OUTPUT AND WELFARE GAINS FROM NON-RECURSE MORTGAGES: THE ROLE OF GENERAL EQUILIBRIUM EFFECTS

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Abstract

Personal bankruptcy protection in the form of recourse laws can affect output through changes in bank credit allocation. But equilibrium outcomes depend on the strength of general equilibrium effects, the dominant source of risk in the economy and the starting level of bankruptcy protection. The paper shows that when the positive effect on output arises, it is an important driver of welfare gains from non-recourse mortgages. Small output gains from milder recourse result, if the initial value of protection is sufficiently high, because the response of mortgage demand to higher protection is weak enough and lenders shift from mortgage lending to capital loans. However, even for low values of bankruptcy protection, when demand for mortgages is high, capital crowding-out can be mitigated by general equilibrium effects into housing prices. This makes non-recourse mortgages output increasing for most of the considered cases in the model. The optimal level of protection is higher than half of earnings even for strategic household default.

Keywords: Personal bankruptcy, recourse, household debt, housing, general equilibrium, bank portfolio choice;

JEL codes: E44, G11, G21, K35, R21

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1 Introduction

This paper analyzes the macroeconomic effects of recourse laws that occur through residential mortgage credit and bank asset portfolio allocation. Recourse laws provide personal bankruptcy protection to mortgage holders. I show that macroeconomic and welfare outcomes may depend on the dominant source of risk in the economy and the starting level of protection. But increasing the level of personal bankruptcy protection provided by recourse laws is welfare increasing for the most valid calibration.

Mortgage credit is a secured debt. However, housing price depreciation can result in cases when the house value falls below the borrowed amount. Recourse laws define what are the losses for the lender when the collateral (housing) is insufficient to repay the mortgage fully. Non-recourse mortgages or mortgages with mild recourse limit the borrower’s liability to her collateral, i.e. the mortgaged house. This implies a high level of personal bankruptcy protection and further I use milder recourse and a higher level of personal bankruptcy protection interchangeably. I abstract from other types of bankruptcy protection that would matter for unsecured debt. If mortgages are recourse loans, mortgage debt of an insolvent individual would not be forgiven but the individual can be sued to collect other assets. Insolvent debtor’s wage income or other assets can be potentially seized to compensate the creditor for the depreciation of the house value. From a creditor’s perspective, a low level of personal bankruptcy protection (recourse mortgages) increases his returns in bad states compared to a high level of protection (mild recourse mortgages). Thus on average lenders may be incentivized to extend more mortgage credit at the expense of other loans, including corporate loans, if mortgages are recourse. If creditors’ response is strong enough, then one would expect crowding out of other credit than mortgage credit. I call the spillover effect of personal bankruptcy protection on other types of assets the portfolio allocation effect. The portfolio allocation effect relates the change in mortgage credit to the change in physical capital, therefore, the resulting credit allocation may have far-reaching implications for the macroeconomy and welfare.

Empirical literature has not explored whether personal bankruptcy protection can generate spillover effects to other types of assets than residential mortgage credit. Suggestive empirical evidence in favor of the portfolio reallocation effect is presented in Table 1. I use Federal Deposit Insurance Corporation (FDIC) bank-level data to compute average bank credit shares. The sample features 7795 banks over more than four years (2001Q1-2005Q3). Ghent and Kudlyak (2011) classify 11 US states as non-recourse states and I use this classification to group banks into two groups where I compute averages of bank credit shares for each category of credit in each group. I compute t-statistics to check whether differences in credit shares across the two groups of US states are statistically significant and find that all the shares differ significantly across the groups except for the ratio of consumer loans to total loans. States that allow recourse tend to
Table 1: Bank credit shares in the US 2001Q1-2005Q3.

<table>
<thead>
<tr>
<th>Credit as a ratio to total bank assets</th>
<th>Recourse states</th>
<th>Non-recourse states</th>
<th>t-stat</th>
</tr>
</thead>
<tbody>
<tr>
<td>Residential real estate loans</td>
<td>22.0</td>
<td>16.7</td>
<td>1.85</td>
</tr>
<tr>
<td>Commercial and industrial loans</td>
<td>8.5</td>
<td>10.8</td>
<td>-2.29</td>
</tr>
<tr>
<td>Consumer loans</td>
<td>12.1</td>
<td>11.2</td>
<td>0.14</td>
</tr>
<tr>
<td>Commercial real estate loans</td>
<td>11.7</td>
<td>16.6</td>
<td>-2.30</td>
</tr>
</tbody>
</table>

Note: The table presents average bank credit shares in the US, computed for individual banks over the period 2001Q1-2005Q3 from FDIC data. The distinction between recourse states and no-recourse states is based on the classification of Ghent and Kudlyak (2011). They classify Alaska, Arizona, California, Iowa, Minnesota, Montana, North Carolina, North Dakota, Oregon, Washington, and Wisconsin as non-recourse states. The last column present t-statistic for the hypothesis of no difference between credit shares in recourse states and non-recourse states.

have on average higher residential real estate mortgage ratios, lower commercial and industrial loans ratios and lower commercial real estate ratios compared to non-recourse states. This suggests that in recourse states banks issue more residential mortgages at the expense of other credit. This evidence supports the hypothesis that the bank response to potentially higher returns on mortgages in recourse states can lead to higher mortgage credit and lower corporate credit. Increasing the level of protection, i.e.

The aim of this paper is to provide a theoretical analysis of what could be the mechanism behind the portfolio allocation effect of recourse laws and whether the effect could be large enough to matter for aggregate outcomes. Also, if the theory suggests different predictions than the correlations presented in Table 1, are any explanations for that? For simplicity, I focus on corporate loans (physical capital loans) as the only available other option for lenders and I develop a theoretical framework with a bank portfolio choice between mortgage credit and corporate credit and two types of risk, housing value shocks and earnings shocks. The model structure is such that the mortgage default risk is priced in and determines the supply side of the credit equilibrium.

In the setup with housing value shocks only, I show that personal bankruptcy protection in the form of milder recourse has benefits even in case of resulting more frequent (strategic) household defaults. Strategic default occurs if the indebted individual chooses to default because the housing value is below the loan value but not because she would not have resources left for consumption upon repayment. Milder recourse increases financial benefits from default and in equilibrium strategic default becomes more frequent. In such a setup milder recourse results in higher equilibrium mortgage credit. However, due to general equilibrium feedback effects on housing prices lenders do not have to increase mortgage credit at the expense of corporate loans but can lend more in total. Since higher supply of corporate loans means more finance available for credit-constrained firms and in turn higher capital stock, wage income in the economy rises. Both borrowing and saving household benefit from higher income.

The crowding-out of capital does not take place because the general equilibrium effect into housing prices increases savings and more funds become available for intermediation. I assume that the saving
household is the owner of the housing stock, so when more generous bankruptcy protection increases borrowers’ demand for housing, the saver experiences income gains from higher housing prices and saves more. Thus milder recourse can substantially increase welfare depending on the strength of feedback effects into housing prices. This channel (through housing prices) would not be present with unsecured credit only and motivates the development of a theoretical model for my research question in particular rather than extrapolating the existing findings for unsecured credit (e.g. as in Li and Sarte (2006)).

The portfolio reallocation effect is mitigated by the general equilibrium effect into housing prices even if the model has more uncertainty. If I allow for earnings shocks to household income, personal bankruptcy protection becomes a consumption insurance tool in bad states. Also the presence of low earnings shocks can now justify personal bankruptcy protection as consumption insurance in forced default cases. The added dimension of earnings uncertainty especially strengthens borrower’s demand for mortgages. For a low starting level of personal bankruptcy protection a marginal increase in the level of protection would lead to a strong positive demand effect on mortgages and higher mortgage credit. This however would still not crowd out capital but rather increase it due to general equilibrium feedback to housing prices.

The general equilibrium effect is the strongest for very low levels of protection and for sufficiently high values of earnings exemptions it is weak enough to be offset by the conventional portfolio allocation effect. For high values of personal bankruptcy protection (or almost non-recourse mortgages), higher borrowing costs offset higher demand for mortgages and mortgage credit even declines, crowding in capital further. Thus equilibrium outcomes depend not only on the dominant source of risk in the economy but the starting level of protection as well.

My findings can be reconciled with the data presented in Table 1, if borrower’s housing demand has sufficiently small effects on housing prices. This could be the case due to several reasons, for instance, if housing supply is especially elastic or borrower’s constitute a small number of house buyers, to name a few. Also, in my model the general equilibrium effect is the strongest for very low levels of protection. For high levels of protection, a marginal increase in the level of protection makes mortgage credit decline and capital loans increase, resulting in the conventional crowding in of capital. If the minimum consumption that the court in the US guarantees to insolvent borrowers in case of recourse is already sufficiently high, this would imply high implicit level of protection. Then, as my model shows, the general equilibrium effect is weak and the conventional portfolio allocation prevails as it is suggested by Table 1.

