Abstract

I develop a theoretical model to investigate the effect of simultaneous regulation with a leverage ratio and a risk-weighted ratio on banks’ risk taking and banking market structure. Regulators face a trade-off between the efficient allocation of resources and financial stability. In an oligopolistic market, risk-weighted requirements incentivize banks with high productivity to lend to low-risk firms. When a leverage ratio is introduced, these banks lose market shares to less productive competitors and react with risk-shifting into high-risk loans. While average productivity in the low-risk market falls, market shares in the high-risk market are dispersed among new entrants with high as well as low productivity.

JEL Classification: G11, G21, G28.

Keywords: Banking Regulation, Heterogeneous Banks, Banking Competition, Capital Requirements, Leverage Ratio, Basel III.
1 Introduction

Since the introduction of Basel III, banks are constrained by competing minimum capital requirements. Banks are subject to the revised risk-based capital framework of Basel II and the non risk-based leverage ratio. The intention of this dual approach was to curb model risk inherent in applied risk-weights and to counteract their pro-cyclicality (BCBS, 2010). This paper sheds light on unintended consequences, especially on the allocation of market shares.

Although the new rules equally apply to all banks, competing capital requirements favor some banks at the expense of others. The simultaneity of both rules implies that the leverage ratio constraint binds only for some banks (BCBS, 2016). The question is, what kind of banks are affected. The rationale of capital requirements is to favor safe banks and charge risky banks. But being risky can be a feature of many traits. Still the question is, what kind of banks are risky.

To address this question, I develop a model with heterogeneous banks where differences in productivity determine banks’ optimal strategies under competing capital constraints and hence riskiness. This paper leans on the idea, forwarded in trade theory by Melitz (2003), that productivity differences play an important role in shaping firms’ optimal strategies. I extend a portfolio choice model by adding heterogeneity in productivity among banks in the form of differences in marginal costs. Banks choose their strategy in a high-risk and a low-risk credit market with Cournot competition. I find that risk-weighted capital requirements incentivize banks with high productivity to specialize on low-risk loans. When the leverage ratio is introduced, these banks lose market shares in the low-risk market to less productive competitors and react with risk-shifting into high-risk loans as in Koehn and Santomero (1980), and Kim and Santomero (1988).

Theoretical work on capital requirements so far ignored the role of productivity in banks’ decision about risk because studies focused on models with representative banks (VanHoose, 2007). Nevertheless, the relationship between productivity and risk taking received much attention in empirical work although the evidence is yet inconclusive. On the one hand, the efficiency-risk hypothesis\(^1\) claims that more productive banks expect higher future profits and thus need a smaller capital buffer. Hence, they can afford a riskier strategy (Berger and di Patti, 2006; Altunbas, Carbo, Gardener and Molyneux, 2007). On the other hand, the charter-value hypothesis claims that more productive banks protect these higher profits by choosing less risky strategies (Fiordelisi, Marques-Ibanez and Molyneux, 2011). Therefore, it is unclear from the perspective of financial stability whether market shares should be allocated to the most productive banks. Due to frictions, e.g. asymmetric information and entry barriers, the banking industry is already prone to allocative inefficiency and X-inefficiency causing welfare losses (Vives, 2001a; Berger, Hunter and Timme, 1993). If more productive banks were also safer banks, regulation should reallocate market shares to their favor. If not, a social planner might face a trade-off between an efficient allocation of resources and financial stability when setting new regulatory guidelines (Allen and Gale, 2004).

\(^1\)Note that empirical studies prefer the term efficiency over productivity, since most of them estimate the distance of a bank to the efficient production frontier. Nevertheless, it would be confusing to talk about efficiency in a theoretical context, since in a model every production decision is the result of an individual optimization.
In this model, productivity creates positive charter value and market power. In the unregulated equilibrium, market shares are allocated according to productivity. The bank with the highest productivity is the market leader in the market for high-risk loans and the market for low-risk loans. Since productivity differences are exogenous to the model, it can be categorized in the light of Efficient Structure theory pioneered by Demsetz (1973). The presence of risk-weighted capital requirements, however, introduces a complementarity between both types of loans (Repullo and Suarez, 2004). As a consequence, banks with high productivity specialize on low-risk loans, and hence average productivity in the high-risk market is rather low. Banks with lower productivity do not have to provide more equity for taking the same risks, yet their default probabilities are higher due to lower charter values. The Basel II equilibrium is therefore characterized by concentration of high-risks in low-productivity banks. The introduction of the leverage ratio affects both markets differently and tends to ameliorate this unwanted concentration. In the low-risk market, the most productive banks lose market shares to competitors with lower productivity so that average productivity falls. In the high-risk market, however, banks with low productivity enter but also the most productive banks gain market shares so that the market is less concentrated.

I rely on the model of Kiema and Jokivuolle (2014) and extend it by introducing heterogeneity and an oligopolistic market. Kiema and Jokivuolle (2014) model banks’ optimal portfolio choice with Basel III capital requirements. As in Repullo and Suarez (2004) and this paper, banks specialize under Basel II. After the leverage ratio is introduced, low-risk banks choose a mixed portfolio so that, overall, bank portfolios are more alike. They study the role of the leverage ratio as a backstop to model risk and find that this role is impeded by less diverse portfolio choices. A recent paper by Smith, Grill and Lang (2017) also examines banks’ risk choices under the competing rules and evaluates whether the leverage ratio effectively reduces the probability of insolvency. They contrast the risk-taking incentives of the leverage ratio with the increase of loss absorbing capital and show that the positive effect of higher capital outweighs the negative effect of increased risk-taking. They test their implications empirically and find that banks become more stable after the announcement of the leverage ratio. I find a similar result which indicates that the leverage ratio can contribute to financial stability. I find that, in switching from the Basel II to the Basel III equilibrium, default probabilities of most banks decline, at least as long as realizations of a common systematic risk-factor not exceed a threshold. Beyond this threshold, default rates in the high-risk market are so high that even the most productive banks are closer to default.

Thus my work contributes to the literature on capital requirements and risk, in particular to the recent literature on the interaction of competing capital requirements. Wu and Zhao (2016) and Blum (2008) show that the leverage ratio complements the risk-weighted ratio given that banks are opaque and able to misreport their actual risk level to the regulators. Brei and Gambacorta (2016) and Gambacorta and Karmakar (2016) study the joint effect of both requirements and demonstrate the countercyclical quality of the leverage ratio. Furthermore, I contribute
to the literature which is using heterogeneous banks. Apart from macroeconomic models with heterogeneous agents, e.g. Choi, Eisenbach and Yorulmazer (2015), only few microeconomic banking models consider heterogeneity. Barth and Seckinger (2013) show how heterogeneous monitoring costs introduce a selection problem in the banking market. Other studies consider two distinct types of banks. Hakenes and Schnabel (2011) find that smaller banks take more risks if big banks have a competitive advantage by choosing the internal ratings-based over the standardized approach in the Basel II framework.

The remainder of this paper is organized as follows. Section 2 introduces the main assumptions and setting of the model. Section 3 gives the baseline equilibrium without regulation. In section 4 banking regulation is introduced and the equilibria with risk-weighted and competing capital requirements are derived. Section 5 discusses the results and possible limitations. Section 6 concludes.

2 The model

Consider a Cournot-Nash game with \( N \) banks competing in two markets. There is a market for low-risk loans and a market for high-risk loans. Banks have different unit costs and no fixed costs. Unit costs of bank \( i \) are denoted as \( c_i \). In what follows, we rank banks according to their costs such that the bank with the lowest unit costs is denominated as bank 1 whereas bank \( N \) has the highest unit costs.

\[
c_1 < c_2 < \cdots < c_N
\]  

Each market represents one of two types of entrepreneurs, risky and less risky entrepreneurs. Once in the game, there is perfect information about types but these costs can be interpreted as screening costs that banks have to incur in order to discern high- and low-risk entrepreneurs. Further, these costs reflect monitoring and administrative costs, such as employment of loan officers, back-office administration of the loan portfolio, or maintenance of monitoring processes. Therefore, low costs represent a more efficient production technology. Banks that are able to operate their loan portfolio at lower costs are more productive. The model introduces productivity differences of banks in the simplest form of differing cost functions. This leads to asymmetric Nash-equilibria where optimal strategies depend on marginal costs.

Let the strategy of bank \( i \) be \( q_i = (q_{h,i}, q_{l,i}) \). Let \( Q_{-i} = (Q_{h,-i}, Q_{l,-i}) \) denote aggregate quantities of all banks except bank \( i \) and \( Q = (Q_h, Q_l) \) the total aggregate supply of loans in the respective markets. Aggregate supply determines inverse demand \( r_{\eta}(Q_{\eta}) \) from entrepreneurs.
of type \( \eta = \{h, l\} \). Inverse demand functions are continuous, monotone, and concave.

\[
\begin{align*}
  r_h(Q_h) &= r_h \left( \sum_{i=1}^{N} q_{h,i} \right) , & r_l(Q_l) &= r_l \left( \sum_{i=1}^{N} q_{l,i} \right) \\
  r'_h(Q_h) &< 0 , & r'_l(Q_l) &< 0 \\
  r''_h(Q_h) &\leq 0 , & r''_l(Q_l) &\leq 0
\end{align*}
\]

(2)

Entrepreneurs demand a loan of size 1 if the interest rate is lower than their expected payoff. I assume expected payoffs are distributed such that it entails inverse demand functions of the described kind. Entrepreneurs, however, have limited liability. They repay the interest rate only if their projects are successful. If their project defaults, entrepreneurs pay nothing to the bank, i.e. loss given default is 1. Banks use average probabilities of success for each type of loan to take this into account.

To determine success probabilities of entrepreneurs, I use the representation by Repullo and Suarez (2004) and Kiema and Jokivuolle (2014) of the Vasicek model (Vasicek (1987),Vasicek (2002)). This risk model underpins the framework of risk-sensitive capital requirements of the Basel II accord. There is a common risk factor captured in \( z \) as well as idiosyncratic risk \( \epsilon_j \) that are both standard normally distributed. Successes of high- and low-risk projects are correlated and \( \rho \) is the correlation parameter. The project of entrepreneur \( j \) is successful if a latent random variable \( x_j \leq 0 \), where

\[
\begin{align*}
  x_j &= \zeta_\eta + \sqrt{\rho} z + \sqrt{1 - \rho} \epsilon_j \\
  z &\sim N(0, 1), & \epsilon_j &\sim N(0, 1).
\end{align*}
\]

(3)

The two types differ in \( \zeta_\eta \) which represents the financial vulnerability of entrepreneurs of type \( \eta \) and \( 0 < \zeta_l < \zeta_h \). If banks know the types of entrepreneurs, they know \( \zeta_l \) and \( \zeta_h \). Consequently, the unconditionally expected probability to default of loans of type \( \eta \) is \( PD_\eta = \Phi(\zeta_\eta) = Pr(\zeta_\eta + \sqrt{\rho} z + \sqrt{1 - \rho} \epsilon_j > 0) \), where \( \Phi \) is the cumulative distribution function of the standard normal distribution. Let the expected probability of success be \( p_\eta = 1 - PD_\eta \), respectively. Note that \( p_h < p_l \) since low-risk entrepreneurs are less likely to default. Assume that investing in the riskier project has a higher expected yield so that

\[
1 < p_l r_l(Q_l) < p_h r_h(Q_h)
\]

(4)

I assume depositors are insured and consequently ignorant of bank risk. They supply an inexhaustible amount of savings at an interest rate \( r_d \). The deposit rate could be the value of an outside option of depositors, e.g. holding cash or a safe asset instead of investing their endowment in a bank. Depositors will then invest in banks whenever these offer a deposit rate at least as high as their outside option. For simpler notation, I define marginal costs as

\[
MC_i = c_i + r_d.
\]

(5)

Each banker is equally endowed with an amount of equity \( e \). Let \( r_c \) denote the opportunity costs
of equity capital and let it be higher than the opportunity costs of depositors, s.t. \( r_d < r_e \). Banks are only operated if expected profits from intermediation are higher than the outside option of bankers. Therefore, I assume that bankers have to invest their equity in the bank in order to employ the banking technology. Banks’ balance sheet constraint is given by

\[
e + d_i = q_{h,i} + q_{l,i}.
\]

Let expected payoff of bank \( i \) be expected profits of intermediation minus opportunity costs given as

\[
\Pi_i(q_i, d_i, e) = p_h r_h(Q_h)q_{h,i} + p_l r_l(Q_l)q_{l,i} - c_i(q_{h,i} + q_{l,i}) - r_d d_i - r_e e.
\]

In addition, each bank has a capacity limit \( W_i \) which is finite but arbitrarily high so it cannot produce more than \( W_i \) in any market. This assumption ensures that banks’ strategy sets are bounded in the unregulated case and is not crucial once regulation is introduced. Furthermore, banks are not allowed to take short positions in neither loans nor deposits, so that \( q_i \geq 0 \) and \( d_i \geq 0 \).

