Innovation, trade and multi-product firms

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This draft: August, 2017

Abstract

This paper contributes to the literature on multi-product exporters by developing a model that emphasizes the role of innovation in explaining heterogeneity in export behavior across both firms and products within the same company. We propose a dynamic model of the Luttmer (2011) type where firms invest to maintain and increase their portfolio of goods: the process of product innovation by incumbent firms is such that the probability to capture new business opportunities at home and abroad is a function of the number of goods already sold. This cumulative growth process gives rise to a heavy tail in the distribution of the number of products exported by each firm, which is consistent with the data. Such a markedly heterogeneous behavior across firms occurs even if we do not assume any heterogeneity in productivity to start with. Differences in export sales across products stem from the demand-side of the model, in the form of a product-specific preference attribute. We show that the model is consistent with several empirical regularities that characterize multi-product firms, such as the strict hierarchy in the sales of products across markets, the weak correlation between export scale and scope within each firm, and the substantial degree of product churning.

Keywords: international trade, extensive margin, innovation, preferential attachment, multi-product firms.

JEL classification: F14, F43, L11, L25, O3

Financial support received through the project ‘The international trade network: empirical analyses and theoretical models’ funded by the Italian Ministry of Education, University and Research (PRIN 2009) is gratefully acknowledged. This paper represents a revised version of a manuscript previously circulated under the title “Innovation, trade and the size of exporting firms”. We thank Michele Bernini, Mauro Caselli, Francesco Di Comite, Andrea Fracasso, Michele Imbruno and Simone Salotti for helpful comments on earlier drafts of the paper. All the usual disclaimers apply.

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1 Introduction

Increasing availability of firm-level data has forcefully shown that firms’ engagement in international markets differs widely. The empirical evidence suggests that this cross-sectional heterogeneity is primarily explained by the extensive margin, i.e. the difference in the number of products sold and/or destinations served (Bernard et al, 2009), rather than by differences in export sales (the intensive margin). In fact, trade is dominated by multi-product companies selling a wide range of different commodities in many destination markets (Bernard et al, 2007). On the other hand, innovative activity correlates strongly with export participation (see for instance Altomonte et al, 2013), with product, rather than process, innovation exerting a dominant effect on the evolution of the extensive margin of trade (Becker and Egger, 2013). Although these empirical regularities seem to provide a natural mechanism to explain the behavior of multi-product exporters, so far the trade literature has not exploited this link. This paper bridges the gap by proposing a model of multi-product exporting firms that emphasizes the role of product innovation in determining the observed heterogeneity both across firms and across products within the same company.

Since the seminar paper by Melitz (2003), trade models have focused on mechanisms able to explain selection based on productivity. Hence, innovation is mainly modeled as a productivity-enhancing investment or, in other words, as process innovation (Costantini and Melitz, 2008; Lileeva and Trefler, 2010). The few papers that do incorporate product innovation use it to describe (random) firm entry (Andrew Atkeson and Ariel Tomás Burstein, 2010), but there is no systematic treatment of product innovation as the mechanism driving the dynamics of a firm’s portfolio of goods.

Early models featuring firm heterogeneity (such as Melitz, 2003; Bernard et al, 2003; Chaney, 2008) focus on explaining differences in export sales across firms, but they are unable to make any prediction on the cross-sectional distribution of the extensive margin (Chaney, 2014), as each firm is assumed to sell a single product. Such restrictive assumption is relaxed by a subsequent generation of models, which therefore allow for differences in the extensive margin of trade (Nocke and Yeaple, 2006; Eckel and Neary, 2010; Bernard et al, 2010; Arkolakis and Muendler, 2010; Bernard et al, 2011).

Within this literature on multi-product firms, two main approaches have come to dominate, which explain within-firm heterogeneity in export sales in different ways. Bernard et al (2010) and Bernard et al (2011) combine firm-specific attributes, such as productivity, with product-specific characteristics (consumer tastes) to generate a non-uniform distribution of sales across a firm’s products. Following a different direction, Eckel and Neary (2010) develop a purely supply-side mechanism, which is then further refined in Arkolakis and Muendler (2010); Arkolakis et al (2016) and Eckel et al (2016). In particular, the core competence model by Eckel and Neary (2010) assumes that firms can
produce multiple products by using a flexible production technology that does not depend on output (firm size) but differs across products (firm scope). The firm has a core product that it can produce at the lowest cost. The firm incurs adaptation costs to produce other products. Firms differs in the cost of production of their core product but they have the same adaptation costs. As a result the profile of sales revenues is differentiated across products as a consequence of increasing production costs (Eckel et al, 2016).

In both types of models, exogenous productivity differences determine firm scope. In fact, in Bernard et al (2010) and Bernard et al (2011) higher productivity can compensate for lower product-specific appeal on consumers to generate enough sales and make a product viable, while in Eckel and Neary (2010) more productive firms can afford to move further away from their competences before hitting the threshold that makes it unprofitable to sell a given product. Hence, in both cases more productive firms feature a wider scope.

Another interesting finding which emerges from firm-level data is the lack of a (positive) relationship between the intensive margin and the extensive margin of trade, especially in close markets (Bernard et al, 2011; Arkolakis et al, 2016). Although this empirical regularity has not received much attention, most models of multi-product firms do have predictions on the interaction between export scale and scope. The core competence model for example predicts a positive relationship between firms’ extensive and intensive margins. This positive correlation is the result of increased competition, firms’ market power and a cannibalization effect which induce firms to become leaner by dropping some marginal products while expanding sales of their core products. A similar result is in Arkolakis et al (2016) who find that, on average, product sales are increasing in the number of products (see also similar results in Bernard et al, 2011). In their model, they introduce destination specific scope economies to account for varying destination-specific correlations between the two margins, with little or no correlation in nearby markets.

The lack of correlation between the intensive margin and firm productivity (and thus the extensive margin) can be derived also in the Bernard et al (2011) setting, but only in the special case of a Pareto distribution of product attributes, and fixed export costs that are independent of product attributes. Overall, multi-product firm models come to different conclusions on the relationship between firms’ extensive and intensive margins depending on the combined effect of product adaptation costs, destination specific scope economies and organizational capabilities. Absence of correlation is obtained just in very specific settings (Bernard et al, 2011).

Our paper contributes to the literature on multi-product exporters and on firm heterogeneity in both the extensive and intensive margin by drawing on recent theoretical research in the industrial organization literature (Klette and Kortum, 2004; Buldyrev

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\(^1\)Nocke and Yeaple (2006) generate a negative relationship between firms’ extensive and intensive margins for firms which produce multiple goods thanks to higher organizational capabilities.
et al, 2007; Luttmer, 2011). In particular, we build upon Klette and Kortum (2004) and Luttmer (2011) to examine an innovation-driven model of multi-product exporting firms.

We employ a dynamic model in which incumbent firms must invest in formal or informal research and development (R&D) to enlarge the portfolio of goods they produce and export. In particular, the more products a firm has, the more resources it can devote to research and develop new products.\(^2\) This cumulative growth process governs the evolution of the extensive margin of trade and, together with the dynamics of firm entry and exit, gives rise to a skewed distribution of the number of products exported by each firm with a heavy right tail. Such a distribution closely matches the empirical evidence, as we show by looking at comprehensive data on French manufacturing firms.

One of the remarkable features of our setting, is its ability to generate large heterogeneity in firm behavior (most notably in the number of product exported across multiple product categories) despite firms are \textit{ex ante} identical and homogeneous in their abilities. In fact, we do not have to assume a skewed productivity distribution to obtain the result of firms producing and selling a different number of products, as in existing models. Rather, as in Klette and Kortum (2004) and Luttmer (2011), it is the stochastic nature of the innovation process which allows bigger firms to secure more business opportunities: this is the key driver for the heterogeneity in the extensive margin in our model.\(^3\)

To account for heterogeneity in the intensive margin within the firm, we borrow from Bernard et al (2011) and Eaton et al (2011) the idea that each good is characterized by an “attribute” that represents consumers’ taste for that product. Hence, we introduce heterogeneity in the sales from the demand rather than the supply side of the model. The importance of demand-side influences in determining dispersion in firm size and revenue-based productivity is not new to the literature: it has been emphasized by Syverson (2004, p. 549), and recently exploited by Di Comite et al (2014) to explain differentiation in export markets.

The inclusion of destination-specific fixed export costs implies a strict within-firm hierarchy among products, so that best-sellers in one destination are more likely to be exported to many markets, and will sell a lot in every location. This additional implication of the model is consistent with the recent evidence put forward by Arkolakis and Muendler (2010) and Mayer et al (2014).

Differently from the core competence model, our framework implies no correlation between firms’ sales (intensive margin of trade) and the number of products exported by each firm (extensive margin of trade). This prediction, which matches recent empirical

\(^2\)This cumulative process is consistent with the empirical finding that innovation features a strong persistent component, as reported for instance by Flaig and Stadler (1994) and Peters (2007).

\(^3\)We are not postulating that differences in productivity are not relevant and should be disregarded. Rather, we propose an additional mechanism to generate heterogeneity in firm performance. Ideally, the two mechanisms could be integrated to compound their effects: this is a possible modeling strategy that we do not pursue in the present paper.
The work is organized as follows: the next section provides a quick glance at the related literature and motivates the paper, Section 3 presents the model, whereas Section 4 discusses its ability to match the relevant empirical evidence on firm-level exporting behavior and multi-product firms. Finally, Section 5 discusses some further implications of the model and concludes.

2 Innovation and multi-product firms: background and motivations

Multi-product exporters dominate international trade. As reported by Bernard et al (2007), despite multi-product firms represented only 57.8 percent of US exporters in 2000, they accounted for 99.6 percent of total export. Similarly, in France 66.3 percent of exporters were multi-product firms in 2003, but they accounted for 99.4 percent of export (Bee et al, 2017). Moreover, the portfolio of goods produced and commercialized by exporting firms experiences a constant and rapid turnover due to product churning, switching and the launch of new products. Hence, product innovation is a major determinant of firm scope. Bernard et al (2010) find that about 40 percent of US firms introduce new products outside their span of 4-digit industries between census years. However, the vast majority of export spells are short-lived: based on a unique dataset of Mexican firms, Iacovone and Javorcik (2010, p. 483) document that “a vast percentage of export goods do not survive for more than a year in the foreign market”. They also show that the survival rate grows with the goods’ tenure in the export market. This finding is consistent with a tradition of quality ladder models in industrial organization in which innovative multi-product firms launching the most recent product takes over the market for that good (Grossman and Helpman, 1991; Klette and Kortum, 2004). To the best of our knowledge such a mechanism of product churning has not been exploited yet in the debate on multi-product exporters.

