Macroeconomic Implications of Long-Term Care Policies

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This version: August 2013

Abstract
Governments in countries with aging populations consider and implement various policies in response to an increasing number of elderly in need of care. Using data from the Health and Retirement Study we find that in the U.S. caregivers are primarily family members and that economic variables are predictors in determining whether care is provided informally at home or in a nursing facility. We argue that policy analysis needs to take into account the response of families: how will different families react to long-term care policies? Will these policies provide additional insurance or will they merely crowd out informal insurance? Which family members will benefit from different policies? We address these questions in an overlapping-generations economy with heterogeneous, imperfectly-altruistic agents. In the model, a frail elderly person prefers to obtain care from a family member; the provision of care is determined by a bargaining process between generations. Potential caregivers take into account the other’s welfare, foregone wages and the cost of nursing homes. We plan to calibrate the model to data from the Health and Retirement Study in order to assess costs and benefits of tax-financed government policies such as subsidizing informal caregivers or formal care.

* We would like to thank Andrew Caplin, Matthieu Chemin, Fabian Lange, and Stijn van Nieuwerburgh as well as conference participants at the SED, 2013. For excellent research assistance we thank Sam Gyetvay and Paul Cipriani.
1 Introduction

Long-term care is defined as becoming dependent on assistance from another person due to functional limitations such as having difficulties with activities of daily living (e.g. getting in and out of bed, getting dressed, showering, and eating) or with instrumental activities of daily living (e.g. buying groceries, going to the doctor, and going for a walk). Persons with functional limitations typically reside in the community long before moving to a nursing home and it is often family members (spouse or children) who provide long-term care (LTC). Among children, it is primarily daughters in their prime working-age years who take on the role of the main caregiver with possibly adverse consequences on their labor supply, retirement decision, and human capital.\(^1\)

The increase in female labor force participation together with changes in family structure (e.g. increasing divorce rates, fewer children, etc.) puts growing pressure on governments to play a more active role in LTC provision, especially in countries with aging populations.\(^2\) For the U.S. it is projected\(^3\) that the amount of elderly requiring LTC as a fraction of the 25 to 64 year old population will increase from 6.4% in 2010 to 7.4% in 2020 and 9.6% in 2030. Adding urgency to these developments is the fact that LTC is one of the major uninsured financial risks for elderly Americans (see, for example, Brown & Finkelstein, 2011) – means-tested Medicaid and small private LTC insurance markets leave the elderly largely uninsured.

In this paper we want to understand the potential benefits and costs which arise from tax-financed government policies (e.g. subsidizing nursing homes or informal care). In order to do this we argue that it is essential to take into account the response of informal caregivers. Subsidizing formal care may merely crowd out existing informal care and thus provide only limited additional insurance. A positive effect, however, may be that it crowds in informal caregivers into the labor force or retains them in the labor force; after all, taxes are paid on formal and not informal work. Subsidizing informal care may be expensive and ineffective if it goes primarily to infra-marginal families, e.g. retired spouses. Also, potential caregivers with low wages may choose to provide care at the cost of not working. Finally, any kind of subsidy will have to be financed by distortionary taxation, which is costly.

\(^1\)see Johnson & Sasso (2006) and Van Houtven et al. (2013).
\(^2\)In Germany and Japan, for example, the government has already stepped in; Germany has universal LTC insurance and Japan has universal LTC insurance for ages 65 and above; see Gleckman, 2010.
\(^3\)See Johnson et al. (2007).
A further issue we will assess is how policies affect the old and young generations differently. The elderly may prefer subsidies for informal caregiving whereas the young may favor government-financed nursing homes. Subsidizing informal caregivers allows the frail elderly to stay longer in the community which they may deem more desirable (Ameriks et al., 2011) while a subsidy for nursing homes would do the opposite. For the young, however, a nursing-home subsidy could mean that they can stay working and/or do not face the risk of supporting the parent financially.

We first document the importance of family-provided care in the United States using the Health and Retirement Study (HRS). Our key findings are the following. About 82% of respondents with functional limitations reside in the community. The bulk of hours of care these individuals receive stems from informal caregivers, primarily from the (retired) spouse or from a (working-age) daughter; formal home care plays a minor role. The availability of informal caregivers (spouse, child, or sibling) makes it more likely that a frail elderly resides in the community. Furthermore, families with low-earning children are more likely to receive informal care, whereas, families with high-earning children receive care more often from paid sources such as nursing homes. Finally, a majority of children (53%) who provides an intensive level of care (at least 19 weekly hours) does not work.

Armed with the key features of the data we build a model in order to study various tax-financed government policies. In order to accommodate the importance of family-provided care we accommodate a life-cycle model populated by overlapping-generations (OLG) with imperfectly-altruistic families. Each family consists of a young and an old household, which interact strategically. In the young household there is a high-productivity and a low-productivity worker. The high productivity worker always provides work in the market place, whereas

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4Spillman (2004) using the 1999 National Long-Term Care Survey finds that 70% of frail elderly were in the community and 30% in institutions. The discrepancy between these and our numbers is likely due to how stringent disability is defined. Our sample contains individuals that have at least one limitation either with an ADL or an IADL and have a helper due to these. This relatively lax definition of disability helps to account for all the help hours which we observe in the data.

5See also Stoller & Martin, 2002; Wolff & Kasper, 2006; etc. Another way of looking at the importance of informal care is by considering imputations of its economic value. P S Arno & Memmott (1999) provide an estimate of the economic value of informal caregiving of $196 billion in 1997. In contrast, national spending for formal home-health care was $32 billion and for nursing-home care it was $83 billion. Thus, the economic value of informal caregiving was equivalent to approximately 18 percent of total national health care spending ($1,092 billion) in 1997; the AARP estimates that in 2009 the economic value of informal care was $450 billion.

6See also Johnson (2008).

7Here, we build on previous work by Barczyk & Kredler (2012) and Barczyk & Kredler (2013).
the low-productivity worker faces a choice between market work and informal care. There are also two members in the retired household. When one member suffers an LTC shock it is assumed, as observed in the data, that the spouse provides care. This has currently no implication for nursing-home use but will matter when considering subsidizing informal care. Once the LTC shock has been realized the individual faces a mortality hazard. Upon his death, the other member may experience the LTC shock. Now, there is the option that a member of the young household provides care at home. For the young household, there is a trade-off between market work and providing care. We assume that the old can make financial transfers to the young in exchange for care; informal care is provided if it is beneficial for both households, and the size of the transfer is determined through Nash bargaining.

Depending on how altruistic the young and the old households in a family are they internalize the other’s situation to some extent. The decision to provide informal care by the member of the young household does not only depend on her own wage but also on the elderly’s financial resources, the price of a nursing home, and the elderly’s preference for being taken care of at home by a family member. Vice versa, the decision to demand informal care depends in addition to one’s own financial resources, the price of the nursing home and one’s dislike for nursing homes also on the young’s earnings capability and her other financial resources. In the calibration, we plan to identify the preference parameter to be taken care of at home by a family member such that the model replicates the informal-care decision that we find in logistic regressions in the data.

Relative to the existing literature which studies the importance of health expense risk for savings (e.g. Palumbo, 1999; DeNardi et al., 2010), nursing-home expense risk for savings (Kopecky & Koreshkova, 2012), and the implications of financing Medicare (e.g. Borger & Won, 2008; Attanasio et al., 2010), our main contribution is that we study macroeconomic outcomes when informal caregiving arises endogenously within a family. Furthermore, we contribute to this literature by having a family made up of two decision units, namely, a working household and a retired household who are imperfectly-altruistic towards each other. Modelling a family in this way allows us to make statements about how policies affect the old and young generations differently. The interaction between these households is conceptualized using a Markov game; see also Barczyk (2012) and Kaplan (2012).
2 Empirical Facts

Our modelling choice is motivated by empirical findings from the 2002 wave of the Health and Retirement Study (HRS). Here we provide a relatively concise overview of our data findings and refer the reader to the data appendix for a lengthier discussion.

Advantages of using the HRS
A major advantage of using the 2002 wave of the HRS is that it is nationally representative of both the non-institutionalized and the nursing-home populations. Furthermore, the HRS collects information about respondents’ informal caregivers. Importantly, it provides data on the frequency and intensity of care provided by various informal caregivers, such as, the spouse, children, relatives, and friends. This allows us to measure the relative importance among the various informal caregivers as well as across informal and formal care.

Measuring care needs
In the context of LTC, care does not refer to medical services which require medically-trained workers, e.g., a physical therapist or a nurse. If a nurse, for example, provides care it would be counted as formal home-care but if the nurse provides a service requiring medical expertise it is not counted at all. As is common in the medical literature on LTC, we measure the need for care using an index that counts the number of functional limitations a respondent declares (the index ranges from 1-10). It is made up of limitations that pertain to activities of daily living (ADLs), such as having difficulties with dressing, bathing, and going to bed, as well as difficulties that are instrumental in nature, called instrumental activities of daily living (IADLs), which includes having difficulties with grocery shopping, taking medication, and managing money.

Respondents in need of care
If a survey respondent has functional limitations the HRS inquires about helpers available for each limitation. Correspondingly, it establishes a file for each helper (helper-level files)
to document the time each helper provides for each limitation with an (I)ADL. Table 1 in the data appendix documents some basic facts about respondents who have at least one helper or live in a nursing home. About 82% reside in the community (we will refer to community residents as CR) and 18% in a nursing home (we will refer to nursing-home residents as NHR). NHR are older than CR, are more likely to be female, and suffer from more limitations with (I)ADLs.

Table 2 in the data appendix presents estimated odds ratios from a logistic regression on what factors explain why an individual with LTC needs resides in a nursing home. Respondents which have a partner or have children are significantly more likely to reside in the community than in a nursing home. The availability of a sibling also has a statistically significant negative relationship, but less so than having a spouse or children, with nursing-home admission. Unsurprisingly, the number of limitations with (I)ADLs and age are important predictors of nursing home residency. Finally, having more wealth correlates with a higher incidence of community residency.

Who cares?

We now aim to sort informal caregivers into the following three categories: those who are of working age (and thus face an opportunity cost from care) belong to the category “Young” (Y); informal helpers beyond working age belong to the caregiver category “Old” (O) and those for which we don’t know whether they are of working age belong to the category “Other” (Ot). The Y-category includes children, step-children, children-in-law, and grandchildren. The spouse/partner, sisters and brothers of the respondent make up the O-category. The Ot-category consists of other relatives and friends. Formal helpers, other than nursing-home helpers, do not play a substantial role in providing care and we therefore combine all formal helpers into one category “Formal” (F).

Table 3 provides an overview of the prevalence of the various caregiver types. It tells us that a large part of the helping population is made up of informal caregivers (Y+O+Ot) and that Y and O are especially common in providing help. It neglects, however, the intensity of care provided.\footnote{If, for example, a child provides one hour of help per month it would be counted as a helper in the Y category but for our purposes we would deem its importance to be negligible. On the other hand, while formal helpers are relatively infrequent they may be very important in how much time they devote in providing LTC.} Table 5 summarizes statistics on monthly hours of care provided by informal and formal caregivers, given that they provide a positive number of hours. Informal caregivers provide the lion’s share of all hours of care. Among informal caregivers, Y and O
are the most significant contributors in terms of care hours. For NHR formal caregivers are most crucial.

We now construct help-intensity categories. A helper who provides 0-7.5 hours per week falls into the help-intensity category “light”, one who provides 7.5-19 weekly hours into “medium”, and care of more than 19 hours per week qualifies as “heavy”. The idea behind the heavy-helper intensity category is that weekly hours of care is equivalent to at least a part-time job. Medium helpers are those that provide at least one hour of help per day and light helpers provide less.

Table 6 shows that the vast majority of all caregiving hours (85.5%) is provided by about one-third of heavy helpers (32.3%). About two-thirds of heavy helpers are informal caregivers and one-third are formal caregivers. Among informal caregivers Y and O are most important and are of roughly the same importance as measured in terms of frequency and hours contributed. Since the fraction of young heavy helpers is larger than that of old heavy helpers, the mean hours provided by a young heavy helper (243.6 hours) is lower than that of an old heavy helper (264.9 hours). The averages are dramatically lower for medium helpers; a young medium helper provides an average of 43.2 hours per month and an old medium helper a monthly average of 43.2 hours.\(^{10}\) Light helpers are for our purposes negligible.

**Caregiving children vs. non-caregiving children**

Table 7 shows that almost half (47.9%) of individuals with children have at least one child caregiver and most (79.5%) of these have exactly one. Among the respondents’ children 17% are caregivers to at least one parent and almost all of these provide care to only one parent. This suggests that the caregiving task is not shared among children but is primarily the responsibility of one child. What, if anything, distinguishes a caregiving child from those that do not engage in caregiving activities?

Table 8 provides an overview of some pertinent characteristics of helpers and non-helpers. Caregiving children tend to be older, female, without work, slightly more educated, and have a somewhat lower household income than non-caregiving children.

Table 9 shows that among caregiving children roughly half are light helpers (< 7.5 hours per week) and the other half are medium (7.5-19 weekly hours) or heavy helpers (> 19 hours per week). Table 10 is a counterpart to table 8, except that it also breaks down the helper category by the intensity of care. Perhaps unsurprisingly, differences between children who

\(^{10}\)Part of the reason that this average is low is that there is clustering around 30 hours, presumably, because many respondents simply answer that they obtain help every day for about one hour.
are heavy helpers and non-helpers as well as between heavy helpers and helpers in general (except median age) are even more pronounced than differences between helpers in general and non-helpers as documented in table 8. The most noteworthy differences between heavy helpers and other and non helpers are as follows. Most (79%) are women. They are more likely to co-reside with their parent(s), less likely to work and a substantial fraction has a fairly low household income (< 35k).

A surprising finding of table 6 was that young heavy helpers provide a high monthly average of help hours and their importance in terms of total hours provided is similar to the spouse/partner of the respondent. For our purposes, we would like to know the employment status of young heavy helpers (table 10 has information on this but here we break the heavy-helper category further down).

Table 11 shows that on average, a heavy-helper child which does not work provides 282 monthly hours of care; this average is 226 monthly hours for those who work part-time and 213 for full-time workers. The median monthly hours of care are substantially below the mean values: 180 for those not working, 124 for part-time, and 150 for full-time heavy-helper children. For 50% of out of work heavy-helper children, providing care amounts to a full-time job (i.e. they give at least 45 hours of care per week); 25% of these provide at least 90 hours of weekly care. Even among those who work full-time, there are 50% who provide care which amounts to almost another full-time job; more than 35 hours per week and 25% help more than 60 hours per week.

**Nursing home or home care?**

The empirical facts presented so far show that informal helpers play a major role in caregiving. It is their availability, as opposed to formal home-care helpers, that seems to be a crucial factor for an elderly in need of care to stay in the community, see table 2. Children (for single respondents) and the spouse (for partnered respondents) are the most important types of informal caregivers. Thus, studying LTC has to take into account the presence of these informal caregivers.

Economic variables, however, also matter in whether home or formal care takes place. Table 12 provides the results of a logistic regression of the binary variable “nursing-home status” on various characteristics of respondents and their children. The explanatory variable “kid income” is the household income of the child with the lowest household income among all children in the family. The intuition is that the child with the lowest household income has the smallest opportunity cost in providing care. The household income of children is
a categorical variable which indicates whether the household income is $<10k$ (category=1), $10k-35k$ (category=2), $35k-70k$ (category=3), or $>70k$ (category=4). Since children’s household income is a categorical variable the estimated odds ratio is relative to a baseline level of household income which is chosen to be the lowest household income ($<10k$).

The results show that respondents with higher household-income children are more likely to be in a nursing home controlling for other pertinent factors. For example, if a respondent’s child has household income of $35k-70k$ she is three times more likely to be in a nursing home compared to when the respondent’s child household income is $<10k$. In contrast is the respondent’s level of wealth: having more wealth makes it less likely that the respondent is a NHR controlling for other pertinent factors. For example, a respondent who is in the third wealth quartile is almost three times less likely to be in a nursing home compared to a respondent in the lowest wealth quartile.

Table 13 provides estimates of odds ratio of a similar logistic regression replacing household income with the child’s education. Education is a categorical variable which indicates whether the kid has less than a high school degree (category=1), a high school degree (category=2), more than a high school degree but less than four years or post-secondary education (category=3), or at least four years of post-secondary education (category=4). If the respondent has a child which has at least four years of post-secondary education the respondent is two and a half times more likely to be in a nursing home compared to a respondent who has a child with less than a high school degree. The estimates in terms of respondent’s wealth are similar to those before.