### 3 Unregulated equilibrium

Consider the optimization problem of a bank without capital requirements. By inserting eq. (5), and eq. (6) in eq. (7) and rearranging, the problem of bank \( i \) is

\[
\text{Max } \Pi_i(q_i) \quad \text{s.t. } \Pi_i(q_i) \geq (r_e - r_d)e \quad \text{and } 0 \leq q_i \leq W_i \quad \text{where}
\]

\[
\Pi_i(q_i) = (p_h r_h(Q_h) - MC_i)q_{h,i} + (p_l r_l(Q_l) - MC_i)q_{l,i}
\]

Because of the flat deposit rate due to the deposit insurance and the fact that debt financing is cheaper than equity financing, banks have strong incentives to increase their balance sheet size through levering if these are not balanced by regulation or market forces.

In a Cournot game though, competition ensures that bank size stays limited. If any bank expands its loan business the interest rates decrease for all banks so that competitors reduce their loan business. All in all, the lower interest rates fall, the less attractive is an expansion strategy. Furthermore, the lower interest rates fall, the fewer banks are able to participate in the loan market because some banks’ marginal costs would be too high to make a profit. Consequently, the least productive banks do not provide loans in equilibrium and some less productive banks only provide loans in the high-risk market where expected revenues are higher. The unregulated equilibrium is summarized in following proposition.

**Proposition 1** (Unregulated equilibrium).

In an unregulated equilibrium, optimal aggregate supply in the high-risk market is \( Q_h^* \) provided by a subset \( \{1, \ldots, \nu_h\} \) with \( \nu_h \in \{1, \ldots, N\} \) of banks at interest rate \( r_h(Q_h^*) \), and aggregate supply in the low-risk market is \( Q_l^* \) provided by a subset \( \{1, \ldots, \nu_l\} \) with \( \nu_l < \nu_h \) and \( \nu_l \in \{1, \ldots, N\} \) of banks at interest rate \( r_l(Q_l^*) \).

---

\(^6\)This assumes that equity is costly contrary to the discussion in Admati, DeMarzo, Hellwig and Pfleiderer (2013).
Aggregate supplies are a result of best-response correspondence of optimal strategies where strategy $q^*_i$ of bank $i$ is $(q^*_{h,i}, q^*_{l,i})$ with

$$q^*_i = \max \left[0, \frac{p_{\eta} r_{\eta}(Q^*_{\eta}) - MC_i}{\nu_{\eta} p_{\eta} r_{\eta}(Q^*_{\eta}) - \sum_{i=1}^{\nu_{\eta}} MC_i Q^*_{\eta}} \right].$$

(9)

More productive banks gain higher market shares and are bigger than less productive banks.

**Proof.** Proof is in the appendix.

The fraction in eq. (9) represents the market share of bank $i$ in market $\eta$. It equals the ratio of the rent that bank $i$ can earn on a loan of type $\eta$ relative to total rents earned in the market. All banks weight their revenue with the same unconditional success probabilities $p_{\eta}$ and earn in equilibrium the same market interest rates. Therefore, bank 1 with the lowest marginal costs $MC_1$ will have the highest market share in the market for low-risk loans and the market for high-risk loans, whereas bank $\nu_h$ has the lowest market share in the market for high-risk loans and its marginal costs $MC_{\nu_h}$ are only slightly smaller than or equal to the market interest rate $r_h(Q_h)$.

Therefore, Cournot competition with heterogeneous cost functions gives reasonable implications of how productivity advantages translate into scale and market power advantages. Since productivity differences are exogenous to the model, it can be categorized in the light of Efficient Structure theory pioneered by Demsetz (1973).

4 Regulating heterogeneous banks

4.1 Necessity to regulate and determinants of bank default

Given that bank defaults result in high social cost for the economy, the objective of a social planner is to avoid any bank default. A bank $i$ defaults if the realization of systematic risk $z$ is higher than the critical value $z_{i,\text{crit}}$ defined as

$$\pi_i(c_i, q_i, r(Q), z_{i,\text{crit}}) - r_d d_i(q_i, \epsilon) = 0$$

(10)

where

$$\pi_i(c_i, q_i, r(Q), z) = (1 - PD_l(z)) r_l(Q_l) q_{l,i} +$$

$$((1 - PD_h(z)) r_h(Q_h) q_{h,i} - c_i(q_{h,i} + q_{l,i})$$

and $PD_{\eta}(z)$ is the default probability of projects of type $\eta$ conditional on the realization of systematic risk $z$. In a portfolio with many loans of type $\eta$ with roughly equal size, the fraction of defaulting loans in such a portfolio converges to $PD_{\eta}(z)$ (Elizalde et al., 2005). Rearranging $Pr(\zeta_{\eta} + \sqrt{\rho} z + \sqrt{1 - \rho} \epsilon_j > 0)$ gives

$$PD_{\eta}(z) = Pr \left( \epsilon_j > -\frac{\zeta_{\eta} + \sqrt{\rho} z}{\sqrt{1 - \rho}} \right) = \Phi \left( \frac{\zeta_{\eta} + \sqrt{\rho} z}{\sqrt{1 - \rho}} \right).$$

(11)

Under any distribution of risk, here it is the standard normal distribution, extreme realizations of systematic or idiosyncratic risk are possible, so that default cannot be prevented with absolute
certainty no matter how much loss absorbing capital is available to a bank. The micro-prudential approach of the Basel Committee is to set a maximal admissible default probability. It is a well known shortcoming of portfolio models that they do not provide an innate explanation for regulation. Nevertheless, portfolio models mirror best the approach chosen by the current regulator. I therefore have to assume that regulation is necessary to tame banks leveraging.

**Assumption 1** (Necessity of regulation). In the unregulated equilibrium, all banks show unacceptable high default probabilities. The regulator implements capital requirements to lower default probabilities.

Based on eq. (10), I distinguish three channels that determine banks’ default probabilities. First, critical value \( (z_{i, \text{crit}}) \) depends directly on banks’ heterogeneous costs \( (c_i) \). Lower costs create higher charter values and hence resilience. Second, critical value \( (z_{i, \text{crit}}) \) depends on the chosen strategies \( (q_i) \) which in turn depend on marginal costs. This channel is more comprehensible if split into two: A portfolio allocation channel and a leverage channel. The former concerns the share of high-risk loans to total loans in the portfolio, i.e. \( \gamma_i = \frac{q_{h,i}}{q_{h,i} + q_{l,i}} \). A higher portfolio share \( \gamma_i \) is riskier. This channel is addressed by the risk-weighted ratio. The leverage channel concerns the size of the portfolio, i.e. \( q_{h,i} + q_{l,i} \), under the assumption that equity is fixed for all banks. Risks that arise through this channel are limited by the leverage ratio. Third, critical value \( (z_{i, \text{crit}}) \) depends on market interest rates \( (r(Q)) \) which are indirectly also functions of marginal costs.\(^7\) Higher interest rates increase charter values and therefore loss absorbing capacity of banks.\(^8\)

Since marginal costs are difficult to measure once we leave the simple model world, it would hardly be feasible to write regulatory rules contingent on productivity. Furthermore, it is unclear ex-ante whether the regulator should tax or relieve more productive banks taking into account the indirect effects of productivity on portfolio strategy, market power, and size. The regulator so far conditions capital requirements on the portfolio strategy and size but not on productivity. This is reasonable since portfolio allocations and leverage are easily observable.

### 4.2 Basel II equilibrium

The Basel II accord introduced risk-sensitive capital requirements to avoid the risk-shifting phenomenon described by Koehn and Santomero (1980), Kim and Santomero (1988) and others. They show that if capital requirements are not risk-sensitive, banks have incentives to shift their portfolio towards riskier assets. Following the Basel II approach for credit risk, banks must categorize their assets with respect to their riskiness into different buckets for which different risk-weights are applied. In the Standard Approach these weights are set by the regulator. In the Internal Ratings based Approach banks are allowed to use internal risk-models to provide expected default probabilities or more inputs, e.g. loss given default, for the calibration of the weights.

This model describes the IRB approach where default probabilities of loans of a certain type are used to calculate capital requirements. The model is static so that the maturity of

---

\(^7\)One could write \( \pi_i(c_i, q_i(c_i), r(Q \sum q_i(c_i))), z_i) \).

\(^8\)Formal derivation of these channels are in the proof of lemma 4 in the appendix.
all loans is one. The risk-weighted requirement is constructed such that the probability that unexpected losses of the asset portfolio exceed available equity is lower than a threshold \( \alpha \), i.e. the admissible probability of default set by the regulator. However, the regulator implicitly ignores heterogeneity here. As is shown later, banks can have default probabilities above \( \alpha \) if heterogeneity is taken into account.

Let us assume the regulator sets \( \alpha \) for some representative bank. As a result, equity is insufficient to cover unexpected losses with probability \( \alpha \) for that bank.