I find that the optimal level of personal bankruptcy protection is achieved when the recourse law exempts approximately between 58 and 72 percent of borrower’s earnings. Beyond this point, the capital increase levels off while borrowing costs continue increasing, thus reducing expected utility for both the saver and the borrower. However, the model is not calibrated to match the US mortgage market data, so this finding
should be taken with caution.

My findings should be firstly considered in the light of trade-offs presented by the theory of personal bankruptcy protection. Athreya (2002) explains the main trade-off as the competing demand and supply effects. A higher level of personal bankruptcy protection encourages demand for mortgage credit: borrower’s consumption is better insured against bad states of nature. The negative supply effect would manifest in tighter borrowing conditions. Empirical evidence on this matter is not conclusive either. Gropp et al. (1997), Pence (2006) and Lin and White (2001) provide evidence in favor of the dominating supply effect. However, Severino and Brown (2016) find a decline in unsecured credit after BAPCPA\(^1\) made filing for bankruptcy more expensive and subject to tighter conditions in 2005. In my model, crowding-in of capital can occur only if the supply effect dominates the demand effect. I indeed show that it is the case for sufficiently high values of bankruptcy protection only and argue that the demand effect dominates if there is a high value of consumption insurance.

My main contribution to the literature is the analysis of recourse policies and the evaluation of welfare outcomes while acknowledging general equilibrium effects. Most of quantitative studies with housing markets and equilibrium mortgage default pursued the analysis of different government policies (e.g. Jeske et al. (2013)) or the causes of the foreclosure crisis (e.g. Chatterjee and Eyigungor (2015), Corbae and Quintin (2015), Garriga and Schlagenhauf (2009), etc.) in the US but did not discuss recourse policies that could mitigate defaults. The exceptions are Mitman (2016), Corbae and Quintin (2015) and Hatchondo et al. (2015). Mitman (2016) finds that recourse has little effect on mortgage default rates and Corbae and Quintin (2015) and Hatchondo et al. (2015) find that recourse creates welfare gains. Differently from Mitman (2016) they also assume that seeking recourse is costless for the lender and that mortgages are long-term assets. None of the studies mentioned consider recourse policies with feedback to production and this is potentially one of the reasons why, in contrast to my findings, they do not find welfare gains from non-recourse.

In contrast to Mitman (2016), my model lacks the detail to be calibrated to the US mortgage and housing markets which could partially explain why I find negative recourse effects on mortgage default rates. However, for high values of protection (almost non-recourse) these effects are also small suggesting that, if the US has a high implicit level of protection to begin with, my findings could be reconciled with Mitman (2016). A high implicit level of earnings exemptions would prevail if the court uses a measure of minimum consumption which is sufficiently higher than zero. Whereas in my model the official (and the imposed) level of earnings exemptions can be very low (even zero).

\(^{1}\)The Bankruptcy Abuse Prevention and Consumer Protection Act of 2005 (BAPCPA) is a legislative act that reformed the bankruptcy code in the US. It was passed on April 14, 2005. It made more difficult to file bankruptcy under Chapter 7 so that some consumers may instead use Chapter 13.
Section 2 elaborates on the connections of this paper to the literature. Section 3 presents the model. Section 4 discusses the results. I split section 4 to overview the effects on credit allocation and welfare outcomes separately. Section 5 provides robustness checks. Section 6 lists future extensions and section 7 concludes.

2 Related literature

I contribute to the personal bankruptcy literature by highlighting the role of the bank portfolio choice between mortgages and corporate loans in the presence of mortgage return uncertainty and the ensuing general equilibrium effects. So far only models with unsecured consumer debt took into account the portfolio allocation effects. Li and Sarte (2006) shows that a decrease in the level of personal bankruptcy protection can be followed by an increase in capital stock Li and Sarte (2006) because unsecured credit becomes lower. Chatterjee and Gordon (2012), however, argue that capital stock would be unaffected because of a reduction in savings. However, as it follows from my findings, the results from the unsecured credit literature cannot be extrapolated to residential mortgage credit because the interplay between housing prices and household investment decisions play a large role in aggregate outcomes.

Most of the studies that model equilibrium mortgage default abstract from recourse vs. no-recourse considerations. Several quantitative studies develop general equilibrium models that are capable of describing keys characteristics of the housing and mortgage market in the US. Jeske et al. (2013) use such a model to study the effects of government housing market policy on the housing market. Corbae and Quintin (2015) focus on the role of high leverage mortgage in contributing to the foreclosure crisis in the US and Chatterjee and Eyigungor (2015) analyze different shocks that led to the foreclosure crisis in the US and assign the highest contribution to the financial friction shock. Garriga and Schlagenhauf (2009) do not consider endogenous housing prices and distinguish from other papers by modeling mortgages as long-term assets. They demonstrate that a house price shock can account for most of the spike in foreclosure rates in the US.

The exception is Corbae and Quintin (2015), although they express doubts to what extent recourse is actually practiced in the US, and Mitman (2016). Corbae and Quintin (2015) have a section on the implications of non-recourse mortgages on the foreclosure crisis. They find that making all mortgages recourse and assuming zero recourse transaction costs would have limited the mortgage default spike by approximately 20 percent. Therefore, similarly to my findings, the non-recourse aspect matters for housing demand and housing prices. In contrast to my findings, Corbae and Quintin (2015) finds that introducing non-recourse mortgages costs 0.5 percent of agents’ lifetime consumption. The likely reason behind this discrepancy is that I consider endogenous housing prices and allow for physical capital accumulation and Corbae and
Quentin (2015) do not. These two assumptions, as I show, particularly make recourse welfare-decreasing in my setup. The strength of their study is the analysis of distributional implications. They find that recourse makes mortgages more affordable to low-income agents while my model assumes ex-ante identical households and potentially overstates the losses from recourse. My contribution is the introduction of physical capital and analyzing portfolio allocation effects explicitly. Mitman (2016) broadens the household asset portfolio and shows how an endogenous household choice between unsecured debt and secured debt allows to explain variation of bankruptcy rates and foreclosure rates in the US. The paper features recourse vs. non-recourse differences and analyze the effects that the changes in bankruptcy and foreclosure policies in the US had on foreclosures and bankruptcy rates. It abstracts from physical capital too.

The studies mentioned focus on developments in the housing and mortgage markets and naturally they feature endowment economies with the exception of Jeske et al. (2013). The feedback effect of personal bankruptcy regulation into production is not analyzed. My framework comes close to one of the model extensions in Jeske et al. (2013) though. A general equilibrium model with foreclosures in Jeske et al. (2013) is extended with physical capital accumulation to show that the main results still hold and housing subsidies are even more welfare-reducing than in an endowment economy because they limit the accumulation of physical capital. The paper abstracts from non-recourse mortgages and assumes that borrower's liability is limited to the collateral though. The effects of earnings exemptions which I analyze in my paper can be understood as an expected subsidy to borrowers paid by banks in bad states of nature. Similarly to Jeske et al. (2013), I find that in cases when higher earnings exemptions ('subsidies to borrowers') lead to lower capital stock this has creates substantial welfare losses. In contrast to Jeske et al. (2013), my study lacks in a quantitative aspect as I do not match model parameters to statistics from the mortgage market but I treat the question of interest from a more general perspective and show that earnings exemptions have non-linear effects on the credit allocation in the economy depending on the source of risk and the level of protection.

3 Model

The main goal of this model is to illustrate the mechanism through which personal bankruptcy protection impacts the credit allocation in the economy. The model assumes only a few actors in the economy: two households, a perfectly competitive production firm and a bank. By assumption the two households differ in their discount factors which allows for the positive supply and positive demand for savings. Importantly, intermeditation of savings requires a special technology that is available to the banker only. The bank collects deposits and lends to one of the households and the financially constrained production firm. The banker is a
part of the saving household and thus transfers profits or losses to this household at the end of every period. The saving household also owns the housing stock so he can sell some of it to the borrowing household and generate additional income in this way.

The economy exists for two periods. It is a closed economy with production and fixed housing supply. The borrowing household takes a mortgage to finance housing purchases in period 1 and can default in period 2. There are two sources of uncertainty in the model: the housing value shock $\zeta$ and the earnings shock $z$ with commonly known probabilities. The shocks are independent and occur in period 2 only. Their distributions are discussed in more detail in the parameters section.

After default the household loses her housing (foreclosure) and incurs fixed rental costs. The rental market is exogenous to the model. The housing price clears the housing market and the risk-free rate clears the savings market.

The results distinguish between strategic default and non-strategic default. Default is strategic if the household defaults although she could repay the loan out of her wage and still enjoy consumption that is strictly higher than $c_{min}$. In the calibration $c_{min} = h^r$ which implies that the household would have higher than zero consumption after paying the exogenous rent cost $h^r$.

The timing of events ensures that bankruptcy institutions can affect the credit allocation ex-ante and the borrower’s default decision ex-post.

