The Fisher equation reconsidered

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October 2017

No: 37
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October 6, 2017

\textsuperscript{1}We want to thank David Arseneau, Pierpaolo Beningo, Christoffer Koch, Nikolaos Kokonas, David Rappoport, Anna Sokolova, Dimitrios Tsomocos, Alexandros Vardoulakis, Kieran Walsh and Xuan Wang and participants of the 6th LFE/ICEF Conference and a seminar at the Board of Governors of the Federal Reserve System that have improved our paper. Peiris’ work on this study has been supported by the Russian Academic Excellence Project ‘5-100’. First Version: October, 2016

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Abstract

According to Alvarez et al. (2001) “Interest rates and inflation,” there exists systematic evidence that increases in average rates of money growth are associated with equal increases in average inflation rates and in interest rates; which conforms to the quantity theory or, alternatively, the Fisher equation. And this is a puzzle, as there is a consensus that inflation can be reduced by increasing short term interest rates.

Here, we develop an argument to resolve the conundrum. The Central Bank sets the interbank rate, that we identify with the short term rate; commercial banks trade in the interbank market to accommodate flows in balances. The interbank rate, thus, poses a wedge between the return to deposits and the cost of borrowing; which accounts for a negative relation between the short term rate and inflation or, alternatively, between the short term rate and economic activity with no contradiction either with the quantity theory or the Fisher equation.
1 Introduction

Modern monetary policy influences the realised rate of inflation through adjusting interest rates by changing the quantity of reserves. One approach in economic theory has shown that this channel is feasible if there are sufficient nominal rigidities; the New Keynesian model and the associated Taylor Rule.\footnote{The traditional Keynesian mechanism by which Monetary Policy operates, the “Interest Rate Channel”, is described by a contraction in money supply resulting, in the presence of sufficient price stickiness, in a rise in real (policy and market) rates, raising the cost of capital and thereby contracting investment and output. This mechanism is also at the heart of the “New Keynesian” models of Rotemberg and Woodford (1998) and Clarida et al. (1999), among others.} In the absence of nominal rigidities it is more difficult to obtain such a channel. Nominal interest rates depend, roughly speaking, on expected inflation which in turn depends on the expected growth rates in reserves. Current expansions or contractions in reserves can only affect nominal rates of interest through affecting the real interest rates. Alvarez et al. (2001) show that heterogeneity among households, specifically distinguishing between households that access bond markets and those that do not, allow monetary policy to affect the real interest rate, thereby altering both nominal interest rates and realised inflation rates. Though appealing, this approach does not prescribe a role for interest rates \textit{per se}. They matter only because realised inflation affects the distribution of real wealth among households.\footnote{Literature along the lines of Dubey and Geanakoplos (2003) and Tsomocos (2003) disentangle the liquidity effects of monetary policy from the inflation effects by segmenting markets as being either intra-period or inter-period. In these models, monetary expansions affect intra-period liquidity and reduce policy rates, independent of expected money growth or inflation. However the channel through which this rate operates is by affecting the real value of net sales and thereby the wedge between the effective sale and purchase prices and the efficiency of spot market transactions.}

In this paper we show that the monetary policy transmission channel can be decomposed into one that is driven purely by interest rates and one that is driven by wealth effects. Our model provides a transmission channel for monetary policy absent wealth effects. In nominal terms, our results are similar to Alvarez et al. (2001). Our central result is that monetary policy has real effects because it determines the wedge between borrowing and lending in the private sector. This wedge then affects inter-temporal allocations even when it is wealth neutral. As a consequence the path of inflation is not reflected in the levels of policy rates but their rate of change. Put another way, in a multi-period version of our argument, allocations are determined by the path of inflation and the path of policy rates. This is in contrast to workhorse neoclassical models such as Lucas and Stokey (1987) where...
the path of inflation is directly reflected in ‘the’ interest rate via the Fisher equation. In our framework policy rates determine the spread between real private sector loan and deposit rates while the rate of inflation determines their nominal value through the Fisher equation.