3 The Model

Setting

SHOULD PROBABLY MOVE TO INDEXING HOUSEHOLDS BY “Parents/Kids” or A, B HERE SINCE BOTH CAN BE OLD AND YOUNG.

Time is continuous. We model a life-cycle economy populated by families. A family consists of two distinct decision units: one parent household and a collection of children households. We assume that a children household is $\Delta T = 25$ years younger than the parent household and use the parent household’s age $j$ to keep track of a family’s life-cycle stage. In order to capture changing demographic forces, we assume that the children generation consists of one marginal household and $\nu$ infra-marginal households.
We index old households by $j$ (age or cohort), where $j \in [T_o, T]$, and implicitly assume that the young generation is a constant number of years, $\Delta G$, younger. The maximum age a household can attain is $T$. An old household enters the economy at age $T_o$ and a young household at age $T_y = T_o - \Delta G$.

There are three primary sources of idiosyncratic risks: health uncertainty in the form of requiring LTC, mortality hazard, and labor income uncertainty. Each of these follow a Poisson process.

A household starts to face LTC risks beginning at certain age, $T_{\sigma}$, with age-dependent Poisson rate $\sigma_j$; once the need for LTC arises it persists until death. Mortality hazard sets in at age $T_{\delta}$, and the hazard rate, $\delta_{j}^{ltc}$, depends on the age and the LTC status of the household. We assume that prior to the retirement age, $T_R$, there are neither mortality nor LTC risks.

Households face an idiosyncratic labor income process throughout their working lives, $j < T_R$, with Poisson rate $\xi$. The initial productivity realization of the young generation at age $T_y$ depends on the productivity of the parents at age $T_o$. Labor earnings display a hump-shaped life-cycle profile. Upon becoming retired a household receives social security...
benefits that depend on the earnings realization just prior to retirement.

Markets to insure against idiosyncratic risks are absent. Households can save in a riskless asset subject to a no-borrowing constraint. Upon death of an old household, any wealth left is transferred to the young generation in the family.

**Sources of care**

While the need for long-term care is exogenous, an afflicted person can choose one of three ways to obtain care: informal care, privately financed formal care, or publicly financed formal care, i.e. Medicaid (MA).

We assume that in a partnered household the spouse automatically (and freely) takes on the role of the informal caregiver.\footnote{The inclusion of such households is important for two reasons: first, it may be illegal for the government to target informal-care subsidies to families where caregivers are of a certain age. Second, the number of such natural low-opportunity-cost caregivers is likely to decrease in coming decades, most importantly due to high divorce rates. We will later explore how to incorporate such changes into our model.} For a single household, however, obtaining informal care is more delicate. The parent and the child have to come to an agreement whether the home-care arrangement is in their own best interest. In order to induce the child to provide care the parent can provide a non-negative transfer, $Q$. For a given $Q$ the home care decision is as follows: the old may or may not demand care, $h_d \in \{0, 1\}$, and the young may or may not supply care, $h_s \in \{0, 1\}$; only if $h_d = h_s = 1$ does home care take place. The transfer $Q$ is the result of symmetric Nash bargaining and so any surplus from home care is split equally. Home care can be decided upon in each instant, and there is no cost of switching from home to other forms of care.

The frail elderlies’ outside option to informal care is to purchase formal care provided in a nursing home. We have seen that empirically formal home care plays a minor role so that we neglect it. A representative nursing home provides care services at a price $q$ measured in terms of the consumption good. This price includes only the value of care. Measuring the cost of a nursing home by using the cost of care is important for two reasons. When an individual compares whether to obtain home care or formal care a key determinant is the cost of care. Comparing the price of a nursing home with the price $Q$ of informal care would be misleading since the price of a nursing home also includes room and board, amenities, services, and so on which still have to be purchased under home care. Furthermore, by using the price of care for the cost of a nursing home individuals can choose differing levels of nursing home quality consistent with what happens in reality. Formal care can be decided
upon in each instant, and there is no cost of switching from formal to other forms of care.

The third option is to obtain publicly financed care in a nursing home. In order to qualify for Medicaid all assets and social-security benefits have to be handed to the government. While all nursing home residents receive the same quality of care, MA residents cannot choose the level of comfort they obtain from the nursing home. Instead, they obtain a predetermined consumption floor $c_{ma}$. The expenditures a government must incur to finance a nursing home resident may be much higher than the consumption equivalent consumption floor. There might be inconveniences in receiving Medicaid, individuals may have Medicaid aversion and there might be a stigma associated with receiving Medicaid. We assume that being on Medicaid is an absorbing state.

Following Barczyk & Kredler (2013), we assume that households can give voluntary transfers (gifts) $g^y_t \geq 0$ and $g^o_t \geq 0$ to each other. For example, a young household may subsidize the purchase of formal care by the old when the young is relatively rich and has a high opportunity cost of providing care. In equilibrium, gifts will occur only when one household is in a very favorable situation and the poor household is borrowing-constrained. We omit these transfers from the model presentation in order to focus on the elements that are new in our model and will discuss the constrained case later on.\footnote{Following Barczyk & Kredler (2012), we also introduce a noise term into the model that ensures equilibrium existence; we omit its representation in the presentation in order to economize on notation.}

**Firms**

There are two goods in the economy: consumption goods, $G$, and formal care, $F$. Both of these goods are produced using only labor. Firms in the goods sector have production technology $G = AL^G$, where $A \geq 1$, and firms producing formal care face production function $F = L^F$. Firms take wage rates $w^G$ and $w^F$ as given and demand labor $L^G$ and $L^F$ to maximize profits $L^G(A - w^G)$ and $L^F(q - w^F)$, respectively. All markets are assumed to be competitive and because the flow of labor between the two sectors is unrestricted there is one equilibrium wage rate given by $w = q = A$. Note that we have implicitly chosen the normalization that one efficiency unit of labor produces one unit of formal care.

**Old household’s income**

We follow Kopecky & Koreshkova (2012) and model the Social Security benefit function as
follows:

\[ S(\bar{E}_e) = \begin{cases} 
0.9\bar{E}_e, & \text{if } \bar{E}_e < 0.2\bar{E}, \\
0.9(0.2\bar{E}) + 0.33(\bar{E}_e - 0.2\bar{E}), & \text{if } 0.2\bar{E} \leq \bar{E}_e \leq 1.25\bar{E}, \\
0.9(0.2\bar{E}) + 0.33(1.25\bar{E} - 0.2\bar{E}) + 0.15(\bar{E}_e - 1.25\bar{E}), & \text{if } 1.25\bar{E} \leq \bar{E}_e \leq 2.46\bar{E}, \\
0.9(0.2\bar{E}) + 0.33(1.25\bar{E} - 0.2\bar{E}) + 0.15(2.46\bar{E} - 1.25\bar{E}), & \text{if } \bar{E}_e > 2.46\bar{E}, \\
\end{cases} \]

The marginal replacement rates are 0.9, 0.33 and 0.15. The cutoff values are 20%, 125% and 246% of the average economy-wide labor earnings.

We use Gouveia and Strauss (1994) parametric form of a progressive income tax function given by

\[ \tau(y) = b \left[ 1 - (sy^p + 1)^{-1/p} \right]. \]

The parameter estimates are taken from Guner et al (2012) (for all households: \( b = 0.264, s = 0.013, \) and \( p = 0.964 \)).

After-tax income of the old household is given by

\[ Y^p(j, \epsilon; h^*) = [1 - \tau(y_2)] y_2 - h^*(Q - s^h). \]  \hspace{1cm} (2)

An old household has initially two members. Prior to retirement both have labor income given by \((1 + \beta_0)e^w\). An old household’s income tax rate is based on

\[ y_2 = ra + (1 - \tau^{SS})(1 + \beta_0)e^w, \]

where \(\tau^{SS}\) is the Social Security tax rate.

Post retirement the household obtains Social Security benefits according to schedule (1). Pension income \(P^p\) of the household is given by

\[ P^p(e_{Tr}^p, n^p) = (1 - \tau(ra^p))\tau(ra^p) + \left( \frac{n^p}{2} \right) S(\bar{E}_e). \]  \hspace{1cm} (3)

The function \(n^p\) decays from two over time to one. When there are two household members they obtain the Social Security benefit which is based on household income as explained previously. As \(n^p\) decays the household moves gradually to a single household. A single household obtains half of the Social Security benefits.
The function $n_p$ helps us to model the for-our-purposes pertinent realities of a two-member household in a tractable way. Firstly, males have a lower life expectancy than females; secondly, if the husband requires care the wife typically provides it; and thirdly, we can study the effects of having fewer partnered households due to higher divorce rates.

The decay of $n_p$ sets in once mortality hazard is turned on. Then, $n_p$ decreases in such a way to capture the fact that the husband is the first to die. Furthermore, some fraction of $n_p$ requires LTC which is interpreted as care provided by the wife. The purpose of this spousal technology is to ensure that a subsidy for informal caregivers also goes to spouses who provide care. To do so in a tractable way (in order to avoid having an additional state variable) the husband gradually vanishes from the old household. If the LTC shock hits the household the husband dies, and the only person in need of care is the elderly mother. If the mortality shock hits, the entire household dies.

**Child generation’s income**

We model the child generation as a collection of one *marginal household* and $\nu \geq 0$ *infra-marginal households*. The marginal household consists of one infra-marginal and one marginal worker. The infra-marginal worker always works in the labor market; the marginal worker faces the choice between working and providing informal care. Members of the infra-marginal households always engage in market work.

After-tax income of the child generation during their working lives – including home-care transfers and subsidies for home care – is given by

$$Y^k(j, \epsilon; h^*) = \left[1 - \tau(y(h^*))\right] y(h^*) + h^*(Q + s_h) + \nu \left[1 - \tau(y(0))\right] y(0),$$

where $y$ is taxable income of a household, and $h^*$ is an indicator variable equal to one if home care takes place and zero otherwise.

A household’s taxable income is given by

$$y(h^*) = \frac{ra}{1+\nu} + (1 - \tau_{SS})(1 + \beta_0 - h^*) \epsilon w.$$  

For the marginal household, labor income depends on the home care choice and is given by $(1 + \beta_0 - h^*) \epsilon w$, while an infra-marginal household’s labor income is always $(1 + \beta_0) \epsilon w$. The function $\epsilon = \epsilon(j, \epsilon)$ maps age $j$ and productivity shock $\epsilon$ into efficiency units. To account for the fact that heavy help is provided by primarily one child, we assume that one household
member has an edge in the labor market over the other and therefore always works. This worker’s effective wage rate is \( \beta_0 \)-times that of the marginal worker and can be thought of as arising due to higher efficiency units or a gender-wage gap. By varying \( \beta_0 \) we can study, for example, the consequences changes in the gender-wage gap have on the provision of informal care as the opportunity cost of the marginal worker changes.

The factor \( \nu \) allows us to target the right number of average children and to study the effects of a decrease in the total fertility rate. With a decrease in the average number of children the number of potential informal caregivers decreases which would be reflected in a smaller \( \nu \). The total fertility rate for women who are of age 45 to 95 in 2000 has been approximately 3 children during their prime child-bearing age, whereas, currently it stands at about 2 children per woman.\(^{13}\)

When the marginal worker provides LTC, the marginal household’s labor income reduces to \( \beta_0 w \). Progressive taxation of household income leads to a tax benefit of providing care because the tax rate decreases – this tax saving is similar to a home-care subsidy and would be especially pronounced if the marginal worker is taxed at a high rate (e.g. spouses in Germany are taxed at a marginal rate that builds on the existing labor earnings in the household). The household obtains a non-negative transfer \( Q \) and subsidy \( s_h \) on which no taxes have to be paid.

When the child generation is retired, it receives interest payments and Social Security benefits. Social Security benefits depend on average lifetime labor earnings. In order to avoid carrying around an additional state, we use as a proxy for average lifetime labor earnings the average over the labor-earnings profile which corresponds to the productivity realization just prior to retirement denoted by \( \bar{e}_{T_R} \). Given this productivity realization we assume that the household has always had this productivity. Household labor earnings just prior retirement (in the absence of home care) are given by \( (1 + \beta_0)\bar{e}(T_R, e_{T_R}^{y})w \) and it is straightforward to calculate the average over the number of working periods, \( \bar{E}_x = \bar{e}(1 + \beta_0)w \). We assume that time taken off for informal care does not influence the average lifetime labor earnings relevant to calculate Social Security benefits. In other words, if the marginal worker provides informal care the Social Security system still treats her as if she is working on the labor market. This is obviously not the case in reality but eases the computational burden by not

having to keep track of the informal-care history.

The child generation’s after-tax income is

\[
P^k(\epsilon^y_{TR}, n^k; h^*) = (1 + \nu) \left[ (1 - \tau(y))y + \frac{n^k}{2} S(\bar{E}_c) \right] + h^*(Q + s_h). \tag{5}
\]

When the child generation is retired we have that taxable per-household income of the marginal and the infra-marginal households are given by

\[
y = \frac{ra}{1 + \nu},
\]

and each households’ pension income is \( \frac{n^k}{2} S(\bar{E}_c) \). The reason for the term \( n^k/2 \) is that initially the household consists of two adults, \( n^k = 2 \), and together they receive Social Security benefit \( S(\bar{E}_c) \). The number of household members gradually decreases to 1 and with it the Social Security benefit. Note, when the young generation is retired providing informal care does not come with a loss of labor income and so there are no opportunity costs of providing care.

Once the child generation reaches the age at which it faces the LTC hazard (which in our calibration only happens once the parent generation is dead with certainty, i.e. after time \( T \)) the child generation’s after-tax income is \( Y^y(\epsilon^y_{TR}, n^k; h^*) \) when healthy but reduces to

\[
P^k(\epsilon^y_{TR}, 1; 0) = (1 + \nu) \left[ (1 - \tau(y))y + \frac{1}{2} S(\bar{E}_c) \right],
\]

when the children are hit by the LTC shock since there is only one member per household left.

We estimate efficiency units \( \hat{e}(\cdot) \) in terms of dollars. However, in the model we have chosen the normalization that one efficiency unit of labor produces one unit of formal care. Hence, in order to get the model counterpart, we have to divide \( \hat{e}(\cdot) \) by \( q_{sq} \), where \( q_{sq} \) is the price of formal care in the status quo (\( sq \)). For the marginal household income is given by

\[
y_0 = \frac{ra}{1 + \nu} + (1 - \tau^{SS})(1 + \beta_0 - h^*) \frac{\hat{e}}{q_{sq}} w_{sq},
\]
and for the infra-marginal household it is

\[ y_1 = \frac{ra}{1 + \nu} + (1 - \tau^{SS})(1 + \beta_0)\frac{\hat{e}}{q_{sq}} w_{sq}. \]

In the status quo \( q = w \) and so cancel each other out. When we study changes to labor factor productivity in the goods sector, i.e. changes to \( A \), the wage rate will differ from \( w_{sq} \) reflecting the fact that production in the goods market has become more efficient relative to the production of formal care (technology intensive versus labor intensive). But, because of free flow of labor between the sectors the wage rate in the formal care sector changes accordingly. Thus, an increase in \( A \) has merely the effect of increasing the opportunity cost of providing informal care.

**Preferences**

A child household consists of \( n^k \) household members and a parent household of \( n^p \) members. To account for economies of scale within a household we adjust consumption expenditure using an equivalence scale. We use the OECD equivalence scale which is \( \phi_{OECD}(N_{ad}, N_{ch}) = 1 + 0.7(N_{ad} - 1) + 0.5N_{ch} \), where \( N_{ad} \) is the number of adults and \( N_{ch} \) is the number of children, see Bick and Choi (2013); see Villaverde and Krueger (2007), Table 1, for different household equivalence scales. So for our households that consist of at most two adults we obtain the scale

\[
\phi(n) = \begin{cases} 
1 + 0.7(n - 1) & n \in [1, 2], \\
1.7 + 0.5(n - 2) & n > 2.
\end{cases}
\]

Per-period utility of the kid generation, \( u^k \), of consumption expenditure, \( c^k \), is given by

\[
u^k(c^k, n^k) = n^k(1 + \nu)u\left(\frac{c^k}{(1 + \nu)\phi(n^k)}\right).
\]

Consider the argument of function \( u \). There are \( 1 + \nu \) households in the child generation and so \( c^k/(1 + \nu) \) is per-household consumption expenditure. This per-household consumption expenditure is divided by the effective number of household members \( \phi(n^k) \) which yields per-kid effective consumption units. Because there are \( n^k(1 + \nu) \) persons in the child generation, \( u \) is multiplied by this number to aggregate individual utilities to obtain \( u^k \).