• **Period 1**
  1. The saver and the borrower receive an income endowment.
  2. The saver receives a housing stock endowment which he can sell to the borrower.
  3. The firm is set up and plans its production in the next period.
  4. The bank receives deposits from the saver and faces demand for mortgage credit and corporate credit.
  5. Demand for credit is satisfied at the price that ensures a zero profit for the bank.
  6. Households consume, the borrower buys housing.

• **Period 2**
  1. Uncertainty with respect to earnings shocks and housing value shocks is resolved.
  2. Households supply labor at the predetermined wage inelastically.
  3. The borrower chooses whether to default and the lender transfers realized returns to the saver.
  4. The firm transfers profits/losses to the saver and exits.
  5. Households consume out of their wages and proceeds from housing.
6. The defaulted household incurs rental costs.

I solve model with backwards induction using terminal values indicated in the model description. The solution algorithm and the baseline set of parameters are given in the appendix.

3.1 Households

Households are of two types. The household with a higher discount factor $\beta_S$ is the saver as opposed to the borrowing household ($\beta_B < \beta_S$).

3.1.1 Saver

In the beginning of period 1 the saver receives an endowment $y^S$. Also he owns the fixed housing stock $h$. The household chooses how much of this utility-generating asset he wants to sell to the other household at the market price $q$ and how much does he want to keep for himself. Next to the housing stock, this household has access to saving through putting deposits $d_1$ to the bank. Labor supply is inelastic. The household chooses consumption $c^S_1$, housing $h^S_1$ and how much to save by putting deposits $d_1$ such that her value function is given by

$$V^S_1 = \max_{\{c^S_1, h^S_1, d_1\}} \left( \frac{(c^S_1)^{1-\sigma} (h^S_1)^{\sigma}}{1-\sigma} - 1 + \beta_S V^S_2 \left( d_1, h^S_1; \zeta, z \right) \right)$$

s.t. $c^S_1 + q \cdot h^S_1 + d_1 \leq y^S + q \cdot h$

Note that the value function depends on future shocks to the household’s income and housing value. Although housing shocks are aggregate, households can receive different earnings shocks.

In period 2 the saver receives labor income $z_2 w_1 n^S$ that is affected by the realization of the earnings shock $z_2$, gross returns on deposits $(1 + r_1) d_1$ and a lump-sum profit transfer from firms and the bank $\pi_2$. The saver owns all firms and the bank, thus, all profits/losses get attributed to him at the beginning of period 2. The saver simply consumes everything in period 2:

$$V^S_2 = \max_{\{c^S_2\}} \left( \frac{(c^S_2)^{1-\sigma} (h^S_2)^{\sigma}}{1-\sigma} - 1 \right)$$

s.t. $c^S_2 \leq z_2 w_1 n^S + (1 + r_1) d_1 + \zeta_2 h^S_1 + \pi_2$

and $h^S_2 = h^S_1$

The budget constraint reflects that in period 2 housing becomes a value-generating asset: it takes its terminal
value determined by the realization of the housing value shock $\zeta$ and contributes to the saver’s income. (Only in period 1 the housing is purchased at the market-clearing price $q$.)

### 3.1.2 Borrower

Household default may arise because of two reasons. The housing value shock is the direct source of uncertainty that the collateral value and can trigger default. The setup allows for the household to default if her financial benefits from personal bankruptcy are high enough: the collateral value falls below the debt value and the costs associated with default are not high enough to make up for the difference between the collateral value and the debt value. This would resemble the housing market bust.

The second driver of the default decision can be an earnings shock. A sufficiently negative shock to the borrower’s earnings may make the indebted household incapable of meeting the payment. Although earnings shocks reduce income that household can use to repay the mortgage, the earnings shock cannot trigger default without housing shocks because in the framework with two periods only the household could always repay by selling the housing asset. However, in the setup where I allow for both type of shocks, a realization of low earnings can strengthen incentives to default and also affect the borrowing decision significantly. This largely resembles mortgage default due to earnings shocks (e.g. job loss, sickness) combined with low housing prices. Therefore, in the model results section I consider the case with only housing shocks present and the case with both earnings and housing value shocks.

I formalize the setup by letting the borrowing household choose consumption $c^B_1$, housing stock $h^B_1$ and how much to borrow from the bank ($m_1$) such that her value function $V^B_1$ is maximized:

$$V^B_1 = \max \left\{ c^B_1, h^B_1, m_1 \right\} \frac{(c^B_1)^{1-\sigma} (h^B_1)^{\sigma h}}{1 - \sigma} - 1 + \beta^B V^B_2 (h^B_1, m_1; \zeta, z)$$

subject to the budget constraint:

$$c^B_1 + q \cdot h^B_1 \leq y^B + m_1$$

For simplicity I assume that the household borrows the maximum allowed amount given by $\rho = \tilde{\zeta} h^B_1 / (1 + r^m_1)$. $\rho$ ($0 < \rho < 1$) is an exogenous LTV ratio, $\tilde{\zeta}$ denotes an expected value of the housing shock $\zeta$ and $r^m_1$ stands for the mortgage net interest rate. The household pledges her housing $h^B_1$ which can be seized in case of default in period 2. Low values of the housing shock would reduce the value of the housing asset possibly triggering household default (though not necessarily). It is important to note that housing value in period 2 is exogenous for the sake of simplicity.

Higher earnings exemptions imply a higher level of personal bankruptcy protection. Earnings exemptions matter in case of insolvency because, if household income, comprised of wage income and the value of
housing assets, is insufficient to repay the mortgage, the bank can claim non-exempt earnings. Then higher earnings exemptions mean that the borrower can keep more of her earnings in case of default.

The maximum level of earnings that the household can keep in case of default is denoted by $\kappa$. Then $\max \{ w_1 n^B - \kappa, 0 \}$ from insolvent household's earnings is seized by the creditor together with housing assets in exchange for forgiving mortgage debt. A positive parameter $\kappa$ implies personal bankruptcy protection: if the borrower's wage income is below the maximum exemption level $\kappa$, the household keeps her wage. Otherwise, the borrower has to give wage above the exemption level to the bank. The setup loosely corresponds to recourse laws in the US: if the value of the house is insufficient to repay the mortgage, the bank can seize some but not all of other assets (including cash) to cover the losses. $\kappa = 0$ approximately corresponds to recourse laws in some states in the US, assuming that the minimum consumption level that has to be guaranteed is zero.

Further I formalize maximization problems for different decisions of the borrower. If the borrower does not default, the value function for period 2 conditional on repayment $V^{BP}_{2}$ is the following:

$$V^{BP}_{2} = \max \left\{ c^B_2 \right\} \frac{\left( c^B_2 \right)^{1 - \sigma} \left( h^B_2 \right)^{\sigma} \left( 1 - \sigma \right)}{1 - \sigma}$$

s.t. the budget constraint which gives residual consumption equal to the household wage income which is left after repaying the mortgage and receiving the housing value:

$$c^B_2 \leq z_2 w_1 n^B + \zeta_2 h^B_1 - (1 + r^m_1) m_1$$

and

$$h^B_2 = h^B_1$$

The value for the borrower after default is given by utility from consumption and rental services. Since the household loses his claim on housing value, she has to incur fixed renting costs $h^r$. It follows that the borrower’s value in period 2 conditional on default is given by $V^{BD}_{2}$:

$$V^{BD}_{2} = \max \left\{ c^B_2 \right\} \frac{\left( c^B_2 \right)^{1 - \sigma} \left( h^r \right)^{\sigma} \left( 1 - \sigma \right)}{1 - \sigma}$$

s.t. $c^B_2 \leq \min \left\{ z_2 w_1 n^B, \kappa \right\} - h^r$

The household's choice whether to default or not is determined by the maximum value:

$$V^B_2 = \max \left\{ V^{BP}_{2}, V^{BD}_{2} \right\}$$
3.2 Production

Firms live for two periods in total. They buy capital \( k_1 \) in period 1 but produce in period 2 only. For simplicity, labor supply \( n_1 \) is inelastic. It follows that both capital and wage is predetermined. I assume that firms need to finance capital expenditure by taking a loan from the bank. Earnings shocks in this economy are idiosyncratic, so firm’s labor supply and production revenue are not affected and firms always repay their loans. Firms use a perfectly competitive technology:

\[
y_2 = k_1^n n_1^{1-\alpha}
\]

Every firm solves the following maximization problem in the first period of their lives:

\[
\max_{k_1, n_1} E_1 \left\{ y_2 + (1 - \delta)k_1 - w_1 n_1 - \left(1 + r_1^l\right) l_1 \right\}
\]

s.t. \( l_1 = k_1 \), \( n_1 = n \)

\( l_1 \) denotes loan taken to finance capital. For simplicity we assume that capital depreciates fully \((\delta = 1)\). It follows that competitive wages and returns on capital would satisfy

\[
\alpha k_1^n n_1^{1-\alpha} - 1 = r_1^l
\]

and

\[
(1 - \alpha) k_1^n n_1^{-\alpha} = w_1
\]

3.3 Banking sector

A representative competitive bank decides on the volume of loans to households and firms and deposits such that it breaks even in expectation. The bank is owned by the saving household so realized profits/losses are transferred to the household in the beginning of the second period.