The regulator infers the critical value of systematic risk \( z_\alpha = \Phi^{-1}(1-\alpha) \) from eq. (11) such that \( \Pr(z \leq z_\alpha) = 1-\alpha \). Consequently, if the representative bank holds at least \( PD_\eta(z_\alpha) \) equity for each loan of type \( \eta \), it is able to cover losses with probability \( 1-\alpha \). In detail, the capital requirement has two components: loan loss provisions for expected losses \( PD_\eta \) and equity capital for unexpected losses \( PD(z) - PD_\eta \). In this model the risk-adequate capital requirement for a loan of type \( \eta \) simplifies to

\[
\beta_\eta = PD_\eta(z_\alpha) = \Phi \left( \frac{\zeta_\eta + \sqrt{\rho} \Phi^{-1}(1-\alpha)}{\sqrt{1-\rho}} \right).
\] (12)

The requirement is additive for both types of loans given that banks hold a well-diversified portfolio within each class of loans (Vasicek, 2002). Since high-risk firms have a higher financial vulnerability \( (\zeta_h > \zeta_l) \), the capital requirement for high-risk loans is higher than for low-risk loans. The risk-weighted capital constraint of Basel II is given by

\[
e \geq \beta_h q_{h,i} + \beta_l q_{l,i} \quad \text{where} \quad 0 < \beta_l < \beta_h < 1.
\] (13)

Adding the risk-weighted capital constraint to bank \( i \)'s optimization problem and introducing \( \mu_i \) as the shadow price of being constrained by the requirement gives

\[
\begin{align*}
\text{Max}_{q_{i},\mu_i} \quad & \Pi_i(q_i, \mu_i) = (p_h r_h(Q_h) - MC_i)q_{h,i} + (p_l r_l(Q_l) - MC_l)q_{l,i} \\
& - \mu_i (\beta_h q_{h,i} + \beta_l q_{l,i} - e) \\
\text{s.t.} \quad & \Pi_i(q_i, \mu_i) \geq (r_e - r_d)e, \quad 0 \leq q_i \leq W_i, \quad 0 \leq \mu_i
\end{align*}
\] (14)

Whereas in the unregulated equilibrium competitive pressures are the main force limiting bank size and determining the bank portfolio composition, under assumption 1 capital requirements pose much stricter limits on size and composition. They introduce complementarity between both types of loans. Because the requirement in eq. (13) is additive, banks enjoy no immediate advantage by diversifying their portfolio between asset classes. Therefore, a specialized portfolio is always better than a mixed portfolio strategy if it is feasible (Repullo and Suarez, 2004; Kiema and Jokivuolle, 2014). Moreover, whenever

\[
p_l r_l(Q_l) - MC_i > \frac{\beta_l}{\beta_h} (p_h r_h(Q_h) - MC_i)
\] (15)

bank \( i \) has incentives to fully specialize on low-risk loans. Let \( \Pi_i^s(q_{i,i}, q_{l,i}) \) denote the expected payoff of bank \( i \) implementing strategy \( s \) where \( s = h \) when bank \( i \) specializes on high-risk loans.

\( ^9 \) Confer Kiema and Jokivuolle (2014) for a detailed account of how default probabilities are effectively restricted by Basel II capital requirements in a representative bank model.
and \( s = t \) when the bank specializes on low-risk loans. Solving eq. (15) for \( MC_i \) gives the cutoff marginal costs of the bank with the lowest productivity which specializes on low-risk loans. It is therefore the cutoff of the low-risk market, denoted as \( \tilde{MC}^l \), and defined s.t.

\[
\Pi^l_i(0, \frac{e}{\beta_l}) \geq \Pi^h_i(\frac{e}{\beta_h}, 0) \quad \forall \: i \in \{1, ..., N\} : MC_i \leq \tilde{MC}^l
\]

where

\[
\tilde{MC}^l = \frac{\beta_h p_l r_l(Q_l) - \beta_l p_h r_h(Q_h)}{\beta_h - \beta_l}.
\]

(16)

An equilibrium can only exist if this cutoff is positive and there are banks that specialize on low-risk loans as well as banks that specialize on high-risk loans. It follows that in equilibrium capital requirements pose an upper bound on the interest rate on high-risk loans relative to the interest rate of low-risk loans, i.e.

\[
p_l r_l(Q_l^*) < p_h r_h(Q_h^*) < \frac{\beta_h}{\beta_l} p_l r_l(Q_l^*).
\]

(17)

Nevertheless, not all banks are active in equilibrium. Of all banks with marginal costs above the cutoff \( \tilde{MC}^l \) only banks with marginal costs below expected revenue \( p_h r_h(Q_h) \) are profitable. Let the cutoff marginal costs for the high-risk market be denoted as

\[
\tilde{MC}^h = p_h r_h(Q_h^*).
\]

(18)

Some of the banks in the high-risk market are constrained by the risk-weighted ratio, i.e. \( e = \beta_h q_{h,i}^* \), while others are not constrained, i.e. \( e > \beta_h q_{h,i}^* \). The constrained strategy \( h \) is only feasible for banks with non-negative shadow prices, i.e. \( \mu_i \geq 0 \) according to eq. (14). There is a negative relation between \( MC_i \) and \( \mu_i \). More productive banks are able to produce the highest quantities in an unregulated equilibrium, hence they face higher shadow prices of being constrained by capital requirements. Let the cutoff marginal costs between constrained and unconstrained banks in the high-risk market be denoted as

\[
\tilde{MC}^{\mu_h} = p_h r_h + \frac{e}{\beta_h} p_h r_h'.
\]

(19)

The equilibrium is illustrated in the upper half of fig. 2 and is summarized in Proposition 2.

**Proposition 2 (Basel II equilibrium).**

With additive risk-weighted capital requirements, if eq. (17) holds and

\[
-(p_h r_h(Q_h^*) - p_l r_l(Q_l^*)) < \frac{e}{\beta_h} p_l r_l'(Q_l^*) < 0,
\]

\[
-\frac{\beta_l}{\beta_h - \beta_l} (p_h r_h(Q_h^*) - p_l r_l(Q_l^*)) < \frac{e}{\beta_l} p_l r_l'(Q_l^*) < 0,
\]

(20)

more productive banks specialize on low-risk loans while less productive banks specialize on high-
risk loans, i.e. optimal strategies in equilibrium are

\[
(q_{h,i}^*, q_{l,i}^*, \mu_i^*) = \begin{cases} 
(0, q^l_i, \mu^l_i(MC_i)) & \text{if } MC_i \leq M^lC^l, \\
(q^h_i, 0, \mu^h_i(MC_i)) & \text{if } M^lC^l < MC_i \leq M^hC^l, \\
(q^u_i(MC_i), 0, 0) & \text{if } M^hC^l < MC_i \leq M^hC^h, \\
(0, 0, 0) & \text{if } \tilde{M}^lC^l < MC_i 
\end{cases}
\]

(21)

where a subset \( \{1, \ldots, \nu_l\} \) of banks with \( MC_i \leq \tilde{M}^lC^l \) for all \( i \in \{1, \ldots, \nu_l\} \) offer aggregate supply of low-risk loans \( Q^*_l \) at interest rate \( r^*_l(Q^*_l) \), and a subset \( \{\nu_l + 1, \ldots, \nu_h\} \) of banks with \( \tilde{M}^lC^l < MC_i \leq \tilde{M}^lC^h \) for all \( i \in \{\nu_l + 1, \ldots, \nu_h\} \) offer aggregate supply of high-risk loans \( Q^*_h \) at interest rate \( r^*_h(Q^*_h) \).

**Proof.** It follows from eq. (16) and the arguments above. Derivation of the conditions is in the appendix.

Given these equilibrium strategies, it is possible to determine default probabilities. The direct effect of productivity advantages on the critical value of systematic risk \( z_{i,crit} \) which is defined in eq. (10) is positive, i.e. banks with lower marginal costs ceteris paribus have higher profits. Positive profits constitute positive charter value and add to loss absorbing capacity. Therefore, when comparing banks that specialize on the same type of loans, the relationship between productivity and default probability is straightforward. These banks have the same strategy and earn the same interest rate. Hence, banks with lower marginal costs have lower default probabilities than banks with higher marginal costs that are active in the same loan market. When comparing specialists on the high-risk and low-risk market, the relationship between productivity and default probabilities is not straightforward. The portfolio allocation channel and leverage channel take opposite directions. On the one hand, high-risk specialists have a riskier investment strategy and higher costs. On the other hand, they are less levered. Additionally, the interest rate channel works in favor of banks specializing on high-risk loans. If we impose a stricter limit on the upper bound of the high-risk market interest rate than eq. (17) and therewith limit the influence of the interest rate channel, a relationship can be clearly stated. In that case, the direct cost channel and the portfolio channel outweigh the leverage channel, so that banks with higher productivity are definitely less likely to default. Lemma 1 summarizes.

**Lemma 1.** In equilibrium, more productive banks have lower default probabilities than less productive banks in the same market, i.e.

\[
\forall i \in \{1, \ldots, \nu_l\} : q^*_l > 0 \\
\forall i \in \{\nu_l + 1, \ldots, \nu_h\} : q^*_h > 0
\]

(22)

If \( p_h r_h(Q^*_h) < \frac{\beta_h}{\beta_l} p_l r_l(Q^*_l) \), more productive banks have lower default probabilities even across markets, i.e.

\[
\forall i \in \{1, \ldots, N\} : q^*_i > 0
\]

(23)

**Proof.** Proof is in the appendix.
4.3 Basel III equilibrium

Among other measures aimed at capital adequacy, the Basel III accord introduced the leverage ratio. The motives of the regulator were driven by macro- as well as micro-prudential considerations. In order to comply, banks need to back up 3% of their total exposure with Tier 1 equity capital. Total exposure includes on-balance as well as off-balance sheet assets. The leverage ratio capital constraint of Basel III is given by

\[
\beta \text{ according to } \geq \beta (q_{h,i} + q_{l,i}) - e
\]

where \(0 < \beta_l < \beta < \beta_h < 1\). (24)

Adding the leverage ratio to the risk-weighted capital constraint in bank \(i\)'s optimization problem and introducing \(\lambda_i\) as the shadow price of being constrained by the leverage ratio gives

\[
\begin{align*}
\text{Max}_{q_i, \mu_i, \lambda_i} & \quad \Pi_i(q_i, \mu_i, \lambda_i) = (p_h r_h(Q_h) - MC_{h,i}) q_{h,i} + (p_l r_l(Q_l) - MC_{l,i}) q_{l,i} \\
& \quad - \mu_i (\beta_h q_{h,i} + \beta_l q_{l,i} - e) - \lambda_i (\beta (q_{h,i} + q_{l,i}) - e)
\end{align*}
\]

s.t. \(\Pi_i(q_i, \mu_i, \lambda_i) \geq (r_e - r_d) e, \quad 0 \leq q_i \leq W_i, \quad 0 \leq \mu_i, \quad 0 \leq \lambda_i\) (25)

The additional constraint reduces the set of feasible strategies. The shaded area including the bounding line segments in fig. 1 illustrates the set of feasible strategies of bank \(i\). Since the leverage ratio poses extra costs on banks specializing on low-risk loans, it sets incentives to shift the portfolio toward riskier assets. Therefore, a mixed strategy is better for banks that previously specialized on low-risk loans. These banks change their strategy to strategy \(v\) which is the mixed portfolio exactly on the vertex in fig. 1 where both constraints are binding. For the remainder of banks it is still optimal to specialize on high-risk loans as long as it is feasible. Let \(\tilde{MC}_{l}^{i}\) denote the cutoff marginal costs between banks choosing strategy \(v\) and banks choosing strategy \(h\). Since only banks that choose strategy \(v\) offer loans to low-risk entrepreneurs, \(\tilde{MC}_{l}^{i}\) defines the marginal costs of the bank with the lowest productivity that still participates in the

Figure 1: Feasible quantities under both capital requirements.
Let \( \overline{MC}^l \) be defined by

\[
\Pi^h_i(q^h, q^l, \mu^h, \lambda^h(MC_i)) \geq \Pi^l_i(\frac{e}{\beta}, 0) \quad \forall i \in \{1, \ldots, N\} : MC_i \leq \overline{MC}^l
\]

where 

\[
\overline{MC}^l = \frac{\beta_h q_h(r(Q_l) - \beta_l r_h(Q_h))}{\beta_h - \beta_l}.
\]  

Furthermore, only banks with non-negative shadow prices \( \mu_i \) are able to choose strategy \( h \). The cutoff marginal costs for constrained banks in the high-risk market is therefore given as \( \overline{MC}^{\mu h} \) defined in eq. (19). Banks with marginal costs above \( \overline{MC}^{\mu h} \) but below expected revenue \( p_h r^*(Q^*_h) \) still specialize on high-risk loans. They can offer only small quantities, s.t. \( e < \beta_h q^*_h \). The cutoff marginal costs for these unconstrained banks in the high-risk market is defined as

\[
\overline{MC}^h = p_h r^*(Q^*_h).
\]

The Basel III equilibrium is illustrated in the lower half of fig. 2 and is summarized in the following proposition.

