Combining Factor Models and External Instruments to Identify Uncertainty Shocks

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October 23, 2017

PRELIMINARY, DO NOT CIRCULATE

Abstract

The empirical structural VAR literature which aims at recovering the effect of movements in macroeconomic uncertainty on the economy relies on two prerequisites: (i) An informationally sufficient model, which is a necessary condition for being able to recover structural from reduced form shocks, and (ii) a valid identification strategy. Commonly-used small-scale recursively identified VAR models are likely to fail both these conditions. I propose a Dynamic Factor Model (DFM) identified via an External Instrument (EI) to address both issues jointly. First, I find that, contrary to recursively identified models, the EI identification suggests an abrupt decline an rebound of the stock market. Second, contrary to small-scale models, the DFM suggests an increase in nominal quantities following an uncertainty shocks. Third, I show evidence that small-scale models are, indeed, informationally deficient, which could explain the differences in findings.

1 Introduction

Since Bloom (2009) there has been a growing evidence from empirical vector-autoregressive (VAR) models that macroeconomic uncertainty, i.e. difficulties in accurately predicting the future of macroeconomic quantities, has effects on the real economy. A correct assessment of these effects relies on two prerequisites: An informationally sufficient model and a valid identification scheme. Without the former, it is impossible to recover structural shocks from reduced form shocks even if the identification scheme is credible. Without the latter, the mapping between reduced-form to

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structural shocks is incredible. Progress has been made in both directions. However, these two issues have for now only been addressed in isolation.

In this paper, I will address them jointly by employing a Dynamic Factor Model (DFM), which is believed to alleviate informational insufficiency issues (see e.g. Bernanke et al., 2005) together with an identification strategy based on external instruments (EI), which has been argued to be more credible than the commonly used recursive identification (see e.g. Stock and Watson, 2012 and Piffer and Podstawski, 2017).

I find that recursively identified models (both small- and large-scale) fail to capture the adverse stock-market reaction which follows an uncertainty shock because they exclude on-impact reactions of the stock market index. This issue can be resolved using EI identification and I find a severe drop and quick recovery of the stock market as Piffer and Podstawski (2017). Furthermore, small-scale models (both recursively identified and via EI) suggest that uncertainty shocks resemble aggregate demand shocks, i.e. they decrease both economic activity and nominal quantities such as goods prices and nominal wages, as argued by Leduc and Liu (2016). My large-scale DFM, however, suggests that a decline in economic activity is accompanied by a short-term rise in nominal quantities.

Several papers have employed larger-scale models to model the impact of uncertainty shocks. These are for example Kamber et al. (2016) or Mumtaz et al. (2016). A couple of recent contributions have employed identification strategies other than recursive identification in the context of uncertainty shocks. Examples are Piffer and Podstawski (2017) and Ludvigson et al. (2015) who both use EI and Mumtaz and Theodoridis (2017) who identify uncertainty shocks via realized volatility within their model.

A paper that is close to mine in terms of methodology is Kerssenfischer (2017). He employs a dynamic factor model (DFM) combined with an external instrument to investigate whether commonly found puzzles in the responses of the economy to monetary policy shocks can best be explained by informational or by identification issues.

The remainder of the paper is organized as follows: Section 2 introduces the model setup. Section 3 explains the identification of an uncertainty shock. Section 4 presents the data, the estimation strategy and discusses the results. The last section concludes.

2 Model Setup

The empirical model is a Dynamic Factor Model (DFM). It is based on the assumption that the dynamics of a large set of observable series, $X_t$, can be explained by linear combinations of a small set of factors, $F_t$. The mapping from factors into observable series can be described by the following observation equation:
\[ X_t = \Lambda F_t + \xi_t, \]  
where \( X_t \) is a \( N \times 1 \) vector of observable series, \( \Lambda \) is a \( N \times K \) matrix of factor loadings, \( F_t \) is a \( K \times 1 \) vector of latent factors and \( \xi_t \) is a \( N \times 1 \) vector of idiosyncratic errors. \( \xi_t \) can in general be serially correlated, i.e. \( \text{Cov}(\xi_t, \xi_{t-1}) \neq 0 \), but they are uncorrelated across series, i.e. \( \text{Cov}(\xi_{t,i}, \xi_{t,j}) = 0 \) \( \forall i, j \).

In addition, I assume that the factors themselves follow a VAR-process with lag length \( P \), which can be described by the transition equation:

\[ F_t = A(L)F_{t-1} + u_t, \]  
where \( A(L) \) includes a constant and \( P \) lag operators. The \( K \times 1 \) vector of reduced form errors is serially uncorrelated, i.e. \( \text{Cov}(u_t, u_{t-1}) = 0 \) \( \forall t \). Also, \( u_t \) are uncorrelated with all leads and lags of the idiosyncratic errors, \( \xi_t \), i.e. \( \text{Cov}(u_t, \xi_{t-k}) = 0 \) \( \forall k \). They can be expressed as a linear combination of structural shocks, \( \epsilon_t \), as \( u_t = B\epsilon_t \), where \( B \) is a \( K \times K \) matrix containing the structural impact effects. Furthermore, the structural errors are assumed to be uncorrelated and have variance normalized to one: \( \text{Var}(\epsilon_t) = I_K \). Note that this DFM is written in static form, i.e. lags of the factors do not enter the observation equation (1). As pointed out by Stock and Watson (2016), an equivalent dynamic form can be derived, so this is not a restrictive assumption.