Exploiting data collected by the French Customs (the used by Mayer et al, 2014; a more detailed description is offered in Section 4), we find that French companies are also widely diversified across different 2-digit product chapters of the Combined Nomenclature (see Table 1). Sales dispersion is mostly across broad product categories (2-digit level) than within narrowly defined markets. On average, a French firm exports 41.82 products (defined as CN 8-digit subheadings) spanning more than 7 broad product categories (i.e. CN 2-digit chapters). To achieve such a wide range of good exported, firms must discover that different products can be profitably exported to destinations whose consumers’ pre-
ferences are not knowable ex ante (Hausmann and Rodrik, 2003). New product discovery is thus a key engine of firm diversification and growth.

Table 1: Diversification of multi-product firms. Number of product chapters (2-digit Combined Nomenclature, CN2) by firms classified according to the number of 8-digit products (CN subheadings), summary statistics. The total number of CN2 is 98.

<table>
<thead>
<tr>
<th>NC8 - Products</th>
<th>NC2 - Chapters</th>
</tr>
</thead>
<tbody>
<tr>
<td>scope</td>
<td>avg scope</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2 – 5</td>
<td>3.19</td>
</tr>
<tr>
<td>6 – 10</td>
<td>7.69</td>
</tr>
<tr>
<td>11 – 20</td>
<td>14.78</td>
</tr>
<tr>
<td>21 – 50</td>
<td>32.55</td>
</tr>
<tr>
<td>51 – 100</td>
<td>71.10</td>
</tr>
<tr>
<td>101 +</td>
<td>295.91</td>
</tr>
<tr>
<td>Total</td>
<td>41.82</td>
</tr>
</tbody>
</table>

Authors’ calculation based on data for 2003 collected by the French Customs Service

While the relevant role played by product innovation in shaping firm size and scope is central to recent models of firm dynamics (Klette and Kortum, 2004; Fu et al, 2005; Buldyrev et al, 2007; Luttmer, 2007, 2011), somehow surprisingly, it has received little attention in the literature on multi-product exporters. In fact, in the theoretical trade literature, innovation is mostly discussed in terms of productivity-enhancing investment (i.e. process, rather than product innovation), such as in Costantini and Melitz (2008).

While process innovation can help firms to internationalize by increasing their productivity, here we focus on the role of product innovation, which determines the scope of the firm by modifying the range of products new to the firm and new to the market that a firm commercializes. The empirical evidence suggests that innovative firms tend to reach wider and more distant markets than non-innovative counterparts and that firms tend to jointly make innovation and export participation decisions (Aw et al, 2007, 2011; Cassiman and Golovko, 2011). Indeed, a number of recent empirical studies has found that product, rather than process, innovation crucially determines firm-level export participation (Cassiman and Martínez-Ros, 2007; Becker and Egger, 2013; Altomonte et al, 2013). Moreover, innovation statistics show that about one quarter of European firms have introduced a new or significantly improved product during the period 2012-2014 and the share of product innovators becomes progressively higher as firms internationalize (Eurostat, 2017). Only about 40% of firms whose largest market is local or regional are innovative whereas almost 67% of firms whose market is international and outside Europe are product innovators.\(^4\)

\(^4\)As for France, around 80% of international firms are innovators, versus only 46% of firms whose market is local or regional (Eurostat, 2017).
In our model we let firms invest to develop ideas for new products. As in Luttmer (2011), ideas for final goods are costly to produce and replicate. Replication generates ideas for new and diverse final products potentially to be sold in different markets. As usual, we assume that firms face monopolistic competition; however, final demand for their products depends on consumers’ tastes. Even though the literature about multi-product exporters has recognized that there can be alternative mechanisms to generate product heterogeneity, such as innovation (see for instance Bernard et al, 2011), this approach has not been explored yet.

3 The Model

In our model, firms are distributed over a finite set of $C$ countries, populated by a continuum of identical consumers of measure $H_t = H e^{\eta t}$, where $\eta \geq 0$ is the growth rate of the population. Time is continuum and denoted by $t$, with initial time $t = 0$.

3.1 Households

The intertemporal utility of the representative consumer is

$$U_t = E_t \left[ \int_t^\infty ln(X_t)e^{-\rho t} dt \right]$$

where $\rho > 0$ is the discount factor and $X_t$ denotes aggregate consumption. $X_t$ is a CES composite of all available goods

$$X_t = \left[ \sum_{j=1}^{N_t} a_j^{\frac{1}{\sigma}} (x_{j,t})^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$$

where $x_{j,t}$ is consumption of product $j$, $\sigma > 1$ is the elasticity of substitution across products, and $N_t$ is total mass of products at time $t$, and $a_j$ is an exogenous product-specific attribute that captures consumer tastes, in a way similar to Bernard et al (2011). The preference attribute $a_j$ comes from a time-invariant distribution $\Gamma(a)$ which is continuous in the domain $0 < a < \infty$ with mean $\bar{a}$. We assume that $\Gamma(a)$ is common to all goods and firms in each country. Furthermore, in every country, the number of products that firms produce may vary as the result of a stochastic innovation process.

A similar assumption is made by Eaton et al (2011) who assume that an exogenous demand shock specific to product $i$ in a given market $n$ affects consumer demand for that good. Moreover, the choice to model firm heterogeneity as coming from consumers’ tastes is consistent with the findings by Syverson (2004) who shows that demand-side conditions play an important role in explaining persistent firm-level dispersion.
The representative household maximizes utility subject to the standard budget constraint. The resulting demand for a product \( j \) is

\[
x_{j,t} = a_j \left( \frac{p_{j,t}}{P_t} \right)^{-\sigma} Y_t \frac{Y_t}{P_t} \tag{3}
\]

where \( Y_t = \sum_{j=1}^{N_t} p_{j,t} x_{j,t} \) is total household expenditure on the composite good \( X \) and \( P_t \) is the price index defined as

\[
P_t = \left[ \sum_{j=1}^{N_t} a_j (p_{j,t})^{1-\sigma} \right]^\frac{1}{1-\sigma}.
\]

Here, as in the original formulation by Dixit and Stiglitz (1977), we are assuming that \( N_t \) is large enough so that each product has no effect on the aggregate price index. Aggregate consumption growth and the interest rate are related via the standard Euler equation

\[
\frac{\dot{Y}}{Y} = r - \rho \tag{4}
\]

with \( r \) being the interest rate and \( \rho \) the discount factor.

\subsection*{3.2 Firms}

Labor is the sole factor of production, and each product \( j \) is produced according to the following technology:

\[
x_{j,t} = z_{j,t} l_{j,t} \tag{5}
\]

where \( l_{j,t} \) is the amount of labor used in the production process and \( z_{j,t} \) is labor productivity. We assume that firms must pay a fixed entry cost \( f_c \) to sell in a foreign market, with \( c = 1, \ldots, C - 1 \) indexing destinations. We also assume that foreign countries can be ranked on the basis of these fixed entry costs such that \( f_1 < f_2 < \cdots < f_{C-1} \). To ship goods abroad, firms incur trade costs of the standard iceberg type: \( \tau > 1 \) units shipped result in 1 unit reaching the destination market. Firms set the price of each product \( j \) in order to maximize (static) profits, given the wage rate \( w_t \), yielding the standard result that the optimal price is given by a mark-up over marginal cost:

\[
p_{j,t} = \frac{\sigma}{\sigma - 1} \frac{w_t}{z_{j,t}} \tau. \tag{6}
\]

Indicating with \( p_{j,t}^D \) and \( p_{j,t}^F \) the domestic and foreign price respectively, and given that \( \tau = 1 \) for domestic sales, we can write \( p_{j,t}^F = p_{j,t}^D \tau = \frac{\sigma}{\sigma - 1} \frac{w_t}{z_{j,t}} \tau \) (see Appendix B for details on the derivations). Revenues from sales of product \( j \) in the domestic and in a foreign
market are given by:

\[ r_{j,t}^D = a_j \left( \frac{p_{j,t}^D}{P_t^D} \right)^{1-\sigma} Y_t^D \]  

(7)

\[ r_{j,t}^F = a_j \left( \frac{p_{j,t}^F}{P_t^F} \right)^{1-\sigma} Y_t^F \]  

(8)

Due to country-specific entry costs, the number of goods available to consumers differ across countries, which implies \( Y_t^D \neq Y_t^F \) and \( P_t^D \neq P_t^F \). As a consequence, product \( j \) is associated with different revenues in each foreign destination. Total revenues from sales of product \( j \) are equal to \( r_{t,j} = r_{t,j}^D \) if product \( j \) is sold only domestically and to \( r_{t,j} = r_{t,j}^D + \sum_{c=1}^{k} r_{t,j}^F \) with \( k \in [1, C-1] \) if product \( j \) is also exported to foreign markets.

Profits from selling product \( j \) into the domestic market are given by

\[ \pi_{t,j}^D = a_j \left( \frac{\sigma w_t}{(\sigma - 1) P_t^D} \right)^{1-\sigma} Y_t^D t_{z,j}^{\sigma-1} = \frac{r_{t,j}^D}{\sigma} \]  

(9)

whereas profits from selling product \( j \) into a foreign market with entry cost \( f_c \) are

\[ \pi_{t,j}^F = a_j \left( \frac{\sigma w_t \tau}{(\sigma - 1) P_t^F} \right)^{1-\sigma} Y_t^F t_{z,j}^{\sigma-1} - f_c = \frac{r_{t,j}^F}{\sigma} - f_c. \]  

(10)

As for revenues, total profits from a product \( j \) are given by \( \pi_{t,j} = \pi_{t,j}^D + \sum_{c=1}^{k} \pi_{t,j}^F \), with

\[ \sum_{c=1}^{k} \pi_{t,j}^F = 0 \]  

if product \( j \) is not exported.