Our results highlight the distinction between inside and outside money. Inside money are private sector (commercial) bank liabilities arising from household deposits and are balanced by commercial bank assets that correspond to household loans. At the aggregate level an imbalance between liabilities and assets generates a demand for outside money, or Central Bank reserves, as would occur in the event of seigniorage transfers or lump sum money-financed fiscal transfers. The classic cash in advance framework of Lucas and Stokey (1987) and McMahon et al. (2015), among others, describes this. Additionally, demand for outside money may arise if commercial banks are disaggregated and heterogeneous. Payments between banks need to be intermediated by a specie that both parties have confidence in. In a world where contracts may not be enforceable, commercial bank liabilities will be dominated as a specie by central bank reserves. In our model we describe a disaggregated banking system which transfers a proportion of deposit liabilities between banks and needs to be intermediated by reserves obtained at cost from the central bank. That only deposits are transferred implies an additional cost relative to loans. As a result, deposits command a lower rate of return than loans. The wedge between the rate on loans and deposits is then related to the cost of obtaining reserves. Importantly, this cost generates a pure price effect, driving a wedge between the feasible allocations of borrowers and lenders. We show that, in contrast to Alvarez et al. (2001), this wedge is independent any wealth effects that monetary policy generates through inflation fluctuations.

The textbook view of intermediation states that deposits pre-exist loans and are therefore outside money. The aggregate volume of loans is then generated through money circulation increasing the volume of deposits. This view can be termed the “Intermediation of Loanable Funds” (ILF) model of banking. The alternate view, termed the “Financing Through Money Creation” (FMC) model, states that loans can pre-exist deposits has existed since the time of Macleod (1856) and Wicksell (1906). Much of modern banking centres around the FMC model. According to the FMC view, outside money is still needed as a means of settling payments between banks and for facilitating the demand for physical currency. The distinction between

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3The distributional effects of monetary policy are emphasised in Auclert (2017), though we obtain this channel through directly affecting the real borrowing and lending rates in the economy.

4For an overview of the use in practice see McLeay and Thomas (2014).
the models can be summarised as being whether outside money (reserves) determine deposits (the ILF view) or whether deposits drive the quantity of reserves (the FMC view). Our paper is closely related to Jakab and Kumhof (2015) and Kumhof et al. (2016) however in the later paper reserves (outside money) enters implicitly through bank capital and they focus on financial stability issues. In our framework inside and outside money co-exist but are not perfect substitutes: outside money is needed to settle transactions across banks.

The interbank market, or the federal funds market in the US, is the primary mechanism by which banks settle transactions, satisfy reserve requirements and manage liquidity. Although the market is an ‘over the counter’ one, Central Banks set targets for it and attempt to achieve it by supplying reserve balances throughout the day. Our focus on interbank payments is in a similar spirit to Poole (1968), though we analyse the general equilibrium effects of bank payments in outside money. In a similar spirit Bianchi and Bigio (2014) present a rich framework studying the interaction of outside money and the creation of inside money while Piazzesi and Schneider (2017) examine an economy where inside money is used by households but outside money is used for interbank payments and focus on the implications of the supply of each for asset prices; we focus on the role that central bank policy rates play in determining real economic activity through private sector deposit and loan rates. Afonso and Lagos (2015) examines the market within a search and matching framework where banks bargain over both the volume and return on the loans. In our framework we assume that the interbank rate is set exactly at the target level, or equivalently, that the reserves supplied by the central bank are at the desired level. This greatly simplifies the economic environment and allows us to focus on the pass through effects of monetary policy.

There has been increased interest in the role of inside money from the viewpoint of financial stability and macro-prudential policy. The focus of these papers is that, in addition to government liabilities (outside money) backed by fiscal policy, the banking system creates inside money backed by risky investments. Economies with only outside-money provide inefficient levels of liquidity, while the introduction of inside money introduces risk.\(^\text{7}\) We

\(^5\)See Goodhart (2017) for a discussion of the theories of money demand, particularly with respect to the view presented in this paper.