### 3 Identification of an Uncertainty Shock

The DFM has two main advantages compared to small-scale VAR models: First, the DFM is less likely to suffer from informational deficiency (see e.g. Bernanke et al., 2005 or Forni and Gambetti, 2014). Because the information set in \( X_t \) is large, it is more likely that the reduced form errors, \( u_t \), span the space of structural innovations \( \epsilon_t \). This renders it possible to construct \( \epsilon_t \) from \( u_t \). Second, in the context of a DFM, the researcher does not have to take a stand on which measure to use for an economic concept of interest such as ”economic activity”, ”the price level” or ”uncertainty”. These can be treated as latent factors, which can be estimated.

However, informational sufficiency and an appropriate measure of economic concepts are necessary, not sufficient conditions to recover \( \epsilon_t \). As in small-scale VAR models, one needs to find credible restrictions for \( B \) in order to obtain \( \epsilon_t \) in a DFM. In addition, the DFM allows the researcher to impose restrictions on the mapping from factors to observable series through restrictions on \( \Lambda \).

I employ two different sets of restrictions to identify an uncertainty shock. The baseline identification scheme A relies on an external instrument. This strategy has recently been combined with dynamic factor models by Kerssenfischer (2017) in the context of monetary policy shocks. The
alternative identification scheme B relies on short-run exclusion restrictions.

3.1 Baseline: Identification via External Instruments

Identification of the uncertainty shock via an external instrument is possible if there exists an instrument $m_t$ such that

- $E(m_t \epsilon_t^{unc}) = \phi, \quad \phi \neq 0$
- $E(m_t \tilde{\epsilon}_t) = 0,$

where $m_t$ is an instrument, $\epsilon_t^{unc}$ is the uncertainty shock and $\tilde{\epsilon}_t$ are the remaining $K-1$ structural shocks. This says that the instrument is uncorrelated with all structural shocks (exogeneity) except for the shock of interest (relevance).

In order to implement the identification scheme, the observation and transition equations rewrite as:

$$
\begin{bmatrix}
\tilde{X}_t \\
unc_t
\end{bmatrix}
= \begin{bmatrix}
\tilde{\Lambda} \\
0 \\
1
\end{bmatrix}
\begin{bmatrix}
\tilde{F}_t \\
unc_t
\end{bmatrix}
+ \begin{bmatrix}
\xi_t \\
0
\end{bmatrix},
$$

(3)

where $\tilde{X}_t$ are all observable series except for one of the uncertainty measures, $unc_t$ is one of the uncertainty measures, which is imposed as an observable factor. $\tilde{F}_t$ are $REI$ latent factors. The vector $\xi_t$ contains a zero as the last element because $unc_t$ is measured without error.

The transition equation is now specified as:

$$
\begin{bmatrix}
\tilde{F}_t \\
unc_t
\end{bmatrix}
= A(L)
\begin{bmatrix}
\tilde{F}_{t-1} \\
unc_{t-1}
\end{bmatrix}
+ \begin{bmatrix}
\tilde{B} \\
b_{unc}
\end{bmatrix}
\begin{bmatrix}
\tilde{\epsilon}_t \\
\epsilon_t^{unc}
\end{bmatrix},
$$

(4)

where, without loss of generality, $b_{unc}$ is the last column of $B$ corresponding to the uncertainty shock and $\tilde{B}$ are the remaining $K-1$ columns corresponding to non-identified structural shocks.

Note that we do not restrict $\Lambda$ except for the fact that we impose one of the uncertainty measures to be an observable factor. All factors and all observable series are allowed to contemporaneously react to an uncertainty shock.

3.2 Scheme B: Recursive Identification

In order to implement an identification scheme which mimics the commonly used recursive identification, rewrite (1) and (2) as:

$$
\begin{bmatrix}
X^f_t \\
unc_t \\
X^s_t
\end{bmatrix}
= \begin{bmatrix}
\Lambda^f \\
\Lambda^u \\
\Lambda^s
\end{bmatrix}
\begin{bmatrix}
F^f_t \\
F^unc_t \\
F^s_t
\end{bmatrix}
+ \xi_t,
$$

(5)
where we distinguish three distinct sets of observed series: \( X_t = \begin{bmatrix} X_{ft} & X_{unc} & X_{st} \end{bmatrix}' \). \( X_{ft} \) is of dimension \( N_f \times 1 \), \( X_{unc} \) is of dimension \( N_{unc} \times 1 \) and \( X_{st} \) is of dimension \( N_s \times 1 \). We also distinguish three distinct sets of factors: \( F_t = \begin{bmatrix} F_{ft} & F_{unc} & F_{st} \end{bmatrix}' \) of dimension \( R_f \times 1, R_{unc} \times 1 \) and \( R_s \times 1 \), respectively.

