ON HABIT AND UTILITY-ENHANCING GOVERNMENT CONSUMPTION*

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Abstract

Bayesian estimation is used to investigate whether (i) deep (DH) as opposed to superficial habit (SH) and (ii) utility-enhancing as opposed to wasteful government consumption improve the fit of a DSGE model. If the stock of SH features the additional persistence typical of DH, the two specifications are virtually as good. Introducing either type of habit in public consumption or including the latter in the utility function do not improve the fit. However, when government consumption is utility-enhancing, private-public consumption complementarity is found. Robustly across specifications, the confidence band of the impact government consumption multiplier of output includes one.

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1 Introduction

Over the last decade, the macroeconomic literature has exerted a lot of effort in an attempt to reconcile the predictions of dynamic stochastic general equilibrium (DSGE) models with structural vector-autoregression (SVAR) results on the effects of government spending shocks. This task has turned more difficult owing to the fact that the empirical literature itself is far from having reached a consensus and, hence, it is not obvious what DSGE models should or should not replicate. Part of the structural VAR literature finds (i) a fiscal multiplier greater than one; and that, in response to an increase in public consumption, (ii) private consumption and (iii) the real wage increase; while (iv) the price mark-up falls (see Blanchard and Perotti, 2002; Monacelli and Perotti, 2008; Gali et al., 2007; Pappa, 2009; Monacelli et al., 2010; Canova and Pappa, 2011; Fragetta and Melina, 2011, among others). However, for instance, Ramey (2011b), by using a narrative identification approach, finds evidence for a crowding out of private consumption, while Nekarda and Ramey (2011) and Nekarda and Ramey (2013) find evidence for a positive or at least neutral response of the mark-up. On the fiscal multiplier, after reviewing a large number of empirical and theoretical contributions, Ramey (2011a) concludes that the government spending multiplier is likely to be in a neighborhood of one.

On the contrary, canonical DSGE models predict much lower fiscal multipliers compared to the empirical estimates (see e.g. Cogan et al., 2010). In these models the fall in the price mark-up and the increase in the real wage is usually obtained by including price and/or wage stickiness. However, they manage to predict only an initial positive response in the real wage, and the fall in the mark-up is not generally enough to push aggregate supply upward to such an extent that the fiscal multiplier is materially magnified. Private consumption is still crowded out unless (i) a non-additively separable utility function is adopted and the intertemporal elasticity of substitution of consumption is set to be low in order to foster a strong intratemporal substitution effect between consumption and leisure (see for example Linnemann, 2006; Monacelli et al., 2010); or (ii) it has to be assumed that an implausibly high share of consumers show a “rule-of-thumb” non-optimising behaviour (Gali et al., 2007); or (iii) agents form habit on consuming categories of goods (deep habit, DH henceforth) as opposed to forming habit on a composite good (superficial habit, SH henceforth) (see Cantore et al., 2012; Ravn et al., 2012; Cantore et al., 2014; Jacob, 2014; Melina and Villa, 2014; Zubairy, 2014, among others); or (iv) public consumption enters

\[1\] Woodford (2011) shows that the government spending multiplier is (i) necessarily below one in a neoclassical Real Business Cycle (RBC) model and exactly the same both in an RBC with monopolistic competition and in a sticky-price New-Keynesian (NK) model with strict inflation targeting; (ii) exactly one in an NK model with fixed real interest rate; (iii) somewhere between the two values in a model featuring a Taylor rule.
the representative household’s utility function with an elasticity of substitution between
public and private spending sufficiently low, to deliver a certain degree of complementarity
between the two (Bouakez and Rebei, 2007, Linnemann and Schabert, 2004 and Melina
and Villa, 2014).  

This paper focuses on these last two features, takes an empirical perspective, and uses
Bayesian estimation techniques to determine the extent to which assuming (i) deep as opposed to superficial habit and (ii) utility-enhancing as opposed to wasteful government consumption enhance an otherwise standard DSGE model’s ability to fit US data.

The assumption that agents form habit in consumption has become a standard feature
of DSGE models (see e.g. the canonical models of Christiano et al., 2005; Smets and
Wouters, 2007). Such a feature was introduced on empirical grounds to enable DSGE models to match the hump-shaped response of consumption obtained in empirical exercises employing VARs. Ravn et al. (2006) introduce in the DSGE literature the idea that agents may form habit not on the overall consumption level, but separately over a continuum of varieties of goods. Whether agents form habit on a composite good or on categories of goods has potentially important consequences for the propagation mechanism of macroeconomic shocks. In fact, whereas in the symmetric equilibrium both habit specifications affect the demand side of the model in the same indistinguishable way, DH also alters the supply side of the model. This occurs because firms incorporate in their decisions that the demand they will face tomorrow is partly a function of the current firm-specific demand they are able to attract today. Regardless of the presence of price stickiness, this in turn implies that the price mark-up exhibits a counter-cyclical behaviour.

The implications of utility-enhancing government consumption have been largely un-

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2 Also the zero-lower-bound for the nominal interest rate has been found to be a determinant for higher output multipliers (Christiano et al., 2011), but we see this more as a special circumstance rather than a feature able to explain business cycle patterns in normal times.

3 There exist several microfoundations of habit, which in turn affect the model’s equilibrium conditions in different ways. While internal habit captures inertia in household’s consumption decisions, external habit captures preference interdependence across households (i.e. keeping up with the Joneses). However, as shown by Dennis (2009), up to a first-order approximation of the model, whether habit is internal or external has empirically little effect on its business cycle characteristics. This is true both for the additive and multiplicative version of habit.

4 Mark-ups are counter-cyclical due to the action of two contemporaneous effects: an intra-temporal effect (or price-elasticity effect) and an inter-temporal effect. The intra-temporal effect arises because the price elasticity of demand becomes procyclical and this represents a determinant for the mark-up to become countercyclical, as long as the latter is inversely related to the former. The inter-temporal effect is brought about by the expectation of future sales coupled with the notion that consumers form habit at the variety level. In response to, say, an expansionary demand shock, firms are inclined to give up some of the current per-unit profits – by temporarily lowering their mark-up – in order to expand their customer base and make higher profits in the future. This distinctive feature of deep habit has recently been exploited in several contributions. For instance, Di Pace and Faccini (2012) introduce it in a DSGE model with labor search-match frictions to generate amplification in the response of labour market variables. Leith et al. (2009, 2012) study the implications of deep and superficial habit for optimal monetary policy.
derstudied. However, we deem that this is indeed appealing because it essentially assumes that agents gain utility also from publicly-purchased goods; that these, to a certain extent, are complementary to their private consumption bundle; and they are not necessarily a waste of resources, as implicitly assumed by the vast majority of DSGE models.

Assessing these features, however, requires particular care as DSGE models incorporating DH typically assume, on one hand, that this is present also in government consumption and, on the other hand, that there is an additional persistence in the stock of habit. Moreover, a utility-enhancing government consumption may exhibit various degrees of complementarity with private consumption. Thus, in order to evaluate the individual and joint contribution of each of these issues, and for the sake of robustness, we estimate a battery of eight different permutations of a DSGE model with two alternative datasets, and two alternative data transformations, for a total of thirty-two estimation rounds.

We organize our research plan around the following four main questions:

1. Does DH fit the data better than SH?
2. Does habit (of either form) in government consumption improve the fit of the model?
3. Should government spending be in the households’ utility function?
4. Are private and public consumption complements or substitutes in households’ preferences?

The answers to these questions are not only important per se, as they shed light on the usefulness and consequences of each individual feature for the transmission of shocks, but they may also help shed light on important and still debated empirical questions through the lens of an estimated DSGE model. For instance, various alternative combinations of these features and their associated parameter values may in principle deliver a government spending multiplier from small to high. In addition, private consumption can be both crowded out and crowded in by public consumption.

In the empirical literature there are only a few studies that are tangent to these questions. As far as DH is concerned, in the microeconometric literature Verhelst and Van den Poel (2013) find some evidence of DH formation by estimating a spatial panel model using scanner data from a large European retailer. Ravn et al. (2006) estimate the DH parameters via Generalized Method of Moments (GMM) methods. Zubairy (2014) estimates the DH parameters within the Bayesian estimation of a medium-scale DSGE model. As regards utility-enhancing government consumption, Bouakez and Rebei (2007) apply minimum-distance and the maximum-likelihood methods on US data and find complementarity between private and public spending. Perhaps the closest contribution to ours is that
of Kormilitsina and Zubairy (2013), who compare various mechanisms that may deliver the private consumption crowding-in result. However, they compare, among other things, a model with DH jointly in private and public consumption (including the additional persistence in the stock of habit) with a canonical model with SH only in private consumption and no persistence in the stock of habit; and a model with utility enhancing government consumption and private-public consumption complementarity with the canonical model. Differently, we disentangle the individual contributions of habit (of either form) in private and (separately) in government consumption; the additional persistence in the stock of habit, which we find to be an important driver of the results; and we study how each form of habit interacts with utility enhancing government consumption. In other words our Bayesian estimation provides a contribution that the existing literature lacks: a systematic likelihood-race-based assessment of each individual model feature. We first fill in this gap and then assess the effects that such features have on the transmission mechanism of seven standard structural shocks, with an emphasis on government consumption shocks.

A number of noteworthy results emerge. First, if the model accounts for an additional persistence also in the stock of SH, as implemented in the seminal paper of Fuhrer (2000) and widely used in the DH specification, there is no empirical support in favour of the deep over the superficial form of habit. In other words, analogously to what Dennis (2009) finds for internal versus external habit, also DH is virtually as good as SH at fitting the data. Second, the additional persistence in the stock of habit, statistically improves the model’s marginal likelihood. Third, introducing either type of habit in public consumption or including government consumption in the utility function do not improve (nor worsen) the fit of the model. However, when consumption does enter households’ utility, there is evidence in favour of a complementarity between private and public consumption and, although we find a weaker complementarity than Bouakez and Rebei (2007), the estimated elasticity of substitution between private and public consumption is well identified by the data and the largest portion of its probability density is located below one.

Inspecting impulse response functions helps us rationalize these results and provide also a contribution to the debate regarding the response of private consumption to a government spending shock, and the size of the fiscal multiplier. At the posterior mean of parameter values, all model variants produce very similar responses to all demand and supply shocks with the exception of the government spending shock, which has a small role in the model’s variance decomposition. This explains why changing these features does not materially affect the model’s marginal likelihood, which is jointly affected by seven structural shocks. Across the various specifications, private consumption either responds negatively or positively but insignificantly to a government spending shock, while the confidence band of the government consumption multiplier of output always includes one. The model
would predict a higher fiscal multiplier and a clear-cut private consumption crowding-in only by assuming higher-than-estimated levels of DH formation both in private and public consumption or greater-than-estimated private-public consumption complementarity.

These results suggest that if the effects of fiscal policy do not concern the DSGE modeler, a canonical DSGE model with Smets and Wouters (2007)-type of frictions is not dominated by a model including more peculiar features such as DH or utility-enhancing government consumption. SH or DH in government consumption are possible determinants of the private consumption crowding-in, while private-public consumption complementarity represents an amplification mechanism. However, if their relevant parameters are set equal to their posterior estimates, the amplification is not large.

The remainder of the paper is structured as follows. Section 2 presents the model. Section 3 presents the empirical strategy using Bayesian methods. Section 4 presents the results of the empirical analysis. Finally, Section 5 concludes. Technical details and robustness exercises are appended to the paper.

2 Model

The model builds on the standard DSGE literature. It is a New-Keynesian model with Rotemberg price and wage stickiness, convex investment adjustment costs, variable capital utilization and (deep or superficial) habit formation only in private or also in government consumption. Government consumption expenditures may be wasteful or utility-enhancing. In the latter case they are allowed to exhibit a certain degree of complementarity with private consumption.

