_t)} = 0, \]  
(19)

the government’s intertemporal budget constraint is

\[ d^j_0 = \sum_{t=0}^{\infty} \frac{1}{\prod_{s=0}^{t} (1 + r^j_s)} \left[ \tau^j_r c^j_t + \tau^k_j r^k_j k^j_t + \tau^w_j w^j_l - g^j_t \right]. \]  
(20)
Therefore, the equilibrium conditions can be summarized by (4),(6), (8), (17)-(18), and (20). I solve the model following a FOC approach and numerical methods. The system (4),(6), (8), and (17)-(18) involves an infinite number of equations and unknowns. To make the computation of the equilibrium feasible, I assume that the economy converges to steady state at some date \( t = T \), where \( T \) is 50 periods ahead. From the Euler equations of bonds and capital at the steady state,

\[
\frac{1}{\beta^j} = 1 + r_{ss}^j
\]

we see that steady state level of capital is a function of the parameters of the model, hence it is unique. On the other hand, the steady state levels of bond holdings and consumption depend on initial conditions, such as the initial level of bond holdings or debt itself as showed Schmitt-Grohe and Uribe (2003).

Hence, given the initial conditions, \( d_t^j, b_t^j, b_0^{t,j}, \) and \( k_0^j \), where \( 0 = T_0 = 2009 \), and the terminal condition \( T \), I use the Newton-Raphson algorithm to solve the square system of equations (4),(6), (8), and (17)-(18) for all \( t \in (2009, T) \) where, \( \{c_t^j, k_t^{j+1}, d_{t+1}^j, b_{t+1}^j, b_{t+1,0}^{t,j}\}_{2009}^T \) are the unknowns. In addition, the government’s intertemporal budget constraint (20) must be satisfied, thus one of the government instruments is freely chosen to fulfill this condition, I specify which instrument when explaining the calibration, estimation, and data.

### 2.2 Eurobonds scheme

Now, the policy is implemented to the baseline model at initial period \((T_0)\). Each of our economies starts with the same initial conditions as the baseline case, \( k_{T_0}^j, d_{T_0}^j, b_{T_0}^j, \) and \( b_{T_0}^{t,j} \), but now the government, in order to balance budget, is able to issue bonds \((d_{t+1}^j \text{ for } t = T_0, ..., T)\) of two types in the following periods: eurobonds and national bonds. The eurobonds yields are represented by \( r_{e,t}^j \) and the national bonds by \( r_{n,t}^{t,j} \). The eurobonds have a different financial cost than national bonds but can be issued only until a debt-to-GDP ratio of 60\%\(^4\).

Thus, I define the government budget constraint as

\[
g_t^j - d_{t+1}^j = \tau_t^{kj} r_t^{kj} k_t^j + \tau_t^{cj} c_t^j + \tau_t^{wj} w_t^{t,j} - d_t^j (1 + r_t^j) \text{ for } t = T_0
\]

\(^4\)Threshold of financial sustainability according to the Maastricht treaty.
\[ g_t^j - d_{t+1}^j = r_t^{kj} k_t^j + r_t^{cj} c_t^j + r_t^{wj} w_t^j - \min \left\{ d_t^j, \phi y_t^j \right\} (1 + r_{s,t}^e) - \max \left\{ 0, d_t^j - \phi y_t^j \right\} (1 + r_{s,t}^m) \text{ for } t > T_0, \]

where \( \phi = 60\% \), and \( s = \{l, m, u\} \) are the several scenarios that are considered for the eurobonds yields. The debt position in eurobonds of government is \( \min \left\{ d_t^j, \phi y_t^j \right\} \) and \( \max \left\{ 0, d_t^j - \phi y_t^j \right\} \) is the quantity of national bonds, which corresponds to the debt issued beyond the 60% of GDP.

With respect to the eurobonds yields, I consider three scenarios for \( r_{s,t}^e \) because there has been a large debate on this part of the policy. I consider the lower and upper bound that have been mentioned more, and also the middle point of these two extremes. This rate should be understood as an indicator of the quality of the Euro-area institutions, as it is mentioned in the introduction. I define the quality of institutions as an addition of two elements: degree of integration and of the automaticity of fiscal mechanisms. In this context of public debt analysis, degree of integration refers to pool or not debts under a joint and several guarantee. Degree of automaticity depends on having or not automatic mechanisms, for this concrete policy we refer to have or not an orderly planned mechanism of default (eurobonds as senior debt and national bonds as junior). In this line, \( r_{l,t}^e \) represents the optimal quality of institutions, \( r_{m,t}^e \) and \( r_{u,t}^e \) the intermediate and the worst quality, respectively.

The lower bound, \( r_{l,t}^e \), considers that the eurobonds is an extremely safe asset equivalent to the US treasury bonds or benchmark German bonds due to two motives: liquidity and safety. The first because pooling debt under joint and several guarantee would mean that eurobonds would be a very high liquid asset for world investors, and the second, because the 60% of GDP as senior debt is an easily sustainable level that would make that the default risk was very low. On the contrary, the upper bound (\( r_{u,t}^e \)) consists in a weighted average\(^5\) of bonds yields of all the Euro-area members. In this case, the policy is implemented without the optimal institutional reforms that would force a qualitative change from the uncertainty of the status quo. And, \( r_{m,t}^e \) is the middle point of these opposite boundaries.