\[
\begin{bmatrix}
F_{ft} \\
F_{unc} \\
F_{st}
\end{bmatrix}
= A(L)
\begin{bmatrix}
F_{ft-1} \\
F_{unc-1} \\
F_{st-1}
\end{bmatrix}
+ \begin{bmatrix}
b_{11} & \cdots & 0 \\
\vdots & \ddots & \vdots \\
b_{N1} & \cdots & b_{NN}
\end{bmatrix}
\begin{bmatrix}
\epsilon_{ft} \\
\epsilon_{unc} \\
\epsilon_{st}
\end{bmatrix}
\tag{6}
\]

where \( B \) is now restricted to be lower-triangular.

Note that we impose short-run exclusion restrictions in two places: In the transition equation (6) and on \( A \) through (5). \( A \) is restricted such that financial variables react only to financial factors, the uncertainty measures react to financial and uncertainty factors and real variables react to all three sets of factors.

These restrictions on \( A \) are chosen in order to mimic the identification scheme often employed in small-scale VAR models (see, e.g. Bloom, 2009 and Caggiano et al., 2014). It labels a shock an uncertainty shock if (i) it is orthogonal to other structural shocks and (ii) it is the only shock with a contemporaneous impact on the uncertainty measure, except for financial shocks.

In small-scale VARs, this scheme is implemented by ordering a stock-market index (e.g. S & P 500) first, followed by an uncertainty indicator (e.g. the VXO) and a number of variables measuring economic activity (e.g. CPI, industrial production, unemployment). The main shortcoming of this identification strategy is the implausible assumption that financial variables do not contemporaneously react to an uncertainty shock.

In order to mimic this scheme in the DFM context, I order financial factors first. They explain the movement in series such as stock market indices or bond spreads. They are followed by an uncertainty factor constructed from various uncertainty indeces, and real factors, which explain the variance in series such as industrial production, CPI or unemployment. The federal funds rate is also included in the set of real series to account for the fact that the Central Bank might contemporaneously react to movements in uncertainty by adjusting the federal funds rate.

This identification scheme shares the shortcoming with recursively identified small-scale VAR models, which is that financial variables are not allowed to contemporaneously react to an uncertainty shock.
4 Data, Estimation and Results

4.1 Data and Transformations

The point of departure is the monthly FRED dataset by McCracken and Ng (2016), which uses US Data from 1959M4 to 2017M2.  

I add three uncertainty indeces from Ludvigsson, Ma and NG (2016): Macro-uncertainty, financial uncertainty and real uncertainty.  These are available from 1960M7 to 2017M6.

One concern is the measurement of the monetary policy stance. Traditionally, the effective federal funds rate is considered the policy tool of the central bank. However, given that it was constrained by the zero lower bound in the period 2009M1 to 2015M11, this variable cannot serve as an indicator of the policy stance. That is why, for this period, I replace the effective federal funds rate by the shadow rate as computed by Wu and Xia (2016). Figure 7 in the Appendix shows the shadow rate.

All series are transformed to have zero mean and unit variance. All series are transformed to induce stationarity as proposed by McCracken and Ng (2016). The data is then shortened so as to ensure that less than 5 series from the original dataset have missings. In a subsequent step, missings are replaced by zeros, which is the unconditional mean of the standardized series. These missing values occur mostly at the beginning of the dataset and amount to less than 1 percent of total observations. Therefore, the joint dynamics should not be overly affected by this imputation. The resulting dataset has coverage from 1962M8 to 2016M10.

In addition, the gold price variation around selected events from Piffer and Podstawski (2017) is used as an external instrument (EI). It is available from 1979M1 to 2015M7.

4.2 Determining the number of factors

The number of factors are determined based on two criteria: First, a scree plot, which plots the marginal contributions of the \( r \)-th principal component to the average \( R^2 \) of the \( N \) regressions of \( X_t \) against the first \( r \) principal components. It is the average additional explanatory value of the \( r \)-th factor. Second, the Bai and Ng (2002) (BN) criterion function, which consists of the sum of squared idiosyncratic errors plus a penalty term. It writes as:

\[
BN(R) = \log\left( \frac{1}{NT} \sum_{t=1}^{T} \hat{\xi}_t^\prime \hat{\xi}_t \right) + R \frac{N + T}{NT} \log(min(N,T)),
\]

\[\tag{7}\]

1It is available at https://research.stlouisfed.org/econ/mccracken/fred-databases/

2These are available at https://www.sydneyludvigson.com/data-and-appendixes/

3It is available at https://sites.google.com/site/michelepiffereconomics/home/research-1
where $\hat{\xi}_t$ are the estimated idiosyncratic errors from equation (1). When these two criteria contradict each other, judgement is required.