\(^6\)See Ashcraft and Duffie (2007) and Afonso and Lagos (2015) for a description of the over the counter nature of the fed funds market and references to related literature.

\(^7\)See Sargent (2011) and Gorton (2016) for a historical perspective. Important recent papers in this literature include Caballero (2006), Brunnermeier and Sannikov (2016) and Benigno and Robatto (2016).
abstract from these important considerations and focus on the transmission mechanism for monetary policy in the presence of inside money.

Our results are related to the “net interest margin” (NIM) that banks charge. This is defined as the difference between interest income on assets less interest expense on liabilities divided by total interest-earning assets. The common view is that NIMs and bank profitability move in tandem with key policy rates; a view that implicitly assumes that assets are more sensitive to policy rates. However, as liabilities tend to have a shorter maturity, there are reasons to believe that liabilities are also sensitive to changes in policy rates. The evidence concerning the relationship between the NIM and policy rates is mixed (Ennis et al. (2016)). English (2002) finds that the assets are more sensitive than liabilities to long term rates across 10 industrialised countries while in the UK Alessandri and Nelson (2015) show the existence of a positive long run relationship between bank profitability and the term structure of interest rates while there is evidence that loan rates move with policy rates.

In our model bank profits are zero when money financing costs are included. Nevertheless, the spread between real deposit and loan rates move with the key policy rate. We show that real loan rates move together with nominal policy rates while real deposit rates move in the opposite direction. Empirically, recent evidence finds that in the US deposit rates are downwards-flexible and upwards-sticky (Driscoll and Judson (2013)).

We commence our analysis in Section 2 where we outline the structure of the economic environment and timing of events and then present the model and results. In the Appendix we show a simplified version of Alvarez et al. (2001) and Dubey and Geanakoplos (2003) to highlight the key difference with our framework.

2 Model

The economy is inhabited by two types of households, borrowers and lenders. Households do not lend directly to each other but via a banking-financial system. The banking-financial system accepts deposits and supplies loans elastically at competitive rates. There is a monetary-fiscal authority that provides (or accepts) liquidity to (or from) the banking system and transfers profits to the borrower households.

There are two types of commercial banks, Red and Blue, and at the beginning of date 0, half of each of the two types of households belong to either of the bank and are assigned the colour of their bank. Households transact in the financial and product markets with banks and households
of the same colour. At midday of date 0, each household is reassigned a bank/colour.

Figure 1 describes the sequence of events. Each period is composed of 3 distinct sub-periods, termed Morning, Midday and Afternoon. In the Morning households sell their endowment, purchase commodities and either borrow or deposit with the banking system. The banking-financial system repays any overnight loans and can obtain liquidity from the monetary-fiscal authority.

At midday households are assigned a new type, either Red or Blue. Households that are originally Red and are assigned a new type ‘Red’ make no changes, but households originally ‘Red’ and assigned the new type ‘Blue’ switch their deposits to the Blue bank. Transferring deposits across banks requires cash (liquidity) and banks can obtain this by borrowing in the overnight loan market from the monetary-fiscal authority. In the afternoon, incoming deposits are received and saved as excess reserves and accrue an interest rate also.

Importantly our banks do not maximise profits per se but are required to have zero period by period profits. In effect one can think of them as market clearing conditions, and as they are competitive, the budget constraints of the banks allow private sector borrowing and lending rates to adjust to policy rates.

2.1 Inside-Outside Money

When a Red borrower borrows from a Red Bank \( x \) units of currency, the Red bank simultaneously creates assets and liabilities on its balance sheet of \( x \) units. The loan is an asset, while the available funds to Red Firm is a liability. The borrower uses the wealth from loans together with current period transfers to purchase commodities (from the deposit households). Deposit

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\*This restricts the demand for outside money to interbank transfers.
households give the bank the difference between their gross income and gross purchases as deposits.\footnote{The implicit cash-in-advance structure of transactions described here is no consequential to the model.}

Outside money enters through transfers to the households and the transfers of deposits between banks that need to be backed by cash. Banks cannot, and Households do not, discriminate between inside money and outside money.