Similarly, per-period utility for a parent household, \( u^p \), of consumption expenditure, \( c^p \),
is given by

\[ u^p(c^p, n^p) = n^p u \left( \frac{c^p}{\phi(n^p)} \right). \]

Consumption expenditure is adjusted to account for within-household scale effects to obtain per-parent effective consumption units, \( \frac{c^p}{\phi(n^p)} \). Household utility is obtained by summing individual utilities.

Children’s flow utility is

\[ u^k + \alpha^k [u^p + \eta h], \]

and the parent household’s flow utility is

\[ u^p + \eta h + \alpha^p u^k. \]

The parameters \( \alpha^k, \alpha^p \in [0, 1] \) represent the children’s and the parent’s respective degrees of altruism. The parameter \( \eta \) captures the preference for home care versus residing in a nursing home, and \( h \) is an indicator variable equal to one if home care takes place and zero otherwise. If the old obtains Medicaid her flow utility is given by \( u(c_{ma}) \). The consumption equivalent \( c_{ma} \) captures the consumption value of Medicaid but also all inconveniences and potential stigma associated with it.

Spending an additional dollar on \( c^k \) yields marginal utility to the kid generation of

\[ u^k(c^k, n^k) = \frac{n^k}{\phi(n^k)} u_c \left( \frac{c^k}{(1 + \nu)\phi(n^k)} \right). \]

An additional dollar spent means that each child obtains from it \( 1/(1 + \nu)\phi(n^k) \) effective units which each child values at:

\[ u_c \left( \frac{c^k}{(1 + \nu)\phi(n^k)} \right) \frac{1}{(1 + \nu)\phi(n^k)}. \]

Summing over the number of children \((1 + \nu)n^k\) gives us the expression for the kid generation’s marginal utility.

Marginal utility of the parent household is

\[ u^p_c(c^p, n^p) = \frac{n^p}{\phi(n^p)} u_c \left( \frac{c^p}{\phi(n^p)} \right). \]
Each parent obtains \(1/\phi(n^p)\) effective units from the additional dollar spend and values it at \(u_c(c^p/\phi(n^p))\). Summing \(n^p\) times yields the expression for the parents’ marginal utility.

We now derive the expression for optimal unconstrained consumption. When parents are unconstrained their optimal consumption is characterized by

\[
u^p_{c}(c^p, n^p) = V^p_{a^p}.
\]

Substituting CES utility yields

\[
u^p_{c}(c^p, n^p) = \frac{n^p}{\phi(n^p)} \left( \frac{c^p}{\phi(n^p)} \right)^{-\gamma} = V^p_{a^p}.
\]

Simplifying yields

\[n^p \phi(n^p)^{\gamma-1} (c^p)^{-\gamma} = V^p_{a^p},\]

and solving for \(c^p\) yields

\[c^p = \left[ n^p \phi(n^p)^{\gamma-1} \right]^{1/\gamma} (V^p_{a^p})^{-1/\gamma}.
\]

For logarithmic utility this simplifies to

\[c^{p,\text{log}} = n^p (V^p_{a^p})^{-1}.
\]

Apparently, for log-utility scale effects have no direct effect on optimal consumption and only the size of the household matters.

When children are unconstrained their optimal consumption is characterized by

\[u^k_{c}(c^k, n^k) = V^k_{a^k}.
\]

Substituting marginal utility for CES utility yields

\[u^k_{c}(c^k, n^k) = \frac{n^k}{\phi(n^k)} \left( \frac{c^k}{(1+\nu)\phi(n^k)} \right)^{-\gamma} = V^k_{a^k}.
\]

Simplifying by first taking out the denominator from the round brackets

\[n^k(1+\nu)^{\gamma} \phi(n^k)^{\gamma-1} (c^k)^{-\gamma} = V^k_{a^k}.
\]
Simplifying further

\[(c^k)^\gamma = (1 + \nu)^\gamma \left[n^k \phi (n^k)^{\gamma - 1}\right] (V_{a_k}^k)^{-1},\]

and solving for \(c^k\)

\[c^k = (1 + \nu) \left[n^k \phi (n^k)^{\gamma - 1}\right]^{1/\gamma} (V_{a_k}^k)^{-1/\gamma},\]

For logarithmic utility this simplifies to

\[c_{log} = n^k (1 + \nu)(V_{a_k}^k)^{-1}.\]

**Government**

The government finances a pay-as-you-go social security system, a Medicaid program, and non-valued purchases of size \(G\). It has two channels of influencing the choice families make regarding formal and informal care. The first one is to subsidize formal care with subsidies \(s_f\). The second is a subsidy on informal care \(s_h\). Government financing is done solely by levying a labor income tax \(\tau\).

Medicaid expenditures are given by:

\[MA = \int \left[\kappa I(ma_j) - I(ma_j)(SS_j + a_{o_j})\right] dj,\]

where \(\kappa\) is the total cost per public resident and \(I(ma(j))\) is an indicator variable equal to one if individual \(j\) relies on Medicaid; \(SS\) are social security benefits and \(a_{o_j}\) is wealth obtained from the old in order to qualify for Medicaid.

In each period, the government’s budget is balanced:

\[\tau_t(A_tL + F) = MA + SS + G + s_f F + s_h \int h_j dj,\]

where \(h_j\) is an indicator variable equal to one if individual \(j\) receives home care.

**Decision problems**

The payoff-relevant state is given by children’s level of wealth, \(a^k\), and productivity, \(e^k\), the parent’s level of wealth, \(a^p\), productivity, \(e^p\), LTC status, \(ltc \in \{0, 1\}\), and age, \(j\). We denote the state more compactly by \((\omega, ltc)\), where \(\omega\) summarizes the variables \(\omega = (j, a^k_j, a^p_j, e^k_j, e^p_j)\). When the parent receives Medicaid, or after the death of the parents, the payoff-relevant state is given by \(\bar{\omega} = (j, a^k_j, e^k_j)\).
In what follows we first ignore the choice of altruistically-motivated transfers (gifts) in order to streamline our discussion on elements which are new to our model. It turns out that in the equilibrium, gifts only flow to borrowing-constrained households and we will discuss the constrained case separately below.

Children and parents’ problems prior to retirement

Consider the households’ problems before the old household retires. The old neither faces the risk of death nor of LTC because of our assumption that $T_R < T_\delta < T_\sigma$.

The following Hamilton-Jacobi-Bellman equation (HJB) characterizes the decision problem of the young generation. It takes the policies of the parent household as given and chooses $c^k$ so that

$$
\rho V^k(\omega, 0) = \max_{c^k} \left\{ u^k(c^k, n^k) + \alpha^k u^p(c^p, n^p) + \eta h^* + \dot{\alpha}^k(c^k, h^*) V^k_{\alpha^k} + \dot{\alpha}^p(c^p, h^*) V^p_{\alpha^p} \right\} +
$$

$$
+ \xi [V^k(c^k, \cdot) - V^k] + \xi [V^k(c^k, \cdot) - V^k] := (7)
$$

For as long as the old is unconstrained and faces no LTC risk the young faces a standard consumption-smoothing problem. The young’s optimal consumption equates its marginal utility with the marginal value of savings, $u_c^k = V^k_{\alpha^k}$. The significance of this standard result is that the decision maker does not have to contemplate all the possible consumption-savings choices the other household makes and so her best response is constant in the contemporaneous actions of the other.

The parent household takes the young generation’s policies as given and chooses $c^p$ in the following way

$$
\rho V^p(\omega, 0) = \max_{c^p} \left\{ u^p(c^p, n^p) + \eta h^* + \alpha^p u^k(c^k, n^k) + \dot{\alpha}^p(c^p, h^*) V^p_{\alpha^p} + \dot{\alpha}^k(c^k, h^*) V^k_{\alpha^k} \right\} +
$$

$$
+ \xi [V^p(c^p, \cdot) - V^p(\omega)] + \xi [V^p(c^p, \cdot) - V^p(\omega)] := (8)
$$

The old’s optimal choice of consumption is such that it equalizes the consumption and savings margins, $u_c^p = V^p_{\alpha^p}$.

Each household is subject to the laws of motion of both the young and the old wealth
given by
\[
\dot{a}^k(c^k, h^*) = Y^k(j, c^k; 0) - c^k,
\]
\[
\dot{a}^p(c^p, h^*) = Y^p(j, c^p; 0) - c^p,
\]
where \(Y^k\) is given by (4) and \(Y^p\) by (2).

Over a small increment of time the change in wealth is determined by how much a household consumes out of income from interest payments and labor income. Changes in the wealth position are valued with the appropriate marginal values. In equation (7), \(V^k_{a_k}\) is the marginal value of the children with respect to the parent’s savings; \(V^p_{a^k}\) in equation (8) is the analogue for the parents.

Finally, value functions of either generation depend not only on one’s own idiosyncratic income risk but also on that of the other. Idiosyncratic income risk in the form a Poisson process for labor productivity shows up as the jump terms shown in the square brackets. Conditional on a productivity shock the jump is given by the difference between the valuation of the state with the new wage realization and the current state. The prime ‘ indicates a new realization of household productivity.

**Children and parents’ problems after retirement: no LTC**

We now turn to the case in which the old is retired, faces mortality and LTC risks, and is currently healthy. The young’s HJB – leaving out the terms describing the wage uncertainty – is given by

\[
\rho V^k(\omega, 0) = \max_c \left\{ u^k(c^k, n^k) + \alpha^k [u^p(c^p, n^p) + \eta h^*] + \dot{a}^k(c^k, h^*)V^k_{a_k} + \dot{a}^p(c^p, h^*)V^p_{a^k} \right\} + \\
+ \sigma[Z^k(\omega, 1) - V^k] + \delta^0[W(\bar{\omega}) - V^k],
\]

where

\[
Z^k(\omega, 1) = I_{ma}(\omega, 0) MA^k(\bar{\omega}) + (1 - I_{ma}(\omega, 0)) V^k(\omega, 1),
\]

\[
I_{ma} = 1 \text{ if } MA^o(\bar{\omega}) > V^o(\omega, 1).
\]

With the introduction of mortality, \(\delta\), and LTC hazards, \(\sigma\), the young’s value depends on functions \(W\) and \(Z^k\). The value \(W\) is the child generation’s continuation value upon the parent household’s death. In that case the state is given by \(j\), the young’s wealth and income.
If the parents experience the LTC shock, the children’s value function is given by $Z_k^k(\omega, 1)$. In this event the parent can choose whether or not to rely on Medicaid. If she does the children’s value is $MA_k^k(\omega)$, and if not it is $V_k^k(\omega, 1)$. The parent chooses Medicaid only if her Medicaid value exceeds that of not receiving Medicaid, $MA_k^k(\omega) > V_k^k(\omega, 1)$, and $\mathcal{I}_{ma}$ equals one if this is case.

For the parent household we have the following HJB

$$\rho V_p^{\omega}(\omega, 0) = \max_{\epsilon_p} \left\{ u^p(c_p, n_p) + \eta h^* + \alpha_p u^y(c_k, n_k) + \alpha_p (c_p, h^*) V_{ap}^p + \alpha_k (c_k, h^*) V_{ck}^p \right\} +$$

$$+ \sigma [Z_p^p(\omega, 1) - V_p^p] + \delta^0 [\alpha_p W(\omega) - V_p^p].$$

(10)

Because of altruism the parent has a continuation value which goes beyond her own lifetime given by $\alpha_p W$.

Each household is subject to the laws of motion of both the young and the old wealth given by

$$\dot{a}_k^k(c_k, h^*) = Y^k(j, e_k^k; h^*) - c_k,$$

$$\dot{a}_o^o(c_k, h^*) = P_p^p(\epsilon_T, n_p) - c_p,$$

where $Y^k$ is given by (4) and $P_p^p$ by (3).

Children and parents’ household problems after retirement: LTC

Recall that the LTC shock also implies that that only one member remains in the parent household. Thus, the per-period utility of the old household is simply given by $w^p(c_p, 0) = u(c_p)$.

Our timing protocol within a period is that (1) the parent decides whether to make use of publicly-financed formal care; (2) transfer $Q$ is announced; (3) kinds and the parent decide on home care; (4) the transfer is exchanged; (5) altruistically-motivated transfers flow; and (6) consumption choices are made. Suppose that in the future period $t + \Delta t$ the old does not choose Medicaid. We now go $\Delta t$ backwards in time and consider the HJBs in stage (3) of our timing protocol in the current period $t$. Taking as given the transfer $Q$, the young and the old decide on whether care should be provided at home or in a nursing home.

Specifically, children choose consumption $c_k^k$ and whether or not to provide home care
such that

$$\rho V^k(\omega, 1) = \max_{c^k, h^S \in \{0, 1\}} \{ u^k(c^k, n^k) + \alpha^k [u(c^p) + \eta h^*] + \dot{\alpha}^k(c^k, h^*) Z_{a^p}^k + \dot{\alpha}^k(c^k, h^*) Z_{a^p}^k \} +$$

$$+ \delta^1 [W(\bar{\omega}) - V^k],$$

where

$$h^* = h^D h^S, \quad Z_{a^p}^k = V_{a^p}^k(\omega, 1), \quad \text{and} \quad Z_{a^p}^k = V_{a^p}^k(\omega, 1).$$

The marginal values of savings are evaluated using the value function $Z$. This is because the marginal value of savings depends on whether next period the parent household will be a Medicaid recipient or not. If so $Z_{a^p}^k = V_{a^p}^k(\omega, 1) = 0$ because of means testing and $Z_{a^k}^k = M A_{a^k}^k > 0$ because the child still faces a consumption-savings problem. Because we have assumed that she has decided against it we have that $Z_{a^k}^k = V_{a^k}^k(\omega, 1)$ and $Z_{a^p}^k = V_{a^p}^k(\omega, 1)$.

Similarly, the parent household chooses consumption $c^p$ and whether or not to demand home care $h^D$ to satisfy

$$\rho V^p(\omega, 1) = \max_{c^p, h^D \in \{0, 1\}} \{ u^p(c^p) + \eta h^* + \alpha^p u^k(c^k, n^k) + \dot{\alpha}^p(c^p, h^*) Z_{a^p}^p + \dot{\alpha}^k(c^k, h^*) Z_{a^p}^k \} +$$

$$+ \delta^1 [\alpha^k W(\bar{\omega}) - V^p],$$

where

$$h^* = h^D h^S, \quad Z_{a^p}^p = V_{a^p}^p(\omega, 1), \quad \text{and} \quad Z_{a^p}^k = V_{a^p}^k(\omega, 1).$$

As for the children, the parent’s choice of consumption under either home or formal care is identical because of the independence between the care choices, i.e. $u_c(c^p) = V_{a^p}^p$ irrespective of the source of care (this will not typically be the case in borrowing-constrained states).

Both households face the laws of motion for wealth given by

$$\dot{\alpha}^k(c^k, h^*) = Y^k(j, c^k; h^*) - c^k;$$

$$\dot{\alpha}^p(c^p, h^*) = P^p(j^p_{TR}, 1) - h^* Q - (1 - h^*) q - c^p;$$

where $Y^k$ is given by (4) and $P^p$ by (3). When the child generation is retired $Y^k$ is replaced by $P^k$ given by equation (5). The child generation faces mortality and LTC risks only after
\(T = 95\). We postpone the discussion about the determination of home care to section 4.

In step (1) of the timing protocol, the old chooses whether to opt for a Medicaid-financed nursing home spot; recall that this is an absorbing state. If she does, children’s HJB is given by

\[
\rho MA_k(\tilde{\omega}) = \max_{c_k}\left\{ u^k(c^k, n^k) + \alpha^k u(c_{ma}) + \dot{a}^k(c^k, h^*) MA^k_a + \delta^1 [W - MA^k] \right\} + \delta^1 [W - MA^k].
\]

The old household’s HJB is given by

\[
\rho MA_k(\tilde{\omega}) = u(c_{ma}) + \alpha^p u^k(c^k, n^k) + \dot{a}^k(c^k, h^*) MA^p_a + \delta^1 [\alpha^p W - MA^p].
\]

In the Medicaid state the parent has no choices. Her value is still changing, however, because of her link to the children who face a dynamic-decision problem. Both take the children’s law of motion for wealth into account:

\[
\dot{a}^k(c^k, h^*) = Y^k(j, c^k, h^*) - c^k,
\]

where \(Y^k\) is given by (4) or replaced by \(P^k\) – equation (5) – if the children are retired.