For the baseline identification, I extract $R^{EI}$ factors from all series contained in $X_t$. I refer to this as the "Full Set". For the alternative scheme, I extract $R^s$ real factors from $X^s_t$, $R^{unc}$ uncertainty factors from $X^{unc}_t$ and $R^f$ financial factors from $X^f_t$.

Table 1 shows the BN criterion. For the number of factors extracted under the baseline identification scheme, I follow the BN criterion and choose $R^{EI} = 7$.

For the alternative identification scheme, the scree plot (figure 1) suggests that 3 real factors explain a large share of the variance in real variables, while the fourth real factor has a smaller marginal explanatory power. The BN criterion points towards the use of 5 real factors. I choose $R^s = 3$. The scree plot suggest 2 financial factors, which together explain about half the variance in financial factors. The BN criterion is not informative in this case, so I choose $R^f = 2$. One uncertainty factor explains about 60% of the variance in uncertainty measures, so I choose $R^{unc} = 1$.

This choice gives 7 latent factors in Scheme A and 6 latent factors in Scheme B. This has the advantage that the dimension with which the transition equation (2) is estimated, is similar in size to that of a small-scale VAR model, which typically uses 8 variables. This ensures that differences in results are unlikely to be driven by estimation efficiency.

A potential concern is that the idiosyncratic disturbances $\xi_t$ are correlated across series, i.e. $\text{Corr}(\xi_{t,i}, \xi_{t,j}) \neq 0$ for some $i \neq j$. In that case, an approximate DFM as in Chamberlain and Rothschild (1982) would be appropriate. In my case, the correlation across series exceeds 0.2 in less than 5 percent of cases. This suggests that cross-series correlation is not a quantitatively important issue and we can proceed with standard estimation procedures.

<table>
<thead>
<tr>
<th>Nb of factors</th>
<th>Real Factors</th>
<th>Financial Factors</th>
<th>Uncertainty Factors</th>
<th>Full Set</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>-0.1582</td>
<td>-0.2014</td>
<td>-0.7909</td>
<td>-0.1305</td>
</tr>
<tr>
<td>2.0</td>
<td>-0.2157</td>
<td>-0.5142</td>
<td>-1.705</td>
<td>-0.1913</td>
</tr>
<tr>
<td>3.0</td>
<td>-0.2738</td>
<td>-0.7561</td>
<td>undefined</td>
<td>-0.2389</td>
</tr>
<tr>
<td>4.0</td>
<td>-0.2865</td>
<td>-0.9651</td>
<td>-68.9</td>
<td>-0.2741</td>
</tr>
<tr>
<td>5.0</td>
<td>-0.2932</td>
<td>-1.122</td>
<td>undefined</td>
<td>-0.2982</td>
</tr>
<tr>
<td>6.0</td>
<td>-0.2917</td>
<td>-1.228</td>
<td>undefined</td>
<td>-0.3156</td>
</tr>
<tr>
<td>7.0</td>
<td>-0.288</td>
<td>-1.33</td>
<td>undefined</td>
<td>-0.3255</td>
</tr>
<tr>
<td>8.0</td>
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<td>-1.449</td>
<td>undefined</td>
<td>-0.3242</td>
</tr>
<tr>
<td>9.0</td>
<td>-0.2701</td>
<td>-1.535</td>
<td>undefined</td>
<td>-0.3215</td>
</tr>
<tr>
<td>10.0</td>
<td>-0.2619</td>
<td>-1.661</td>
<td>undefined</td>
<td>-0.3158</td>
</tr>
</tbody>
</table>

Table 1: *Bai and NG (2002)* criterion
4.3 Factors in the two Schemes

For scheme A, factors are estimated and the observable factor is imposed using the iterative procedure suggested by Boivin et al. (2009). For scheme B, this procedure is adapted to reflect the restrictions that are imposed. See the appendix for a step-by-step explanation of this modified algorithm.

The factors from the two identification schemes are correlated suggesting that they yield similar fits to the data. For each factor from Scheme B, there is at least one factor from Scheme A such that the Null hypothesis of no correlation can be rejected at conventional significance levels (see tables 4 and 5 in the Appendix for details). Unsurprisingly, the uncertainty factor from Scheme B is highly correlated with the the uncertainty measure, which is imposed as an observable factor in Scheme A (a correlation coefficient of 0.75 with a corresponding p value of less than 0.01). However, the other factors from Scheme A also have counterparts in Scheme B: The first and third factor from Scheme A have a 99 and 94 percent correlation with the first and second real factor in Scheme B, respectively, suggesting that they capture movements in the real business cycle. The second factor in Scheme A has a 86 percent correlation with the first financial factor in Scheme B suggesting that it captures mostly movements in financial variables. These two observations are also confirmed by
a visual inspection of Figures 2 and 3, which plot the factors from the two schemes.