2.2 Static Model

Households are denoted by superscript $b$ for borrowers and $d$ for depositors while deposits at banks are denoted with a superscript $R$ for red and $B$ for blue. We focus on symmetric equilibria, so the distinction between household colours is not consequential and we ignore it for our analysis.

2.2.1 Banking System

Deposits are denoted $d$ and total banking system deposits are $d_R + d_B = d$. Each bank receives half of total deposits. We only show the budget constraints of the Red bank as the Blue bank budget constraint is symmetric. We then show the constraints of the combined banking system.

Red Bank

In the morning, banks receive deposits from Red depositors (offering net interest $\rho_0$) and extend loans to Red borrowers (at $\rho_0 + \phi_0$). In the afternoon, a fraction of deposits switch to the Blue bank. Excess cash from the morning together with loans from the monetary-fiscal authority (at $r_0 + f_0$) then back the value of the transferred assets. In the next moment deposits transferring to the Blue bank that are transferred out, while incoming deposits from the Blue bank are received in the form of cash (outside money). The cash is deposited with the Central Bank and receives a net interest rate $r_0$.

\[
\begin{align*}
\hat{m}_0^R + \frac{1}{2} b_0 & \leq d_0^R \\
\gamma d_0^R & \leq \frac{1}{2} B_0^{CB} + \hat{m}_0^R \\
m_0^R & \leq \gamma d_0^R \\
\frac{1}{2} B_0^{CB}(1 + r_0 + f_0) + (1 + \rho_0)d_0^R & \leq \frac{1}{2}(1 + \rho_0 + \phi_0)b_0 + m_0^R(1 + r_0)
\end{align*}
\]
In the morning red banks receive deposits $d_0^R$ and extend loans $\frac{1}{2}b_0$ and keep any additional money $\hat{m}_0^R$ till midday. The second budget constraint is at midday when a proportion $\gamma$ of deposits are transferred out of the bank. These are financed by obtaining outside money from the central bank $\frac{1}{2}B_0^{CB}$. The third budget constraint shows incoming transfers in the form of reserves (outside money) $m_0^R$. The final budget constraint occurs the second period where deposits are paid a net interest of $\rho_0$ while loans pay $\rho_0 + \phi_0$. As a consequence $\phi_0$ is the spread of private sector loans over deposits. The net interest on reserves is $r_0$ while the net interest on monetary-fiscal loans is $r_0 + f_0$. To be clear $f_0$ is the premium of monetary-fiscal authority debt over reserves. The interbank rate can be considered $r_0 + f_0$ while the opportunity cost of obtaining new reserves (outside money) is $f_0$. This can be thought of as the cost of interbank transfers.

**Combined Banking System**

The budget constraints of the banking system can be combined as follows.

$$\hat{m}_0^R + \hat{m}_0^B + b_0 \leq d_0$$

$$\gamma d_0 \leq B_0^{CB} + \hat{m}_0^R + \hat{m}_0^B$$

$$m_0^R + m_0^B \leq \gamma d_0$$

$$B_0^{CB}(1 + r_0) + (1 + \rho_0)d_0 \leq (1 + \rho_0 + \phi_0)b_0 + m_0^R + m_0^B.$$  

Note that the transfers between banks do not net out as cash obtained from the monetary-fiscal authority must back the transfers. The combined budget constraint simplifies, in real terms to

$$\hat{m}_0 + \hat{b}_0 - \hat{d}_0 \leq \hat{B}_0^{CB}$$  

$$\hat{B}_0^{CB}(1 + \hat{r}_0 + \hat{f}_0) + (1 + \hat{\rho}_0)\hat{d}_0 \leq (1 + \hat{\rho}_0 + \hat{\phi}_0)b_0 + \hat{m}_0(1 + \hat{r}_0)$$  

$$\hat{m}_0 \geq \gamma \hat{d}_0$$

Date 0 variables are normalised as $m_0 = P_0\hat{m}_0$, $B_0^{CB} = P_0\hat{B}_0^{CB}$ while nominal rates are normalised as $1 + \rho_0 = \frac{P_1}{P_0}(1 + \hat{\rho}_0)$ and $1 + r_0 + f_0 = \frac{P_1}{P_0}(1 + \hat{r}_0 + \hat{f}_0)$ and so on where $P_0$ is the initial price level and $\frac{P_1}{P_0}$ is the rate of (expected) inflation.