In period 1 the banks expects a net return \( r_m^2 \) on the mortgage and a return \( r_2^l \) on the corporate loan. The indicator \( I(\text{repaid}) \) denotes the borrowing household’s decision to repay debt. The firm always repays. The expected return on the mortgage reflects that in case of default the bank will seize the non-exempt earnings together with housing:

\[
E_1 \left(1 + r_m^2\right) m_1 = E_1 \{I(\text{repaid}) \left(1 + r_m^n\right) m_1\} + E_1 \left\{(1 - I(\text{repaid})) \left(\max\{z_2 w_1 n^R - \kappa, 0\} + \zeta h_1^B\right)\right\}
\]
The expected return on the corporate loan is simply the interest rate times the borrowed amount:

\[ E_1 \left( 1 + r_2^l \right) l_1 = \left( 1 + r_1^l \right) l_1 \]

It can be showed that in equilibrium required returns on both mortgage credit and corporate credit should account for both the bank funding costs \( r_1 \):

\[ E_1 \tilde{r}_m^m = r_1, \quad r_1^l = r_1 \]  \hspace{1cm} (2)

### 3.4 Market clearing

In equilibrium total savings in the economy have to equal total demand for loans:

\[ m_1 + l_1 = d_1 \]

The housing market clears at the price \( q \) such that when

\[ h_1^B + h_1^S = h \]

The rest two conditions ensure labor market clearing:

\[ n_1 = n \]

\[ n = n^B + n^S \]

### 4 Results

Results are obtained by calibrating the model and solving it with backward induction. Most of the parameters are standard and reported in the appendix, section A. There I also describe the chosen shock values and the associated probabilities. Robustness of the results with respect to key parameters is explored in the next section.

This section explores the effects of increasing the level of personal bankruptcy protection on the credit allocation. The section is divided into two subsections to describe the credit allocation effects first and continue with the description of welfare effects. Initially the credit allocation is discussed while assuming that the economy experiences housing value shocks only so that the general equilibrium effects could be presented without delving into consumption insurance or the non-linearities in the mortgage credit response.
After, I include earnings risk as well and exhibit the resulting differences in the credit allocation. Welfare outcomes are discussed in the second subsection.

An exogenous earnings exemption level ($\kappa$) proxies for the level of personal bankruptcy protection. The bank can seize earnings which are above the exemption level $\kappa$, i.e. $\max \{ w_1 n^B - \kappa, 0 \}$. Hence, higher $\kappa$ implies a higher level of personal bankruptcy protection.

### 4.1 Effect on the credit allocation

The resulting credit allocation depends on the first-order effects and the second-order effects. An increase in the level of personal bankruptcy protection (higher $\kappa$) directly translates into an additional expected loss to the bank because the bank can claim less of insolvent borrower’s earnings. This would create a negative first-order supply effect on mortgage credit. Another first-order effect, the demand effect of higher personal bankruptcy protection, is positive because the individual is insured from bad states better thus would like to borrow more. The equilibrium outcome in the mortgage credit market will depend on which of the two effects will dominate.

Changes in mortgage credit will create the second-order effect on corporate loans and thus output. This is what I would call the portfolio effect: higher (lower) mortgage credit would crowd out (crowd in) corporate credit.

#### 4.1.1 Housing value shocks

This section shows that in the model with housing value shocks the demand effect of personal bankruptcy protection can be strong enough to result in higher mortgage credit. However, higher mortgages do not have to crowd-out capital if rising housing prices increase savers’ income sufficiently. This is the general equilibrium effect that I explain in detail below.

Figure 1 plots the values of macroeconomic variables across different levels of earnings exemptions $\kappa$. The maximum $\kappa$ considered here is 0.6 because wages given baseline calibration are always below 0.6 and increasing the level further than that do not change the model dynamics. The figure plots default rates (total $\delta$ and non-strategic $\delta^n$), the mortgage interest rate spread $r^m_1 - r_1$, mortgage volume $m_1$, borrower’s housing stock $h^B_1$, housing price $q$ and capital $k_1$.

When the level of personal bankruptcy protection becomes high enough, the household finds it beneficial to default in bad states. Figure 1 shows how the default rate $\delta$ in period 2 turns positive for $\kappa > 0.57$ which is almost 98 percent of borrower’s wages. Below this value, the household always repays because the earnings level is insufficiently generous and bankruptcy is too costly. Aggregate variables are inactive as
Figure 1: Mortgage total default rate, non-strategic default rate, the mortgage spread, mortgages, housing, housing price and capital stock.

Note: The first row plots total mortgage default rate $\delta$ and non-strategic mortgage default rate $\delta^n$ (left panel) and the mortgage spread (right panel). Both are expressed in percentage points. The mortgage spread is defined as a difference between the mortgage rate and the risk-free rate. The second row plots mortgages $m_1$ and borrower’s housing $h^B_1$. The third row plots housing price $q$ (relative to the consumption price) and capital $k_1$. The horizontal axis measures different levels of earnings exemptions $\kappa$.

long as default risk is zero because earnings exemptions can affect aggregate variables only through the default risk.

All defaults are strategic defaults ($\delta^n = 0$). Housing value shocks do not affect household’s wage income and given this calibration the household could repay out of her wage income and still enjoy positive consumption even in cases with the least valuable housing stock. The highest default risk corresponds to the maximum level of $\kappa$ but it does not increase in $\kappa$ monotonically because the number of housing shock realizations is only two and the household defaults only for the lower value of the housing value shock. This why the maximum default rate is equal to the probability of the lower value of the housing shock ($p_L^c=0.05$). Banks respond to higher default risk and thus higher expected losses by lending less. Tightening borrowing conditions in response to higher default risk are evident from the increasing interest rate spread for mortgages. The spread is defined as the difference between the mortgage rate and the risk-free rate, ($r_m^1 - r_1$).

In equilibrium, the demand effect turns out to be the dominant force in the dynamics of mortgages. Higher interest rates do not prevent the equilibrium mortgage credit from increasing in the level of personal
bankruptcy protection and mortgages increase in volume for the right end of the chosen interval for $\kappa$, see Figure 1. The rise in mortgage credit starts at the level of earnings exemption at which default risk becomes positive ($\kappa > 0.57$), so mortgage credit starts increasing when the household actually makes use of the exemption in bad states. The earnings exemption level $\kappa > 0.57$ implies that more than 80 percent of earnings are shielded from the lender (in this calibration equilibrium $w_{1n}^n$ is always below 0.6) in case of bankruptcy. Thus the household chooses to default on her mortgage only under very generous personal bankruptcy protection.

With rising mortgage credit, the borrowing household affords to purchase more housing stock. Figure 1 shows how borrower's housing purchases approximately follow the mortgage dynamics. The housing price increases to accommodate higher borrower's housing demand and clear the housing market.

The increasing generosity of personal bankruptcy rules affects not only the mortgage market equilibrium but the credit allocation as well. However, it turns out that although the increasing level of personal bankruptcy protection increases mortgage credit, capital loans are not crowded out. I find the opposite to the conventional portfolio effect. Figure 1 shows how capital increases in the level of personal bankruptcy protections for $\kappa > 0.57$.

The portfolio allocation effect is resumed if the housing market does not clear. Figure 2 compares the values of mortgages, capital, housing and deposits to the values they would take if the housing price was fixed. The remaining of the subsection discusses why given a fixed housing price higher mortgage credit leaves fewer funds to corporate lending and thus capital and why this is not the case with the endogenous housing price.

If the housing market clears, higher borrower's demand for housing results in a higher housing price because the saving household sells more housing to the borrower and keeps less housing stock to himself. Figure 2) shows how housing stock at the savers declines in $\kappa$. Higher housing price affects saver's wealth positively and allows the saver to increase savings in order to smooth his consumption over time. Deposits rise, as depicted Figure 2. Consequently, the bank does not have to shift from corporate lending due to limited funds but can actually increase lending to all borrowers, both households and firms. Therefore, if the housing market clears, this gives rise to the general equilibrium effect which, if strong enough, dominates the portfolio allocation effect.

If the housing price was fixed, saver would not sell more housing to the borrower and his wealth would remain unchanged. Deposits supply remains constant across exemption levels, see Figure 2. Then the traditional portfolio effect would stay: capital loans would have to decrease in response to increasing mortgage credit.

The relevance of this result depends on how strong the income effect could be. If borrowers constitute
a small share of total housing demand then their ability to influence housing prices would be limited. With housing production, the housing supply would adjust to satisfy higher housing demand dampening the effect on prices too. Both of these characteristics could be explored in a more detailed model.