The interest rate of national debt is

\[ r_{n,t}^{e,j} = r_{s,t}^e + \psi_t^j (e^{\frac{\phi}{2}} - \phi - 1), \]

where \( \psi_t^j \geq 0 \). \( r_{n,t}^{e,j} \) is simply the interest rate of the eurobonds in case plus an interest-rate premium. The second term of the sum reflects that beyond the 60% of GDP, the country issue national debt, what the authors called “red tranche”. This interest-rate premium is taken from Schmitt-Grohé and Uribe (2003) although with public debt and for other purpose, I argue that having an interest-rate premium, that is an increasing function with respect to the debt-to-GDP ratio, is a good theoretical representation of having an increasing borrowing cost at the margin, which is a key element in the

\(^5\)Weighing by pooled debt, debt below or equal to 60% of GDP.
definition\(^6\) of the eurobonds. In the words of the authors, “In case of a partial default, the red tranche will be hit first and the blue tranche will only be affected by that part of the default (if any) that is not absorbed by the junior tranche. In other words, any government funds used to service and repay government debt will always first be used to satisfy the claims of the blue bond holders...From an investor’s perspective, the prospect of a less disruptive default on the junior tranche increases the risk of default, thereby calling for an additional risk premium”. As I said in the introduction, the perceived default risk of national bonds is present as part of an endogenous interest-rate premium, which is dependent on debt-to-GDP ratio. This formulation implies that when a country follows an unsustainable fiscal path, with a debt moving away from the eurobonds threshold, the default risk increases and so the borrowing rate at the margin.

In this set-up, the households’ budget constraint is

\[
(1 + \tau_i^{c,j})c_i^j + g_i^j + b^{c,j}_{i+1} + b^{n,j}_{i+1} = (1 - \tau_i^{k,j})k_i^j + (1 - \tau_i^{u,j})u_i^j t_i^j + (1 + r_i^j)b_i^j \quad \text{for } t = T_0 \tag{25}
\]

\[
(1 + \tau_i^{c,j})c_i^j + g_i^j + b^{c,j}_{i+1} + b^{n,j}_{i+1} = (1 - \tau_i^{k,j})k_i^j + (1 - \tau_i^{u,j})u_i^j t_i^j + b^{c,j}_i(1 + r_i^{c,j}) + b^{n,j}_i(1 + r_i^{n,j}) \quad \text{for } t > T_0, \tag{26}
\]

where \(b^{c,j}_i\) and \(b^{n,j}_i\) are the quantity of eurobonds and national bonds bought by domestic investors, respectively.

The market clearing conditions are,

- **Feasibility constraint:**
  \[
  c_i^j + g_i^j + i_i^j - (b_i^{c,j} + b_i^{n,j}) = y_i^j - b_i^{f,j} (1 + r_i^j) \quad \text{for } t = T_0 \tag{27}
  \]
  \[
  c_i^j + g_i^j + i_i^j - (b_i^{c,j} + b_i^{n,j}) = y_i^j - b_i^{f,j} (1 + r_i^{c,j}) - b_i^{f,m,j} (1 + r_i^{n,j}) \quad \text{for } t > T_0, \tag{28}
  \]
  where \(b_i^{c,j}\) and \(b_i^{n,j}\) are the quantity of eurobonds and national bonds bought by foreign investors, respectively.

- **Bonds:**
  - Eurobonds:
    \[
    \min \{d_i^j, \phi y_i^j\} = b_i^{c,j} + b_i^{f,c,j} \quad \text{for } t > T_0 \tag{29}
    \]
  - National bonds:
    \[
    \max \{0, d_i^j - \phi y_i^j\} = b_i^{n,j} + b_i^{f,n,j} \quad \text{for } t > T_0. \tag{30}
    \]

\(^6\)See the definition of the policy in the Introduction, page 7.
Equilibrium conditions

Now, we have some different equilibrium conditions with respect to the baseline model. The rest of equilibrium conditions are equivalent to the baseline model and are also satisfied: optimal labor choice, equilibrium prices of capital (14) and labor (15), Euler equation of capital (11), and Transversality conditions ((12)-(13) and (19)), among others.

From the point of view of an investor, the national bonds dominate the eurobonds because they offer a higher return without uncertainty, therefore the domestic investors always prefer to buy national bonds than eurobonds, but they can only do it when the debt-to-GDP ratio is higher than $\phi$. Since the domestic agents buy bonds at the highest possible interest rate, the Euler equation of bonds takes the following form

$$
\frac{u_s(c^j_t, g^j_t)}{(1 + \tau^j_t)} = \beta^s (1 + r^j_{s,t+1}) \frac{u_s(c^j_{t+1}, g^j_{t+1})}{(1 + \tau^j_{t+1})} \quad \text{for all } s \text{ and } t, \tag{31}
$$

where $\beta^s$ is the subjective discount rate and $r^j_{s,t+1}$ is the interest rate at the margin for government borrowing,

$$
r^j_{s,t+1} = r^s_{s,t+1} + \max \left\{ 0, \psi(t) \left( e^{r^j_{s,t+1} - \phi} - 1 \right) \right\}. \tag{32}
$$

The interest rate at the margin is equal to eurobonds yields ($r^s_{s,t}$) when $d^j_t/y^j_t \leq \phi$ and, it is equal to national bonds ($r^n_{s,t}$) when the government has the necessity to issue national debt ($d^j_t/y^j_t > \phi$), the function $\max$ represents this fact.

I define $b^j_{t+1}$ as the quantity of bonds bought by the residents which is equal to

$$
b^j_{t+1} = \begin{cases} b^c_{t+1} & \text{if } \frac{d^j_{t+1}}{y^j_{t+1}} \leq \phi \\ b^n_{t+1} & \text{if } \frac{d^j_{t+1}}{y^j_{t+1}} > \phi \end{cases}. \tag{33}
$$

It reflects that residents buy eurobonds or national bonds depending on the marginal interest rate.

Again, from the Euler equation of capital (11) together with the Euler of bonds (31), we can write the no-arbitrage condition as

$$
1 + r^s_{s,t+1} + \max \left\{ 0, \psi(t) \left( e^{r^j_{s,t+1} - \phi} - 1 \right) \right\} = \left( 1 + \tau^k_{t+1} \right) r^k_{t+1} - \delta^j. \tag{34}
$$
This equation shows that if the government has the necessity to issue debt beyond the 60% of GDP, the domestic investor will buy national bonds instead of eurobonds, therefore the return of investment must be such that this agent is indifferent between investing and purchasing national bonds. On the other hand, when government debt is below the 60% of GDP, the return of investment is such that this agent is indifferent between investing and purchasing eurobonds.