**Monetary-Fiscal Authority**

The Monetary Fiscal Authority creates reserves $M_0$, backed by transfers $\tau_0$ and purchases of loans to the banking system. Reserves are paid an interest
rate of $r_0$ while loans accrue $r_0 + f_0$.

\[ M_0 = B_0 + \tau_0 \\
B_0(1 + r_0 + f_0) = M_0(1 + r_0) \]

\[ \tau_0 = \frac{M_0 f_0}{1 + r_0 + f_0} \] (4)

Monetary policy fixes $\tau_0$ and $r_0$ in nominal terms and uses $M_0$ as a policy variable to affect $f_0$ which is the key policy rate. Expansionary monetary policy reduces the key policy rate.\textsuperscript{10} In real terms

\[ \tilde{M}_0 = \tilde{B}_0 + \tilde{\tau}_0 \] (5)

\[ \tilde{B}_0(1 + \tilde{r}_0 + \tilde{f}_0) = \tilde{M}_0(1 + \tilde{r}_0) \] (6)

\[ \tilde{\tau}_0 = \tilde{M}_0 \frac{\tilde{f}_0}{1 + \tilde{r}_0 + \tilde{f}_0} \] (7)

\textbf{2.2.2 Households}

\textbf{Borrowers}

Borrowers are endowed with 1 unit of labour in the second period, supplied inelastically, in exchange for income in the future. Consumption is denoted by $c$.

\[ P_0 c_0^b \leq b_0 + \tau_0 \]

\[ P_1 c_1^b + (1 + \rho_0 + \phi_0)b_0 \leq P_1 \]

In real terms

\[ c_0^b \leq \tilde{b}_0 + \tilde{\tau}_0 \] (8)

\[ c_1^b + (1 + \tilde{\rho}_0 + \tilde{\phi}_0)\tilde{b}_0 \leq 1 \] (9)

\[ c_0^b + \frac{c_1^b}{(1 + \tilde{\rho}_0 + \tilde{\phi}_0)} \leq \frac{1}{(1 + \tilde{\rho}_0 + \tilde{\phi}_0)} + \tilde{\tau}_0 \] (10)

Preferences are given by

\[ log(c_0^b) + \beta log(c_1^b). \]

\textsuperscript{10}This is identical to that obtained in a non-Ricardian cash-in-advance model. We will later show that the effect of monetary policy on the key policy rate is independent of expected inflation.
Optimality requires

\[ \frac{\beta c^b_0}{c^b_1} = \frac{1}{(1 + \tilde{\rho}_0 + \phi_0)}. \]  
(11)

This implies

\[ c^b_0 = \frac{1}{1 + \beta} \left\{ \frac{1}{1 + \tilde{\rho}_0 + \phi_0} + \tilde{\tau}_0 \right\} \]  
(12)

\[ b_0 = \frac{1}{1 + \beta} \frac{1}{1 + \tilde{\rho}_0 + \phi_0} - \frac{\beta}{1 + \tilde{\tau}_0} \]  
(13)

**Depositors**

Depositors have a similar decision problem but are endowed with labour in the first period. The budget constraints in nominal terms are

\[ P_0 c^d_0 + d_0 \leq P_0 \]

\[ P_1 c^d_1 \leq (1 + \rho_0) d_0. \]

In real terms

\[ c^d_0 + \tilde{d}_0 \leq 1 \]  
(14)

\[ c^d_1 \leq (1 + \tilde{\rho}_0) \tilde{d}_0 \]  
(15)

which is

\[ c^d_0 + \frac{c^d_1}{(1 + \tilde{\rho}_0)} \leq \tilde{1}. \]  
(16)