4.1.2 Both housing value and earnings shocks

Adding uncertainty to household wage earnings makes mortgages more risky. Consequently, the borrowing household who receives a low earnings shock may choose to default at housing value states that did not trigger default in the setup without earnings risk. In this section I show that introducing more uncertainty does not extinguish the income effect and rising mortgages can prevail with higher capital as before. Differently from the setup with housing value shocks only, earnings shocks increase the non-strategic default rate. Earnings shocks combined with housing value shocks can reduce household’s income significantly and repaying the mortgage now can result in negative consumption. Higher rate of forced defaults underscores the role of personal bankruptcy protection in providing consumption insurance. The increased importance of consumption insurance provided by earnings exemptions results in even higher demand effect and the rise in mortgage credit is higher than if wage income is risk-free.
Figure 3: Mortgage total default rate, non-strategic default rate, the mortgage spread, mortgages, housing, housing price and capital stock.

Note: The first row plots total mortgage default rate $\delta$ and non-strategic mortgage default rate $\delta^n$ (left panel) and the mortgage spread (right panel). Both are expressed in percentage points. The mortgage spread is defined as a difference between the mortgage rate and the risk-free rate. The second row plots mortgages $m_1$ and borrower’s housing $h^B_1$. The third row plots housing price $q$ (relative to the consumption price) and capital $k_1$. The horizontal axis measures different levels of earnings exemptions $\kappa$.

Figure 3 plots the values of macroeconomic variables in the economy with earnings risk across different levels of earnings exemptions $\kappa$. The figure plots default rates (total $\delta$ and non-strategic $\delta^n$), the mortgage interest rate spread $r^m_1 - r_1$, mortgage volume $m_1$, housing $h^B_1$, housing price $q$ and capital $k_1$.

Introducing earnings shocks restricts household’s ability to repay in bad states and makes household default more frequent in economies with low levels of personal bankruptcy protection. The increased total default risk is presented in Figure 3. As previously, default risk increases in the earnings exemption level $\kappa$, and the mortgage rate spread follows it, but different exemption levels now corresponds to higher default risk than without earnings shocks. For a comparison, $\kappa = 0.2$ corresponded to zero default risk in the previous setup but given the presence of earnings uncertainty there is already 2 percent default risk at $\kappa = 0.2$. The right panel shows that the borrowing costs (the mortgage spread) start rising for lower exemption levels as compared to the previous setup: previously it became positive only for $\kappa > 0.57$ and now it exceeds zero when $\kappa > 0.17$.

Despite the clear tendency of the increasingly more expensive mortgage credit, for lower values of earn-
ings exemptions demand for mortgages still dominates the supply effect. In Figure 3 equilibrium mortgage credit rises in the level of personal bankruptcy protection for the first half of the κ interval. The jump in the equilibrium mortgage credit corresponds to larger than zero default risk, i.e. when the indebted household actually uses consumption insurance in bad states. Higher risk significantly increases demand for mortgages, because this is the only tool to insure against low consumption in bad states. In the economy with risky earnings mortgages rise by almost 100 percent, up from the increase of 11 percent in the setup with housing shocks only (Figures 1 and 3).

For higher values of earnings exemptions (κ > 0.23) mortgage credit remains almost stable and even declines a bit suggesting that the supply effect almost counteracts higher demand for mortgages when κ is sufficiently high. Under the chosen parametrization, mortgages decline by 0.2 percent when κ increases from 0.23 to 0.46 and remain at that level. Ignoring the dynamics in the mid-interval and comparing the zero level of earnings exemptions with the maximum level of earnings exemptions would lead to the wrong conclusion that increasing the level of personal bankruptcy protection always results in the dominating demand effect. The non-linear responses of mortgages to the earnings exemptions level thus prove to be important for the results.

The increase in borrower’s demand for housing would be possibly lower if the borrowing household had access to unsecured credit. Unsecured credit would help her insure consumption and secured credit would partially lose its importance as the only way to insure consumption against bad states.

The results on credit allocation from the previous section prevail in the setup with risk earnings too. I use Figure 4 to show that the credit allocation again crucially depends on the strength of general equilibrium effects. Figure 4 compares the values of mortgages, capital, housing and deposits to the values they would take if the housing price was fixed.

The conventional portfolio allocation effect is again overshadowed by suddenly higher saver’s income and his decision to smooth consumption by saving more through deposits. Then, although mortgages sharply increase in the first half of the earnings exemption interval, capital is not crowded out. If the housing price clears the housing market, higher saver’s wealth increases deposits and allows the bank to increase lending to all borrowers, both households and firms. Fixed housing prices crowd-out capital because housing prices do not affect the saver’s wealth and his savings remain stable across the interval for earnings exemption values. Then the bank has to shift from corporate loans to mortgage credit in order to satisfy its resource constraint.

Notably, the general equilibrium effect is the strongest for very low levels of protection, therefore, if the starting level of personal bankruptcy protection is already high (in my setup that is κ > 0.23), the effect on prices would be limited. For sufficiently high levels of earnings exemptions, the conventional
portfolio effect is present suggesting that the effect on income is too small to offset direct effects on the bank portfolio composition. I observe that for high values of earnings exemptions ($\kappa > 0.23$), a marginal increase in earnings exemptions leads to a marginal decrease in mortgages combined with an increase in capital by 0.6 percent. This finding is qualitatively in line with the presented US data in Table 1.

4.2 Welfare effects

In this section I show that increasing the level of earnings exemptions can be welfare increasing, particularly due to the associated general equilibrium effect in housing prices. Further, the section discusses how personal bankruptcy protection affects main contributors to higher welfare, namely higher consumption and higher consumption insurance.

The borrowing household benefits from higher personal bankruptcy protection because of better consumption insurance and higher consumption opportunities. Consumption insurance is proxied here with a variance of consumption in period 2 where lower variance relative to income implies higher insurance. Personal bankruptcy protection has a positive effect on consumption insurance for two reasons. Firstly, a positive exemption level protects borrower's consumption in bankruptcy. If the household repays, her
consumption is lower due to higher interest rates associated with higher earnings exemptions. Higher consumption in bad states and lower consumption in good states contribute to lower variance of consumption and thus better insurance.

Personal bankruptcy protection also leads to higher equilibrium consumption. The general equilibrium effect into housing prices boosts savers’ wealth and in equilibrium leads to higher savings and higher equilibrium production. The saving household does not consume the whole increase in income but saves too because of losses associated with the potential bankruptcy his consumption in period 2 is lower on expectation. Although higher consumption contributes to higher welfare, some of the welfare gains may be offset because if consumption rises unevenly across states, for instance, if consumption in good states increases more. In the further discussion, the general equilibrium effect that drives higher consumption is disentangled to understand welfare outcomes better.

Figure 5 presents evidence of higher consumption and better consumption insurance for the borrower. To separate the general equilibrium effect, I again distinguish between the values of variables under the endogenous housing price and the values if the fixed housing price was fixed. The first row presents variables for the economy with housing value shocks only. Expected borrower’s consumption in period 2 $E_{2}^{B_{h}}$ (left panel) and the variance of borrower’s consumption in period 2 $\text{var}(c_{2}^{B_{h}})$ (right panel) are compared with their counterparts if the housing price is fixed, $E_{2}^{B_{q}}$ and $\text{var}(c_{2}^{B_{q}})$. The second row provides similar comparison but for the economy with both housing value shocks and earnings risk. It plots expected borrower’s consumption in period 2 $E_{2}^{B}$ together with consumption when the housing price is fixed, $E_{2}^{B_{q}}$ (left panel). The variance of borrower’s consumption in period 2 $\text{var}(c_{2}^{B})$ is plotted together with variance when the housing price is fixed, $\text{var}(c_{2}^{B_{q}})$, in the right panel.

The left panel in the first row demonstrates that under housing value shocks the expected borrower’s consumption $E_{2}^{B_{h}}$ in period 2 rises. The rise can be attributed to a higher housing price and higher wages. The variance of consumption declines for higher values of $\kappa$, exactly when the household default becomes an equilibrium outcome. Therefore, as expected higher exemptions provide better consumption insurance. If the general equilibrium effect is silenced by assuming a fixed housing price, expected borrower’s consumption in period 2 does not increase suggesting that all equilibrium gains in consumption are entirely through resulting higher savings and higher capital. Switching off the general equilibrium effect, however, does not affect consumption insurance as showed in the right panel.

When earnings are risky, allowing for a higher the level of exemptions boosts the borrower’s expected consumption in period 2 $E_{2}^{B}$ (the second row, left panel). The rise again can be entirely attributed to the general equilibrium effect into housing prices because if the housing price is fixed, consumption even declines, see $E_{2}^{B_{h}}$. When the housing price increases to clear the housing market, the rise in expected

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Figure 5: Borrower’s consumption in period 2 and variance of consumption with housing value shocks and with both housing value shocks and earnings shocks.