2.1 Households

A continuum of identical households \( j \in [0, 1] \) has preferences over differentiated consumption varieties \( i \in [0, 1] \) and derive utility from \( (X_t)^j = X((X_t^c)^j, X_t^g) \), i.e. a habit-adjusted composite of differentiated private and public consumption goods similar to that in Pappa (2009) and Cantore et al. (2012), which allows \( (X_t)^j = (X_t^c)^j \) as a special case and has further properties that are explained in Section 2.6.

Households exhibit external habit formation in consumption, i.e. they catch up with the Joneses, either on the consumption level of each variety of good (deep habit) in the spirit of Ravn et al. (2006), or on the overall level of consumption (superficial habit) as in Fuhrer (2000), which is now quite a standard feature of DSGE models.
The private component of \((X_t)^j\) is

\[
(X_t)^j = \begin{cases} 
\int_0^1 (C_{it}^j - \tilde{\theta}^c S_{it-1}^c)^{\frac{1}{1-\epsilon^c \eta}} \, di \quad \text{under deep habit}, \\
\int_0^1 (C_{it}^j)^{\frac{1}{1-\epsilon^c \eta}} \, di \quad - \hat{\theta}^c S_{it-1}^c \quad \text{under superficial habit},
\end{cases}
\]

where \(\tilde{\theta}^c \in (0, 1)\) is the degree of deep habit formation on each variety, \(\hat{\theta}^c \in (0, 1)\) is the degree of superficial habit formation on aggregate consumption, \(\eta\) is the intratemporal elasticity of substitution, \(\epsilon^P\) is a price mark-up shock, \(S_{it-1}^c\) denotes the stock of habit in the consumption of good \(i\), and \(S_{t-1}^c\) denotes the stock of habit in aggregate consumption.

The stocks of habit evolve over time according to

\[
S_{it}^c = \tilde{\theta}^c S_{it-1}^c + (1 - \tilde{\theta}^c) C_{it} \quad \text{under deep habit},
\]

\[
S_t^c = \hat{\theta}^c S_{t-1}^c + (1 - \hat{\theta}^c) C_t \quad \text{under superficial habit},
\]

where \(\tilde{\theta}^c \in (0, 1)\) and \(\hat{\theta}^c \in (0, 1)\) imply persistence in the stocks of habit. While Fuhrer (2000) allows for persistence in the stock of superficial habit in the way described in equation (2), in the vast majority of DSGE models published in the last decade the implicit assumption is that \(\hat{\theta}^c = 0\) and hence \(S_t^c = C_t\). On the contrary those DSGE models employing deep habit (see e.g. Zubairy, 2014) allow for persistence in the stock of deep habit formation.

The optimal level of demand for each variety, \(C_{it}^j\), for a given composite is obtained by minimizing total expenditure \(\int_0^1 P_{it} C_{it}^j \, di\) over \(C_{it}^j\), subject to (1). This leads to

\[
C_{it}^j = \begin{cases} 
\left(\frac{P_{it}}{P_t}\right)^{-\epsilon^c \eta} (X_t^c)^j + \tilde{\theta}^c S_{it-1}^c \quad \text{under deep habit}, \\
\left(\frac{P_{it}}{P_t}\right)^{-\epsilon^c \eta} (X_t^c)^j \quad \text{under superficial habit},
\end{cases}
\]

where \(P_{it}\) is the price of variety \(i\), and \(P_t \equiv \left[\int_0^1 P_{it}^{\frac{1}{1-\epsilon^c \eta}} \, di\right]^{\frac{1}{1-\epsilon^c \eta}}\) is the nominal price index. The main difference about deep habit relative to superficial habit is that while in the former case the good-specific demand has a price-elastic component, \(\left(\frac{P_{it}}{P_t}\right)^{-\epsilon^c \eta} (X_t^c)^j\), and a price-inelastic component, \(\tilde{\theta}^c S_{it-1}^c\) – which imply a counter-cyclical effect on the price mark-up also in the absence of price stickiness – in the latter case the price-inelastic component is absent, hence the price mark-up is constant in the absence of price stickiness. Multiplying (3) by \(P_{it}\) and integrating, real consumption expenditure, \(C_t^j\), can be written
as
\[ C_t^j = (X_t^c)^j + \Omega_t, \quad (4) \]
where
\[
\Omega_t = \begin{cases} 
\theta^c \int_0^1 \frac{P^c_t}{P^c_{t-1}} S^c_u d\bar{u} & \text{under deep habit,} \\
0 & \text{under superficial habit.}
\end{cases}
\]

Each household \( j \) is a monopolistic provider of a differentiated labour service and supplies labour \( H_t^j \) to satisfy demand,
\[ H_t^j = \frac{w_t^j}{w_t} (w_t^j - e_t^W) H_t, \quad (5) \]
where \( w_t^j \) is the real wage charged by household \( j \), \( w_t \) is the average real wage in the economy, \( \tilde{\eta} \) is the intra-temporal elasticity of substitution between labour services, \( e_t^W \) is a wage mark-up shock, and \( H_t \) is average demand of labour services by firms. Similarly to Zubairy (2014), let us also assume that there is a Rotemberg quadratic cost of adjusting the nominal wage, \( W_t^j \), appearing in the households’ budget constraint, which is zero at the steady state, and that this is proportional to the average real value of labour services as in Furlanetto (2011),
\[
\frac{\xi^W}{2} \left( \frac{W_t^j}{W_{t-1}^j} - \bar{\Pi} \right)^2 w_t H_t = \frac{\xi^W}{2} \left( \frac{w_t^j}{w_{t-1}^j} \Pi_t - \bar{\Pi} \right)^2 w_t H_t, \quad (6)
\]
where \( \xi^W \) is the wage adjustment cost parameter, \( \Pi_t = \frac{P_t}{P_{t-1}} \) is the gross inflation rate, \( \bar{\Pi} \) is its value at the steady state and \( w_t \) is the average real wage.

Households hold \( K_t^j \) capital holdings, evolving according to
\[ K_{t+1}^j = (1 - \delta) K_t^j + e_t^I I_t \left[ 1 - S \left( \frac{I_t}{I_{t-1}} \right) \right], \quad (7) \]
where \( \delta \) is the capital depreciation rate, \( I_t^j \) is investment, \( S(\cdot) \) represents an investment adjustment cost satisfying \( S(1) = S'(1) = 0 \) and \( S''(1) > 0 \), and \( e_t^I \) is an investment-specific technology shock. Households can also control the utilisation rate of capital. In particular using capital at rate \( u_t^j \) entails a cost of \( a \left( u_t^j \right) K_t^j \) units of composite good, satisfying \( a(u) = 0 \), where \( u \) is the steady-state utilisation rate, conventionally assumed to be equal to unity. Investment is also a composite of goods, i.e. \( I_t^j = \left[ \int_0^1 \left( \frac{P^j_t}{P_{t-1}^j} \right)^{1 - \frac{1}{\alpha}} d\bar{u} \right]^{-\frac{1}{1 - \frac{1}{\alpha}}}, \) but does not feature habit formation. Expenditure minimisation leads to the optimal level of
demand of private investment goods for each variety \( i \),

\[
I^j_{it} = \left( \frac{P_{it}}{P_t} \right)^{-\epsilon^P} I^j_t. \tag{8}
\]

Households buy consumption goods, \( C^j_t \); pay a lump-sum tax net of government transfers, \( \tau^L_t \); invest in investment goods, \( I^j_t \) and nominal private bond holdings, \( B^j_t \); bear the wage adjustment cost defined in equation (6) as well as the capital utilisation cost \( a \left( u^j_{it} \right) K^j_t \); and receive the hourly wage, \( w^j_t \), the rental rate \( R^K_t \) on utilised capital \( u^j_{it} K^j_t \), the return on nominal private bond holdings, \( R_t \), and firms’ profits, \( \int_0^1 J_{it} di \), hence their budget constraint reads as

\[
\left( X^j_t \right)^j + \Omega_t + I^j_t + \tau^L_t + \frac{\epsilon^W}{2} \left( \frac{u^j_{it}}{u^j_{it-1}} - \Pi \right)^2 H_t + a \left( u^j_{it} \right) K^j_t + \frac{B^j_t}{P_t} = w^j_t H^j_t + R^K_t u^j_{it} K^j_t
\]

\[
+ \frac{R_t-1B^j_{i-1}}{P_t} + \int_0^1 J_{it} di, \tag{9}
\]

Household’s inter-temporal utility maximisation problem is

\[
\max \left\{ \left( X^j_t \right)^j, K^j_{i+1}, u^j_{it}, I^j_{i+1}, B^j_{i+1}, w^j_t \right\} E_t \sum_{s=0}^{\infty} \epsilon^P_{t+s} \beta^{t+s} U((X_{t+s})^j, 1 - H^j_{t+s}),
\]

where \( \beta \in (0, 1) \) is the discount factor, \( \epsilon^P_t \) is a preference shock, and \( U(\cdot) \) is a well-behaved instantaneous utility function, subject to constraints (5), (7) and (9).

At the symmetric equilibrium, the first-order condition (FOC) with respect to (w.r.t.) the private consumption composite \( (X^j_t)^j \) implies that the Lagrange multiplier on the household’s budget constraint (9) is equal to \( \Lambda^j_t = U_{X^j,t} \), where \( U_{X^j,t} \) is the marginal utility of the private consumption composite. Let \( \Lambda^j_t Q^j_t \) be the multiplier on the capital accumulation equation (7), and \( Q^j_t \) represent Tobin’s Q. Then, the FOC w.r.t. capital, \( K^j_{t+1} \), implies

\[
Q_t = E_t \left\{ D_{t,t+1} \left[ u_{t+1} R^K_{t+1} - a (u_{t+1}) + (1 - \delta) Q_{t+1} \right] \right\}, \tag{10}
\]

where \( D_{t,t+1} \equiv \beta E_t \left[ \frac{\epsilon^P_{t+1} U_{X^j,t+1}}{\epsilon^P_t U_{X^j,t}} \right] \) is the stochastic discount factor. The FOC w.r.t. \( u_t \) implies that the cost of marginally increasing the utilisation rate of capital is equal to the
return of capital itself, $a'(u_t) = R^K_I$, while the FOC w.r.t. investment, $I_t^j$, yields

$$e^l_tQ_t \left( 1 - S \left( \frac{I_t}{I_{t-1}} \right) - S' \left( \frac{I_t}{I_{t-1}} \right) \frac{I_t}{I_{t-1}} \right) + E_t \left( e^l_{t+1}D_{t,t+1}Q_{t+1}S' \left( \frac{I_{t+1}}{I_t} \right) \left( \frac{I_{t+1}}{I_t} \right) \right) = 1,$$

and the FOC w.r.t. private bond holdings delivers the Euler equation,

$$1 = E_t \left[ \frac{R_t}{\Pi_{t+1}} \right].$$

Finally the FOC w.r.t. $w_t$ delivers the wage setting equation, which at the symmetric equilibrium reads as

$$(e_t^W \eta - 1) w_t - e_t^W \eta \frac{u_t}{\mu_t} + \xi^W (\Pi_t^W - \bar{\Pi}) w_t \Pi_t^W = E_t \left[ D_{t,t+1} \xi^W (\Pi_{t+1}^W - \bar{\Pi}) w_{t+1} \Pi_{t+1}^W H_{t+1} \right],$$

where $\mu_t = \frac{u_t}{MRS_t}$ is the wage mark-up and $MRS_t = -\frac{U_{H,t}}{U_{C,t}}$ is the marginal rate of substitution between leisure and consumption.