4.4 Gold Price Variation as External Instrument

Piffer and Podstawski (2017) argue that a proxy for the uncertainty shock could be based on the price of gold. The intuition behind this idea is that gold is considered a safe haven asset which investors refuse to in times of heightened uncertainty. This will generate movements in the price of gold. The challenge consists in finding price variations which are not correlated with structural shocks other than the uncertainty shock (exogeneity condition). In order to achieve this, Piffer and Podstawski (2017) collect a series of 38 events, which are considered to be associated with movements in uncertainty (e.g. the fall of the Berlin wall or the 9/11 terrorist attacks). They then compute the variation of the gold price around these events. In addition they show that this proxy has a low correlation with other structural shocks as computed by Stock and Watson (2012), which is further evidence for exogeneity to their system. They assess the relevance of this proxy via an F-test (see below for details).

Figure 4 shows the proxy. It peaks in well-known events such as the 9/11 terrorist attacks in 2001 or the bankruptcy of Lehman brothers in 2008.
Figure 3: **Estimated Factors (Scheme B):** Latent factors from Scheme B.

Figure 4: **Uncertainty Proxy.** Gold price variation around selected events. The sample period is 1979M1 to 2015M7.
4.5 Lag Length Selection

In order to determine the lag length, I consider the AIC, the BIC and the HQ criterion presented in table 2. I will follow the HQ criterion and choose $P = 3$.

4.6 Test for weak instruments

In order to determine the strength of the instrument I will follow Piffer and Podstawski (2017) and use a t- and an F-test. These are based on the following regression:

$$\hat{u}_{ti} = \alpha + \beta_i m_t + v_{ti}, \quad (8)$$

where $\hat{u}_{ti}$ is the residual of the $i$-th equation of the VAR in equation (4). The Null is $\beta_i = 0$ suggesting that the the instruments has no explanatory power for $u_{ti}$. The resulting t-test and F-test are:

Table 3 presents the t-test and F-test results. The Null of no correlation between the EI and the reduced form residual is rejected for the last row corresponding to the uncertainty measure.
(the observable factor). This suggests that the EI correlates with the residual of the uncertainty equation, which is suggestive evidence of instrument relevance.

4.7 Impulse Responses from Baseline (Scheme A)

Figure 5 shows the impulse responses obtained from 4 models: A small-scale, recursively identified VAR model (column 1); a recursively identified DFM (column 2, Scheme B), a small-scale VAR model using EI (column 3) and a DFM using EI (column 4, Scheme A). The response variables are the ones employed by Bloom (2009), Piffer and Podstawski (2017) and other studies: \( \Delta \log(S&P500) \), VXO, federal funds rate, \( \Delta \log(wages) \), \( \Delta \log(CPI) \), hours, \( \Delta \log(employment) \), and \( \Delta \log(IP) \). IRF confidence bands are computed point-wise at the 1 and 16 percent significance level. The shock is normalized to generate an increase of 2.5 in the VXO, which is comparable in magnitude to Piffer and Podstawski (2017). The number of bootstrap repetition is 500. A wild bootstrap scheme is employed (see appendix for details).

All four models replicate the main findings of Bloom (2009): A rapid drop and subsequent rebound of employment, production and hours worked.

The two recursively identified models (column 1 and 2) exclude an on-impact effect of the uncertainty shock on the stock market index by assumption. Relaxing this assumption in the EI schemes (columns 3 and 4) leads to a severe drop in this index for 4 months and a subsequent quick recovery, in line with Piffer and Podstawski (2017).

The small-scale models (columns 1 and 3) find an initial slight decrease in prices and nominal wages. This is in line with Leduc and Liu (2016) who argue that uncertainty shocks resemble aggregate demand shocks, which decrease both economic activity and nominal quantities. Monetary policy reacts with a tightening (i.e. a decrease in the federal funds rate). The large-scale models (columns 2 and 4), however, point in a different direction: both prices and nominal wages increase on impact and monetary policy reacts with an initial tightening (an increase in the federal funds rate) and only after 3 months begins to loosen its policy. This could be an indication that, similar to the price puzzle for the case of monetary policy shocks described by Sims (1992) and Bernanke et al. (2005), small-scale VAR models might miss information observed by economic agents. DISCUSS ROBUSTNESS

4.8 Test for Informational Sufficiency of the small-scale model

A formal assessment of the informational sufficiency of the small-scale model can be performed by employing a testing procedure proposed by Forni and Gambetti (2014). The test is based on the concept of informational sufficiency. The information contained in a set of variables included in a VAR is globally sufficient for a set of structural if the reduced form errors \( u_t \) span the space of structural shocks \( \epsilon_t \).
Figure 5: **Impulse Responses:** The sample is 1962M8 - 2016M10. The first column shows (non-cumulated) IRFs from a recursively identified small-scale VAR. The second column shows IRFs from a recursively identified DFM. The third column shows IRFs from a small-scale VAR identified via EI. The last column shows IRFs from a DFM identified via EI. The bands are computed as point-wise quantiles at the 1 percent and 16 percent confidence level. The number of bootstrap repetitions is 500.
The intuition behind the test is that if a small-scale VAR model is informationally sufficient then factors extracted from a large set of data should not contain information about the variables included in this small-scale VAR model. This can be tested via a multivariate Granger causality test proposed by Gelper and Croux (2007). The Null and alternative hypothesis write as:

\[ H_0 : \quad y_{PP}^{PP} = \Phi(L)y_{PP}^{PP-1} + v_r^t \]
\[ H_1 : \quad y_{PP}^{PP} = \Phi(L)y_{PP}^{PP-1} + \Psi(L)F_{t-1} + v_f^t, \]

where \( y_{PP}^{PP} \) are the variables in PP2017, \( v_r^t \) are the residuals in the restricted model, \( v_f^t \) are the residuals in the unrestricted, full model and \( \Phi(L) \) and \( \Psi(L) \) are lag polynomials.

Under the Null, the variables in the VAR are explained by their own lags. Under the alternative, factors \( F_{t-1} \) contain relevant information, i.e. the lag polynomial \( \Psi(L) \) consists of non-zero coefficient. The factors are said to be Granger-causal for the variables in the small-scale VAR model. The test statistic relates the residuals from the restricted to the residuals from the unrestricted model.

\[
test^{FG} = \log\left(\frac{\left|\sum_{t=1}^{T} v_r^t v_r^t\right|}{\left|\sum_{t=1}^{T} v_f^t v_f^t\right|}\right)
\]

Figure 6 shows the distribution of the bootstrap test statistic under the Null of no Granger causality together with the actual value. The lag length is left at \( P \), as in the main model. The Null can be rejected suggesting that the factors contain information about the variables in Bloom (2009) and Piffer and Podstawski (2017). Their models are therefore likely to be informationally deficient.

5 Conclusion

This paper combines a Dynamic Factor Model (DFM) with External Instruments (EI).
Figure 6: Test for Informational Sufficiency

References

Bai, J. and S. Ng (2002). Determining the number of factors in approximate factor models. *Econometrica* 70(1), 191–221.


6 Appendix

A1: Estimation of Factors and Loadings

In the recursive identification scheme A I distinguish three distinct sets of factors: $F^f_t$, $F^{unc}_t$ and $F^s_t$. These map into three distinct sets of observable series, $X^f_t$, $X^{unc}_t$ and $X^s_t$, according to (3). Boivin et al. (2009) propose an iterative restricted PC estimation procedure of which I implement the following variation:

1. • Extract $R^f$ financial factors $F^{f(0)}_t$ from $X^f_t$
   • Extract $R^{unc}$ uncertainty factors $F^{unc(0)}_t$ from $X^{unc}_t$
   • Extract $R^s$ real factors $F^{s(0)}_t$ from $X^s_t$

2. • Run the regression

$$X^f_t = \alpha_f + \Lambda^{ff} F^{f(0)}_t + \Lambda^{uf} F^{unc(0)}_t + \Lambda^{sf} F^{s(0)}_t + v^f_t$$

and extract the associated coefficients. Compute

$$\tilde{X}^{f(0)}_t = X^f_t - \hat{\Lambda}^{uf} F^{unc(0)}_t - \hat{\Lambda}^{sf} F^{s(0)}_t$$

. This is the part of $X^f_t$ that cannot be explained by $F^{unc(0)}_t$ or $F^{s(0)}_t$.
   • Run the regression

$$X^{unc}_t = \alpha_u + \hat{\Lambda}^{fu} F^{f(0)}_t + \hat{\Lambda}^{uu} F^{unc(0)}_t + \hat{\Lambda}^{su} F^{s(0)}_t + v^u_t$$

and extract the associated coefficients. Compute

$$\tilde{X}^{unc(0)}_t = X^{unc}_t - \hat{\Lambda}^{su} F^{s(0)}_t$$

. This is the part of $X^s_t$ that cannot be explained by $F^{s(0)}_t$.
   • Run the regression

$$X^s_t = \alpha_s + \hat{\Lambda}^{fs} F^{f(0)}_t + \hat{\Lambda}^{us} F^{unc(0)}_t + \hat{\Lambda}^{ss} F^{s(0)}_t + v^s_t$$

and extract the associated coefficients.

3. • Extract $R^f$ financial factors $F^{f(1)}_t$ from $\tilde{X}^{f(0)}_t$
   • Extract $R^{unc}$ uncertainty factors $F^{unc(1)}_t$ from $\tilde{X}^{unc(0)}_t$

17
4. Repeat steps 2-3 a sufficient number of times

5. Construct $\hat{\Lambda}$ from the regression coefficients of the last repetition.

This procedure yields factors, which are uncorrelated within the same set, but correlated across sets.