Solving for \( r^j_{t+1} \) in the no-arbitrage condition (34), by using (14), we get that capital next period is

\[
k^j_{t+1} = \left( \frac{\alpha^j(1 - \tau^k_{t+1})}{r^e_{s,t+1} + \max \left\{ 0, \psi^j_t e^{\delta^j_{t+1}} - \phi \right\}} \right)^{\frac{1}{1-\alpha^j}} \quad \text{for all } t. \tag{35}
\]

Using the household budget constraints ((25)-(26)), the distinction between the kind of bonds purchased by domestic investors (33) and the prices of productive factors ((14)-(15)), I solve for \( c^j_t \), \( r^j_t \) and \( w^j_t \), respectively, in the Euler equation of bonds (31) which can be rewritten, for \( t = T_0 \), as

\[
(1 - \tau^k_{t+1})^{\alpha^j} k^{a^j}_{t+1} + (1 - \tau^w_{t+1})^{(1 - \alpha^j)} k^{a^j}_{t+1} + b^j_{t+1}(1 + r^j_{s,t+1}) + (1 - \delta^j) k^j_{t+1} - k^j_{t+2} - b^j_{t+1} = \beta^e \left( 1 + r^j_{s,t+1} \right)
\]

and, for \( t > T_0 \), as

\[
(1 - \tau^k_{t+1})^{\alpha^j} k^{a^j}_{t+1} + (1 - \tau^w_{t+1})^{(1 - \alpha^j)} k^{a^j}_{t+1} + b^j_{t+1}(1 + r^j_{s,t+1}) + (1 - \delta^j) k^j_{t+1} - k^j_{t+2} - b^j_{t+1} = \beta^e \left( 1 + r^j_{s,t+1} \right)
\]

As in the baseline model, the government’s intertemporal budget constraint is

\[
d^j_0 = \frac{1}{1 + \tau^k_{0}} \left[ \tau^c_j c^j_0 + \tau^k_j k^j_0 + \tau^w_j w^j_0 - g^j_0 \right] +
\]

\[
\sum_{t=1}^{\infty} \frac{1}{\prod_{n=1}^{T_0} (1 + r^e_{s,t})} \left[ \tau^c_j c^j_t + \tau^k_j k^j_t + \tau^w_j w^j_t - g^j_t + \phi g^j_t \max \left\{ 0, \psi^j_t e^{\delta^j_{t+1}} - \phi \right\} \right]. \tag{38}
\]

This equation is computed in the same way as in the baseline model, substituting forward the government budget constraint ((23)-(24)) and satisfying the transversality condition (19).
Thus, the equilibrium in this case can be summarized by (23)-(24), (27)-(30), and (35)-(38).

Given the same initial conditions and the steady state condition at $T$, (23)-(24), (27)-(30), and (35)-(37) for $t \in (2009, T)$, where \( \{c_t^j, b_t^{j+1}, d_{t+1}^j, b_t^j, c_t^{j+1}, b_{t+1}^j, b_{t+1}^{j+1}\}_t^{2009} \) are the unknowns, is a square system that it is solved with the same method that it is used to get the baseline solution.

The steady state in this case has the same properties as the baseline model, but now the Euler equations of bonds and capital are

$$
\frac{1}{\beta_s^e} = 1 + r_s^e \max \left\{ 0, \psi^j_s \left( e^{\sigma^j_s} - \phi \right) - 1 \right\} 
$$

(39)

$$
\frac{1}{\beta_s^e} = 1 + \left( 1 - \tau_s^{kj} \right) \alpha k_s^{kj} - \delta^j.
$$

(40)

The Euler equation of bonds (39) at steady state must be also satisfied for the eurobonds case

$$
\frac{1}{\beta_s^e} = 1 + r_s^e \phi.
$$

(41)

Then, the debt position at steady state, $d_{ss}^j$, is lower or equal to the 60% of GDP, $d_{ss}^j \leq \phi y_{ss}^j$, in order to satisfy the Euler equation of bonds (39) in both cases: eurobonds and national bonds.

Again, one of the government instruments is freely chosen to fulfill the condition (38), I specify which instrument in the next subsection.

Calibration, estimation, and data

In this subsection, I explain how all the parameter values are computed, what are the values of data for the exogenous processes, and how effective tax rates are estimated. I use data of Spain, Italy, Finland and France in order to calibrate, estimate, and give values to parameters, effective tax rates, and exogenous processes in the small open economies I, II, III and IV, respectively. All the data used is from Eurostat for the period 2009-2012: government bond yields, National Accounts (NA) and government finance statistics. The NA and government finance statistics are expressed in constant 2005 prices.

Effective tax rates and exogenous processes

I followed Mendoza, Razin and Tesar (1994) in order to obtain estimates of the sequences of effective average tax rates \( \{\tau_{kj}^j, \tau_{kj}^w, \tau_{kj}^m\}_t^{2012} \). Mendoza (1994) use the government revenue statistics and NA from OECD (SNA93), here equivalent data from Eurostat (ESA95) is used. Figure 6 shows the effective tax rates and primary government expenditure of all groups.
The exogenous government bond yields $r_j^t$ of the baseline model are taken from the data (Figure 1) but since $t$ represents years in the model, the average annual rate is computed. The lower, middle and upper bounds for the eurobonds yields are: $r_{el}^t$ is the benchmark German bond, $r_{eu}^t$ is the weighted average by pooling debt below or equal to 60% of GDP of all Euro-area countries, and $r_{m}^t$ is the middle rate of $r_{el}^t$ and $r_{eu}^t$. Figure 7 shows the average annual rates of all groups ($r_j^t$) for the baseline model and the three different interest rates of eurobonds.
After our period of interest, the sequence of fiscal policies, \( \{ g^j_t, r^j_t, \gamma^j_t \}_{t=2013} \), and the sequence of exogenous interest rates, \( \{ r^j_t, r^*_t \}_{t=2013} \), are constant in their values of 2012. As mentioned above, a government instrument must be chosen to satisfy the government’s intertemporal budget constraint (20) and (38), in this case \( \{ \gamma^j_t \}_{2013} \).