Note: The first row plots variables in the economy with housing shocks only. The left panel in the first row plots expected borrower’s consumption in period 2 $E_c^{ Bh}$ and expected consumption in period 2 $E_c^{ Bhq}$ when the housing price is fixed. The right panel provides the variance of borrower’s consumption in period 2 when the housing market clears, $\text{var}(c^{ Bh}_2)$, and the one with the fixed housing price, $\text{var}(c^{ Bhq}_2)$. The second row provides the same plots but for the economy with both housing value shocks and risky earnings, that’s why the superscript $h$ disappears. Again in each panel the variables are compared to the ones that result if the housing price was fixed. The horizontal axis measures different levels of earnings exemptions $\kappa$.

Consumption is even sharper and occurs already for lower levels of exemptions than with housing shocks only. The reason is that when earnings are risky, equilibrium default occurs for lower $\kappa$ and thus the general equilibrium effect is triggered for lower $\kappa$ than with housing value shocks only. This raises equilibrium income and in turn equilibrium consumption for lower $\kappa$ too.

The variance of consumption, however, either does not decline in the level of exemptions as demonstrated by the series $\text{var}(c^q_2)$. However, if the general equilibrium effect is silenced by imposing the fixed housing price, the variance of consumption in the economy with risky earnings decreases. This is demonstrated by the dynamics of $\text{var}(c^{ Bhq}_2)$. The result is in line with the idea that personal bankruptcy protection increases consumption in bad states and decreases consumption in good states. The general equilibrium effect mitigates this result by increasing consumption in goods states. Higher interest rate under higher level of exemptions would decrease consumption in good states (states when the borrower repays), however, the general equilibrium effect raises income to such a level that consumption rises even in good states despite higher borrowing costs. The general equilibrium effect does not necessarily increase consumption in bad states as well because in some cases the borrower’s consumption in bankruptcy is given by the exemption.
level \( \kappa \) and thus the the general equilibrium effect into prices does not have an even effect on consumption across states. Higher consumption in good states rather than in all states inflates the variance of consumption. Therefore, although a higher earnings exemption level decreases the variance of consumption for a given level of consumption, increasing the level of consumption increases the resulting variance. The case with housing value shocks is an exception in this sense, because, as we saw, the general equilibrium effect did not matter for the variance. There are two reasons for that. First, the general equilibrium is especially strong with risky earnings and in other cases it does not raise consumption enough to inflate the variance of consumption. Second, in the economy with risk-free earnings household default occurs for very high levels of exemptions only and in all cases the borrower's earnings fall below the exemption level (i.e. \( w^B n < \kappa \)). This keeps the one-to-one mapping between household's consumption and wage and thus the general equilibrium effect increases borrower's consumption in all states without affecting the variance of consumption. When earnings are risky, the borrower defaults for very low \( \kappa \) too and in those cases her consumption is capped by the exemption level and income excess of \( \kappa \) is garnished by the creditor. Although there are just a few instances of such cases, this is enough to mitigate a decrease in variance.

To sum up this discussion, the borrowing household benefits from higher personal bankruptcy protection in the form of higher consumption and lower variance of consumption given the level of consumption. However, if the general equilibrium effect is strong, the variance of consumption does not necessarily decline. Then welfare gains for the borrower come mainly through higher consumption rather than better consumption insurance.

The rest of the section discusses welfare outcomes in the model relating them to the results on consumption and consumption insurance. It will show that, despite more opportunities to insure consumption, personal bankruptcy protection is not necessarily welfare-increasing. In the economy with risk-free earnings, personal bankruptcy protection is welfare-increasing only if it does not result in crowding out of capital stock.

Figure 6 plots expected utility for the agents. The figure exhibits expected utility for the borrower, for the saver and total expected utility for two types of models: the model with an endogenous housing price and the model with a fixed housing price. The first row corresponds to the economy with housing value shocks only and the second row shows values computed for the economy with risky earnings and risky housing values.

With housing value shocks only, the welfare outcome depends highly on the general equilibrium effect. If the housing price is fixed and thus saver's income is not positively affected by higher housing demand, capital is crowded out by higher mortgage credit. In such an environment the expected borrower's utility does not increase in the level of personal bankruptcy protection despite better consumption insurance, as
Figure 6: Expected utility for the borrower, the saver and in total with housing value shocks and both housing value shocks and earnings shocks.

Note: The first row concerns the economy with housing value shocks. The borrower’s expected utility $E_u^{Bh}$ is compared to the borrower’s expected utility if the housing price is fixed $E_u^{Bhq}$. The analogous comparison is done for the saver ($E_u^{Sh}$ vs. $E_u^{Shq}$) and for total utility ($E_u^h$ vs. $E_u^{hq}$). The second row concerns the economy with both housing value shocks and earnings shocks. The borrower’s expected utility $E_u^B$ is compared to the borrower’s expected utility if the housing price is fixed $E_u^{Bq}$. the analogous comparison is done for the saver ($E_u^S$ vs. $E_u^{Sq}$) and for total utility ($E_u$ vs. $E_u^q$). The horizontal axis measures different levels of earnings exemptions $\kappa$.

presented in the first row of Figure 6. Higher borrowing costs offset the benefits of smaller fluctuations in income. Total expected utility approximately follows the expected borrower’s utility suggesting that the economy with housing value shocks does not benefit from personal bankruptcy protection substantially unless it raises the capital stock. Notably, when the housing price is endogenous all agents experience utility gains: higher capital increases wages and all agents afford more consumption.

In the model with risky earnings, however, the consumption insurance provided by higher personal bankruptcy protection becomes sufficiently valuable to create welfare gains even if capital is crowded out. When a fixed housing price prevents capital from rising and even crowds it out as was showed in Figure 4, increasing earnings exemptions can still create welfare. Figure 6 (second row, left panel) exhibits that the borrower’s expected utility increases in $\kappa$ when $\kappa > 0.2$ even under a fixed housing price. However, the borrower experiences welfare losses if the protection is increased from very low levels. For $\kappa \in (0, 0.2)$, the increase in earnings exemptions results in the sharp crowding out of capital and the borrower suffers from shrinking income. Total expected utility follows the dynamics of the borrower’s expected utility except that it is shifted marginally up by the saver’s utility. The saver experiences welfare gains from higher equilibrium income as well and these gains seemingly offset lower profits from banks when defaults become
Figure 7: Mortgages and capital for different relative risk aversion $\sigma_c$.

Note: The figure plots mortgages $m_1$ and capital $k_1$ in the economy with housing shocks and earnings shocks for different relative risk aversion $\sigma_c$. Mortgages and capital are measured in consumption units. The horizontal axis measures different levels of earnings exemptions $\kappa$.

more frequent. If the housing price is allowed to clear the housing market, a higher exemption level unambiguously leads to welfare gains, the reason again being a higher resulting capital stock and higher equilibrium income.

What is an optimal level of personal bankruptcy protection? An endogenous housing price brings welfare gains to all agents and expected total utility rises for the larger part of the earnings exemptions interval, as showed in Figure 6. However, when $\kappa > 0.43$ the increase in expected utility levels off and even declines suggesting that the optimum is reached in the neighborhood of $\kappa \in (0.35, 0.43)$. That is approximately between 58 and 72 percent of borrower’s wage income. However, I do not match the US mortgage market data, so this finding should be interpreted with caution.

5 Robustness checks

This section explores the robustness of results with respect to several key parameters. I choose a risk aversion parameter, the loan-to-value ratio and banker’s preferences as key characteristics that can affect the strength of the mortgage response to the level of earnings exemptions.
Note: The figure plots mortgages $m_1$ and capital $k_1$ in the economy with housing shocks and earnings shocks for different LTV ratios $\rho$. Mortgages and capital are measured in consumption units. The horizontal axis measures different levels of earnings exemptions $\kappa$.

5.1 Risk aversion

The role of personal bankruptcy protection in providing consumption insurance governs the mortgage response to the level of earnings exemptions, as discussed in the previous section. The risk aversion parameter $\sigma_c$ is varied to capture the value of consumption insurance and its importance for the results. Higher $\sigma_c$ means higher relative risk aversion of both the saver and the borrower.

Figure 7 plots mortgages and capital under different assumptions about the value of $\sigma_c$. Lower relative risk aversion indeed makes the mortgage response to the level of earnings exemptions weaker. Since under lower risk aversion the value of consumption insurance is lower, mortgage demand would be lower as well. The capital increase is also lower as the strength of the general equilibrium effects that increases capital depend is directly related to mortgage demand and equilibrium mortgage credit.

5.2 Loan-to-value ratio

The loan-to-value ratio for mortgages is exogenous and thus the non-monotonic relationship between default rates and the level of protection cannot be generated. A higher level of exemptions in this model unambiguously leads to higher default. In Hatchondo et al. (2015) the household can adjust her leverage to overcome the higher default possibility and a higher level of exemptions may even decrease the default...
risk because households would be borrowing less relative to their assets.