**Proposition 3.** With additive risk-weighted capital requirements and a leverage ratio, if eq. (17) holds and

\[
\frac{\beta(\beta_h - \beta_l)}{\beta_h (\beta_h - \beta_l)} (\beta_h p_h r_h(Q^*_h) - \beta_l p_l r_l(Q^*_l)) + \frac{\beta(\beta - \beta_h)}{\beta_h (\beta_h - \beta_l)} p_h r'_h(Q^*_h) e < p_l r'_l(Q^*_l) e
\]

\[
-\beta_h (p_h r_h(Q^*_h) - p_l r_l(Q^*_l)) < p_h r'_h(Q^*_h) e < -\beta_h (p_h r_h(Q^*_h) - p_l r_l(Q^*_l))
\]

more productive banks hold a mixed portfolio while less productive banks specialize on high-risk loans, i.e. optimal strategies in equilibrium are

\[
(q^h, q^l, \mu^h, \lambda^h(MC_i)) = \begin{cases} 
(q^h, q^l, \mu^h, \lambda^h(MC_i)) & \text{if } MC_i \leq \overline{MC}^l \\
(q^h, 0, \mu^h(MC_i), 0) & \text{if } \overline{MC}^l < MC_i \leq \overline{MC}^{\mu h} \\
(q^h, \mu^h(MC_i), 0, 0, 0) & \text{if } \overline{MC}^{\mu h} < MC_i \leq \overline{MC}^h \\
(0, 0, 0, 0) & \text{if } \overline{MC}^h < MC_i
\end{cases}
\]

where a subset \( \{1, \ldots, \nu_l\} \) of banks with \( MC_i \leq \overline{MC}^l \) for all \( i \in \{1, \ldots, \nu_l\} \) offer an aggregate supply \( Q^*_h \) of low-risk loans at interest rate \( r^*_l(Q^*_l) \), and a subset \( \{1, \ldots, \nu_h\} \) of banks with \( MC_i \leq \overline{MC}^{\mu h} \) for all \( i \in \{1, \ldots, \nu_h\} \) offer aggregate supply \( Q^*_h \) of high-risk loans at interest rate \( r^*_h(Q^*_h) \).

**Proof.** Proof is in the appendix. \( \square \)

Note that the cutoffs defined above are only formally the same as in eq. (16), (19), and (18). Because the interest rates in both equilibria are not necessarily the same, the values of these cutoffs differ between the Basel II and Basel III equilibrium. In fact, the number of banks in the low-risk market can only increase and therefore the number of active banks in the high-risk market increases as well.
Corollary 1. Comparing the portfolio choices in the Basel II and Basel III equilibrium, the cutoffs for marginal costs increase, i.e.

$$\tilde{MC}^{\text{Basel II}}_l < \tilde{MC}^{\text{Basel III}}_l$$  \hspace{1cm} (31)

and

$$\tilde{MC}^{h} = p_h r_h(Q_h^{*\text{Basel II}}) < p_h r_h(Q_h^{*\text{Basel III}}).$$ \hspace{1cm} (32)

Proof. Proof is in the appendix.

Lemma 2. By tightening capital requirements through the introduction of the leverage ratio, aggregate loan supply decreases and interest rates increase in both markets.

Proof. Follows directly from corollary 1.

The results of corollary 1 are illustrated in fig. 2. Taking the order of $N$ banks according to their marginal costs, I distinguish six groups of banks according to whether they are affected or unaffected by the leverage ratio (i.e. whether they change their strategies between the Basel II and Basel III equilibrium) and whether they are constrained or unconstrained: (i - solid line segment) low-risk market incumbents, (ii - dashed) affected constrained high-risk market incumbents, (iii - solid) unaffected constrained high-risk market incumbents, (iv - dashdotted) affected unconstrained high-risk market incumbents, (v - solid) unaffected unconstrained high-risk market incumbents, and (vi - dotted) new entrants.

The most productive banks are the low-risk market incumbents (i). Their business model is affected directly by the leverage ratio. They react by shifting their portfolio and choosing the mixed strategy $v$. Thereby they reduce their supply of low-risk loans in order to compensate the additional cost of being constrained with higher loan rates which are available in the high-risk market. This in turn makes the low-risk market attractive for less productive banks that shift from a specialized high-risk into a mixed portfolio strategy (ii). The high-risk market gets more competitive as more productive banks enter it. In a Cournot-equilibrium with asymmetric costs, an increase in the number of banks in a market implies that supply is reduced and prices increase. This phenomenon is termed “anti-competitive” behavior by Amir and Lambson (2000). Some specialized banks in the high-risk market are unaffected by the leverage ratio and do not change their strategy (iii), although they profit from the increase in the high-risk interest rate. Formerly unconstrained banks are able to increase their supply of loans so that some of them grow to point where they are constrained by the risk-weighted ratio (iv) and others grow as well but less (v). Finally, since expected revenue in the high-risk market is higher in the new equilibrium, new banks enter the high-risk market (vi). As a result, market shares are reallocated between heterogeneous banks. More productive banks lose market shares in their home market but gain shares in the other market. Less productive high-risk markets incumbents lose market shares.

To rationalize this, consider that the competitive outcome is achievable in this model if the most productive bank 1 chooses to push every other bank out of the market by producing very high quantities at its marginal costs. Therefore, the more banks are active in equilibrium, the closer market outcomes are to monopoly outcomes. See sec. 5 for a discussion on how crucial the Cournot market is for the results.
Figure 2: Optimal strategies and cutoff marginal costs in the Basel II equilibrium (upper line) and the Basel III equilibrium (lower line). Roman numbers on the bottom indicate groups of banks according to their change in strategy from the Basel II to Basel III equilibrium.

**Lemma 3.** By tightening capital requirements through the introduction of a leverage ratio, market shares in the low-risk market are reallocated towards less productive banks while market shares in the high-risk market are reallocated towards more productive banks and less productive new entrants.

**Proof.** Proof follows directly from proposition 3 and corollary 1.

The reallocation of market shares in the low-risk market implies that the average productivity of banks participating in that market decreases. On the other hand average productivity in the high-risk market might increase, i.e. if the number of new entrants is relatively small. In the unregulated equilibrium, the most productive banks dominate both markets. Hence, any capital requirement indirectly protects market shares of less productive banks in the affected market. This is of course even more visible when considering regulations which directly pose entry barriers to the banking market. The model shows that productivity advantages in an oligopolistic market add to the charter value of a bank which protects against individual failure in any kind of systemic crisis. A regulator concerned with financial stability should therefore take these side-effects on the distribution of market shares into account.

In terms of solvency, the effect of the leverage ratio differs between the categories defined above. First of all, new entrants (vi) have rather low critical values $z_{i,crit}$ because they focus on the high-risk market and their charter values are rather low, since they are closest to producing at marginal costs with zero profits. Unaffected constrained high-risk market incumbents (iii) neither change their portfolio nor their size but benefit from the rise of the interest rate. Therefore, their critical values increase which means that they become more resilient. Unconstrained high-risk market incumbents (iv,v) benefit from the rise of the high-risk interest rate as well. But these banks grow and have higher leverage ratios in the new equilibrium. In contrast to the other groups, banks with higher productivity (i,ii) change their portfolio composition in the new equilibrium. The direction in which the portfolio allocation channel takes effect depends on the realization of systematic risk. Banks with the highest productivity (i) increase their share of high-risk loans. For realizations of systematic risk below a threshold (defined in the appendix in eq. (97)) this decreases their probability to default because of the positive effect of higher earnings from the high-risk market on their charter values. I term this *normal times.* When
systematic risk realizes above the threshold, termed as crisis, the fraction of defaulting loans in the high-risk market gets prohibitively high so that the diversification in the mixed portfolio strategy turns out to have a negative effect on banks’ resilience. For affected constrained high-risk market incumbents (ii) the reverse holds: In normal times their higher share of low-risk loans, which yield only low revenue, increases their default probabilities while in times of crisis the share of low-risk loans decreases their default probabilities.

**Lemma 4.** Default probabilities of the most productive banks (affected low-risk market incumbents) decrease in normal times. In times of high realizations of systematic risk, the portfolio reallocation of these banks has a negative effect on their default probabilities.

Default probabilities of less productive constrained banks (unaffected high-risk market incumbents) decrease. Default probabilities of affected high-risk market incumbents may increase due to increasing interest rates or decrease due to their portfolio reallocation and higher leverage. In times of high realizations of systematic risk, the portfolio reallocation of these banks has a positive effect on their default probabilities.

**Proof.** Formal proof is in the appendix. \( \square \)

5 Discussion

The model highlights how regulation naturally interferes with regular market forces and thus creates side effects on financial stability. Productivity –irregardless of whether it stems from advantages in technology or information– influences banks’ strategies and price setting. And ultimately, it influences market structure.

Regulators face a trade-off between assuring safety in the banking system and distorting competition. Banks should internalize risk-taking which is defined in various dimensions. Banks have different exposures to these dimensions. The model shows that these differences arise systematically due to the heterogeneity between banks. Therefore, as the regulator aims at confining risky banks it might as well narrow profitability of productive banks. Although unpleasant for a bank on its own, it can be seen as an exchange of intangible charter value into observable regulatory capital, both of which have a loss absorbing function.

A limitation to the model surely is the assumption that equity is fixed and the same amount for all banks. This serves to make banks comparable at some level. When in fact, productivity advantages and intangible charter value should be priced on the equity market in a way that more productive banks find it easier to refinance themselves. Increasing equity is an alternative strategy to risk-shifting as a reaction to the leverage ratio. Indeed, banks raised equity ever since the ratio was announced and monitored (Basel Committee on Banking Supervision, 2016; Smith et al., 2017) but investors should have been aware that the capital was needed to comply to tightened regulatory guidelines. However, for this model it would mean that the problem for more productive banks is just moving from the product to the equity market. Loosening constraints by raising equity allows banks to move closer to an unregulated equilibrium where productivity sponsors market shares and size. Consequently, if a leverage ratio were to be binding for any bank at all, it still were binding for the more productive banks even if they do not change their portfolio composition as a response.
Another critical assumption is Cournot competition. While it plausibly implies that productivity produces market power in the form of market shares and profits, it implies that lower concentration comes along with less competitive outcomes. Therefore, the set-up of the model is related to Efficient Structure theories. Such a relation between concentration and loan rates is confirmed by some studies (Jayaratne and Strahan, 1998), yet it is challenged by as many (cf. VanHoose (2007) for a comprehensive literature review).

The focus of my work lies on the evaluation of capital requirements. In this light, you may note that the positive effect on less productive high-risk market incumbents’ default probabilities hinges on exactly this anti-competitive behavior. In other settings, if banks had some price setting power –irrespective of the question of entry and exit– it is reasonable if they reacted by passing on costs to customers by increasing loan rates. As long as excessive risk-taking is associated with high quantities, the regulator cannot avoid increasing financial stability at the expense of credit rationing.

In a competitive setting where banks cannot influence market loan rates, less productive banks would exit the market if new regulation causes additional costs. In fact, this is what happens when moving from the unregulated equilibrium to the Basel II equilibrium. But since banks are already constrained when the leverage ratio is introduced, they can circumvent incurring the costs of being regulated by adapting their business model and entering the high-risk market.