At each point in time, for a firm \( i \) with \( n_{i,t} \) products, total revenues and profits equal

\[ r_{i,t}^{Tot}(n_{i,t}) = \sum_{j=1}^{n_{i,t}} r_{t,j} = \sum_{j=1}^{n_{i,t}} \left( r_{t,j}^D + \sum_{c=1}^{k} r_{t,j}^F \right). \]  

(11)

\[ \pi_{i,t}^{Tot}(n_{i,t}) = \sum_{j=1}^{n_{i,t}} \pi_{t,j} = \sum_{j=1}^{n_{i,t}} \left( \pi_{t,j}^D + \sum_{c=1}^{k} \pi_{t,j}^F \right). \]  

(12)

with \( \sum_{c=1}^{k} \pi_{t,j}^F \) and \( \sum_{c=1}^{k} \pi_{t,j}^F \) being zero for non-exporting firms.\(^6\) We assume that productivity is common across all firms and goods, i.e. \( z_{j,t} = z_t \), so that products are all sold at the same price \( (p_{j,t} = p_t) \).\(^7\)

\(^6\)Equations (11) and (12) show that there is no “cannibalization” effect within the firm, in line with the model by Bernard et al (2011) and with the notion that each product is small relative to the market or, alternatively, that a firm’s products are in different categories of goods.

\(^7\)In Appendix D we show an alternative version of our model where heterogeneity on the demand side is obtained using the more common assumption of productivity heterogeneity across firms. As shown there, our main results are not affected by the chosen approach to model heterogeneity in the intensive margin because CES preferences and monopolistic competition firm productivity and consumer tastes have similar effects on revenues (Bernard et al, 2011).
The common productivity $z_t$ evolves exogenously over time according to the following law of motion: $z_t = ze^{\theta t}$. Homogeneous productivity levels across firms and goods implies that whether a product is exported (at least to the destination with the lowest entry cost $f_1$) solely depends on its preference attribute $a$. Let us define $a^*$ as the attribute cut-off level which makes profit from selling product $j$ in the foreign market with the lowest entry cost $f_1$ equal to zero: $a^*_t = \sup\{a : \pi^{F(f_1)}_t(a) = 0\}$ where $F(f_1)$ is the foreign market with the lowest entry cost $f_1$. Heterogeneous fixed entry costs imply that there is a hierarchy among products whereby top-selling products are more likely to be shipped to many destinations (see Arkolakis and Muendler, 2010, for evidence along these lines). This is in contrast with early work on multi-product firms (Bernard et al, 2011), that assume sales across markets are uncorrelated, but appears to be consistent with recent empirical evidence.

In order to describe the balanced-growth path of the economy, it is convenient to express average revenues and profits for a firm with $n_{i,t}$ products as a function of the average preference attribute characterizing its products $\bar{a} = \frac{1}{n_t} \sum_{j=1}^{n_t} a_j$:

$$r_{i,t}(\bar{a}, n_{i,t}) = \frac{1}{n_t} \sum_{j=1}^{n_t} r_{t,j} = r_{D}^{t,j} + \sum_{c=1}^{k} r_{F}^{t,j}; \quad (13)$$

$$\pi_{i,t}(\bar{a}, n_{i,t}) = \frac{1}{n_t} \sum_{j=1}^{n_t} \pi_{t,j} = \pi_{D}^{t,j} + \sum_{c=1}^{k} r_{F}^{t,j}; \quad (14)$$

Since the distribution of the preference attribute $\Gamma(a)$ is constant and common to all products and firms, and all firms face the same (asymmetric) fixed entry costs to foreign market, by the law of large numbers, as a firm’s portfolio of products grows large the average preference attribute will tend to the population mean $\bar{a} \to \bar{a}$. This implies that the average revenues and profits per product will also tend to an average value, $r_{i,t}(\bar{a}) \simeq r_{i,t}(\bar{a})$ and $\pi_{i,t}(\bar{a}) \simeq \pi_{i,t}(\bar{a})$, which is common to all firms.

This assumption is crucial to allow us to characterize the balanced growth path and to derive a closed-form solution for the distribution of the number of products sold by each firm (see Section 3.5 below). In fact, since a firm’s ability to invest in R&D and innovate depends on its profits, in order to have a single distribution for all products sold and exported by each firm, firms’ investment choices must only depend on the number of products in their portfolio (as in Klette and Kortum, 2004 or Luttmer, 2011), disregarding the possibility that each product may generate a different level of revenues and profits. Intuitively, we are assuming that —on average— firms get the same amount of profits from each product: best-sellers will compensate for the existence of “flops”, i.e. products that generate below-average sales. If, on the other hand, firms exporting many products were more likely to produce a “best selling” product, so that average profits per product were positively associated with the number of products sold, the cumulative process of
preferential attachment that underlies the model’s dynamics would be magnified and our conclusions would be reinforced.

3.3 New products

We follow Luttmer (2011) to model the entry/exit dynamics of products in our economy. New products can either be produced by incumbent firms as a result of innovation activities, or produced from scratch by new entrepreneurs. In equilibrium, new products are introduced by incumbent firms at the rate $\lambda$ and by new entrepreneurs at the rate $\nu$. Existing products can also be lost when competitors innovate over existing products. The rate at which this occurs in equilibrium is $\mu$. Finally, firms with only one product exit the market when some other firm innovates over the good they are currently producing. Thus, the number of products evolves according to the following law of motion:

$$\dot{N} = (\nu_t + \lambda_t - \mu_t)N_t$$

An initial condition determines $N_0$, which however is irrelevant for the equilibrium outcome.

3.3.1 Innovation by incumbents

To increase and maintain its portfolio of products, a firm must invest in R&D activities. We assume that new innovations arrive following a Poisson process with exponentially distributed waiting time of the form

$$\lambda_t = f(i_t)$$

where $i_t$ represents the resources (in this case labor) invested in the innovation process. We assume that $f(.)$ is increasing and exhibits strictly decreasing returns to scale. Each firm faces also the probability that some firm will innovate over a product it is currently producing. We assume that a firm innovating over an existing product can produce a better type of the same good without changing the product-specific attribute that captures consumer tastes. When this event occurs, the incumbent producer loses that product from its portfolio. An existing product is lost with an exponentially distributed waiting time with mean

$$\mu_t = g(h_t)$$

where $h_t$ is labor used to “maintain” existing products and $g$ is strictly decreasing and convex.\(^8\) Given the constant markup $\sigma/(\sigma - 1)$ of prices over marginal costs, firm’s average

\(^8\)This is one of the main departures from Klette and Kortum (2004), who assume a constant and homogeneous degree of “creative destruction” $\mu$.  

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profit per product $\pi_t(\tilde{a})$ can be rewritten as

$$w_t \left( \frac{l_t}{\sigma - 1} \right) + \sum_{c=1}^{k} \left( \frac{l_t}{\sigma - 1} \tau - f_c \right)$$

with $k \in [1, C - 1]$ and $\tilde{a} >= a_t^*$ (when $\tilde{a} < a_t^*$ average profits per product reduce to $w_t(\frac{h}{\sigma - 1})$, see Appendix C for derivations). The value of a firm as a function of the average preference attribute $v_t(\tilde{a})$ must satisfy the Bellman equation

$$r_t v_t(\tilde{a}) = \max_{\lambda, \mu} \left[ w_t \left( \frac{l_t}{\sigma - 1} + \sum_{c=1}^{k} \left( \frac{l_t}{\sigma - 1} \tau - f_c \right) \right) - (i_t + h_t) \right] + v_t(\tilde{a})(\lambda - \mu) + \dot{v}_t(\tilde{a})$$

with $k \in [1, C - 1]$ and average profits becoming $w_t(\frac{h}{\sigma - 1})$ for non-exporting firms. From an economic point of view, the first term inside the square brackets of equation (19) corresponds to profits coming from selling product $j$ in the domestic and in the foreign markets at time $t$ minus the costs associated to innovation ($i$) and imitation ($h$) activities for that particular product. The second term captures the expected gain from innovating over product $j$ and the expected loss from having another firm innovating over product $j$. The optimal levels of labor used in innovation and maintenance activities are given by

$$\lambda_t = f(i_t) \quad \mu_t = g(h_t) \quad v_t(\tilde{a}) f'(i_t) = -v_t(\tilde{a}) g'(h_t) = w_t$$

Since the distribution of the product-specific preference attribute $\Gamma(a)$ is constant and common to all products and all firms face the same fixed entry costs to foreign destinations, the law of large numbers implies that, as a firm’s number of products grows large, the average preference attribute $\tilde{a}$ characterizing its portfolio of goods, tends to the population mean $\tilde{a}$, that is common to all firms. Hence, abstracting for the moment from deviations pertaining to firms with low $n_{i,t}$, along the balanced growth path, there will be a unique innovation rate $\lambda$ and maintenance rate $\mu$ common to all firms.

### 3.3.2 Innovation by entrants

New products can also be introduced by agents acting as entrepreneurs. Following Luttmer (2011) we assume that, at each point in time, agents are endowed with one unit of effort that can be used to supply labor to existing firms or to produce a new product. Each agent has a skill vector $(x, y)$, where $x$ corresponds to the rate at which a new product is generated and $y$ is the amount of labor supplied per unit of time. Agents with skill vectors that satisfy $v_t(\tilde{a}) x > w_t y$ become entrepreneurs, while the other agents choose to supply labor to incumbent firms. Let $T$ be a time-invariant talent distribution defined over the set of all possible skill vectors with finite mean and density $\psi$. The per capita
supply of effort devoted to entrepreneurial activities is defined by

\[ E(v_t(\bar{a})/w_t) = \int_{v_t(\bar{a})x > w_ty} x dT(x, y) \]  

(21)

The per capita labor supply is

\[ L(v_t(\bar{a})/w_t) = \int_{v_t(\bar{a})x < w_ty} y dT(x, y). \]  

(22)

The rate \( \nu_t \) at which new entrepreneurs introduce a new product can be derived as follows

\[ \nu_t N_t = H_t E(v_t(\bar{a})/w_t). \]  

(23)

where \( N_t \) is the stock of products in a given point in time.