### 2.2 Government

Habit can be present also in government consumption. From a technical point of view this is entirely analogous to how these are introduced in private consumption. As shown in Section 2.1 for households’ demand of consumption goods, while superficial habit affects only the demand side of the economy, deep habit affects also the supply side of the economy because the firm-specific demand incorporates a price-inelastic term, which is a function of deep habit parameters. As we show below, this is the case also for government’s demand. Therefore, deep habit in government consumption affect dynamics regardless of whether government consumption enters the utility function linearly or multiplicatively or even if it does not enter at all. On the contrary, superficial habit affect dynamics only if the habit-adjusted government consumption composite, $X_t^g$, enter the utility function multiplicatively. In fact, if government consumption does not enter the utility function – or if it enters linearly – households’ first-order conditions will not depend on $X_t^g$ as well as those of the firms. Only aggregate government consumption, $G_t$ (typically an exogenous process), will affect dynamics through the economy’s resource constraint and the government budget constraint.

From an intuitive point of view, Ravn et al. (2006) justify the use of deep habit in government consumption by assuming that private households view government spending in goods in a way analogous to private consumption and that households derive habit on consumption of government-provided goods. Alternatively, as in Ravn et al. (2012)
and Leith et al. (2009), one can also argue that public goods are local in nature and households care about the provision of individual public goods in their constituency relative to other constituencies. For example, controversies over “post-code lotteries” in health care and other local services (Cummins et al., 2007) and comparisons of regional per capita government spending levels (MacKay, 2001) suggest that households care about their local government spending levels relative to those in other constituencies.

In each period \( t \), the government allocates spending \( P_tG_t \) over differentiated goods sold by firms in a monopolistic market to maximize the quantity of a habit-adjusted composite good:

\[
X_t^g = \begin{cases} 
\left[ \int_0^1 (G_{it} - \tilde{\theta}^g S_{it-1}^g) \frac{1}{1 - \epsilon^g \eta} \, di \right]^{1 - \frac{1}{1 - \epsilon^g \eta}} & \text{under deep habit}, \\
\left[ \int_0^1 (G_{it} \frac{1}{1 - \epsilon^g \eta} \, di \right]^{1 - \frac{1}{1 - \epsilon^g \eta}} - \tilde{\theta}^g S_{it-1}^g & \text{under superficial habit},
\end{cases}
\]  

(12)

subject to the budget constraint \( \int_0^1 P_{it}G_{it} \, di \leq P_tG_t \), where \( \tilde{\theta}^g \) and \( \hat{\theta}^g \) are the degrees of deep and superficial habit formation in government spending and \( S_{it-1}^g \) and \( S_{it-1}^g \) denote the good-specific and the average stock of habit for this expenditure, which evolve as

\[
S_{it}^g = \tilde{\rho}^g S_{it-1}^g + (1 - \tilde{\rho}^g)G_{it} \quad \text{under deep habit},
\]

\[
S_t^g = \hat{\rho}^g S_{t-1}^g + (1 - \hat{\rho}^g)G_t \quad \text{under superficial habit},
\]  

(13)

and exhibits persistence \( \tilde{\rho}^g \) or \( \hat{\rho}^g \). At the optimum,

\[
G_{it} = \begin{cases} 
\left( \frac{P_{it}}{P_t} \right)^{-\epsilon^g \eta} X_t^g + \tilde{\theta}^g S_{it-1}^g & \text{under deep habit}, \\
\left( \frac{P_{it}}{P_t} \right)^{-\epsilon^g \eta} X_t^g & \text{under superficial habit}.
\end{cases}
\]  

(14)

Aggregate real government consumption, \( G_t \), is an exogenous process and the government budget constraint equates government spending to lump-sum taxes, \( G_t = \tau_t^L \).

2.3 Firms

A continuum of monopolistically competitive firms indexed by \( i \in [0,1] \) rents capital services, \( \tilde{K}_{it} \), and hires labour, \( H_{it} \), to produce differentiated goods \( Y_{it} \) with convex technology \( F \left( A_t, H_{it}, \tilde{K}_{it} \right) \), where \( A_t \) is a labour-augmenting technology shock, which are sold at price \( P_{it} \). Firms face quadratic price adjustment costs \( \frac{\epsilon^p}{2} \left( \frac{P_{it}}{P_{it-1}} - 1 \right)^2 Y_t \), as in Rotemberg (1982) – where parameter \( \xi \) measures the degree of price stickiness – and maximize the
flow of discounted profits,

\[ J_{it} = E_t \left\{ \sum_{s=0}^{\infty} D_{it,t+s} \left[ \frac{P_{it+s}}{P_{t+s}} (C_{it+s} + G_{it+s} + I_{it+s}) - \frac{W_{it,s+s}}{P_{t+s}} H_{it,s+s} - R^K_{it+s} K_{it+s} - \frac{\xi}{2} \left( \frac{P_{it+s}}{P_{t+s-1}} - 1 \right)^2 \right] \right\}. \]  

(15)

Under deep habit, the discounted profits in equation (15) have to be maximised w.r.t. \( \tilde{K}_{it}, H_{it}, C_{it}, S^c_{it}, G_{it}, S^g_{it} \) and \( P_{it} \) subject to the following firm-specific demands for good \( i \):

\[ C_{it} = \left( \frac{P_{it}}{P_t} \right)^{-\epsilon_t^P \eta} X_t^c - \bar{\theta}^c S^c_{it-1}, \]

(16)

\[ G_{it} = \left( \frac{P_{it}}{P_t} \right)^{-\epsilon_t^P \eta} X_t^g - \bar{\theta}^g S^g_{it-1}, \]

(17)

\[ I_{it} = \left( \frac{P_{it}}{P_t} \right)^{-\epsilon_t^P \eta} I_t, \]

(18)

obtained integrating equations (3), (14), and (8), respectively across \( j \), along the law of motions of the stocks of habit (2) and (13), and the firm’s resource constraint,

\[ C_{it+s} + G_{it+s} + I_{it+s} = F \left( A_t, H_{it}, \tilde{K}_{it} \right) - FC = Y_{it}, \]

(19)

where \( FC \) are fixed production costs, set to ensure that the free entry condition of long-run zero profits is satisfied. The corresponding first-order conditions for this problem, evaluated at the symmetric equilibrium, are:

\[ R^K_t = MC_t F_{\tilde{K},t}, \]

(20)

\[ W_t = MC_t F_{H,t}, \]

(21)

\[ \nu^c_t = 1 - MC_t + (1 - \bar{\theta}^c)\lambda^c_t, \]

(22)

\[ \lambda^c_t = E_t D_{t,t+1}(\theta^c \nu^c_{t+1} + \bar{\theta}^c \lambda^c_{t+1}), \]

(23)

\[ \nu^g_t = 1 - MC_t + (1 - \bar{\theta}^g)\lambda^g_t, \]

(24)

\[ \lambda^g_t = E_t D_{t,t+1}(\theta^g \nu^g_{t+1} + \bar{\theta}^g \lambda^g_{t+1}), \]

(25)

\[ C_{it} + G_{it} - \epsilon_t^P \eta (\nu^c_t X_t^c + \nu^g_t X_t^g) + (1 - \epsilon_t^P \eta) I_t + \epsilon_t^P \eta MC_t I_t \]

\[ - \xi^P (\Pi_t - 1) \Pi_t Y_t + \xi^P E_t \left\{ D_{t,t+1} \left[ (\Pi_{t+1} - 1) \Pi_{t+1} \right] Y_{t+1} = 0 \right\}. \]  

(26)
Variables $MC_t$, $\nu^c_t$, $\lambda^c_t$, $\nu^g_t$, $\lambda^g_t$ are the Lagrange multipliers associated with constraints (19), (16), (2), (17) and (13), respectively.

Under superficial habit, the evolution of the stocks of habit is not relevant for firms as it can be seen by integrating equations (3) and (14) across $j$. In addition, $C_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\epsilon^{P}_t \eta} X^c_t = \left(\frac{P_{it}}{P_t}\right)^{-\epsilon^{P}_t \eta} C_t$ and $G_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\epsilon^{P}_t \eta} X^g_t = \left(\frac{P_{it}}{P_t}\right)^{-\epsilon^{P}_t \eta} G_t$. As a result, equations (16), (17) and (18) collapse to the standard Dixit-Stiglitz firm-specific demand,

$$Y_{it} = C_{it+s} + G_{it+s} + I_{it+s} = \left(\frac{P_{it}}{P_t}\right)^{-\epsilon^{P}_t \eta} Y_t. \quad (27)$$

In this case, the discounted profits in equation (15) can simply be maximised w.r.t. $K^p_{it+s}$, $H_{it+s}$, and $P_{it+s}$ subject to the resource constraint (19), taking (27) into account. As a result, equilibrium conditions (20) and (21) remain unchanged relative to the deep-habit case, while (22)-(26) collapse to a standard price-setting equation under Rotemberg adjustment costs,

$$1 - \epsilon^P_t \eta + \frac{\epsilon^P_t \eta}{\mu_t} - \xi^P \left(\Pi_t - 1\right) \Pi_t + \xi^P E_t \left[D_{it+1} \left(\Pi_{t+1} - 1\right) \Pi_{t+1} \frac{Y_{t+1}}{Y_t}\right] = 0. \quad (28)$$

In both cases of deep and superficial habit, $MC_t$ is the shadow value of output and represents the firm’s real marginal cost. Let $MC^n_t$ denote the nominal marginal cost. The gross mark-up charged by final good firm $i$ can be defined as $\mu_{it} \equiv P_{it}/MC^n_t = \frac{P_{it}}{P_t}/\frac{MC^n_t}{P_t} = p_{it}/MC_t$, where $p_{it} = \frac{P_{it}}{P_t}$. In the symmetric equilibrium all final good firms charge the same price, $P_{it} = P_t$, hence the relative price is unity, $p_{it} = 1$. It follows that, in the symmetric equilibrium, the mark-up is simply the inverse of the marginal cost. In Appendix (A) we analytically show the effects that deep habit have on countercyclical responses of the price mark-up to aggregate demand shocks in a simplified version of the model.\footnote{From (A.1) the demand function facing each individual firm features an additive price-elastic. It follows that, in addition to the internal optimum we use in our set-up, there arises the possibility of an alternative equilibrium in which the monopolistic firm sets an infinite price. However Schmitt-Grohé and Uribe (2007) show that if consumers have good-specific subsistence points, this strategy is suboptimal for the firm. But in the absence of such subsistence points (as in our model), whether one can rule out this alternative equilibrium remains an open question.}

### 2.4 Monetary policy

Monetary policy is set according to a Taylor-type interest-rate rule,

$$\log\left(\frac{R_t}{R}\right) = \rho_r \log\left(\frac{R_{t-1}}{R}\right) + \left(1 - \rho_r\right) \left[\rho_{\Pi} \log\left(\frac{\Pi_t}{\Pi}\right) + \rho_y \log\left(\frac{Y_t}{Y_t}\right)\right] + \epsilon^M_t, \quad (29)$$

\begin{itemize}
  \item $\rho_r$: long-run interest rate coefficient.
  \item $\rho_{\Pi}$: response to the output gap.
  \item $\rho_y$: response to inflation.
  \item $\epsilon^M_t$: monetary policy shock.
\end{itemize}
where \( Y^f_t \) is the level of output that would prevail in the flexible-price benchmark, \( \rho_r \) is the interest rate smoothing parameter, \( \rho_\pi \) and \( \rho_y \) are the monetary responses to inflation and the output gap, respectively, and \( \epsilon^M_t \) is a mean zero, i.i.d. monetary policy shock with standard deviation \( \sigma^M \).

### 2.5 Equilibrium

In equilibrium all markets clear. The model is completed by the resource constraint,

\[
Y_t = C_t + I_t + G_t + \frac{\xi^P}{2} (\Pi_t - 1)^2 Y_t + \frac{\xi^W}{2} (\Pi^W_t - \bar{\Pi})^2 w_t H_t + a(u_t) K_t,
\]

and the following autoregressive processes for exogenous shocks:

\[
\log \left( \frac{\kappa_t}{\bar{\kappa}} \right) = \rho_\kappa \log \left( \frac{\kappa_t}{\bar{\kappa}} \right) + \epsilon^\kappa_t,
\]

where \( \kappa = \{ A, G, e^P, e^W, e^I \} \), \( \rho_\kappa \) are autoregressive parameters and \( \epsilon^\kappa_t \) are mean zero, i.i.d. random shocks with standard deviations \( \sigma^\kappa \). The symmetric equilibrium of identical households and firms is set out in Appendix B.