6 Conclusion

My work studies the optimal portfolio choice under competing capital requirements for heterogeneous banks. It points to the fact that productivity differences might influence banks’ exposures to risk systematically so that regulation indirectly affects certain types of banks. Capital requirements therefore have repercussions on market structure.

The model shows that if bank size is taxed by the newly introduced leverage ratio, then banks with high productivity are directly affected and react with risk-shifting. However, this higher share of high-risk loans does not increase their default probabilities, at least not as long as systematic risk is moderate. It induces a reallocation of market shares from more to less productive banks in the low-risk market. Average productivity in the low-risk market falls.

As the regulatory toolbox is filling up, it is important to consider the differential treatment caused by the interplay of different measures. The results could apply to other measures. For example, capital requirements on operational risk charge banks based on their gross income. While gross income is used as a proxy of risk caused by complexity, it is reasonable to assume that gross income depends on productivity as well. Productivity is hard to measure. Yet it can create positive charter value in an imperfect competitive environment. Since it might be a difficult to
impossible task to formulate any requirements contingent on productivity in order to regulate heterogeneous banks, capital regulation should at least contemplate possible channels between productivity and risk. If risk measures are positively correlated to productivity measures, regulating these risks turns intangible charter value into observable capital. Generally, the banking market would be more transparent but not necessarily safer and market shares might be reshuffled. If on the other hand risk measures are negatively correlated to productivity, regulating these risks is more than called for. By using approaches with heterogeneous instead of representative banks, further theoretical work could systematically address the complex relationship between risk, capital, and productivity.

References


Appendix

A.1 Proof of Proposition

Before I start the proof of the characteristics of any equilibrium in the following, let me state that they indeed exist.

**Lemma 5** (Existence of equilibria). The unregulated game, the game with a risk-weighted regulation, and the game with a leverage ratio and risk-weighted regulation have at least one Nash-equilibrium in pure strategies.

**Proof.** Proof in appendix A.2.

**Proof.** The First-order conditions to the optimization problem given in eq. (8) for bank $i$

$$\frac{\partial \Pi_i}{\partial q_{i,h}} \leq 0 \quad \text{and} \quad \frac{\partial \Pi_i}{\partial q_{i,l}} \leq 0,$$

$$q_{i,h} \frac{\partial \Pi_i}{\partial q_{i,h}} = 0 \quad \text{and} \quad q_{i,l} \frac{\partial \Pi_i}{\partial q_{i,l}} = 0$$

where $\frac{\partial \Pi_i}{\partial q_{i,\eta}} = p_{\eta}(r_{\eta}(Q_{\eta}) + r'_{\eta}(Q_{\eta})q_{i,\eta} - MC_i$. From eq. (34) and the non-negativity constraint on quantities, we know that banks either produce nothing or, if they supply a positive amount of loans, marginal profits must be zero. Further, if $MC_i > p_{\eta}r_{\eta}(Q_{\eta})$ for any bank $i$ given the strategies of all other banks, i.e. it cannot make a profit in the market at the given interest rate because its marginal costs are too high, then its marginal profits are negative for any non-negative amount of loans in that market. Note that

$$MC_i > p_{\eta}r_{\eta}(Q_{\eta}) > p_{\eta}r'_{\eta}(Q_{\eta})q_{i,\eta} \quad \forall q_{i,\eta} \geq 0.$$

Therefore, the best strategy for such a bank is not to participate.

There are two markets to cater to, so banks decide on their participation and the extent of it in both markets. They do this separately, since the extent to which they choose to produce in one market does not affect their actions or the actions of other banks in the other market. As a result, there can be three types of banks: First, banks that participate in both markets because their marginal costs are lower than both expected returns. Second, banks that participate only in the high-risk market, because their marginal costs are lower than expected return in the high-risk market but higher than expected return in the low-risk market. And third, banks that cannot participate in any market.\(^{[11]}\)

\(^{[11]}\)Note that $p_h r_h(Q_h) > p_l r_l(Q_l)$ by assumption.
Solving eq. (33) for \( q_{\eta,i} \), we get the best response function for bank \( i \) as

\[
\hat{q}_i(Q_{-i}) = \begin{cases} 
(\hat{q}_{h,i}, \hat{q}_{l,i}) & \text{if } MC_i \leq p r_1(Q_l) \\
(\hat{q}_{h,i}, 0) & \text{if } p r_1(Q_l) < MC_i \leq p h r_h \\
(0, 0) & \text{if } p h r_h(Q_h) < MC_i
\end{cases}
\]

where

\[
\hat{q}_{\eta,i} = \frac{p r_\eta(Q_{\eta,i}) - MC_i}{p r_\eta(Q_{\eta,-i})}.
\]

Summing the FOCs for marginal profits in eq. (33) over all banks gives

\[
p_h(N r_h(Q_h) + r_h'(Q_h) Q_h) - \sum_{i=1}^N MC_i \leq 0
\]

\[
p_l(N r_l(Q_l) + r_l'(Q_l) Q_l) - \sum_{i=1}^N MC_i \leq 0.
\]

Let \( \nu_h \) denote the bank with the highest marginal costs that is still able to supply high-risk loans at a profit and let \( \nu_l \) denote the bank with the highest marginal costs that is still able to supply low-risk loans. We can rewrite eq. (37) as

\[
p_h(\nu_h r_h(Q_h) + r_h'(Q_h) Q_h) - \sum_{i=1}^{\nu_h} MC_i = 0
\]

\[
p_l(\nu_l r_l(Q_l) + r_l'(Q_l) Q_l) - \sum_{i=1}^{\nu_l} MC_i = 0.
\]

Solving eq. (38) for the first derivative of the inverse demand function and inserting this into the best response function, we get

\[
\hat{q}_{\eta,i}(Q_{\eta}) = \frac{p r_\eta(Q_{\eta}) - MC_i}{\nu_h p r_\eta(Q_{\eta}) - \sum_{i=1}^{\nu_h} MC_i Q_h}.
\]

From Lemma [5] we know an equilibrium must exist. An equilibrium is characterized by best-response correspondence such that

\[
q_i^* = \arg \max \Pi_i(q_i, Q_{-i}) \quad \forall i \in \{1, \ldots, N\}.
\]

Hence, if there is an equilibrium, optimal strategies of banks must be defined as

\[
q_i^* = (q_{h,i}^*, q_{l,i}^*) \\
q_{h,i}^* = \max \left[ 0, \frac{p h r_h(Q_{h,i}^*) - MC_i}{\nu_h p h r_h(Q_{h,i}^*) - \sum_{i=1}^{\nu_h} MC_i Q_h} \right]
\]

\[
q_{l,i}^* = \max \left[ 0, \frac{p l r_l(Q_{l,i}^*) - MC_i}{\nu_l p l r_l(Q_{l,i}^*) - \sum_{i=1}^{\nu_l} MC_i Q_l} \right].
\]

**Corollary 2** (Market shares without capital requirements). In the unregulated equilibrium, more productive banks, i.e. banks with lower marginal costs, gain higher market shares than
less productive banks.

**Proof.** Let \( \kappa_{\eta,i} = (q_{\eta,i}/Q_{\eta}) \) denote the market share of bank \( i \) in market \( \eta \). Then

\[
\frac{p_{\eta}r_{\eta}(Q_{\eta}) - MC_i}{-p_{\eta}r'_{\eta}(Q_{\eta})} > \frac{p_{\eta}r_{\eta}(Q_{\eta}) - MC_{i+1}}{-p_{\eta}r'_{\eta}(Q_{\eta})}
\]

holds in both markets. \( \Box \)

**Corollary 3** (Bank size without capital requirements). In the unregulated equilibrium, more productive banks are bigger than less productive banks.

**Proof.** See that

\[
q_{h,i}^* + q_{l,i}^* > q_{h,i+1}^* + q_{l,i+1}^*
\]

\[
\kappa_{h,i}Q_{h}^* + \kappa_{l,i}Q_{l}^* > \kappa_{h,i+1}Q_{h}^* + \kappa_{l,i+1}Q_{l}^*
\]

\[
(\kappa_{h,i} - \kappa_{h,i+1})Q_{h}^* > (\kappa_{l,i} - \kappa_{l,i+1})Q_{l}^*
\]

is always true, because the left-hand side of the last inequality is positive while the right-hand side is always negative due to corollary 2. \( \Box \)

**Corollary 4** (Portfolio shares without capital requirements). In the unregulated equilibrium, more productive banks have a higher share of riskier loans in their portfolio than less productive banks.

**Proof.** Let \( \gamma_i = \frac{q_{h,i}}{q_{h,i} + q_{l,i}} \) denote the share of high-risk loans to total loans of bank \( i \). Then

\[
\gamma_i < \gamma_{i+1}
\]

\[
\frac{q_{h,i}^*}{q_{h,i+1}^*} < \frac{q_{l,i}^*}{q_{l,i+1}^*}
\]

\[
MC_i < MC_{i+1}
\]

\( \Box \)

### A.2 Proof of Lemma 5

**Proof.** This proof applies the results of Vives (2001b) and checks whether the conditions formulated therein are met in all games. According to Vives (2001b) Theorem 2.1, a Nash equilibrium for a game with strategy set \( \Omega_i \), payoffs \( \Pi_i \), and players \( i \in \{1, \ldots, N\} \) exists, if

a) strategy sets \( \Omega_i \) are non-empty, convex, and compact subsets of Euclidean space, and

b) payoff \( \Pi_i \) is continuous in the actions of all firms and

c) quasi-concave in its own action.
a) The strategy set of bank $i$ consists of all possible quantities of loans. The model facilitates the view of a bank to a simple loan generating and deposit taking intermediary and therefore abstracts from other financial products where negative positions would be attainable. A potential strategy is therefore non-negative and the strategy set focuses on the upper right quadrant of $\mathbb{R}^2$ which is a non-empty convex set and subset of Euclidean space. Since zero is included in the strategy set, it is closed. Given a capacity limit $0 \leq q_i \leq W_i$, the set is bounded. The Heine-Borel theorem states that any bounded and closed subset of Euclidean space is also compact. Consequently, the first condition is met by an unregulated market.