The following labour market clearing condition must hold

\[ N_t(l_t + i_t + h_t) = H_t L(v_t(\bar{a})/w_t). \]  

(24)

### 3.4 Balanced growth

As \( v_t(\bar{a}) \) is the same across firms, along the balanced growth path, firms will allocate (on average) the same fraction of labor \((i, h, l)\) per product. Here is where the fact that, as the portfolio of products grows large, \( \bar{a} \to \bar{a} \) and profits per product tend to an average value for all firms \( \pi_{i.t}(\bar{a}) \to \pi_{i.t}(\bar{a}) \), plays a crucial role. In other words, the assumption that the intensive and extensive margins are independent of each other implies that firm profits solely depend on the number of products sold.\(^9\) As a result, on average, firms can devote the same amount of resources to all products: intra-firms adjustments may well occur as long as they do not impact on the total amount of resources used by each firm (conditional on the number of products sold \( n_{i,t} \)). Best-selling products, which require higher levels of production will absorb more labor, compensating the existence (within the firm) of “below-average” products that use fewer resources.\(^10\) This in turn implies that the measure of products will grow at the same rate of population \( \eta \). From the consumer’s problem, wages \( w_t \) and per capita consumption \( c_t = X_t H_t \) grow at the rate \( k = \theta + \frac{\eta}{(\sigma - 1)} \) which is larger when goods are less substitutable. The resulting interest rate is \( r = \rho + k \).

\(^9\)From an empirical point of view, we find that the number of product exported by a firm barely correlates with average export per product: considering firms selling at least two products, the correlation coefficient is 0.03 (an analogous lack of substantial correlation between the intensive and extensive margins of trade is reported by Bernard et al, 2007 for US firms and by Bernard et al, 2014 for Belgian ones).

\(^10\)This may not be true for firms with a small number of products for which the average preference attribute may be different from \( \bar{a} \). The existence of those firms has important implications for the shape of the distribution of the number of products by firm that we will discuss in Section 5.
From the Bellman equation (19) one can see that wages and the value of a firm as a function of average preference attribute must satisfy the present-value condition:

\[
v(\tilde{a}) = \frac{w_t}{w} \left[ \frac{l_t}{\sigma - 1} + \sum_{c=1}^{k} \left( \frac{l_t^{\sigma - 1} - f_c}{\sigma - 1} - (i_t + h_t) \right) \right] - (i_t + h_t) - (i_t + h_t) + \left( \lambda - \mu \right) - \left( \lambda - \mu \right)
\]

where \((i, h)\) and \((\lambda, \mu)\) satisfy (20).

As the total number of products grows at rate \(\eta\), new entrepreneurs must add new products at the non-negative rate \(\eta - [\lambda - \mu]\). For a positive \(E(v(\tilde{a})/w)\), the entrepreneurial steady-state supply of products can be derived using (23)

\[
\frac{N}{H} = \frac{E(v(\tilde{a})/w)}{\eta - [\lambda - \mu]}
\]

Differently, \(E(v(\tilde{a})/w) = 0\) and \(\eta = \lambda - \mu\).

Along the balanced growth path, labour market clearing implies

\[
\frac{N(l + i + h)}{H} = L(v, w).
\]

Luttmer (2011) shows that if \(\rho > \eta\) and \(\eta > f(0) - g(0)\), for a positive \(E(v, w)\), then equations (20), (25), (26) and (27) define the unique balanced growth path and \(\eta > \lambda - \mu\).

A balanced growth path can arise with \(E(v, w) = 0\) if the talent distribution has bounded support. In this case, new products are only produced by existing firms.

### 3.5 The distribution of the number of products exported

Let us define \(M_{n,t}\) the mass of firms with \(n\) products at time \(t\). The aggregate measure of products is

\[
N_t = \sum_{n=1}^{\infty} n M_{n,t}.
\]

The number of firms with only one product evolves according to

\[
\dot{M}_{1,t} = \mu 2 M_{2,t} + v N_t - (\mu + \lambda) M_{1,t}
\]

where \(\lambda\), \(\mu\) and \(\nu = \eta - [\lambda - \mu]\) are constant along the balanced growth path. The number of single-product firms increases when firms with two products lose one or as a result of entry by new entrepreneurs. The number may decrease because firms with one product either add a new good to their portfolio or exit the market. The evolution of the number of firms with more than one product is described by

\[
\dot{M}_{n,t} = \lambda (n - 1) M_{n-1,t} + \mu (n + 1) M_{n+1,t} - (\mu + \lambda) n M_{n,t}
\]
for all \( n - 1 \in \mathbb{N} \).

A stationary distribution for the number of goods produced by each firm exists if (29) and (30) have a solution for which \( \frac{M_{n,t}}{N_t} \) is constant over time. Since along the balanced growth path \( N \) grows at the rate \( \eta \), then \( \dot{M}_t = \eta M_{n,t} \) for all \( n \in \mathbb{N} \). As \( N \) and \( M_n \) grow at the same rate \( \eta \), we can define

\[
P_n = \frac{M_{n,t}}{\sum_{n=1}^{\infty} M_{n,t}}
\]

(31)

for all \( n \in \mathbb{N} \).

Equation (31) gives the fraction of firms with \( n \) commodities. We can also define the fraction of all commodities produced by firms of size \( n \) as

\[
Q_n = \frac{nM_{n,t}}{\sum_{n=1}^{\infty} nM_{n,t}}
\]

(32)

for all \( n \in \mathbb{N} \).

Using (32) we can rewrite (29) and (30) as

\[
\eta Q_1 = \mu Q_2 + v - (\lambda + \mu)Q_1
\]

(33)

\[
\frac{1}{n} \eta Q_n = \lambda Q_{n-1} + \mu Q_{n+1} - (\lambda + \mu)Q_n.
\]

(34)

Luttmer (2011) provides a solution for (33)–(34), which we report in Appendix E. In particular, Luttmer (2011) shows that, under some parameter restrictions, a stationary distribution for the number of commodities produced by a firm exists and features a Pareto tail with a shape parameter greater than unity. In particular, \( \nu > 0 \) assures that a stationary distribution exists. Then, if \( \eta > 0 \), \( \lambda > \mu \) and \( \eta > \lambda - \mu \), then the right tail probabilities \( R_n = \sum_{k=n}^{\infty} P_k \) of the stationary distribution of products satisfy

\[
\lim_{n \to \infty} \left( 1 - \frac{R_{n+1}}{R_n} \right) = \xi
\]

(35)

where \( \xi = \frac{\eta}{(\lambda - \mu)} \) and \( R_n \) is a regularly varying sequence with index \(-\xi\) and \( \xi > 1 \).\(^{11}\)

As in Luttmer (2011), our model predicts that the distribution of products by each firm features a Pareto tail. However, in an open economy framework, some further interesting predictions can be derived for exporting firms. As the number of products sold by each firm and the preference attribute which determines whether a product is exported or not are independent, the number of products exported by each firms is simply a random sample

\[^{11}\text{When the rate } \nu = \eta - (\lambda - \mu) \text{ goes to zero, the limiting tail index } \xi = 1 \text{ associated with Zipf’s law arises.}\]
from the overall population of products sold domestically.\textsuperscript{12} Since a random sample taken from a power-law follows the same distribution, we can still conclude that the distribution of the number of exported products is power-law.

4 Empirical Evidence

In recent years, a large literature has documented a series of regularities that characterize international trade flows at various levels of aggregation, and the different dimensions in which trade can be decomposed, namely the intensive and the extensive margin (see for instance Bernard et al, 2009). How does the model developed in the previous section relate to this empirical evidence?

In this section we discuss the main implications of the model in light of the relevant stylized facts pertaining especially to the extensive margins of export and the behavior of multi-product firms. In so doing, we will highlight the main differences between existing models and our own setup, and to what extent these differences allow us to accommodate the empirical evidence. We concentrate on a number of key features that we consider relevant for any model of multi-product firms: the distributions of the number of products exported by each firm (which is characterized by substantial heterogeneity), the hierarchy among the products exported by each firm in different markets, the relationship between export scale and scope at the firm level.

The evidence presented in this Section refers to more than 100,000 French (manufacturing) exporting firms in the year 2003.\textsuperscript{13} The data are collected by the French Customs Service and are based on compulsory custom declarations filed by firms, and include, for each firm, export sales for each 8-digit product (of the Combined Nomenclature) by destination. For export outside the EU, firms are exempted from filing a custom declaration whenever export value is less than 1,000 euros (or volume is less than a ton); for trade within the EU there is an annual threshold of 100,000 euros, below which firms report a limited set of information if any. In practice, we observe a large number of firms reporting annual combined foreign sales below 100,000 so that the coverage is almost complete (see also Mayer et al, 2014).

4.1 Extensive margin: the number of products exported by each firm

Besides being a relatively rare phenomenon, export participation is also marked by a lot of heterogeneity: Bernard et al (2007) report that, in 2000, 40% of US exporters shipped

\textsuperscript{12}In fact, as it is common in this class of models, we also have that the range of exported products is a subset of products available domestically. In other words, there are no products that are exported but not consumed at home.

\textsuperscript{13}Data for other years yield analogous results and are available upon request.
a single product to just one foreign destination, while only 12% of exporting firms sold more than 5 products to more than 5 destinations.

Table 2 provides summary statistics characterizing the large heterogeneity in the number of products exported by French firms. In fact, while the number of products exported by each firm ranges between 1 and 770, 26.91% of firms export a single product, the median value is 4 and less then a quarter of firms export more than 10 products.

Table 2: Number of products exported by each firm: summary statistics. Data refers to 2003 and are classified according to the 8-digit CN (Combined Nomenclature).

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>9.67</td>
</tr>
<tr>
<td>Std. dev</td>
<td>20.96</td>
</tr>
<tr>
<td>Min</td>
<td>1.00</td>
</tr>
<tr>
<td>25th percentile</td>
<td>1.00</td>
</tr>
<tr>
<td>Median</td>
<td>4.00</td>
</tr>
<tr>
<td>75th percentile</td>
<td>10.00</td>
</tr>
<tr>
<td>Max</td>
<td>770.00</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>% of firms selling:</th>
</tr>
</thead>
<tbody>
<tr>
<td>only 1 product</td>
</tr>
<tr>
<td>1–5 products</td>
</tr>
<tr>
<td>&gt;10 products</td>
</tr>
<tr>
<td>&gt;100 products</td>
</tr>
<tr>
<td>&gt;500 products</td>
</tr>
</tbody>
</table>

Figure 1 provides a pictorial representation of the phenomenon: irrespective of whether we look at the number of products exported or at the combination of product-destination pairs (meaning that we treat the same product shipped to different destinations as different products), the distributions appear very skewed, and a power-law fit provides a good approximation of the data.