Preferences of borrowers are given by

\[ \log(c^d_0) + \delta \log(c^d_1). \]

Optimality requires

\[ \frac{\delta c^d_0}{c^d_1} = \frac{1}{(1 + \tilde{\rho}_0)} \]  
(17)

In equilibrium, demands are

\[ c^d_0 = \frac{1}{1 + \delta} \]  
(18)

\[ \tilde{d}_0 = \frac{\delta}{1 + \delta} \]  
(19)
2.2.3 Equilibrium

We first show that real effects are obtained in the model, independent of corresponding nominal adjustments.

Combined Household Budget Constraints

Adding 8 and 14, and setting excess demands to zero each period gives

\[ \tilde{d}_0 - \tilde{b}_0 = \tilde{\tau}_0 \]  
\[ 0 = (1 + \tilde{\rho}_0)\tilde{d}_0 - (1 + \tilde{\rho}_0 + \tilde{\phi}_0)\tilde{b}_0. \]

(20)

(21)

Implies

\[ \frac{\tilde{\phi}_0}{1 + \tilde{\rho}_0} \frac{\tilde{b}_0}{\tilde{\phi}_0} = \tilde{\tau}_0 \]

(22)

\[ \frac{\tilde{\phi}_0}{1 + \tilde{\rho}_0 + \phi_0} \frac{\tilde{d}_0}{\tilde{\phi}_0} = \tilde{\tau}_0. \]

(23)

Substituting the above into supply of debt by borrower household, equation 13:

\[ \tilde{b}_0 = \frac{1 + \tilde{\rho}_0}{[(1 + \tilde{\rho}_0) + \beta(1 + \tilde{\rho}_0 + \tilde{\phi}_0)](1 + \tilde{\rho}_0 + \phi_0)} \]

(24)

Using equations 19, 21 and 24 we obtain

\[ (1 + \tilde{\rho}_0) = \frac{1 + \delta}{\beta\delta} \left( \frac{\phi}{1 + \rho} + \frac{1 + \beta}{\beta} \right)^{-1}. \]

(25)

From the Commercial Banks budget constraint, substituting 1 and 3 into 2 and using 21 gives

\[ \frac{\tilde{\phi}_0}{1 + \tilde{\rho}_0 + \phi_0} = \frac{\gamma f_0}{1 + r_0 + f_0} \]

(26)

\[ \frac{\tilde{\phi}_0}{1 + \tilde{\rho}_0} = \frac{\gamma f_0}{1 + r_0 + (1 - \gamma)f_0} \]

(27)

Both sides of the above expressions can be equivalently written as either nominal or real variables. Policy selects money supply, \( r_0 \) and nominal transfers, thereby determining \( f_0 \). As a consequence nominal policy choices determine
the real wedge between private sector borrowing and lending rates. Substituting 27 into 25 allows us to solve for \( \tilde{\rho} \) and \( \tilde{\phi} \).

\[
1 + \tilde{\rho}_0 = \frac{1 + \delta}{\beta \delta} \frac{1}{1 + r_0 + (1 - \gamma) f_0} + \frac{1 + \beta}{\beta} \tag{28}
\]

\[
1 + \tilde{\rho}_0 + \tilde{\phi}_0 = \frac{1 + \delta}{(1 + \beta) \delta} \left\{ 1 + \frac{\gamma f_0}{1 + \beta f_0} \right\} \tag{29}
\]

From equation 4, expansionary monetary policy reduces the key policy rate \( f_0 \), reduces \( \tilde{\phi}_0 (1 + \tilde{\rho}_0) \), increases \( 1 + \tilde{\rho}_0 \) and reduces \( 1 + \tilde{\rho}_0 + \tilde{\phi}_0 \). Equivalently, expansionary monetary policy reduces the real wedge between borrowing and lending rates in the economy.