The capital requirements essentially lower the upper bound on the strategy set. Both constraints are linear and define a triangle in $\mathbb{R}^2$, which is convex. Figure 1 illustrates both constraints. In the case of joint regulation with both constraints, the strategy set is an intersection of the two strategy sets of the preceding games which are both convex. Hence, their intersection is convex as well. In all constrained cases, they include the upper bound and zero as the lower bound. Consequently, strategy sets of the constrained games are non-empty, convex, and compact subsets of Euclidean space. Let the strategy set $\Omega_i$ be defined as

\[
\begin{align*}
\text{(without constraints)} & \quad \Omega_i = \{q_i \mid 0 \leq q_i \leq W_i\} \\
\text{(risk-weighted)} & \quad \Omega_i = \{q_i \mid 0 \leq \beta_h q_{h,i} + \beta_l q_{l,i} \leq \epsilon\} \\
\text{(both constraints)} & \quad \Omega_i = \{q_i \mid 0 \leq \max[\beta_h q_{h,i} + \beta_l q_{l,i}, \beta(q_{h,i} + q_{l,i})] \leq \epsilon\}
\end{align*}
\]

b) The payoff function of bank $i$ is given as

\[
\Pi_i(q_i) = (p_h r_h(Q_h) - MC_i)q_{h,i} + (p_l r_l(Q_l) - MC_i)q_{l,i}
\]

where continuity follows from the continuity of its components. The inverse demand functions in $r(Q)$ are continuous by definition and $q_i$ itself is continuous. Hence their product and difference is. Adding constraints was shown to alter the strategy space but not the payoff function. Therefore, the second condition for the existence of an equilibrium is fulfilled in all scenarios.

c) Profits are quasi-concave with respect to banks’ own strategy choices, if all principal minors of the bordered Hessian matrix of $\Pi_i(q_i)$ are of alternating signs. Bordered Hessian of $\Pi(q_i)$ holding $Q_{-i}$ constant is

\[
H = \begin{pmatrix}
0 & \frac{\partial \Pi_i}{\partial q_{i,l}} & \frac{\partial \Pi_i}{\partial q_{i,h}} \\
\frac{\partial \Pi_i}{\partial q_{l,i}} & \frac{\partial^2 \Pi_i}{\partial q_{l,i}^2} & 0 \\
\frac{\partial \Pi_i}{\partial q_{h,i}} & 0 & \frac{\partial^2 \Pi_i}{\partial q_{h,i}^2}
\end{pmatrix}.
\]

The first principal minor is

\[
-\left(\frac{\partial \Pi_i}{\partial q_{i,l}}\right)^2 \leq 0,
\]

which is non-positive by construction of $H$. The second principal minor is equal to the deter-
minant of $H$ which is

$$\frac{\partial^2 \Pi_i}{\partial q_{i,l}^2} \left( \frac{\partial \Pi_i}{\partial q_{i,h}} \right)^2 - \frac{\partial^2 \Pi_i}{\partial q_{i,h}^2} \left( \frac{\partial \Pi_i}{\partial q_{i,l}} \right)^2 \geq 0.$$ 

This is non-negative since

$$\frac{\partial^2 \Pi_i}{\partial q_{i,l}^2} = 2p_{q_i} \frac{\partial q_{i}}{\partial q_{i,h}} + p_{q_i} q_{i,h} \frac{\partial^2 r}{\partial q_{i,h}^2}$$

and inverse demand is concave so that $\frac{\partial q_{i}}{\partial q_{i,h}} < 0$ and $\frac{\partial^2 r}{\partial q_{i,h}^2} < 0$ (See assumption in eq. (2)). Therefore, $\Pi_i$ is quasi-concave with respect to $q_i$. Constraints on the strategy set in form of capital requirements do not alter the profit function, hence the third condition for existence is fulfilled in all scenarios. We conclude that at least one Nash-equilibrium must exist in each game.

A.3 Proof of Proposition 2

Proof. The proof is structured as follows. First, I compare all possible strategies to eliminate dominated strategies. Then, I derive the conditions for feasibility of the dominating strategies.

From the FOCs of eq. (14) we derive five possible strategies depending on whether the risk-weighted constraint in eq. (13) is binding. Let $s$ denote the strategy where $s \in \{l, rw, h, uc, 0\}$, and $q_{i,s}^\eta$ the optimal quantity of bank $i$ in market $\eta$, $\Pi_i^s$ its payoff, and $\mu_{i,s}^\eta$ its slack parameters if it implements strategy $s$.

$$\begin{align*}
(q_{i,s}^\eta, q_{i,s}^l, \mu_{i,s}^\eta) &= \begin{cases} 
(0, q_{i,s}^l, \mu_{i,s}^l) & \text{low-risk specialist} \\
(q_{i,s}^\eta, q_{i,s}^l, \mu_{i,s}^l) & \text{mixed risk-weighted constrained} \\
(q_{i,s}^\eta, 0, \mu_{i,s}^l) & \text{high-risk specialist} \\
(q_{i,s}^\eta, q_{i,s}^l, 0) & \text{unconstrained} \\
(0, 0, 0) & \text{not participating}
\end{cases} \quad (45)
\end{align*}$$

where $q_{i,s}^\eta$ is defined in eq. (9), and $q_{i,s}^l, q_{i,s}^w, \mu_{i,s}^l, \mu_{i,s}^w$ depend on $MC_i, r$, and $r'$, while $q_{i,s}^l$ and $q_{i,s}^h$ are independent of $MC_i$, and

$$q_l^l = \frac{e}{\beta_i}, \quad q_l^h = \frac{e}{\beta_h}. \quad (46)$$

If feasible, constrained strategies dominate the unconstrained strategy and clearly the non-participating strategy, since in the unconstrained strategy banks are left with unused equity. If no constrained strategy is feasible for a bank, but still marginal costs are lower than expected revenue from any loan (i.e. $MC_i \leq MC_i^h$, see eq. (18)), banks participate (see eq. (35)) with an unconstrained strategy.

From eq. (16) we know that banks with marginal costs below the cutoff prefer strategy $l$
over $h$. Comparing $l$ and $rw$ gives

$$\Pi^w_r(q^w_i) < \Pi^l_l(q^l_i)$$

$$(p_h r_h - MC_i) q^w_{h,i} + (p_{rl} - MC_i) q^w_{r,i} < (p_{rl} - MC_i) q^l_{r,i}$$

$$(p_h r_h - MC_i) < \frac{q^l_{r,i} - q^w_{r,i}}{q^w_{h,i}}$$

$$(p_h r_h - MC_i) < \frac{\beta_h}{\beta_l}$$

which is the same condition as in eq. (15). For the last step, I used the fact that eq. (13) holds with equality for strategy $l$ and $rw$. Consequently, whenever strategy $l$ dominates $h$, $l$ dominates $rw$ as well. One can show in a similar way, that whenever strategy $h$ dominates $l$, it dominates $rw$ as well. Hence, banks would never choose a mixed portfolio strategy if a specialization strategy is available.

Now, I derive conditions for feasibility of all strategies. First, the cutoff $\tilde{MC}^l$ has to be positive. Otherwise all banks find it optimal to specialize on high-risk loans with can never be a Nash-equilibrium. Then supply of high-risk loans would be very high and the loan rate falls whereas there is no supply of low-risk loans so that the interest rate on low-risk loans rises and ultimately $p_h r_h > p_{rl}$ is violated or $\tilde{MC}^l > 0$. The first condition is therefore

$$p_{rl}(Q^l) < p_h r_h(Q^h) < \frac{\beta_h}{\beta_l} p_{rl}(Q^l).$$

Secondly, a strategy $s$ is only feasible if $\mu^s_i \geq 0$. The shadow prices are functions of marginal costs and market prices ($\mu^s_i(MC_i, r(Q))$) which imply cutoffs $\tilde{MC}^{\mu^s}$ which themselves have to be positive to be meaningful, s.t.

$$\mu^s_i \geq 0 \quad \forall i \in \{1,...,N\} : MC_i \leq \tilde{MC}^{\mu^s} \quad \text{where} \quad \tilde{MC}^{\mu^s} > 0$$

For strategies $l$ and $h$ this means that

$$\mu^l_i \geq 0 \quad \forall i \in \{1,...,N\} : MC_i \leq \tilde{MC}^{\mu^l} \quad \text{where} \quad \tilde{MC}^{\mu^l} = p_{rl} + \frac{\epsilon}{\beta_l} p_{rl} > 0$$

$$\mu^h_i \geq 0 \quad \forall i \in \{1,...,N\} : MC_i \leq \tilde{MC}^{\mu^h} \quad \text{where} \quad \tilde{MC}^{\mu^h} = p_h r_h + \frac{\epsilon}{\beta_h} p_h r_h > 0$$

Thirdly, the following conditions ensure that there is a certain order between feasibility cutoffs $\tilde{MC}^{\mu^s}$ and dominance cutoffs $\tilde{MC}^l$ and $\tilde{MC}^h$. All banks with $MC_i \leq \tilde{MC}^l$ can choose $l$ only if

$$\tilde{MC}^l < \tilde{MC}^{\mu^l}$$

$$-\frac{\beta_l}{\beta_h - \beta_l} (p_h r_h(Q^h) - p_{rl}(Q^l)) < \frac{\epsilon}{\beta_l} p_{rl}(Q^l) < 0. \quad (52)$$
All banks with $MC_i \leq \overline{MC}^{\mu_h}$ can choose $h$ only if

$$\overline{MC}^{\mu_w} < \overline{MC}^{\mu_h}$$

$$(pr_t(Q^t_i) - ph_r(Q^*_h)) < \frac{e}{\beta_h} ph_r(Q^*_h) < 0.$$ (53)

Eq. (53) usefully implies that

$$pr_t(Q^t_i) < ph_r(Q^*_h) + \frac{e}{\beta_h} ph_r(Q^*_h)$$

$$pr_t(Q^t_i) < \overline{MC}^{\mu_h}$$ (54)

so that if banks choose the unconstrained strategy, they specialize on high-risk loans and are not able to supply low-risk loans profitably.

Furthermore, condition (52) is always stricter than condition (50), and condition (53) is always stricter than condition (51). Hence, given eq. (17), (52), and (53) optimal strategies in equilibrium are

$$(q_{h,i}, q_{l,i}^*, \mu_{i}^*) = \begin{cases} 
(0, q_{l}^i, \mu_{i}^{l}(MC_i)) & \text{if } MC_i \leq \overline{MC}^{l} \\
(q_{h}^i, 0, \mu_{i}^{h}(MC_i)) & \text{if } \overline{MC}^{l} < MC_i \leq \overline{MC}^{\mu_h} \\
(q_{h,i}^{uc}(MC_i), 0, 0) & \text{if } \overline{MC}^{\mu_h} < MC_i \leq \overline{MC}^{h} 
\end{cases}$$ (55)

A.4 Proof of Lemma 1

Proof. First, I show that within each strategy, banks with lower marginal costs have higher critical values and therefore lower default probabilities. Then, I show that within the same bank and given $ph_r(Q^*_h) < \frac{\beta_h}{\alpha_h} ph_r(Q^*_l)$, strategies with a higher share of high-risk loans have a higher default probability.

For the specialized strategies, we can solve eq. (10) for $z_{i,\text{crit}}^\eta$ which is the critical value of bank $i$ if it specializes on strategy $\eta$. Given equilibrium strategies and outcomes we get

$$\left(1 - PD_{\eta}(z_{i,\text{crit}}^\eta)\right) r_{\eta}(Q^*_\eta)q_{\eta}^{n_s} - MC_i q_{\eta}^{n_s} + r_de = 0$$

$$\left(1 - \Phi \left(\frac{\zeta_{\eta} + \sqrt{p} z_{i,\text{crit}}^\eta}{\sqrt{1 - \rho}}\right)\right) r_{\eta}(Q^*_\eta) \frac{e}{\beta_{\eta}} - MC_i \frac{e}{\beta_{\eta}} + r_de = 0.$$ (56)

Rearranging gives

$$z_{i,\text{crit}}^\eta = \frac{\sqrt{1 - \rho}}{\sqrt{\rho}} \Phi^{-1} \left(1 - \frac{MC_i - r_d \frac{e_{\eta}}{\eta_{\eta}}}{r_{\eta}(Q^*_\eta)}\right) - \frac{\zeta_{\eta}}{\sqrt{\rho}}.$$ (57)

Except $MC_i$, all parameters in eq. (57) are equal for banks with the same constrained equilib-
rium strategy. Taking the derivative with respect to $MC_i$ gives

$$
\frac{\partial z_i^{\eta,crit}}{\partial MC_i} = (-1) \frac{\sqrt{1 - \rho}}{\sqrt{\rho} \phi \left( \Phi^{-1} \left( 1 - \frac{MC_i - r_d q^i_{\eta}}{r\eta(Q^i_{\eta})} \right) \right)} < 0
$$

(58)

where $\phi(x)$ is the PDF of the standard normal distribution. Therefore, if $MC_i < MC_{i+1}$, then $z_i^{\eta} > z_{i+1}^{\eta}$ for $\eta = \{h, l\}$.