![Figure 1: Complementary cumulative distribution of the number of products (blue) and product-destination pairs (red) exported by firm in double logarithmic scale, with power-law fit. The power-law fit is obtained using the methodology described in Clauset et al (2009).]
As discussed in Section 3.5, the model predicts that the distribution of the number of products produced by each firm features a Pareto tail with a shape parameter greater than unity. Since the number of products exported by each firm is a random sample (based on the preference attribute) from the set of products sold domestically, and since a random sample taken from a power-law follows the same distribution, we can conclude that the distribution of the number of exported products is also power-law, in accordance with the empirical evidence. Moreover, an estimation of the shape parameter performed using the different available methodologies delivers values that range between 1.5 and 2.3, well above one as predicted by the model.\footnote{The shape parameter of a power-law tail can be computed by means of the Hill estimator, via regression analysis as proposed by Gabaix and Ibragimov (2011), or as a result of a Maximum Entropy estimation (see Bee et al, 2011 for details on the different approaches and a comparison of their respective virtues). Results are qualitatively similar and available upon request.}

### 4.2 Product hierarchy within multi-product firms

A number of recent papers present evidence suggesting a rather strict hierarchy among the products exported by each firm in different markets. In particular, Mayer et al (2014) compute the global ranking of each product sold by each firm (based on total export sales) and compare it with the local ranking, i.e. the ranking within each destination market. They report an average correlation of 0.68, which appears not driven by firms exporting one product to a single destination, but rather reflect a broader phenomenon: the correlation is still 0.59 for firms exporting more than 50 products to more than 50 countries. Arkolakis and Muendler (2010) take a slightly different route to look at the same phenomenon, namely the persistence of product ranking across destination markets. Using a large sample of Brazilian firms, they focus on two reference markets, the US and Argentina (Brazil’s two main export destinations), and compare export behavior there with export to all other markets. They find that, within a firm, the best-selling products in the reference market have higher sales in all other markets as well. Indeed, the rank-correlation between sales in the US (Argentina) and sales in the rest of the world is as high as 0.837 (0.860). Furthermore, lower ranked products tend to be shipped to fewer destinations.

Here we add some further evidence based on French data. When we replicate the approach by Mayer et al (2014) we find results that are comparable to theirs, as reported in Table 3. The average rank correlation across the 199 destinations for which we have at least 20 observations is 0.60. Interestingly, the largest values refer to France main trading partners (in 2003), namely Germany, Spain and the UK. This suggests that in large (and closer) markets the ranking of products is more strongly correlated, whereas marginal destinations (where fewer products are exported) tend to drive down the correlation.

Table 4 looks more systematically to the issue by using France’s three main trading
partners as reference markets. In so doing, we are able to replicate some of the analyses described in Arkolakis and Muendler (2010) —results are indeed very similar to theirs—and add further interesting information. For each reference market, column 1 of Table 4 reports the average number of destinations served by products with a given rank in the reference market. So, for instance, we see that the top selling product in Germany ships on average to 12.46 markets, whereas a firm’s 32nd most popular product in Spain is exported to roughly 10 countries. Column 2 displays the average number of foreign markets to which a firm with at least as many products as the corresponding rank exports. Clearly, firms with more products tend to serve more markets. However, among these destinations, higher ranking products cover a higher proportion: column 3 reports the share of destinations served by the firm, which are covered by products of a given ranking. This percentage goes down rather quickly: while the top product is shipped to 67% of all destinations covered by firms serving at least one destination other than Germany, the share goes down to 48% for the second product and 34% for the fourth. A very similar pattern is found for the other two reference markets, namely Spain and the UK.

Moreover, column 1 of Table 4 suggests a positive correlation between a product’s rank in the main export market and the number of destinations served. Although the relationship is not monotone, we do observe lower ranking products being shipped to fewer markets. To look more precisely to this issue, we compute the rank correlation between a product’s sales in a reference market and the number of destinations covered. Results (see Table 5) show a positive and significant correlation ranging between 0.34 and 0.42, which is robust to using all firms rather than focusing on enterprises exporting more than one product and/or serving more than one foreign market.

The presence of country-specific fixed export costs, together with the assumption that preference attributes are product-specific (but common across destinations), imply that our model predicts a (strict) hierarchy of export sales across destinations, similarly to what happens for models of multi-product firms based on core-competences (Eckel and Neary, 2010; Arkolakis and Muendler, 2010; Mayer et al, 2014; Arkolakis et al, 2016), and

<table>
<thead>
<tr>
<th>destination</th>
<th>observations</th>
<th>Spearman’s ρ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Germany</td>
<td>69,188</td>
<td>0.874</td>
</tr>
<tr>
<td>Spain</td>
<td>59,868</td>
<td>0.872</td>
</tr>
<tr>
<td>UK</td>
<td>53,666</td>
<td>0.872</td>
</tr>
<tr>
<td>overall</td>
<td>mean</td>
<td>0.599</td>
</tr>
<tr>
<td></td>
<td>st. dev.</td>
<td>0.126</td>
</tr>
<tr>
<td></td>
<td>min</td>
<td>0.200</td>
</tr>
<tr>
<td></td>
<td>2nd quarter</td>
<td>0.520</td>
</tr>
<tr>
<td></td>
<td>median</td>
<td>0.596</td>
</tr>
<tr>
<td></td>
<td>3rd quarter</td>
<td>0.676</td>
</tr>
<tr>
<td></td>
<td>max</td>
<td>0.874</td>
</tr>
</tbody>
</table>
Table 4: Overlap between reference country and rest of the world by product rank

<table>
<thead>
<tr>
<th>rank*</th>
<th>Reference market: Germany</th>
<th>Reference market: Spain</th>
<th>Reference market: UK</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1) (2) (3) (4)</td>
<td>(1) (2) (3) (4)</td>
<td>(1) (2) (3) (4)</td>
</tr>
<tr>
<td>1</td>
<td>12.46 18.68 67% 11,097</td>
<td>13.04 19.62 66% 10,246</td>
<td>13.79 20.72 67% 9,437</td>
</tr>
<tr>
<td>2</td>
<td>11.39 23.85 48% 6,275</td>
<td>12.31 25.51 48% 5,662</td>
<td>12.83 26.60 48% 5,260</td>
</tr>
<tr>
<td>4</td>
<td>10.04 29.76 34% 3,340</td>
<td>10.96 31.76 34% 2,973</td>
<td>11.38 33.03 34% 2,747</td>
</tr>
<tr>
<td>8</td>
<td>9.43 36.96 26% 1,572</td>
<td>9.72 38.75 25% 1,411</td>
<td>10.74 40.53 26% 1,280</td>
</tr>
<tr>
<td>16</td>
<td>9.19 44.34 21% 679</td>
<td>9.65 45.38 21% 634</td>
<td>9.78 47.70 20% 582</td>
</tr>
<tr>
<td>32</td>
<td>9.11 53.83 17% 259</td>
<td>10.02 53.54 19% 240</td>
<td>9.09 52.71 17% 233</td>
</tr>
<tr>
<td>64</td>
<td>9.00 61.83 15% 92</td>
<td>10.45 60.56 17% 89</td>
<td>9.55 59.91 16% 82</td>
</tr>
<tr>
<td>128</td>
<td>9.14 71.09 13% 22</td>
<td>7.55 78.18 10% 22</td>
<td>10.13 60.26 17% 23</td>
</tr>
</tbody>
</table>

* rank indicates the product rank for the firm in the specific reference market. The analysis is restricted to firms-products shipping to the reference market and at least one other destination. Columns: (1) average number of destinations served by products with a given rank in the reference market; (2) average number of foreign markets served by firms with at least as many products as the corresponding rank; (3) share of destinations that a product of given rank reaches relative to the total number of destinations in column 2; (4) number of firms.

Table 5: Correlation between export sales in top destination and number of destinations served.

<table>
<thead>
<tr>
<th>destination</th>
<th>GER</th>
<th>SPA</th>
<th>GBR</th>
<th>Top by firm</th>
</tr>
</thead>
<tbody>
<tr>
<td>All firms</td>
<td>0.339</td>
<td>0.358</td>
<td>0.384</td>
<td>0.407</td>
</tr>
<tr>
<td>Firms serving more than 1 dest.</td>
<td>0.351</td>
<td>0.373</td>
<td>0.389</td>
<td>0.423</td>
</tr>
<tr>
<td>Firms exporting more than 1 prod.</td>
<td>0.347</td>
<td>0.369</td>
<td>0.391</td>
<td>0.400</td>
</tr>
<tr>
<td>Firms exporting more than 1 prod. to more than 1 dest.</td>
<td>0.354</td>
<td>0.378</td>
<td>0.393</td>
<td>0.420</td>
</tr>
</tbody>
</table>

Spearman’s rank correlation coefficient; (number of observations in parentheses).
contrary to the version of the Bernard et al (2011) model where the authors assume that product attributes are independent across countries. In this latter case, in fact, there is not relation between sales in different markets and no hierarchy among products.

4.3 Relationship between firm scale and scope

Another relevant empirical observation concerns to the relationship between the intensive and the extensive margin of trade. Although this phenomenon has not received much attention so far, most models of multi-product firms do have predictions about it. Indeed, when a single mechanism drives both firm scale and scope (as firm productivity does in Eckel and Neary, 2010 and Bernard et al, 2011), a relationship between the two trade margins is warranted.

Bernard et al (2011) show that among US exporters to Canada there is a very small (albeit significant) positive correlation between scale (average export by firm) and scope (number of products exported). Arkolakis et al (2016) distinguish between regional (Latin American) and other destinations for Brazilian exporters and show that only in the latter case does average export increase with firm scope. Our data confirm that there is no relationship between export scale and scope for trade within the EU (which accounts for more than 60% of French exports in 2003), while the correlation is positive and significant (even if noisy and country-specific) for extra-European destinations. Figure 2 summarizes the results, while column (1) of Table 6 provides the coefficients from a regression of (log) scale on (log) scope, augmented by an interaction between scope and a dummy for extra-EU destinations, plus partner-country fixed effects. The regression confirms that no relationship exists between firm scale and scope for exports within the EU, while the coefficient of the interaction term with extra-EU destinations is positive and significant.