Note that the elements contained in $\Lambda_{uf}^*, \Lambda_{sf}^*$ and $\Lambda_{su}^*$ will approach zero if this loop is repeated often enough, as required by the imposed identification scheme. To see why, consider $\hat{\Lambda}_{uf}^{i(i)}$ in repetition $i$: It measures the effect of $F_u^i(t)$ on $\tilde{X}_f^i(t)$. But $F_u^i(t)$ are extracted from a set of variables $\tilde{X}_t^{f(i-1)}$ from which the effect of $F_u^{i-1}(t)$ has been removed. It is therefore not surprising that the explanatory power of $F_u^i(t)$ for $\tilde{X}_f^i(t)$ is low and that the associated coefficient, $\hat{\Lambda}_{uf}^{i(i)}$, tends to zero with increasing $i$.

**A3: Bootstrap Confidence Intervals**

When constructing confidence intervals for IRFs, one has to choose whether to ignore or account for the estimation uncertainty in the latent factors.

Bai and Ng (2006) show that the former strategy is appropriate if $\sqrt{\frac{T}{N}} \to 0$ when $N$ and $T$ tend to infinity. That is, if the cross-sectional dimension, $N$, is not much smaller or even larger than the time-series dimension, $T$, one can treat the factors as data and proceed in the usual way in constructing confidence intervals. That is, one can either use asymptotic confidence intervals or use bootstrap methods, resampling only $u_t$, not $\xi_t$. This reduces the computational costs dramatically and is therefore often used in applications.

However, when $N$ is a lot smaller than $T$, one should account for the factor estimation uncertainty. The reason is that, while asymptotically equivalent, in finite sample, ignoring factor uncertainty leads to under-coverage in this case. No consensus has been reached how to do this, but 3 methods are often used: Stock and Watson (2016), Kilian and Lütkepohl (2016) or Yamamoto (2012) (method A).

Here, the bootstrap procedure to obtain IRF confidence bands assumes factor loadings to be fixed across bootstrap repetitions, i.e. it does not account for estimation uncertainty of the factors.

A second consideration when computing bootstrap IRFs is the design of the bootstrap. A standard choice in the VAR literature is to use residual bootstrap where one resamples the residuals by drawing with replacement from $u_t$ and $\xi_t$. This method is appropriate in the recursive identification scheme. When using identification via EI however, one has to resample not only the residuals and idiosyncratic errors, but also the EI using the same mechanism in order to preserve the correlation pattern between $m_t$ and $u_t$. As Morten and Ravn (2013) pointed out, when the EI contains many zeros, drawing with replacement yields a zero vector with positive probability. This should be avoided. Therefore, they suggest to use a fixed-design wild bootstrap for $u_t$, $\xi_t$ and $m_t$. 
This is implemented in the following steps: First, generate a random variable \( e_t \) which follows a Rademacher distribution:

\[
e_t = \begin{cases} 
+1 & \text{with } p = \frac{1}{2} \\
-1 & \text{with } p = \frac{1}{2}
\end{cases}.
\]

Generate a bootstrap sample \( u_t^* = e_t u_t, \xi_t^* = e_t \xi_t \) and \( m_t^* = e_t m_t \).

The entire bootstrap mechanism then follows the following steps:

1. Estimate equation (1) as described above. Obtain idiosyncratic shocks \( \xi_1, ..., \xi_T \) and estimate \( \hat{\Lambda} \) and \( \hat{F}_1, ..., \hat{F}_T \). Estimate the VAR in equation (2) and obtain reduced form residuals \( u_1, ..., u_t \) and parameter estimates \( \hat{A}_1, ..., \hat{A}_p \).

2. Resample the VAR residuals to obtain a bootstrap sample \( u_1^*, ..., u_T^* \). Subtract the mean.

3. Generate a bootstrap sample of the factors \( F_1^*, ..., F_T^* \) recursively using equation (2) and \( \hat{A}_1, ..., \hat{A}_p \) and \( u_1^*, ..., u_T^* \).

4. Generate a bootstrap sample of data \( X_1^*, ..., X_T^* \) using (1) and \( F_1^*, ..., F_T^*, \hat{\Lambda} \) and \( \xi_1^*, ..., \xi_T^* \).

5. Optional: Reestimate \( \hat{\Lambda}^* \) and \( \hat{F}_t^* \) via PC using the bootstrap sample of data.

6. Reestimate the model (2) using the bootstrap factors \( F_1^*, ..., F_T^* \) and construct IRFs imposing the relevant identification restrictions:

   - For the recursive identification: Reestimate \( \hat{\Sigma}_u \), Cholesky-decompose to obtain the lower-triangular \( \hat{B}^C \).
   - For the EI identification: Resample the instrument \( m_1^*, ..., m_T^* \) keeping the same sampling structure as with \( u_t \) to conserve the correlation structure. Estimate the relative impulse vector, \( \mu^* = \frac{E(m_t u_t^*)}{E(m_t u_t^{*\text{std}})} \).