For high-risk specialists that are not constrained (strategy uc), the parameters $MC_i$ and $q_{h, i}^{uc}$ change in eq. (57). Simplifying $z_{i,crit}^{uc} > z_{i+1,crit}^{uc}$ yields

$$
(MC_{i+1} - MC_i)q_{h, i+1}^{uc} q_{h, i}^{uc} > r_d e(q_{h, i+1}^{uc} - q_{h, i}^{uc})
$$

(59)

which is always true since $q_{h, i+1}^{uc} - q_{h, i}^{uc} < 0$.

Hence, when comparing different banks with the same strategy, we find that within each market banks with lower marginal costs have higher critical values and therefore lower default probabilities.

Let us now compare default probabilities of different strategies for one bank $i$. If $p_h r_h(Q^i_h) < \frac{\partial}{\partial \beta} p_h r_l(Q^i_l)$, then

$$
1 - \frac{MC_i - r_d \beta_l}{r_l(Q^i_l)} > 1 - \frac{MC_i - r_d \beta_h}{r_h(Q^i_h)}
$$

(60)

and hence

$$
\Phi^{-1} \left( 1 - \frac{MC_i - r_d \beta_l}{r_l(Q^i_l)} \right) > \Phi^{-1} \left( 1 - \frac{MC_i - r_d \beta_h}{r_h(Q^i_h)} \right)
$$

(61)

so that the right hand side in the following is negative which ensures that it is true that

$$
\frac{\zeta_h - \zeta_l}{\sqrt{1 - \rho}} > \Phi^{-1} \left( 1 - \frac{MC_i - r_d \beta_h}{r_h(Q^i_h)} \right) - \Phi^{-1} \left( 1 - \frac{MC_i - r_d \beta_l}{r_l(Q^i_l)} \right)
$$

(62)

and thus

$$
z_{i,crit}^{h} > z_{i,crit}^{l}.
$$

(63)

Since we know that $z_{i,crit}^{h} > z_{i+1,crit}^{h}$, we can compare the default probabilities of the least productive bank in the low-risk market $\nu_l$ (which has marginal cost just below or at the cutoff: $MC_{\nu_l} \leq MC^l$) with the next bank $\nu_{l+1}$ that is the most productive bank in the high-risk market with $MC_{\nu_{l+1}} > MC^l$, and state that

$$
z_{l,crit}^{l} > \cdots > z_{\nu_{l},crit}^{l} > z_{\nu_{l+1},crit}^{h} > \cdots > z_{\nu_{h},crit}^{uc}.
$$

(64)

A.5 Proof of Proposition 3

Proof. The proof is structured as follows. First, I compare all possible strategies to eliminate dominated strategies. Then, I derive conditions for feasibility of dominating strategies.

From the FOCs of eq. (25) we can derive seven possible strategies depending on which or if
any constraint is binding. Let \( s \) denote the strategy where \( s \in \{ l, lr, v, rw, h, uc, 0 \} \), and \( q^s_{h,i} \), the optimal quantity of bank \( i \) in market \( \eta \), \( \Pi^s_i \) its payoff, and \( \mu^h_i \) and \( \lambda^r_i \) its slack parameters if it implements strategy \( s \).

\[
(q^s_{h,i}, q^s_{l,i}, \mu^h_i, \lambda^r_i) = \begin{cases} 
(0, q^l_i, 0, \lambda^r_i) & \text{low-risk specialist} \\
(q^v_{h,i}, q^v_{l,i}, 0, \lambda^r_i) & \text{mixed lr-constrained} \\
(q^w_{h,i}, 0, \mu^h_i, \lambda^r_i) & \text{mixed lr- and rw-constrained} \\
(q^w_{h,i} + q^w_{l,i}, q^w_{l,i}, 0, 0) & \text{mixed rw-constrained} \\
(0, 0, 0, 0) & \text{unconstrained} \\
\end{cases}
\]

where \( q^w_{h,i} \) is defined in eq. (65), \( q^w_{h,i} + q^w_{l,i}, \mu^h_i, \lambda^r_i \), and \( \mu^h_i \) depend on \( MC_i \), \( r \), and \( r' \), while \( q^l_i, q^r_i \) are independent of \( MC_i \) and depend on \( r \), and \( r' \), and

\[
q^l_i = \frac{e}{\beta}, \quad q^h_i = \frac{e}{\beta_h}, \quad q^v_i = \frac{(\beta - \beta_h)e}{\beta(\beta_h - \beta)}, \quad q^w_i = \frac{(\beta_h - \beta)e}{\beta(\beta_h - \beta)}. 
\]

If feasible, banks choose constrained over the unconstrained or the non-participating strategy. If a bank is constrained by the leverage ratio in equilibrium, it has strong incentives to increase the share of high-risk loans as much as possible (Kim and Santomero, 1988). Comparing the payoff of strategies \( v \) and \( l \) gives

\[
\Pi^v_i(q^v) > \Pi^l_i(q^l) \\
p_h r_h q_h^v + p_v r_v q_v^v - MC_i(q_h^v + q_v^v) > p_v r_v q^l - MC_i q^l \\
\quad p_h r_h q_h^v - p_v r_v (q^l - q^v) > 0 \\
\quad (p_h r_h - p_v r_l) q_h^v > 0. 
\]

Note that for all strategies constrained by the leverage ratio eq. (24) holds with equality so that bank \( i \)’s costs are equal for strategies \( l, lr, \) and \( v \). Furthermore, since \( q^l = \frac{e}{\beta_h} \), from eq. (24) follows that \( q^l - q^v = q^v \). Comparing the payoff of strategies \( v \) and \( lr \) gives

\[
\Pi^v_i(q^v) > \Pi^l_i(q^{lr}) \\
p_h r_h q_h^v + p_v r_v q_v^v - MC_i(q_h^v + q_v^v) > p_h r_h q_{h}^{lr} + p_v r_v q_v^{lr} - MC_i(q_{h}^{lr} + q_v^{lr}) \\
\quad p_h r_h(q_h^v + q_h^{lr}) - p_v r_v(q_h^{lr} - q_v^{lr}) > 0 \\
\quad (p_h r_h - p_v r_l)(q_h^v - q_h^{lr}) > 0. 
\]

For the last step, reckon that the leverage ratio constraint in eq. (24) holds with equality for strategies \( v \) and \( lr \). Equation (67) and eq. (66) are true for all banks irregardless of \( MC_i \). Hence, strategy \( v \) dominates strategies \( l \) and \( lr \).

\[
\Pi^v_i(q^v) > \Pi^l_i(q^l) \quad \forall MC_i \\
\Pi^v_i(q^v) > \Pi^{lr}_i(q^{lr}) \quad \forall MC_i
\]

(68)
Comparing strategy \( v \) to \( h \) gives the cutoff defined in eq. (26), and comparing it to strategy \( rw \) gives

\[
\Pi_i^v(q_i^{rw}) < \Pi_i^v(q^v)
\]

\[
(p_h r_h - MC_i)q_{h,i}^{rw} + (p_r r_l - MC_i)q_{l,i}^{rw} < (p_h r_h - MC_i)q_{l,i}^{v} + (p_r r_l - MC_i)q_{l,i}^{v}
\]

\[
\frac{(p_h r_h - MC_i)}{(p_r r_l - MC_i)} > \frac{q_{i,i}^{rw} - q_{l,i}^{v}}{q_{h,i}^{rw} - q_{h,i}^{v}}
\]

which gives the same cutoff as in eq. (26). For the last step, note that eq. (13) holds with equality for both strategies. Hence, strategy \( v \) only dominates strategies \( h \) and \( rw \) if marginal costs are below the cutoff, i.e.

\[
\Pi_i^v(q^v) > \Pi_i^h(q^h) \quad \forall MC_i : MC_i \leq \overline{MC}^l
\]

\[
\Pi_i^v(q^v) > \Pi_i^{rw}(q^{rw}) \quad \forall MC_i : MC_i \leq \overline{MC}^l
\]

Comparing strategies \( h \) and \( rw \) gives

\[
\Pi_i^{rw}(q_{i,i}^{rw}) < \Pi_i^h(q^h)
\]

\[
(p_h r_h - MC_i)q_{h,i}^{rw} + (p_r r_l - MC_i)q_{l,i}^{rw} < (p_h r_h - MC_i)q_{h}^{h}
\]

\[
\frac{(p_h r_h - MC_i)}{(p_r r_l - MC_i)} > \frac{q_{i,i}^{rw} - q_{h,i}^{h}}{q_{h,i}^{rw} - q_{h,i}^{v}}
\]

which again gives the same cutoff as in eq. (26). Hence,

\[
\Pi_i^h(q^h) > \Pi_i^{rw}(q_{i,i}^{rw}) \quad \forall i \in \{1,...,N\} : MC_i > \overline{MC}^l
\]

\[
\Pi_i^h(q^h) > \Pi_i^v(q^v) \quad \forall i \in \{1,...,N\} : MC_i > \overline{MC}^l
\]

Now, I derive conditions for feasibility of all strategies. Firstly, we need condition (17) to ensure that the cutoff \( \overline{MC}^l \) separating strategy \( v \) and \( h \) is positive.

Secondly, a strategy \( s \) is only feasible if \( \mu_s^i \geq 0 \) and \( \lambda_s^i \geq 0 \). Some shadow prices are functions of marginal costs and market prices (\( \mu_s^i(MC_i,r(Q)) \) or \( \lambda_s^i(MC_i,r(Q)) \)) which imply cutoffs \( \overline{MC}^{\mu_s} \) or \( \overline{MC}^{\lambda_s} \) which themselves have to be positive to be meaningful, s.t.

\[
\mu_s^i \geq 0 \quad \forall i \in \{1,...,N\} : MC_i \leq \overline{MC}^{\mu_s} \quad \text{where} \quad \overline{MC}^{\mu_s} > 0
\]

For strategies \( v \) and \( h \) this means

\[
\mu^h \geq 0 \quad \forall i \in \{1,...,N\} : MC_i \leq \overline{MC}^{\mu_h} \quad \text{where} \quad \overline{MC}^{\mu_h} > 0
\]

\[
\lambda^v \geq 0 \quad \forall i \in \{1,...,N\} : MC_i \leq \overline{MC}^{\lambda_v} \quad \text{where} \quad \overline{MC}^{\lambda_v} > 0
\]

\[
\mu^v \geq 0 \quad \forall i \in \{1,...,N\}
\]
where

\[
\widetilde{MC}^{\mu h} = p_h r_h + \frac{e}{\beta_h} p_h r_h'
\]

\[
\widetilde{MC}^{\lambda v} = \frac{\beta_h p_h r_l - \beta_l p_h r_h}{(\beta_h - \beta_l)} + \frac{\beta_h (\beta_h - \beta_l) p_h r_l'}{\beta (\beta_h - \beta_l)} - \frac{\beta_l (\beta_h - \beta_l) p_h r_h'}{\beta (\beta_h - \beta_l)} p_h r_h e
\]

\[
\mu^v = \frac{p_h r_h - p_l r_l}{(\beta_h - \beta_l)} - \frac{\beta_l (\beta_h - \beta_l) p_h r_l'}{\beta (\beta_h - \beta_l)} p_h r_l e + \frac{\beta_h (\beta_l - \beta_l) p_h r_h'}{\beta (\beta_h - \beta_l)} p_h r_h e
\]

Thirdly, strategies \( v \) and \( h \) should be viable for all banks for whom these strategies are profit maximizing. That is the case if

\[
\underline{\widetilde{MC}}^{\lambda v} < \widetilde{MC}^{\mu h} < \widetilde{MC}^{\mu v}
\]

\[
\widetilde{MC}^{\mu h} > \max \left[ \widetilde{MC}^{\mu v}, \widetilde{MC}^{\lambda v}, \widetilde{MC}^{\lambda v}, p_r r_l \right].
\]

where

\[
\widetilde{MC}^{\mu v} =
\]
\[
\widetilde{MC}^{\lambda v} =
\]
\[
\widetilde{MC}^{\lambda v} =
\]

The conditions given in eq. (74), (75), (76), (80), and (81) simplify to eq. (28) and (29) in the following way: Given (74) and (75), \( \underline{MC}^{\lambda v} < \widetilde{MC}^{\lambda v} \) in (80) is true. Given \( \widetilde{MC}^{\lambda v} < \widetilde{MC}^{\mu h} \) in (80), (74) is true. If (75) and

\[
-\beta_h (p_h r_h - p_l r_l) < p_h r_h e,
\]

then \( \widetilde{MC}^{\mu h} > p_r r_l \) in (81), which itself implies \( \widetilde{MC}^{\mu h} > \widetilde{MC}^{\lambda v} \), and \( \widetilde{MC}^{\mu h} > \widetilde{MC}^{\mu v} \) in (81). If (85) and

\[
p_h r_h e < -\beta (p_h r_h - p_l r_l),
\]

then \( \widetilde{MC}^{\mu h} > \widetilde{MC}^{\lambda v} \) in (81). To sum up, condition (28) is equal to eq. (75), and eq. (85) and (86) combine to condition (29) which is stricter than (76) and \( \widetilde{MC}^{\lambda v} < \widetilde{MC}^{\mu h} \) in (80).