**Young’s problem: parent household dead**

In order to complete the description of the decision problems we still need to discuss the case that arises for the young generation after the death of the parent household. In terms of our research agenda these households play a very minor role and so a simpler decision problem suffices. The aim is to obtain a reasonable continuation value \(W\), that is, one that encodes plausible marginal values of savings.

In order to simplify matters we assume that the young generation is not linked to any children households. It solves a standard consumption-savings problem and depending on its age faces LTC and mortality risks. When requiring LTC the young generation can either choose privately or publicly financed formal care. Furthermore, the continuation value beyond the young generation’s lifetime is zero. We augment the state \(\tilde{\omega}\) with a 0 when the young does not require care and with a 1 when LTC is required.
The child generation’s HJB in the absence of requiring LTC is given by

$$\rho W(\tilde{\omega}, 0) = \max_{c^k} \left\{ u^k(c^k, n^k) + \dot{A}(c^k) W_A \right\} - \delta^0 W + \sigma [Z(\tilde{\omega}, 1) - W],$$  \hspace{1cm} (15)

s.t.

$$\dot{A}(c^k) = Y^k(j, \epsilon_k; 0) - c^k,$$

$$Z(\tilde{\omega}, 1) = \max \{ W(\tilde{\omega}, 1), MA \},$$

where $Y^k$ is given by (4) or replaced by $P^k$ – equation (5) – if the children are retired. We denote the child generations’s wealth by $A$ to emphasize that wealth includes any assets left over by the parent household. The household can choose to enter Medicaid conditional on suffering an LTC shock; it would do so if $MA > W(\tilde{\omega}, 1)$.

Suppose the children require LTC and have chosen not to make use of Medicaid in period $t + \Delta t$. The HJB is given by

$$\rho W(\tilde{\omega}, 1) = \max_{c^k} \left\{ u(c^k) + \dot{A}(c^k) Z_A \right\} - \delta^1 W,$$  \hspace{1cm} (16)

s.t.

$$\dot{A}(c^k) = P^k(\epsilon_{Tr}^k; 1; 0) - (1 + \nu)q - c^y,$$

$$Z_A = W_A(\tilde{\omega}, 1),$$

where $P^k$ is given by equation (5) evaluated for $n^k = 1$ since the there is only one member per household once the LTC shock hits. Each household must pay $q$ for nursing-home care. Here, we assume that “next period” the young household does not choose Medicaid which is why $Z_A = W_A$; if the household chooses Medicaid its marginal value of saving is zero because savings will have no use.

A household in Medicaid has the following HJB

$$\rho MA_j = (1 + \nu)u(c_{ma}) - \delta^1_j MA_j.$$  \hspace{1cm} (17)

There are no decisions to be made. The value $MA_j$ decreases in $j$ because mortality hazard increases in $j$.

**Equilibrium Definition**

A recursive equilibrium is given by value functions for the young, $(V^y_j, MA^y_j, W_j, MA_j)$,
and the old, \((V_o^j, MA_o^j)\), policy rules for the young, \((c_y^j, h_y^S^j, MA \text{ choice}_i)\), and the old, \((c_o^j, h_o^D^j, MA \text{ choice}_j)\), densities \(\lambda_j\) of families over the state space, and a home-care pricing function \(Q_j\), such that, given an initial density \(\lambda_{T_yT_o}\), prices \((w, r, q)\), and a government policy \((\tau, s_h, s_f, G)\):

1. For the young: the value function \(V_y^j\) satisfies equation (7) for ages prior to the old’s retirement, equation (9) for ages after the old is retired and healthy, and (11) when the parent is sick. When the old is in Medicaid \(MA_o^j\) satisfies equation (13). When the parent household is dead, \(W_j\) is the young’s value function and satisfies equation (15) when healthy and equation (16) when sick and in nursing home, and when in Medicaid, \(MA_j\) satisfies (17); maximum values are attained by the policies \((c_y^i, h_y^S^i, MA \text{ choice}_i)\), given policy rules of the old household \((c_o^j, h_o^D^j, MA \text{ choice}_j)\), and the home-care pricing function \(Q_{ij}\).

2. For the old: the value function \(V_o^j\) satisfies equation (8) for ages prior to her retirement, equation (10) for ages after her retirement when she is healthy, and (12) when sick, and when she is on Medicaid value function \(MA_o^j\) satisfies equation (14); the maximum is attained by the policies \((c_o^i, h_o^D^i, MA \text{ choice}_i)\), given policy rules of the young household \((c_y^j, h_y^S^j, MA \text{ choice}_j)\), and the home-care pricing function \(Q_{ij}\).

3. Firms: decisions are optimal taking prices \((w, r, q)\) as given.

Determination of prices and the tax rate:

1. the home-care pricing function \(Q_j\) is the symmetric-Nash-bargaining solution between child and parent household \(j\) for all \(\omega\);

2. the labor markets clear;

3. the tax rate \(\tau\) is such that the government’s budget is balanced, i.e. (6) holds.

Finally:

1. the densities \(\lambda_j\) are obtained from the initial density \(\lambda_{T_yT_o}\) and the above given equilibrium objects;

2. the exogenous parameters are \((\lambda_{T_yT_o}, A, r, s_h, s_f, G)\).
4 Nursing home or home care?

We now study the choice between home and formal care. When both households are unconstrained the characterization is straightforward. When one (or both) household(s) is (are) constrained the analysis is slightly more involved. Thus, we treat the unconstrained and constrained cases separately.

Recall the timing protocol of the game over a short time horizon $dt$:

1. The parent decides if to enter Medicaid or not.
2. Nature chooses a transfer $Q \geq 0$ (which equals the Nash bargaining solution in equilibrium) and households decide if to supply or demand home care.
3. If home care takes place, $Q$ is transferred from parent to child; whether or not home care takes place, both may give non-negative altruistic gifts $g^k$ and $g^p$.
4. After all transfers are handed over, players decide on consumption $c^k$ and $c^p$.

Given knowledge of the value functions, we will now use backward induction to determine the solution of this game. To focus the discussion on home and formal care we assume that the parent does not choose Medicaid in stage 1.

Unconstrained case

We start with the consumption stage 4. From HJBs (11) and (12) we see that the consumption and the home care choice are independent of each other. Thus, no matter whether home or formal care takes place consumption is the same. This simplifies matters significantly. Denote optimal unconstrained consumption by $c^k = (u^k_c)^{-1}(V^k_{ak})$ and $c^p = (u^p_c)^{-1}(V^p_{ap})$, where $u^p_c = u_c$ because the parent household consists of only one member.

In stage 3, equilibrium gifts are zero, $g^k = 0 = g^p$, for any $Q$. In equilibrium, gifts only flow to constrained recipients. We will turn to the constrained case shortly.

In stage 2, parents and children decide whether home or formal care takes place. If home care generates a surplus for the family it takes place. The way the surplus is split among the parents and the children is determined through Nash bargaining. For any transfer $Q$ the
surplus for the kid generation can be calculated using HJB (11):

$$\rho V^k = \max \begin{cases} \begin{aligned} & u^k(c^k, n^k) + \alpha^k[u(c^p) + \eta] + \lambda_k(c_p, 1)V^k_{a_k} + \lambda_p(c_p, 0)V^k_{a_p} , \\
& \text{young’s flow value from home care} = H^{k|h} \\
\end{aligned} \end{cases} + \text{jump terms},$$

$$\begin{aligned} & u^k(c^k, n^k) + \alpha^k u(c^p) + \lambda_k(c_p, 0)V^k_{a_k} + \lambda_p(c_p, 0)V^k_{a_p} , \\
& \text{young’s flow value from formal care} = H^{k|f} \end{aligned}$$

where we substitute optimal consumption and gift choices from stages 3 and 4 and the laws of motion for wealth:

$$\begin{aligned} & \dot{a}_k(c_k, h^*) = Y^k(j, e^k; h^*) - c_k , \\
& \dot{a}_p(c_p, h^*) = P^p(\epsilon_t^p, 1) - h^*Q - (1 - h^*)q - c_p. \end{aligned}$$

Children’s surplus from home care is $S^k(Q) = H^{k|h}(Q) - H^{k|f}$ and it is non-negative if

$$V^k_{a_k}(Q + s_h) + V^k_{a_p}(q - s_f) + \alpha^p \eta \geq V^k_{a_k}[Y^k(j, e^p; 0) - Y^k(j, e^p; 1)] + V^k_{a_p} Q.$$

The marginal benefit to the young household of providing care is given by the transfer $Q$ and the home-care subsidy $s_h$; the young values these using her marginal value of saving, $V^k_{a_k}$ (the only factors a selfish agent would consider). Additionally, the parent does not have to purchase formal care so that children benefit from the foregone expenditures, $q - s_f$, valuing them with $V^k_{a_p}$. Finally, it internalizes a fraction of the parent’s preference for family-provided care, $\alpha^p \eta$.

The costs children incur from providing home care contains the transfer $Q$ because it comes out of the parent’s wealth which is valued at $V^k_{a_p}$. It also includes the marginal worker’s labor income and tax benefits (the only factors a selfish agent would consider) because she uses her time endowment for home care instead of working in the market sector. We denote the opportunity costs of providing care, $Y^k(j, e^p; 0) - Y^k(j, e^p; 1)$, by $oc(j, e^p)$. The opportunity cost of providing home care is increasing in the marginal worker’s labor income and decreasing in the tax benefits. When the children are retired the opportunity cost equals zero.
The reservation transfer for the kid generation is given by:

\[ Q^k = \frac{V^k_{ah}(oc - sh) - V^k_{ap}(q - sf) - \alpha^k \eta}{V^p_{ah} - V^p_{ap}} \equiv -\mu^k. \]  

(18)

Thus, for any \( Q \geq Q^k \) the kid generation’s surplus from home care is positive. The difference in marginal valuations \( \mu^k \) is the transfer motive of the children. In equilibrium it is negative throughout the state space which says that children have a higher valuation for their own wealth than for the wealth of the parent household, i.e. \( V^k_{ah} > V^k_{ap} \). Note that our continuous-time framework allows us to obtain this reservation value in a very simple form when we are given the value function.

Analogously, we can calculate the parent household’s surplus from home care given any price \( Q \) using HJB (12):

\[
\rho V^p = \max \left\{ \frac{u(c^p) + \eta + \alpha^p u^k(c^k, n^k) + \dot{a}^p(a^p, 1)V^p_{ah} + \dot{a}^k(c^k, 1)V^p_{ak}}{V^p_{ah} - V^p_{ap}} \right. \\
\left. \text{parent’s flow value from home care} \equiv H^{P|h} \right. \\
\left. \frac{u(c^p) + \alpha^p u^k(c^k, n^k) + \dot{a}^p(a^p, 0)V^p_{ap} + \dot{a}^k(c^k, 0)V^p_{ak}}{V^p_{ah} - V^p_{ap}} \right. \\
\left. \text{parent’s flow value from formal care} \equiv H^{P|f} \right. \\
+ \text{jump terms,} \\
\right\}
\]

where again optimal consumption and gifts from the previous stages are substituted. Parent’s surplus from home care is \( S^p(Q) = H^{p|h}(Q) - H^{p|f} \) and it is non-negative if:

\[
\frac{V^p_{ah}(Q + s_h) + V^p_{ap}(q - s_f) + \eta \geq V^p_{ah} oc + V^p_{ap} Q}{\text{marginal benefit}} \geq \frac{V^p_{ah} oc + V^p_{ap} Q}{\text{marginal cost}}.
\]

The left-hand side of this equation shows the benefits and the right-hand side the costs to the parent household of obtaining home care. The parent internalizes the fact that the children get \( Q + s^h \) using \( V^p_{ah} \). She values the fact that she does not have to spend \( q - s_f \) on nursing-home care and has additional utility, \( \eta \), from getting family-provided home care. On the cost side, she has to pay for home care and takes into account the care-provider’s opportunity cost.
The maximum the parent is willing to transfer to obtain home care is given by

\[
\bar{Q}_p = \frac{V_{p}^{a}(oc - s_h) - V_{p}^{a}(q - s_f) - \eta}{V_{a}^{p} - V_{a}^{p}} \equiv \mu^p.
\]  

(19)

Thus, as long as \( Q \leq \bar{Q}_p \) the parent’s surplus from home care is positive. The difference in marginal valuations \( \mu^p \) is the transfer motive of the parent household which is negative throughout the state space, i.e. \( V_{a}^{p} > V_{a}^{p} \).

Home care will take place if the maximum transfer the parent is willing to provide exceeds the children’s reservation price, i.e. \( \bar{Q}_p \geq Q_k \). If this is the case then any \( Q \in [Q_k, \bar{Q}_p] \) induces home care. According to Nash-bargaining the share of the surplus obtained by each generation equals their bargaining power. Symmetric Nash-bargaining, for example, picks the average of the reservation prices, \( Q^* = \frac{1}{2}(Q_k + \bar{Q}_p) \). This expression is simple because the payoffs from home care occur over an instant of time are linear.

Figure 1 illustrates a situation in which home care is efficient. The old’s reservation value is above the one from the young. The distance between them is the total surplus that arises from home care. With symmetric Nash-bargaining this surplus is shared equally.

Figure 2: Home-care decision

The home-care supply and demand functions. Home-care is a binary choice. It takes place only if the maximum willingness to pay exceeds the reservation price. Because of symmetric Nash-bargaining, the equilibrium price is such that the surplus is equally shared.

Constrained case
We now turn to the determination of the transfer when one (or both) generations are at the no-borrowing constraint.

Once again we begin with stage 4. The optimal consumption strategy is either the unconstrained level whenever enough resources are available, or, if not, to consume all available resources (see Barczyk & Kredler, 2012). So players’ realized consumption levels \( c_p^* \) and \( c_k^* \) (under the assumption that home care takes place) are

\[
\begin{align*}
    c_p^* &= \begin{cases} 
        c^p & \text{if } a_p > 0 \\
        \min\{c^p, P^p - Q - g^p + g^k\} & \text{if } a_p = 0
    \end{cases}, \\
    c_k^* &= \begin{cases} 
        c^k & \text{if } a_k > 0 \\
        \min\{c^k, s_h + Q + Y^k + g^p - g^k\} & \text{if } a_k = 0.
    \end{cases}
\end{align*}
\]

Note that when informal care takes place, the children receive the government subsidy \( s_h \) and after-tax labor earnings \( Y^k \) excluding the marginal worker’s labor income (or \( P^p \) when the children are retired). The old has the pension \( P^p \) as income.