**Parameter values**

The capital and labor income shares are calculated in the standard way with data of 2009 from the NA, particularly following Gollin (2002). The depreciation rate and initial capital-output ratio are calibrated so that the equilibrium of the model in 2009 replicates the data in the NA. Following Kehoe and Ruhl (2009), multiplying the no-arbitrage condition (16) at 2009 by \( k^j_{2000} \) in both sides,

\[
k^j_{2009}r^j_{2009} = (1 - \gamma^j_{2009})y^j_{2009}^{-1}k^j_{2009} - \delta k^j_{2009},
\]

and using the equilibrium price of capital (14), where at 2009 is \( \omega^j_{2009}y^j_{2009} = \gamma^j_{2009}k^j_{2009} \), then, we get that initial capital and depreciation rate are

\[
k^j_0 = \frac{k^j_{2000}}{k^j_{2000}} = \frac{(1-\gamma^j_{2000})\omega^j_{2000}GDP^j_{2000} - CFC^j_{2000}}{\gamma^j_{2000}},
\]

\[
\delta^j = \frac{CFC^j_{2000}}{k^j_{2000}},
\]

where \( GDP^j_{2000} \) is the gross domestic product and \( CFC^j_{2000} \) is the consumption of fixed capital in NA.

The discount rate is computed from Euler equation of bonds (21) at steady state \( \beta^j = 1/(1+r^*_ss) \).
The adjustment parameter of the debt-elastic interest-rate premium, $\psi_j^s$, is estimated, for each economy $j$, using real data of debt levels and spreads with respect to the eurobonds yields in case ($r_{s,t}^e$). A simple linear regression model is computed,

$$(r_t^d - r_{s,t}^e) = \psi_j^s X_t^j \quad X_t^j = \max \left\{ 0, (e^{\psi_j^s} - \phi - 1) \right\},$$

using the Newey-West Standard Errors to study the significance of the coefficient because there is a problem of autocorrelation. We take the largest possible sample since the introduction of the Euro, monthly values from 2002 to 2012 (132 observations). The following table summarizes all the results:

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Group</th>
<th>I: Spain</th>
<th>II: Italy</th>
<th>III: Finland</th>
<th>IV: France</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\psi_j^s$</td>
<td></td>
<td>0.2757*** (0.0301)</td>
<td>0.0172** (0.0066)</td>
<td>-</td>
<td>0.0235*** (0.0030)</td>
</tr>
<tr>
<td>$\psi_m^s$</td>
<td></td>
<td>0.2033*** (0.0179)</td>
<td>0.0124** (0.0020)</td>
<td>-</td>
<td>-0.0267*** (0.0031)</td>
</tr>
<tr>
<td>$\psi_u^s$</td>
<td></td>
<td>0.1310*** (0.0076)</td>
<td>0.0076** (0.0015)</td>
<td>-</td>
<td>-0.0016 (0.0013)</td>
</tr>
</tbody>
</table>

Note: Newey-West Standard Errors are reported in parentheses. *, **, *** indicate significance at the 10%, 5%, and 1% level, respectively.

It is worth nothing that there is probably another issue relative to the omission of relevant variables such as primary deficit or level of confidence of investors, among others. This would make that our estimates present an upward biased, this estimation would assign more spread variability to the debt level than an unbiased estimator but for our purpose it is not a problem from a point of view of prudence. $\hat{\psi}_{u}^{IV}$ and $\hat{\psi}_{m}^{IV}$ are negative so that I set $\psi_{u}^{IV}, \psi_{m}^{IV} = 0$ in order to be again conservative, implying that no country when issue national debt could get a lower borrowing rate than the eurobonds. For the case of group III, we can not estimate these parameters because the variable $X_{t}^{III}$ is always zero. Therefore, we assume that $\psi_{u}^{III} = \hat{\psi}_{t}^{I}$, the largest estimated value, and that, for the upper and the middle scenarios this group has $\psi_{m}^{III}, \psi_{u}^{III} = 0$ for the same reason as group IV.

The initial proportion of sovereign debt holdings between residents and the non-residents has been taken from the “Bruegel Dataset of Sovereign debt holdings” by Merler (2014).

This is the summary of the parameter values:
The interest rate of eurobonds must be consistent with the Euler equation of bonds at steady state (41), thus the subjective discount rate in the several scenarios of eurobonds is $\beta^e = 1/(1+r_{t,su}^e)$. So, we get $\beta^e_u = 0.9637$, $\beta^e_m = 0.9744$, and $\beta^e_l = 0.9853$.

### 3 Simulations and results

All the analysis consists in studying the competitive equilibrium outcomes with and without eurobonds during the sovereign debt crisis (2009-2012). Before implementing the eurobonds, I briefly study the baseline model. Afterwards, with the eurobonds, the quantitative analysis consists in the policy evaluation.

The results plotted in all figures are the baseline model (red line) and the three scenarios of the eurobonds yields: lower (blue dashed line), middle (blue pointed line), and upper bound (blue line). In addition, the data appears represented by a black line in some figures.

#### Baseline model

First of all, I evaluate the replication capability of the baseline model. Concretely, I look for a theory that is able to do a reasonable replication of the government debt and the lending behavior of resident investors between productive investment and bond holdings during the sovereign debt crisis (2009-2012).

Figure 8 presents the debt-to-GDP paths of the data and baseline model, as we can see in this figure the baseline debts are also soaring as in the data. Groups I and IV end up with a final debt-to-GDP ratio lower than the data. For group II and III, the model generates a higher debt ratio in the last two periods.