The price level is obtained from the cash-constraint, equation 3,\(^{11}\) using the equilibrium value of real debt (equation 19).

\[
P_0 = \frac{M_0}{\gamma} \frac{1 + \delta}{\delta} \tag{30}
\]

Expanding money supply increases the current price level. However the expected rate of inflation is left undetermined. In multiperiod extensions, the rate of inflation will then depend on the growth rate of real deposits and nominal money balances (reserves supplied) independent of the nominal interest rates chosen by the central bank. The independence of the two instruments is derived from the role of nominal transfers to households in determining the path of reserve balances (money supply) given the path of nominal interest rates set by the central bank.

The distinction between real and nominal interest rates do not matter here as it is ratios (implicitly indexed) that drive results. As a consequence nominal policy interest rates have real effects directly through the impact of liquidity driving a wedge between private sector borrowing and lending rates. This contrasts with Alvarez et al. (2001) where real effects are obtained through the distributional role that inflation plays. In the Appendix we derive a simplified version of the Alvarez et al. (2001) model highlighting the role that the wealth effects of inflation plays in their paper. To see that that monetary policy is wealth-neutral in our paper, substitute the equilibrium value of the transfer into the borrower budget constraint to obtain:

\(^{11}\)As aggregate money balances equals money supply
Thus the effective present value budget constraint of borrowers is identical to that of lenders.

3 Concluding Remarks

We have shown that monetary expansions result in higher realised inflation, reduce the policy interest rates, and are independent of changes in expected money supply. Importantly, our results depend not on wealth effects but on the way policy rates create a wedge between private sector borrowing and lending rates.

Reserves and interest rates are, here, independent tools. In the model presented, they are connected through the initial nominal balances of households, while in a multi period setting the path of reserves, and hence inflation, will be determined by the path of nominal transfers to households.

We have not made any assumption or restriction on the nominal rates charged by the bank. Furthermore the expected rate of inflation is left indeterminate. Nevertheless, expansions in money supply affect nominal policy rates. If $\gamma$ is zero, we obtain the classical result that nominal rates depend only on expected inflation.

Real deposits $\frac{1}{1+\delta}$ are linearly related to the total endowment each period.

To reconcile our model with the quantity theoretic world, we only need to let velocity be defined as $\frac{1}{\gamma_d} = \frac{1+\delta}{\gamma_d}$.

We have assumed that the rate of deposit transfers is independent of both the key policy rate and the deposit rate. In reality these can be expected to be connected. Furthermore, policy rates move endogenously with economic conditions compounding the difficulty with empirical validation.
References


Appendix


The economy consists of traders (T) and non-traders (N). Traders can participate in the bond market while non-traders cannot. The model imposes a requirement that only a fraction of current income \( y_t \) can be used to finance consumption expenditure \( c_t \). As a consequence, unspent current period income is carried over to the subsequent period in the form of nominal balances and can be used to finance next period expenditure. The proportion of current period expenditure that can be financed by current period income is \( v_t \). Traders receive a nominal transfer from the central bank \( (T) \). Time is infinite and we focus on a deterministic path economy to highlight the mechanism of the model.

The cash-flow equation provides the link between money growth and inflation.

\[
\begin{align*}
    p_t &= \frac{M_t}{y_t} \frac{1}{1 - v_t} \\
    \frac{p_{t+1}}{p_t} &= \frac{M_{t+1}}{M_t} \frac{1 - v_t}{1 - v_{t+1}} \frac{y_t}{y_{t+1}} \\
    &= (1 + \mu_{t+1}) \frac{1 - v_t}{1 - v_{t+1}} \frac{y_t}{y_{t+1}}
\end{align*}
\]

The real demands of traders is given by

\[
c_t^T = (1 - v_{t-1}) \frac{p_{t-1}}{p_t} y_{t-1} + v_t y_t \quad (34)
\]

Market clearing gives

\[
y_t = \lambda c_t^T + (1 - \lambda) c_t^N
\]

and using this and 34 we obtain the demands of non-traders

\[
c_t^T = \frac{1}{\lambda} \left\{ y_t - (1 - \lambda)(1 - v_{t-1}) \frac{p_{t-1}}{p_t} y_{t-1} - (1 - \lambda)v_t y_t \right\} \quad (35)
\]