7. Repeat steps 2-6 a sufficient number of times and compute point-wise confidence intervals.

Since all series have been standardized prior to estimation, these impulse responses are in standard-deviations units of the respective series. In order to obtain the original variable-scaling, we multiply by its standard-deviation. Note that having subtracted each variable’s mean prior to estimation does not affect the impulse responses and therefore does not have to be undone.
### A4: Correlations between factors in Scheme A and B

#### Scheme B

<table>
<thead>
<tr>
<th>Factor $^{E1}$</th>
<th>FinancialFactor</th>
<th>FinancialFactor</th>
<th>UncertaintyFactor</th>
<th>RealFactor</th>
<th>RealFactor</th>
<th>RealFactor</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E11$</td>
<td>-0.001749</td>
<td>0.3227</td>
<td>0.593</td>
<td>-0.9889</td>
<td>-0.01211</td>
<td>0.02981</td>
</tr>
<tr>
<td>$E12$</td>
<td>0.86</td>
<td>0.1522</td>
<td>0.344</td>
<td>0.05373</td>
<td>0.2955</td>
<td>0.6217</td>
</tr>
<tr>
<td>$E13$</td>
<td>0.1673</td>
<td>-0.1155</td>
<td>0.139</td>
<td>0.02948</td>
<td>-0.9359</td>
<td>0.2164</td>
</tr>
<tr>
<td>$E14$</td>
<td>0.331</td>
<td>-0.6598</td>
<td>0.2074</td>
<td>-0.9469</td>
<td>0.06045</td>
<td>-0.5575</td>
</tr>
<tr>
<td>$E15$</td>
<td>-0.1595</td>
<td>-0.6369</td>
<td>-0.07622</td>
<td>-0.07921</td>
<td>0.09438</td>
<td>0.4948</td>
</tr>
<tr>
<td>$E16$</td>
<td>-0.1019</td>
<td>0.00416</td>
<td>0.3745</td>
<td>0.06144</td>
<td>0.01621</td>
<td>0.008605</td>
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<tr>
<td>$E17$</td>
<td>0.1069</td>
<td>0.01258</td>
<td>-0.1228</td>
<td>-0.05911</td>
<td>-0.1228</td>
<td>-0.01721</td>
</tr>
<tr>
<td>Uncertainty</td>
<td>0.07926</td>
<td>0.07125</td>
<td>0.7519</td>
<td>-0.406</td>
<td>-0.08808</td>
<td>-0.001889</td>
</tr>
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</table>

Table 4: Correlations among factors

#### Scheme A

<table>
<thead>
<tr>
<th>Factor $^{E1}$</th>
<th>FinancialFactor</th>
<th>FinancialFactor</th>
<th>UncertaintyFactor</th>
<th>RealFactor</th>
<th>RealFactor</th>
<th>RealFactor</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E11$</td>
<td>0.9645</td>
<td>3.049 $\cdot 10^{-17}$</td>
<td>4.528 $\cdot 10^{-63}$</td>
<td>0</td>
<td>0.7578</td>
<td>0.4477</td>
</tr>
<tr>
<td>$E12$</td>
<td>8.812 $\cdot 10^{-192}$</td>
<td>9.641 $\cdot 10^{-5}$</td>
<td>1.614 $\cdot 10^{-19}$</td>
<td>0.1709</td>
<td>1.385 $\cdot 10^{-14}$</td>
<td>6.861 $\cdot 10^{-71}$</td>
</tr>
<tr>
<td>$E13$</td>
<td>1.782 $\cdot 10^{-5}$</td>
<td>0.003159</td>
<td>0.0003752</td>
<td>0.4528</td>
<td>2.99 $\cdot 10^{-296}$</td>
<td>2.446 $\cdot 10^{-8}$</td>
</tr>
<tr>
<td>$E14$</td>
<td>4.141 $\cdot 10^{-18}$</td>
<td>1.363 $\cdot 10^{-82}$</td>
<td>9.355 $\cdot 10^{-8}$</td>
<td>0.2331</td>
<td>0.1234</td>
<td>1.91 $\cdot 10^{-54}$</td>
</tr>
<tr>
<td>$E15$</td>
<td>4.367 $\cdot 10^{-5}$</td>
<td>2.385 $\cdot 10^{-75}$</td>
<td>0.05193</td>
<td>0.04334</td>
<td>0.01601</td>
<td>1.682 $\cdot 10^{-41}$</td>
</tr>
<tr>
<td>$E16$</td>
<td>0.009307</td>
<td>0.9156</td>
<td>4.165 $\cdot 10^{-23}$</td>
<td>0.1173</td>
<td>0.6798</td>
<td>0.8265</td>
</tr>
<tr>
<td>$E17$</td>
<td>0.006344</td>
<td>0.7488</td>
<td>0.001688</td>
<td>0.1319</td>
<td>0.001689</td>
<td>0.6613</td>
</tr>
<tr>
<td>Uncertainty</td>
<td>0.04321</td>
<td>0.06925</td>
<td>1.609 $\cdot 10^{-119}$</td>
<td>3.152 $\cdot 10^{-27}$</td>
<td>0.02461</td>
<td>0.9616</td>
</tr>
</tbody>
</table>

Table 5: P values
A5: Shadow Rate by Wu and Xia (2016)

Figure 7: **Shadow Rate**: Wu and Xia (2016) shadow rate available from 1960M1 to 2015M11