### 2.6 Functional forms

The utility function specializes as

\[
U(\bar{X}_t, 1 - H_t) = h(\bar{X}_t) (1 - \varrho_t) (1 - H_t) \varrho_t \frac{1}{1 - \sigma_c - 1} \frac{1}{1 - \sigma_c},
\]

where \( \sigma_c > 0 \) is the coefficient of relative risk aversion, and \( \omega \) is a preference parameter that determines the relative weight of leisure and the consumption composite in utility.

In order to allow for complementarity between private and public consumption we specialize the consumption composite as a constant-elasticity-of-substitution (CES) aggregate,

\[
X^c(X^c_t, X^g_t) = \left\{ \nu_x \left[ \frac{X^c_t}{\sigma_x} \right]^{\frac{\sigma_x - 1}{\sigma_x}} + (1 - \nu_x) \left[ \frac{X^g_t}{\sigma_x} \right]^{\frac{\sigma_g - 1}{\sigma_x}} \right\}^{\frac{\sigma_x}{\sigma_x - 1}},
\]

where \( \nu_x \) is the weight of private goods in the aggregate and \( \sigma_x \) is the elasticity of substitution between private and public consumption. Such a specification is desirable in that it encompasses (i) the case of perfect substitutability between private and public consumption (when \( \sigma_x \rightarrow \infty \)); (ii) the Cobb-Douglas case of imperfect substitutability (when \( \sigma_x \rightarrow 1 \)); (iii) the Leontief case of perfect complementarity (when \( \sigma_x \rightarrow 0 \)); and (iv) the standard case in DSGE modelling of non-utility-enhancing public consumption (when \( \nu_x = 1 \)). If \( \nu_x < 1 \) and \( \sigma_x < \infty \) then public consumption affects the marginal utility of
public consumption,

\[ U_{X_t} = \nu_x \frac{1}{\sigma_x} \left( 1 - \rho \right) X_t^{(1 - \varphi)(1 - \sigma_c)} - (1 - H_t)^{\varphi(1 - \sigma_c)} \left( \frac{X_t}{X_c} \right)^{\frac{1}{\sigma_x}}, \]

and the marginal disutility of labour,

\[ U_{H}\sigma = -\rho X_t^{(1 - \varphi)(1 - \sigma_c)} (1 - H_t)^{\varphi(1 - \sigma_c)} - 1, \]

thus influencing consumption/saving decisions and the labour supply.

Investment adjustment costs are quadratic as in Christiano et al. (2005):

\[ S \left( \frac{I_t}{I_{t-1}} \right) = \frac{\gamma}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right)^2, \]

\( \gamma > 0 \), which satisfy \( S(1) = S'(1) = 0 \) and \( S''(1) = \gamma > 0 \), where \( \gamma \) represents the elasticity of the marginal investment adjustment cost to changes in investment.

The cost of capital utilisation is \( a(u_t) = \gamma_1 (u_t - 1) + \frac{\gamma_2}{2} (u_t - 1)^2 \). Following the literature, we normalize the steady-state utilisation rate to unity, \( u = 1 \). It follows that \( a(u) = 0, a'(u) = \gamma_1, a''(u) = \gamma_2 \) and the elasticity of the marginal utilisation cost to changes in the utilisation rate is \( \frac{a''(u)}{a'(u)} = \frac{\gamma_2}{\gamma_1} \equiv \sigma_u \), which is what we estimate.

The production function is a conventional Cobb-Douglas:

\[ F \left( A_t, H_t, \tilde{K}_t \right) = \left( A_t H_t \right)^{\alpha} \tilde{K}_t^{1-\alpha}, \]

where, \( \tilde{K}_t \equiv u_t K_t \), and \( \alpha \) represents the labour share of income.

3 Bayesian estimation

3.1 Data

The model is log-linearised around a non-stochastic steady state and estimated by Bayesian methods using US quarterly data over the Great Moderation period 1984:Q1-2008:Q3. Although observations on all variables are available at least from 1955 onwards, we focus on this period because it is characterized by a single monetary policy regime. Extending the sample period to include the Great Recession may yield biased estimates due to the nonlinearities induced by the fact that the nominal interest rate in the US reached the zero lower bound (Galí et al., 2011). Seven observables correspond to the seven structural shocks present in the model: hours of work, private consumption, private investment, government spending, real wage, inflation, nominal interest rate. Data sources and transformations are discussed in Appendix C. Here we want to point out that, with the exception of private consumption and investment, the construction of the remaining five observables is standard.

\[^6\text{We prefer to opt for Bayesian estimation as opposed to impulse-response matching as identification issues are more likely to arise with the latter. In particular Canova and Sala (2009) demonstrate that even in the absence of invertibility problems, identification deficiencies may make impulse response matching exercises problematic and inference erratic.}\]
and closely follows the dataset of Smets and Wouters (2007). The construction of private consumption and investment varies in the literature. While Smets and Wouters (2007) and Mountford and Uhlig (2009), amongst others, use private consumption expenditure (BEA Table 1.1.5) and private fixed investment (BEA Table 5.3.5) for consumption and investment, respectively; other authors in the literature, such as Galí et al. (2007) and Zubairy (2014), amongst others, define private consumption only on non-durable goods and services and include consumption of durables in the series of private investment. Hence, in addition to the “standard” (STD) dataset à la Smets and Wouters (2007), we construct an "alternative" (ALT) dataset where the data for private investment include both gross private domestic investment (BEA Table 1.1.5) and private consumption expenditure in durable goods, while private consumption only features consumption expenditure in non-durables and services. Given that one of our aims is to compare different specifications of habit formation for consumption we deem appropriate to use both data specifications in order to check the robustness of our results.

3.2 Data filtering and measurement equations

Various authors (see Canova, 2013, Canova and Ferroni, 2011, Ferroni, 2011, Castelnuovo, 2013, Gorodnichenko and Ng, 2010 and Delle Chiaie, 2009, amongst others) have discussed and showed how the arbitrariness of the choice of the statistical filter, applied to detrend macroeconomic times series, might have strong effects on the structural estimation of DSGE models. Therefore, for the sake of robustness, we apply two different filters to the observables in the estimation. In particular, as far as the real variables in the observables are concerned, we consider (i) the first difference filter (FD), which emphasizes high frequency movements and dampen medium run and business cycles fluctuations, and (ii) the Hodrick-Prescott (HP) (1600) filter that wipes out the fluctuations with periodicity larger than 32 quarters and leave fluctuations with shorter horizons unchanged. Our baseline filtering technique is the FD, following Smets and Wouters (2007). In the appendix we also present parameter estimates obtained using the HP(1600) filter. Hence, for each model variant presented below, we have performed 4 different estimations using 2 dataset (STD, ALT) and two filters (FD, HP(1600)).
The corresponding measurement equations, for the FD case, are:

\[
\begin{align*}
\Delta c_{\text{obs}, t}^i &= \Delta \log(c_t^i) + t, \\
\Delta i_{\text{obs}, t}^i &= \Delta \log(i_t^i) + t, \\
\Delta g_{\text{obs}, t} &= \Delta \log(g_t^i) + t, \\
\Delta w_{\text{obs}, t} &= \Delta \log(w_t^i) + t, \\
\Delta h_{\text{obs}, t} &= \log(h_t^i), \\
\Delta \pi_{\text{obs}, t} &= \log(\pi_t^i) + t_{\pi}, \\
\Delta \tau_{\text{obs}, t} &= \log(\tau_t^c) + t_{\tau}, \\
\Delta \tau_{\text{obs}, t} &= \log(\tau_t^c) + t_{\tau},
\end{align*}
\]

where \( i = \text{STD}, \text{ALT} \), \( x_{\text{obs}, t} \) is the observable corresponding to variable \( X \) in the DSGE model, and \( \log(x_t^i) \) corresponds to the log-deviation from steady state of variable \( X \) in the model, where \( t, t_{\pi} \) and \( t_{\tau} \) are constants that capture the mean of the observables. The equation of hours does not feature a constant as we demean the series prior to estimation.

For the HP filter case, calling \( \tilde{x}_{\text{obs}} \) the HP filtered series for \( x \), the measurement equations are:

\[
\begin{align*}
\tilde{c}_{\text{obs}, t}^i &= \log(c_t^i), \\
\tilde{i}_{\text{obs}, t}^i &= \log(i_t^i), \\
\tilde{g}_{\text{obs}, t} &= \log(g_t^i), \\
\tilde{w}_{\text{obs}, t} &= \log(w_t^i), \\
\tilde{h}_{\text{obs}, t} &= \log(h_t^i), \\
\tilde{\pi}_{\text{obs}, t} &= \log(\pi_t^i) + t_{\pi}, \\
\tilde{\tau}_{\text{obs}, t} &= \log(\tau_t^c) + t_{\tau},
\end{align*}
\]

3.3 Model battery

As discussed in the introduction, the aims of our estimation exercise can be summarised in four interrelated research questions:

1. Does deep habit fit the data better than superficial habit?

2. Does habit (of either form) in government consumption improve the fit of the model?

3. Should government spending be in the households’ utility function?

4. Are private and public consumption complements or substitutes in households’ preferences?
In order to provide an answer to these questions we set-up and estimate a battery of modifications of the model in order to compare log-likelihoods and their predictions. The various model specifications considered are summarised in Table 1.

Models A to C have in common the fact that there is no government spending in the utility function, hence only private consumption increases the wellbeing of agents. They differ in the specification of habit formation in consumption. Model A presents superficial habit in private consumption, Model B features deep habit only in private consumption and Model C has deep habit in both private and public consumption. Model D to G have in common the fact that government spending enters the utility function, through the CES aggregator presented above, and in all of these specification both $\nu_x$ and $\sigma_x$ are estimated. The four specifications consider superficial and deep habit just for private consumption or in both private and public consumption. Model H is a variant of the model with superficial habit in private consumption (without government consumption in the utility function) where we set the persistence in the stock of habit $\hat{\varrho}^c = 0$ as common in recent standard specifications of superficial habit.

### 3.4 Estimation procedure

The joint posterior distribution of the estimated parameters is obtained in two stages. First, the posterior mode and the Hessian matrix are obtained via standard numerical optimization routines. The Hessian matrix is then used in the Metropolis-Hastings (MH) algorithm to generate a sample from the posterior distribution. Two parallel chains are used in the Monte-Carlo-Markov-Chain Metropolis-Hastings (MCMC-MH) algorithm. For each chain, 150,000 random draws from the posterior density are obtained via the MCMC-MH algorithm (although the first 20% ‘burn-in’ observations are discarded), with the variance-covariance matrix of the perturbation term in the algorithm being adjusted in order to

---

7 We do not consider the case in which there is superficial habit in both private and public consumption because it can be showed that, in absence of government consumption in the utility function, this would be equivalent to model A as explained in Section 2.2.

8 We use the Sims solver available in Dynare.
<table>
<thead>
<tr>
<th>Calibrated parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subjective discount factor</td>
<td>$\beta$</td>
<td>0.99</td>
</tr>
<tr>
<td>Capital depreciation rate</td>
<td>$\delta$</td>
<td>0.025</td>
</tr>
<tr>
<td>Labour share of income</td>
<td>$\alpha$</td>
<td>0.67</td>
</tr>
<tr>
<td>Government-expenditure-output ratio</td>
<td>$g_y$</td>
<td>0.20</td>
</tr>
<tr>
<td>Intratemporal elasticity of substitution between labour services</td>
<td>$\tilde{\eta}$</td>
<td>21</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Targeted steady state relationship</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Hours worked</td>
<td>$H$</td>
<td>0.33</td>
</tr>
<tr>
<td>Price mark-up</td>
<td>$\mu$</td>
<td>1.20</td>
</tr>
</tbody>
</table>

Table 2: Calibrated Parameters

obtain reasonable acceptance rates (between 20%-40%). For each model the marginal likelihood is calculated using the modified harmonic mean estimator.