Whenever parents give altruistic transfers, they implement the consumption rate for the child – desired consumption \( \tilde{c}^k \) – which is in their best interest. It must satisfy the FOC

\[
u_p^c(c^p, n^p) = \alpha_p u_c^k(\tilde{c}^k, n^k).
\]

Using CES utility for \( u \), substitute the marginal utilities into this FOC

\[
\frac{n^p}{\phi(n^p)} \left( \frac{c^p}{\phi(n^p)} \right)^{-\gamma} = \alpha_p \frac{n^k}{\phi(n^k)} \left( \frac{c^k}{(1 + \nu)\phi(n^k)} \right)^{-\gamma}.
\]

Simplify by taking out the denominator from the round brackets

\[
n^p \phi(n^p)^{-1}(c^p)^{-\gamma} = \alpha_p n^k (1 + \nu)^{\gamma} \phi(n^k)^{\gamma-1}(c^k)^{-\gamma}.
\]

Multiply through by \( (c^k)^{\gamma} \) and \( (c^p)^{\gamma} \)

\[
n^p \phi(n^p)^{-1}(c^k)^{\gamma} = \alpha_p n^k (1 + \nu)^{\gamma} \phi(n^k)^{\gamma-1}(c^p)^{\gamma}.
\]
Solving for $c^k$ yields $\tilde{c}^k$

$$\tilde{c}^k = (1 + \nu) \left[ \alpha_p \frac{n^k}{n^p} \left( \frac{\phi(n^k)}{\phi(n^p)} \right)^{\gamma - 1} \right]^{1/\gamma} c^p.$$ 

For log-utility this simplifies to

$$\tilde{c}^{k, \log} = \frac{(1 + \nu)n^k}{n^p} \alpha_p c^p.$$ 

Let’s define

$$\hat{\alpha}_p(n^k, n^p) = \alpha_p (1 + \nu)^{\gamma} \frac{n^k}{n^p} \left( \frac{\phi(n^k)}{\phi(n^p)} \right)^{\gamma - 1}$$

and so desired consumption by the parents for the children is given by:

$$\tilde{c}^k = \hat{\alpha}_p^{1/\gamma} c^p.$$ 

Desired consumption by the children for the parents is given by:

$$\tilde{c}^p = \hat{\alpha}_k^{1/\gamma} c^k,$$

where

$$\hat{\alpha}_k(n^p, n^k) = \alpha_k (1 + \nu)^{-\gamma} \frac{n^p}{n^k} \left( \frac{\phi(n^p)}{\phi(n^k)} \right)^{\gamma - 1}.$$ 

For log-utility this simplifies to

$$\tilde{c}^{p, \log} = \frac{n^p}{(1 + \nu)n^k} \alpha_k c^k.$$ 

Given these strategies, in the gift-giving stage 3 each agent strives to lift the other agent up to the level of consumption that she desires for the other if the other is below this level.
These desired consumption levels are

\[
\tilde{c}^o = \begin{cases} 
\alpha_k^{1/\gamma} c^k & \text{if } a_k > 0, a_p = 0, \\
\min \left\{ \alpha_k^{1/\gamma} c^k, \frac{\alpha_k^{1/\gamma}}{1+\alpha_k^{1/\gamma}} (p^p + y^k + s_h) \right\} & \text{if } a_k = 0, a_p > 0 
\end{cases}
\]

\[
\tilde{c}^k = \begin{cases} 
\alpha_p^{1/\gamma} c^p & \text{if } a_k = 0, a_p > 0 \\
\min \left\{ \alpha_p^{1/\gamma} c^p, \frac{\alpha_p^{1/\gamma}}{1+\alpha_p^{1/\gamma}} (p^p + y^k + s_h) \right\} & \text{if } a_k = a_p = 0
\end{cases}
\]

As long as one household in the family has assets available, the desired levels are just a fraction of the household’s own unconstrained consumption, their size being governed by the intensity of altruism and the number of household members. When both have run out of assets, the desired levels are given by the agent’s desired allocation of the dynasty’s flow income \((P^p + y^k + s_h)\) under informal care, where \(s_h\) is the government subsidy and \(y^k\) is after-tax labor earnings of children excluding the marginal worker’s labor income (or \(P^k\) when the children are retired).

Consider first the gift-giving decision of the parent. Whenever the transfer \(Q\) from the second stage is not sufficient to lift the children to the desired consumption \(\tilde{c}^k\), the parent will make up the difference by a subsequent gift \(g^p\). Indeed there is an optimal transfer level \(Q_p^*\) for the parent: if \(Q < Q_p^*\), the parent will give a gift \(g^p = Q_p^* - Q\) to implement her preferred consumption for the children. If \(Q \geq Q_p^*\), the gift is zero. We have

\[
g^p = \max \{ Q_p^* - Q, 0 \},
\]

where

\[
Q_p^* = \begin{cases} 
-\infty & \text{if } a_k > 0 \\
\min \{ \tilde{c}^k, c^k \} - s_h - y^k & \text{if } a_k = 0
\end{cases} \tag{22}
\]

We define \(Q_p^* = -\infty\) whenever \(a_k > 0\) since the parent would like to receive an infinitely high flow of transfers since the transfer motive is always negative \((\mu^p < 0)\). When \(a_k = 0\), we need to make the adjustment \(\min \{ \tilde{c}^k, c^k \}\) since the donor never wants gifts to go into savings, again since \(\mu^p < 0\) (see again Barczyk & Kredler, 2012). We note that \(Q_p^*\) may become negative if the children’s income \(s_h + y^k\) is high, which would imply that the old would actually like to take away from the young if she could; the optimal gift is then \(g^p = 0\).

Following the same logic for the children, we define their desired transfer level as \(Q_k^*\)
and write

\[ g^k = \max\{Q - Q^*_k, 0\}, \]

where

\[ Q^*_k = \begin{cases} \infty & \text{if } a_p > 0 \\ Pp - \min\{\tilde{c}^p, c^p\} & \text{if } a_p = 0 \end{cases}. \]  

(23)

The kids would like to receive unbounded transfer flows as long as \( a_p > 0 \), and would lift the parent up to their desired consumption level (again making sure that transfers do not go into savings since \( \mu^k < 0 \)) if the exchange-motivated transfer \( Q \) is above the maximum that the children accept from the parent.

From this it follows that \( Q^*_k > Q^*_p \) must hold. Whenever one of the households has positive wealth, this statement is obvious because at least one of the desired transfers is unbounded. When both households are broke, imperfect altruism implies that each player would choose the other to consume less than herself, resulting in the ideal transfer being higher for the young.

We now argue that we only have to consider transfers \( Q \in [Q^*_p, Q^*_k] \). To see that we need not consider \( Q < Q^*_p \), observe that the parent would react to such a low transfer by a gift in the gift-giving stage, lifting up the total amount given to the young to \( Q + g^p = Q^*_p \). Thus any transfer \( Q < Q^*_p \) will lead to the same consumption-savings allocation and the same surplus as \( Q = Q^*_p \), so we may consider these transfers as equivalent and restrict the analysis to \( Q \geq Q^*_p \). Similarly, any \( Q > Q^*_k \) would be “undone” by a gift from the children, leading to the same allocation and surplus as \( Q = Q^*_k \).

A key advantage of restricting the analysis to the interval \( Q \in [Q^*_p, Q^*_k] \) is that both agents’ surpluses are monotone on this range: the parent strictly prefers lower transfers and children prefer higher transfers, the bounds of the interval being their respective bliss points. Now taking into account the non-negativity constraint on \( Q \), we define the following bounds on the equilibrium transfer:14 THERE SEEMS TO BE A TYPO HERE??? SHOULD

\[ \text{In our computations, we also impose an upper bound } Q_{max} < \infty \text{ on } Q^*_k \text{ for computational purposes. In regions where the parent is wealth-rich but faces only a short time to live, children can essentially count on possessing all dynasty wealth within little time and thus the timing of transfers becomes inessential for players. In such regions players are essentially pooling their wealth, transfer motives being close to zero. This can lead equilibrium transfers to reach very high levels (see equation (19)), which has no implications on the allocation but slows down our algorithm considerably.} \]
THERE BE A MAX ON $Q_{UB}$

\[ Q_{lb} = \max\{0, Q_p^*\}; \quad Q_{ub} = \min\{0, Q_k^*\}. \tag{24} \]

If $Q_p^* < 0$ then the parent’s ideal transfer is negative and so taking into account the non-negativity constraint on the transfer the parent’s constrained ideal transfer is zero. If $Q_k^* < 0$ then the parent cannot afford the level of consumption which the kids deem desirable and they provide a gift in order for the parent to consume this level. In this situation any positive transfer, $Q \geq 0$, would be undone and we set $Q = 0$.

The home-care decision is then characterized as follows:

**Proposition (general characterization of home-care decision):** Let $Q_p^*$ and $Q_k^*$ be defined as in (22) and (23), and let $Q_{lb}$ and $Q_{ub}$ be as defined in (24). Then $Q_p^* < Q_k^*$, and in equilibrium the following hold:

1. (bliss points are undesirable) If $S_p(Q_{lb}) < 0$ or $S_k(Q_{ub}) < 0$, then $h^* = 0$.

2. (bliss points are desirable) If $S_p(Q_{lb}) \geq 0$ and $S_k(Q_{ub}) \geq 0$, then there exist thresholds $Q_k^* \in [Q_{lb}, Q_{ub}]$ and $Q_p^* \in [Q_{lb}, Q_{ub}]$ such that $S_k(Q) \geq 0$ iff $Q \geq Q_k^*$ and $S_p(Q) \geq 0$ iff $Q \leq Q_p^*$.

   (a) (excessive reservation transfer) If $Q_k^* > \bar{Q}_p$, then $h^* = 0$.

   (b) (bargaining solution) If $Q_k^* \leq \bar{Q}_p$, then $h^* = 1$ and

   \[ Q^* = \max_{Q \in [Q_k^*, \bar{Q}_p]} \{S_k(Q)\alpha S_p(Q)(1-\alpha)\}. \]

   For the parent $g^p = 0$ and for the kids $g^k = 0$ if $Q_k^*\geq 0$. If $Q_k^* < 0$, then $g^k = -Q_k^* > 0$ and $Q^* = 0$. Consumption is as given in (20) and (21) using $Q = Q^*$.

We now go in detail over the different cases covered by the proposition:

1. If the parent is not willing to demand home care even for the lowest-possible transfer (i.e. $S_p(Q_{lb}) < 0$), or the child is not willing to provide care for the highest-possible transfer ($S_k(Q_{ub}) < 0$), then no home care takes place.
2. If both are willing to consider home care under some transfer, we can find the child’s reservation transfer \( Q^k \in [Q_{lb}, Q_{ub}] \) above which \( h^k(Q) = 1 \). Note that this reservation transfer may be equal to \( Q_{lb} \) and/or to zero if \( S^k(Q_{lb}) \geq 0 \). Also, the parent’s willingness to pay is \( \bar{Q}^p \in [Q_{lb}, Q_{ub}] \) below which the \( h^p(Q) = 1 \). This willingness to pay may equal \( Q_{ub} \) if \( S^p(Q_{ub}) \geq 0 \). We can distinguish the following two cases according to the ordering of \( Q^k \) and \( \bar{Q}^p \):

(a) \( Q^k > \bar{Q}^p \): there is no \( Q \) such that both agents have a positive surplus and thus \( h^* = 0 \).

(b) \( Q^k \leq \bar{Q}^p \): the surplus is positive for both agents on \( Q \in [Q^k, \bar{Q}^p] \), thus \( h^* = 1 \).

We can find the Nash-bargaining solution \( Q^* \) by evaluating its first-order condition for \( Q \) on \( Q \in [Q^k, \bar{Q}^p] \), which can be shown to be decreasing on \( [Q^k, \bar{Q}^p] \).

The following sub-cases are of interest:

i. \( Q_{lb} = Q_{ub} = 0 \): This case arises when the kids are not willing to accept a transfer \( Q > 0 \) from the parent and would undo this by an altruistic gift, i.e. \( Q^*_k < 0 \). In this case we only have to check if both agents prefer home care to formal care for \( Q = 0 \), in which case home care takes place.

ii. \( Q_{lb} = 0 < Q_{ub} \): The parent’s bliss point is such that she would prefer not to give any transfer, i.e. \( Q^*_p = 0 \). In this case a corner solution \( Q^* = 0 \) may arise, which is characterized by the Nash-bargaining FOC being negative at \( Q = 0 \).

iii. \( 0 < Q_{lb} < Q_{ub} \): In this case we typically find an interior solution, which may be identified by finding the root of the Nash-bargaining FOC on \((Q_{lb}, Q_{ub})\).\(^{15}\)

Finally, we note that the case where both players are unconstrained is included as a special case covered in point 2 of the proposition.

\(^{15}\)If the donor is able to implement the preferred consumption for the recipient, which amounts \( \alpha^{1/\gamma}c_0 < c_y \) for the old, then a corner solution cannot occur. The reason is that the derivative of the surplus \( S^o(Q) \) is zero at \( Q^l = Q^*_o \). At this point the surplus of the young could be increased without lowering the surplus of the old, thus ruling it out as the bargaining solution if \( \alpha < 1 \).
5 Calibration

An individual dies with certainty at age $J = 95$. Beginning at age $J_{\delta} = 70$, an agent faces mortality risk and from age $J_{ltc} = 70$ on, the agent runs the risk of requiring LTC. The mandatory retirement age is set to $J_{ret} = 65$. A member of the old cohort is of age $j^o \in [45, 95]$ and a member of the young cohort is of age $j^y \in [20, 70]$. A family consists of one member of the old cohort aged $j^o$ and one member of the young cohort aged $j^y = j^o - 25$.

Mortality & LTC hazards

Our policy experiments surround the frail-elderly population that require substantial amounts of care. We define an individual to require LTC if the hours of care the individual obtains corresponds to a part-time job or more (i.e., care $> 19$ weekly hours); we also include in this category individuals residing in a nursing home. Instead of using monthly hours of care as the cutoff to determine the LTC status we use a certain number of functional limitations since these are more reliably measured (this cutoff is 6, see the data appendix for details).

Making use of the longitudinal dimension of the HRS (using waves 1996-2010) we estimate conditional mortality probabilities separately for non-LTC individuals, $\pi^0_j$, and for individuals requiring LTC, $\pi^1_j$, as functions of age, $j$. We also need an estimate of the probability of requiring LTC at a certain age, $\lambda_j$. Using the estimates $\pi^0_j$, $\pi^1_j$ and $\lambda_j$ allows us to back out the conditional LTC probabilities $\phi_j$.\(^\text{16}\)

Finally we transform the conditional probabilities into yearly hazard rates; $\delta^0_j$ denotes the mortality hazard for an age-$j$ individual who does not require LTC and $\delta^1_j$ for an individual requiring LTC; $\sigma_j$ denotes the age-$j$ LTC hazard. Table 1 provides an overview of the estimated conditional probabilities of death and requiring LTC.

\(^{16}\)We do not estimate $\phi_j$ directly from the data since the LTC status according to our measure displays too much variation for an individual over time.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
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<td>$\pi_j^0$</td>
<td>pr. of death by age if healthy</td>
<td>$[1 + \exp(-(-3.43 + 0.11j))]^{-1}$</td>
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<tr>
<td>$\pi_j^1$</td>
<td>pr. of death by age if ltc</td>
<td>$[1 + \exp(-(-1.05 + 0.057j))]^{-1}$</td>
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<tr>
<td>$\lambda_j$</td>
<td>fraction of ltc population by age</td>
<td>$[1 + \exp(-(-3.74 + 0.134j))]^{-1}$</td>
</tr>
<tr>
<td>$\phi_j$</td>
<td>pr. of ltc by age</td>
<td>back out using $\pi_j^0, \pi_j^1, \lambda_j$</td>
</tr>
</tbody>
</table>

Table 1: Mortality and LTC probabilities

Logistic-regression estimates based on the HRS waves 3-10 (1996-2010); $\lambda_j$ is estimated in order to back out $\phi_j$ together with $\pi_j^0$ and $\pi_j^1$. The bi-annual probabilities are converted into annual hazard rates: $\pi_j^0 \rightarrow \delta_j^0$, $\pi_j^1 \rightarrow \delta_j^1$, and $\phi_j \rightarrow \sigma_j$.

Figure 3 shows the estimated yearly hazard rates for individuals of ages 70-95. The blue-dashed line is the LTC hazard rate, $\sigma_j$. The solid-blue line is the mortality hazard without LTC needs, $\delta_j^0$, and the red-solid line is the mortality hazard of an LTC individual, $\delta_j^1$.

Figure 3: Hazards

Yearly hazard rates based on estimating conditional probabilities using the HRS waves 3-10.

Once an individual has LTC needs the mortality hazard increases dramatically. The LTC hazard is fairly small prior to age 80 but then increases rapidly. For healthy individuals the mortality hazard is of a similar magnitude as the LTC hazard.

Figure 4 shows the fraction of individuals requiring LTC at a given age (conditional on being alive). In the data it is the case that at age 70 almost 3% of individuals fall into the
LTC category.

Figure 4: $\lambda$

Estimated fraction of LTC individuals out of population (sick and healthy) aged 70-95.

Table 2 summarizes the baseline parameters of the model economy. We will now discuss the baseline calibration.
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<thead>
<tr>
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<td>$\alpha^o$</td>
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<td>own calibration</td>
</tr>
<tr>
<td>$\alpha^y$</td>
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<td>own calibration</td>
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<tr>
<td>$\eta$</td>
<td>home-care preference</td>
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<td>own calibration</td>
</tr>
<tr>
<td>$\sigma_{\epsilon}$</td>
<td>st. dev. log-earnings</td>
<td>0.78</td>
<td>Hintermaier &amp; Koeniger (2011)</td>
</tr>
<tr>
<td>$\sigma_{a^o}$</td>
<td>st. dev. log-wealth (old)</td>
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<td>Hintermaier &amp; Koeniger (2011)</td>
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<tr>
<td>$\xi$</td>
<td>earnings hazard</td>
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<td>standard</td>
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<tr>
<td>$\rho_{\epsilon\epsilon}$</td>
<td>intergen. earnings elasticity</td>
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<td>Solon (handbook)</td>
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<td>$\rho_{a^o a^o}$</td>
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<td>Budria-Rodriguez et. al (2002)</td>
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<tr>
<td>$q$</td>
<td>private exp. on formal care</td>
<td>$25k$</td>
<td>policy reports</td>
</tr>
<tr>
<td>$q_{ma}$</td>
<td>government exp. on MA</td>
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<tr>
<td>$c_{ma}$</td>
<td>MA consumption floor</td>
<td>$2.5k$</td>
<td>own calibration</td>
</tr>
</tbody>
</table>

Table 2: Parameters in baseline calibration

The young’s altruism parameter is chosen so that the average fraction of young (ages 20-45) providing transfers to parents (5.5%) is roughly in the ballpark from what we know from the HRS (5.4% in the HRS 2002, see table 13 in TransData). The old’s altruism parameter is chosen so that the average fraction of old (ages 45-70) providing transfers to children (13.6%) is roughly in the ballpark from what we know from the HRS (14% in the HRS 2002, see table 11 in TransData).