Note that the realised rate of inflation, and hence realised money growth, appears in 35. The bond pricing equation is

\[
\frac{1}{1 + r_t} = \beta \frac{c_t^T}{c_{t+1}^T} \frac{p_t}{p_{t+1}} \quad (36)
\]

The right hand side of the above expression only includes \( \mu_t \) if \( y_{t-1} \) is non-zero.
Wealth Neutralising Transfers

Suppose $T_i^N = (p_t - p_{t-1})(1 - v_{t-1})y_{t-1}$ and $T_i^T = r_{t-1}(1 - v_{t-1})p_{t-1}y_{t-1}$. The sum of the two transfers is exactly the per period seigniorage profits of the central bank. The household flow budget constraints

$$p_t c_t^N = (1 - v_{t-1})p_{t-1}y_{t-1} + v_t p_t y_t + T_t^N$$
$$p_t c_t^T = (1 - v_{t-1})p_{t-1}y_{t-1} + v_t p_t y_t + B_t - \frac{B_{t+1}}{1 + r_t} + T_t^T$$

In real terms, and imposing market clearing, we obtain

$$c_t^N = (1 - v_{t-1})y_{t-1} + v_t y_t$$
$$c_t^T = \frac{1}{\lambda} \{ y_t - (1 - \lambda)(1 - v_{t-1})y_{t-1} - (1 - \lambda)v_t y_t \}$$

The demands of the traders now do not include a term related to realised money growth. As inflation depends only on expected money growth, then from 36, interest rates will not change in response to current monetary expansions (that hold expected growth rates constant). Importantly, under wealth neutralising transfers, monetary policy does not determine allocations and is purely inflationary.

Dubey Geanakoplos (2003)

There is one period, two agents ($h = \{ \alpha, \beta \}$), each endowed with two distinct goods ($e^h$), and who have preferences over both goods ($l = \{1, 2\}$). Central Bank profits are transferred ex-ante to households and there is a cash in advance requirement on purchases. Households can choose how much of their endowment to sell (there is no “sell all” requirement as in Lucas and Stokey (1987)). As a consequence, households must purchase goods before they receive income from the goods they sell. The central bank stands willing to supply money $M$ in exchange for bonds $B$ at a market determined intra-period interest rate $r$. Subscripts denote goods and superscripts, agents.

The budget constraint of household $h$, endowed with good $l$ and purchasing good $l'$$p_l c_{il}^h \leq \frac{b^h}{1 + r} + \tau^h \quad (37)$
$$b^h \leq p_l (e_{il}^h - c_{il}^h) \quad (38)$$

where $b$ is debt. These budget constraints can be combined to obtain

$$p_l c_{il}^h \leq \frac{p_l (e_{il}^h - c_{il}^h)}{1 + r} + \tau^h. \quad (39)$$
The monetary-fiscal authority budget constraint is

\[
M \frac{r}{1 + r} = \tau^\alpha + \tau^\beta
\]  

(40)

If transfers are fixed in nominal terms, then expansionary monetary policy reduces the policy rate \( r \). Preferences of households are

\[
U^h = u(c^h_t) + u(c^h_{t+1}).
\]

Optimality requires

\[
\frac{p^r}{p^l} (1 + r) = \frac{\partial u(c^h_t)}{\partial c^h_t} \frac{\partial u(c^h_{t+1})}{\partial c^h_{t+1}}
\]

(41)

and as a consequence

\[
(1 + r) \frac{\partial u(c^h_{t+1})}{\partial c^h_t} = \frac{1}{(1 + r)} \frac{\partial u(c^h_t)}{\partial c^h_{t+1}}
\]

(42)

As \( r \to 0 \) the economy reaches Pareto efficiency. Importantly, the policy rate \( r \) reflects inter-temporal allocations in a multi-period model if and only if it affects intra-temporal allocations.