3.5 Calibrated parameters

A number of structural parameters are kept fixed in the estimation procedure, in accordance with the usual practice in the literature (see Table 2). In particular, conventional values are used for the subjective discount factor, $\beta = 0.99$, which implies an annual real interest rate of 4%; the capital depreciation rate, $\delta = 0.025$, which implies an annual depreciation of 10%; the Cobb-Douglas parameter, $\alpha = 0.67$, which corresponds to a labour share of income of 2/3; and the steady-state share of government consumption in GDP, $g_y = 0.20$. As regards the intra-temporal elasticity of substitution between labour services, we follow Zubairy (2014) and set $\tilde{\eta} = 21$.

The steady-state values of hours worked, $H$, and the price mark-up, $\mu$, are jointly determined by the relative weight of leisure in the utility function, $\varrho$, the intra-temporal elasticity of substitution in the goods market, $\eta$, and by whether habit is deep or superficial (together with the degree of habit formation). Hence, we set fixed steady-state targets of $H = 0.33$ and $\mu = 1.20$, such that households’ members work on average 1/3 of their time and firms earn a price mark-up is 20% over the marginal cost, while $\varrho$ and $\eta$ adjust to accommodate these targets. A similar strategy applies to fixed costs in production, $FC$, which are set to satisfy the zero-profit free entry condition of in the long run.

3.6 Priors

The choice of priors for estimation of structural parameters and shocks is presented in Table 3. For parameters commonly found in DSGE models we use priors in line with Smets and Wouters (2007). In particular, as regards the shocks, we assume a beta distribution for the autoregressive parameters and an inverse gamma distribution for the standard deviations.
<table>
<thead>
<tr>
<th>Parameters</th>
<th>Prior</th>
<th>Functional Form</th>
<th>Mean</th>
<th>Std</th>
</tr>
</thead>
<tbody>
<tr>
<td>AR technology</td>
<td>$\rho_A$</td>
<td>beta</td>
<td>0.5</td>
<td>0.2</td>
</tr>
<tr>
<td>AR government spending</td>
<td>$\rho_G$</td>
<td>beta</td>
<td>0.5</td>
<td>0.2</td>
</tr>
<tr>
<td>AR investment specific</td>
<td>$\rho_I$</td>
<td>beta</td>
<td>0.5</td>
<td>0.2</td>
</tr>
<tr>
<td>AR preference</td>
<td>$\rho_B$</td>
<td>beta</td>
<td>0.5</td>
<td>0.2</td>
</tr>
<tr>
<td>AR price mark-up</td>
<td>$\rho_P$</td>
<td>beta</td>
<td>0.5</td>
<td>0.2</td>
</tr>
<tr>
<td>AR wage mark-up</td>
<td>$\rho_W$</td>
<td>beta</td>
<td>0.5</td>
<td>0.2</td>
</tr>
<tr>
<td>Investment adjustment costs</td>
<td>$\gamma$</td>
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<td>0.5</td>
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<tr>
<td>Variable capital utilization</td>
<td>$\sigma_u$</td>
<td>normal</td>
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<td>0.5</td>
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<tr>
<td>Relative risk aversion</td>
<td>$\sigma_c$</td>
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<td>0.375</td>
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<td>Persistence of habit in G</td>
<td>$\tilde{\varrho}^G$ or $\hat{\varrho}^G$</td>
<td>beta</td>
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<td>0.1</td>
</tr>
<tr>
<td>Degree of habit in G</td>
<td>$\tilde{\theta}^G$ or $\hat{\theta}^G$</td>
<td>beta</td>
<td>0.7</td>
<td>0.1</td>
</tr>
<tr>
<td>Persistence of habit in C</td>
<td>$\tilde{\varrho}^C$ or $\hat{\varrho}^C$</td>
<td>beta</td>
<td>0.7</td>
<td>0.1</td>
</tr>
<tr>
<td>Degree of habit in C</td>
<td>$\tilde{\theta}^C$ or $\hat{\theta}^C$</td>
<td>beta</td>
<td>0.7</td>
<td>0.1</td>
</tr>
<tr>
<td>EoS between private and public C</td>
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<td>gamma</td>
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<td>0.5</td>
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<tr>
<td>Weight of private consumption in U</td>
<td>$\nu_x$</td>
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<td>0.1</td>
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<tr>
<td>Rotemberg prices</td>
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<td>normal</td>
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<td>0.5</td>
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<td>Rotemberg wages</td>
<td>$\tilde{\xi}_W$</td>
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<td>Inflation weight in Taylor rule</td>
<td>$\rho_{\pi}$</td>
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<td>0.25</td>
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<tr>
<td>Interest rate smoothing</td>
<td>$\rho_{\pi}$</td>
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<tr>
<td>Output gap weight in Taylor rule</td>
<td>$\rho_y$</td>
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<td>Average real $t$</td>
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<tr>
<td>Average inflation</td>
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<tr>
<td>Average interest rate</td>
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<td>0.1</td>
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</tbody>
</table>

**Table 3:** Priors used in the estimation

For investment adjustment costs, variable capital utilization, Rotemberg price and wage adjustment costs we assume a normal distribution centered around values found in the literature. For the deep and superficial habit parameters we assume a beta distribution centered around the common value of 0.70. The parameters of the Taylor rule have a normal prior for the inflation weight and a beta prior for the interest rate smoothing and the output gap, following again Smets and Wouters (2007). The constants in the measurement equations have a normal prior centered around the mean of the corresponding observable
in the sample period.\(^9\)

For the elasticity of substitution between private and public consumption in households' utility, \(\sigma_x\), and the weight associated to private consumption, \(\nu_x\), the choice of priors is more problematic due to mixed evidence and/or the shortage of previous empirical estimates and studies. The question of whether private consumption and public spending are complements or substitute has been examined by several studies in a partial equilibrium framework.\(^10\) The results are usually mixed and inconclusive and usually depend on the specification of the utility function and on the measurement of the interest rates. With US data, Bouakez and Rebei (2007) have estimated the elasticity of substitution between private and public consumption in a general equilibrium setting and they found strong complementarity. Although they do estimate the elasticity of substitution, they calibrate the weight of private relative to aggregate consumption (\(\nu_x\) in our model) to 0.8. Here we prefer to estimate these two parameters jointly, especially because Linnemann and Schabert (2004) and Bouakez and Rebei (2007) discuss and show how complementarity between private and public consumption and the presence of public spending in the utility function is a sufficient condition for a crowding in of consumption after a fiscal stimulus (in the form of a positive government consumption shock), no matter the price setting assumption in the model. Hence for \(\nu_x\) we choose a beta prior centered around 0.8 which is not far from the standard case of no government spending in utility (\(\nu_x = 1\)) and indeed we deem that the prior of assigning more weight to private consumption is a reasonable one. For \(\sigma_x\) we opt to use a prior defined by a gamma distribution with mean centered around the case of unitary elasticity of substitution between private and public consumption. This implies a prior Cobb-Douglas aggregator of private and public consumption goods and hence imperfect substitutability. However, a standard deviation of 0.5 for the prior distribution allows encompassing both the case of substantial complementarity and that of greater substitutability.

### Results

In what follows we first use marginal likelihoods comparisons across model variants, data sets and data filtering, to answer the four main questions highlighted in Section 3.3 and then we present the estimates of structural parameters and shocks. While in this section of the paper we present parameter estimates with the STD dataset and the FD filter, in Appendix D we report those of each model variant using the two sets of data (STD and

\(^9\)For the real variables we assume the same constant in the measurement equations and we take the average of the four series (consumption, investment, real wages and government spending) as prior mean.

ALT) and the two different filters (FD and HP).

4.1 Does deep habit fit the data better than superficial habit?

In this section we want to compare deep habit against the more common superficial habit formulation present in the literature. The comparison is based on a likelihood race across versions of the model differing only by the presence of the two different habit formulations. As reported Section 2, in order to do a meaningful comparison, we introduce some persistence in the stock of habit also in the SH case.\footnote{This is needed in order to rule out the possibility that the DH formulation may perform better only because it introduces more persistence in consumption.}

Hence we compare the three different model specifications, i.e. A, B and H, in order to see if the introduction of deep habit improves the model fit. All three models have no public spending in utility and have habit only in consumption. Model A and H present SH in consumption and the only difference between the two is the presence (A) or not (H) of the persistence in the stock of SH, $\hat{\rho}_c$. Model B has instead DH in consumption and the usual specification with persistence in the stock of DH. In Table 4 we present the log-likelihood density of the three model variants under the two data sets and two filtering procedures.

To interpret the marginal log-likelihood (LL) differences we appeal to Jeffries (1996) who judges that a Bayes Factor (BF) of 3-10 is “slight evidence” in favour of model $i$ over $j$. This corresponds to a LL difference in the range $[\ln 3, \ln 10] = [1.10, 2.30]$. A BF of 10-100 or a LL range of $[2.30, 4.61]$ is “strong to very strong evidence”; a BF over 100 (LL over 4.61) is “decisive evidence”.

Three main points can be highlighted from this first set of results. There is clear evidence (more pronounced with the STD dataset) that DH are preferred to SH in their standard formulation (i.e. when $\hat{\rho}_c = 0$) as Model B has a significantly higher LL than Model H. The second point is that Model A and B cannot be ranked in terms of LL as their difference is always less than significant with all four datasets/data transformations. The last point is a consequence of the first two, noticing that the SH specification with

<table>
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<th>H</th>
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</thead>
<tbody>
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<td>ALT HP</td>
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</table>

Table 4: Deep vs superficial habit
persistence in the stock of habit (A) is strictly preferred to the one without it (H). Hence the first novel finding of this paper is about the empirical comparison of the two habit specifications in a model without public spending in utility and hence abstracting from any complementarity between private and public consumption. In particular, we find that DH does indeed fit the data better than the standard SH specification, but the main reason seems to be the presence of the extra inertia introduced in consumption by the additional persistence in the stock of habit instead of the habit specification itself. Indeed when we introduce the same additional persistence in the SH specification the difference in data fitting becomes statistically insignificant.

4.2 Does habit (of either form) in government consumption improve the fit of the model?

The next question we explore is whether introducing habit in government consumption improves the fit of the model. We answer this question by performing three model comparisons of six different model specifications presented in Table 1. We compare: the DH model variants with no government consumption in the utility function with and without DH in government spending (Models C and B); the SH model variants with government consumption in the utility function with and without SH in government spending (Models E and D) and the DH model variants with government consumption in the utility function with and without DH in government spending (Models G and F). Again we present the results with both datasets and filtering techniques in Table 5. Results are very robust across datasets and filters and show that having habit in government spending does not improve (nor worsen) the model’s fit both under SH and under DH.

4.3 Should government spending be in the utility function?

In other words, we are now interested in whether the introduction of government spending in the utility function improves the fit of the model. In order to check this we compare model variants with the same habit assumption but with and without public spending in
the utility function. In the case in which there is no public spending in the utility we simply calibrate $\nu_x = 1$ and $\sigma_x = 10000$, otherwise we estimate these two parameters together with the rest of parameters and structural shocks. Hence, we compare model A with D (SH only in consumption), B with F (DH only in consumption) and C with G (DH in both private and public consumption). Table 6 presents the results which show that the inclusion of government spending in the utility function does not strongly affect the log-likelihood. Therefore, in order to shed more light on this question and the next (about the estimated complementarity between private and public consumption), in the following subsection, we present the estimated structural parameters and shock across model variants to see, besides other issues, what the data say about the two key parameters $\nu_x$ and $\sigma_x$.