**Earnings**

Annual labor earnings of a $j$-year old individual are given by:

$$w_j = \exp \left( \beta_0 + \beta_1 j + \beta_2 j^2 + \beta_3 j^3 \right) \exp (\epsilon_j), \quad j = 0, \ldots, 44,$$

where $j$ proxies experience. We assume that labor earnings are log-normally distributed so
that log-labor earnings are normally distributed.

Idiosyncratic log-labor productivity, $\epsilon_j$, follows an AR(1) process:

$$
\epsilon_j = \rho \epsilon_{j-1} + \zeta_j,
$$

where $\rho = \text{corr}(\epsilon_{t+1}, \epsilon_t)$ is the autocorrelation coefficient of log-productivity. Innovations $\zeta$ to the young’s and the old’s labor productivity are assumed to be uncorrelated, i.e. $\text{corr}(\zeta^o_t, \zeta^y_t) = 0, \forall t > 0$.

It follows that log-earnings of old and young at time $t$ are given by:

$$
\log(w^o_t) = f^o_t + \epsilon^o_t = f^o_t + \rho^t \epsilon^o_0 + \sum_{s=1}^{t} \rho^{s-1} \zeta^o_s,
$$

$$
\log(w^y_t) = f^y_t + \epsilon^y_t = f^y_t + \rho^t \epsilon^y_0 + \sum_{s=1}^{t} \rho^{s-1} \zeta^y_s,
$$

where $\text{corr}(\epsilon^o_0, \epsilon^y_0) = 0.5$ is the intergenerational elasticity of labor earnings.$^{17}$

We approximate the AR(1) process using a discrete random variable $\tilde{\epsilon}$ following a discrete-state Markov chain. This random variable can take on one of three values from the set $\{-d, 0, d\}$. The worker has low productivity when $\tilde{\epsilon} = -d$, medium productivity when $\tilde{\epsilon} = 0$ and high productivity when $\tilde{\epsilon} = d$. We assume that the marginal distribution of the young’s and the old’s productivity are the same and approximate it by matching the first and second moments.

Denote the probabilities (fractions of households) associated with the discrete-random variable by $\{p^l, p^m, p^h\}$; we impose the restriction that $p^l = p^h \equiv p$. The unconditional

$^{17}$We can see that despite productivity innovations being uncorrelated, the young and the old earnings are still correlated over the lifetime due to the initial (heritability) correlation. The covariance is given by:

$$
\text{cov}(\log(w^o_t), \log(w^y_t)) = \text{cov}\left(\rho^t \epsilon^o_0 + \sum_{s=1}^{t} \rho^{s-1} \zeta^o_s, \rho^t \epsilon^y_0 + \sum_{s=1}^{t} \rho^{s-1} \zeta^y_s\right) = \rho^{2t} \text{cov}(\epsilon^o_0, \epsilon^y_0),
$$

and the correlation by:

$$
\text{corr}(\log(w^o_t), \log(w^y_t)) = \frac{\text{cov}(\log(w^o_t), \log(w^y_t))}{\sqrt{\text{Var}(\log(w^o_t))} \sqrt{\text{Var}(\log(w^y_t))}} = \rho^{2t} \text{corr}(\epsilon^o_0, \epsilon^y_0) = 0.5 \rho^{2t}.
$$

After, for example, 20 years the correlation between the old and the young’s log-earnings is $0.5(0.99)^{40} = 0.3345$ when using autocorrelation $\rho = 0.99$ (here, a very persistent process).
mean $\mu_\epsilon$ and variance $\sigma_\epsilon^2$ are matched if:

$$p(-d) + p(d) = \mu_\epsilon,$$
$$\sqrt{2pd^2} = \sigma_\epsilon.$$ 

The mean equals zero and the standard deviation of log-productivity we take from the data, $\sigma_\epsilon = 0.78$. According to Gaussian quadrature in the case of the univariate standard normal distribution with mean zero and variance one, $d = \sqrt{3}$. With our standard deviation $d = \sigma_\epsilon \sqrt{3} = 1.351$. Using these values provides us with the approximated marginal distribution:

$$p = \frac{\sigma_\epsilon^2}{2d^2} = \frac{1}{6}, \quad \text{and} \quad p_m = 1 - 2p = 1 - \frac{\sigma_\epsilon^2}{d^2} = \frac{2}{3}.$$ 

We assume that the marginal distribution is stationary so at each age there are the same fractions of households with various productivities. Households may experience changes to their productivity and we will return to the time-series properties of the idiosyncratic productivity after discussing the economy’s initial earnings distribution.

**Initial labor earnings distribution**

In order to start the economy off we need to specify the joint probabilities of families’ earnings. These joint probabilities arise due to the intergenerational transmission of ability as measured by the intergenerational elasticity of earnings, $corr(\epsilon_0^y, \epsilon_0^y) = 0.5$ (see Solon, Handbook chapter section 4.2). We assume that initial log-productivity in a family $\epsilon_0^y$ and $\epsilon_0^y$ is jointly normally distributed with mean $\mu_\epsilon = 0$, standard deviation $\sigma_\epsilon^y = \sigma_\epsilon^y = \sigma_\epsilon$, and correlation $corr(\epsilon_0^y, \epsilon_0^y) = 0.5$.

Let’s denote the corresponding joint probabilities (fractions of dynasties with various productivity combinations) by

$$\begin{pmatrix}
p & p_m & p \\
q_{lu} & q_{mu} & q_{lu} & |p \\
q_{lm} & q_{mm} & q_{lm} & |p^m \\
q_{ll} & q_{ml} & q_{ul} & |p
\end{pmatrix},$$

where the first subscript denotes the productivity realization of the young and the second of the old. The approximated marginal distribution is added at the margins for convenience.
Summing rows and columns (which have to add up to the corresponding fraction of the marginal distribution) provides us with 5 linearly-independent equations for 9 unknowns. Pinning down the intergenerational correlation yields an additional restriction. Finally, we impose the following three restrictions: (1) \( q_{ll} = q_{uu} \), i.e. the fraction of dynasties in which both have low productivity equals the one in which both have high productivity; (2) \( q_{lu} = q_{ul} \), i.e. there is an equal fraction of dynasties in which either the young or the old has the low productivity and the other has high productivity; and (3) the conditional variance is homoscedastic.

The joint probabilities are given by

\[
\begin{pmatrix}
\frac{1}{6} & 2/3 & 1/6 \\
0 & 1/12 & 1/12 & 1/6 \\
1/12 & 0.5 & 1/12 & 2/3 \\
1/12 & 1/12 & 0 & 1/6
\end{pmatrix}.
\]

*Earnings hazard*

In order to pin down the transition probabilities we match the value of the autocorrelation \( \rho = 0.99 \) and impose additional restrictions. We impose that transitioning from the low to the high and from the high to the low state in one shot is impossible, i.e. \( \pi_{h|l} = \pi_{l|h} = 0 \). Furthermore, the probabilities of reaching the medium state from either the low or the high state are set equal, i.e. \( \pi_{m|l} = \pi_{m|h} \).

Next, since the marginal distribution of productivities is stationary we have to make sure that the transition probabilities preserve this stationarity. Thus, we need to have that \( p_{t+1} = p_t \) and \( p_{t+1}^m = p_t^m \) (recall, fraction \( p \) is of low productivity, \( p^m \) have medium productivity, and \( p \) have high productivity):

\[
\begin{pmatrix}
p \\
p^m \\
p
\end{pmatrix} = \begin{pmatrix}
\pi_{l|l} & \pi_{l|m} & 0 \\
\pi_{m|l} & \pi_{m|m} & \pi_{m|h} \\
0 & \pi_{h|m} & \pi_{h|h}
\end{pmatrix}\begin{pmatrix}
p \\
p^m \\
p
\end{pmatrix}, \quad \text{where} \quad \pi_{m|l} = \pi_{m|h} \equiv h.
\]

Furthermore, the columns of the transition matrix have to sum to unity \( \pi_{l|l} = 1 - h, \pi_{h|h} = \)
1 - h, and $\pi_{m|m} = 1 - \pi_{l|m} - \pi_{h|m}$.

\[
\begin{pmatrix}
  p \\
  p^m \\
  p
\end{pmatrix} = 
\begin{pmatrix}
  1 - h & \pi_{l|m} & 0 \\
  h & 1 - \pi_{l|m} - \pi_{h|m} & h \\
  0 & \pi_{h|m} & 1 - h
\end{pmatrix}
\begin{pmatrix}
  p \\
  p^m \\
  p
\end{pmatrix}.
\]

From the first and third equation we have that

\[(1 - h)p + \pi_{l|m}p^m = (1 - h)p + \pi_{h|m}p^m\]

so that $\pi_{l|m} = \pi_{h|m}$. The second equation tells us that

\[p^m = 2hp + (1 - \pi_{l|m} - \pi_{h|m})p^m,\]

and because $\pi_{l|m} = \pi_{h|m} \equiv q$ it says that

\[qp^m = hp.\]

Thus, we are left with two unknowns $h$ and $q$. The final restriction is given by the autocorrelation, $\rho$:

\[\rho = \frac{Cov(\tilde{\epsilon}_t, \tilde{\epsilon}_{t-1})}{Var(\tilde{\epsilon}_t)}\]

The autocovariance is given by:

\[Cov(\tilde{\epsilon}_t, \tilde{\epsilon}_{t-1}) = E(\tilde{\epsilon}_t\tilde{\epsilon}_{t-1}) = 2p(1 - h)d^2.\]

The variance is given by $Var(\tilde{\epsilon}_t) = 2pd^2$ and so the autocorrelation is $\rho = 1 - h$ which gives us the value for $h = 1 - \rho = \pi_{m|l} = \pi_{m|h}$. We can now solve for $q$:

\[q = \frac{(1 - \rho)p}{p^m} = \pi_{l|m} = \pi_{h|m}.\]

From the transition probability matrix it is easy obtain the hazard matrix for earnings uncertainty.

*Replacement rates*
Upon retirement we assume that social security pays a fraction of last-period labor earnings. This replacement rate is progressive in the sense that individuals with higher labor earnings in their last working period receive a smaller fraction. Replacement rates corresponding to the three different levels of wage earnings are chosen to be \{0.6, 0.5, 0.4\}.

Figure 5 portrays the life-cycle profile of labor earnings and social security.

![Figure 5: Wage/Social-Security profiles](image)

Calibrated labor earnings and social security profiles.

*Wealth*

In our economy the initial wealth distribution is not endogenously generated by the model but taken as given from the data. We now explain the calibration of the initial wealth distribution.

*Initial wealth distribution*

We calibrate a conditional wealth distribution for the old at age 45 and assume that the young have no wealth. The old’s labor earnings are informative about wealth holdings, but, we assume that the young’s labor earnings have no additional information on old’s wealth once we take into account old’s labor earnings. We assume that \(a^o\) and \(w^o\) are jointly log-normally distributed. In order to parameterize the conditional wealth distribution we need to find \(\beta\) of the following regression

\[
\log(a^o) - \mu_{a^o} = \beta(\log(w^o) - \mu_{w^o}) + \epsilon,
\]
where
\[
\beta = \frac{\text{cov}(\log(a^o), \log(w^o))}{\text{var}(\log(w^o))} = \frac{\text{Corr}(\log(a^o), \log(w^o))\sqrt{\text{Var}(\log(a^o))}}{\sqrt{\text{var}(\log(w^o))}}.
\]

From before we have the standard deviation of log-earnings. We need to obtain values for the correlation between log-wealth and log-labor earnings as well as the standard deviation of log-wealth. Hintermaier and Koeninger (2011, RED) report Gini coefficients of wealth for many years of the SCF, categorized by age range and for a restricted sample that excludes the top 10%. For the age category 46-55 they find that the Gini coefficient for wealth is \(\approx 0.55\). We use this value and back out \(\sigma(\log(a^o)) = 1.07\).

Budria-Rodriguez, Diaz-Gimenez, Quadrini, Rios-Rull (2002, Updated Facts...) report quite different correlation coefficients between earnings and wealth based on the 1992 and 1998 waves of the SCF. Correlation between earnings and wealth in the 1992 SCF is 0.23 and correlation between earnings and wealth in the 1998 SCF is 0.47. We use the average of both years, \(\text{Corr}(a^o, w^o) = 0.35\); adjusting for logarithm we get \(\text{Corr}(\log(a^o), \log(w^o)) = 0.2922\).

Now we are in a position to approximate the conditional normal distribution for \(\log(a^o)\) – the initial wealth distribution for the 45-year old. Since \(\log(w^o)\) and \(\log(a^o)\) are normally distributed, \(\log(a^o)\) conditional on \(\log(w^o)\) is also normally distributed:
\[
\log(a^o) \mid \log(w^o) \sim N(\mu, \sigma^2)
\]
where \(\mu\) is the conditional mean and \(\sigma^2\) is the conditional variance. We can obtain the conditional mean and the conditional variance from the following regression equation
\[
\log(a^o) - \mu_{a^o} = \beta(\log(w^o) - \mu_{w^o}) + \epsilon.
\]
Thus, the conditional mean and conditional variance are given by:
\[
\mu = \mu_{a^o} + \beta(\log(w^o) - \mu_{w^o}), \quad \sigma^2 = \sigma_{a^o}^2 - \beta^2\sigma_{w^o}^2 = 1.044
\]
\[
\Rightarrow \log(a^o) \mid \log(w^o) \sim N(10.7947 + 0.4(\log(w^o) - 9.9625), 1.044).
\]

Now, using the conditional mean and variance as inputs to the normal CDF we can find the proportion of dynasties over our grid for wealth.


Costs

We now discuss the choice for the cost of formal care \( q \). The per unit cost the representative nursing home incurs is \( \kappa \) and consists of the cost of providing nursing care, i.e., it does not include the cost of room, board and medical care. One unit of labor produces one unit of nursing care so the cost to the nursing home is \( \kappa = w^f \). One unit of nursing care generates \( q \) in revenue. The nursing home demands labor input \( L^f \) to maximize \((q - w^f)L^f\) and so \( w^f = q \).

What is a realistic value for \( q \)? We cannot just take the price of a nursing home since we need to have the cost of providing care. One way to go about this is to start with the cost of a nursing home and then deduct the value for room and board and medical services.\(^\text{18}\) It should be relatively straightforward to find the cost of only room and board. Then we can deduct this cost from the cost of a nursing home to get an upper bound for the value of care; it’s an upper bound since in the price of a nursing home there are many other services included such as medical assistance, entertainment programs etc. Another way of obtaining the value of care is by using the cost of a formal-home care nurse.

We found information on the components of estimated nursing home costs per resident day (in 1994 for the upper midwest). The categories nursing and other care-related account for approximately 45% of the estimated daily rate per resident day. A very old paper (1972, Ruchlin and Levey) reports that nursing accounts for 35.6% (Table 1) of the total cost (an average over nursing homes in Massachusetts). A reasonable approximation seems to be 40%, and assuming that the yearly cost of a nursing home is upwards of 60k we choose the cost of formal care to be \( q = 25k \). This price includes only the cost of nursing care. Any extras are captured by the consumer’s choice of consumption.

We still need to find how much it costs the government to pay per resident in a nursing home. Wouldn’t the 25k not also be reasonable to assume that it costs the government when it finances a nursing-home resident entirely? No, the government still needs to pay for room and board.

Preference

The altruism parameters are chosen in such a way that the model produces the right amount

\(^{18}\) We could also calibrate this parameter using the model. This could be an interesting result since the value of care provided by a nursing home is difficult to measure. But, calibrating this parameter is complicated by the fact that we cannot separately identify the home-care preference parameter. Also, it is better to use all available information.
of inter-vivos transfers. Preferences are logarithmic.