Hence, given eq. (75), (85), and (86) optimal strategies in equilibrium are

\[
(q^v_{h,i}, q^v_l, \mu^v, \lambda^v_{i}) =
\]

\[
\begin{cases}
(q^h_{i}, q^v_{i}, \mu^v_{i}, \lambda^v_{i}(MC_i)) & \text{if } MC_i \leq \widetilde{MC}^{\mu h} \\
(q^h_{i}, 0, 0, 0) & \text{if } \widetilde{MC}^{\mu h} < MC_i \leq \widetilde{MC}^{\mu h} \\
(q^h_{i}, 0, 0, 0) & \text{if } \widetilde{MC}^{\mu h} < MC_i \leq \widetilde{MC}^{\mu h}
\end{cases}
\]

☐
A.6 Proof of Corollary 1

Proof. I proof corollary 1 by contradiction. Assume the cutoff \( \tilde{MC_l} \) decreases. It implies that the number of banks participating in low-risk market decreases. Then fewer banks produce a smaller quantity each so that the total supply of low-risk loans decreases. Note that these banks previously produced \( q_l^i = \frac{e^{\beta_l}}{\hat{\beta}_l} \) and now produce \( q_l^i = \frac{(\beta_h - \beta_l)e^{\beta_h}}{\beta_l(\beta_h - \beta_l)} < q_l^i \). Hence, the interest rate on low-risk loans increases. From eq. (26) follows that the interest rate on high-risk loans must increase as well (and even more) otherwise the cutoff would not decrease as was assumed.

Due to eq. (2) the interest rate on high-risk loans only increases if total supply decreases. On the other hand an increase of \( r_h \) implies that the cutoffs \( \tilde{MC_h} \) and \( \tilde{MC_{\mu_h}} \) both increase while \( \tilde{MC_l} \) decreases. Thus, the number of specialized banks in the high-risk market increases and more productive banks with strategy \( v \) enter the high-risk market. All in all, this implies that the aggregate supply of high-risk loans must increase which contradicts the necessary decrease of aggregate supply such that the interest rate could rise. Hence, the cutoff \( \tilde{MC_l} \) cannot decrease but has to increase.

Assume further the cutoff \( \tilde{MC_h} \) decreases. Then the interest rate on high-risk loans necessarily decreases and aggregate supply increases. That is

\[
Q_h^{*B_2} < Q_h^{*B_3} \Rightarrow \nu_l^{B_3} (1 + \frac{q_v}{q_h}) - \nu_l^{B_2} < (\nu_l^{B_3} - \nu_l^{B_2})
\]

which cannot be true since the right hand side is negative if the cutoff decreases, as was assumed, while the left hand side is positive because the cutoff in the low-risk market increase as was shown earlier. Hence, the cutoff in the high-risk market must increase as well.

A.7 Proof of Lemma 4

Proof. I am interested in the change in default probabilities from the Basel II to Basel III equilibrium for each bank, i.e. the change in the root of \( \Pi_i(c_i, q_i^{*B_2}, r(Q^{*B_2}), z) \) and \( \Pi_i(c_i, q_i^{*B_3}, r(Q^{*B_3}), z) \). Since costs \( c_i \) stay constant for each bank, I separate the effect of the change in interest rates \( r(Q) \) on \( z_{i, crit} \) and the effect of the changing strategy \( (q_i \) on \( z_{i, crit} \). I divide the latter into a portfolio reallocation effect due to changing share of high-risk to low-risk loans in the bank portfolio \( (\gamma_i \) on \( z_{i, crit} \) and a leverage effect due to bigger or smaller size relative to equity \( (\beta_i \) on \( z_{i, crit} \).

Before considering these three channels, I show that \( \Pi_i(c_i, q_i, r(Q), z) \) has a unique root for mixed strategies as well. Note that in case of mixed strategies, eq. (10) cannot be solved for \( z_{i, crit} \). But

\[
\frac{\partial \Pi_i}{\partial z} = -r_h(Q_h)q_h^v \frac{\partial PD_h}{\partial z} - r_l(Q_l)q_l^v \frac{\partial PD_l}{\partial z} < 0
\]

with

\[
\frac{\partial PD_n}{\partial z} = \sqrt{\frac{\rho}{1 - \rho}} \left( \frac{\zeta_g + \sqrt{\rho z}}{\sqrt{1 - \rho}} \right) > 0
\]

so that \( \Pi_i(c_i, q_i, r(Q), z) \) is a decreasing function. Further it is monotone due to the monotonicity
of the CDF in $PD_\eta(z)$. We know from optimality conditions of an equilibrium solution that $\Pi_i(c_i, q_i, r(Q), 0) \geq 0$ (note that $1 - PD_\eta(0) = p_\eta$). Therefore, $\Pi_i(c_i, q_i, r(Q), z) - rd_i$ has a unique root at $z_{i,crit} \geq 0$.

Now, I derive how each of the three channels affects the critical value of realization of systematic risk. First, consider unaffected constrained high-risk market incumbents that do not change their optimal portfolio strategy when the leverage ratio is introduced. For these banks only the market interest rate on high-risk loans changes. Since they are specialized on high-risk loans, we can solve eq. (10) for $z_{i,crit}^h$ and get eq. (57). Taking the derivative with respect to $r_h(Q_h)$ gives

$$\frac{\partial z_{i,crit}^h}{\partial r_h(Q_h)} = -\left(-MC_i + rd_i \frac{v}{Q_h}\right) \frac{\sqrt{1 - \frac{1}{r_h(Q_h)} (MC_i - rd_i \frac{v}{Q_h})}}{r_h(Q_h)^2 \phi \left(\Phi^{-1} \left(1 - \frac{1}{r_h(Q_h)} (MC_i - rd_i \frac{v}{Q_h})\right)\right)} > 0 \quad (91)$$

Given corollary [1], we know interest rate on high-risk loans increases. Hence, $z_{i,crit}^h$ increases and default probabilities of unaffected constrained high-risk market incumbents decrease.

Second, consider affected low-risk and high-risk market incumbents. In order to separate the effects caused through risk-shifting in the portfolio choice and deleveraging, we rewrite eq. (10) by expanding with $\frac{d_i + e_i}{d_i + e_i}$, defining the share of high-risk loans in bank $i$’s portfolio as $\gamma_i = \frac{q_i}{d_i + e_i}$, and defining bank $i$’s leverage ratio as $\beta_i = \frac{\epsilon_i}{\gamma_i}$ as

$$\Pi_i(c_i, \gamma_i, d_i, r(Q), z) = (p_h r_h(Q_h) \gamma_i + pr_l(Q_l)(1 - \gamma_i) - MC_i)(d_i + e)$$

$$+ \beta_i r_d(d_i + e). \quad (92)$$

The effect of deleveraging is

$$\frac{\partial \Pi_i}{\partial \beta_i} = (d_i + e) r_d > 0 \quad (93)$$

Affected low-risk market incumbents reduce their total size (changing from strategy $l$ to $v$), which reduced their default probability according to eq. (93), while affected high-risk market incumbents (changing from strategy $h$ to $v$) increase their size, which increases their default probabilities.

The effect of a higher share of high-risk loans is

$$\frac{\partial \Pi_i}{\partial \gamma_i} = (q_i^h + q_i^l) (r_h(Q_h)(1 - PD_h(z)) - r_l(Q_l)(1 - PD_l(z))) \quad (94)$$

which could be either negative or positive depending on $z$ in the following way:

$$\lim_{z \to -\infty} r_h(Q_h)(1 - PD_h(z)) - r_l(Q_l)(1 - PD_l(z)) = r_h(Q_h) - r_l(Q_l)$$

$$\lim_{z \to \infty} r_h(Q_h)(1 - PD_h(z)) - r_l(Q_l)(1 - PD_l(z)) = 0$$

$$r_h(Q_h)(1 - PD_h(0)) - r_l(Q_l)(1 - PD_l(0)) = p_h r_h(Q_h) - p_l r_l(Q_l) \quad (95)$$

This means that the effect is positive for non-positive $z$ and vanishes for very high $z$. But the
effect can be negative, because \( \frac{\partial \Pi_i}{\partial \gamma_i} \) has a local minimum given at \( \hat{z} \) defined by

\[
\frac{\partial^2 \Pi_i}{\partial \gamma_i \partial z} = 0 \iff \hat{z} = \frac{-\zeta_h^2 + \zeta_l^2 + 2 \ln(\frac{\hat{\rho}}{\hat{\nu}})(1 - \rho)}{2 \sqrt{\hat{\rho}(\zeta_h - \zeta_l)}}.
\]

(96)

Therefore, as \( z \to \infty \), \( \frac{\partial \Pi_i}{\partial \gamma_i} \) must approach the limit 0 from below implying

\[
\exists \tilde{z} : 0 < \hat{z} < \tilde{z} \quad s.t. \quad \begin{cases} \frac{\partial \Pi_i}{\partial \gamma_i} &\geq 0 \text{ if } z \leq \tilde{z} \\ \frac{\partial \Pi_i}{\partial \gamma_i} &< 0 \text{ if } z > \tilde{z} \end{cases}
\]

(97)

Affected low-risk market incumbents increase their share of high-risk loans (changing from strategy \( l \) to \( v \)), which increases (decreases) their default probability according to eq. (97) if the realization of systematic risk is below (above) \( \tilde{z} \), while affected high-risk market incumbents (changing from strategy \( h \) to \( v \)) reduce their share of high-risk loans, which decreases (increases) their default probabilities if the realization of systematic risk is below (above) \( \tilde{z} \).

Overall, if \( z \leq \tilde{z} \), increasing interest rates and delevering reduce affected low-risk market incumbents’ default probabilities and the reallocation of portfolio shares towards the riskier asset increases them. For affected high-risk market incumbents the reverse holds: Higher leverage and a higher engagement in the low yielding asset increase their default probabilities while the reallocation towards less risky loans decreases their default probabilities.