### 4.4 Estimation results and private-public-consumption complementarity

In this subsection we present the estimated parameters and shocks across model variants A to H. Table 7 presents the mean of the posterior estimates with the baseline data specification (STD) using FD of the real observables.

A number of noteworthy results emerge from these estimates. First, all parameters and shock estimates are pretty robust across different model specifications. In Appendix D we also present the same table using the HP(1600) filter and the ALT dataset and we show that the estimates are also quite robust across datasets and filters. Second, the immediate impact of deep habit captured by parameters $\theta^C$ and $\theta^G$ is similar for private and public consumption, but the persistence of the stocks of habit (and therefore the long-run impact) is slightly higher for public consumption. Third, for standard structural parameters and shocks we obtain results in line with available estimates in the literature.

Therefore, we prefer to devote the remainder of this section to the two key parameters that can help us answer questions 3 (should government spending be in the utility function?) and 4 (are private and public consumption complements or substitutes?).

As regards the weight of private consumption on aggregate consumption, $\nu_x$, we obtain a posterior mean of around 0.75 across models. Figure 1 (top-right) shows the posterior
mean and 95% confidence bands for $\epsilon_t$ across model variants in which this parameter is estimated. By looking at the figure, it is easy to see that the 95% confidence band is below the data seem to statistically prefer the case in which government consumption delivers some utility to households. However, by inspecting the top-left panel of figures, one, i.e. the data seem to statistically prefer the case in which government consumption

Table 7: Estimated parameters and structural shocks across model variants. STD data

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<tr>
<th>Parameters</th>
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<th>C</th>
<th>D</th>
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Figure 1: Estimated weight of private consumption in the utility function ($\nu_x$) and estimated elasticity of substitution between private and public consumption ($\sigma_x$). Prior and posterior distributions across models on the LHS and 95% confidence bands on the RHS.

which shows the prior and posterior densities of the parameter across models, we can infer that the data are not strongly informative on the parameter. In fact, the posterior density differ from the prior only to a small extent. Such a finding, coupled with the result that the log-likelihoods of models featuring government spending in the utility function is similar to those of models excluding it (see Section 4.2) is a warning that we should use caution in drawing a definite conclusion on whether government spending should or should not be in the utility function.

As regards the elasticity of substitution, $\sigma_x$, its point estimate is below one, around 0.7 across models. The bottom-left panel of figures shows that the data seem to be quite informative about this parameter across models, given that the posterior distributions are visibly different from the prior and they become tighter around the posterior mean. In the bottom-right figure, we again show the posterior mean and 95% confidence bands for the parameter across model variants in which this is estimated. At a 95% confidence level, the parameter is below one in two out of four models and in all four models the vast majority
of the distribution lies around a value compatible with a certain complementarity between private and public consumption (results are robust when we use different data and filters, see appendix D). This result goes in the direction of the findings of Bouakez and Rebei (2007), although they find a lower point estimate for the value of this elasticity. Therefore, as far as question 4 is concerned, if public consumption is allowed to be utility enhancing, then it does exhibit a certain degree of complementarity with its private counterpart.

4.5 Impulse response functions

A straightforward way to assess the extent to which the model features under investigation matter from a practical viewpoint is inspecting how macroeconomic variables respond to structural shocks across model specifications. In Figure 2 we report the posterior median of impulse response functions (IRFs) of key macroeconomic variables to all structural shocks we employed in the Bayesian estimation of all model variants (using the baseline dataset and filter).\footnote{The IRFs obtained using the other three combinations of dataset/filters do not differ qualitatively and are available upon request}

In order to allow comparability of the results we normalise IRFs in order the shocks to be of size 1% in all cases but the case of the government spending shock, in which these are normalised so as to the shock to be of size 1% of steady-state output and IRFs to represent fiscal multipliers.

In all cases, whether or not the persistence in the stock of superficial habit is assumed visibly affects the persistence, and hence the shape, of the responses of private consumption and, to a somewhat smaller extent, that of private investment and real output. This visually reflects the fact that the marginal likelihood of models featuring the additional persistence in the stock of habit is significantly higher and that this feature – accounted for by seminal papers such as Fuhrer (2000) but somewhat neglected by later papers – is important to capture the persistence present in the data.

Whether habit is superficial or deep and whether government spending enhances the utility function of private agents or not affect the IRFs in the case of a government spending shock but leaves the IRFs in the cases of all other shocks virtually unaffected. Such a finding is also helpful in clarifying the likelihood-race results of the previous section. The fact that, once the persistence in the stock of habit is accounted for, the marginal likelihood of a model with deep habit does not significantly differ from that of a model with superficial habit is the consequence of the fact that model dynamics are very similar across the two habit specifications for the vast majority of shocks, namely all but the government spending shock.
Figure 2: Posterior median impulse responses of selected macroeconomic variables to all structural shocks (the size of shocks is 1% in all cases but in the case of the government spending shock, in which it is one percent of steady-state output to allow interpreting the responses as fiscal multipliers)
A similar argument apply to the fact of whether government spending is in the utility function or not.

In Table 8 we report the unconditional variance decomposition of all observable variables for model G (results for all other models are very similar and available upon request). The table unveils that the government spending shock has a limited role in explaining the business cycle fluctuations of the observable variables relative to the other shocks, in particular the price and wage mark-up shocks, as demonstrated by Smets and Wouters (2007).

Therefore, results so far suggest that if government spending shocks are not the main concern of a NK model, then a standard model with Smets and Wouters (2007)-type of frictions is not dominated by a model with more peculiar features such as deep habit formation or utility-enhancing government consumption, because these do not significantly affect its marginal likelihood and dynamics. Only the additional persistence in the stock of superficial habit emerges as a useful feature in the case of all shocks.

### 4.6 A closer look at the effects of a government spending shock

Issues such as the type of habit formation or utility-enhancing government consumption become relevant if government spending shocks are indeed the subject matter of economic analysis, because not only the shape and persistence, but also the magnitude and the sign of some IRFs is affected by these modeling choices. Indeed the size of the output multiplier of government spending and the sign of the response of private consumption to a government spending shock have been at the centre of long debate both in the DSGE and in the SVAR literature and a clear consensus does not seem to have emerged on these issues. To illustrate these aspects in greater detail, in Figure 3 we magnify the responses of four key macroeconomic variables (real output, private consumption, private investment and the inflation rate) to a government spending shock of size 1% percent of steady-state output. In particular we report posterior median impulse responses within 90% confidence.
Figure 3: Bayesian impulse responses of real output, private consumption and the inflation rate to a government spending shock of size 1% of steady-state output (line represents median responses; dashed lines represent the 5th and 95th percentiles.

In all cases a positive government spending shock has a significantly positive effect on real output for at least five years. While the median impact multiplier varies across models, in all cases we do not rule out an impact multiplier of one. In all models but model G (deep habit in private and government consumption coupled with utility-enhancing government consumption) inflation responds positively behaving as in response to a standard aggregate demand shock. In the case of model G the government spending shock determines a pronounced fall in the price mark-up (that makes aggregate supply shift temporarily more than aggregate demand), which in turn causes an impact fall in the rate of inflation. Such a fall is, however, not significantly different from zero.\(^{13}\)

\(^{13}\)The same effect may also be obtained even without utility-enhancing government consumption but with a degree of deep habit formation in private and government consumption higher than our posterior
In the standard NK model with superficial habit formation, no additional persistence in the stock of habit, and useless government consumption (model H) the median output multiplier is below one and private consumption is crowded out by government spending, as widely known in the literature. The consumption crowding out becomes increasingly less severe, which is mirrored by an increasingly higher output multiplier, with the addition of (i) deep habit in private consumption (model B); (ii) the additional persistence in the stock of superficial habit (model A); utility-enhancing government consumption (models D and F); and deep habit also in government consumption (model C). A median crowding-in effect on private consumption is obtained with either a model with deep or superficial habit in both private and government consumption with persistent stocks of habit in which government consumption is utility enhancing (models E and G) or with a model with superficial habit in private and public consumption, where the latter is utility-enhancing and exhibits a certain degree of complementarity with the former (model G). In no case, however, the response of private consumption is significantly above zero. Private investment is crowded out by the expansion in government spending, but its fall is not statistically significant (except for model B).

In Figure 4 we compare the median responses of output and private consumption obtained with the posterior estimates of models B-E with alternative values of some key parameters. The first row (Model B) shows that while imposing deep habit only on private consumption is enough to generate an output multiplier greater than one, this is not enough to generate a crowding-in effect on private consumption. The second row (Model C) shows that imposing deep habit both in private and in government consumption enhances the size of the output multiplier and may deliver a small but positive response of private consumption. Note, however, that model C, at the point estimate of the deep habit parameters ($\hat{\theta}_C \approx \hat{\theta}_G \approx 0.66$), still delivers a crowding out. In particular, in a NK model, the degree of deep habit formation necessary to generate a consumption crowding-in is higher than the level needed in a RBC model as in Ravn et al. (2006) as price stickiness dampen the deep-habit effect (see Jacob, 2014). The third row shows that in a model with superficial habit only in private consumption, in which government consumption enhances the private sector’s utility, the output multiplier is around one but private consumption is still crowded out. The fall in private consumption is less severe with higher levels of complementarity (lower $\sigma_x$) between private and government consumption. Lastly, the fourth row (Model E) shows that once superficial habit is assumed both in private and government consumption, then private-public consumption complementarity becomes a much more powerful feature in enhancing the output multiplier and the private consumption mean (Cantore et al., 2014; Jacob, 2014; Melina and Villa, 2014).
Figure 4: Impulse responses of real output and private consumption to a government spending shock of size 1% of steady-state output.

Consumption crowding-in. In particular while with the Cobb-Douglas aggregator ($\sigma_x = 1$) private consumption is crowded out, the CES aggregator with the point estimate of $\sigma_x = 0.69$ delivers a consumption crowding-in.

5 Concluding remarks

In this paper we use Bayesian estimation techniques to empirically assess the extent to which assuming (i) DH as opposed to SH and (ii) utility-enhancing as opposed to wasteful government consumption enhance an otherwise standard DSGE model's ability to fit US data. In particular, we disentangle the individual contributions of habit (of either form) in private and (separately) in government consumption; the additional persistence in the stock of habit; and we study how each form of habit interacts with utility-enhancing government consumption.
The analysis is conducted, first via the estimation of a battery of eight different permutations of a DSGE model with two alternative datasets, and two alternative data transformations; second, via a systematic and robust likelihood-race-based evaluation of each individual model feature; third, via the inspection of the transmission mechanism of seven standard structural shocks.

We find that DH does indeed seem to fit the data better than the standard SH specification, but we argue that the main reason is the presence of the extra inertia introduced in consumption by the additional persistence in the stock of habit that is typically assumed in the DH specification itself. In fact we demonstrate that, if the model accounts for an additional persistence also in the stock of SH, the two forms of habit are virtually as good at fitting the data. The additional persistence in the stock of habit *per se* statistically improves the model’s goodness of fit. We acknowledge that a microeconometric approach to this research question – e.g. similar to that of Verhelst and Van den Poel (2013) – may yield different answers. Our analysis employs a macroeconometric approach and hence investigates on whether such model features work in a significant different way when it comes to explaining business cycle fluctuations of aggregate variables.

Introducing either type of habit in public consumption or including government consumption in the utility function do not improve (nor worsen) the fit of the model. However, when government consumption enters households’ utility, there is evidence in favour of a complementarity between private and public consumption.

At the posterior mean, all model variants produce very similar responses to all demand and supply shocks with the exception of the government spending shock. However, robustly across specifications, the confidence band of the impact government consumption multiplier always includes one. In addition, the response of private consumption following a government spending shock is either negative or does not statistically differs from zero.