6  Preliminary results (full model)

6.1  Comparative statics: changes in $q$

An interesting observation is that if we decrease the price of formal care there is a pronounced increase in Medicaid participation. Our intuition was that a reduction in the price of formal care should lead to a strong increase in the use of formal care: the formal care region should expand and individuals choose formal care over Medicaid.

As expected, the formal care region does expand and crowds out the home care region. What we however did not take into account is the fact that the children decrease their gift to the parent under formal care. The kids control the gift and adjust it to their liking. When the parent makes her Medicaid decision she knows her level of consumption under formal care and decides accordingly. Thus, even if the price of formal care is relatively low it can still be too high for the parent to afford a higher consumption level than she obtains under Medicaid.

We illustrate this situation in figures 6 and 7. Consider the diagram in which kids and parents both have the medium productivity, the plot in the centre. When the price of formal care is at its benchmark value of $25k$ then home care takes place almost everywhere. In contrast, reducing the price of formal care to $14k$ leads to a much larger formal care region.

Considering the second stage of our timing protocol this diagram tells us that formal care takes place as long as the wealth of the children is roughly below the fifth grid point. In the first stage of the decision tree the figure indicates that the parent chooses Medicaid (because of the black region). The reason is simply that the parent depends on the children to pay for formal care. But, the children set their gift to their liking. In this case it happens to be that the gift is not large enough to enable consumption under formal care that is larger than Medicaid-provided consumption and hence the elderly opts for Medicaid.

In terms of the Proposition what happens is that the surplus of the children is negative at the feasible largest transfer that they accept from the parent. In other words, $S_k(Q_{ub}) < 0$ (part 1. of the Proposition) when the price of formal care is low. When the price is high it is positive.
The home-care region is in white, the formal-care region in grey, and the Medicaid region in black. The age of the parent is about 86.
The home-care region is in white, the formal-care region in grey, and the Medicaid region in black. The age of the parent is about 86.
7 Policy experiments (uses a simplified version of the model)

We begin by studying a simpler version of the model to help us understand the key mechanisms of the actual model. To this end we consider two infinitely-lived agents referred to as young and old. Both have deterministic income streams. The only differences between the young and the old agent are that the old may have to acquire a source of LTC and the young may provide home care; the old has a preference for home care; the degrees of altruism may differ.

The young faces a standard consumption-savings problem with the possibility of gifts. When the old requires LTC she also has the choice between working at the market wage \( w \) or provide LTC in exchange for a transfer \( T \) from the old. The old also faces a standard consumption-savings problem with the possibility of gifts. When she requires LTC, the old either obtains informal care in exchange for a transfer \( T \), purchases formal care at a per unit price \( p \) or gives up assets and income forever and enters an absorbing state in which consumption \( c_{ma} \) is provided; this state is interpreted as residing in a Medicaid-financed nursing home (MA).

In this modified setting, the state for a household is given by the young’s level of wealth \( (a_y) \), the old’s level of wealth \( (a_o) \) and whether the old requires LTC (1) or not (0).

For ease of presentation we show the HJBs neglecting the choice of gifts. Gifts only flow to constrained individuals and so the HJBs as presented here represent restrictions which need to be satisfied by unconstrained choices. When the old does not require LTC, the old’s and young’s HJBs are given by:

\[
p V^o(\omega, 0) = \max_c \left\{ u(c^o) + \alpha^o u(c^y) + \dot{a}^o V^o_{a^o}(\omega, 0) + \dot{a}^y V^o_{a^y}(\omega, 0) \right\} + \\
+ \sigma \left[ \max_{Z^o(\omega)} \left\{ V^o(\omega, 1), W^o(\omega^y) \right\} - V^o(\omega, 0) \right].
\]

\[
p V^y(\omega, 0) = \max_c \left\{ u(c^y) + \alpha^y u(c^o) + \dot{a}^y V^y_{a^y}(\omega, 0) + \dot{a}^o V^y_{a^o}(\omega, 0) \right\} + \\
+ \sigma \left[ \max_{Z^y(\omega)} \left\{ V^y(\omega, 1), W^y(\omega^y) \right\} - V^y(\omega, 0) \right].
\]

s.t.
\[
\dot{a}^o = r a^o + P - c^o,
\]
\[
\dot{a}^y = r a^y + w - c^y.
\]
The term that is multiplied by $\sigma$ captures the change in value from the state in which the old does not require LTC and the state in which she does require LTC. For the old this change is given by $[\max \{V^o(\omega, 1), W^o(a^y)\} - V^o(\omega, 0)]$. If the old does require LTC she faces the choice between entering the MA state valued at $W^o(a^y)$ or not. Not entering the MA state is valued at $V^o(\omega, 1)$. The young’s value changes to $W^y(a^y)$ if the old chooses the MA option and to $V^y(\omega, 1)$ if not.

$V^0(\cdot, 1)$ and $V^y(\cdot, 1)$ satisfy the following HJBs:

$$
\rho V^o(\omega, 1) = \max_{c^o, h^o \in \{0, 1\}} \{u(c^o) + \eta h^* + \alpha^o u(c^y) + \dot{a}^o(h^*)Z^o_{ao}(\omega) + \dot{a}^y(h^*)Z^y_{ao}(\omega)\}
$$

$$
\rho V^y(\omega, 1) = \max_{c^y, h^y \in \{0, 1\}} \{u(c^y) + \alpha^y [\eta h^* + u(c^o)] + \dot{a}^y(h^*)Z^y_{ao}(\omega) + \dot{a}^o(h^*)Z^o_{ao}(\omega)\}
$$

s.t.

$$\dot{a}^y(h^*) = ra^y + h^*(T + s_h) + (1 - h^*)w - c^y,$$

$$\dot{a}^o(h^*) = ra^o + P - h^*T - (1 - h^*)(p - s_f) - c^o.$$

Once the old requires LTC there are no sources of uncertainty. Recall that $h^* = 1$ if home care takes place and 0 otherwise. In addition to the transfer $T$ the young’s law of motion now includes a subsidy $s^h \geq 0$ for informal care and the old’s law of motion includes a subsidy $s^f \geq 0$ for formal care.

When the old enters the Medicaid-financed nursing home the old and young’s value functions satisfy:

$$\rho W^o(a^y) = u(c_{ma}) + \alpha^o u(c^y) + \dot{a}^y W^o_{ao}(a^y),$$

$$\rho W^y(a^y) = \max_{c^y} \{u(c^y) + \alpha^o u(c^y) + \dot{a}^y W^o_{ao}(a^y)\},$$

s.t.

$$\dot{a}^y = ra^y + w - c^y.$$

The old faces no more choices but her value still changes due to the presence of the young. The young faces a standard consumption-savings problem.
The young’s reservation transfer to supply home care is:

\[ T^S = \frac{V^{y}_{ah}(w - s_h) - V^{y}_{ah}(p - s_f) - \alpha^y \eta}{V^{y}_{ah} - V^{y}_{ah}} \equiv -\mu^y, \]

The old’s maximum transfer she is willing to pay for home care is:

\[ \bar{T}^D = \frac{\eta + V^{o}_{ao}(p - s_f) - V^{o}_{ah}(w - s_h)}{V^{o}_{ao} - V^{o}_{ah}} \equiv -\mu^o. \]

Table 8 summarizes the baseline parameter values used for the numerical example.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
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<tr>
<td>(\alpha^o)</td>
<td>0.3</td>
<td>(w)</td>
<td>22</td>
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<tr>
<td>(\alpha^y)</td>
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<td>(P)</td>
<td>20</td>
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<td>(\gamma)</td>
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<td>(p)</td>
<td>18</td>
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<td>(\rho)</td>
<td>3%</td>
<td>(\sigma)</td>
<td>15%</td>
</tr>
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<td>(r)</td>
<td>2%</td>
<td>(c_{ma})</td>
<td>6</td>
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<tr>
<td>(s_h, s_f)</td>
<td>0</td>
<td>(\eta)</td>
<td>0.007</td>
</tr>
</tbody>
</table>

The wage of the young is \(w\) and the pension of the old is \(P\). The price of formal care is \(p\) and Medicaid consumption is \(c_{ma}\). The hazard of requiring LTC is \(\sigma\) and home-care preference is \(\eta\). The home-care subsidy is denoted by \(s_h\) and the formal-care subsidy by \(s_f\).

The parameter values are chosen in such a way as to generate an instructive laboratory for a variety of cases which the model can generate. Nonetheless, the chosen values are not unreasonable. In particular, the degrees of altruism are in line with the calibration by Barczyk (2012) who studies an OLG model with young and old imperfectly-altruistic agents. The size of the coefficient of relative risk aversion \(\gamma\) is a common choice in macroeconomics. The Medicaid-financed consumption floor has been chosen so that Medicaid is chosen in at least one state (when both are broke) in the benchmark case without subsidies. The hazard of requiring LTC is on the larger side. In this simplified version of the model, however,
qualitatively the results would not change when changing this hazard rate. The main leap of faith is taken with respect to the size of the home-care preference parameter. A priori we know very little about what would constitute a reasonable magnitude for this preference parameter. In future versions of the paper we will tackle the calibration of this parameter in a careful manner.

We begin by considering the laws of motion for the young’s and the old’s wealth in the benchmark economy without subsidies. Figure 9 shows $\dot{a}^y$ and $\dot{a}^o$ when the old does not require LTC (blue) and when she does require LTC (red). The horizontal axis traces out the wealth of the old; along the vertical axis is the young’s wealth. An arrow emanates from a particular point in the discretized state space $(a^o, a^y)$. If an arrow points horizontally to the right ($\rightarrow$) it indicates that the old is saving ($\dot{a}^o > 0$) and the young consumes her cash-on-hand ($\dot{a}^y = 0$) for the given combination of wealth levels. If an arrow points vertically up ($\uparrow$) it means that the young is saving ($\dot{a}^y > 0$) and the old consumes her cash-on-hand ($\dot{a}^o = 0$) for the given combination of wealth levels. The length of an arrow signifies the magnitude of the savings behavior; e.g. a longer arrow means that the economy moves more quickly in the direction indicated by the arrow. The area in white corresponds to the region of the state space in which home care takes place; the grey area is the region in which the old makes use of formal care. When both are broke ($a^o = 0 = a^y$) the old enters the Medicaid state. These regions, of course, only exist when the old requires LTC.

First focus on the law of motion in the case when the old does not require LTC (the arrows in blue). We see that along the horizontal axis the old is saving while the young consumes hand-to-mouth. The old displays typical precautionary savings behavior building up a buffer of wealth in anticipation of the LTC shock. After all, the LTC shock is akin to a permanent reduction in the old’s income. Furthermore, the old wants to save herself away from the Medicaid state which occurs when both are broke, i.e. when $a^o = 0 = a^y$. The young does not share these concerns. When she has no wealth, she does not, and will not, provide gifts to the old; if she would, she would share the concern for precautionary saving. Also, the young does not expect her income to drop in case the LTC shock hits the old; in contrast, she might expect her income to rise if home care takes place at a transfer which is larger than her wage. Thus, when the young is broke, she simply consumes what she has on hand.

As we move away from the horizontal axis into the state space the same type of consumption-saving behavior persists: the old engages in precautionary savings while the young dis-saves.
As a matter of fact, the young dis-saves throughout the entire state space when the old does not require LTC (recall that the rate-of-time preference \( \rho \) is larger than the interest rate \( r \)). The old’s saving behavior is, however, more intricate.

Consider the upper-left corner where the young is rich and the old is getting poorer, say, the area to the left of \( a^o = 900 \) and above \( a^y = 2000 \). In this part of the state space the old’s saving behavior changes from saving to dis-saving. This is what Barczyk & Kredler (2013) refer to as the dynamic Samaritan’s dilemma. The old expects that she will eventually obtain gifts from the young, namely, when she is broke and he is very rich. In anticipation of this she consumes down her resources at an inefficiently high rate (figure 15 shows that the young provides gifts to the old when she is broke and he is rich). Furthermore, she knows that if the LTC shock hits she can rely on the young to subsidize her purchase of formal care (again, figure 15 shows that the young provides gifts to the old when she is broke and requires LTC). Thus, her precautionary savings motive is overshadowed in this region of the state space. In essence, the young is doing the precautionary savings on behalf of the old.

Figure 9: Laws of Motion: \( w = 22 \)

The blue arrows represent the law of motion for \( a^y \) and \( a^o \) when the old does not require LTC and the red arrows represent the law of motion when she requires LTC. The area in white corresponds to the region of the state space in which home care takes place; the grey area is the region in which the old makes use of formal care. When both are broke \((a^o = 0 = a^y)\) the old enters the Medicaid state.
As we lower the young’s wealth note that the old’s savings behavior stirs gradually away from the vertical axis. This happens because for once gifts are decreasing as the young becomes poorer and, second, the time to Medicaid is shortened. Both forces act as an incentive for the old to save.

Now focus on the law of motion when the old requires LTC (the arrows in red). In the vast majority of the state space both the young and the old now dis-save. Since there is no “coming back” from the LTC state and the rate-of-time preference exceeds the interest rate this consumption-savings behavior should not come as a big surprise.

It is noteworthy that in regions of the state space in which the intra-family wealth distribution is tilted in favor of the old the young does not stir the economy towards states where she has no wealth. In contrast, when the intra-family distribution is such that the young is relatively rich the economy does head rapidly to states where the old has no wealth. This feature is, once again, the dynamic Samaritan’s dilemma. Interestingly, for the old the introduction of exchange-motivated transfers has removed this dilemma; at least in the sense that incentives for the young are removed to stir the economy towards her bankruptcy. The young is still confronted with the dynamic Samaritan’s dilemma as can be seen from the arrows pointing towards the old’s bankruptcy when the young is rich. The old overconsumes in anticipation of obtaining gifts from the young. This overconsumption is similar to the inefficiency which arises in the tragedy-of-the-commons. However, this overconsumption occurs long before the young and old consume out of common resources, namely, the young’s wealth.

The consumption functions shown in figure 10 help to reinforce some of the equilibrium features just discussed. The upper panel shows the old’s consumption when she does not require LTC (left) and when she requires LTC (right). The bottom panel is the counterpart for the young’s consumption. The x-y plane corresponds to the old’s and the young’s wealth.

Barczyk & Kredler (2013) thoroughly discuss the qualitative features of the consumption functions in a model with income risk. These are qualitatively the same as those in the upper panel as well as the young’s consumption function when the old does not require LTC. The most striking feature of these consumption functions is that in parts of the state space where one of the agents has no wealth and gifts flow there is a discrete drop in consumption. For example, consider the old’s consumption function when she requires LTC. When the old has no wealth, the young provides gifts to the old to subsidize the purchase of formal care for all levels of the young’s wealth except when she is also broke in which case the old relies only
on Medicaid (see figure 15). Upon entering this region, the old’s consumption jumps down discretely; this is the counterpart to the well-known Samaritan’s dilemma from two-period models. Prior to this happening is what Barczyk & Kredler (2013) refer to as the dynamic Samaritan’s dilemma. Distortions from the borders of the state space feed back into the state space causing consumption decisions to be distorted long before actual gifts
The top panel displays the old’s consumption function when she does not require LTC (left) and when she requires LTC (right). The bottom panel is the counterpart for the young’s consumption.
are observed. This also explains the non-monotonicity of the old’s consumption as a function of her own wealth (the "bump" in her consumption function).

A novel feature of the current setting can be gleaned from comparing the young’s consumption functions when the old does not require and requires LTC. When the old does not require LTC she provides gifts to the young when her wealth level is roughly above 1500 and the young has no wealth. In this case we see that the young’s consumption displays the downward jump. In contrast, when the old requires LTC, this downward jump disappears. The reason is that in this part of the state space home care takes place. Consumption for the young does not have to decrease since the young still has some control over the allocation. Without home care the old chooses gifts in such a way which is in her best interest and temporarily becomes the family dictator. With exchange-motivated transfers this is no longer the case.

7.1 Comparative Statics

7.1.1 An Increase in the Young’s Wage

An important feature of the data is that home care is less likely the more income a potential caregiver has. The model’s prediction is consistent with this empirical observation. To demonstrate this we increase the young’s wage from $w = 22$ to $w = 30$. Figure 11 shows the laws of motion and the care regions for this case.