<table>
<thead>
<tr>
<th>Group</th>
<th>Parameter</th>
<th>$\alpha^j$</th>
<th>$\delta^j$</th>
<th>$k^j_o/y^j_o$</th>
<th>$\beta^j$</th>
<th>$\psi^j_l$</th>
<th>$\psi^j_m$</th>
<th>$\psi^j_u$</th>
<th>$d^j_o/y^j_o$</th>
<th>$b^j_o/d^j_o$</th>
</tr>
</thead>
<tbody>
<tr>
<td>I:Spain</td>
<td></td>
<td>0.3578</td>
<td>0.0584</td>
<td>2.6704</td>
<td>0.9448</td>
<td>0.2757</td>
<td>0.2033</td>
<td>0.1310</td>
<td>40.2%</td>
<td>54.1%</td>
</tr>
<tr>
<td>II:Italy</td>
<td></td>
<td>0.4342</td>
<td>0.0446</td>
<td>3.8223</td>
<td>0.9479</td>
<td>0.0172</td>
<td>0.0124</td>
<td>0.0076</td>
<td>106%</td>
<td>50.1%</td>
</tr>
<tr>
<td>III:Finland</td>
<td></td>
<td>0.3726</td>
<td>0.0536</td>
<td>3.1609</td>
<td>0.9815</td>
<td>0.2757</td>
<td>0</td>
<td>0</td>
<td>33.9%</td>
<td>15.4%</td>
</tr>
<tr>
<td>IV:France</td>
<td></td>
<td>0.3381</td>
<td>0.0576</td>
<td>2.4205</td>
<td>0.9753</td>
<td>0.0235</td>
<td>0</td>
<td>0</td>
<td>68.2%</td>
<td>41%</td>
</tr>
</tbody>
</table>
As it is explained in the introduction, there is evidence (consequences 2 and 3) that public debt was crowding out productive investment in GHIPS during the European sovereign debt crisis. Therefore, the ratio of bond holdings over capital \((b_j^t/k_j^t)\) is computed in order to see what is the evolution of the credit allocation between the government and the private productive sector during this period. In this direction, we collect data from the domestic banks in order to get a proxy, computing the ratio of sovereign debt holdings over the private productive credit\(^7\), since capital is

\(^7\)The data on government debt holdings and private productive credit of domestic banks is taken from the ECB statistics [MFI balance sheets], and are outstanding amounts [stocks]. Private productive credit refers to loans to NFC and households, excluding credit for consumption and lending for house purchase.
equivalent to private productive credit in the model. Figure 9 shows the evolution of these ratios. First of all, we notice that the volatility is much higher in the model than in the data. Furthermore, we observe that this ratio increases a lot in GIIPS for the period 2009-2012 compared to previous years in the data. In fact, there is a clear change in tendency. The results of the model also show that the agents reallocate to sovereign bonds their part of income devoted to savings when interest rate increases, crowding out or displacing productive investment, since firms could not borrow credit or capital from abroad, their level of capital in use falls. Concretely, for group I and II, this ratio increases in 14.7 and 10.9 % points for the period 2009-2012 in the data, respectively. In the model, these changes are 27.1 and 48.4 % points, respectively.

On the other hand, the same empirical evidence (Figure 4) has showed that the credit reallocation did not take place, or much less, in the rest of countries. Figure 9 shows that domestic banks of group III increased the ratio of public over private credit from from 1.8% at 2009 to 4.3% at 2012. But, this ratio falls from 19% at 2009 to 11% at 2012 for the group IV, following the tendency of the previous years. In the model, group III experiences a reduction to the minimum of this ratio (0 %) in an opposite direction to the data, and for the group IV, we can see that there is
a changing behavior of this ratio much more noticeable between the data and model, it increases in 9.2% points during the crisis. Therefore, we can see that, for the rest of countries, the data and the results are not so clearly marked in the same direction compared to the GIIPS.

In this line, Figure 10 presents the proportion of public debt bought by residents, values of the data and baseline model. The model presents much more volatility of this measure compared to the data. It shows how the domestic investors significantly increased their bond holdings of public debt of the country in GIIPS during the period 2009-2012 as the interest rate were increasing, both in the data and in the model. Concretely, in the data, group I goes from the 54.1% of debt in domestic hands at 2009 to 66.9% at 2012, and group II from 50.1% to 60.3%. Although, they reach much higher proportions in the model, for example: residents of group I and II buy all public debt in 2011 and 2012 (100%), increasing their holdings in 55.9 and 49.9% points during the crisis (2009-2012), respectively.

Figure 10. Resident and foreign holdings of government debt

---

8 Government debt holdings of residents are taken from the “Bruegel Dataset of Sovereign debt holdings” by Merler (2014).
On the contrary, as we can see in figure 10, the proportion of public debt in domestic hands in the rest of countries does not experience a change in tendency as it does in GIIPS, it continues falling or stabilizes in the period 2009-2012. In fact, it falls in -6.3 % points for group III and is slightly increasing (4 % points) for group IV during this period. The changes of these proportions are -15.3 and 23.3 % points during the same period in the model.

According to the previous results, despite the high volatility of debt holdings and capital to bond yields, we can conclude that the crowding-out effect is much weaker in the rest of countries compared to the GIIPS.

Hence, I consider that is a good model to do policy evaluation because it generally captures the fiscal performance, summarized by debt-to-GDP ratio, and the investment or credit behavior of residents of the several groups.

Model with Eurobonds

Henceforth, all the analysis is based on using the artificial economies to do policy evaluation for all the groups of countries. All the outcomes are presented for all scenarios, $s = \{l, m, u\}$, of eurobonds yields.

The first result is about the fiscal policy implications of eurobonds. I contrast the key hypothesis of the eurobonds proposal, the authors define a policy that mean a saving to taxpayers and an incentive-driven policy (that enhances fiscal discipline) at the same time. They argue that eurobonds accomplish these two objectives thanks to a resulting lower average borrowing costs, which would afford lower taxes, and an improved fiscal discipline due to increasing borrowing rates at the margin, respectively. Figure 11 illustrates the total annual interest rate payments. We can see that GIIPS win from European solidarity because their total borrowing costs are reduced in all scenarios of eurobonds yields.
But, at the same time, the increasing interest-rate premium with respect to debt level guarantee an increasing borrowing cost at the margin, Figure 12 shows that countries with high debt-to-GDP ratios (group II) have higher borrowing rates at the margin than the actual yields (baseline) at 2010 when $r_{n,t}$. Therefore, they would have the strongest incentive to undertake fiscal adjustments. In addition, it increases the effectiveness of fiscal policy in the sense that when taxes and expenditure become more sustainable the interest rate at the margin reduces. For example, the same figure for group II shows that the marginal interest rates are increasing for $r_{n,t}$ and $r_{m,t}$ at 2011 but then become decreasing at 2012, due to fiscal policy measures on $g_j$ and taxes.