We look more carefully at this phenomenon by focusing at the top 5 destination markets (in terms of export sales) for each group of countries (EU Vs non-EU), running a separate regression of firm scale on firm scope (both in logs) for each destination. Estimated coefficients for the logarithm of firm scope are reported in column 2 of Table 6. We find a positive and significant coefficient only for 2 out of 5 EU destinations, whereas the coefficient are all significant for extra-EU markets.

To investigate this issue further, we run a small simulation where, for each firm and product sold in a given destination, we substitute actual export sales with 100 random draws from sales of single-product exporters to the same destination, chosen within the same CN 2-digit chapter. We then compute a simulated version of firm scale and compare our results in column (2) of Table 6 with those obtained from a regression of this “randomized scale” on firm scope. In this way, each firm maintains the number of products it actually exports, but export sales are randomly assigned. This process is very similar to the one postulated in our theoretical framework and does a fairly good job in replicating
Table 6: Scale and scope of multi-product exporters

<table>
<thead>
<tr>
<th></th>
<th>(1) all destinations</th>
<th>(2) real</th>
<th>(3) simulated</th>
</tr>
</thead>
<tbody>
<tr>
<td>ln(scope)</td>
<td>0.004 (0.006)</td>
<td>0.096*** (0.020)</td>
<td>0.030*** (0.008)</td>
</tr>
<tr>
<td>ln(scope) × non-EU</td>
<td>0.536*** (0.009)</td>
<td>-0.017 (0.020)</td>
<td>-0.057*** (0.009)</td>
</tr>
<tr>
<td>destination fixed effects</td>
<td>YES</td>
<td>Italy</td>
<td>0.020 (0.022)</td>
</tr>
<tr>
<td>R²</td>
<td>0.080</td>
<td>Belgium</td>
<td>-0.100*** (0.015)</td>
</tr>
<tr>
<td>Observations</td>
<td>284,842</td>
<td>US</td>
<td>0.830*** (0.022)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Japan</td>
<td>0.703*** (0.031)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>China</td>
<td>0.792*** (0.053)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Algeria</td>
<td>0.647*** (0.030)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Turkey</td>
<td>0.654*** (0.046)</td>
</tr>
</tbody>
</table>

Standard errors in parentheses; ***p < 0.01, **p < 0.05, *p < 0.1

Column (1): log(scale_{id}) = \beta_0 + \beta_1 \log(scope_{id}) + \beta_2 \log(scope_{id}) × non-EU + \delta_d + \epsilon_{id}; constant and destination dummies not reported.

Columns (2)-(3) report the coefficient on firm scope (in log) from the following regression: log(scale_{i}) = \gamma_0 + \gamma_1 log(scope_{i}) + \epsilon_i. Scale is average export per product, computed either on actual data (col. (2)) or as the average from 100 random draws from sales of single-product exporters in the same destination and NC-2digit chapter (col. (3)). The number of observations range between 2,258 (China) and 19,862 (Belgium).
Figure 2: Export scale and scope for French manufacturing firms. The vertical axis is in log scale and represents average export sales by firms exporting a given number of products. Sales are normalized by the average sales of single-product exporters to the same destination. Firm scope is the number of different CN8 codes exported, computed in each destination. The dashed lines represent a linear fit (on the levels of the variables).

the empirical results (see column 3 of Table 6). In fact, only six out of 10 regressions yield a positive and significant coefficient for firm scope, and these positive coefficients are concentrated among extra-EU destinations.

Figure 3 presents another way to look at the same phenomenon: the relationship between firm scope and the range of export sales. If the number of products exported increases with productivity, and the latter also determines export sales, we expect the distance between the revenues generated by each firm’s “best seller” and its marginal product to increase with scope (in the core-competence model, for instance, more productive firms have more room for adding product that sell progressively less). In a stochastic framework as ours, on the contrary, there is no relationship between the sales of different products within the firm, so that a blockbuster can coexist with products selling very little. Hence, the range of export sales should be uncorrelated with firm scope (above and beyond statistical nuisance). Figure 3 displays the relationship for two major export destinations, namely Germany and the US. Along with the information derived from the data (the log of the average export range for firms selling a given number of products), we also report the average value across 500 replications where actual sales are replaced by random draws from sales of single-product firms to the same destination. The 1st and 99th percentiles from the 500 replications provide us with a simulated confidence interval. The Figure suggests that the data behave no differently from a completely random
allocation of business opportunities.\textsuperscript{15}

Overall, at least for exports to nearby markets (which typically represent the bulk of foreign sales), we do not find a strong evidence in favor of the positive relationship between scale and scope implied by the core-competence model (Eckel and Neary, 2010) where firm productivity drives both the number of products exported by each firm and export sales associated to each product. In our model, on the contrary, the intensive and extensive margin of trade are assumed to be independent: investment in innovation determines the probability to develop and introduce new products, whereas the amount of export sales is given by the preference parameter associated with each product and is therefore unaffected by firm-specific characteristics. This simple setup represents a reasonable benchmark, at least to model export to regional markets.

Existing models deal with this lack of a strong correlation between scale and scope that is observed in the data by means of additional restrictions or model structure. In Bernard et al (2011), for instance, average exports per product are uncorrelated with the number of products exported only when consumer tastes are distributed Pareto, and the product-

\textsuperscript{15}It is worth noting that in Figure 3 it is a “nearby” destination such as Germany that displays a larger deviation from a purely stochastic benchmark. In fact, the left panel of the figure shows that the observed range of export values is often below the 95\% lower bound, and thus even less correlated with firm scope than one would expect under a random allocation of sales. Summing up all the evidence, there seems not to be a clear-cut pattern linking firm scale (however measured) and scope.
specific fixed costs of exporting are independent of product attributes (tastes). On the other hand, to make sense of this empirical regularity, Arkolakis et al (2016) introduce scope economies in their model: they assume that firms face marketing or distribution costs when penetrating a new market, and this cost can be spread to different variates sold by the same firm. Then, by assuming that scope economies are more important in nearby markets, they manage to neutralize the positive relationship between scale and scope implied by the “core competence” part of their model.

While the evidence presented in this Section does not represent a full-fledged test of any specific model, nonetheless it suggests that scope economies are not necessary to generate a lack of correlation between scale and scope. In fact, since our “randomized scale” comes from single-product firms, it is designed to neutralize synergies of any sort among products sold by the same firm. The fact that results are similar to those obtained from real data then implies that a process of random allocation of export sales, similar to the one postulated in our model, is sufficient to break a correlation between the intensive and extensive margin of trade. In other words, the data display a large degree of randomness in export sales, which is consistent with our stochastic setup, which we claim stems from product innovation, and more difficult to reconcile with (the baseline version of) existing models where productivity drives both firm scale and scope.

5 Discussion and Conclusions

The paper has presented an open-economy model of firm dynamics à la Luttmer (2011) that emphasizes the role of innovation to explain the heterogeneous export behavior by firms, and the large degree of concentration in export flows. The central role assigned to R&D and product innovation fills a gap in the literature on multi-product exporters and, more generally, in the trade literature, where innovation is mainly considered as a productivity enhancing mechanism. We have shown that the model does a good job replicating a number of empirical regularities that characterize multi-product exporters, such as the hierarchy in product sales across destinations, the large difference in the number of products exported by firms, with a large fraction of single-product firms and a small number of highly diversified companies, and the absence of any significant correlation between scale and scope observed in nearby markets.

In this section we further the discussion about some of the implications of the model, compare our setup with existing models of multi-product firms, and illustrate some explanations for possible departures from the model predictions.

A crucial assumption of the model is that the product-specific attributes representing consumers’ taste come from the same time-invariant distribution \( \Gamma(\bar{a}) \). This implies that, as firm scope grows larger (the number of products increases), the average preference attribute \( \bar{a} \) will better approximate the mean \( (\bar{a}) \) of the distribution \( \Gamma(\bar{a}) \) by virtue of the
law of large numbers. Hence, average revenues and profits per product will approximately be the same across firms, allowing us to characterize the balanced growth path and to derive a unique distribution of the number of products by firm.

On the other hand, the existence of firms exporting a small number of products can explain the fact that the lower tail of the distribution of $P(n)$ does not follow a power-law. In fact, as already anticipated in Section 3.3.1, the model assumes that all firms invest as if they were getting “average” profits on all their products. While this is not a constraint for firms with a large portfolio of products —as poor performance by some products can be compensated by other items featuring above-average sales— it is possible that firms selling only a few “unpopular” items are constrained in their ability to invest in innovation, since their revenues may be too low.\footnote{The model does not consider the financial system and the possibility that firms can access external resources. However, it is well known that financing innovation activities is particularly difficult given the intrinsic uncertainty of the process and of the associated returns.} If this is the case, then the cumulative process that lies at the core of our model may not properly work for firms in the lower tail of the distribution, leading to some departures from the power-law distribution. This is indeed what we observe for the distribution of exported products in Figure 1.

While the paper aims primarily at explaining firm-level differences in the number of products exported, it can provide some interesting insights on the intensive margin of export: in fact, export sales are also characterized by a high degree of heterogeneity, both across and within firms. The model presented in Section 3 leaves the distribution of the preference attribute $\Gamma(a)$ unspecified: by simply assuming that $\Gamma(a)$ follows a lognormal, we obtain that export sales of each product are also lognormally distributed, a feature that closely matches the data (see figure Figure A-1 in Appendix F, and also the evidence presented by Head et al, 2014 and Bee and Schiavo, 2017).\footnote{Most papers featuring heterogeneous firms postulate a Pareto productivity distribution which, under the standard assumption of a CES demand structure, translates into a Pareto distribution of sales and exports. More recent evidence, however, suggests that the Pareto distribution does a poor job when applied to the whole size/productivity distribution, and fits reasonably well only to the upper tail (see for instance Combes et al, 2012; Freund and Pierola, 2015).}

While this is not —strictly speaking— a result of the model, but rather follows directly from an (arbitrary) distributional assumption, the same happens in the existing literature, where the Pareto distribution of export sales comes directly from the assumption on the distribution of firm productivity. A similar path is taken by Eaton et al (2011), who assume that the logarithm of demand shocks follow a normal distribution with zero mean and finite variance, and show that this specific distributional choice matches data on sales by French manufacturing firms.