Comparing this figure with figure 9 we can quickly see that the formal care region has become substantially larger. Whereas the cutoff value of the old’s wealth for home care to take place is roughly $a^o = 900$ when $w = 22$ this cutoff value is now around $a^o = 1700$. The primary reason for the smaller home-care region is that the young’s reservation transfer for home care increases when her wage increases; her opportunity cost of providing care has increased. Furthermore, the old’s maximum willingness to pay decreases since the old internalizes the young’s increased opportunity cost from providing care. The intuition behind the economy’s dynamics is the same as discussed for the benchmark economy.
7.1.2 Home-care vs. Formal-care Subsidies

We now return to the baseline parameterization of the economy and study two simple policy experiments. In the first one we introduce a subsidy to only home care and in the second one we introduce a subsidy to only formal care. These experiments are too simple in order to be taken quantitatively seriously (e.g. we neglect the way they are financed for now). Nonetheless they are useful to help us understand the workings of the model. Furthermore, they provide a first approximation as to which policy the young and the old generations would prefer if they were given a choice between the two.
Figure 12: Laws of Motion: Home-care Subsidy

The blue arrows represent the law of motion for $a^y$ and $a^o$ when the old does not require LTC and the red arrows when she requires LTC.

Figure 12 shows the laws of motion and the care regions for the case in which a home-care subsidy of $s_h = 2.5$ is introduced. The home care region increases substantially compared to the one from the benchmark economy. Two main effects account for this dramatic change in the size of the regions. First, the young’s reservation transfer for care decreases; providing home care is now associated with a lower opportunity cost. Second, the old’s maximum willingness to pay for home care increases since once again she internalizes the reduction in the young’s opportunity cost. Both of these changes act in the direction of making home care more a more attractive option. The economy’s dynamics are qualitatively unchanged.

Figure 13 presents the laws of motion and the type-of-care regions for the case in which a formal-care subsidy of $s_f = 2.5$ is introduced. The formal care region becomes larger. When formal care is subsidized the old’s maximum willingness to pay for home care decreases simply because formal care has become cheaper. The young now knows that when the old purchases formal care less of her resources will be exhausted; the young becomes less willing to provide home care. Both forces act in the direction of making home care less attractive. The economy’s dynamics are qualitatively unchanged.
Figure 13: Laws of Motion: Formal-care Subsidy

Figure 14 portrays the exchange-motivated transfers along the state space where the young is broke. The vertical-dashed lines demarcate cutoff values of the old’s wealth between home and formal care. The solid line traces out transfers without subsidies; the triangles are transfers with a home-care subsidy, the red triangles are what the young receives for providing home care including the subsidy. The stars are transfers under the formal-care subsidy.

The most striking feature of this figure is that the cutoff value of the old’s wealth decreases significantly with the introduction of a home-care subsidy. When there are no subsidies the cutoff value for home care to take place is roughly when the old has a level of wealth of 900. This is in contrast to a cutoff value of the old’s wealth of about 400 when home care is subsidized and 1100 when formal care is subsidized.

In terms of magnitudes we see that the transfer amount is smallest with a home-care subsidy. In this simple experiment this is unsurprising since the subsidy is financed out of nowhere and therefore has no general equilibrium effects.
Figure 14: Exchange-Motivated Transfers: Young Broke

The vertical-dashed lines demarcate cutoff values of the old’s wealth between home and formal care. The solid line traces out transfers without subsidies; the triangles are transfers with a home-care subsidy, the red triangles are what the young receives for providing home care, which includes the subsidy. The stars are transfers under the formal-care subsidy.

We now consider the part of the state space where the old is broke. Here, a crucial feature is that the young provides gifts to the old. Figure 15 shows the flow of gifts from young to old when the old does not require LTC (black) and when she requires LTC (red). When the old requires LTC the young provides gifts to the old in order to subsidize the purchase of formal care. The circles indicate gifts in the presence of the formal-care subsidy, the triangles when formal care is subsidized and the solid line are gifts without subsidies.
Figure 15: Gifts From Young to Old: Old Broke

The vertical-dashed lines demarcate the onset of gifts from young to old. Red highlights gifts when the old requires LTC and black when she does not. The circles indicate gifts in the presence of the formal-care subsidy, the triangles when formal care is subsidized and the solid line are gifts without subsidies.

Figure 16 portrays the exchange-motivated transfer functions without subsidies (left), with a home-care subsidy (upper right) and with a formal-care subsidy (lower right). The figure also shows planes to highlight the type of care the old receives. Along the x-y plane are the combinations of the young’s and the old’s wealth. We know that home care takes place if the transfer policy prescribes a non-zero transfer. The size of the exchange-motivated transfer is given by the policy.

Consider first the transfer policy from the baseline case (the figure on the left). A noticeable feature of the transfer policy is its non-monotonicity, in particular, its shape in the upper-left corner. The dominant effect in this part of the state space is due to the directional derivative $V^y_{a^y} - V^y_{a^o}$. It becomes small relative to its size in the remainder of the state space.

The meaning of this directional derivative is the value to the young of taking away one unit of resources from the old ($-V^y_{a^o}$) and adding it to her own resources ($V^y_{a^y}$). The difference between these two values is positive throughout the state space, i.e. $V^y_{a^y} > V^y_{a^o}$. As the young, however, becomes rich relative to the old this difference between marginal valuations of the other’s and own wealth decreases.
Figure 16: Exchange-Motivated Transfers

The left-hand graph is the transfer policy without subsidies, $s_h = 0 = s_n$. In the upper-right corner is the transfer policy when home care is subsidized, $s_h = 2.5$, and in the lower-right corner when formal care is subsidized, $s_n = 2.5$. Home care takes place when the transfer policy is non-zero.
From figure 9 we know that in this part of the state space the economy is heading rapidly to the part of the state space where the old is broke (recall the red horizontal arrows pointing to the left). The reason is that the old has incentives to deplete her wealth since she can count on gifts from the young. Thus, the economy is heading towards a region in which the young and the old consume out of the young’s resources. The fact that the young’s marginal value for the old’s wealth is close to her marginal value for her own wealth is a reflection of this eventual pooling of resources. This pooling will however never take place if the economy is far enough removed from the old’s bankruptcy, say, below the 45-degree line (see again figure 9). This mainly explains why the monotonicity of the transfer function does not continue throughout the state space.

Finally, we take a peek at the consumption policies when one (or both) of the generations is (are) broke in the case when the old does not require LTC and when she does. Figure 17 shows these policies. The top panel displays consumption of the old (left) and the young (right) when the young is broke. The bottom panel shows consumption of the old (left) and the young (right) when the old is broke. The solid line traces out consumption without subsidies; the triangles are consumption under a home-care subsidy. The stars are consumption under the formal-care subsidy. Red signifies consumption when the old requires LTC and black when she does not.

The figures suggest that consumption does not change dramatically with the onset of LTC as long as an agent is not broke (see the first and the fourth graph). The differences in consumption levels when the old requires LTC are more stark when an individual is broke (see the second and the third graph).

Consider the young’s consumption when she is broke (the second graph). When both are broke the young consumes his income $w = 22$ before and after the LTC shock either when there are no subsidies or with a home-care subsidy. The young consumes less than her income and provides a gift to the old in case of the formal-care subsidy. Only in the case of this subsidy does the old not choose Medicaid when both are broke. She is better off not entering this state since formal care is subsidized and she obtains a gift from the young. In the other two cases she would also obtain a gift from the young if she would not choose Medicaid. But she chooses Medicaid since formal care is too expensive for her and the gift from the young is insufficient to keep her out of Medicaid.
Figure 17: Consumption: One or the Other Broke

The top panel displays consumption of the old (left) and the young (right) when the young is broke. The bottom panel shows consumption of the old (left) and the young (right) when the old is broke. The solid line traces out consumption without subsidies; the triangles are consumption under a home-care subsidy. The stars are consumption under the formal-care subsidy. Red signifies consumption when the old requires LTC and black when she does not.

When the wealth of the old surpasses 400 home care takes place in the case in which it is subsidized. Also, as the old’s wealth increases the young’s consumption increases because she obtains a larger transfer for providing home care. The young’s consumption follows a similar pattern when there are no subsidies and when there is a formal-care subsidy with the difference that home care takes place at higher levels of the old’s wealth. When the old does not require LTC (shown in black) the young’s consumption eventually exceeds her income since she obtains gifts from the old when the old becomes sufficiently wealthy (approximately above 1500).
Now consider the old’s consumption when she is broke (the third graph in figure 17). Let’s begin with consumption when she requires LTC and both are broke. When there are no subsidies or when there is a home-care subsidy she enters Medicaid and obtains a consumption level of $c_{ma} = 6$. With the formal-care subsidy she does not enter Medicaid and her consumption is slightly above that provided by Medicaid (6.4). As mentioned above this is because the subsidy and the gift from the young enable her to have a higher level of consumption then she would obtain from Medicaid. In either case the economy is stuck; from then on the old’s consumption is given by either $c_{ma} = 6$ or $c = 6.4$ and the young’s consumption by either $c = w = 22$ or $c = w - g^y = 20.13$.

While the old’s consumption when both are broke is highest in the case of the formal-care subsidy this is not the case as long as the young’s level of wealth is positive and less than about 800. In all three scenarios the old obtains formal care in this part of the state space. Additionally, the young provides gifts to the old in order to help purchase this formal care (recall figure 15). The young’s gifts are, however, least generous in the case of the formal-care subsidy. The reason is that the young overconsumes in the cases of no subsidies and a home-care subsidy in anticipation of the old entering the Medicaid state; see the fourth figure which shows that the young’s consumption is larger in these scenarios. Since the gift amount is directly related to the donor’s level of consumption gifts are also relatively large compared to the situation in which the old does not enter Medicaid. Once the young’s wealth is large enough (i.e. above 800) the Medicaid state is far enough removed to be relevant.

A related observation is the old’s consumption behavior in this part of the state space (i.e. $w^o = 0$ and $0 < w^y < 800$) when she does not require LTC (shown in black). The old consumes less than her income and is therefore saving in anticipation that the LTC shock might hit. In this part of the state space the shock would be particularly unpleasant to her because the low-consumption state would not be too far off. Once the young is wealthy enough (i.e, $w^y > 800$) the low-consumption state is far enough and so she increases her consumption by consuming hand-to-mouth. When the young is very wealthy (i.e. when $w^y > 2000$) she provides gifts to the old which goes into the old’s consumption.

### 7.2 Generations’ Policy Preference

In our model generations may differ as to which policy option they prefer if given a choice. The old may prefer a home-care subsidy because it would enable her to remain in her home
which yields additional utility. The young may prefer a formal-care subsidy because it enables her to continue working without having to worry about caring for the old.

On the other hand, the old may prefer the formal-care subsidy. She dislikes the fact that the young has to give up her wage and a formal-care subsidy would allow her to obtain care more cheaply and not having the child forego her wage. The young may prefer the home-care subsidy since she does not want the old to spend her resources on formal care but rather on her. Additionally, she also internalizes the old’s preference for staying at home.

In what follows we will compare by how much the young and the old would have to be compensated in the baseline parameterization in order to be indifferent to either the formal-care subsidy or the home-care subsidy. The more a generation needs to be compensated in terms of consumption in the baseline scenario the more valuable the policy option is deemed.

We compute the generations’ consumption-equivalent variations (CEV) for each point in the state space. A rough sketch on how we compute CEV is as follows. Write the status-quo value function for the old as:

\[ V^o(\omega) = E \int_0^\infty (u(c_t^o) + \alpha^o u(c_t^y) + \eta h_t^*) dt = E \int_0^\infty (u(c_t^o) + \alpha^o u(c_t^y)) dt + E \int_0^\infty \eta h_t^* dt. \]

Denote the old’s value function when home care is subsidized by \( V^{o, hc} \). Then \( \beta^{hc}(\omega) \) has to satisfy:

\[
\hat{V}^o(\omega) = (1 + \beta^{hc}(\omega))^{-1} \cdot \]

\[
(1 + \beta^{hc}(\omega))^{-1} \cdot \]

Denote the old’s value function when home care is subsidized by \( V^{o, hc} \). Then \( \beta^{hc}(\omega) \) has to satisfy:

\[
\hat{V}^o(\omega) = V^{o, hc}(\omega),
\]

\[
(1 + \beta^{hc}(\omega))^{-1} V^{o, c}(\omega) + V^{o, h}(\omega) = V^{o, hc}(\omega),
\]
and we can solve for $\beta$:

$$
\beta^{hc}(\omega) = \left( \frac{V^{o,hc}(\omega) - V^{o,h}(\omega)}{V^{o,c}(\omega)} \right)^{\frac{1}{\gamma^*}} - 1.
$$

We can get the CEV when formal care is subsidized $\beta^f$ in the exact same way and can do the analogous calculations for the young. If the $\beta$ associated with a policy is larger than the one associated with the other policy we will say that a generation prefers the former policy over the latter.

Figure 18 partitions the state space into regions according to which policy the young and the old prefer. Additionally, it portrays the type-of-care regions from the baseline parameterization without subsidies shown in figure 9.

The solid and the dashed curves trace out the points in the state space along which the young and the old are indifferent between the formal- and the home-care subsidies, respectively, e.g. along the solid line the old’s $\beta$ for the home-care subsidy equals the old’s $\beta$ for the formal-care subsidy. In states which are to the right of the two major curves a generation prefers the home-care subsidy e.g. the old’s $\beta$ for the home-care subsidy exceeds the old’s $\beta$ for the formal-care subsidy; in states which are to the left, the formal-care subsidy is preferred. This is not the case for the region $E$. Within this region the young prefers the home-care subsidy over the formal-care subsidy; to the right of it the young prefers the formal-care subsidy.

In region A both prefer the formal-care subsidy over the home-care subsidy and both prefer any subsidy over none. Around the origin formal care takes place even in the presence of the home-care subsidy; refer back to figure 12 which shows the formal-care region in this case. Thus, the home-care subsidy is irrelevant in this part of the state space since home care will never be obtained. A formal-care subsidy is therefore much more valuable. Moving further away from the origin home care does take place eventually. But, the economy is not too far removed from the formal-care region. Since the economy is moving there the formal-care subsidy is preferred by both generations.

The old prefers the formal-care subsidy all the way to the point where both are broke. Only with this policy she never enters Medicaid and thus has a higher consumption level in the long run. For the young the situation around the origin is different. In region E the young would actually prefer no policy or the home-care subsidy (which is essentially no policy since home care does not take place) over the formal-care subsidy. The reason is that her
The type-of-care regions are those from the baseline parameterization without subsidies shown in figure 9. The solid and the dashed curves trace out the points in the state space along which the young and the old are indifferent between the formal- and the home-care subsidies, respectively. In states which are to the right of the two major curves a generation prefers the home-care subsidy; in states which are to the left, the formal-care subsidy is preferred. This is not the case for the region E. Within this region the young prefers the home-care subsidy over the formal-care subsidy; to the right of it the young prefers the formal-care subsidy. A quick summary is the following: (A) Both prefer the formal-care subsidy; (B) both prefer the home-care subsidy; (C) the old prefers the home-care subsidy while the young prefers the formal-care subsidy; (D) the old prefers the formal-care subsidy while the young prefers the home-care subsidy; (E) the old prefers the formal-care subsidy while the young prefers the home-care subsidy.

consumption is somewhat lower when the old does not enter Medicaid; the young actually wants the old to enter Medicaid but cannot force her to do so. For the young it is not credible to threat that she will not provide gifts if the old makes use of formal care instead. Given that
the old does not choose Medicaid it is in the young’s best interest to provide a gift in order to help the old finance formal care. To summarize, in region E the old prefers the formal-care subsidy while the young prefers the home-care subsidy or simply no subsidy at all.

Region B is characterized by the fact that both prefer the home-care subsidy. In much of this part of the state space there is always home care whether or not it is subsidized. Thus, a formal-care subsidy does not do anything here.

In regions C and D the young and the old have differential preferences with respect to the subsidy. In region C the old prefers the home-care subsidy while the young prefers the formal-care subsidy; in D the old prefers the formal-care subsidy while the young prefers the home-care subsidy.

In region C the old’s preference for home care dominates. When home care is subsidized the formal-care region becomes substantially smaller. Considering figure 12 we see that the home-care region even extends along the vertical axis where the old is broke up until the young’s wealth is about 1200. The economy is far from the formal-care region and so for the old the benefits of the home-care subsidy outweigh the benefits of the formal-care subsidy in this part of the state space. The young prefers the formal-care subsidy over the home-care subsidy which allows her to obtain the wage rate while knowing that her parent is taken care off.

Finally, in D the old prefers the formal-care subsidy while the young prefers the home-care subsidy. The reason why the old prefers the formal-care subsidy is the same as we discussed for region A above. The young prefers the home-care subsidy in this region since the old becomes relatively wealthier and therefore the old provides higher exchange-motivated transfers.

References