These findings lead to the conclusion that if the effects of fiscal policy do not concern the DSGE modeler, a canonical DSGE model with Smets and Wouters (2007)-type of frictions is not dominated by a model including more peculiar features such as deep habit or utility-enhancing government consumption. SH or DH in government consumption are possible determinants of the private consumption crowding-in, private-public consumption complementarity represents an amplification mechanism. However, if their relevant parameters are set equal to their posterior estimates for the US economy, the amplification is not large.

Given that the transmission of the fiscal shock may differ substantially across models, future research may focus on the design of robust fiscal policy in the context of an optimal pooling of competing models as in Geweke (2010).
References


**Appendix**

A Deep habit and more strongly countercyclical mark-ups

Under deep habit the mark-up is more strongly counter-cyclical relative to standard sticky-price models with superficial habit due to the co-existence of two effects: an *intra-temporal*
effect and an inter-temporal effect. To understand how the mechanism works, let us consider, without loss of generality, a stripped-down version of the model with flexible prices, no capital and no persistence in the stock of external habit. Then, let us derive an analytical expression of the price mark-up in the symmetric equilibrium.

In this setting, the aggregate demand faced by firm \( i \) is 
\[
Y_{it} = C_{it} + G_{it},
\]
where
\[
C_{it} = \left( \frac{P_{it}}{P_t} \right)^{-\eta} X_t^c + \bar{\theta}^c C_{it-1}, \tag{A.1}
\]
\[
G_{it} = \left( \frac{P_{it}}{P_t} \right)^{-\eta} X_t^g + \bar{\theta}^g G_{it-1}, \tag{A.2}
\]
Profits of firms \( i \) can be expressed as
\[
J_{it} = E_t \sum_{j=0}^{\infty} D_{t,t+j} \left( \frac{P_{it+j}}{P_{t+j}} - MC_{t+j} \right) Y_{it+j} = E_t \sum_{j=0}^{\infty} D_{t,t+j} \frac{\mu_{it+j} - 1}{\mu_{t+j}} Y_{it+j}.
\]
Maximizing profits \( J_{it} \) with respect to \( C_{it}, G_{it} \) and \( \mu_{it} \) subject to the firm-specific demands for private and public goods (A.1) and (A.2) yields the following first-order conditions:
\[
\nu_{it}^c = \frac{\mu_{it} - 1}{\mu_t} + \bar{\theta}^c E_t D_{t,t+1} \nu_{it+1}^c, \tag{A.3}
\]
\[
\nu_{it}^g = \frac{\mu_{it} - 1}{\mu_t} + \bar{\theta}^g E_t D_{t,t+1} \nu_{it+1}^g, \tag{A.4}
\]
\[
C_{it} + G_{it} = \eta \left( \frac{\mu_{it}}{\mu_t} \right)^{-\eta-1} (\nu_{it}^c X_t^c + \nu_{it}^g X_t^g), \tag{A.5}
\]
where \( \nu_{it}^c \) and \( \nu_{it}^g \) are the Lagrange multipliers associated to constraint (A.3) and (A.4) and represent the shadow value of selling an extra unit in period \( t \) to households and the government, respectively. In the symmetric equilibrium, equations (A.1)-(A.5) become
\[
X_t^c = C_t - \bar{\theta}^c C_{t-1}, \tag{A.6}
\]
\[
X_t^g = G_t - \bar{\theta}^g G_{t-1}, \tag{A.7}
\]
\[
\nu_{it}^c = \bar{\theta}^c E_t D_{t,t+1} \nu_{it+1}^c + 1 - \frac{1}{\mu_t}, \tag{A.8}
\]
\[
\nu_{it}^g = \bar{\theta}^g E_t D_{t,t+1} \nu_{it+1}^g + 1 - \frac{1}{\mu_t}, \tag{A.9}
\]
\[
C_t + G_t = \eta (\nu_{it}^c X_t^c + \nu_{it}^g X_t^g), \tag{A.10}
\]
respectively. Combining equations (A.6)-(A.10), the mark-up can be written as
\[
\mu_t = \left[ 1 - \frac{1}{\eta \left( 1 - \frac{\beta^c C_{t-1} + \beta^g G_{t-1}}{C_t + G_t} \right)} + \frac{X_t^c}{X_t^c + X_t^g} \tilde{\beta}^c E_t D_{t,t+1} \nu_{t+1}^c + \frac{X_t^g}{X_t^c + X_t^g} \tilde{\beta}^g E_t D_{t,t+1} \nu_{t+1}^g \right]^{-1}.
\]

It is easy to show that term \(\eta \left( 1 - \frac{\beta^c C_{t-1} + \beta^g G_{t-1}}{C_t + G_t} \right) \equiv \epsilon_t\) represents the price elasticity of demand in the symmetric equilibrium. In fact, \(\epsilon_{it} = -\frac{\partial X_t}{\partial P} \frac{P_t}{Y_t} = \eta \left( 1 - \frac{\beta^c C_{it-1} + \beta^g G_{it-1}}{C_{it} + G_{it}} \right)\).

Under deep habit, \(\epsilon_t\) is less than the intra-temporal elasticity of substitution, \(\eta\), and an increase in aggregate demand, for instance due to an increase in \(G_t\), relative to habitual demand \(\beta^c C_{it-1} + \beta^g G_{it-1}\) makes \(\epsilon_t\) increase. In other words, \(\epsilon_t\) displays a procyclical behaviour. This feature is known as price-elasticity (or intra-temporal) effect and it is one determinant for the mark-up being counter-cyclical.

Terms \(\tilde{\beta}^c E_t D_{t,t+1} \nu_{t+1}^c\) and \(\tilde{\beta}^g E_t D_{t,t+1} \nu_{t+1}^g\) represent the present value of future per-unit profits induced by a unit increase in current sales towards households and the government, respectively. The two terms enter the expression of the mark-up as a weighted average, in which the weights, \(\frac{X_t^c}{X_t^c + X_t^g}\) and \(\frac{X_t^g}{X_t^c + X_t^g}\), are the shares of habit-adjusted demand of households and the government in aggregate habit-adjusted demand. If future per-unit profits \(\nu_{t+1}^c\) and \(\nu_{t+1}^g\) are expected to be high, the current mark-up falls. This is known as inter-temporal effect. Intuitively, the awareness of high future profits coupled with the notion that consumers form habit at the variety level, makes firms inclined to give up some of the current profits – by temporarily lowering their mark-up – in order to lock-in new consumers into customer/firm relationships and charge them higher mark-ups in the future.

Clearly, in the absence of deep habit, i.e. if \(\tilde{\beta}^c = \tilde{\beta}^g = 0\), the price elasticity of demand is constantly equal to the intratemporal elasticity of substitution, \(\epsilon_t = \eta\), and the mark-up, in this flexi-price version of the model, is also constant and equal to \(\mu_t = \frac{\eta}{\eta - 1}\).

**B Symmetric equilibrium**

**B.1 Utility function and marginal utilities**

\[
U_t = \left[ X_t^{(1-\phi) \left( 1 - H_t \right)^\phi} \right]^{\frac{1-\sigma_c}{1-\sigma_c}} - 1
\]

\[
X_t = \left\{ \nu_x^{\frac{1}{\sigma_x}} \left[ X_t^c \right]^{\frac{\sigma_x - 1}{\sigma_x}} + (1 - \nu_x)^{\frac{1}{\sigma_x}} \left[ X_t^g \right]^{\frac{\sigma_x - 1}{\sigma_x}} \right\}^{\frac{\sigma_x}{\sigma_x - 1}}
\]

\[
U_{X^c,t} = \nu_x^{\frac{\sigma_x}{\sigma_x - 1}} (1 - \phi) X_t^{(1-\phi)(1-\sigma_c)-1} (1 - H_t)^{\phi(1-\sigma_c)} \left( \frac{X_t^c}{X_t} \right)^{\frac{1}{\sigma_x}}
\]
\[ U_{H,t} = -\varrho X_t^{(1-\varrho)(1-\sigma_c)} (1 - H_t)^{\varrho(1-\sigma_c)-1} \]  

(B.4)

**B.2 Euler equation**

\[ 1 = E_t \left[ D_{t,t+1} \frac{R_t}{\Pi_{t+1}} \right] \]  

(B.5)

\[ D_{t,t+1} = \beta e^{B_{t+1}} U X_{t+1} \]  

(B.6)

**B.3 Wage-setting equation**

\[ (e_t^{W} - 1) w_t - e_t^{W} \frac{w_t}{\tilde{\mu}_t} + \xi_t^{W} (\Pi_t^{W} - \bar{\Pi}) w_t \Pi_t^{W} = E_t \left[ D_{t,t+1} \xi_t^{W} (\Pi_{t+1}^{W} - \bar{\Pi}) w_{t+1} \Pi_{t+1}^{W} \frac{H_{t+1}}{H_t} \right] \]  

(B.7)

\[ \tilde{\mu}_t = \frac{w_t}{MRS_t} \]  

(B.8)

\[ MRS_t = -\frac{U_{H,t}}{U_{C,t}} \]  

(B.9)

**B.4 Capital accumulation and investment decisions**

\[ K_{t+1} = (1 - \delta)K_t + e_t I_t \left[ 1 - S \left( \frac{I_t}{I_{t-1}} \right) \right] \]  

(B.10)

\[ Q_t = E_t \left\{ D_{t,t+1} \left[ u_{t+1} R_t^K - a (u_{t+1}) + (1 - \delta)Q_{t+1} \right] \right\} \]  

(B.11)

\[ e_t^l Q_t \left( 1 - S \left( \frac{I_t}{I_{t-1}} \right) \right) - S' \left( \frac{I_t}{I_{t-1}} \right) \frac{I_t}{I_{t-1}} \right) + E_t \left( D_{t,t+1} e_t^l Q_{t+1} S' \left( \frac{I_{t+1}}{I_t} \right) \left( \frac{I_{t+1}}{I_t} \right)^2 \right) = 1 \]  

(B.12)

\[ a' (u_t) = R_t^K \]  

(B.13)

\[ S \left( \frac{I_t}{I_{t-1}} \right) = \frac{\gamma}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right)^2 \]  

(B.14)

\[ S' \left( \frac{I_t}{I_{t-1}} \right) = \gamma \left( \frac{I_t}{I_{t-1}} - 1 \right) \]  

(B.15)

\[ a (u_t) = \gamma_1 (u_t - 1) + \frac{\gamma_2}{2} (u_t - 1)^2 \]  

(B.16)

\[ a' (u_t) = \gamma_1 + \gamma_2 (u_t - 1) \]  

(B.17)

**B.5 Habit dynamics**

\[ X_t^c = \begin{cases} C_t - \tilde{\vartheta} e S_{t-1}^c & \text{under deep habit} \\ C_t - \tilde{\vartheta} e S_{t-1}^c & \text{under superficial habit} \end{cases} \]  

(B.18)
\[ S_t^c = \tilde{\varrho}^c S_{t-1}^c + (1 - \tilde{\varrho}^c)C_t \quad \text{under deep habit} \quad (B.19) \]
\[ S_t^c = \check{\varrho}^c S_{t-1}^c + (1 - \check{\varrho}^c)C_t \quad \text{under superficial habit} \]

\[ X_t^g = \begin{cases} C_t - \tilde{\varrho}^g S_{t-1}^g & \text{under deep habit} \\ C_t - \check{\varrho}^g S_{t-1}^g & \text{under superficial habit} \end{cases} \quad (B.20) \]

\[ S_t^c = \tilde{\varrho}^g S_{t-1}^g + (1 - \tilde{\varrho}^g)G_t \quad \text{under deep habit} \quad (B.21) \]
\[ S_t^c = \check{\varrho}^g S_{t-1}^g + (1 - \check{\varrho}^g)G_t \quad \text{under superficial habit} \]