The assumption of lognormality at the level of product sales, combined with the model implication of a heavy-tailed distribution for the number of products exported by each firm, yield another interesting insight on the intensive margin of trade. Bee et al (2017) show that the distribution of export revenues tends to develop a Pareto upper tail once
sales are aggregated at the level of the firm (and further up). In fact, the upper tail of the distribution is significantly thicker than a lognormal for the class of highly-diversified, multi-product companies (Bee et al, 2017). This feature can be explained by the interaction of the two margins of export. More precisely, if the sales of each product follow a lognormal distribution and the number of products sold by each firm follows a power-law, the distribution of total firm sales is a lognormal distribution multiplied by a stretching factor which increases in firm scope (Growiec et al, 2008).\footnote{No closed form solution exist for the sum of lognormal distributions: this implies that, if product sales follow a lognormal distribution, aggregate exports (by firm, sector or country) cannot be characterized precisely.} When the number of products is small, the stretching factor is negligible and the distribution is close to a lognormal; on the contrary, the distribution displays a significant Pareto upper tail when the number of products gets larger.

The possibility that a firm may lose products when a competitor innovates over an existing good allows the model to rationalize the presence of significant churning observed in the data (Iacovone and Javorcik, 2010), and thus represents a further interesting feature of the our theoretical framework. Changes in the variable or fixed cost of exporting will induce firms to adjust their scope, so that within firm product churning acts as an additional margin of adjustment. Moreover, as pointed out by Iacovone and Javorcik (2010), gross churning will be larger than its net effect since firms may simultaneously add and drop products. Indeed, equation (15) shows that the equilibrium number of available goods results from the combination of new products being introduced (either by incumbents or new firms) and existing products being dropped.

The emphasis we give to innovation as the main mechanism behind firms’ ability to capture new business opportunities is not new in the recent trade literature, where several authors have long recognized the role that innovation may have in explaining the productivity premium associated with export participation (Cassiman et al, 2010; Golovko and Valentini, 2011; Esteve-Pérez and Rodríguez, 2013; Damijan and Kostevc, 2015). In our framework, however, innovation does not act as a mechanism to lower costs or improve quality, as it is customary in models of heterogeneous firms and trade, but rather as a way to develop new products (\textit{blueprints} in Luttmer’s original words). One could summarize the difference by saying that the focus shifts from process to product innovation.

This distinction is useful to understand the difference between our setup and the core-competence model introduced by Eckel and Neary (2010), and to rationalize the different implications they generate in terms of the correlation between firm scale and scope. The two approaches are not mutually exclusive, but rather complement each other in explaining different types of innovation: our mechanism pertains to \textit{diversification}, i.e. the introduction of new products, possibly in different markets or industries; the core-
competence view refers more to a strategy of product *differentiation* within the same market (aka “versioning” in the management literature) where the risk of cannibalization is much more relevant, sales of different products influence each other and can be traced down to a common ability by the firm.

Hence, in the real world the two strategies are likely to coexist: the prevalence of one or the other can then lead to observe a more or less pronounced correlation between scale and scope. In other words, the dependency between the intensive and extensive margins implied by the Eckel and Neary (2010) model can be moderated or completely eliminated by the presence of a diversification strategy that brings firms to expand in different market-segments or in different sectors. As long as distant markets feature higher fixed export costs (as one could easily conjecture in the case of extra-EU versus EU destinations) the probability of a “lucky draw”, i.e. a large product-specific preference attribute that makes export to those markets profitable, is lower (than in the case of nearby markets). Therefore, one can imagine that firms have a higher incentive to differentiate (rather than diversify) when operating in more difficult markets, as they know that the development of a new product faces a lower probability of success. Such a mechanisms could explain why we tend to observe a positive correlation between scale and scope only at more remote destinations.

A promising avenue for future research is to integrate the two approaches in a single theoretical framework, allowing for country- of even firm-specific features to determine the optimal degree of differentiation and diversification chosen by firms, and observed in different markets.

**References**


Appendices

A  Household’s problem

The representative household maximizes the following intertemporal utility function subject to an intertemporal budget constraint

\[ U_t = E_t \left[ \int_t^{\infty} \ln(X_t) e^{-\rho t} dt \right] \]

\[ \dot{A}_t = r_t A_t + w_t - X_t P_t \]

where \( X_t \) is a composite good, \( P_t \) is the price of the composite good, \( w_t \) is the wage rate, and \( A_t \) is the value of the household’s asset holdings. At any period \( t \), the representative consumer is endowed with one unit of labor. Total spending in final good at \( t \) is \( Y_t = P_t X_t \). The consumer’s problem is solved in two steps.

A.1  First step: dynamic consumption problem

The current value Hamiltonian is

\[ H(X_t, A_t, v_t) = \ln(X_t) + v_t [r_t A_t + w_t - X_t P_t] \]

The first order conditions are

\[ X_t : \ \frac{v_t}{X_t} = \frac{1}{X_t P_t} = \frac{1}{Y_t} \]

(A-1)

\[ A_t : \ \frac{\dot{v}_t}{v_t} = \rho - r_t \]

(A-2)

Taking the time derivative of (A-1), we get

\[ \frac{\dot{v}_t}{v_t} = -\frac{Y_t}{Y_t} \]

using (A-2), we get the standard Euler equation

\[ \frac{\dot{Y}_t}{Y_t} = r_t - \rho \]
A.2 Second step: static choice across products

The representative household chooses the optimal bundle of goods to consume \((X_t)\) given its budget constraint

\[
\max_{x_{j,t}}\; \; \; X_t = \left[ \sum_{j=1}^{N_t} a_j^{\frac{1}{\sigma}} (x_{j,t})^{\frac{\sigma - 1}{\sigma}} \right]^{\frac{\sigma}{\sigma - 1}}
\]

subject to \( \sum_{j=1}^{N_t} x_{j,t} p_{j,t} = Y_t. \)

The first order condition (FOC) for any product \(j\) is given by:

\[
\frac{\sigma}{\sigma - 1} \left[ \sum_{j=1}^{N_t} a_j^{\frac{1}{\sigma}} (x_{j,t})^{\frac{\sigma - 1}{\sigma}} \right]^{\frac{1}{\sigma - 1}} a_j^{\frac{1}{\sigma}} \left( \frac{\sigma - 1}{\sigma} \right) x_j^{-\frac{1}{\sigma}} = \lambda p_j
\]

where \(\lambda\) is the relevant Lagrangian multiplier. Taking the ratio between the FOCs for any two products \(i,j\) we obtain

\[
x_i = \frac{Y_t a_i p_i^{-\sigma}}{\sum_{j=1}^{N_t} a_j p_j^{-\sigma}}.
\]

The price index \(P_t\) can be defined as the level of income necessary to buy 1 unit of the bundle \(X_t\). Setting \(X_t = 1\) and solving for the associated expenditure level one gets

\[
P_t = \left[ \sum_{j=1}^{N_t} a_j p_j^{1-\sigma} \right]^{-\frac{1}{\sigma}}.
\]

Hence, demand for product \(j\) can be written as

\[
x_{j,t} = a_j \left( \frac{p_{j,t}}{P_t} \right)^{-\sigma} \frac{Y_t}{P_t}.
\]

B Firm’s optimal pricing rule

Let \(p_t^D\) and \(x_t^D\) be price and quantity of product \(j\) for the domestic market and \(p_t^F, x_t^F\) prices and quantities of product \(j\) for any of the \((C-1)\) foreign markets (we omit \(j\) to simplify notation).

Firms maximize the following profit function to find the optimal price of product \(j\) in the domestic and in the foreign market:

\[
\pi_t = \left( p_t^D x_t^D - \frac{w_t x_t^D}{z_t} \right) + \sum_{F=1}^{C-1} \left( p_t^F x_t^F - \frac{w_t x_t^F}{z_t} \tau - f_c \right)
\]
subject to
\[ x_i^D = a \left( \frac{p_t^D}{\bar{p}_t^D} \right)^{-\sigma} \frac{Y_t^D}{\bar{p}_t^D} \]
\[ \sum_{F=1}^{C-1} x_i^F = a \left( \frac{p_t^F}{\bar{p}_t^F} \right)^{-\sigma} \frac{Y_t^F}{\bar{p}_t^F}. \]

The resulting first order conditions are
\[ p_t^D = \frac{\sigma}{(\sigma - 1)} \frac{w_t}{z_t} \]
\[ p_t^F = \frac{\sigma}{(\sigma - 1)} \frac{w_t}{z_t} \tau = p_t^D \tau. \]

C Profit per product: the Bellman equation

Abstracting from the time index \( t \) for the sake of simplicity, profits from an average (exported) product \( j \) are given by
\[ \pi_j = x_j^D (p_j^D - \frac{w}{z}) - \sum_{j=1}^{k} (x_j^F (p_j^F - \tau \frac{w}{z}) - f_c) \]
with \( k \in [1, C - 1] \); using \( x_j^D = z_j l_j, x_j^F = z_j l_j, p_j^D = \frac{\sigma}{(\sigma - 1)} \frac{w}{z}, \) and \( p_j^F = \frac{\sigma}{(\sigma - 1)} \frac{w}{z} \tau, \) we can rewrite profits as
\[ \pi_j = x_j^D \left( \frac{\sigma}{(\sigma - 1)} \frac{w}{z} \right) + \sum_{j=1}^{k} \left[ x_j^F \left( \frac{\sigma}{\sigma - 1} \frac{w}{z} - \frac{w}{z} \tau \right) - f_c \right] = \]
\[ = x_j^D \frac{w}{z} \left( \frac{\sigma}{(\sigma - 1)} - 1 \right) + \sum_{j=1}^{k} \left[ x_j^F \frac{w}{z} \tau (\frac{\sigma}{\sigma - 1} - 1) - f_c \right] = \]
\[ = \frac{x_j^D w}{z(\sigma - 1)} + \sum_{j=1}^{k} \left[ \frac{x_j^F w}{z(\sigma - 1)} \tau - f_c \right] = \]
\[ = \frac{lw}{z(\sigma - 1)} + \sum_{j=1}^{k} \left[ \frac{lw}{z(\sigma - 1)} \tau - f_c \right] \]