Group I is able to reduce the interest payments (Figure 11) in all cases, and the interest rate at the margin is also below the baseline model because the debt-to-GDP ratio does not surpass the 60% of GDP.
On the other hand, groups III and IV share risk from GIIPS if the eurobonds yields are not so low, Figure 11 and 12 show that if eurobonds yields are in its upper ($r_{u,t}^e$) and middle ($r_{m,t}^e$) yields both interest payments and borrowing rates at the margin are higher than the actual rates. If eurobonds are in the lower bound the contrary occurs, thus the direction of the results on these countries could reverse depending on the valuation of pooled debt.

**Figure 12. Comparison: interest rates at the margin**

In order to close this point on fiscal implications we should study how the eurobonds affect the implementation of fiscal policy. That is, to corroborate if, apart of being an incentive-driven policy, the eurobonds will reduce the fiscal pressure. The Table 1 delivers the resulting consumption tax rates, $\{\tau_t^{G}\}_{2013}^T$, that fulfill the government’s intertemporal budget constraints, (20) and (38), respectively. We can see that eurobonds ease the feasibility of fiscal policy because this consumption...
tax rates are decreasing with the eurobonds yields, even being negative (subsidy) for some scenarios and groups. Taxes are below the baseline case for all groups when eurobonds yields are $r_{f,t}$. On the other hand, the rest of countries need higher taxes than the baseline for $r_{m,t}$ and $r_{u,t}$. Therefore, we see that the policy means a saving to taxpayers of GIIPS for all $s$.

Table 1. Consumption tax

<table>
<thead>
<tr>
<th>Group</th>
<th>Consumption Tax ($r_{f,t}^2$, $t = 2013, ..., T$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$r_{f,t}^2$</td>
</tr>
<tr>
<td>I</td>
<td>0.046</td>
</tr>
<tr>
<td>II</td>
<td>-0.228</td>
</tr>
<tr>
<td>III</td>
<td>0.783</td>
</tr>
<tr>
<td>IV</td>
<td>-0.001</td>
</tr>
<tr>
<td>Average</td>
<td>0.150</td>
</tr>
</tbody>
</table>

The second main result is about fiscal performance. The debt-to-GDP ratios of all the groups are represented in Figure 12, the policy allows GIIPS to reach lower debt-to-GDP ratios with respect the baseline model in all scenarios. The cause is a reduction of the debt burden together with an increase of tax collection due to a growing tax bases of capital and labor income. Table 2 summarizes the differences in percentage points of the final debt-to-GDP for each scenario with respect to the baseline model. Group I gets a minimum (maximum) reduction of -9.9 (-22.9) percentage points. Group II gets a minimum (maximum) reduction -9.1 (-40.6).

Figure 13. Comparison: debt-to-GDP ratios
On the contrary, groups III and IV end up with a debt-to-GDP ratio 12.4 and 9.9 (4.6 and 0.98) % points higher than the baseline model when eurobonds yield the upper bound (middle rate), respectively. They reduce debt level in -3.6 and -4.5 % points for the lower bound eurobonds yields, respectively.

<table>
<thead>
<tr>
<th>Group</th>
<th>Final Debt-to-GDP</th>
<th>Difference (% points)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$r_{lt}$</td>
<td>$r_{mt}$</td>
</tr>
<tr>
<td>I</td>
<td>41.18%</td>
<td>47.79%</td>
</tr>
<tr>
<td>II</td>
<td>98.01%</td>
<td>117.05%</td>
</tr>
<tr>
<td>III</td>
<td>48.45%</td>
<td>56.69%</td>
</tr>
<tr>
<td>IV</td>
<td>71.88%</td>
<td>77.38%</td>
</tr>
<tr>
<td>Total</td>
<td>68.27%</td>
<td>78.78%</td>
</tr>
</tbody>
</table>

The third main result is that the implementation of the policy would help to solve the crowding-out effect on productive investment in GIIPS. Figure 14 shows the effect of the policy on the credit allocation through the bond holdings-capital ratio. We observe that public credit over productive credit is much lower for GIIPS with respect to the baseline case. This ratio is always decreasing for GIIPS with the policy for $r_{lt}$, no crowding-out effect would take place. For group I, the government debt only displaces investment in 2011 for the upper bound scenario, but ending up with a bonds-capital ratio lower at the end than the initial value. For group II, the crowding-out effect would be reduced when eurobonds yields are higher than $r_{m,t}$. This group ends up with a bonds-capital ratio at 2012 higher than the value at the beginning of the crisis but much lower than the baseline model when the middle or upper bound scenario are the case. If we observe this ratio in 2010 in
the upper bound case, for the group II, it is above the baseline due to a higher marginal interest rate in this period compared to the actual rate (Figure 12).

**Figure 14. Comparison: Bond Holdings-Capital ratio**

In the rest of countries, when eurobonds path is the middle or upper bound the bonds-capital ratio is higher than the baseline model. On the contrary, this ratio is always lower when the pooled debt yields the lower bound. For $r_{t,f}^j$, it is decreasing for group III and falls with the exception of 2011 for group IV, but both groups reach a significantly lower bonds-capital ratio at the end (2012).

Consequently, the output is in accordance to the evolution of productive investment. Figure 15 presents the GDP for all groups. It points out that GIIPS reach a higher GDP all periods with the exception of group II in 2010 for $r_{t,f}^j$, as it is pointed out above for credit allocation. Table 3 presents the difference in percentage of the output in the three scenarios with respect to the baseline.
model. Group I experiences a minimum (maximum) increase of 7.24% (18.85%) on average, and Group II, of 3.93% (17.64%).