I vary the exogenous LTV ratio to exhibit that given the relationship between the exemption level and default rates does not reverse in my setup for realistic exogenous LTV values but becomes weaker if the LTV ratio is sufficiently low. Figure 8 plots mortgages and capital under different assumptions about the value of $\rho$. When the exogenously imposed LTV ratio becomes low enough, household default is not present anymore because the debt is low enough to be repaid. This results in non-response mortgage credit and thus stable capital across different values of $\kappa$. For higher values of LTV ratio the mortgage response to the level of exemptions is positive but weaker if $\rho$ is lower than in the baseline parametrization and thus the household leverage is lower.

### 5.3 Bank manager’s risk aversion

Finally, I investigate whether different assumptions about the banking problem affect the strength of the supply effect of personal bankruptcy protection. Allowing for the banker to be risk averse modifies the banking problem as described in the Appendix, section C.1. The key change is that the banker now takes into account not only expected returns but also the uncertainty associated with those returns. Increasing the level of personal bankruptcy protection makes bank returns on mortgages not only lower in bad states but also increases the variance of returns on mortgages relative to the case when the bank could get compensated
for (almost) all unpaid mortgage debt by garnishing wages. Under a higher variance of returns the bank lends less to the household at a given interest rate because the banker does not get compensated for the risk enough. The bank’s response would make the first-order supply effect of personal bankruptcy protection stronger.

Figure 9 plots the resulting mortgages and capital. The banker’s risk aversion parameter \( A = 0 \) corresponds to the case when the banker is risk-neutral and \( A = 50 \) implies risk aversion. Indeed the bank shrinks the credit supply to the household stronger if the bank manager is risk averse. However, this happens for higher values of earnings exemptions only, i.e. when the demand effect is already weaker. Notably, for higher values of earnings exemptions uncertainty of returns on mortgages is also stronger because the bank loses more in bad states as compared to low levels of personal bankruptcy protection when the bank can get compensated for (almost) all debt. Since decreasing mortgage credit crowds-in corporate loans for high levels of exemptions, the risk aversion assumption makes mortgages decline by more and capital increase by more. Thus, the supply effect is indeed stronger if the bank manager accounts for the uncertainty of mortgage returns.

6 Future extensions

The paper assumes that the household borrows the maximum allowed amount that is given by the exogenous loan-to-value ratio. Thus the relationship between the level of protection and mortgage default rates is monotonic in contrast to the findings in Hatchondo et al. (2015). Allowing the household to choose her leverage is an important extension to validate the strength of the demand effect of personal bankruptcy protection.

As it is showed in Mitman (2016), considering the household portfolio choice is crucial to match some key facts about bankruptcy rates and foreclosure rates in the US. Including unsecured debt would create another tool for consumption insurance and potentially lessen the demand for mortgages. Therefore, expanding the model to this direction would be necessary.

The consumption insurance provided by personal bankruptcy protection is a driver behind higher mortgage demand in the economy with earnings shocks compared to the economy with housing value shocks only. Exploring the role of the utility function for the results would be useful to strengthen the consumption insurance argument in the paper, especially, if I considered a utility function that allows to separate a risk aversion parameter and an intertemporal substitution parameter, Epstein-Zin preferences.

Employing households to produce housing goods may have a quantitative effect on my findings. Once the increase in the level of personal bankruptcy protection reduces mortgage demand, for some values of
earnings exemptions housing demand would decrease too. This would have a negative effect on wage income from working at the housing sector and act as an opposite effect to the increase in wage income received from financially constrained firms. However, given that the share of housing production in total output is usually less than half, the decrease in wage income received from the housing sector would be unlikely to offset the increase in wage income from the rest of the economy.

The model assumes exogenous rental market. The interaction between rental market and the housing market could be very enriching and help targeting the data.

7 Conclusions

I develop a general equilibrium model with mortgage default to illustrate how an increase in the level of personal bankruptcy protection can create benefits even in the case of strategic household default. If household default is forced, increasing the level of personal bankruptcy protection is welfare-increasing for most of earnings exemptions values even if it leads to crowding out of capital stock.

The welfare results depend on the general equilibrium effect which manifests in the positive feedback effect into savers’ wealth and higher savings. I explore it under two scenarios. First, I consider an economy with housing value shocks only and then the economy with risky earnings and housing value shocks. I show that with housing value shocks the general equilibrium effect determines that an increase in the level of personal bankruptcy protection does not necessarily crowd out corporate lending but can even increase it. Higher lending to financially constrained firms increases capital stock and leads to the positive feedback effect into production. This results in total output and welfare gains for both the borrower and the saver despite that household default is strategic.

Introducing earnings uncertainty does not extinguish the general equilibrium effect and rising mortgages can prevail with higher capital as before. Differently from the setup with housing value shocks only, earnings shocks increase the non-strategic default rate highlighting the role of personal bankruptcy protection in providing consumption insurance against income shocks. I show that the increased importance of consumption insurance provided by earnings exemptions results in even higher demand effect and the rise in mortgage credit is higher than if wage income is risk-free. Notably, the general equilibrium effect is the strongest for very low levels of protection and for sufficiently high values of earnings exemptions it is weak enough to result in the conventional portfolio allocation effect.

The highest welfare is achieved when the level of earnings exemptions reaches approximately between 58 and 72 percent of borrower’s earnings. Beyond this point, the capital stock levels off while borrowing costs continue increasing and reducing the value of consumption insurance.
References


A Parameters

Model parametrization is presented in Table 2. I take parameters for the utility function from Mitman (2016). Discount factors are common in the business cycles literature which distinguishes between savers and borrowers, e.g. the discussion on this in Iacoviello (2005). Endowment values, stock values are ad-hoc. The rest of the parameters are standard except for the parametrization of shocks which I describe below.

The housing value shock $\zeta$ can take two values: $\zeta \in \{\zeta_L, \zeta_H\}$. The probability of a bad state $\zeta_L$ occurring is denoted by $p_L$ and is commonly known. The number of the states is arbitrary but comes without a loss of generality. The lower value of the housing value shock is chosen low enough such that household default would sometimes occur in bad states even in the absence of earnings shocks. Corbae and Quintin (2015) set it higher (to 0.7) to model the drop in house prices in the US, however, given my model setup, this value is too high to make household default occur in the absence of earnings shocks.

The earnings shock takes five states with commonly known probabilities. One can think of it as an idiosyncratic shock that affects a continuum of households in period 2 when all households are identical in period 1. Assuming a continuum of households, the probability of a particular value of earnings shock occurring would correspond to a share of households that get hit with the earnings shock of this size.

My measure of the variance of earnings value shock comes from Singh and Stoltenberg (2017). Singh and Stoltenberg (2017) use Consumer Expenditure Interview Survey (CEX) data 1999-2003 to compute within group inequality. They follow Krueger and Perri (2006) and Blundell et al. (2008), and regress the logs of household consumption and income on a cubic function of age and a set of dummies that include region, marital status, race, education, experience, occupation and sex. The unobserved risk amounts to idiosyncratic income risk used to compute the income risk. Further they calibrate an ergodic earnings distribution for an incomplete market model by specifying permanent and transitory shocks to yield a transition matrix. I follow this methodology closely as I choose a permanent income shock with an autoregressive coefficient of 0.9899 to compute the earnings states and the corresponding probabilities. The number of the states is ad-hoc.

I assume that earnings shocks and housing value shocks are independent.
I use two grids, for savings and housing. Both have the size of 60.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
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<tbody>
<tr>
<td>Saver’s discount factor</td>
<td>( \beta_S )</td>
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<tr>
<td>Borrower’s discount factor</td>
<td>( \beta_B )</td>
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<tr>
<td>CRRA parameter</td>
<td>( \sigma )</td>
</tr>
<tr>
<td>Housing parameter in the utility function</td>
<td>( \sigma_h )</td>
</tr>
<tr>
<td>Endowment for the saver</td>
<td>( y^S )</td>
</tr>
<tr>
<td>Endowment for the borrower</td>
<td>( y^H )</td>
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<tr>
<td>Housing stock</td>
<td>( h )</td>
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<tr>
<td>Capital depreciation</td>
<td>( \delta )</td>
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<tr>
<td>Capital share of output</td>
<td>( \alpha )</td>
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<tr>
<td>Number of housing value states</td>
<td>( s^h )</td>
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<td>( (p_L^\zeta; 1 - p_L^\zeta) )</td>
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<td>Housing value shock: expected value</td>
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</tr>
<tr>
<td>Housing value shock: variance</td>
<td>( \text{var}(\zeta) )</td>
</tr>
<tr>
<td>Number of earnings states</td>
<td>( s^z )</td>
</tr>
<tr>
<td>Earnings value shock: probabilities</td>
<td>( (0.2039; 0.2000; 0.1921; 0.2000; 0.2039) )</td>
</tr>
<tr>
<td>Earnings value shock: states</td>
<td>( (0.3591; 0.5488; 0.8387; 1.2818; 1.9590) )</td>
</tr>
<tr>
<td>Earnings shock: expected value</td>
<td>( E_z )</td>
</tr>
<tr>
<td>Earnings value shock: variance</td>
<td>( \text{var}(z) )</td>
</tr>
<tr>
<td>LTV ratio</td>
<td>( \rho )</td>
</tr>
<tr>
<td>Rent</td>
<td>( h^r )</td>
</tr>
<tr>
<td>Minimum consumption level</td>
<td>( c_{\text{min}} )</td>
</tr>
</tbody>
</table>

Table 2: Baseline parameters.