For a firm whose products are sold only domestically, average profits per product reduce to \( \frac{lw}{z(\sigma - 1)} \)

D Heterogeneous productivity levels

In this section, we abandon the assumption that products are hit by exogenous preference shock and assume that each product is produced with a different level of productivity
from a time-invariant distribution common across goods and firms. Everything else being equal to the model presented in Section 3, the main mechanisms are still at work and the main predictions still valid. As before, the intertemporal utility of the representative consumer is

\[
U_t = E_t \left[ \int_t^\infty \ln(X_t)e^{-\rho t}dt \right] \tag{A-3}
\]

where \(X_t = \left( \sum_{j=1}^{N_t} x_{j,t}^{\sigma-1} \right)^{\frac{1}{\sigma}}\). Products are distinguished only by their productivity levels indexed by \(z > 0\) and are produced according to the following production technology

\[
x_{j,t} = z_j l_t \tag{A-4}
\]

Productivity levels \(z\) come from a distribution \(\Gamma(z)\) which is continuous in the domain \(0 < z < \infty\) with mean \(\bar{z}\). The resulting demand for a product \(j\) is now given by

\[
x_{j,t} = \left( \frac{p_{j,t}}{P_t} \right)^{-\sigma} Y_t \tag{A-5}
\]

where \(P_t = \left[ \sum_{j=1}^{N_t} (p_{j,t})^{1-\sigma} \right]^{1/\sigma}\) is the price index. As before, the static profit maximization problem for a product yields prices

\[
p_{j,t}^D = \frac{\sigma}{\sigma - 1} \frac{w_t \tau}{z_j} \tag{A-6}
\]

\[
p_{j,t}^F = \frac{\sigma}{\sigma - 1} \frac{w_t \tau}{z_j} \tag{A-7}
\]

for the domestic market a foreign market respectively. Revenues from sales of product \(j\) in the domestic market and in a foreign market are

\[
r_{j,t}^D = \left( \frac{p_{j,t}^D}{P_t^D} \right)^{1-\sigma} Y_t^D \tag{A-8}
\]

\[
r_{j,t}^F = \left( \frac{p_{j,t}^F}{P_t^F} \right)^{1-\sigma} Y_t^F \tag{A-9}
\]

As before, due to asymmetric entry costs, the number of products available to consumers differ across countries, implying \(Y_t^D \neq Y_t^F\) and \(P_t^D \neq P_t^F\). It follows that product \(j\) is associated to different revenues in each foreign market. Total revenues generated by product \(j\) are \(r_{t,j} = r_{t,j}^D\) if product \(j\) is sold only domestically, and become \(r_{t,j} = r_{t,j}^D + \sum_{c=1}^{k} r_{t,j}^F\) with \(k \in [1, C - 1]\) if product \(j\) is also exported to foreign markets.

Profits from selling product \(j\) into the domestic market and into a foreign market with
entry cost $f_c$ are

$$\pi_{t,j}^D = \left( \frac{\sigma w_t}{(\sigma - 1)P_t^D} \right)^{1-\sigma} Y_{t,j}^D \sigma^{\sigma-1} = \frac{r_{t,j}^D}{\sigma}$$  (A-10)

$$\pi_{t,j}^F = \left( \frac{\sigma w_t^T}{(\sigma - 1)P_t^F} \right)^{1-\sigma} Y_{t,j}^F \sigma^{\sigma-1} - f_c = \frac{r_{t,j}^F}{\sigma} - f_c$$  (A-11)

Total profits generated by product $j$ are given by $\pi_{t,j} = \pi_{t,j}^D + \sum_{c=1}^k \pi_{t,j}^F$, with $\sum_{c=1}^k \pi_{t,j}^F = 0$ if product $j$ is not exported.

At each point in time, for a firm $i$ with $n_{i,t}$ products, total revenues and profits equal

$$r_{i,t}^{Tot}(n_{i,t}) = \sum_{j=1}^{n_{i,t}} r_{t,j} = \sum_{j=1}^{n_{i,t}} \left( r_{t,j}^D + \sum_{c=1}^k r_{t,j}^F \right).$$  (A-12)

$$\pi_{i,t}^{Tot}(n_{i,t}) = \sum_{j=1}^{n_{i,t}} \pi_{t,j} = \sum_{j=1}^{n_{i,t}} \left( \pi_{t,j}^D + \sum_{c=1}^k \pi_{t,j}^F \right)$$  (A-13)

with $\sum_{c=1}^k r_{t,j}^F$ and $\sum_{c=1}^k \pi_{t,j}^F$ being zero for non-exporting firms.

As in the model presented in Section 3, we can express average revenues and profits of a firm with $n_{i,t}$ products as a function of a summary measure. However, this summary measure is now given by a firm’s average productivity $\bar{z} = \sum_{j=1}^{n_{i,t}} \frac{1}{n_{i,t}} z_j$

$$r_{i,t}(\bar{z}, n_{i,t}) = \sum_{j=1}^{n_{i,t}} \frac{1}{n_{i,t}} r_{t,j} = r_{t,j}^D + \sum_{c=1}^k r_{t,j}^F$$  (A-14)

$$\pi_{i,t}(\bar{z}, n_{i,t}) = \sum_{j=1}^{n_{i,t}} \frac{1}{n_{i,t}} \pi_{t,j} = \pi_{t,j}^D + \sum_{c=1}^k \pi_{t,j}^F$$  (A-15)

As the portfolio of goods $n_{i,t}$ that a firm produces increases, by the law of large number we expect $\bar{z} \to \bar{z}$, so that $r_{i,t}(\bar{z}) \simeq r_{i,t}(\bar{z})$ and $\pi_{i,t}(\bar{z}) \simeq \pi_{i,t}(\bar{z})$, where $\bar{z}$ is the average productivity of the distribution $\Gamma(z)$. Thus, for a large enough $n$, the average revenues and profits per product will be approximately the same for all firms. As in the model with product-specific attributes, this assumption guarantees the existence of a balance growth path and allows us to derive a unique distribution of the number of products produced by firms as shown in Sections 3.3–3.5. Moreover, as productivity is now constant over time, in the balanced growth path wages $w_t$ and per capita consumption $c_t$ grow at rate $k = \eta/(\sigma - 1)$.  

37
E Solution of the system (33)–(34)

Luttmer (2011) shows that if $\lambda$, $\mu$, $\eta$ and $\nu = \eta - (\lambda - \mu)$ are positive, the sequence $\{\beta_n\}_{n=0}^\infty$ defined by the recursion $\beta_n = 1/(1 - (\lambda \beta_n/\mu) + (\eta + \lambda n)/\mu n)$ and the initial condition $\beta_0 = 0$ is monotone and converges to $\min\{1, \mu/\lambda\}$. In this case, there exists only one non-negative and summable solution to equations (33)–(34), which is (the proof of the solution to the system is provided by Luttmer, 2011 in Appendix B of his paper)

$$Q_n = \frac{\nu}{\lambda} \sum_{k=0}^{\infty} \frac{1}{\beta_{n+k}} \prod_{m=n}^{n+k} \beta_m \prod_{m=n}^{n+k} \frac{\lambda \beta_m}{\mu}$$  \hspace{1cm} (A-16)

For large $n$ and $\lambda \neq \mu$ the distribution satisfies

$$Q_n \sim \frac{\nu}{|\lambda - \mu|} \prod_{m=1}^{n-1} \frac{\lambda \beta_m}{\mu}.$$  \hspace{1cm} (A-17)

If $\nu = 0$, the only non-negative and summable solution to equations (33)–(34) is zero, meaning that a stationary distribution does not exist. If $\nu > 0$, equation (A-16) adds up to 1 by construction and gives a stationary distribution $\{P_n\}_{n=1}^\infty$ by means of $P_n \propto \frac{Q_n}{n}$. The mean number of products of a firm can be written as $1/(\sum_{n=1}^{\infty} Q_n/n)$ which is finite by construction. If $\lambda < \mu$, $Q_n$ is bounded above by a multiple of the geometrically declining sequence $(\lambda/\mu)^n$. When $\lambda > \mu$ then $(\lambda \beta_n/\mu) \uparrow 1$ and (A-17) declines at a rate that is slower than a given geometric rate. Luttmer (2011) shows that under some parameter restrictions the connectivity distribution features a Pareto tail with a shape parameter greater than unity. If $\eta > 0$, $\lambda > \mu$ and $\eta > \lambda - \mu$, then the right tail probabilities $\lim_{n \to \infty} \left(1 - \frac{R_{n+1}}{R_n} \right) = \xi$ (A-18)

where $\xi = \frac{\eta}{(\lambda - \mu)}$. That is, $R_n$ is a regularly varying sequence with index $-\xi$ and $\xi > 1$.

Finally, when the rate $\nu = \eta - (\lambda - \mu)$ goes to zero, the limiting tail index $\xi = 1$ associated with Zipf’s law arises.

F Empirical evidence on the distribution of export sales

Most of the existing papers postulate a Pareto productivity distribution which, under the standard assumption of a CES demand structure, translates into a Pareto distribution of sales and exports. The debate about the proper characterization of firm size dates back at least to Simon and Bonini (1958), though the most common empirical reference to justify the choice of a Pareto distribution is the work by Axtell (2001).

Figure A-1 presents an histogram of (log) export sales by firm and product, and
a (truncated) normal fit. The inset displays a Q-Q plot that compares the empirical quantiles of the data against the theoretical quantiles derived from a truncated lognormal distribution whose parameters are estimated from the data. The plot confirms that the (truncated) lognormal provides a very good approximation of the data.

Figure A-1: Distribution of log export sales by firm and product, with superimposed truncated normal density. Inset: quantile-quantile plot of empirical versus theoretical quantiles. Data for 2003, CN 8-digit classification. The parameters of the (truncated) lognormal distribution are estimated using the EM algorithm discussed in Bee (2006).