### Figure 15. Comparison: GDP.

However, the rest of countries reach a higher GDP in all the periods when \( r_{s,t} \) occurs, from Table 3 we see that the GDP grows 2.74% and 2.31% on average above the baseline for group III and IV, respectively. In the medium (upper) bound, the GDP of group III experiences a reduction on average of -3.22% (-8.22%) with respect to the baseline, and -0.5% (-4.78%) for group IV. Therefore, we observe that in the rest of countries the policy is beneficial in terms of output and productive investment depending on the eurobonds yields, \( r_{s,t} \).

The results on the long-run are in line of those of the transitional dynamics (2009-2012), GIIPS are better off for all \( s \) in terms of GPD, but the rest of countries only with the lower bound eurobonds
yields.

Table 3. Difference in percentage rate of output

<table>
<thead>
<tr>
<th>Group</th>
<th>( r_{2010}^e )</th>
<th>( r_{2010}^m )</th>
<th>( r_{2010}^u )</th>
<th>( r_{2011}^e )</th>
<th>( r_{2011}^m )</th>
<th>( r_{2011}^u )</th>
<th>( r_{2012}^e )</th>
<th>( r_{2012}^m )</th>
<th>( r_{2012}^u )</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>9.44</td>
<td>6.60</td>
<td>3.97</td>
<td>17.48</td>
<td>11.46</td>
<td>6.24</td>
<td>20.6</td>
<td>19.62</td>
<td>11.52</td>
</tr>
<tr>
<td>II</td>
<td>2.37</td>
<td>0.89</td>
<td>-0.2</td>
<td>15.83</td>
<td>8.66</td>
<td>3.90</td>
<td>34.7</td>
<td>17.72</td>
<td>8.09</td>
</tr>
<tr>
<td>III</td>
<td>1.95</td>
<td>-1.02</td>
<td>-3.76</td>
<td>2.94</td>
<td>-2.98</td>
<td>-8.08</td>
<td>3.35</td>
<td>-5.66</td>
<td>-12.8</td>
</tr>
<tr>
<td>IV</td>
<td>0.56</td>
<td>-0.22</td>
<td>-2.51</td>
<td>1.45</td>
<td>-0.69</td>
<td>-4.99</td>
<td>4.93</td>
<td>-6.59</td>
<td>-6.83</td>
</tr>
<tr>
<td>Total</td>
<td>3.39</td>
<td>1.39</td>
<td>-0.66</td>
<td>9.99</td>
<td>4.37</td>
<td>-0.49</td>
<td>18.68</td>
<td>7.71</td>
<td>-0.23</td>
</tr>
</tbody>
</table>

The last result is about welfare. Since \( g^j_t \) is the same before and after the policy, the analysis of welfare can be reduced to the analysis of private consumption. In Table 4, we can see that the long-run consumption is always higher when the policy is implemented for groups I and II. On the other hand, consumption is only higher for group III and IV in the scenario where eurobonds yields are the lower bound, \( r_{f,t}^e \). In order to study the exact changes in welfare the sum of discounted utility is computed. Table 4 also summarizes the welfare change in percentage in the three scenarios of eurobonds with respect to the baseline model.

Table 4. Long-term consumption and welfare change

<table>
<thead>
<tr>
<th>Group</th>
<th>( r_{f,t}^e )</th>
<th>( r_{m,t}^e )</th>
<th>( r_{u,t}^e )</th>
<th>( r_{f,t}^m )</th>
<th>( r_{m,t}^m )</th>
<th>( r_{u,t}^m )</th>
<th>( r_{f,t}^u )</th>
<th>( r_{m,t}^u )</th>
<th>( r_{u,t}^u )</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>0.847</td>
<td>0.778</td>
<td>0.728</td>
<td>0.668</td>
<td>0.634</td>
<td>0.608</td>
<td>0.668</td>
<td>0.634</td>
<td>0.608</td>
</tr>
<tr>
<td>II</td>
<td>1.457</td>
<td>1.296</td>
<td>1.210</td>
<td>1.034</td>
<td>1.028</td>
<td>1.024</td>
<td>1.034</td>
<td>1.028</td>
<td>1.024</td>
</tr>
<tr>
<td>III</td>
<td>0.478</td>
<td>0.390</td>
<td>0.327</td>
<td>0.443</td>
<td>0.423</td>
<td>0.411</td>
<td>0.443</td>
<td>0.423</td>
<td>0.411</td>
</tr>
<tr>
<td>IV</td>
<td>0.529</td>
<td>0.473</td>
<td>0.428</td>
<td>0.477</td>
<td>0.464</td>
<td>0.451</td>
<td>0.477</td>
<td>0.464</td>
<td>0.451</td>
</tr>
<tr>
<td>Total</td>
<td>3.313</td>
<td>2.938</td>
<td>2.696</td>
<td>2.624</td>
<td>2.586</td>
<td>2.547</td>
<td>2.624</td>
<td>2.586</td>
<td>2.547</td>
</tr>
</tbody>
</table>

We can see that groups I and II are better off whatever are the eurobonds yields considered in this paper. The welfare in groups III and IV decreases for \( r_{m,t}^e \) (\( r_{u,t}^e \)) in -34.71% (-58.69%) and -4.34% (-37.65%), respectively, as in the study of the GDP in Table 3 where the policy implies a reduction of -3.22% (-8.22%) and -0.5% (-4.78%) on average, respectively.
4 Conclusions

The results are robust because the analysis is very prudent when computing the return of pooled debt and the debt-elastic interest-rate premium, in the sense that a large range of possible yields are considered for eurobonds and the parameter of the national-debt interest-rate premium is computed in a conservative way, it is estimated, with an upward bias, considering the actual spreads with respect to the eurobonds yields, and besides, we assume that no country can issue national debt at a borrowing rate below the eurobonds. The first may not be the case because spreads could even be reduced below the actual with the new institutional set-up. The latter makes a lot of sense if eurobonds are in its lower bound but, if the eurobonds yields are the middle or upper bound it would be possible that, for example, group IV could even issue debt below these interest rates when issuing national debt because it has an actual bonds yields lower than the eurobonds middle and upper rates all the years of the sovereign debt crisis despite its debt-to-GDP is higher than 60% of GDP. Remember that this group represents countries like Germany, Netherlands, Austria and France where bond yields were lower than $r^{e}_{m,t}$ and $r^{e}_{u,t}$ during the sovereign debt crisis. Hence, setting $\psi_{m}^{IV},\psi_{u}^{IV} = 0$ when their estimates are negative may overstate the negative results for group IV under these scenarios.