**B Solution algorithm**

In the description of the algorithm we introduce uncertainty in the form of the housing quality shock \( \zeta \) and earnings shock \( z \) and allow the borrowing household to default. We employ the backward induction method.

The housing value shock occurs in period 2 and has two \( (s^h = 2) \) realization values \( \{\zeta_L^h, \zeta_H^h\} \). The low value occurs with a probability \( p_L^\zeta \). The productivity shock occurs in period 2 as well and has five \( (s^z = 5) \) realization values with commonly known probabilities. Parameters that concern the solution algorithm are presented in Table 2.

1. Generate two grids: one for savings and one for housing. The housing grid size is \( g^h \) and the savings grid size is \( g^d \).
2. Take \( y^H \), \( y^S \) and \( h \) as given.
3. Make a guess for a risk-free interest rate \( r_1 \), housing price \( q \), a mortgage interest rate \( r_1^m \) and a
corporate interest rate \( r_1 \). Since firms do not default, \( r_1 \) always will be set equal to \( r_1 \).

4. \( w_1 \) and \( k_1/n_2 \) follow.

5. Compute wages \( w_1 \) and capital stock \( k_1 \) by using the firm's first order conditions (equations (3.2)-(3.2)).

6. Express the saver's value function in period 2 \( V_{2S} \) as a function of instantaneous utility in period 2, given the realizations of shocks (\( s^h \) \( \cdot \) \( s^z \) possible states) and all possible combinations of savings values \( d_1 \) and housing values \( h_1^S \) from the respective grids. This yields \( V_{2S} \left( d_1, h_1^S; \zeta, z \right) \).

7. Express the borrower's value functions in period 2 \( V_{2BP} \) and \( V_{2BD} \) as a function of instantaneous utility in period 2, given the realizations of shocks (\( s^h \) \( \cdot \) \( s^z \) possible states) and all possible housing values \( h_1^B \) from the grid for housing. To impute mortgages make use of \( m_1 = \rho \zeta h_1 / (1 + r_{m1}) \). This yields \( V_{2BP} \left( h_1^B; \zeta, z \right) \) and \( V_{2BD} \left( h_1^B; \zeta, z \right) \).

8. Determine the optimal default choice for the borrowing household by computing \( V_{2BP} \left( h_1^B; \zeta, z \right) = \max \left\{ V_{2BP} \left( h_1^B; \zeta, z \right), V_{2BD} \left( h_1^B; \zeta, z \right) \right\} \). This yields decision rules for every possible \( h_1^B \) the agent brings to period 2, given the materialization of shocks in period 2.

9. Express \( V_{1S} \) as a function of instantaneous utility in period 1 for every combination of savings \( s_1 \) and housing \( h_1^S \) and the respective value in period 2. To find the corresponding optimal value in period 2, the realizations of \( V_{2S} \left( d_1, \zeta, z \right) \) have to be not only discounted but also weighted with respective probabilities of each shock occurring. Do an analogical exercise for \( V_{1B} \).

10. Compute the value of the saver in period 1 \( V_{1S} \) for every possible combination of \( d_1 \) and \( h_1^S \). Compute the value of the borrower in period 1 \( V_{1B} \) for every possible \( h_1^B \).

11. Find the maximum value of \( V_{1S} \left( d_1, h_1^S \right) \) and \( V_{1B} \left( h_1^B \right) \). This yields optimal savings \( d_1 \) and optimal housing choices \( h_1^S \) and \( h_1^B \). Given these variables, mortgages can be inputed and borrower's default decisions can be determined for all possible combinations of shocks in period 2.

12. Compute expected returns on the mortgage \( \tilde{r}_2 \) by weighting returns under different combinations of shock with the respective probabilities. variance of returns can also be computed accordingly.

13. Check if \( \tilde{r}_2 - r_1 \) is different from zero.

14. If yes, change \( r_{m1} \) and repeat steps 7-13 until \( \tilde{r}_2 \approx r_1 \) under the set tolerance level \( (fminsearch) \).

15. Check if \( h_1^S + h_1^B - h \) is different from zero.

16. If yes, change \( q \) and repeat steps 6-15 until the housing market clears: \( h_1^S + h_1^B \approx h \) under the set tolerance level \( (fminsearch) \).

17. Check if \( k_1 + m_1 - d_1 \) is different from zero.
18. If yes, change \( r_1 \) and repeat steps 3-17 until the deviation from the savings market clearing condition disappears (fminsearch).

C Model extensions

C.1 Banking sector with a mean-variance utility function for the bank manager

A risk neutral banking sector in the baseline model is extended to account for the risk aversion of a bank manager. For simplicity, it is done by assuming the mean-variance utility for the manager. In this extension a representative competitive bank is operated by the risk averse manager and owned by the saving household. I assume that in the beginning of every period the realized profits/losses from the previous period are transferred to the saving household which allows me to abstract from the dynamic properties of the bank problem and focus on the credit allocation in the current period only. The uncertainty of returns associated with different assets on their balance sheet plays a role in this problem, because the assumed risk-aversion affects the bank manager decision how much to lend to the borrowing household and how much to the firm, i.e. the portfolio choice. Thus interest rates on different types of credit are set not only to make the bank break even in expectation, but also to account for the relative risk the bank manager is taking given the return on the mortgage and the return on the corporate loan. Thus the bank would ask a higher interest rate on mortgages to compensate for otherwise lower expected returns compared to the net deposit rate but also to compensate for the uncertainty.

In period 1 the bank manager expects a net return \( \tilde{r}_m^2 \) on the mortgage and a return \( \tilde{r}_l^2 \) on the loan, where the indicator \( I(\text{repaid}) \) denotes the borrower’s (either household’s or firm’s) decision to repay debt. The expected return on the mortgage reflects that in case of default the bank will seize the non-exempt earnings together with housing:

\[
E_1 (1 + \tilde{r}_m^2) m_1 = E_1 \{ I(\text{repaid}) (1 + r_1^m) m_1 \} + E_1 \left\{ (1 - I(\text{repaid})) \left( \zeta h_1^B + \max \left\{ zw_2 n^B - \kappa, 0 \right\} \right) \right\}
\]

The expected return on the corporate loan reflects that the firm always repays:

\[
E_1 \left( 1 + \tilde{r}_l^2 \right) l_1 = \left( 1 + r_1^l \right) l_1
\]

To determine the optimal asset allocation, I assume that the bank manager discounts expected returns with the scaled variance of the returns. The associated risk is by assumption scaled by a parameter \( A \) which
reflects the degree of bank manager’s risk aversion and can be varied to capture different levels of risk aversion. I assume a utility function for the bank manager that takes this into account and use it to solve the bank profit maximization problem:

$$\max_{m_1, l_1, d_1} E_1 \tilde{r}_m m_1 + E_1 \tilde{r}_l l_1 - r_1 d_1 - \frac{A}{2} \text{var} \left( E_1 \tilde{r}_m m_1 + E_1 \tilde{r}_l l_1 - r_1 d_1 \right)$$

s.t.

$$m_1 + l_1 = d_1$$

Note that the stochastic properties of the housing shock and the productivity shock (more precisely, independence of the two shocks) imply that $\text{cov}(\tilde{r}_2, \tilde{r}_2^m) = 0$. Given this I can simplify the problem:

$$\max_{m_1, l_1} E_1 \tilde{r}_m m_1 + E_1 \tilde{r}_l l_1 - r_1 (m_1 + l_1) - \frac{A}{2} \left( \text{var} (\tilde{r}_2^m) m_1^2 - \text{var} (\tilde{r}_2^l) l_1^2 \right)$$

It follows that in equilibrium returns on credit should account for both the bank funding costs $r_1$ and the associated risk:

$$E_1 \tilde{r}_m = r_1 + A \cdot \text{var} (\tilde{r}_2^m) m_1$$

$$E_1 \tilde{r}_l = r_1 + A \cdot \text{var} (\tilde{r}_2^l) l_1$$

The derived equilibrium conditions show that, the expected variance of returns increases, credit conditions would tighten for that particular type of credit proportionally to the risk aversion parameter $A$. When $A=0$, the bank profit maximization problem features that of a risk-neutral perfectly competitive banking sector.