### B.6 Production function, marginal products and factor demands

\[ F(A_t, H_t, u_t, K_t) = (A_t H_t)\alpha (u_t K_t)^{1-\alpha} \quad (B.22) \]
\[ Y_t = F(A_t, H_t, u_t, K_t) - FC \quad (B.23) \]
\[ F_{H,t} = \alpha \frac{F(A_t, H_t, u_t, K_t)}{H_t} \quad (B.24) \]
\[ F_{K,t} = (1 - \alpha) \frac{F(A_t, H_t, u_t, K_t)}{u_t K_t} \quad (B.25) \]
\[ R_t^K = MC_t F_{K,t} \quad (B.26) \]
\[ W_t = MC_t F_{H,t} \quad (B.27) \]

### B.7 Price-setting under deep habit

\[ \nu_t^c = 1 - MC_t + (1 - \tilde{\varrho}^c)\lambda_t^c \quad (B.28) \]
\[ \lambda_t^c = E_t D_{t,t+1} (\check{\varrho}^c \nu_{t+1}^c + \tilde{\varrho}^c \lambda_{t+1}^c) \quad (B.29) \]
\[ \nu_t^g = 1 - MC_t + (1 - \tilde{\varrho}^g)\lambda_t^g \quad (B.30) \]
\[ \lambda_t^g = E_t D_{t,t+1} (\tilde{\varrho}^g \nu_{t+1}^g + \check{\varrho}^g \lambda_{t+1}^g) \quad (B.31) \]

\[ C_{it} + G_{it} - e_t^P \eta (\nu_t^c X_t^c + \nu_t^g X_t^g) + (1 - e_t^P \eta) I_t + e_t^P \eta MC_t I_t \]
\[ -\xi^P (\Pi_t - 1) \Pi_t Y_t + \xi^P E_t \{ D_{t,t+1} [(\Pi_{t+1} - 1) \Pi_{t+1}] Y_{t+1} = 0 \} \quad (B.32) \]

### B.8 Price-setting under superficial habit

\[ 1 - e_t^P \eta + \frac{e_t^P \eta}{\mu_t} - \xi^P (\Pi_t - 1) \Pi_t + \xi^P E_t \left[ D_{t,t+1} (\Pi_{t+1} - 1) \Pi_{t+1} \frac{Y_{t+1}}{Y_t} \right] = 0 \quad (B.33) \]

### B.9 Taylor rule

\[ \log \left( \frac{R_t}{R} \right) = \rho_R \log \left( \frac{R_t}{R} \right) + (1 - \rho_R) \left[ \rho_{\Pi} \log \left( \frac{\Pi_t}{\Pi} \right) + \rho_Y \log \left( \frac{Y_t}{Y} \right) \right] + e_t^M \quad (B.34) \]
B.10 Resource constraint and autoregressive processes

\[ Y_t = C_t + I_t + G_t + \frac{e^P}{2} (\Pi_t - 1)^2 Y_t + \frac{e^W}{2} (\Pi_t^W - \bar{\Pi})^2 w_t H_t + a (u_t) K_t \]  

(B.35)

\[ \log \left( \frac{G_t}{G} \right) = \rho_G \log \left( \frac{G_{t-1}}{G} \right) + \epsilon_t^G \]  

(B.36)

\[ \log \left( \frac{A_t}{A} \right) = \rho_A \log \left( \frac{A_{t-1}}{A} \right) + \epsilon_t^A \]  

(B.37)

\[ \log \left( \frac{e^t}{e^t} \right) = \rho_e \log \left( \frac{e^{t-1}}{e^t} \right) + \epsilon_t^I \]  

(B.38)

\[ \log \left( \frac{e^P}{e^P} \right) = \rho_e \log \left( \frac{e^{P,t-1}}{e^{P,t}} \right) + \epsilon_t^P \]  

(B.39)

\[ \log \left( \frac{e^B}{e^B} \right) = \rho_e \log \left( \frac{e^{B,t-1}}{e^{B,t}} \right) + \epsilon_t^B \]  

(B.40)

\[ \log \left( \frac{e^W}{e^W} \right) = \rho_e \log \left( \frac{e^{W,t-1}}{e^{W,t}} \right) + \epsilon_t^W \]  

(B.41)

C Data sources and construction

In this section we describe the data sources and how we constructed the observables to be used in the estimation. In table 9 we present the original dataset and the data sources.

<table>
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<tr>
<th>Label</th>
<th>Description</th>
<th>Source</th>
<th>Frequency</th>
</tr>
</thead>
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<tr>
<td>GDP</td>
<td>Nominal GDP</td>
<td>BEA Table 1.1.5</td>
<td>Q</td>
</tr>
<tr>
<td>PCE</td>
<td>Personal Consumption expenditure (total)</td>
<td>BEA Table 1.1.5</td>
<td>Q</td>
</tr>
<tr>
<td>PCE_D</td>
<td>Personal Consumption expenditure on durables</td>
<td>BEA Table 1.1.5</td>
<td>Q</td>
</tr>
<tr>
<td>PCE_ND</td>
<td>Personal Consumption expenditure on non durables</td>
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<td>Q</td>
</tr>
<tr>
<td>PCE_S</td>
<td>Personal Consumption expenditure on services</td>
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<td>Q</td>
</tr>
<tr>
<td>PFI</td>
<td>Private Fixed Investment</td>
<td>BEA Table 5.3.5</td>
<td>Q</td>
</tr>
<tr>
<td>GPDI</td>
<td>Gross Private Domestic Investment</td>
<td>BEA Table 1.1.5</td>
<td>Q</td>
</tr>
<tr>
<td>GCE</td>
<td>Government consumption expenditure and gross investment</td>
<td>BEA Table 1.1.5</td>
<td>Q</td>
</tr>
<tr>
<td>RGDP</td>
<td>Real GDP (base year 2005)</td>
<td>BEA Table 1.1.5</td>
<td>Q</td>
</tr>
<tr>
<td>CNPV16OV</td>
<td>Civilian non-institutional population, over 16</td>
<td>BLS</td>
<td>Q</td>
</tr>
<tr>
<td>CEI6OV</td>
<td>Civilian Employment sixteen years and over</td>
<td>BLS</td>
<td>Q</td>
</tr>
<tr>
<td>LBMINU</td>
<td>Non-farm business hours worked</td>
<td>BLS</td>
<td>Q</td>
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<tr>
<td>LBCPU</td>
<td>Hourly non-farm business compensation</td>
<td>BLS</td>
<td>Q</td>
</tr>
<tr>
<td>FFR</td>
<td>Federal Funds Rate</td>
<td>St. Louis FRED</td>
<td>Q</td>
</tr>
</tbody>
</table>

Table 9: Data sources

From these sources data we constructed 5 common ‘raw’ observables\(^\text{14}\) and two different measures of consumption and investment as showed in table 10 and we considered the

\(^{14}\)Note that the resulting series of hours (as in table 10) is then demeaned before it is used for the estimation.
Table 10: Data transformations - observables

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Construction</th>
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</thead>
<tbody>
<tr>
<td>GDP_Deflator</td>
<td>GDP deflator</td>
<td>GDP_{100}</td>
</tr>
<tr>
<td>index</td>
<td>Population index</td>
<td>CNP16OV_{2002}</td>
</tr>
<tr>
<td>CE16OV_index</td>
<td>Employment index</td>
<td>CE16OV_{2002}</td>
</tr>
<tr>
<td>g_{obs}</td>
<td>Real per capita government spending</td>
<td>LN(\frac{GDP_{Deflator}}{\text{index}})_{100}</td>
</tr>
<tr>
<td>h_{obs}</td>
<td>Per capita hours worked</td>
<td>LN(\frac{LBMNU \cdot CE_{16} \text{OV_{index}}}{GDP_{Deflator}})_{100}</td>
</tr>
<tr>
<td>w_{obs}</td>
<td>Real wage</td>
<td>LN(\frac{LBCPU}{GDP_{Deflator}})_{100}</td>
</tr>
<tr>
<td>r_{n_{obs}}</td>
<td>Quarterly Federal Funds rate</td>
<td>FFR_{4}</td>
</tr>
<tr>
<td>\pi_{obs}</td>
<td>Inflation</td>
<td>\Delta GDP_{Deflator}_{100}</td>
</tr>
<tr>
<td>c_{sw_{obs}}</td>
<td>Real per capita consumption</td>
<td>LN(\frac{PCE_{index}}{GDP_{Deflator}})_{100}</td>
</tr>
<tr>
<td>i_{sw_{obs}}</td>
<td>Real per capita investment</td>
<td>LN(\frac{PFI_{index}}{GDP_{Deflator}})_{100}</td>
</tr>
<tr>
<td>c_{nsw_{obs}}</td>
<td>Real per capita consumption</td>
<td>LN(\frac{PCEND + PCES_{index}}{GDP_{Deflator}})_{100}</td>
</tr>
<tr>
<td>i_{nsw_{obs}}</td>
<td>Real per capita investment</td>
<td>LN(\frac{GPDI + PCED_{index}}{GDP_{Deflator}})_{100}</td>
</tr>
</tbody>
</table>

Hence the two set of observables used in the estimation will be:

\[ obs^{sw} = [g_{obs}, h_{obs}, w_{obs}, r_{n_{obs}}, \pi_{obs}, c_{sw_{obs}}, i_{sw_{obs}}] \]
\[ obs^{nsw} = [g_{obs}, h_{obs}, w_{obs}, r_{n_{obs}}, \pi_{obs}, c_{nsw_{obs}}, i_{nsw_{obs}}] \]

As discussed in the paper for the estimation we apply two statistical filters to both sets of data (only on the real variables). In figures 5-8 we plot the observables with FD and HP(1600) filter.

Figure 5: Observables (FD)
Figure 6: Consumption and investment series (FD)

Figure 7: Observables (HP)

Figure 8: Consumption and investment series (HP)
## D Estimation Results

<table>
<thead>
<tr>
<th>Parameters</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
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<tbody>
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<td>$\rho_A$</td>
<td>0.9709</td>
<td>0.9695</td>
<td>0.9693</td>
<td>0.9702</td>
<td>0.9709</td>
<td>0.9692</td>
<td>0.9684</td>
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<tr>
<td>$\rho_G$</td>
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<td>0.7394</td>
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<td>0.7383</td>
<td>0.7325</td>
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<td>$\rho_B$</td>
<td>0.7226</td>
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<td>0.7068</td>
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<td>0.7271</td>
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<td>0.8687</td>
<td>0.8929</td>
<td>0.8914</td>
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Table 11: Parameters and structural shocks estimated across model variants. STD data and HP(1600) filtering of the real observables.
and HP(1600) filtering of the real observables.

Table 13: Parameters and structural shocks estimated across model variants. ALT data and HP(1600) filtering of the real observables.
D.1 Robustness of estimation of $\nu_x$ and $\sigma_x$  

Figure 9: 95% confidence bands for $\nu_x$ across models  

Figure 10: 95% confidence bands for $\sigma_x$ across models
Figure 11: Prior and posterior distributions of $\nu_x$ across models (STD data and HP filter)

Figure 12: Prior and posterior distributions of $\sigma_x$ across models (STD data and HP filter)

Figure 13: Prior and posterior distributions of $\nu_x$ across models (ALT data and FD)
Elasticity of substitution between private and public consumption, $\sigma_x$

Prior density
Posterior density

Model G

Model F

Model E

Model D

Figure 14: Prior and posterior distributions of $\sigma_x$ across models (ALT data and FD)

Weight of private consumption in the utility function, $\nu_x$

Prior density
Posterior density

Model G

Model F

Model E

Model D

Figure 15: Prior and posterior distributions of $\nu_x$ across models (ALT data and HP filter)

Elasticity of substitution between private and public consumption, $\sigma_x$

Prior density
Posterior density

Model G

Model F

Model E

Model D

Figure 16: Prior and posterior distributions of $\sigma_x$ across models (ALT data and HP filter)