In the same direction, it is worth nothing that the study underestimates additional benefits from the fact that if the eurobonds had been implemented before, the disciplinary benefits would have affected sooner which would have made that the initial debt levels would have been much lower at the beginning of the sovereign debt crisis. This is very important because as we have observed, the initial conditions matter a lot for the transitional implications of the policy.

The aim of the paper was to investigate the implications of eurobonds taking into account that the effects would have been heterogeneous for the members of the Euro area depending on two dimensions: initial debt and actual bond yields. The four-group analysis has done this job in the previous section.

Our policy of interest is a coordinated fiscal policy therefore, we must look at the general results to evaluate it. To do so, the results has also been computed for all the groups as a whole. I study the overall results on fiscal policy, output and welfare, respectively.

The first overall result on fiscal policy is about the implications of the eurobonds on the feasibility of this kind of policies. Table 1, in the last row, shows us the average consumption tax rate that guarantee the implementability of all the government’s intertemporal budget constraint. We can see that the average tax rates in the lower and middle eurobonds yields are lower than the baseline model. Concretely, tax rates can be cut from 0.419 to 0.150 and 0.360, respectively. On the other hand, it should be raised to 0.581 for the upper eurobonds yields.

Secondly, we summarize the overall fiscal performance using the final stock of debt-to-GDP ratio. Overall final debt is lower than the baseline when eurobonds yield the lower and middle interest
rates. We can see in the last row of Table 2 that from the lower to the middle bound the final debt-to-GDP is -17.7 and -7.2 percentage points below the baseline. This is a very appealing result because the policy is not only able to reduce debt for GIIPS, but also gets an overall reduction of debt. On the other hand, final overall debt is 1.9 % points above the baseline model when eurobonds are $r_{u,t}$.

The total value added (GDP or output) is also another interesting measure for our analysis because it maximizes the possibilities of consumption, investment and public expenditure at every period. The differences of the overall GDP (2009-2012) appear in the last row of Table 3 at the bottom panel, we can observe that it grows significantly above the baseline when eurobonds yield the lower and middle rates, it grows a 10.68% and 4.49% more on average, respectively. Under the upper bound, overall output was -0.46% lower on average. In the same row, we can see that the long-run GDP is 23.29%, 11.39% and 2.12% higher than the baseline in the three scenarios, respectively.

The overall welfare is increasing with respect to the baseline model in the medium and lower bound scenarios of eurobonds yields. Table 4 shows, in the last row, that the sum of discounted utility for the lower and middle eurobonds yields are 176.73% and 38.44% higher than the baseline model, respectively. On the other hand, welfare falls in -9.74% for the upper eurobonds yields.

Up to this point, after having analysed all the results, the most important conclusion we can get is that Euro area has the opportunity to implement a policy that could be very profitable for all the groups of countries in order to address times of soaring sovereign spreads. Strictly speaking, following the analysis of total output and welfare, we observe that if eurobonds yields are the lower bound all the groups have a significant higher growth in all the periods, groups I, II, III and IV have an output 18.85%, 17.64%, 2.74% and 2.31% on average bigger than the baseline (Table 3), respectively, and the welfare is 350.95%, 368.04%, 33.94% and 85.23% (Table 4) also higher for groups I, II, III and IV, respectively. In addition, the final debt-to-GDP ratio is -22.9, -40.6, -3.6 and -4.5 % points (Table 2) lower than the baseline for groups I, II, III and IV, respectively. As it is assumed in the theory, this opportunity implies the optimal Euro-area institutions because this scenario ($r_{l,t}$) is only possible if all the members, without exception, commit to a more integration, pooling debt under joint and several liability as if they were a unique country, and to a more automatic Euro-area fiscal mechanisms, that is having an orderly planned mechanism of default (eurobonds as senior debt and national bonds as junior). On the other hand, if the institutional reforms to a more integration and automatic mechanisms are not the case, eurobonds would be the middle or upper bound yields, which are not as good as the previous one or even worse than the baseline model as the results show. We have seen that if this qualitative change in the Monetary Union institutions is not materialised implying $r_{m,t}$ or $r_{u,t}$, the rest of countries lose in terms of GDP, welfare and debt level, for instance.

In order to conclude, the degree of success of this unified policy strongly relies on the commitment
of all groups to undertake the optimal institutional reforms, which are a suitable level of integration and a proper mechanisms, that would mean that eurobonds yields are as close as possible to the benchmark German bonds or the US treasury bonds.

**Extensions**

This paper instead of solving an optimal government problem or an optimal fiscal policy problem (Ramsey equilibrium) is a competitive equilibrium analysis because its purpose was to analyze the overall and heterogenous effects of the eurobonds on the several countries of the Euro area. Hence, it might have potential extensions in future essays. A first extension would be to study the eurobonds in the context of the literature on default and self-fulfilling debt crises conformed by Calvo (1988), Cole and Kehoe (2000), Arellano (2008), Mendoza and Yue (2012), Corsetti and Dedola (2012), Aguiar et al. (2013), and Conesa and Kehoe (2013), among others. A second would go in line with the literature of optimal fiscal policy in small open economies in order to see what would be the role of the eurobonds, the main contributions to this literature are Correia (1996) and Razin and Sadka (1991a), among others. Another extension could model the moral hazard, in a principal-agent problem when a country (agent) belongs to a monetary/fiscal union (principal), in order to study the implications of incentives when there are policies that affect all the members of the union, like the eurobonds.

**References**


